Chapter VI

Effects of Three-Body Force in Nuclear Matter*)

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The contribution of two-pion exchange three-body force from the P-wave π-N scattering to the binding energy of nuclear matter is investigated. The three-body force (TBF) is derived as the effective two-body force under consideration of the correlations between all the pairs of nucleons concerned with TBF. The reaction-matrix equations are solved self-consistently with two- and three-(effective two-)body potential. The binding energy contribution from TBF is about 3.5 MeV attraction for the case of the Hamada-Johnston potential and 4.1 MeV for Tamagaki's OPEH potential. The characteristic effect of TBF is discussed in connection with the binding energy and the saturation density. In nuclear matter calculation with two-nucleon potentials, there seems to be that the energy gain has generally been accompanied by increasing the saturation density, but our results show that TBF gives the energy gain, but does not increase the saturation density so much as two-body force does.

§ 1. Introduction

It is well-known that there is a large difference in binding effect on nuclear matter with realistic nucleon-nucleon potentials which reproduce two-nucleon scattering data equally well. A large number of works have been performed about the saturation problem of nuclear matter, but at the present time it seems to be difficult to draw any definite conclusion whether we can really obtain the binding energy of nuclear matter with realistic two-nucleon potential. These circumstances are attributed firstly to the fact that the two-nucleon potential is not uniquely determined from the deuteron and two-nucleon scattering data, especially with respect to the repulsive core and to the central-tensor ratio in the triplet even state, and secondly to the fact that the framework of the Brueckner theory, on which nuclear matter calculations are based, does not necessarily give the variational results.

On the other hand, on light nuclei, the variational calculations[2] with re-

*) Preliminary results of this work have been reported in Ref. 1).

alistic two-nucleon potentials give the binding energy nearly about the experimental value, even with potentials which give too small binding energy for nuclear matter; for example, with the Hamada-Johnston (H-J) potential,[3] ^{3}H: -6.1 MeV, ^{4}He: -20.6 MeV, nuclear matter: -7.8 MeV/particle. Hence, at the present time it seems to be difficult to explain the binding energy both for light nuclei and for nuclear matter from the same potential. How can we understand these discrepancies on binding energy? In order to answer the question, it is important to investigate the effects of the three-body force (TBF) quantitatively.

The aim of this paper is to estimate to what extent is the contribution to the binding energy of nuclear matter from TBF. Up to present, several studies[4)~7)] of the effects of TBF in nuclear matter have been done and it is known generally that the contribution to the binding energy is about 2 MeV/particle attraction. However, in order to discuss on the results quantitatively, it is important that the effects of TBF are taken into account as accurately as possible both in the method of the binding energy calculation and in the derivation of TBF.

It seems to be natural to consider the correlations between all the pairs of nucleons concerned with TBF, since the repulsive core is known to exist in the two-body forces. Nogami et al. and others [5),6)] have treated these correlations by introducing the cutoff of short-range part of TBF, but we think it is not reasonable to do so, because their results strongly depend on the cutoff distance and at the present time we have no means to determine it. And so far, in the estimations of the effects of TBF on nuclear matter there have been no calculations solving the reaction-matrix equation self-consistently with the potential including TBF as well as two-body potential.

In this paper, we derive TBF as the effective two-body potential under consideration of correlations between all the pairs of nucleons. When the effective two-body potential is added to the realistic two-body potential, we can solve the reaction-matrix equation self-consistently similarly to the case with only two-body potential. The correlation functions are given by the nuclear matter wave functions solved self-consistently.

In §2, we give the representation of TBF with correlations between all the pairs of nucleons as the effective two-body force. In §3, our method of nuclear matter calculation and computational procedure are presented. Results and discussion are given in §4. In §5, summary and conclusions are mentioned.

§ 2. Three-body force with correlations between all the pairs of nucleons

In this section, we represent TBF as the effective two-body potential under consideration of the correlations between all the pairs of nucleons.

As TBF, we consider the two-pion exchange three-body force shown in Fig. 1, whose S-matrix element is given by[5]

$$S=\sum_{\alpha}\sum_{\beta}\frac{4\pi f^2}{\mu^2}\tau_{1\alpha}\tau_{2\beta}\frac{1}{(2\pi)^6}\int d\boldsymbol{q}_1 d\boldsymbol{q}_2 e^{-i\boldsymbol{q}_1\boldsymbol{r}_1}K(q_1^2)(\boldsymbol{\sigma}_1\boldsymbol{q}_1)\frac{K'(q_1)^2}{q_1^2+\mu^2}$$
$$\times\langle\alpha,\ \boldsymbol{q}_1|S_{\pi N}^{(3)}|\beta,\ \boldsymbol{q}_2\rangle\frac{K'(q_2^2)}{q_2^2+\mu^2}(\boldsymbol{\sigma}_2\boldsymbol{q}_2)K(q_2^2)e^{i\boldsymbol{q}_2\boldsymbol{r}_2},\tag{1}$$

where

$$\langle\alpha,\ \boldsymbol{q}_1|S_{\pi N}^{(3)}|\beta,\ \boldsymbol{q}_2\rangle=2\pi i\delta(0)K(q_1^2)e^{i\boldsymbol{q}_1\boldsymbol{r}_3}$$
$$\times[(A\tau_{3\alpha}\tau_{3\beta}+B\tau_{3\beta}\tau_{3\alpha})(\boldsymbol{\sigma}_3\boldsymbol{q}_2)(\boldsymbol{\sigma}_3\boldsymbol{q}_1)$$
$$+(B\tau_{3\alpha}\tau_{3\beta}+A\tau_{3\beta}\tau_{3\alpha})(\boldsymbol{\sigma}_3\boldsymbol{q}_1)(\boldsymbol{\sigma}_3\boldsymbol{q}_2)+D\delta_{\alpha\beta}]K(q_2^2)e^{-i\boldsymbol{q}_2\boldsymbol{r}_3}\tag{2}$$

is the π-N scattering matrix for a zero-energy pion, and A and B are related to the P-wave scattering, D to the S-wave one. Also $f^2(=0.08)$ is the π-N coupling constant, K and K' are the vertex and the propagator form factors respectively. In this paper, we consider only the contribution from the P-wave scattering, so the D-term is neglected. In Eq. (1), we put $S=-2\pi i\delta(0)W$, then W represents TBF.[8]

We now introduce the correlations between all the pairs of nucleons shown in Fig. 2, where ϕ is the correlation function. The correlations between N_1 and N_2 nucleons need not to be considered, because these are counted in solving the reaction-matrix equation in nuclear matter as the pair correlations. When we consider only the single exchange term by taking the diagonal sum in Eq. (1) and integrate with respect to $\boldsymbol{r}_3$, we get the effective two-body potential as follows:

$$U(\boldsymbol{r})=\rho\int d\boldsymbol{r}_3 W(\boldsymbol{r}_1,\ \boldsymbol{r}_2,\ \boldsymbol{r}_3)\phi^2(\boldsymbol{r}_1-\boldsymbol{r}_3)\phi^2(\boldsymbol{r}_3-\boldsymbol{r}_2)$$
$$=-\frac{4\pi f^2\cdot\rho(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{(2\pi)^6\mu^2}\int d\boldsymbol{r}_3\phi^2(\boldsymbol{r}_1-\boldsymbol{r}_3)\phi^2(\boldsymbol{r}_3-\boldsymbol{r}_2)$$
$$\times\int d\boldsymbol{q}_1 d\boldsymbol{q}_2\frac{(\boldsymbol{\sigma}_1\boldsymbol{q}_1)(\boldsymbol{\sigma}_2\boldsymbol{q}_2)}{(q_1^2+\mu^2)(q_2^2+\mu^2)}e^{-i\boldsymbol{q}_1\boldsymbol{r}_1}e^{i\boldsymbol{q}_2\boldsymbol{r}_2}$$
$$\times[2(A+B)(\boldsymbol{q}_1\boldsymbol{q}_2)]e^{i\boldsymbol{q}_1\boldsymbol{r}_3}e^{-i\boldsymbol{q}_2\boldsymbol{r}_3},\tag{3}$$

where $\rho=3/(4\pi r_0^3)$ is the density of nuclear matter.

We put

$$\phi^2(\boldsymbol{r})=\int h(\boldsymbol{q})e^{-i\boldsymbol{q}\boldsymbol{r}}d\boldsymbol{q}\tag{4}$$

and substitute Eq. (4) into Eq. (3), then after integrating with respect to $\boldsymbol{r}_3$ we have

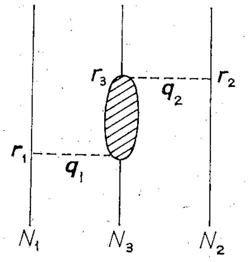

Fig. 1. The diagram for TBF considered in this paper.

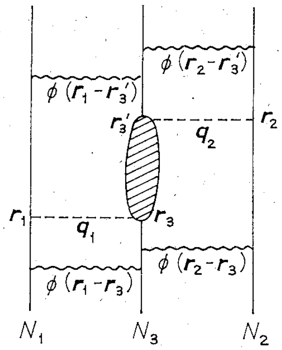

Fig. 2. The diagram for TBF under consideration of the correlations between all the pairs of nucleons.

$$U(\boldsymbol{r})=-\frac{4\pi f^2\cdot\rho\cdot 2(A+B)}{(2\pi)^3\mu^2}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\int d\boldsymbol{q}e^{-i\boldsymbol{q}\boldsymbol{r}}\int d\boldsymbol{q}_1 h(\boldsymbol{q}-\boldsymbol{q}_1)\frac{(\boldsymbol{\sigma}_1\boldsymbol{q}_1)}{q_1^2+\mu^2}$$
$$\times\int d\boldsymbol{q}_2 h(\boldsymbol{q}-\boldsymbol{q}_2)\frac{(\boldsymbol{\sigma}_2\boldsymbol{q}_2)}{q_2^2+\mu^2}(\boldsymbol{q}_1\boldsymbol{q}_2) \tag{5}$$

with $\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2$. To calculate Eq. (5), we introduce

$$S_{\alpha\beta}(\boldsymbol{p})=\int d\boldsymbol{q}\frac{h(\boldsymbol{p}-\boldsymbol{q})q_\alpha q_\beta}{q^2+\mu^2}. \quad (\alpha, \beta=x', y', z') \tag{6}$$

Here (x', y', z')-system have the z'-axis along $\boldsymbol{p}$ direction. Going on with calculations, we can express Eq. (5) as follows:

$$U(\boldsymbol{r})=-\frac{4\pi f^2\cdot\rho\cdot 2(A+B)}{(2\pi)^3\mu^2}(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\int d\boldsymbol{q}e^{-i\boldsymbol{q}\boldsymbol{r}}$$
$$\times\left[(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)S^2_{x'x'}(q)+(\boldsymbol{\sigma}_1\boldsymbol{q})(\boldsymbol{\sigma}_2\boldsymbol{q})\frac{\{S^2_{z'z'}(q)-S^2_{x'x'}(q)\}}{q^2}\right].$$

After all, we have the effective two-body force as follows:

$$U(r)=\frac{(4\pi)^2}{(2\pi)^3}\cdot\frac{f^2}{\mu^2}\cdot\frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\left[(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\int_0^\infty q^2dq j_0(qr)\frac{q^2}{(q^2+\mu^2)^2}\delta\mu_C^2(q^2)\right.$$
$$\left.+S_{12}\int_0^\infty q^2dq\left\{j_0(qr)-3\frac{j_1(qr)}{qr}\right\}\frac{q^2}{(q^2+\mu^2)^2}\delta\mu_T^2(q^2)\right] \tag{7}$$

with

$$\delta\mu_C^2(q^2)=F_C(q^2)\delta\mu_{NC}^2(q^2),$$
$$\delta\mu_T^2(q^2)=F_T(q^2)\delta\mu_{NC}^2(q^2). \tag{8}$$

Here $S_{12}=3(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r})/r^2-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ and

$$F_C(q^2)=\left[1-\frac{1}{3}\frac{q^2+\mu^2}{q^2}\{1-\phi^2(0)\}-S_1(q)\right]^2$$
$$+2\left[-\frac{1}{3}\frac{q^2+\mu^2}{q^2}\{1-\phi^2(0)\}-S_0(q)+\frac{1}{2}S_1(q)\right]^2,$$
$$F_T(q^2)=\left[1-\frac{1}{3}\frac{q^2+\mu^2}{q^2}\{1-\phi^2(0)\}-S_1(q)\right]^2$$
$$-\left[-\frac{1}{3}\frac{q^2+\mu^2}{q^2}\{1-\phi^2(0)\}-S_0(q)+\frac{1}{2}S_1(q)\right]^2. \tag{9}$$

$S_0(q)$ and $S_1(q)$ in Eq. (9) are given by

$$S_0(q)=\frac{q^2+\mu^2}{q^2}\cdot\frac{1}{2\pi^2}\int_0^\infty r^2dr\{1-\phi^2(r)\}\mathcal{S}_0(q,r),$$
$$S_1(q)=\frac{q^2+\mu^2}{q^2}\cdot\frac{1}{2\pi^2}\int_0^\infty r^2dr\{1-\phi^2(r)\}\mathcal{S}_1(q,r), \tag{10}$$

where

$$\mathcal{S}_0(q,r)=-\pi^2\mu^3Y(\mu r)j_0(qr),$$
$$\mathcal{S}_1(q,r)=\frac{2\pi^2\mu^3}{qr}Y(\mu r)\{2Z_T(\mu r)j_1(qr)-qrZ_L(\mu r)j_0(qr)\}, \tag{11}$$

and $Y(x)=e^{-x}/x$, $Z_T(x)=1+(3/x)+(3/x^2)$, $Z_L(x)=1+(2/x)+(2/x^2)$. Moreover, $\delta\mu^2_{NC}(q^2)$ in Eq. (8) means $\delta\mu^2(q^2)$ with no correlation, which is given by

$$\delta\mu^2_{NC}(q^2)=-2\rho(A+B)q^2H^2(q^2),$$
$$H(q^2)=K^2(q^2)K'(q^2). \tag{12}$$

§ 3. The binding energy calculation of nuclear matter

In this section, we mention our calculational method and computational procedure to get the contribution to the binding energy of nuclear matter.

The binding energy of nuclear matter is given by solving the reaction-matrix equation, which represents the nucleon-nucleon scattering in nuclear matter. Many calculations have been performed with two-body potentials. In our calculation, the effective two-body potential $U(r)$ derived from TBF is added to the realistic two-body potential. Then, we can include the estimation of TBF in the whole self-consistent procedure. In the derivation of the effective two-body potential, the correlations between all the pairs of nucleons are under consideration.

3-1 *Outline of calculational method*

The reaction matrix G in nuclear matter is given by

$$G=v+v\frac{Q}{e}G, \tag{13}$$

where v is the nucleon-nucleon potential which is expressed by two- plus three-body (effective two-body) potential in our case. Q is the Pauli operator, and the denominator e means the excitation energy of the two nucleons in the intermediate state. Equation (13) is solved in r-space so as to get the wave functions for soft core potentials as well as for hard core potentials. The formalism in r-space nuclear matter calculation has been given by Brueckner and Gammel[9] in detail. In our case, we set the potential energy for particle states equal to zero according to the hole-line-expansion method,[10] then Eq. (13) is solved only on the energy shell.

The wave function is given by the following integral equation:

$$u_{ll'}^{JS}(r)=krj_l(kr)\delta_{ll'}+4\pi\sum_{l''}\int dr'\mathcal{G}_l(r,r')V_{l'l''}^{JS}(r')u_{ll''}^{JS}(r'), \tag{14}$$

where

$$V_{ll'}^{JS}(r)=\int d\hat{\mathbf{r}}\mathcal{Y}_{JlS}^{M\dagger}v(r)\mathcal{Y}_{Jl'S}^{M}. \tag{15}$$

$\mathcal{G}_l(r,r')$ is a Green's function

$$\mathcal{G}_l(r,r')=\frac{1}{2\pi^2}\int_0^\infty dk'\frac{k'rj_l(k'r)k'r'j_l(k'r')Q(k',P)}{e(k',P;\Sigma)}, \tag{16}$$

where $\boldsymbol{P}$ and $\boldsymbol{k}$ are the total and the relative momenta respectively, and Σ denotes the single particle energy of two nucleons in Fermi sea.

The diagonal reaction-matrix elements which give the contribution to binding energy are expressed as follows:

$$\sum_{m_S}\langle k,S,m_S|G|k,S,m_S\rangle$$
$$=\frac{4\pi}{k^2}\sum_J\sum_{l=J-1}^{J+1}(2J+1)\int dr'kr'j_l(kr')\sum_{l'=J-1}^{J+1}V_{ll'}^{JS}(r')u_{ll'}^{JS}(r'). \tag{17}$$

For a hard core potential, we rewrite the Eqs. (14) and (17) by replacing $v(r)u(r)=\lambda\delta(r-r')$ for $r\leq r_c$, where r_c is a core radius and λ is determined by the condition $u(r_c)=0$.

After all, the one-body potential $V(m_0)$ and the binding energy per nucleon are obtained as follows:

$$V(m_0)=\sum_{n_0}\sum_{T}\sum_{S}\frac{2T+1}{2}\sum_{m_S}\langle m_0n_0|G(S,\,m_S,\,T)|m_0n_0\rangle, \tag{18}$$

$$\frac{B.E}{A}=\int_0^{p_F}m_0^2dm_0\left\{\frac{\hbar^2m_0^2}{2M}+\frac{1}{2}V(m_0)\right\}\Big/\int_0^{p_F}m_0^2dm_0. \tag{19}$$

3-2 *Computational procedure*

In this paper, we take into account states with $l\leq 2$. It is known that the total contribution to binding energy from higher partial waves is small. But for the $^3P_2$ state, we treat the coupled state $^3P_2+^3F_2$, so as to estimate the contribution from the tensor part of interactions as accurately as possible. We adopt H-J, OPEH (with a hard core and the OPEP-tail) and OPEG (with a 2 BeV Gaussian soft core and the OPEP-tail)[11] potentials as typical realistic nucleon-nucleon ones which reproduce the two-nucleon scattering data equally well. $\delta\mu_{NC}^2$ is given by Eq. (12) from σ_{33} experimental value, where

$$C_P=\mu^4f^2(A+B)/4\pi=\frac{\mu^4f^2}{9\pi^2}\int_0^\infty\frac{\sigma_{33}(p)}{p^2+\mu^2}dp=0.45\text{ MeV}. \tag{20}$$

This value of C_p was re-calculated[12] from the most recent experimental value of the total cross section of the π-N scattering in the $I=J=3/2$ state. In our calculation, we treat the case of no pionic form factor, or $H=1$, but the correlations between all the pairs of nucleons are introduced. As the correlation functions ϕ, $^1S_0$-state wave functions with the average momentum are used, which are given by solving the reaction-matrix equation with two- plus three-(effective two-)body potentials.

§ 4. Results and discussion

The binding energies of nuclear matter with two- plus three-(effective two-) body potentials and only with two-body potentials are shown in Table I and in Fig. 3. The contribution of TBF to the binding energy is about 3.5 MeV for the case of H-J and 4.1 MeV for OPEH. These values are not so large as to supply the discrepancy of binding energy between the theoretical and experimental ones, but are about 10% in total potential energy, which are meaningful to understand the saturation properties of nuclear matter. In Fig. 3, we notice the characteristic effect of TBF on the binding energy and saturation density. The results with two-body potentials show that the smaller (or the softer) the repulsive core is and also the weaker the tensor force is, the larger the binding energy is. They also show that the energy gain has a tendency to increase the saturation density, which may be too high to reproduce the experimental value. Therefore there needs something

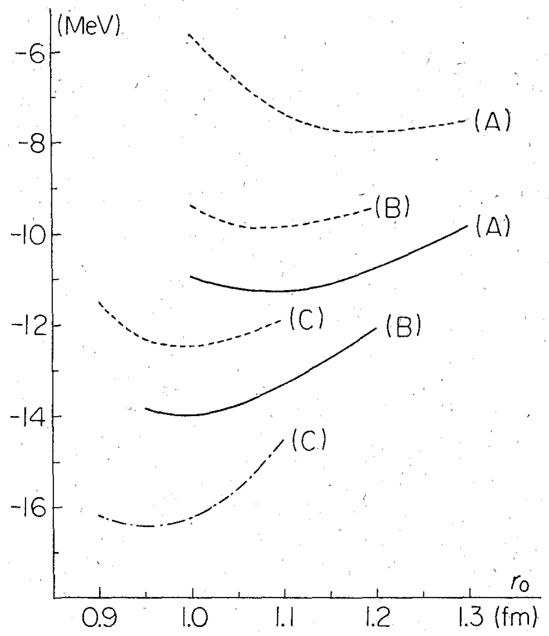

Fig. 3. Binding energy of nuclear matter for each potential: (A) H-J, (B) OPEH, (C) OPEG. The dashed lines denote for only the two-body potential and the solid lines for the potential including $U(r)$. The dot-dashed curve for OPEG is obtained by using 3-range Gaussian parameters in Table III.

Table I. Binding energy of nuclear matter for each potential. The row (a) is for only the two-body potential and (b) is for the potential including $U(r)$.

(in MeV unit)

potential		r_0(fm)				
		0.9	1.0	1.1	1.2	1.3
H-J	(a)		−5.72	−7.42	−7.81	−7.58
	(b)		−10.94	−11.29	−10.77	−9.85
OPEH	(a)		−9.42	−9.87	−9.47	
	(b)		−13.98	−13.29	−12.10	
OPEG	(a)	−11.55	−12.47	−11.88		
	(b)	−16.22	−16.07	−14.62		

to get the energy gain without increasing the density so as to reproduce the experimental value of saturation density as well as the binding energy. Our results suggest that TBF may be one of such candidates. In Fig. 4, we show the effective two-body potential $U(r)$. As seen in this figure, the central part is repulsive as far as a rather large distance ($r \lesssim 1$ fm) and the tensor part is attractive. This is very interesting to understand the feature of TBF. We can say that the tensor part and the central part of TBF have a different role respectively, that is, the former brings mainly the energy gain and the later has an effect to prevent increasing the density.

It is well-known that the tensor force plays an important role for the saturation problem of nuclear matter. In nuclear matter, the effect of tensor force

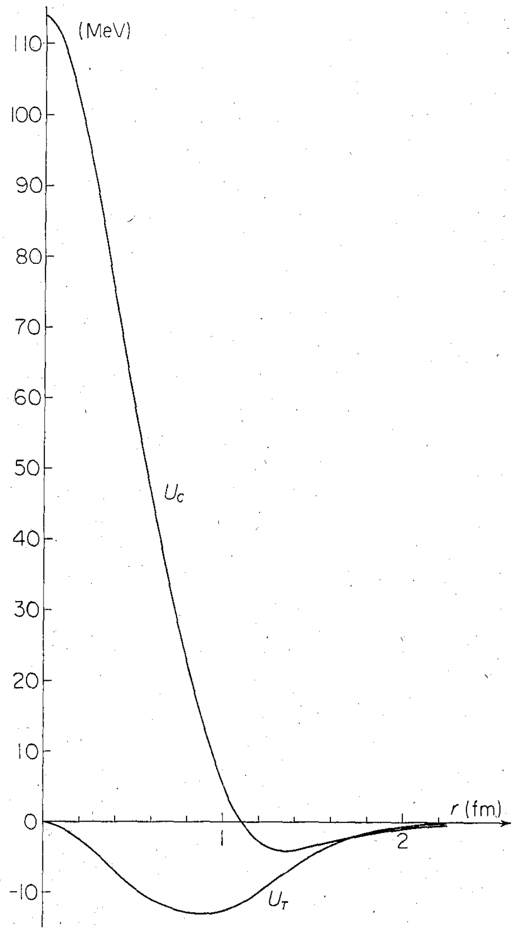

Fig. 4. Effective two-body potential $U(r)$ for the case of H-J at $r_0=1.1$ fm. U_C is the central part for the singlet even and the triplet even states, and U_T is the tensor part for the triplet even state.

is suppressed compared with in free scattering, and the contributions from the ^1S and the ^3S states are nearly equal. TBF contributes to make the pion mass effectively small, then the tensor force in the inner region of nuclear force is enhanced. Therefore, by considering TBF, the effect of tensor force which is suppressed in nuclear matter is recovered. We give the contributions to the potential energy for each partial wave in Table II and in Fig. 5, where also we can see that the binding energy is brought mainly from the ^3S state.

In our calculations, the correlation functions were introduced between N_1 and N_3, N_2 and N_3 nucleons. They are given by solving the reaction-matrix equations self-consistently. These correlation effects are expressed as $F(q^2)$ in Eq. (9), corresponding to the pionic form factors $H(q^2)$ as seen in Eq. (12). Most recently, Blatt and McKellar[7] calculated the contribution of TBF to the binding energy of nuclear matter using correlation functions derived from the Reid soft core potential. They showed that the total contribution from the two-pion exchange TBF was 5.7 MeV attraction, in their calculation the pionic form factor was contained. If we adopt the same form factor as

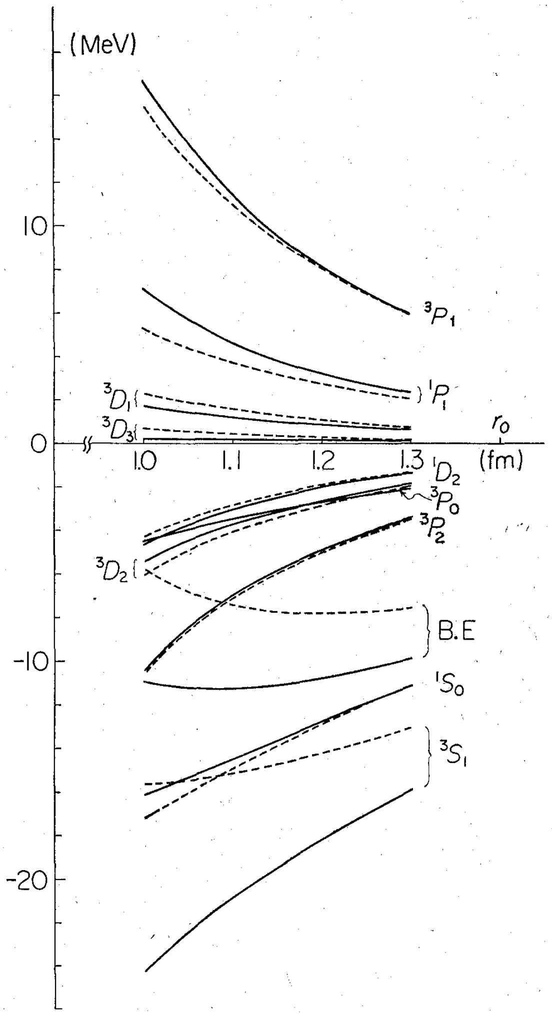

Fig. 5. Potential energy contribution for each partial wave in the case of H-J. The dashed lines are for only the two-body potential and the solid lines for the potential including $U(r)$.

used by them adding to $F(q^2)$ in our calculation, it must be found to give less binding, even for the soft core potential. The large discrepancy between our results and theirs is attribute to the double counting of N_1 and N_2 correlation in their calculation. Therefore, in their calculation the short range repulsive part of effective central force is cut, so that the contribution from the second order term gives too much binding. When this is treated correctly, their results would come near ours. The value $C_P=0.45$ MeV[12] used in our calculations should be compared with $C_P=0.61$ MeV calculated by Nogami et al. In spite of the smaller C_P value used, our calculated contributions of TBF are larger than that estimated by them. Therefore our resultant values with $C_P=0.45$ MeV show that the effect of TBF is rather large.

It is useful to express the effective two-body force as the superposition of 3-range Gaussian type functions, for instance, to study the effect of TBF on finite nuclei; $U(r)=\sum a_i e^{-b_i r^2}$. Our fit parameters for the case of H-J

Table II. The contribution to the potential energy for each partial wave at $r_0=1.1$ fm. The column (a) is for only the two-body potential and (b) is for the potential including $U(r)$.

(in MeV unit)

		H-J		OPEH		OPEG	
		(a)	(b)	(a)	(b)	(a)	(b)
Potential energy	$^1S_0$	−14.89	−14.53	−15.07	−14.70	−16.39	−15.80
	$^3S_1$	−15.19	−20.80	−17.72	−22.65	−18.54	−22.69
	$^1P_1$	3.67	4.57	4.41	5.23	4.54	5.29
	$^3P_0$	−3.47	−3.47	−3.88	−3.90	−3.78	−4.21
	$^3P_1$	10.94	11.36	11.61	12.05	11.36	12.40
	$^3P_2$	−7.10	−7.00	−7.32	−7.26	−7.29	−7.47
	$^1D_2$	−2.91	−2.96	−3.22	−3.29	−3.13	−3.30
	$^3D_1$	1.52	1.20	1.65	1.32	1.63	1.67
	$^3D_2$	−4.19	−3.72	−4.47	−4.00	−4.42	−4.61
	$^3D_3$	0.43	0.20	0.28	0.04	0.27	0.24
	total	−31.28	−35.15	−33.73	−37.14	−35.73	−38.48
Kinetic energy		23.86	23.86	23.86	23.86	23.86	23.86
Total energy		−7.42	−11.29	−9.87	−13.29	−11.88	−14.62

Table III. 3-range Gaussian parameters for the central and tensor part of effective two-body potential $U(r)$ in the case of H-J at $r_0=1.1$ fm.

	a_1	a_2	a_3	b_1	b_2	b_3
^1E- and ^3E-central parts	−90.4	194.6	9.70	1.08	1.70	14.4
^3E-tensor part	−199.6	198.1	1.50	1.31	1.56	25.2

at $r_0=1.1$ fm are given in Table III. The results for OPEG (with a 2 BeV Gaussian soft core)[11] calculated by using these parameters are also plotted in Fig. 1, where ρ-dependence of TBF is introduced only through Eq. (12), and the contribution of TBF is about 3.8 MeV attraction with little increase in the density.

§ 5. Summary and conclusions

The effect of two-pion exchange three-body force in nuclear matter has been investigated under consideration of correlations between all the pairs of nucleons. TBF with correlations has been derived as the effective two-body potential. The reaction-matrix equation has been solved self-consistently with two- plus three-(effective two-) body potential. We found that the total energy of nuclear matter with H-J (OPEH) including TBF is -11.3 (-14.0) MeV at the saturation density $r_0=1.10$ (0.99) fm, though it is far from the empirical value -15.8 MeV at $r_0=1.07$ fm. The contribution of TBF is about 3.5 (4.1) MeV attraction to binding energy. It has been shown that the energy contribution of TBF is mainly due to the tensor force. We have discussed the characteristics of TBF in nuclear matter. TBF gives the energy gain, but does not increase the saturation density so much as two-body force does. Therefore, we conclude that TBF gives a meaningful contribution to binding energy and will be expected to make an important role to understand the saturation properties of nuclear matter.

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