Chapter VI

Many-Body Theoretical Description of Alpha-Like Four-Body Correlations

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§1. Introduction

According to the investigations in the previous chapters, it has been clear that the alpha-cluster structure is considerably well-founded in light nuclei. Nevertheless, we cannot ignore the successes of the shell model in describing the structure of p shell and sd shell nuclei. In the shell model, it is assumed that the nucleons move rather independently in the self-consistent average field and only a few nucleons in the open shell participate to the nuclear low excitations. In the extreme alpha-cluster model, two protons and two neutrons are assumed to be coupled with each other so strongly that they become a subunit of nuclear constituent and the concept of the average field for single nucleon disappears. Although the shell-model aspect and the alpha-cluster aspect, as mentioned above, present a striking contrast to each other, it is natural to consider that both the aspects *coexist* in the light nuclei.

As seen in the previous chapters,*) a typical example of the nucleus in which both the aspects coexist is considered to be ^{20}Ne. Shell-model calculations for low-excited states in ^{20}Ne and its neighboring nuclei have made many successes in reproducing level structures and γ-transition probabilities,[1] but the use of rather large effective charge suggests the presence of other correlations which are not included in the shell model. Among the low-excited states in ^{16}O$\sim^{20}$Ne nuclei, there also exist some levels which can hardly be reproduced by the conventional shell-model calculation. (The first excited 0^+ state in ^{16}O is one of the typical examples.) These facts imply that there exist some correlations which cannot be covered by the simple shell-model treatment. It can be considered that these light nuclei have a dual character, i.e., that some of their phenomena are described by the shell model but others by the alpha-cluster model and, besides, there exist some phenomena which can be described by both of them. Then, it is desirable to construct a unified microscopic picture for the light nuclei, in which both the aspects, the shell-model and the alpha-cluster aspects, are combined. For this purpose, we adopt the following picture: Although there exist very strong correlations which are against the stability of the shell-model-Hartree-Fock field, the Hartree-Fock field is still barely stable in the neighborhood of the ground state to realize the shell structure.

Along this line, one of the present authors (T. M.) and Suzuki[2] have, for the first time, proposed a theory of microscopic description of the alpha-like four-body mode. They have asserted that the four-body correlations are so strong in the light nuclei that the corresponding ground-state correlations should be taken into account properly and these ground-state correlations play an essential role for the alpha-like four-body mode to be well established.

*) See Chapters III and IV.

It should be noticed that these modes are "dressed" four-body modes in the new Tamm-Dancoff sense and are quite different from the "bare" modes treated by the conventional shell model. In addition, they have discussed the relation between the "dressed" four-body mode and the alpha cluster from the viewpoint of "phase transition". Eichler and two of the authors (T. M. and K. T.)[3] have shown an evidence of importance of the ground-state correlations through calculation of the interaction between the four-body mode and the single-hole mode in connection with the weak coupling model by Arima, Horiuchi and Sebe.[4] It has been suggested by Fukushima and two of the authors (M. K. and K. T.)[5] that the alpha-like four-body correlations are characterized not only by the ground-state correaltions but also by some other kinds of correlations. Two of the present authors (M. K. and K. T.) and Matsuse[6] have investigated in detail the structure of another important correlations, the spatial correlations (the higher-shell-mixing effects), in the case of ^{20}Ne.

In this chapter, on the basis of the series of the above works, a unified microscopic theory in which both the aspects, the shell-model and the alpha-cluster aspects, are combined is formulated and, within this framework, the structure of the strong four-body correlations is investigated. In §2, the formulation describing the alpha-like four-body mode is developed on the condition that the shell-model-Hartree-Fock field is stable. In §§3 and 4, we investigate the detailed properties of the four-body correlations, decomposing them into two parts, the spatial correlations and the ground-state correlations. The former is discussed in §3 and the latter in §4. The mutual "phase transition" between the shell-model "phase" and the alpha-cluster "phase" is also discussed in §4.

§2. Alpha-like four-body modes and the Hartree-Fock approximation

In this section, a formalism treating the alpha-like four-body mode on the basis of the Hartree-Fock approximation is presented. We obtain the fundamental equation of motion of the mode in the presence of the "core" including the ground-state correlations.

2.1 Basic assumptions

In the framework of the second quantization, the Hamiltonian under consideration is given by

$$\hat{H}=-\frac{\hbar^2}{2m}\int dx(\nabla\phi^\dagger(x)\cdot\nabla\phi(x)) + \frac{1}{2}\int dx_1dx_2\phi^\dagger(x_1)\phi^\dagger(x_2)v(x_1x_2)\phi(x_2)\phi(x_1), \qquad (2\cdot1)$$

where x stands for a complete labeling of one-particle variables, $x \equiv (\boldsymbol{r}, \sigma, \tau)$ and $v(x_1x_2)(=v(x_2x_1))$ means the two-body interaction potential. The nucleon field operator $\phi^\dagger(x)$ satisfies the anti-commutation relations

$$\begin{aligned} &\{\phi^\dagger(x_1),\ \phi(x_2)\}_+ = \delta(x_1, x_2), \\ &\{\phi(x_1), \phi(x_2)\}_+ = \{\phi^\dagger(x_1), \phi^\dagger(x_2)\}_+ = 0, \end{aligned} \tag{2·2}$$

where

$$\delta(x_1, x_2) = \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(\sigma_1\sigma_2)\delta(\tau_1\tau_2). \tag{2·3}$$

Now, let us define the ground state of the "core" (consisting of A_0 nucleons) with energy eigenvalue $W(A_0;\ 0)$ by

$$\hat{H}|A_0;\ 0\rangle = W(A_0;\ 0)\,|A_0;\ 0\rangle \tag{2·4}$$

and consider only a *special class* of the eigenstates of (A_0+4)-particle system which is written down as

$$\begin{aligned} &|A_0+4; n\rangle \\ &= \frac{1}{\sqrt{4!}}\int\left(\prod_{i=1}^{4} dx_i\right)\Psi_n(x_1x_2x_3x_4)\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\,|A_0;0\rangle, \end{aligned} \tag{2·5}$$

where n means a set of quantum numbers characterizing the state in the special class. Then, $\Psi_n(x_1x_2x_3x_4)$ can be understood as the wave function describing the four particles added to the "core". If we write Eq. (2·5) as

$$|A_0+4;\ n\rangle = X_n^\dagger|A_0;\ 0\rangle, \tag{2·6}$$

the operator $X_n^\dagger$ is a four-body mode. Hence, the special class of eigenstates under consideration is constructed by operating the four-body mode $X_n^\dagger$ to the ground state of the "core" and seems to be essentially different from the states described by the Bloch-Brink type wave function.

Let $W(A_0+4;\ n)$ be the energy eigenvalue of the state $|A_0+4;\ n\rangle$. From the relation

$$\{W(A_0+4;\ n) - W(A_0;\ 0)\}\,|A_0+4;\ n\rangle = [\hat{H}, X_n^\dagger]\,|A_0;0\rangle, \tag{2·7}$$

we have the following equation;

$$\begin{aligned} &\int\left(\prod_{i=1}^{4} dx_i\right)[\{W(A_0+4;n) \\ &\quad - W(A_0;0)\}\Psi_n(x_1x_2x_3x_4)]\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\,|A_0;0\rangle \\ &\quad = \int\left(\prod_{i=1}^{4} dx_i\right)[H^{(0)}\Psi_n(x_1x_2x_3x_4)]\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\,|A_0;\ 0\rangle \\ &\quad + \int\left(\prod_{i=1}^{4} dx_i\right)\int dy\left\{\sum_{i=1}^{4} v(x_iy)\right\}\Psi_n(x_1x_2x_3x_4) \\ &\quad \times\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\phi^\dagger(y)\phi(y)\,|A_0;\ 0\rangle, \end{aligned} \tag{2·8}$$

where

$$H^{(0)}=\sum_{i=1}^{4}\left(-\frac{\hbar^2}{2m}\nabla_i^2\right)+\frac{1}{2}\sum_{i\neq j}v(x_ix_j). \tag{2·9}$$

In the simplest case where the "core" is absent, i.e., $A_0=0$, the second term on the right-hand side of Eq. (2·8) vanishes because of $\phi(y)|A_0=0;\ 0\rangle=0$. Thus the eigenvalue equation for $\Psi_n(x_1x_2x_3x_4)$ simply becomes

$$W(A_0=4;\ n)\Psi_n^{(0)}(x_1x_2x_3x_4)=H^{(0)}\Psi_n^{(0)}(x_1x_2x_3x_4) \tag{2·10}$$

with the normalization $\int|\Psi_n^{(0)}(x_1x_2x_3x_4)|^2(\Pi_{i=1}^{4}dx_i)=1$. Equation (2·10) with the Hamiltonian $H^{(0)}$ defined by Eq. (2·9) is nothing but the Schrödinger equation for the four-particle system of which the lowest bound state is the α-particle.

Now, it is clear that Eq. (2·8) describes the alpha-like four-body mode in the presence of "core" and all complications due to the existence of the "core" come from the second term on the right-hand side of Eq. (2·8). In order to evaluate this term, we must know the structure of the ground state of the "core", $|A_0;\ 0\rangle$, which is determined in principle by Eq. (2·4). It is, however, impossible to solve Eq. (2·4), so that we must assume an approximation for the "core", which should be consistent with the picture discussed in the last section. Then, the basic assumption in evaluating the second term is that $|A_0;\ 0\rangle$ is well-described by the Hartree-Fock type approximation. This means that the following antisymmetric factorization process (with respect to $|A_0;\ 0\rangle$) is possible:

$$\begin{aligned}
&\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\phi^\dagger(y)\phi(y)|A_0;\ 0\rangle\\
&\quad\cong\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)|A_0;\ 0\rangle\rho^*(y,y)\\
&\quad+\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)\phi^\dagger(y)|A_0;\ 0\rangle\rho^*(x_1,y)\\
&\quad-\phi^\dagger(x_1)\phi^\dagger(x_3)\phi^\dagger(x_4)\phi^\dagger(y)|A_0;\ 0\rangle\rho^*(x_2,y)\\
&\quad+\phi^\dagger(x_1)\phi^\dagger(x_2)\phi_\dagger(x_4)\phi^\dagger(y)|A_0;\ 0\rangle\rho^*(x_3,y)\\
&\quad-\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(y)|A_0;\ 0\rangle\rho^*(x_4,y),
\end{aligned} \tag{2·11}$$

where $\rho(x,y)$ is the density matrix of the ground state of the "core" and it is defined by

$$\rho(x,y)\equiv\langle A_0;\ 0|\phi^\dagger(y)\phi(x)|A_0;\ 0\rangle\ (=\rho^*(y,x)). \tag{2·12}$$

It will later be shown that the antisymmetric factorization process corresponds to the Hartree-Fock approximation for the "core" $|A_0;\ 0\rangle$.

2. 2 The alpha-like four-body mode in the presence of the "core"

With the basic assumption represented by Eq. (2·11), we have the eigenvalue equation for $\Psi_n(x_1x_2x_3x_4)$ from Eq. (2·8):

$$
\begin{aligned}
&\{W(A_0+4;\, n)-W(A_0;\, 0)\Psi_n\}(x_1x_2x_3x_4)\\
&=\Big[\sum_{i=1}^{4}\Big(-\frac{\hbar^2}{2m}\nabla_i^2\Big)+\frac{1}{2}\sum_{i\neq j}v(x_ix_j)\Big]\Psi_n(x_1x_2x_3x_4)\\
&+\int dy\,[\delta(x_1,y)\int dz v(x_1z)\rho(z,z)-v(x_1y)\rho(x_1,y)]\Psi_n(yx_2x_3x_4)\\
&+\int dy\,[\delta(x_2,y)\int dz v(x_2z)\rho(z,z)-v(x_2y)\rho(x_2,y)]\Psi_n(x_1yx_3x_4)\\
&+\int dy\,[\delta(x_3,y)\int dz v(x_3z)\rho(z,z)-v(x_3y)\rho(x_3,y)]\Psi_n(x_1x_2yx_4)\\
&+\int dy\,[\delta(x_4,y)\int dz v(x_4z)\rho(z,z)-v(x_4y)\rho(x_4,y)]\Psi_n(x_1x_2x_3y)\\
&-\int dy\,[v(x_1x_2)+v(x_1x_3)+v(x_1x_4)]\rho(x_1,y)\Psi_n(yx_2x_3x_4)\\
&-\int dy\,[v(x_1x_2)+v(x_2x_3)+v(x_2x_4)]\rho(x_2,y)\Psi_n(x_1yx_3x_4)\\
&-\int dy\,[v(x_1x_3)+v(x_2x_3)+v(x_3x_4)]\rho(x_3,y)\Psi_n(x_1x_2yx_4)\\
&-\int dy\,[v(x_1x_4)+v(x_2x_4)+v(x_3x_4)]\rho(x_4,y)\Psi_n(x_1x_2x_3y). \qquad (2\cdot13)
\end{aligned}
$$

From Eq. (2·13), we can write the corresponding Hamiltonian H to Eq. (2·9), which describes the alpha-like four-body mode in the presence of the "core" as follows:

$$
H=\frac{1}{2m}\sum_{i=1}^{4}\boldsymbol{p}_i^2+\sum_{i=1}^{4}U_i+H_{\mathrm{int}}^{(\mathrm{I})}+H_{\mathrm{int}}^{(\mathrm{II})}, \qquad (2\cdot14)
$$

where the operator U_i is the conventional Hartree-Fock field acting on the i-th particle ($i=1, 2, 3, 4$) due to the existence of the "core". The matrix element of U_i is defined by

$$
\begin{aligned}
\langle x_i|\,U_i\,|y_i\rangle&=\langle y_i|\,U_i\,|x_i\rangle^*\\
&=\delta(x_i,y_i)\int dz v(x_iz)\rho(z,z)-v(x_iy_i)\rho(x_i,y_i). \qquad (2\cdot15)
\end{aligned}
$$

The operator $H_{\mathrm{int}}^{(\mathrm{I})}$ represents the internal interactions of the alpha-like four-body mode which correspond to the second term of the right-hand side of Eq. (2·9). The operator $H_{\mathrm{int}}^{(\mathrm{II})}$ mainly describes the complex "blocking effects" due to the presence of the "core": If the effects of the Pauli principle are neglected, $H_{\mathrm{int}}^{(\mathrm{II})}$ vanishes and the effects of the presence of the "core" are just reduced to the average field acting on the each particle composing the alpha-like four-body mode. The matrix elements of $H_{\mathrm{int}}^{(\mathrm{I})}$ and $H_{\mathrm{int}}^{(\mathrm{II})}$ are defined by

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(\rm I)}|y_1y_2y_3y_4\rangle=\left[\frac{1}{2}\sum_{i\neq j}v(x_ix_j)\right]\left[\prod_{i=1}^{4}\delta(x_i,y_i)\right], \tag{2·16}$$

$$\begin{aligned}&\langle x_1x_2x_3x_4|H_{\rm int}^{(\rm II)}|y_1y_2y_3y_4\rangle\\&=-[v(x_1x_2)+v(x_1x_3)+v(x_1x_4)]\rho(x_1,y_1)\delta(x_2,y_2)\delta(x_3,y_3)\delta(x_4,y_4)\\&\quad-[v(x_1x_2)+v(x_2x_3)+v(x_2x_4)]\rho(x_2,y_2)\delta(x_1,y_1)\delta(x_3,y_3)\delta(x_4,y_4)\\&\quad-[v(x_1x_3)+v(x_2x_3)+v(x_3x_4)]\rho(x_3,y_3)\delta(x_1,y_1)\delta(x_2,y_2)\delta(x_4,y_4)\\&\quad-[v(x_1x_4)+v(x_2x_4)+v(x_3x_4)\,[\rho(x_4,y_4)\delta(x_1,y_1)\delta(x_2,y_2)\delta(x_3,y_3).\end{aligned} \tag{2·17}$$

Here, it should be noticed that the operator $H_{\rm int}^{(\rm II)}$ is *not* Hermitian in the usual sense:

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(\rm II)}|y_1y_2y_3y_4\rangle\neq\langle y_1y_2y_3y_4|H_{\rm int}^{(\rm II)}|x_1x_2x_3x_4\rangle^*. \tag{2·18}$$

This non-Hermiticity which is caused by the presence of the "core" makes the alpha-like four-body mode much complicated.

2. 3 Shell-model representation of the fundamental eigenvalue equation for the alpha-like four-body mode

In the last subsection, we have obtained the effective Hamiltonian (2·14) which the alpha-like four-body mode satisfies. In order to study the properties of the Hamiltonian, it is necessary at first to investigate the formal properties of Eq. (2·13). For this purpose, the conventional shell-model representation seems to be convenient.

The shell-model representation is determined as follows: Let $|A_0-1;h\rangle$ be a set of "one-hole" states of the (A_0-1)-particle system defined by the requirement that they have the largest matrix element of the type

$$\langle A_0-1;\,h|\phi(x)|A_0;\,0\rangle\equiv\varphi_h(x). \tag{2·19}$$

Similarly we define

$$\langle A_0+1;\,p|\phi^\dagger(x)|A_0;\,0\rangle\equiv\varphi_p{}^*(x), \tag{2·20}$$

where $|A_0+1;p\rangle$ represents a set of "one-particle" states of the (A_0+1)-particle system. From the equation of motion

$$-[\hat{H},\phi(x)]=-\frac{\hbar^2}{2m}\nabla^2\phi(x)+\int dy v(xy)\phi^\dagger(y)\phi(y)\phi(x), \tag{2·21}$$

we then have

$$\varepsilon_h\varphi_h(x)=-\frac{\hbar^2}{2m}\nabla^2\varphi_h(x)+\int dy v(xy)\langle A_0-1;\,h|\phi^\dagger(y)\phi(y)\phi(x)|A_0;\,0\rangle, \tag{2·22a}$$

$$\varepsilon_p\varphi_p(x)=-\frac{\hbar^2}{2m}\nabla^2\varphi_p(x)+\int dy v(xy)\langle A_0+1;\, p|\phi^\dagger(x)\phi^\dagger(y)\phi(y)|A_0;\, 0\rangle, \tag{2·22b}$$

where

$$\varepsilon_h=W(A_0;\, 0)-W(A_0-1;h),\quad \varepsilon_p=W(A_0+1;\, p)-W(A_0;\, 0). \tag{2·23}$$

By performing the antisymmetric factorization similar to Eq. (2·11) for the second terms on the right-hand side of Eqs. (2·22a) and (2·22b), these equations are reduced to

$$\varepsilon_h\varphi_h(x)=-\frac{\hbar^2}{2m}\nabla^2\varphi_h(x)+\int dy\langle x|U|y\rangle\varphi_h(y), \tag{2·24a}$$

$$\varepsilon_p\varphi_p(x)=-\frac{\hbar^2}{2m}\nabla^2\varphi_p(x)+\int dy\langle x|U|y\rangle\varphi_p(y), \tag{2·24b}$$

where $\langle x|U|y\rangle$ is defined by Eq. (2·15). Equations (2·24a) and (2·24b) are nothing but the self-consistent Hartree-Fock equations for the single-particle states and can be unified in an equation

$$\varepsilon_\alpha\varphi_\alpha(x)=-\frac{\hbar^2}{2m}\nabla^2\varphi_\alpha(x)+\int dy\langle x|U|y\rangle\varphi_\alpha(y). \tag{2·25}$$

Needless to say, the single-particle wave functions $\varphi_\alpha(x)$ must satisfy the orthogonality relation

$$\int\varphi_\alpha^*(x)\varphi_\beta(x)dx=\delta_{\alpha\beta}. \tag{2·26}$$

Now, let us consider the Hartree-Fock approximation for the ground state of the "core", $|A_0;\, 0\rangle$. In this approximation, the state $|A_0;\, 0\rangle$ is generally obtained by choosing the lowest possible A_0 eigenstates of Eq. (2·25) and the density matrix can then be simply written as

$$\begin{aligned}\rho(x,y)&=\langle A_0;\, 0|\phi^\dagger(y)\phi(x)|A_0;\, 0\rangle\\&=\sum_h\varphi_h(x)\varphi_h^*(y)=\sum_\alpha{}'\varphi_\alpha(x)\varphi_\alpha^*(y),\end{aligned} \tag{2·27}$$

where the symbol $\sum_\alpha'$ means the summation over all occupied states. The Hartree-Fock field U is determined by Eq. (2·15) with the use of the density matrix (2·27) and the single-particle wave functions are determined by Eq. (2·25) with U thus obtained. Substituting these single-particle wave functions into Eq. (2·27), we have a new density matrix and then repeat the above process: This is the self-consistency problem of the Hartree-Fock approximation.

The true ground state of the "core" is determined in principle by Eq.

(2·4) but it is impossible to solve it. Instead, the Hartree-Fock ground-state wave function is often used as an approximation for $|A_0; 0\rangle$. However, if the true $|A_0; 0\rangle$ much differs from for Hartree-Fock ground-state wave function, for example, if $|A_0; 0\rangle$ involves the ground-state correlations due to the four-body correlations in the sense of the new Tamm-Dancoff (NTD) approximation, the density matrix $\rho(x, y)$ of the "core" becomes different from the expression (2·27) and includes the particle-state components as well as the hole-state ones. In this case, the both processes of determining the Hartree-Fock field U and getting the alpha-like four-body mode $X_n^\dagger$ are coupled with each other and become more complicated. Since it is quite difficult to treat such complicated problem exactly, we usually neglect the coupling and replace the density matrix $\rho(x, y)$ with Eq. (2·27). Therefore, Eq. (2·27) is regarded as an approximated expression for the true $\rho(x, y)$ when the ground-state correlations in the NTD sense are included in $|A_0; 0\rangle$. One of the present authors (K. T.) has estimated the possibility of such approximation with an simplified model.[7]

In the shell-model representation, the fundamental eigenvalue equation (2·13) is written as

$$\begin{aligned}
&\{W(A_0+4;\, n) - W(A_0;\, 0)\}\Psi_n(\alpha\beta\gamma\delta) \\
&\quad = \sum_{\alpha_1\beta_1\gamma_1\delta_1} \langle\alpha\beta\gamma\delta|H|\alpha_1\beta_1\gamma_1\delta_1\rangle\Psi_n(\alpha_1\beta_1\gamma_1\delta_1) \\
&\quad = \sum_{\alpha_1\beta_1\gamma_1\delta_1} \{(\varepsilon_\alpha+\varepsilon_\beta+\varepsilon_\gamma+\varepsilon_\delta)\delta_{\alpha\alpha_1}\delta_{\beta\beta_1}\delta_{\gamma\gamma_1}\delta_{\delta\delta_1} \\
&\qquad + \langle\alpha\beta\gamma\delta|H_{\mathrm{int}}^{(\mathrm{I})}|\alpha_1\beta_1\gamma_1\delta_1\rangle + \langle\alpha\beta\gamma\delta|H_{\mathrm{int}}^{(\mathrm{II})}|\alpha_1\beta_1\gamma_1\delta_1\rangle\}\Psi_n(\alpha_1\beta_1\gamma_1\delta_1), \quad (2\cdot28)
\end{aligned}$$

where

$$\begin{aligned}
&\langle\alpha\beta\gamma\delta|H_{\mathrm{int}}^{(\mathrm{I})}|\alpha_1\beta_1\gamma_1\delta_1\rangle \\
&\quad = \langle\alpha\beta|v|\alpha_1\beta_1\rangle\delta_{\gamma\gamma_1}\delta_{\delta\delta_1} + \langle\alpha\gamma|v|\alpha_1\gamma_1\rangle\delta_{\beta\beta_1}\delta_{\delta\delta_1} \\
&\qquad + \langle\alpha\delta|v|\alpha_1\delta_1\rangle\delta_{\beta\beta_1}\delta_{\gamma\gamma_1} + \langle\beta\gamma|v|\beta_1\gamma_1\rangle\delta_{\alpha\alpha_1}\delta_{\delta\delta_1} \\
&\qquad + \langle\beta\delta|v|\beta_1\delta_1\rangle\delta_{\alpha\alpha_1}\delta_{\gamma\gamma_1} + \langle\gamma\delta|v|\gamma_1\delta_1\rangle\delta_{\alpha\alpha_1}\delta_{\beta\beta_1}, \quad (2\cdot29)
\end{aligned}$$

$$\begin{aligned}
&\langle\alpha\beta\gamma\delta|H_{\mathrm{int}}^{(\mathrm{II})}|\alpha_1\beta_1\gamma_1\delta_1\rangle \\
&\quad = -\langle\alpha\beta|v|\alpha_1\beta_1\rangle(\theta_{\alpha_1}+\theta_{\beta_1})\delta_{\gamma\gamma_1}\delta_{\delta\delta_1} - \langle\alpha\gamma|v|\alpha_1\gamma_1\rangle(\theta_{\alpha_1}+\theta_{\gamma_1})\delta_{\beta\beta_1}\delta_{\delta\delta_1} \\
&\qquad - \langle\alpha\delta|v|\alpha_1\delta_1\rangle(\theta_{\alpha_1}+\theta_{\delta_1})\delta_{\beta\beta_1}\delta_{\gamma\gamma_1} - \langle\beta\gamma|v|\beta_1\gamma_1\rangle(\theta_{\beta_1}+\theta_{\gamma_1})\delta_{\alpha\alpha_1}\delta_{\delta\delta_1} \\
&\qquad - \langle\beta\delta|v|\beta_1\delta_1\rangle(\theta_{\beta_1}+\theta_{\delta_1})\delta_{\alpha\alpha_1}\delta_{\gamma\gamma_1} - \langle\gamma\delta|v|\gamma_1\delta_1\rangle(\theta_{\gamma_1}+\theta_{\delta_1})\delta_{\alpha\alpha_1}\delta_{\beta\beta_1}. \\
&\quad (2\cdot30)
\end{aligned}$$

In Eq. (2·28) with Eqs. (2·29) and (2·30), we have used the following notations:

$$\Psi_n(\alpha\beta\gamma\delta) \equiv \int\left(\prod_{i=1}^{4} dx_i\right)\varphi_\alpha^*(x_1)\varphi_\beta^*(x_2)\varphi_\gamma^*(x_3)\varphi_\delta^*(x_4)\Psi_n(x_1x_2x_3x_4), \quad (2\cdot31)$$

$$\langle\alpha\beta|v|\alpha_1\beta_1\rangle \equiv \int dx_1dx_2\varphi_\alpha^*(x_1)\varphi_\beta^*(x_2)v(x_1x_2)\varphi_{\alpha_1}(x_1)\varphi_{\beta_1}(x_2), \quad (2\cdot32)$$

$$\theta_\alpha = \begin{cases} 1 \text{ for levels occupied in the Hartree-Fock ground state,} \\ 0 \text{ for levels unoccupied in the Hartree-Fock ground state.} \end{cases} \tag{2·33}$$

Equation (2·28) is nothing but the fundamental equation for describing the four-body correlations proposed by one of the authors (T. M.) and K. Suzuki.[2] Now, it should be noticed that the definition of θ_α represented by Eq. (2·33) is correct only for the case in which the density matrix of the "core" could be approximated by Eq. (2·27) and, in the case where the ground state includes the ground-state correlations (in the NTD sense), Eq. (2·33) is considered as an approximation.

2. 4 Properties of the fundamental eigenvalue equation

Let us decompose the wave function $\Psi_n(\alpha\beta\gamma\delta)$ as

$$\Psi_n(\alpha\beta\gamma\delta) = \sum_{i=1}^{5} \Psi_n^{(i)}(\alpha\beta\gamma\delta), \tag{2·34}$$

where

$$\begin{aligned} \Psi_n^{(1)}(\alpha\beta\gamma\delta) &= Q_{4p}(\alpha\beta\gamma\delta)\Psi_n(\alpha\beta\gamma\delta), \\ \Psi_n^{(2)}(\alpha\beta\gamma\delta) &= Q_{3p\text{-}1h}(\alpha\beta\gamma\delta)\Psi_n(\alpha\beta\gamma\delta), \\ \Psi_n^{(3)}(\alpha\beta\gamma\delta) &= Q_{2p\text{-}2h}(\alpha\beta\gamma\delta)\Psi_n(\alpha\beta\gamma\delta), \\ \Psi_n^{(4)}(\alpha\beta\gamma\delta) &= Q_{1p\text{-}3h}(\alpha\beta\gamma\delta)\Psi_n(\alpha\beta\gamma\delta), \\ \Psi_n^{(5)}(\alpha\beta\gamma\delta) &= Q_{4h}(\alpha\beta\gamma\delta)\Psi_n(\alpha\beta\gamma\delta), \end{aligned} \tag{2·35}$$

and the projection operators Q_{4p}, $Q_{3p\text{-}1h}$ etc. are defined respectively by

$$\begin{aligned} Q_{4p}(\alpha\beta\gamma\delta) &= (1-\theta_\alpha)(1-\theta_\beta)(1-\theta_\gamma)(1-\theta_\delta), \\ Q_{3p\text{-}1h}(\alpha\beta\gamma\delta) &= \theta_\alpha(1-\theta_\beta)(1-\theta_\gamma)(1-\theta_\delta) + (1-\theta_\alpha)\theta_\beta(1-\theta_\gamma)(1-\theta_\delta) \\ &\quad + (1-\theta_\alpha)(1-\theta_\beta)\theta_\gamma(1-\theta_\delta) + (1-\theta_\alpha)(1-\theta_\beta)(1-\theta_\gamma)\theta_\delta, \\ Q_{2p\text{-}2h}(\alpha\beta\gamma\delta) &= (1-\theta_\alpha)(1-\theta_\beta)\theta_\gamma\theta_\delta + (1-\theta_\alpha)\theta_\beta(1-\theta_\gamma)\theta_\delta \\ &\quad + (1-\theta_\alpha)\theta_\beta\theta_\gamma(1-\theta_\delta) + \theta_\alpha(1-\theta_\beta)(1-\theta_\gamma)\theta_\delta \\ &\quad + \theta_\alpha(1-\theta_\beta)\theta_\gamma(1-\theta_\delta) + \theta_\alpha\theta_\beta(1-\theta_\gamma)(1-\theta_\delta), \\ Q_{1p\text{-}3h}(\alpha\beta\gamma\delta) &= (1-\theta_\alpha)\theta_\beta\theta_\gamma\theta_\delta + \theta_\alpha(1-\theta_\beta)\theta_\gamma\theta_\delta + \theta_\alpha\theta_\beta(1-\theta_\gamma)\theta_\delta + \theta_\alpha\theta_\beta\theta_\gamma(1-\theta_\delta), \\ Q_{4h}(\alpha\beta\gamma\delta) &= \theta_\alpha\theta_\beta\theta_\gamma\theta_\delta. \end{aligned} \tag{2·36}$$

By definition, for example, $Q_{2p\text{-}2h}(\alpha\beta\gamma\delta)$ picks up only the components where two among the states $(\alpha, \beta, \gamma, \delta)$ are in unoccupied levels and the other two in occupied levels. Then, by using these definitions, Eq. (2·28) can be written in the form of five-component equation as

$$\{W(A_0+4;\, n)-W(A_0;\, 0)\}\Psi_n^{(i)}(\alpha\beta\gamma\delta)$$
$$=\sum_{j=1}^{5}\{\sum_{\alpha_1\beta_1\gamma_1\delta_1}H^{(ij)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}\Psi_n^{(j)}(\alpha_1\beta_1\gamma_1\delta_1)\},\quad (i=1,2,\cdots,5) \tag{2·37}$$

where the matrix elements $H^{(ij)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}$ $(i,j=1,2,\cdots,5)$ are defined, for example, as follows:

$$H^{(11)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}\equiv Q_{4p}(\alpha\beta\gamma\delta)\langle\alpha\beta\gamma\delta|H|\alpha_1\beta_1\gamma_1\delta_1\rangle Q_{4p}(\alpha_1\beta_1\gamma_1\delta_1),$$
$$H^{(32)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}\equiv Q_{2p-2h}(\alpha\beta\gamma\delta)\langle\alpha\beta\gamma\delta|H|\alpha_1\beta_1\gamma_1\delta_1\rangle Q_{3p-1h}(\alpha_1\beta_1\gamma_1\delta_1),$$
$$H^{(53)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}\equiv Q_{4h}(\alpha\beta\gamma\delta)\langle\alpha\beta\gamma\delta|H|\alpha_1\beta_1\gamma_1\delta_1\rangle Q_{2p-2h}(\alpha_1\beta_1\gamma_1\delta_1),\quad \text{etc.}, \tag{2·38}$$

the matrix elements $\langle\alpha\beta\gamma\delta|H|\alpha_1\beta_1\gamma_1\delta_1\rangle$ being defined by Eq. (2·28).

From the properties of the matrix elements of $H_{\rm int}^{(\rm I)}$ and $H_{\rm int}^{(\rm II)}$ given by Eqs. (2·29) and (2·30), we can find the following properties of $H^{(ij)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}$:

$$H^{(ii)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}=H^{(ii)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta},\quad (i=1,2,\cdots,5) \tag{2·39a}$$

$$\left.\begin{aligned} &H^{(31)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}=-H^{(13)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta},\quad H^{(42)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}=-H^{(24)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta},\\ &H^{(53)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1}=-H^{(35)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta},\end{aligned}\right\} \tag{2·39b}$$

$$H^{(12)}=H^{(14)}=H^{(15)}=H^{(25)}=H^{(41)}=H^{(51)}=H^{(52)}=H^{(54)}=0. \tag{2·39c}$$

Consequently the Hamiltonian can be written in the form of (5×5)-matrix as

$$(H^{(ij)})=\begin{pmatrix} H^{(11)} & 0 & H^{(13)} & 0 & 0\\ H^{(21)} & H^{(22)} & H^{(23)} & H^{(24)} & 0\\ -H^{(13)\dagger} & H^{(32)} & H^{(33)} & H^{(34)} & H^{(35)}\\ 0 & -H^{(24)\dagger} & H^{(43)} & H^{(44)} & H^{(45)}\\ 0 & 0 & -H^{(35)\dagger} & 0 & H^{(55)}\end{pmatrix}. \tag{2·40}$$

Since the norm of the five-component wave function $\Psi_n^{(i)}(\alpha\beta\gamma\delta)$ cannot be defined because of a lack of symmetry property in the Hamiltonian (2·40), the concept of the alpha-like four-body *mode* does not seem to be established without assuming any approximation for the Hamiltonian. Therefore, we make an approximation that the contributions of the matrix elements $\langle\alpha\beta|v|\alpha_1\beta_1\rangle$, in which one among $(\alpha,\beta,\alpha_1,\beta_1)$ is a particle- (or hole-)state and the other three are hole- (or particle-)states, to our four-body correlations are not so important compared with those of the other matrix elements and may be neglected at the first step. The matrix elements neglected in our approximation are graphically shown in Fig. 1. Hereafter we will call them *Y*-type interaction. The *Y*-type interaction is, in essence, what should be renormalized into the single-particle field and then not the interaction con-

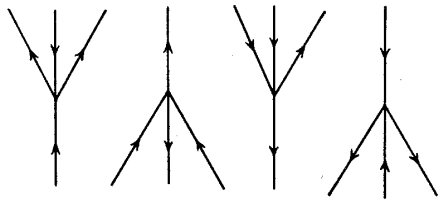

Fig. 1. Graphical representation of the matrix elements of the Y-type interaction.

structing the *modes*. Therefore, the above approximation in which the Y-type interaction is neglected at the first step is consistent with our basic assumption under which the concept of the four-body mode is considered to be well-established and *only* the special class of the eigenstates formed by operating the modes $X_n^\dagger$ to the ground state of the "core" is taken into account.

Under the approximation to neglect the Y-type interaction, the properties

$$H^{(21)}=H^{(23)}=H^{(32)}=H^{(34)}=H^{(43)}=H^{(45)}=0 \tag{2·41}$$

can be found and the Hamiltonian (2·40) is simply reduced to

$$(H^{(ij)})=\begin{pmatrix} H^{(11)} & 0 & H^{(13)} & 0 & 0 \\ 0 & H^{(22)} & 0 & H^{(24)} & 0 \\ -H^{(13)\dagger} & 0 & H^{(33)} & 0 & H^{(35)} \\ 0 & -H^{(24)\dagger} & 0 & H^{(44)} & 0 \\ 0 & 0 & -H^{(35)\dagger} & 0 & H^{(55)} \end{pmatrix}. \tag{2·42}$$

Then the eigenvalue equation (2·37) is devided into two equations

$$\begin{aligned} &\{W(A_0+4;\,n)-W(A_0;\,0)\}\begin{pmatrix} \Psi_n^{(1)}(\alpha\beta\gamma\delta) \\ \Psi_n^{(3)}(\alpha\beta\gamma\delta) \\ \Psi_n^{(5)}(\alpha\beta\gamma\delta) \end{pmatrix} \\ &=\sum_{\alpha_1\beta_1\gamma_1\delta_1}\begin{pmatrix} H^{(11)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} & -H^{(13)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} & 0 \\ -H^{(13)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta} & -H^{(33)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} & H^{(35)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} \\ 0 & H^{(35)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta} & H^{(55)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} \end{pmatrix} \\ &\quad\times\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \Psi_n^{(1)}(\alpha_1\beta_1\gamma_1\delta_1) \\ \Psi_n^{(3)}(\alpha_1\beta_1\gamma_1\delta_1) \\ \Psi_n^{(5)}(\alpha_1\beta_1\gamma_1\delta_1) \end{pmatrix}, \end{aligned} \tag{2·43a}$$

$$\begin{aligned} &\{W(A_0+4;\,n)-W(A_0;\,0)\}\begin{pmatrix} \Psi_n^{(2)}(\alpha\beta\gamma\delta) \\ \Psi_n^{(4)}(\alpha\beta\gamma\delta) \end{pmatrix} \\ &=\sum_{\alpha_1\beta_1\gamma_1\delta_1}\begin{pmatrix} H^{(22)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} & H^{(24)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} \\ -H^{(24)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta} & H^{(44)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} \end{pmatrix}\begin{pmatrix} \Psi_n^{(2)}(\alpha_1\beta_1\gamma_1\delta_1) \\ \Psi_n^{(4)}(\alpha_1\beta_1\gamma_1\delta_1) \end{pmatrix}. \end{aligned} \tag{2·43b}$$

The special class of the eigenstates with eigenvalue $\omega_n \equiv W(A_0+4;\, n) - W(A_0;\, 0)$, which is suitable for our picture (i. e., the alpha-like four-body mode in the presence of the "core"), must have the largest four-particle amplitude $\Psi_n^{(1)}(\alpha\beta\gamma\delta)$. Therefore, the special class of the eigenstates under consideration are completely determined by Eq. (2·43a). For such physical solutions, Eq. (2·43b) only shows that the amplitudes $\Psi_n^{(2)}(\alpha\beta\gamma\delta)$ and $\Psi_n^{(4)}(\alpha\beta\gamma\delta)$ vanish because the determinant of the coefficients of the simultaneous linear equation (2·43b) does, in general, not vanish for those ω_n, i. e.,

$$\begin{vmatrix} H^{(22)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} - \omega_n \delta_{\alpha\alpha_1}\delta_{\beta\beta_1}\delta_{\gamma\gamma_1}\delta_{\delta\delta_1} & H^{(24)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} \\ -H^{(24)*}_{\alpha_1\beta_1\gamma_1\delta_1,\alpha\beta\gamma\delta} & H^{(44)}_{\alpha\beta\gamma\delta,\alpha_1\beta_1\gamma_1\delta_1} - \omega_n \delta_{\alpha\alpha_1}\delta_{\beta\beta_1}\delta_{\gamma\gamma_1}\delta_{\delta\delta_1} \end{vmatrix} \neq 0. \quad (2\cdot44)$$

Thus, all informations on the physical eigenmodes suitable for our picture must be given by Eq. (2·43a).

From the eigenvalue equation (2·43a), we have the following orthonormality condition for the amplitudes $\Psi_n^{(i)}(\alpha\beta\gamma\delta)$:

$$\sum_{\alpha\beta\gamma\delta} (\Psi_m^{(1)*}(\alpha\beta\gamma\delta), \Psi_m^{(3)*}(\alpha\beta\gamma\delta), \Psi_m^{(5)*}(\alpha\beta\gamma\delta)) \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Psi_n^{(1)}(\alpha\beta\gamma\delta) \\ \Psi_n^{(3)}(\alpha\beta\gamma\delta) \\ \Psi_n^{(5)}(\alpha\beta\gamma\delta) \end{pmatrix} = N_n \delta_{mn}, \quad (2\cdot45)$$

where N_n is the normalization constant. This condition is described in another form

$$\sum_{\alpha\beta\gamma\delta} \Psi_m^*(\alpha\beta\gamma\delta) \{Q_{4p}(\alpha\beta\gamma\delta) - Q_{2p-2h}(\alpha\beta\gamma\delta) + Q_{4h}(\alpha\beta\gamma\delta)\} \Psi_n(\alpha\beta\gamma\delta) = N_n \delta_{mn}. \quad (2\cdot46)$$

2. 5 Summary on the fundamental eigenvalue equation and the physical meaning of the alpha-like four-body eigenmodes

In the present subsection, we will give a compact expression of the fundamental eigenvalue equation for describing the alpha-like four-body mode, which has been reduced to the three-component equation (2·43a) in the last subsection, and will also discuss the physical meaning of eigenmodes.

In order to simplify the expression, we define a three-component vector $\boldsymbol{\Psi}_n$ and an operator $\boldsymbol{P}$ as follows:

$$\boldsymbol{\Psi}_n = \begin{pmatrix} \Psi_n^{(1)} \\ \Psi_n^{(3)} \\ \Psi_n^{(5)} \end{pmatrix}, \quad (2\cdot47)$$

$$\boldsymbol{P} = \begin{pmatrix} Q_{4p} & 0 & 0 \\ 0 & -Q_{2p-2h} & 0 \\ 0 & 0 & Q_{4h} \end{pmatrix}, \quad (2\cdot48)$$

where the shell-model representations of $\Psi_n^{(1)}$, $\Psi_n^{(3)}$, $\Psi_n^{(5)}$ are the amplitudes appeared in Eq. (2·43a) and the matrix elements of the operators Q_{4p}, Q_{2p-2h}, Q_{4h} are defined by Eq. (2·36). By using the relation

$$\boldsymbol{P}^2=\begin{pmatrix} Q_{4p} & 0 & 0 \\ 0 & Q_{2p-2h} & 0 \\ 0 & 0 & Q_{4h} \end{pmatrix}, \tag{2·49}$$

the eigenvalue equation (2·43a) is formally written as

$$\omega_n \boldsymbol{P}\boldsymbol{\Psi}_n=\boldsymbol{P}H\boldsymbol{P}^2\boldsymbol{\Psi}_n, \tag{2·50}$$

where the eigenvalue ω_n means $W(A_0+4;\ n)-W(A_0;\ 0)$ and H is the effective Hamiltonian defined by Eq. (2·14). Now, we write the Hamiltonian in the form

$$H=H_0+H_{\rm int}, \tag{2·51a}$$

$$H_0=\frac{1}{2m}\sum_{i=1}^{4}\boldsymbol{p}_i^2+\sum_{i=1}^{4}U_i, \tag{2·51b}$$

$$H_{\rm int}=H_{\rm int}^{(\rm I)}+H_{\rm int}^{(\rm II)}. \tag{2·51c}$$

Here, it should be noticed that $H_{\rm int}$ is not Hermitian but $\boldsymbol{P}H\boldsymbol{P}^2$ appeared on the right-hand side of Eq. (2·50) is Hermitian. We introduce new notations defined by

$$\boldsymbol{H}\equiv\boldsymbol{P}H\boldsymbol{P}^2=\boldsymbol{H}_0+\boldsymbol{H}_{\rm int}, \tag{2·52a}$$

$$\boldsymbol{H}_0\equiv\boldsymbol{P}H_0\boldsymbol{P}^2, \tag{2·52b}$$

$$\boldsymbol{H}_{\rm int}\equiv\boldsymbol{P}H_{\rm int}\boldsymbol{P}^2. \tag{2·52c}$$

Since both H_0 and $\boldsymbol{P}$ are diagonal in the shell-model representation, they are commutable with each other. Then we obtain

$$\boldsymbol{H}_0=H_0\boldsymbol{P}=\boldsymbol{P}H_0. \tag{2·53}$$

The orthonormality condition is simply written in the form

$$(\boldsymbol{\Psi}_m\cdot\boldsymbol{\Psi}_n)=\boldsymbol{\Psi}_m^{\dagger}\boldsymbol{P}\boldsymbol{\Psi}_n=N_n\delta_{mn}. \tag{2·54}$$

For the sake of later convenience, we will write the coordinate representation of the above expressions. The fundamental eigenvalue equation (2·50) is expressed in the representation as

$$\int[\langle x_1x_2x_3x_4|\boldsymbol{H}|x_1'x_2'x_3'x_4'\rangle-\omega_n\langle x_1x_2x_3x_4|\boldsymbol{P}|x_1'x_2'x_3'x_4'\rangle]\times\boldsymbol{\Psi}_n(x_1'x_2'x_3'x_4')\Big(\prod_{i=1}^{4}dx_i\Big)=0. \tag{2·55}$$

By using Eqs. (2·51b), (2·52) and (2·53), Eq. (2·55) is rewritten in the form

$$\int [\langle x_1x_2x_3x_4|\boldsymbol{P}|x_1'x_2'x_3'x_4'\rangle \Big\{\omega_n - \sum_{i=1}^{4}\Big(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}_i'^2 + U(x_i')\Big)\Big\} - \langle x_1x_2x_3x_4|\boldsymbol{H}_{\rm int}|x_1'x_2'x_3'x_4'\rangle]\boldsymbol{\Psi}_n(x_1'x_2'x_3'x_4')\Big(\prod_{i=1}^{4}dx_i'\Big) = 0, \quad (2\cdot56)$$

where we have assumed that the average field is local. Here the matrix elements of $\boldsymbol{P}$ are

$$\langle x_1x_2x_3x_4|\boldsymbol{P}|x_1'x_2'x_3'x_4'\rangle = \begin{pmatrix} \langle x_1x_2x_3x_4|Q_{4p}|x_1'x_2'x_3'x_4'\rangle & 0 & 0 \\ 0 & -\langle x_1x_2x_3x_4|Q_{2p\text{-}2h}|x_1'x_2'x_3'x_4'\rangle & 0 \\ 0 & 0 & \langle x_1x_2x_3x_4|Q_{4h}|x_1'x_2'x_3'x_4'\rangle \end{pmatrix} \quad (2\cdot57)$$

and

$$\left.\begin{aligned} &\langle x_1x_2x_3x_4|Q_{4p}|x_1'x_2'x_3'x'_4\rangle = \prod_{i=1}^{4}\eta(x_i, x_i'), \\ &\langle x_1x_2x_3x_4|Q_{2p\text{-}2h}|x_1'x_2'x_3'x_4'\rangle \\ &\qquad = \frac{1}{4}\sum_{(ijkl)}\rho(x_i, x_i')\rho(x_j, x_j')\eta(x_k, x_k')\eta(x_l, x_l'), \\ &\langle x_1x_2x_3x_4|Q_{4h}|x_1'x_2'x_3'x_4'\rangle = \prod_{i=1}^{4}\rho(x_i, x_i'), \end{aligned}\right\} \quad (2\cdot58)$$

where

$$\eta(x_i, x_i') \equiv \delta(x_i, x_i') - \rho(x_i, x_i') \quad (2\cdot59)$$

and the symbol $\sum_{(ijkl)}$ means the summation over all permutations of $(1, 2, 3, 4)$. The interaction Hamiltonian $\boldsymbol{H}_{\rm int}$ is a (3×3)-component operator and it is expressed in the following form:

$$\boldsymbol{H}_{\rm int} = \begin{pmatrix} H_{\rm int}^{(11)} & H_{\rm int}^{(13)} & 0 \\ H_{\rm int}^{(13)\dagger} & -H_{\rm int}^{(33)} & H_{\rm int}^{(53)\dagger} \\ 0 & H_{\rm int}^{(53)} & H_{\rm int}^{(55)} \end{pmatrix}. \quad (2\cdot60)$$

The coordinate representations of these components are

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(11)}|x_1'x_2'x_3'x_4'\rangle = \frac{1}{4}\sum_{(ijkl)}\eta(x_i, x_i')\eta(x_j, x_j') \times \int dx_k''dx_l''\eta(x_k, x_k'')\eta(x_l, x_l'')v(x_k'', x_l'')\eta(x_k'', x_k')\eta(x_l'', x_l'), \quad (2\cdot61a)$$

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(13)}|x_1'x_2'x_3'x_4'\rangle=-\frac{1}{4}\sum_{(ijkl)}\eta(x_i,x_i')\eta(x_j,x_j')$$
$$\times\int dx_k''dx_l''\eta(x_k,x_k'')\eta(x_l,x_l'')v(x_k''x_l'')\rho(x_k'',x_k')\rho(x_l'',x_l'), \tag{2·61b}$$

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(33)}|x_1'x_2'x_3'x_4'\rangle=\frac{1}{4}\sum_{(ijkl)}\rho(x_i,x_i')\rho(x_j,x_j')$$
$$\times\int dx_k''dx_l''\eta(x_k,x_k'')\eta(x_l,x_l'')v(x_k''x_l'')\eta(x_k'',x_k')\eta(x_l'',x_l')$$
$$-\eta(x_i,x_i')\eta(x_j,x_j')$$
$$\times\int dx_k''dx_l''\rho(x_k,x_k'')\rho(x_l,x_l'')v(x_k''x_l'')\rho(x_k'',x_k')\rho(x_l'',x_l')], \tag{2·61c}$$

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(53)}|x_1'x_2'x_3'x_4'\rangle=\frac{1}{4}\sum_{(ijkl)}\rho(x_i,x_i')\rho(x_j,x_j')$$
$$\times\int dx_k''dx_l''\rho(x_k,x_k'')\rho(x_l,x_l'')v(x_k''x_l'')\eta(x_k'',x_k')\eta(x_l'',x_l'), \tag{2·61d}$$

$$\langle x_1x_2x_3x_4|H_{\rm int}^{(55)}|x_1'x_2'x_3'x_4'\rangle=-\frac{1}{4}\sum_{(ijkl)}\rho(x_i,x_i')\rho(x_j,x_j')$$
$$\times\int dx_k''dx_l''\rho(x_k,x_k'')\rho(x_l,x_l'')v(x_k''x_l'')\rho(x_k'',x_k')\rho(x_l'',x_l'). \tag{2·61e}$$

The alpha-like four-body eigenmodes in (A_0+4)-particle system are obtained by solving the eigenvalue equation (2·56). The modes $X_n^\dagger$ thus obtained can be written as

$$X_n^\dagger=\frac{1}{\sqrt{4!}}\int(\prod_{i=1}^4 dx_idx_i')[\langle x_1x_2x_3x_4|Q_{4p}|x_1'x_2'x_3'x_4'\rangle\Psi_n^{(1)}(x_1'x_2'x_3'x_4')$$
$$+\langle x_1x_2x_3x_4|Q_{2p\text{-}2h}|x_1'x_2'x_3'x_4'\rangle\Psi_n^{(3)}(x_1'x_2'x_3'x_4')$$
$$+\langle x_1x_2x_3x_4|Q_{4h}|x_1'x_2'x_3'x_4'\rangle\Psi_n^{(5)}(x_1'x_2'x_3'x_4')]$$
$$\times\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4)$$
$$=\frac{1}{\sqrt{4!}}\int(\prod_{i=1}^4 dx_idx_i')\langle x_1x_2x_3x_4|\boldsymbol{P}^2|x_1'x_2'x_3'x_4'\rangle\boldsymbol{\Psi}_n(x_1'x_2'x_3'x_4')$$
$$\times\phi^\dagger(x_1)\phi^\dagger(x_2)\phi^\dagger(x_3)\phi^\dagger(x_4). \tag{2·62}$$

These modes $X_n^\dagger$ are classified according to the magnitudes of the amplitudes $\Psi_n^{(1)}$, $\Psi_n^{(3)}$, $\Psi_n^{(5)}$ as follows:

$$X_n^\dagger=\begin{cases}X_{n_+}^\dagger\equiv\alpha_n^\dagger,\\ X_{n_0}^\dagger\equiv\gamma_n^\dagger,\\ X_{n_-}^\dagger\equiv\beta_n.\end{cases} \tag{2·63}$$

Here, the operator $\alpha_n^\dagger$ is the creation of the alpha-like four-body modes which creates four particles with the large amplitudes $\Psi_{n+}^{(1)}(x_1x_2x_3x_4)$ and annihilates four holes with the small amplitudes $\Psi_{n+}^{(5)}(x_1x_2x_3x_4)$ being accompanied by the two-particle creation and two-hole annihilation amplitudes $\Psi_{n+}^{(3)}(x_1x_2x_3x_4)$. In the same way, the operator $\beta_n^\dagger$ is the creation operator of the correlated four-hole mode which creates four holes with large amplitudes $\Psi_{n-}^{(5)}(x_1x_2x_3x_4)$ and annihilates four particles with small amplitudes $\Psi_{n-}^{(1)}(x_1x_2x_3x_4)$ being accompanied by the two-hole creation and two-particle annihilation amplitudes $\Psi_{n-}^{(3)}(x_1x_2x_3x_4)$. It should be noticed that, in the absence of the ground-state correlations $\alpha_n^\dagger$ and $\beta_n^\dagger$ become the operators which create an exact four-particle eigenstate and an exact four-hole eigenstate in the sense of the Tamm-Dancoff (TD) approximation. The appearance of an "unphysical" eigenmode $\gamma_n^\dagger$ whose largest amplitudes are $\Psi_{n0}^{(3)}(x_1x_2x_3x_4)$ is essentially due to the fact that the original interaction of our system is not a four-body interaction but the two-body interaction. It should be noticed that, in the absence of the ground-state correlations, the "unphysical" eigenmode $\gamma_n^\dagger$ does not appear.

The structure of the ground state $|A_0; 0\rangle$ of the "core" (including the ground-state correlations in the NTD sense) is determined within the framework of the NTD approximation by the equations

$$\alpha_n|A_0; 0\rangle=0, \quad \beta_n|A_0; 0\rangle=0, \quad \gamma_n^\dagger|A_0; 0\rangle=\gamma_n|A_0; 0\rangle=0. \tag{2·64}$$

Clearly, the ground state $|A_0; 0\rangle$ incorporates the collective correlations responsible for the four-body modes and is a superposition of $0p$-$0h$, $2p$-$2h$, $4p$-$4h$, $\cdots$ excitations in the sense of the TD method.

The orthonormality condition of the eigenstates of the (A_0+4)-particle system is represented by

$$\left.\begin{aligned}&\langle A_0; 0|\alpha_m\alpha_n^\dagger|A_0; 0\rangle=N_n\delta_{mn},\\&\langle A_0; 0|\beta_m\beta_n^\dagger|A_0; 0\rangle=N_n\delta_{mn}.\end{aligned}\right\} \tag{2·65}$$

As discussed in detail by one of the authors (T. M.) and Suzuki,[2] it can be shown that Eq. (2·65) is equivalent with the orthonormality condition (2·45) or (2·46).

§3. Structure of four-body correlations (I)

—Spatial correlations—

In the present and the next sections, we will study the structure of the four-body correlations in light nuclei on the basis of the formulation obtained in the last section. For this aim, we devide the four-body correlations into two classes, i.e., the *spatial correlations*, which clusterize (or localize) four

particles (two neutrons and two protons) in light nuclei, and the *ground-state correlations*, which is one of the important ingredients bringing about the four-body scattering correlations. These two kinds of correlations must be strongly related with each other. However, in order to investigate the structure of each kind of correlation, we dare to separate them.

In this section, introducing a model space i.e., the *vertically truncated subspace*, which has been shown to be just suitable for investigation of the spatial correlations,[5] we solve the alpha-like four-body modes in ^{20}Ne and study the properties of the correlations. The first half of the description in this section is based on the work of Ref. 6).

3.1 Physical essence of the spatial correlations

As discussed in the previous chapters, the alpha-cluster aspect in light nuclei is characterized by the fact that two neutrons and two protons in nuclei show strong internal binding and weak relative interaction, so that they can be seen to be an unit of the nuclear constituents. From a viewpoint of our picture which has already been discussed in §1, we could understand this as follows: Two neutrons and two protons in the nuclear Hartree-Fock field correlate strongly so as to make spatial localization in some extent. In other words, the amplitude of the alpha-like four-body mode has large mixing of many higher-shell components when it is expanded by the Hartree-Fock single-particle wave functions. Such higher-shell mixing property is extremely different from that taken into account in the conventional shell model, in which only one or two major shells in the neighborhood of the Fermi surface are considered. Thus the concept of the alpha-cluster is very different from one in the shell model.

In order to see how the spatial localization (or higher-shell mixing) of two neutrons and two protons is realized, we investigate the case of ^{20}Ne in the framwork of the formulation represented in the last section. Here, we regard the so-called rotational ground band in ^{20}Ne as the excitations of the alpha-like four-body modes in the presence of the ^{16}O-"core". Since the aim of the present section is to investigate the higher-shell mixing in the modes, it may be reasonable to adopt the Tamm-Dancoff approximation. In this approximation, the ground state of the "core" is assumed to be the Hartree-Fock ground state and, in the present case, it is the doubly closed shell of ^{16}O. Therefore, the two components in the amplitude vector of the four-body eigenmode vanish, i.e.,

$$\Psi_n^{(3)}=\Psi_n^{(5)}=0. \tag{3·1}$$

Then the fundamental eigenvalue equation (2·55) becomes one-component equation

$$\int[\langle x_1x_2x_3x_4|Q_{4p}HQ_{4p}|x_1'x_2'x_3'x_4'\rangle-\omega_n\langle x_1x_2x_3x_4|Q_{4p}|x_1'x_2'x_3'x_4'\rangle]$$
$$\times\Psi^{(1)}(x_1'x_2'x_3'x_4')(\prod_{i=1}^{4}dx_i')=0. \tag{3·2}$$

In the operator H which has been given by Eqs. (2·51), the part $H_{\rm int}^{\rm (II)}$ does not contribute in the Tamm-Dancoff approximation. We can then write

$$Q_{4p}HQ_{4p}=Q_{4p}(H_0+H_{\rm int}^{\rm (I)})Q_{4p}$$
$$=Q_{4p}\Big(\frac{1}{2m}\sum_{i=1}^{4}\boldsymbol{p}_i^2+\sum_{i=1}^{4}U_i+\sum_{i<j}^{4}v_{ij}\Big)Q_{4p}. \tag{3·3}$$

It is natural to consider that the correlation amplitudes $\Psi_n^{(1)}(x_1x_2x_3x_4)$ of the alpha-like four-boby modes under consideration have the [4] symmetry which is one of the most important properties in the usual shell model of light nuclei as well as in the alpha-cluster model. The spin and isospin part of the correlation amplitude may, therefore, be expressed by the Slater determinant coupled to both spin and isospin zero, $\chi_{S=0,T=0}(1234)$ as

$$\Psi_n^{(1)}(x_1x_2x_3x_4)\equiv\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\chi_{S=0,T=0}(1234), \tag{3·4}$$

where L and M mean the total angular momentum and its projection of the four particles, respectively.

3.2 Subspace for describing the spatial correlations and generator-coordinate representation

Since it is almost impossible to solve the eigenvalue equation (3·2), we make an approximation for the correlation amplitude $\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$, by which a new type of truncation of the Hilbert space is defined. The eigenstate $X_n^\dagger|A_0;0\rangle$ under the approximation should be able to involve not only the most important part of the usual shell model at one limit but also the localized alpha-cluster-like character at another limit.

According to the above consideration, we adopt the following type of function as the correlation amplitude $\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$;

$$\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\equiv\exp\Big\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{S})^2\Big\}\langle\boldsymbol{S}|\Phi_{LM}\rangle$$
$$=\exp\Big(-\frac{\alpha^2}{2}\sum_{j=1}^{3}t_j^2\Big)\langle\boldsymbol{S}|\Phi_{LM}\rangle, \tag{3·5a}$$

$$\langle\boldsymbol{S}|\Phi_{LM}\rangle\equiv\widehat{\Phi}_L(S)\,Y_{LM}(\Omega_S), \tag{3·5b}$$

where $\boldsymbol{S}$ is the centre-of-mass coordinate of the outer four particles and $\boldsymbol{t}_1$, $\boldsymbol{t}_2$ and $\boldsymbol{t}_3$ are their internal coordinates in the sense that

$$\boldsymbol{S}=\frac{1}{4}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4),$$

$$\boldsymbol{t}_1=\frac{1}{\sqrt{2}}(\boldsymbol{r}_1-\boldsymbol{r}_2),\quad \boldsymbol{t}_2=\frac{1}{\sqrt{2}}(\boldsymbol{r}_3-\boldsymbol{r}_4),\quad \boldsymbol{t}_3=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2-\boldsymbol{r}_3-\boldsymbol{r}_4). \quad (3\cdot6)$$

The set of the state vector $|\Phi_{LM}\rangle$, whose coordinate representation is $\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$ defined by Eqs. (3·5), forms a subspace $\mathcal{H}_\Phi$ in the Hilbert space $\mathcal{H}$ describing the four-particle states.

The correlation amplitude (3·5a) is equivalently expressed in the form

$$\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)=\int\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R})^2\right\}\langle\boldsymbol{R}|\Phi_{LM}\rangle d\boldsymbol{R}, \quad (3\cdot7a)$$

$$\langle\boldsymbol{R}|\Phi_{LM}\rangle=\Phi_L(R)\,Y_{LM}(\Omega_R), \quad (3\cdot7b)$$

and the relation between $\langle\boldsymbol{S}|\Phi_{LM}\rangle$ and $\langle\boldsymbol{R}|\Phi_{LM}\rangle$

$$\langle\boldsymbol{S}|\Phi_{LM}\rangle=\int\exp\{-2\alpha^2(\boldsymbol{R}-\boldsymbol{S})^2\}\langle\boldsymbol{R}|\Phi_{LM}\rangle d\boldsymbol{R} \quad (3\cdot8)$$

is easily found.

Expression (3·7a) means that $\langle\boldsymbol{R}|\Phi_{LM}\rangle$ is the representation of the state vector $|\Phi_{LM}\rangle$ in the subspace $\mathcal{H}_\Phi$ by means of the generator coordinate $\boldsymbol{R}$. This generator coordinate is connected with the actual centre-of-mass coordinate of the four particles, $\boldsymbol{S}$, through Eq. (3·8). However, it should be emphasized that the coordinate $\boldsymbol{R}$ is nothing but a parameter coordinate. The use of the coordinate $\boldsymbol{R}$ displays great advantages in solving the eigenvalue equation (3·2) in the subspace $\mathcal{H}_\Phi$. Hence it is convenient to carry out the actual calculation with the $\boldsymbol{R}$-coordinate and to discuss the physical situations after transforming into $\boldsymbol{S}$-coordinate.

Substituting the correlation amplitude (3·4) with Eqs. (3·7) into the eigenvalue equation (3·2), we obtain the integral equation in the R-coordinate

$$\int[H_L(R,R')-\omega_L N_L(R,R')]\Phi_L(R')R'^2dR'=0 \quad (3\cdot9)$$

with the overlap kernel and the energy kernel

$$\begin{aligned}\begin{pmatrix}N_L(R,R')\\H_L(R,R')\end{pmatrix}=&\int\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R})^2\right\}Y^*_{LM}(\Omega_R)\chi^*_{S=0,T=0}(1234)\\&\times\begin{pmatrix}\langle x_1x_2x_3x_4|Q_{4p}|x_1'x_2'x_3'x_4'\rangle\\\langle x_1x_2x_3x_4|Q_{4p}HQ_{4p}|x_1'x_2'x_3'x_4'\rangle\end{pmatrix}\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i'-\boldsymbol{R}')^2\right\}\\&\times Y_{LM}(\Omega_R')\chi_{S=0,T=0}(1'2'3'4')d\Omega_Rd\Omega_{R'}\left(\prod_{i=1}^{4}dx_idx_i'\right).\end{aligned} \quad (3\cdot10)$$

From relations (2·46) or (2·54), the normalization condition of $\Phi_L(R)$

$$\int \Phi_L^*(R) N_L(R, R') \Phi_L(R') R^2 R'^2 dR dR' = 1 \tag{3·11}$$

is obtained.

In the present case, since we consider the ^{16}O-core as a doubly closed shell, the density matrix of the core is expressed in the form

$$\rho(x, x') = \rho(\boldsymbol{r}, \boldsymbol{r}') \delta(\sigma, \sigma') \delta(\tau, \tau'). \tag{3·12}$$

Then the projection operator Q_{4p} is written as

$$\begin{aligned} &\langle x_1 x_2 x_3 x_4 | Q_{4p} | x_1' x_2' x_3' x_4' \rangle \\ &\quad = \langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle \prod_{i=1}^{4} \delta(\sigma_i, \sigma_i') \delta(\tau_i, \tau_i') \end{aligned} \tag{3·13}$$

with

$$\langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle = \prod_{i=1}^{4} [\delta(\boldsymbol{r}_i - \boldsymbol{r}_i') - \rho(\boldsymbol{r}_i, \boldsymbol{r}_i')], \tag{3·14}$$

$$\rho(\boldsymbol{r}, \boldsymbol{r}') = \sum_{nlm}{}' \varphi_{nlm}(\boldsymbol{r}) \varphi_{nlm}^*(\boldsymbol{r}'), \tag{3·15}$$

where $\varphi_{nlm}(\boldsymbol{r})$ is the space-coordinate part of the single-particle wave function.

Using Eq. (3·13), we can write the overlap kernel and the energy kernel as follows:

$$\begin{aligned} \begin{pmatrix} N_L(R, R') \\ H_L(R, R') \end{pmatrix} &= \int \exp\left\{ -\frac{\alpha^2}{2} \sum_{i=1}^{4} (\boldsymbol{r}_i - \boldsymbol{R})^2 \right\} Y_{LM}^*(\Omega_R) \\ &\times \begin{pmatrix} \langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle \\ \langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} H Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle \end{pmatrix} \exp\left\{ -\frac{\alpha^2}{2} \sum_{i=1}^{4} (\boldsymbol{r}_i' - \boldsymbol{R}')^2 \right\} Y_{LM}(\Omega_{R'}) \\ &\times d\Omega_R d\Omega_{R'} \left(\prod_{i=1}^{4} d\boldsymbol{r}_i d\boldsymbol{r}_i' \right), \end{aligned} \tag{3·16}$$

where

$$\begin{aligned} \langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} H Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle &= \int \langle \boldsymbol{r}_1 \boldsymbol{r}_2 \boldsymbol{r}_3 \boldsymbol{r}_4 | Q_{4p} | \boldsymbol{r}_1'' \boldsymbol{r}_2'' \boldsymbol{r}_3'' \boldsymbol{r}_4'' \rangle \\ &\times \left\{ \sum_{i=1}^{4} \left(-\frac{\hbar^2}{2m} \nabla_i''^2 \right) + \sum_{i=1}^{4} U(\boldsymbol{r}_i'') + \sum_{i<j} \bar{v}(\boldsymbol{r}_i'', \boldsymbol{r}_j'') \right\} \\ &\times \langle \boldsymbol{r}_1'' \boldsymbol{r}_2'' \boldsymbol{r}_3'' \boldsymbol{r}_4'' | Q_{4p} | \boldsymbol{r}_1' \boldsymbol{r}_2' \boldsymbol{r}_3' \boldsymbol{r}_4' \rangle \left(\prod_{i=1}^{4} d\boldsymbol{r}_i'' \right). \end{aligned} \tag{3·17}$$

Here we have assumed that the residual two-body force in Eq. (3·3), which is the internal interaction acting among the outer four particles, is a central force and

$$\bar{v}(\boldsymbol{r}_i, \boldsymbol{r}_j) = \frac{1}{2} [v_{13}(\boldsymbol{r}_i, \boldsymbol{r}_j) + v_{31}(\boldsymbol{r}_i, \boldsymbol{r}_j)], \tag{3·18}$$

where v_{13} and v_{31} are the potentials for the states of $T=0$, $S=1$ and $T=1$,

$S=0$, respectively.

3.3 Vertically truncated subspace and its basis vectors

Now, it is very convenient to choose a suitable set of the basis vectors of the subspace $\mathcal{H}_\Phi$. For this purpose, we assume the harmonic oscillator potential and a Gaussian interaction for the single-particle potential $U(\boldsymbol{r})$ and the residual interaction $\bar{v}(\boldsymbol{r}_i, \boldsymbol{r}_j)$ as follows:

$$U(\boldsymbol{r}) \equiv \frac{1}{2} m\omega_0{}^2 r^2, \tag{3·19}$$

$$\bar{v}(\boldsymbol{r}_i, \boldsymbol{r}_j) \equiv \frac{1}{2}({}^{13}V + {}^{31}V)\exp(-\mu^2 r_{ij}^2), \quad (r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j|), \tag{3·20}$$

where ${}^{13}V$ and ${}^{31}V$ are the interaction strengths in the states of $T=0$, $S=1$ and $T=1$, $S=0$, respectively. It should be noticed that even if we add a spin-orbit interaction to the single-particle potential (3·19), it has no contribution to the energy kernel $H_L(R, R')$ because of the [4] symmetry property of the subspace $\mathcal{H}_\Phi$.

As is well-known, the harmonic oscillator wave function and the corresponding energy eigenvalue are

$$\varphi_{nlm}(\boldsymbol{r}) = R_{nl}(r)\, Y_{lm}(\Omega_r), \tag{3·21}$$

$$R_{nl}(r) = \left[\frac{\beta^{2l+3}}{\sqrt{\pi}} \frac{2^{n+l+2} n!}{(2n+2l+1)!!}\right]^{1/2} r^l e^{-(\beta^2/2)r^2} L_n^{(l+1/2)}(\beta^2 r^2) \tag{3·22}$$

and

$$\varepsilon_{nl} = (2n+l+3/2)\hbar\omega_0, \tag{3·23}$$

where $\beta = \sqrt{m\omega_0/\hbar}$ stands for the size parameter of the core nucleus and $L_n^{(l+1/2)}(\beta^2 r^2)$ is the associate Laguerre polynomial. Substituting this single-particle wave function into Eq. (3·15), we obtain the following density matrix of the ^{16}O-core;

$$\rho(\boldsymbol{r}, \boldsymbol{r}') = \left(\frac{\beta}{\sqrt{\pi}}\right)^3 e^{-\beta^2(r^2+r'^2)/2}\{1 + 2\beta^2(\boldsymbol{r}\cdot\boldsymbol{r}')\}. \tag{3·24}$$

It is straightforward to get the explicit expressions of the kernels, $N_L(R, R')$ and $H_L(R, R')$, from Eq. (3·16) by using this density matrix. However, as they are rather complicated, we do not write them here.

Now, let $N_L^{(0)}(R, R')$ and $H_L^{(0)}(R, R')$ be the normalization kernel and the energy kernel when we suppose $\rho=0$ (which means neglect of the effect of the Pauli principle from the "core"). These kernels are obtained by putting $Q_{4\rho}=1$ in Eq. (3·16) and written down in the following forms:

$$N_L^{(0)}(R,R')=\frac{2^2\pi^7}{\alpha^{12}}\exp\{-\alpha^2(R^2+R'^2)\}\mathcal{J}_L(2\alpha^2RR'), \tag{3·25a}$$

$$\begin{aligned}H_L^{(0)}(R,R')=&\frac{2^2\pi^7}{\alpha^{12}}\hbar\omega_0\exp\{-\alpha^2(R^2+R'^2)\}\\&\times\left[\left\{\frac{3(\alpha^4+\beta^4)}{\alpha^2\beta^2}-\frac{(\alpha^4-\beta^4)(R^2+R'^2)}{2\beta^2}\right\}\mathcal{J}_L(2\alpha^2RR')\right.\\&\left.+\frac{(\alpha^4+\beta^4)}{\beta^2}RR'\mathcal{J}_L'(2\alpha^2RR')\right]+\frac{2^2\pi^7}{\alpha^{12}}3({}^{13}V+{}^{31}V)\left(\frac{\alpha^2}{\alpha^2+2\mu^2}\right)^{3/2}\\&\times\exp\{-\alpha^2(R^2+R'^2)\}\mathcal{J}_L(2\alpha^2RR'),\end{aligned} \tag{3·25b}$$

where the function $\mathcal{J}_L(z)$ stands for the modified Bessel function with order of half integer

$$\mathcal{J}_L(z)=\sqrt{\frac{\pi}{2z}}I_{L+1/2}(z) \tag{3·26}$$

and $\mathcal{J}_L'(z)$ its derivative.

Simpler eigenvalue equation

$$\int[H_L^{(0)}(R,R')-\omega_L^{(0)}N_L(R,R')]\Phi_L^{(0)}(R')R'^2dR'=0 \tag{3·27}$$

with the kernels defined by Eqs. (3·25) describes the motion of the four particles under an extreme approximation of neglecting the Pauli principle due to the "core", but the analytic property of the solution $\Phi_L^{(0)}(R)$ is very similar to that of the solution of Eq. (3·9). It is easily found that the solutions of Eq. (3·27) are

$$\begin{aligned}\Phi_{NL}^{(0)}(R)=&\left[\frac{2^{N+L+2}N!}{(2N+2L+1)!!}\right]^{1/2}\frac{\alpha^{L+9}}{\pi^4}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)^{L+3/2}\left(\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}\right)^N\\&\times\exp\left(-\frac{2\alpha^2\beta^2}{\alpha^2-\beta^2}R^2\right)R^L L_N^{(L+1/2)}\left(\frac{4\alpha^4\beta^2}{\alpha^4-\beta^4}R^2\right),\quad(\alpha>\beta)\end{aligned} \tag{3·28a}$$

$$\omega_{NL}^{(0)}=\hbar\omega_0\left\{2N+L+6+\frac{9(\alpha^2-\beta^2)^2}{4\alpha^2\beta^2}\right\}+3\left(\frac{\alpha^2}{\alpha^2+2\mu^2}\right)^{3/2}({}^{13}V+{}^{31}V). \tag{3·28b}$$

The eigenfunctions satisfy the following orthonormality condition:

$$\int\Phi_{NL}^{(0)*}(R)N_L^{(0)}(R,R')\Phi_{N'L}^{(0)}(R')R^2R'^2dRdR'=\delta_{NN'}. \tag{3·29}$$

Form of the function $\Phi_{NL}^{(0)}(R)$ is quite similar to the three-dimensional harmonic oscillator wave function except for the difference in arguments of the exponential function and the Laguerre polynomial. Because of this difference, the nodal points of the function locate much nearer to the origin

than those of the harmonic oscillator wave function. At the limit of $\alpha^2 \approx \beta^2$, it shows singular property and, for example, if $\alpha^2=\beta^2$ for $N=L=0$, it is easily seen that $\Phi_{NL}^{(0)}(R) \propto \delta(R)/R^2$. This is not surprising, since the correlation amplitude becomes $(0s)^4$-type configuration as

$$\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4) \propto \exp\left(-\frac{\beta^2}{2}\sum_{i=1}^{4} r_i^2\right) \tag{3·30}$$

after integration over R with the above $\Phi_{N=L=0}^{(0)}(R) \propto \delta(R)/R^2$ and expression (3·7a). This means that a very simple shell-model configuration of the four particles corresponds to a rather singular function in the $\boldsymbol{R}$-coordinate. In addition, it should especially be noticed that the function $\Phi_{NL}^{(0)}(R)$ shows considerably singular behavior even for wide range of the parameter $\beta^2 \leqq \alpha^2 \lesssim 2\beta^2$, in which we take physical interest. Such properties are also proved for the solution $\Phi_L(R)$ of the eigenvalue equation (3·9), in which the effect of the Pauli principle due to the "core" is exactly taken into account.

Considering the above discussion on the analytic property of the solution $\Phi_L(R)$, we choose a usefull set of basis vectors of the subspace $\mathcal{H}_\Phi$. Let $\{|\Phi_{NLM}^{(0)}\rangle\}$ be such set of basis vectors. It is convenient to put for the basis vectors

$$\langle \boldsymbol{R}|\Phi_{NLM}^{(0)}\rangle \equiv \Phi_{NL}^{(0)}(R)\, Y_{LM}(\Omega_R), \tag{3·31}$$

where $\Phi_{NL}^{(0)}(R)$ is defined by Eq. (3·28a). These basis vectors are represented in the $\boldsymbol{S}$-coordinate by the relation (3·8) as

$$\langle \boldsymbol{S}|\Phi_{NLM}^{(0)}\rangle \equiv \widehat{\Phi}_{NL}^{(0)}(S)\, Y_{LM}(\Omega_S) = \int \exp\{-2\alpha^2(\boldsymbol{R}-\boldsymbol{S})^2\}\, \Phi_{NL}^{(0)}(R)\, Y_{LM}(\Omega_R)\, d\boldsymbol{R}. \tag{3·32}$$

The function $\widehat{\Phi}_{NL}^{(0)}(S)$ is a harmonic oscillator eigenfunction satisfying the following equation:

$$\left[-\frac{1}{2}\left(\frac{\hbar^2}{4m}\right)\nabla_S^2 + \frac{1}{2}(4m\omega_0^2)S^2\right]\langle \boldsymbol{S}|\Phi_{NLM}^{(0)}\rangle = (2N+L+3/2)\langle \boldsymbol{S}|\Phi_{NLM}^{(0)}\rangle. \tag{3·33}$$

From Eq. (3·5a), we get the space-coordinate representation of the basis vector

$$\psi_{NLM}^{(0)}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4) = \exp\left(-\frac{\alpha^2}{2}\sum_{j=1}^{3} t_j^2\right)\widehat{\Phi}_{NL}^{(0)}(S)\, Y_{LM}(\Omega_S). \tag{3·34}$$

These basic functions satisfy the orthonormality relation

$$\int \psi_{NLM}^{(0)*}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\,\psi_{N'L'M'}^{(0)}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\left(\prod_{i=1}^{4} d\boldsymbol{r}_i\right) = \delta_{NN'}\delta_{LL'}\delta_{MM'}. \tag{3·35}$$

It is seen from Eq. (3·34) that the basis vectors stand for the states in which the internal distribution of the four particles is of Gaussian type with the size parameter α^2 and their centre-of-mass motion is a harmonic oscillator eigenstate.

It is interesting to see how the basis vectors involve the higher single-particle states. The basic function $\psi^{(0)}_{NLM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$ is expanded as

$$\psi^{(0)}_{NLM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)=\sum_{n_il_im_i}C^{(0)}(n_1l_1m_1,\, n_2l_2m_2,\, n_3l_3m_3,\, n_4l_4m_4)\,[\prod_{i=1}^{4}\varphi_{n_il_im_i}(\boldsymbol{r}_i)]\,, \tag{3·36}$$

where

$$C^{(0)}(n_1l_1m_1,\, n_2l_2m_2,\, n_3l_3m_3,\, n_4l_4m_4)$$
$$=\int\Big[\prod_{i=1}^{4}\int\varphi^{*}_{n_il_im_i}(\boldsymbol{r}_i)\exp\Big\{-\frac{\alpha^2}{2}(\boldsymbol{r}_i-\boldsymbol{R})^2\Big\}d\boldsymbol{r}_i\Big]\Phi^{(0)}_{NL}(R)\,Y_{LM}(\Omega_R)\,d\boldsymbol{R}. \tag{3·37}$$

The integration in the right-hand side of Eq. (3·37) is easily carried out and it is seen that any basic function $\psi^{(0)}_{NLM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$ with $\alpha^2>\beta^2$ involves all single-particle states keeping [4] symmetry. In this sense, we may refer to our subspace $\mathcal{H}_\Phi$ as *vertically truncated subspace* in contrast with the usual one-major-shell-model subspace which we may call *horizontally truncated subspace.*

Now, let us rewrite the eigenvalue equation (3·9) using the set of basis vectors $\{|\Phi^{(0)}_{NL}\rangle\}$. The solution $\Phi_L(R)$ is expanded by the set of basic functions $\{\Phi^{(0)}_{NL}(R)\}$ as

$$\Phi_L(R)=\sum_N C_{NL}\Phi^{(0)}_{NL}(R). \tag{3·38}$$

The corresponding expansion in the S-coordinate is

$$\widehat{\Phi}_L(S)=\sum_N C_{NL}\widehat{\Phi}_{NL}(S), \tag{3·39}$$

where the coefficients C_{NL} in this expansion are just the same as those in the expansion (3·38). It will later be shown that the expansion for the physical solution of Eq. (3·9) is very good and such a $\Phi_L(R)$ is well approximated by several terms. Using expansion (3·38), we can write the eigenvalue equation (3·9) in the following form:

$$\sum_{N'}(\mathcal{H}^L_{NN'}-\omega_L\mathcal{N}^L_{NN'})C_{N'L}=0, \tag{3·40}$$

where the matrix elements $\mathcal{H}^L_{NN'}$ and $\mathcal{N}^L_{NN'}$ are defined by

$$\begin{pmatrix}\mathcal{N}^L_{NN'}\\ \mathcal{H}^L_{NN'}\end{pmatrix}=\int\Phi^{(0)*}_{NL}(R)\begin{pmatrix}N_L(R,R')\\ H_L(R,R')\end{pmatrix}\Phi^{(0)}_{N'L}(R')R^2R'^2dRdR'. \tag{3·41}$$

Now, the eigenvalue equation (3·40) does not possess any difficult numerical property and is, in practice, solved in a usual manner of matrix eigenvalue problem. Using the solution $\hat{\Phi}_L(S)$ thus calculated, we can obtain the wave function of the outer four particles as

$$\Psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)=\int\langle \boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4|Q_{4p}|\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'\boldsymbol{r}_4'\rangle\exp\left(-\frac{\alpha^2}{2}\sum_{j=1}^{3}t_j^2\right)$$
$$\times\hat{\Phi}_L(S')\,Y_{LM}(\Omega_{S'})\left(\prod_{i=1}^{4}d\boldsymbol{r}_i'\right) \qquad (3\cdot42)$$

which is very similar to the form of the wave function in the resonating group method often used in the investigation of the structure of light nuclei.

3. 4 Tamm-Dancoff calculations for low-lying states of ^{20}Ne

The low-lying even-parity states of ^{20}Ne are studied in this subsection by using the model proposed in the previous subsections. Since the ^{16}O nucleus is treated as the inert core with $(0s)^4(0p)^{12}$ configuration, the density matrix $\rho(\boldsymbol{r},\boldsymbol{r}')$ appearing in the projection operator Q_{4p} is composed of the $0s$ and $0p$ single-particle wave function as Eq. (3·24). The size parameter β^2 of the ^{16}O-core and the range parameter μ^2 of the residual interaction (3·20) are assumed to have the same values as used in the *sd* shell model of Ref. 1a): $\beta^2=m\omega_0/\hbar=0.362$ fm^{-2} corresponding to $\hbar\omega_0=15.0$ MeV and $\beta/(\sqrt{2}\,\mu)=0.7$. The strength of the interaction

$$V_0\equiv\frac{1}{2}(^{13}V+{}^{31}V) \qquad (3\cdot43)$$

is left as an adjustable parameter in order to reproduce the relative energy spectrum and the binding energy of the outer four particles. The single-particle Hamiltonian H_0 has been given by Eq. (3·19), in which the spin-orbit force is not written since it never affects our eigenenergy.

Hereafter, we measure the energy of the eigenstate of ^{20}Ne, ω_n, subtracting the total single-particle energy of the $(0d)_p^2(0d)_n^2$ configuration from it as done in Ref. 1a). The observed binding energy of the ground state of ^{20}Ne is then 31. 38 MeV in this definition. The calculated energy corresponding to the above definition, W_n, is derived from ω_n as $W_n=\omega_n-4\varepsilon_{0d}$, where ε_{0d} is given by Eq. (3·23). In the single-particle potential $U(r)$ of Eq. (3·19), we dropped an additional constant energy, because it is canceled out in W_n.

Eigenenergies are then obtained by solving the secular equation (3·40) for each angular momentum L and each variational size parameter α^2. The dimension N_D of the secular matrix can be truncated very well under the condition $2N_D+L=14$ (c.f., §4. 1 of Ref. 6)).

Calculated energies are shown in Fig. 2 and an energy spectrum con-

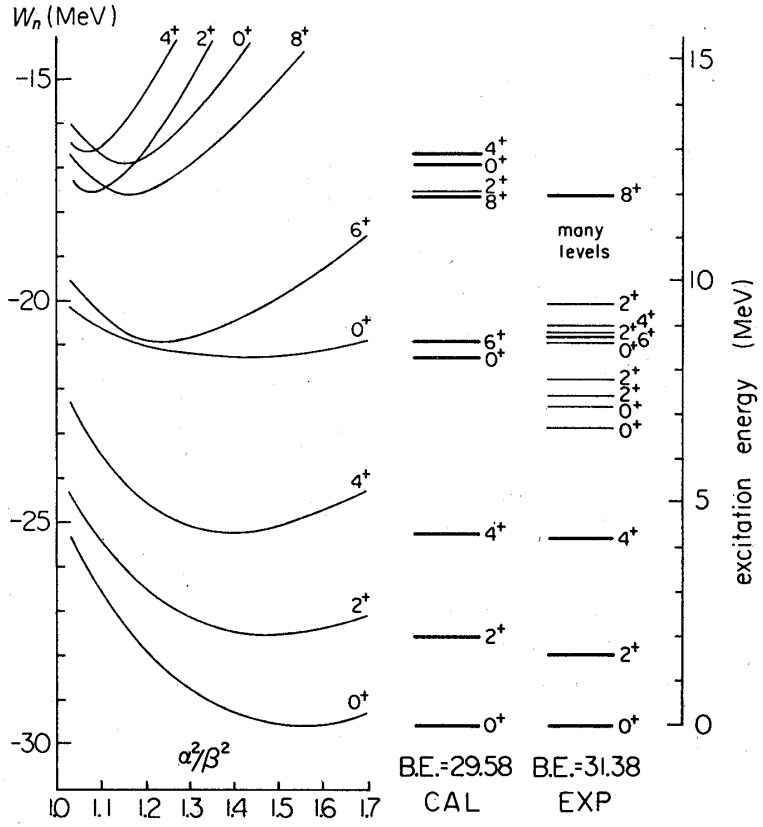

Fig. 2. Level structure of ^{20}Ne. Calculated eigenenergies W_n are plotted against the variational size parameter α^2 divided by β^2. The energy spectrum consisting of the lowest value for each curve is shown on the right together with the experimental data.[8] Here the strength of the residual interaction $V_0=-35$ MeV and no correction in the single-particle energy ε_{nl}. The figure is from Ref. 6).

sisting of the lowest value for each curve is put on the right of the curves. The states which have the same spin but different size parameter α^2 in the spectrum are not strictly orthogonal, but they are almost orthogonal (c.f., §4. 1 of Ref. 6)).

The binding energy and the level structure of the ground band are reproduced very well with the strength $V_0=-35$ MeV. The strength of the residual interaction is much weaker than that used in the *sd* shell model of Ref. 1a): $V_0=-52.5$ or -48.7 MeV in the case of ^{20}Ne. This fact represents evidently the large effect of the higher shells upon the four-body correlation energy. It is interesting to see how the four-body wave functions involve the single-particle states higher than 1*s*-0*d* shell. The detailed values of such probabilities are listed in Table I of Ref. 6), but here we stress the following points. In the ground band, the lower the state comes from 8^+ to 0^+, the more higher-shells mix into the four-body wave function. This feature might correspond to the "anti-stretching" property of the ground-band states.[9] The remaining states other than the ground-

band states shown in Fig. 2 consist almost of $(sd)^4$ configuration and are not the state having excitation in "relative motion".

Since the four-body wave function involves so many higher single-particle states in the present model, a centre-of-mass motion must be excited. However, the spurious components in the low-lying states shown in Fig. 2 are less than $\sim 7\%$ (c.f., §4.3 of Ref. 6)) and the fact does not seem to have so much influence on our physical conclusions.

The size parameter α^2 which gives minimum energy of the ground state in Fig. 2 takes almost the same value as that of free α-particle, $\alpha^2=0.58$ fm^{-2}. However it should be noticed that this correspondence would not mean a free α-particle-like localization of the outer four particles because of the Pauli principle effect Q_{4p}. Actually, concerning a root mean square of the intrinsic coordinate of the particles, with the use of the definition

$$\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle \equiv \int \Psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\frac{1}{4}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{S})^2\Psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\left(\prod_{i=1}^{4}d\boldsymbol{r}_i\right), \quad (3\cdot44)$$

we get

$$\sqrt{\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle}\,/\sqrt{\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle_{sd}}=0.87,$$

while

$$\sqrt{\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle_{\text{free-}\alpha}}\,/\sqrt{\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle_{sd}}=0.52,$$

where $\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle_{sd}$ and $\langle(\boldsymbol{r}-\boldsymbol{S})^2\rangle_{\text{free-}\alpha}$ denote the values (3·44) for the outer four particles of the *sd* shell model and for the free α-particle, respectively.

In order to visualize the behavior of the relative motion between the outer four particles and the ^{16}O-core, we construct the following "relative amplitude" $u_L(S)$ in a similar way to the procedure used in the resonating-groups method:

$$u_L(S)\equiv\int\left(\frac{\alpha}{\sqrt{\pi}}\right)^{9/2}\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{S})^2\right\}Y^*_{LM}(\Omega_S)\Psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\left[\prod_{i=1}^{4}d\boldsymbol{r}_i\right]_S \quad (3\cdot45)$$

with $\boldsymbol{S}=(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4)/4$, where the coordinate $\boldsymbol{S}$ denotes the "relative" coordinate between the centre-of-mass of the four particles and the centre of the single-particle potential due to the ^{16}O-core. In the definition (3·45), the integrations are carried out over all intrinsic variables and angle part of $\boldsymbol{S}$, and the size parameter α^2 takes the same value as in $\Psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)$. The functions $u_L(S)$ for our low-lying states are shown in Fig. 3, being compared with that of the $(sd)^4[4](80)L$ configuration which is the main part of the $(sd)^4$ shell model. For the states in the ground band, the "relative" motion grows up more than the $(sd)^4(80)$ case as seen in the reduction of the inner

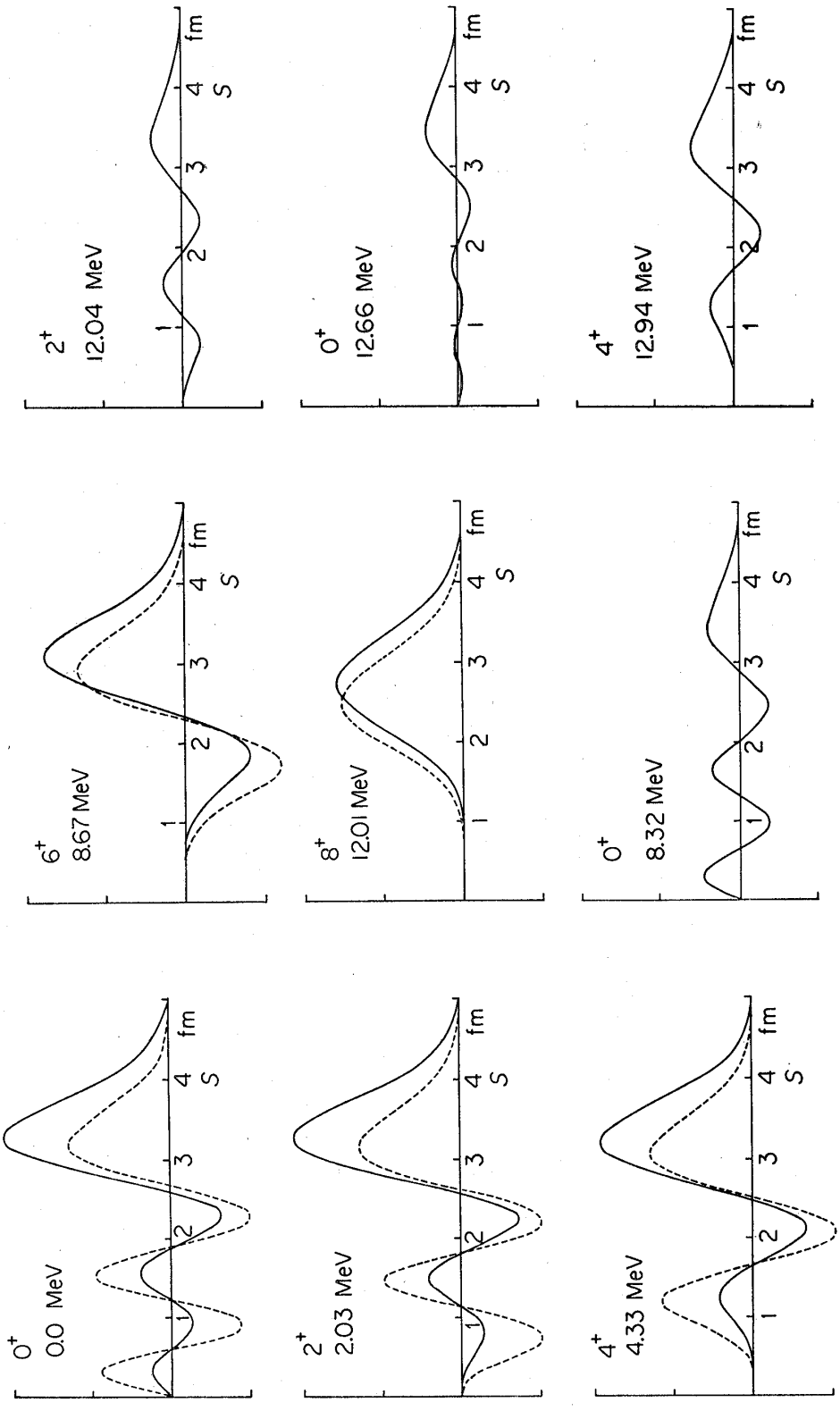

Fig. 3. The amplitude of the "relative motion" between the outer four particles and the ^{16}O-core defined by the expression (3·45) for the given state (the solid line). The quantity for the $(sd)^4[4](80)L$ configuration is also plotted for given L in the ground band (the dashed line). The ordinates show $u_L(S)\times S$ on an arbitrary scale. The figures are from Ref. 6).

oscillation and enhancement of outer part.*) This effect must give the enhancement of α-width of the 6^+ and 8^+ states.[10] The remaining states shown in Fig. 3 have very small component of "relative" motion, because they are almost composed of $(sd)^4$ configuration but with very small (80) amplitude.

Now, we treat the size parameter α^2 as an another generator coordinate in the following way. We extend the variational function (3·7) to the following form with the linear combination with respect to the size parameter α_i^2:

$$\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)=\sum_i\int\exp\left\{-\frac{\alpha_i^2}{2}\sum_{j=1}^{4}(\boldsymbol{r}_j-\boldsymbol{R})^2\right\}\Phi_L(\alpha_i^2,R)\,Y_{LM}(\Omega_R)\,d\boldsymbol{R}$$

$$=\sum_i\sum_N C_{NL}(\alpha_i^2)\int\exp\left\{-\frac{\alpha_i^2}{2}\sum_{j=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R})^2\right\}\Phi_{NL}^{(0)}(\alpha_i^2,R)\,Y_{LM}(\Omega_R)\,d\boldsymbol{R}. \tag{3·46}$$

Here, the second equality corresponds to the before-mentioned expansion of the variational function, (3·38), and basic functions $\Phi_{NL}^{(0)}(\alpha_i^2,R)$ are defined with Eq. (3·28a) by replacing α^2 by α_i^2. Since size parameter α_i^2 in the above expression is not continuum value but discrete one, the variable α_i^2 are not a generator coordinate in the exact sense.

In actual calculation, α_i^2 takes several values between β^2 and $1.7\beta^2$, which are chosen based on α^2-dependence of the curves in Fig. 2. Eigenvalue equation similar to Eq. (3·40) but with larger dimension is solved and resulting energy spectrum are shown in Fig. 4. Relative level structure of the ground band and the binding energy are almost reproduced with the use of the strength $V_0=-34$ MeV.

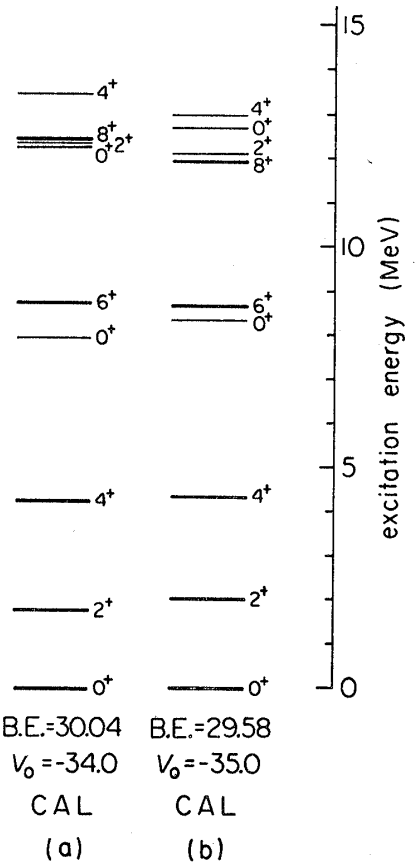

Fig. 4. (a) Energy spectrum obtained by an extension of the degree of freedom of the variational size parameter α^2. (b) Energy spectrum obtained before in Fig. 2, where the size parameter α^2 has fixed value for each state.

3.5 Relation between the vertically-truncated-subspace model and the *sd* shell model

When the size parameter of the outer four particles is equal to that of the ^{16}O-core, i.e., $\alpha^2=\beta^2$, our basic con-

*) This tendency was also obtained by Nemoto and Bando[11] based on other model. (See also Chapter IV.)

figurations

$$\int\langle \boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4|Q_{4p}|\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'\boldsymbol{r}_4'\rangle\psi^{(0)}_{NLM}(\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'\boldsymbol{r}_4')\left(\prod_{i=1}^{4}d\boldsymbol{r}_i'\right) \tag{3·47}$$

with $2N+L<8$ vanish through the Q_{4p} operation, because at least one particle in $\psi^{(0)}_{NLM}$ lies in the $0s$ or $0p$ shell. On the other hand, in the case of $\alpha^2=\beta^2$ and $2N+L>8$, the basic configurations (3·47) always involve the single-particle components higher than the $1s$-$0d$ shell. Thus, only when $\alpha^2=\beta^2$ and $2N+L=8$, the basic configurations (3·47) are composed of pure $(sd)^4$ components and are proved to be exactly $(sd)^4[4](80)$ configurations with $L=0, 2, 4, 6$ and 8 in the SU_3 scheme.

We discuss another relation. Here we define a new projection operator $Q^{(sd)}_{4p}$ instead of Q_{4p} with the same expression as the first one in Eq. (2·36) but with

$$\theta_\alpha=\begin{cases}0 & \text{if the level } \alpha \text{ is in the } 1s\text{-}0d \text{ shell,}\\ 1 & \text{otherwise.}\end{cases}$$

Through the $Q^{(sd)}_{4p}$ operation, all four particles are put in the sd shell. Using the coordinate representation for the $Q^{(sd)}_{4p}$ and the vertically truncated bases $\psi^{(0)}_{NLM}$, we introduce the following type of basic four-particle configurations with [4] symmetry:

$$\Psi^{(sd)}_{NLM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)=\int\langle \boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4|Q^{(sd)}_{4p}|\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'\boldsymbol{r}_4'\rangle\psi^{(0)}_{NLM}(\boldsymbol{r}_1'\boldsymbol{r}_2'\boldsymbol{r}_3'\boldsymbol{r}_4')\left(\prod_{i=1}^{4}d\boldsymbol{r}_i'\right). \tag{3·48}$$

When $\alpha^2=\beta^2$, only configurations with $2N+L=8$ can be alive after the $Q^{(sd)}_{4p}$ operation and they are again proved to be [4] (80) configurations with $L=0, 2, 4, 6$ and 8. In the case of $\alpha^2>\beta^2$, the $Q^{(sd)}_{4p}$ operation makes $\Psi^{(sd)}_{NLM}$ with $2N+L\leq 8$ be alive. The number of linearly independent states among $\{\Psi^{(sd)}_{NL};\ \alpha^2>\beta^2 \text{ and } 2N+L\leq 8\}$ can be counted with the rank of the overlap matrix defined by the matrix elements $\langle\Psi^{(sd)}_{NLM}|\Psi^{(sd)}_{N'LM}\rangle$. The numbers are 4, 4, 3, 2 and 1 for $L=0, 2, 4, 6$ and 8, respectively, while in the SU_3 scheme the numbers of linearly independent states with [4] symmetry are 4, 5, 4, 2 and 1 for $L=0, 2, 4, 6$ and 8, respectively. Thus, our approach with $Q^{(sd)}_{4p}$ and $\alpha^2>\beta^2$ is equivalent to the SU_3 scheme with [4] symmetry for $L=0, 6$ and 8 and resulting eigenenergies are independent on the size parameter α^2. Even for $L=2$ and 4, the lowest two states of each L solved with [4] SU_3 scheme are very closely reproduced by the $\Psi^{(sd)}_{NL}$ scheme, and energy eigenvalues are proved to be independent of the parameter α^2.

Now we introduce other types of operator, $Q^{(sd,pf)}_{4p}$ and $Q^{(sd,pf,sdg)}_{4p}$: The former restricts every one of the four particles within the sd or pf shell (two-major-shell truncation) and the latter within the sd, pf or sdg shell

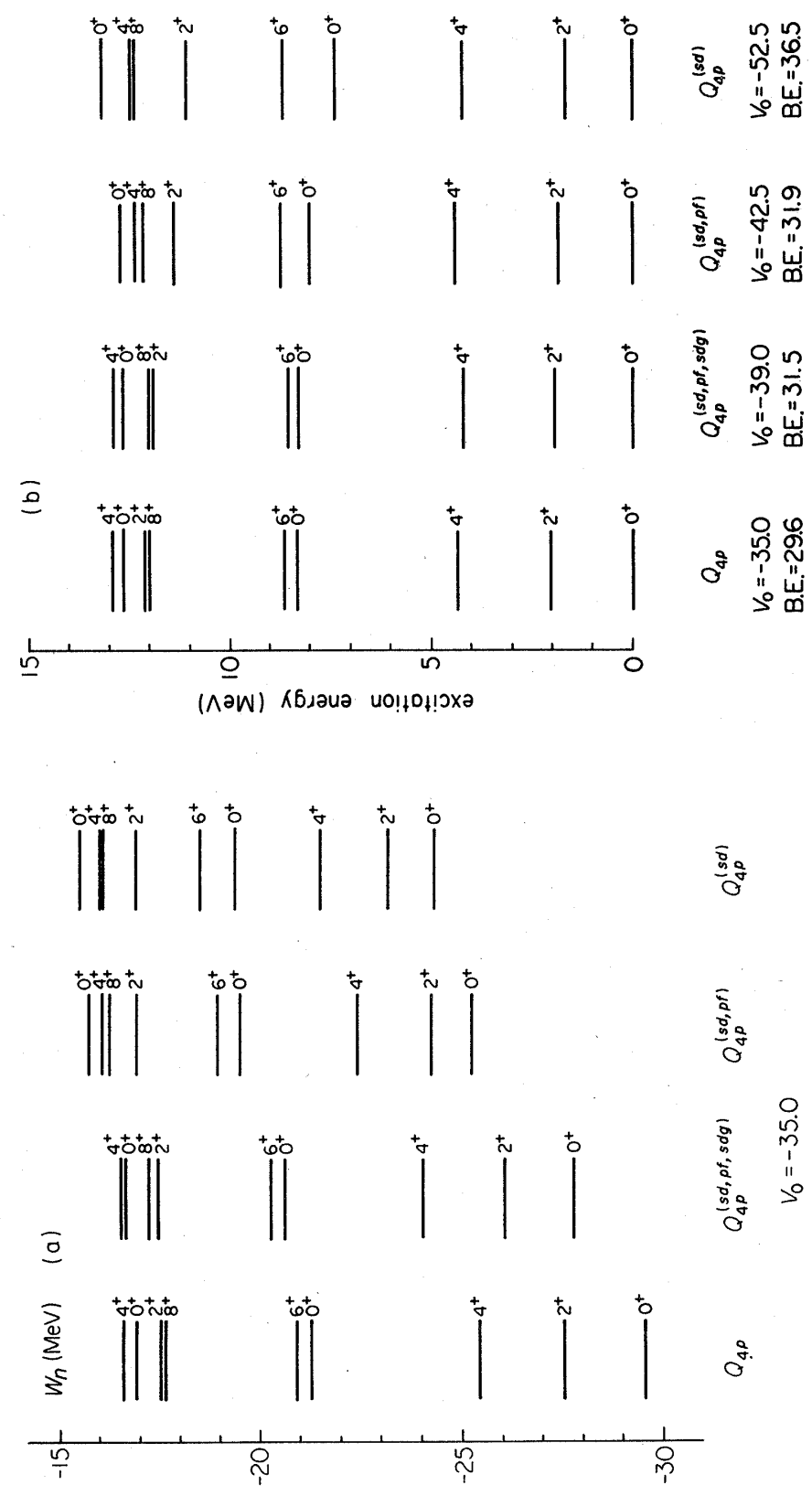

Fig. 5. The level structure of ^{20}Ne calculated by using the vertical truncations within three major shells (sd, pf, sdg), namely by using the projection operators $Q_{4p}^{(sd')}$, $Q_{4p}^{(sd,pf)}$ and $Q_{4p}^{(sd,pf,sdg)}$. The before-mentioned result with Q_{4p} is also shown again. There is no correction in the single-particle energy ε_{nl}. a) $V_0 = -35$ MeV in all cases. b) Most suitable strength V_0 for each case is adopted. The figures are from Ref. 6).

(three-major-shell truncation). The operators are defined in the same manner as the case of the $Q_{4p}^{(sd)}$ but with proper θ_α for each truncation.

By using the $Q_{4p}^{(sd,pf)}$ and $Q_{4p}^{(sd,pf,sdg)}$, we can compose new basic configurations $\Psi_{NLM}^{(sd,pf)}$ and $\Psi_{NLM}^{(sd,pf,sdg)}$ in the same way as Eq. (3·48). In these cases all configurations with

$$2N+L\leq 12 \quad \text{for} \quad (sd, pf)\text{-truncation},$$
$$2N+L\leq 16 \quad \text{for} \quad (sd, pf, sdg)\text{-truncation}$$

are linearly independent and resulting eigenenergies are proved to depend upon the value of the variational size-parameter α^2. Thus, three types of eigenvalue equations with the same form as the expression (3·40) are introduced, but different truncation operators $Q_{4p}^{(sd)}$, $Q_{4p}^{(sd,pf)}$ and $Q_{4p}^{(sd,pf,sdg)}$ are used in defining of the matrix elements $\mathcal{N}_{NN'}^{L}$ and $\mathcal{H}_{NN'}^{L}$.

Energy spectra in the three cases are shown in Fig. 5 together with the result given before by using Q_{4p}. One can clearly see how the truncations affect the eigenenergies and what states are benefited by the higher-shell mixing. However, if a suitable strength of the interaction is taken for each of the three truncations, almost same level structures can be obtained. This seems to suggest that to extend the size of truncation of the single-particle shell model states unoccupied in the "core" may be renormalized almost in the reduction of strength of residual interaction. As mentioned in the last subsection, this fact has already been found for the case in which the size parameter α^2 is treated not as a fixed parameter, but as a generator coordinate. The situation would be the same in the use of the effective charge for the electromagnetic transitions.

Therefore, if one wants to get some definite evidence coming from higher-shell-mixing (or spatial-correlation) effect for the low-lying and non-"core"-excited states, it does not seem to be good way to see energy spectra or electromagnetic transitions. For that purpose, one had better investigate α-reduced width or α-transfer strength. The study of the "relative amplitude" shown in the last subsection would give proper informations for the aim.

3. 6 Spatial correlations of the outer four particles in the finite-well single-particle potential

So far a harmonic oscillator potential has been assumed as a Hartree-Fock field due to the ^{16}O-core. It is desirable to adopt more realistic single-particle potential of finite well for the purpose of studying the spatial correlations of the extra four particles. In this subsection, calculations with an approximated finite-well potential are presented and resulting effects on the localization especially on the "relative" motion are discussed.

Woods-Saxon potential would be suitable for our aim, but the type of potential is simulated extremely well by the following function in this paper

for the sake of simplicity in calculation:

$$\left.\begin{aligned} &U_W(r)=U_0[1+c_1(ar)^2+c_2(ar)^4]e^{-(ar)^2}, \\ &U_0=-48.4\ \text{MeV},\quad c_1=1.11,\quad c_2=0.291, \\ &a=0.48\ \text{fm}^{-1}. \end{aligned}\right\} \tag{3·49}$$

The overlap between the $0s$ (or $0p$) single-particle wave function obtained with this potential and the $0s$ (or $0p$) wave function of harmonic oscillator potential is 99.9% (or 99.6%). Eigenenergies of the $0d$ and $1s$ states are -2.23 MeV and -3.20 MeV, respectively, reproducing well the experimental values in ^{17}O: -2.11 MeV for $0d$ and -3.26 MeV for $1s$ state. Further, Coulomb potential between outer proton and the ^{16}O-core is simply simulated by

$$\left.\begin{aligned} &U_c(r)=U_0'e^{-(br)}, \\ &b=0.15\ \text{fm}^{-1},\quad U_0'=4.0\ \text{MeV}. \end{aligned}\right\} \tag{3·50}$$

Fast dumping of the tail part affects hardly the bound states of ^{20}Ne. Energy shifts of single-particle states caused by $U_c(r)$ are 3.4 MeV for both of $0d$ and $1s$ states and they almost reproduce experimental values.

Now we adopt

$$U_i=U_W(r_i)+\frac{1+\tau_{3i}}{2}U_c(r_i) \tag{3·51}$$

as the single-particle potential U_i in Eq. (3·3) instead of harmonic oscillator potential (3·19). In Fig. 6, our approximated potential for a proton is illustrated together with the harmonic oscillator potential used in the previous

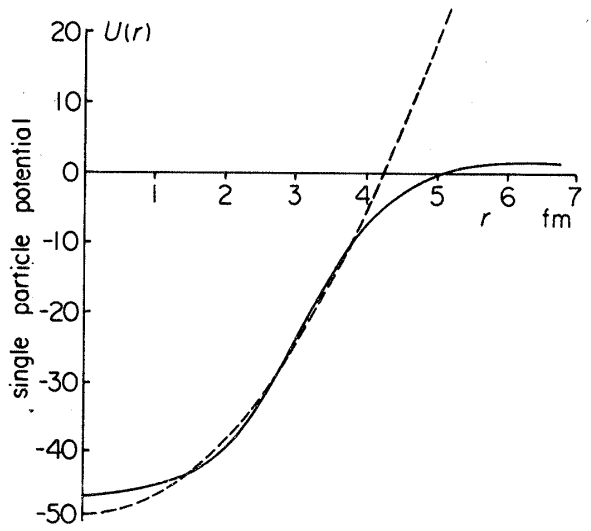

Fig. 6. The approximated finite-well potential for a proton (3·51) is shown by the solid line and the harmonic oscillator potential by the dashed line.

subsections.

Since the $0s$ and $0p$ wave functions of the present potential are almost the same as those of the harmonic oscillator potential, the new density matrix of the ^{16}O-core, which is used in constructing the operator Q_{4p}, is approximated with high accuracy by old one (3·24) made from the $0s$ and $0p$ wave function of the harmonic oscillator potential. After the calculation of $H_L(R, R')$ in Eq. (3·10) with the new single-particle potential U_i is carried out analytically, all procedures to get final solutions are the same as before. Here it should be noticed that we need not prepare the $0d$, $1s$ and higher single-particle wave functions of the finite-well potential in the calculation of $H_L(R, R')$. This is a great advantage of our model.

For the states of ^{20}Ne above α-threshold 4.73 MeV, for instance 6^+ and 8^+ states, a bound-state approximation is assumed in the present calculation for simplicity. Further, the expansion (3·38) or (3·39) of the solution $\phi_L(R)$ or $\hat{\phi}_L(S)$ is restricted to $N=0$ to 15 for each L, which is sufficient for describing the states under consideration. The parameters of the residual interaction (3·20) is slightly changed from the one used before on account of the broader feature of the $0d$, $1s$ and higher continuum orbits, namely an account of new character of the truncated subspace. We take $\lambda=\beta/(\sqrt{2}\mu)=0.75$ instead of $\lambda=0.7$ and keep $V_0=-35.0$ MeV.

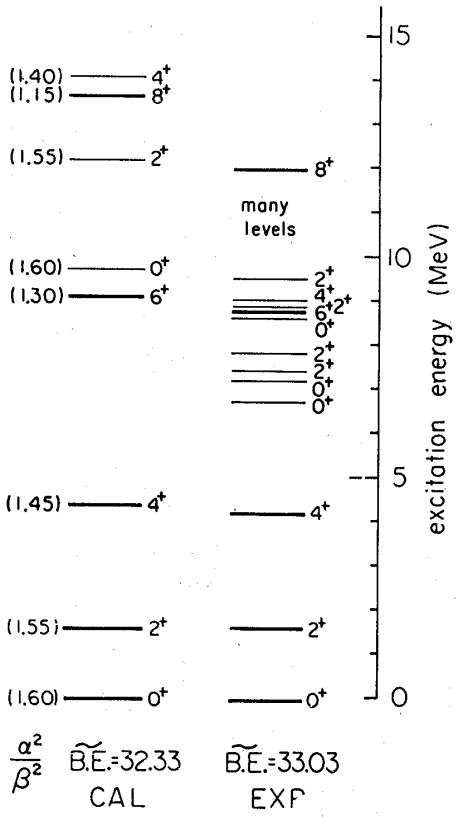

Fig. 7. The energy spectrum calculated with the finite-well single-particle potential is shown together with the experimental one. The values in parentheses are the ratios α^2/β^2 which give the minimum energy for each state. See text about the definition of the binding energy.

Eigenvalues ω_n of Eq. (3·40) should correspond to binding energies of ^{20}Ne relative to the ground state of ^{16}O nucleus. The experimental value ω_n of the ground state of ^{20}Ne is 33.03 MeV. Note that the definition of the binding energy in this subsection differs from that in §3.4.

Calculated energy spectrum is shown in Fig. 7. Binding energy and level structure of the ground band are well reproduced. It is interesting to see in Fig. 8 how their "relative amplitudes" defined by Eq. (3·45) differ from those of the solutions with harmonic-oscillator-single-particle potential. Stronger localization can be seen in the case of the finite-well potential, namely,

outermost peak of "relative amplitude" is further pushed out and enhanced. This effect would give some enhancement of the strength of α-transfer reaction and α-width of 6^+ and 8^+ states. According to Arima and Yoshida,[10] and Yazaki,[12] shell-model wave functions of 6^+ and 8^+ states with usual oscillator parameter give α-width several times as small as experimental values. Our wave functions with finite-well potential might give better agreement for the quantities.

Here it should be noted again that our "relative" coordinate **S** denotes the distance between the centre-of-mass of the extra four particles and the centre of the single-particle potential of the ^{16}O-core (not the centre-of-mass of ^{16}O). Then, our "relative amplitude" cannot be used directly for the estimation of the α-width. Appropriate treatment of the centre-of-mass

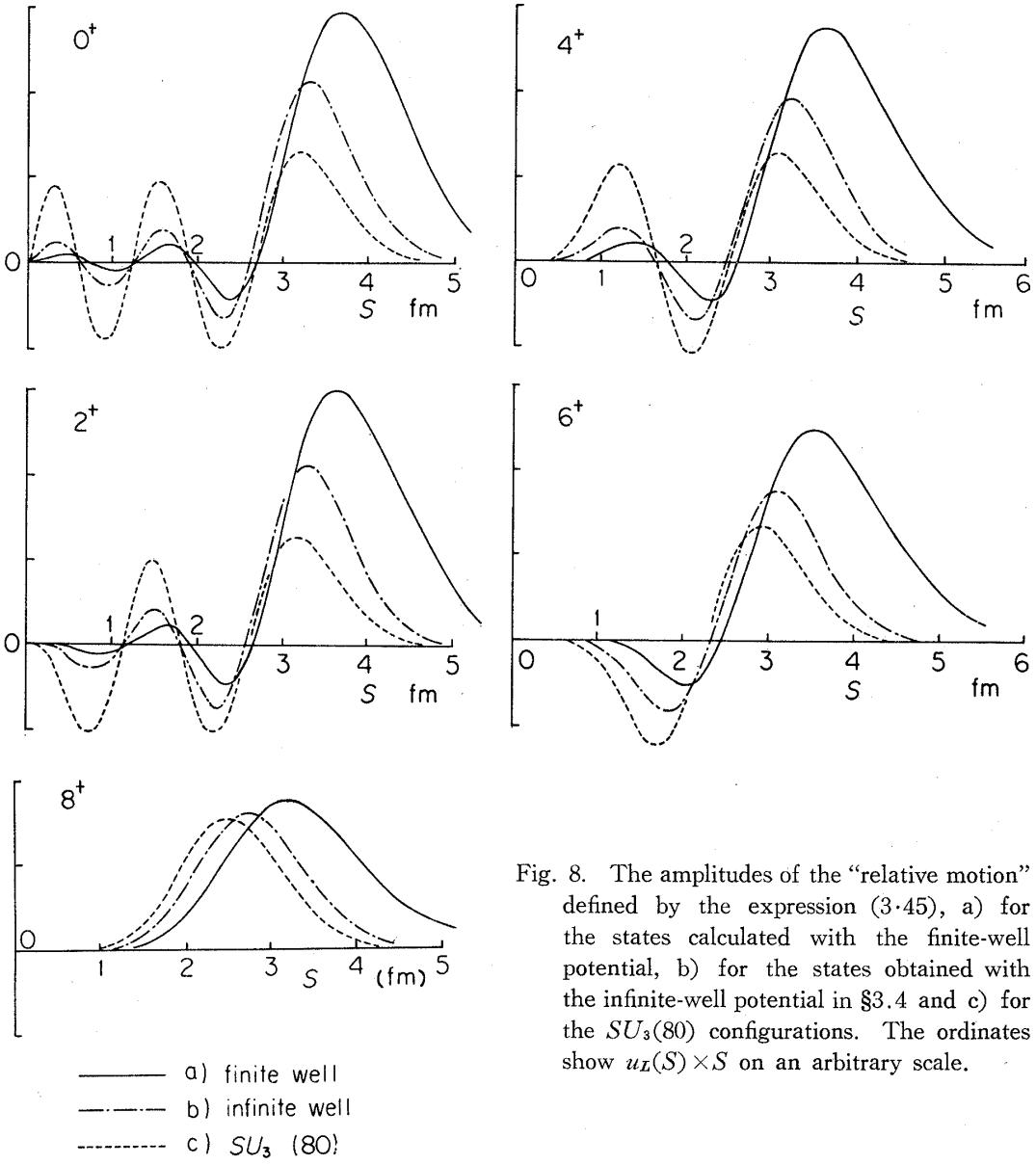

Fig. 8. The amplitudes of the "relative motion" defined by the expression (3·45), a) for the states calculated with the finite-well potential, b) for the states obtained with the infinite-well potential in §3.4 and c) for the $SU_3(80)$ configurations. The ordinates show $u_L(S)\times S$ on an arbitrary scale.

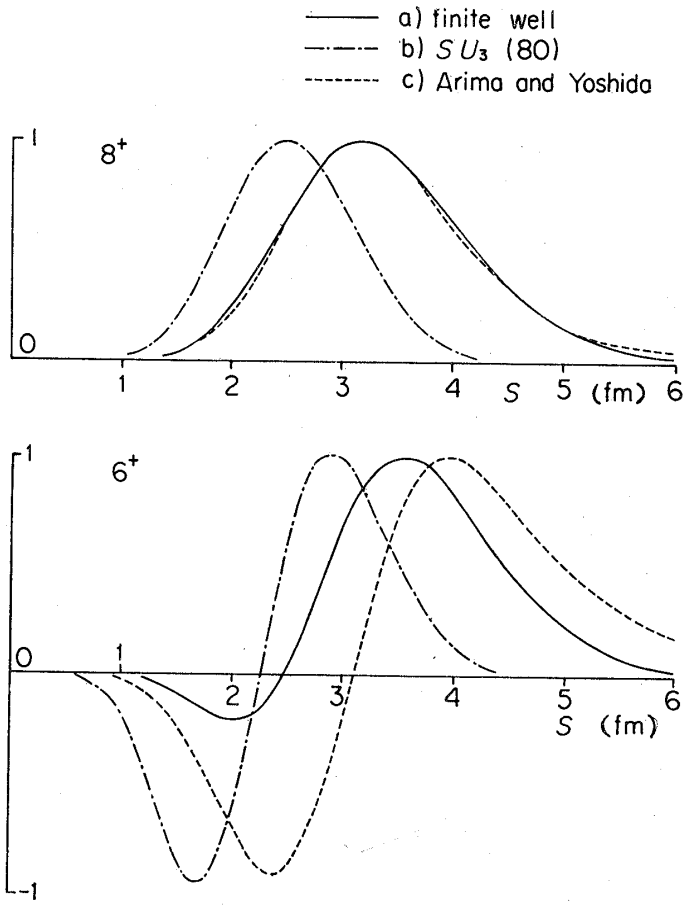

Fig. 9. The "relative amplitudes" of the 6^+ and 8^+ states calculated a) with a finite-well potential and b) with the $SU_3(80)$ configuration. c) Relative wave functions of the 6^+ and 8^+ states calculated with the state-dependent optical potentials given by Arima and Yoshida. All curves are normalized to unity at the peaks.

problem can be carried out by taking the position of the ^{16}O nucleus as a generator coordinate as well as the position of the outer four particles. This approach is in calculation. However, we could give the following qualitative discussion on a mechanism of the α-clusterization even at this stage.

Recently Arima and Yoshida[10] showed that α-decay widths of 6^+ and 8^+ states can be explained based on harmonic-oscillator-shell-model wave functions, if the relative wave functions are replaced by the improved relative wave functions derived from angular-momentum-dependent optical potential between ^{16}O and α-particle. However, the latter wave functions are quite different in the interaction region from those derived from shell-model wave functions with *usual* oscillator size parameter. Large part of the difference must come from the effect of configuration mixing of higher excitation or α-clusterization as pointed out by them. It is very interesting to see in Fig. 9 that our "relative" amplitudes of 6^+ and 8^+ states calculated with a finite-well single-particle potential are very like the relative wave functions of 6^+

and 8^+ states calculated with the above-mentioned L-dependent optical potentials given by Arima and Yoshida. This seems to suggest that the enhancement of α-widths which is one of the essential characteristics of α-clusterization would be explained based on the spatial correlations of the outer four particles moving in a *finite-well* single-particle potential.

As a concluding remark of this section 3, we might say the following. When the outer four particles correlate spatially into an alpha-like mode or cluster, a very essential role may be played by the situation that the particles can occupy the higher orbits than *sd* ones, especially *continuum-energy* orbits having decaying character. The nucleus consisting of a closed shell core and outer four particles may have the predisposition to bring about an alpha-cluster through the composition of the finite-well Hartree-Fock potential.

§4. Structure of four-body correlations (II)

—Ground state correlations—

In this section, we investigate the structure of the ground-state correlations which play a quite important role for the four-body correlations. The importance of the ground-state correlations is for the first time pointed out in connection with the structure of the so-called mysterious 0^+ state in ^{16}O and ^{40}Ca by one of the present authors (T. M.) and Suzuki.[2] In the first half of the present section, on the basis of their discussions, we will give general expressions about the ground-state correlations within the framework of the formulation developed in §2 and, in the remaining part, we apply the NTD method to the case of ^{20}Ne in the vertically truncated subspace introduced in the last section. Finally, the future problems are also discussed.

4.1 Physical essence of the ground-state correlations

We have considered the special class of eigenstates of (A_0+4)-particle system, which is described by expression (2·6). In the TD approximation, —the usual shell-model calculation is a kind of the TD approximation and the discussions developed in the last section are also done in the same approximation—the ground state of the A_0-particle system, $|A_0; 0\rangle$, is assumed to be the Hartree-Fock ground state (for example, as the doubly closed shell configuration). The inconsistency of such a model is evident; while assuming a strong "alpha-like" correlations of the outer four particles we make the "core" contain no correlations at all. Thus the correlations are asymmetrically ascribed only to the outer four particles, but not to the "core". Alternatively, the consistency of the model requires to incorporate the collective correlations for the alpha-like modes also in the A_0-particle "core". In that case the four-body correlations are symmetrically ascribed to the

"core" as the ground-state correlations and to the outer four particles. It is known that the correct ground state of the A_0-particle "core" has a collective predisposition for collective modes. The corresponding modes then are "dressed" four-particle modes in the NTD sense in contrast to the quite different "bare" modes described by the TD treatment. Proper treatment of the three-component eigenvalue equation (2·50) just means the above-mentioned NTD treatment.

Now, let us discuss on the structure of the ground state $|A_0; 0\rangle$ determined within the framework of the NTD method developed in §2. The state $|A_0; 0\rangle$ must be determined from Eq. (2·64). For a while, we use the shell-model representation for simplifying the description. Let $c_\alpha^\dagger$ and c_α be the creation and annihilation operators of a nucleon in the shell-model state α. We can define the particle and hole creation and annihilation operator $(a_\alpha^\dagger, a_\alpha)$ and $(b_\alpha^\dagger, b_\alpha)$, respectively, as

$$\left.\begin{aligned} c_\alpha^\dagger &= (1-\theta_\alpha)c_\alpha^\dagger + \theta_\alpha c_\alpha^\dagger = a_\alpha^\dagger + b_\alpha, \\ c_\alpha &= (1-\theta_\alpha)c_\alpha + \theta_\alpha c_\alpha = a_\alpha + b_\alpha^\dagger, \end{aligned}\right\} \tag{4·1}$$

where

$$\theta_\alpha = \begin{cases} 1 & \text{for states occupied in the Hartree-Fock ground state of the } A_0\text{-particle system,} \\ 0 & \text{for states unoccupied in the Hartree-Fock ground state of the } A_0\text{-particle system.} \end{cases} \tag{4·2}$$

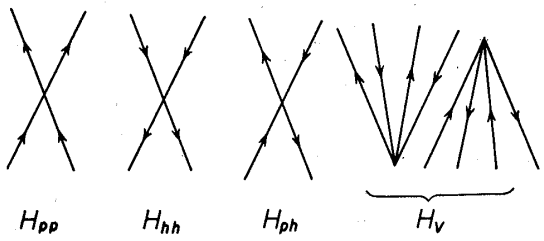

Fig. 10. Graphical representation of the matrix elements of the effective two-body force, H_{pp}, H_{hh}, H_V and H_{ph}.

We can easily see that the matrix elements of the effective two-body force contributing to the effective four-body interaction $\boldsymbol{H}_{\rm int}$ defined by Eq. (2·52c) are H_{pp}, H_{hh} and H_V given in Fig. 10. It is noteworthy that the matrix element H_{ph} in Fig. 10 does not contribute to the four-body modes at all. Needless to say, the ground state $|A_0; 0\rangle$ is a mixture of $0p$-$0h$, $2p$-$2h$, $4p$-$4h$, $\cdots$ excitations in the TD sense and is generally written as

$$\begin{aligned} |A_0; 0\rangle = C_0|\phi_0\rangle &+ \sum_{\alpha\beta\gamma\delta} C_1(\alpha\beta;\gamma\delta)a_\alpha^\dagger a_\beta^\dagger b_\gamma^\dagger b_\delta^\dagger|\phi_0\rangle \\ &+ \sum_{\alpha\beta\gamma\delta}\sum_{\alpha_1\beta_1\gamma_1\delta_1} C_2(\alpha\beta\gamma\delta;\alpha_1\beta_1\gamma_1\delta_1)a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger b_{\alpha_1}^\dagger b_{\beta_1}^\dagger b_{\gamma_1}^\dagger b_{\delta_1}^\dagger|\phi_0\rangle + \cdots, \end{aligned} \tag{4·3}$$

where $|\phi_0\rangle$ is the free ground state (the Hartree-Fock ground state) of the A_0-particle system. After solving the fundamental eigenvalue equation (2·50), we can determine the coefficients $C_0, C_1, C_2, \cdots$ from Eq. (2·64) in the following way: Let us first rewrite the eigenmodes $X_n^\dagger$ as

$$\begin{aligned} X_n^\dagger &= \frac{1}{\sqrt{4!}} \sum_{\alpha\beta\gamma\delta} \sum_{i=1,3,5} \Psi_n^{(i)}(\alpha\beta\gamma\delta) : c_\alpha^\dagger c_\beta^\dagger c_\gamma^\dagger c_\delta^\dagger : \\ &= \frac{1}{\sqrt{4!}} \{ \sum_{\alpha\beta\gamma\delta} \psi_n(\alpha\beta\gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\delta^\dagger + 6 \sum_{\alpha\beta\gamma\delta} \xi_n(\alpha\beta;\ \gamma\delta) a_\alpha^\dagger a_\beta^\dagger b_\gamma b_\delta \\ &\quad + \sum_{\alpha\beta\gamma\delta} \varphi_n(\alpha\beta\gamma\delta) b_\alpha b_\beta b_\gamma b_\delta \}, \end{aligned} \tag{4·4}$$

where $\Psi_n^{(1)}(\alpha\beta\gamma\delta)$, $\Psi_n^{(3)}(\alpha\beta\gamma\delta)$ and $\Psi_n^{(5)}(\alpha\beta\gamma\delta)$ are the shell-model representations of the three components of the correlation-amplitude vector $\boldsymbol{\Psi}_n$ and

$$\xi_n(\alpha\beta;\gamma\delta) = (1-\theta_\alpha)(1-\theta_\beta)\theta_\gamma\theta_\delta \Psi_n^{(3)}(\alpha\beta\gamma\delta). \tag{4·5}$$

Here the symbol : :denotes the normal product with respect to particles and holes. From Eq. (2·64), we have a set of recurrence relations connecting C_{n+1}, C_n and C_{n-1}, the first of which are

$$\begin{aligned} &C_0\varphi_{n+}(\alpha\beta\gamma\delta) - 2 \sum_{P[\text{even}]}{}' \{ \sum_{\alpha_1\beta_1} \xi_{n+}(\alpha_1\beta_1;\ \alpha\beta) C_1(\alpha_1\beta_1;\ \gamma\delta) \} \\ &\qquad + 4! \sum_{\alpha_1\beta_1\gamma_1\delta_1} \psi_{n+}(\alpha_1\beta_1\gamma_1\delta_1) C_2(\alpha_1\beta_1\gamma_1\delta_1;\ \alpha\beta\gamma\delta) = 0, \\ &\qquad\qquad (\text{from} \quad \alpha_n |A_0;\ 0\rangle = 0) \end{aligned} \tag{4·6a}$$

$$\begin{aligned} &C_0\psi_{n-}(\alpha\beta\gamma\delta) - 2 \sum_{P[\text{even}]}{}' \{ \sum_{\alpha_1\beta_1} \xi_{n-}(\alpha\beta;\ \alpha_1\beta_1) C_1(\gamma\delta;\ \alpha_1\beta_1) \} \\ &\qquad + 4! \sum_{\alpha_1\beta_1\gamma_1\delta_1} \varphi_{n-}(\alpha_1\beta_1\gamma_1\delta_1) C_2(\alpha\beta\gamma\delta;\ \alpha_1\beta_1\gamma_1\delta_1) = 0, \\ &\qquad\qquad (\text{from} \quad \beta_n |A_0;\ 0\rangle = 0) \end{aligned} \tag{4·6b}$$

$$\begin{aligned} &C_0\varphi_{n0}(\alpha\beta\gamma\delta) - 2 \sum_{P[\text{even}]}{}' \{ \sum_{\alpha_1\beta_1} \xi_{n0}(\alpha_1\beta_1;\ \alpha\beta) C_1(\alpha_1\beta_1;\ \gamma\delta) \} \\ &\qquad + 4! \sum_{\alpha_1\beta_1\gamma_1\delta_1} \psi_{n0}(\alpha_1\beta_1\gamma_1\delta_1) C_2(\alpha_1\beta_1\gamma_1\delta_1;\ \alpha\beta\gamma\delta) = 0, \\ &C_0\psi_{n0}(\alpha\beta\gamma\delta) - 2 \sum_{P[\text{even}]}{}' \{ \sum_{\alpha_1\beta_1} \xi_{n0}(\alpha\beta;\ \alpha_1\beta_1) C_1(\gamma\delta;\ \alpha_1\beta_1) \} \\ &\qquad + 4! \sum_{\alpha_1\beta_1\gamma_1\delta_1} \varphi_{n0}(\alpha_1\beta_1\gamma_1\delta_1) C_2(\alpha\beta\gamma\delta;\ \alpha_1\beta_1\gamma_1\delta_1) = 0, \\ &\qquad\qquad (\text{from} \quad \gamma_n^\dagger |A_0;\ 0\rangle = \gamma_n |A_0;\ 0\rangle = 0) \end{aligned} \tag{4·6c}$$

where the symbol $\sum'_{P[\text{even}]}$ means a restricted summation of even permutations of $(\alpha, \beta, \gamma, \delta)$ such as

$$\begin{aligned} \sum_{P[\text{even}]}{}' f(\alpha\beta\gamma\delta) \equiv & f(\alpha\beta\gamma\delta) + f(\alpha\gamma\delta\beta) + f(\alpha\delta\beta\gamma) \\ & + f(\beta\gamma\alpha\delta) + f(\gamma\delta\alpha\beta) + f(\beta\delta\gamma\alpha), \end{aligned} \tag{4·7}$$

$f(\alpha\beta\gamma\delta)$ being the arbitrary function of $(\alpha, \beta, \gamma, \delta)$. After determining the coefficients C_1 and C_2 in terms of C_0 from Eqs. (4·6), we can proceed to the next relations connecting C_3 to C_1 and C_2 in order to determine C_3 and so on. If the ground state is "normal" (i.e., non-"bound" alpha-clustering), the following inequality relation should be fulfilled;

$$|C_0|>|C_1|, |C_2|, \cdots. \tag{4·8}$$

Here we should discuss the quasi-boson character of the physical four-body eigenmodes $\alpha_n^\dagger$ and $\beta_n^\dagger$. It is shown by one of the authors and Suzuki[2] that the various eigenstates $\alpha_n^\dagger|A_0; 0\rangle$ and $\beta_n^\dagger|A_0; 0\rangle$ satisfy the orthonormality relations (2·65) and, under a reasonable approximation, these relations are equivalent to Eq. (2·54) which is the orthonormality condition of the correlation amplitudes obtained from the fundamental eigenvalue equation (2·50). On the other hand, the ground state of the A_0-particle "core" is, as mentioned above, determined from Eq. (2·64). Therefore we have the following relations:

$$\left.\begin{aligned}\langle A_0; 0|\,[\alpha_m, \alpha_n^\dagger]\,|A_0; 0\rangle &= N_n\delta_{mn},\\ \langle A_0; 0|\,[\beta_m, \beta_n^\dagger]\,|A_0; 0\rangle &= N_n\delta_{mn},\end{aligned}\right\} \tag{4·9}$$

which imply that the physical four-body eigenmodes $\alpha_n^\dagger$ and $\beta_n^\dagger$ possess a quasi-boson character. It will later be shown that we can put $N_n=1$. (See Eq. (4·26).)

4. 2 A schematic model

In order to study the properties of the solution of the three-component eigenvalue equation (2·50) and the ground state of the A_0-particle "core", we use a schematic model in which the explicit solution of the eigenvalue equation can be obtained. For simplicity, we consider a model consisting of two levels with equal angular momenta $j_a=j_b=j$.*)

The alpha-like eigenmode $X_0^\dagger$ defined by Eq. (4·4) (with $n=0$) is now written as

$$\begin{aligned}X_0^\dagger=\frac{1}{\sqrt{4!}}\sum_{JM}\sum_{TZ}&\langle JJM-M|00\rangle\langle TTZ-Z|00\rangle\\ &\times\{\psi_0(JT)A^\dagger_{JM,TZ}A^\dagger_{J-M,T-Z}+6\xi_0(JT)A^\dagger_{JM,TZ}B_{J-M,T-Z}\\ &+\varphi_0(JT)B_{JM,TZ}B_{J-M,T-Z}\},\end{aligned} \tag{4·10}$$

where

*) Detailed discussions of the physical situation with the use of such schematic models have been given in Refs. 2), 3), 5) and 7).

$$\left.\begin{aligned} A^{\dagger}_{JM,TZ} &= \sum_{\substack{m_\alpha m_{\alpha'} \\ \tau_\alpha \tau_{\alpha'}}} \langle jjm_\alpha m_{\alpha'} | JM \rangle \left\langle \frac{1}{2}\frac{1}{2}\tau_\alpha \tau_{\alpha'} | TZ \right\rangle a_\alpha^{\dagger} a_{\alpha'}^{\dagger}, \\ B_{JM,TZ} &= \sum_{\substack{m_\beta m_{\beta'} \\ \tau_\beta \tau_{\beta'}}} \langle jjm_\beta m_{\beta'} | JM \rangle \left\langle \frac{1}{2}\frac{1}{2}\tau_\beta \tau_{\beta'} | TZ \right\rangle b_\beta b_{\beta'}. \end{aligned}\right\} \quad (4\cdot11)$$

We write the matrix element of the effective two-body force in the form

$$\langle \alpha\beta | v | \gamma\delta \rangle = -\frac{1}{2}\sum_{JT} G(abcd; JT) \langle j_a j_b m_\alpha m_\beta | JM \rangle \left\langle \frac{1}{2}\frac{1}{2}\tau_\alpha \tau_\beta | TZ \right\rangle \times \langle j_c j_d m_\gamma m_\delta | JM \rangle \left\langle \frac{1}{2}\frac{1}{2}\tau_\gamma \tau_\delta | TZ \right\rangle . \quad (4\cdot12)$$

Considering the idea of Danos and Jillet's "stretch coupling scheme",[13] in this model we assume the following simplified approximation:

$$\left.\begin{aligned} &G(aaaa; JT) = G(bbbb; JT) = \delta_{JJ_m}\delta_{T0} G, \\ &G(aabb; JT) = \delta_{JJ_m}\delta_{T0} G', \end{aligned}\right\} \quad (4\cdot13)$$

where J_m means $J_{\max}(=2j)$.

With the assumption (4·13), we have the eigenvalue equation

$$\left.\begin{aligned} &\{\omega_0 - 4\varepsilon_a\}\psi_0(J_m 0) = -(2 - 4a_{J_m J_m} t_{00})\{G\psi_0(J_m 0) - G'\xi_0(J_m 0)\}, \\ &\{\omega_0 - 2(\varepsilon_a + \varepsilon_b)\}\xi_0(J_m 0) = -G'\psi_0(J_m 0) + G'\varphi_0(J_m 0), \\ &\{\omega_0 - 4\varepsilon_b\}\varphi_0(J_m 0) = -(2 - 4a_{J_m J_m} t_{00})\{G'\xi_0(J_m 0) - G\varphi_0(J_m 0)\}, \end{aligned}\right\} \quad (4\cdot14a)$$

corresponding to Eq. (2·50) together with the following linear equations for the amplitudes $\psi_0(JT)$, $\xi_0(JT)$ and $\varphi_0(JT)$ with $J \neq J_m$ and $T \neq 0$:

$$\left.\begin{aligned} &\{\omega_0 - 4\varepsilon_a\}\psi_0(JT) = 4a_{JJ_m} t_{T0}\{G\psi_0(J_m 0) - G'\xi_0(J_m 0)\}, \\ &\xi_0(JT) = 0, \\ &\{\omega_0 - 4\varepsilon_b\}\varphi_0(JT) = 4a_{JJ_m} t_{T0}\{G'\xi_0(J_m 0) - G\varphi_0(J_m 0)\}, \end{aligned}\right\} \quad (4\cdot14b)$$

where

$$\left.\begin{aligned} a_{JJ'} &= \sqrt{(2J+1)(2J'+1)}\, W(jJJ'j; jj), \\ t_{TT'} &= \sqrt{(2T+1)(2T'+1)}\, W\left(\frac{1}{2}TT'\frac{1}{2}; \frac{1}{2}\frac{1}{2}\right). \end{aligned}\right\} \quad (4\cdot15)$$

After solving Eq. (4·14a), we can determine the amplitudes with $J \neq J_m$ and $T \neq 0$ from Eq. (4·14b) as follows:

$$\psi_0(JT) = -\frac{4a_{JJ_m} t_{T0}}{K}\psi_0(J_m 0), \quad \xi_0(JT) = 0, \quad \varphi_0(JT) = -\frac{4a_{JJ_m} t_{T0}}{K}\varphi_0(J_m 0) \quad \text{for} \quad J \neq J_m \quad \text{and} \quad T \neq 0, \quad (4\cdot16)$$

where

$$K=2-4a_{J_mJ_m}t_{00}. \tag{4·17}$$

With the aid of the relations (4·16), the alpha-like eigenmode $X_0^\dagger$ can be written down as

$$X_0^\dagger=\frac{1}{\sqrt{4!}}\frac{6}{K}\sum_M\langle J_mJ_mM-M|00\rangle\{\psi_0A^\dagger_{J_mM,00}A^\dagger_{J_m-M,00} +\eta_0A^\dagger_{J_mM,00}B_{J_m-M,00} +\varphi_0B_{J_mM,00}B_{J_m-M,00}\}, \tag{4·18}$$

where

$$\psi_0=\psi_0(J_m0),\quad \eta_0=\sqrt{K}\,\eta_0(J_m0),\quad \varphi_0=\varphi_0(J_m0). \tag{4·19}$$

The eigenvalue equation (4·14a) can be written in the following matrix form:

$$\omega_0\begin{pmatrix}\psi_0\\ \eta_0\\ \varphi_0\end{pmatrix}=\begin{pmatrix}4\varepsilon_a-B_4 & -G_V & 0\\ -G_V & -2(\varepsilon_a+\varepsilon_b) & G_V\\ 0 & G_V & 4\varepsilon_b+B_4\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\psi_0\\ \eta_0\\ \varphi_0\end{pmatrix}, \tag{4·20}$$

where

$$B_4\equiv KG \quad\text{and}\quad G_V\equiv\sqrt{K}G'. \tag{4·21}$$

The quantity B_4 is the binding energy of four particles (or holes) in the TD sense and the quantity G_V is just the interaction bringing about the ground-state correlations.

From Eq. (4·20), we obtain three solutions for ω_0;

$$\begin{aligned}\omega_{0+}&=2(\varepsilon_a+\varepsilon_b)+\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2},\\ \omega_{00}&=2(\varepsilon_a+\varepsilon_b),\\ \omega_{0-}&=2(\varepsilon_a+\varepsilon_b)-\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2},\end{aligned} \tag{4·22}$$

where ω_{0+} and ω_{0-} are the solutions corresponding to the creation operator for the alpha-like four-body mode $\alpha_0^\dagger$ and to the annihilation operator for the "correlated four holes" β_0, respectively. Here ω_{00} is the solution corresponding to the unphysical eigenmode $\gamma_0^\dagger$. Explicit expressions of the correlation amplitudes ψ_0, η_0 and φ_0 are given, respectively, by

$$\begin{aligned}\psi_{0+}&=\sqrt{N}\,\frac{\{2(\varepsilon_a-\varepsilon_b)-B_4\}+\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2}}{2\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2}},\\ \eta_{0+}&=\sqrt{N}\,\frac{-2G_V}{2\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2}},\\ \varphi_{0+}&=\sqrt{N}\,\frac{\{2(\varepsilon_a-\varepsilon_b)-B_4\}-\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2}}{2\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V^2}},\end{aligned} \tag{4·23}$$

for the "alpha-like" four-body mode $\alpha_0^\dagger$,

$$\begin{aligned}\psi_{0-}&=\varphi_{0+},\\ \eta_{0-}&=\eta_{0+},\\ \varphi_{0-}&=\psi_{0+},\end{aligned} \tag{4·24}$$

for the "correlated four holes" β_0,

$$\begin{aligned}\psi_{00}&=\eta_{0+},\\ \eta_{00}&=\sqrt{N}\,\frac{2(\varepsilon_a-\varepsilon_b)-B_4}{2\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V{}^2}},\\ \varphi_{00}&=\eta_{0+},\end{aligned} \tag{4·25}$$

for the unphysical eigenmode $\gamma_0^\dagger$,

where the normalization constant N is given by $N=K/6$. They satisfy the normalization conditions

$$\begin{aligned}&|\psi_{0+}|^2-|\eta_{0+}|^2+|\varphi_{0+}|^2=N,\\ &|\psi_{0-}|^2-|\eta_{0-}|^2+|\varphi_{0-}|^2=N,\\ &|\psi_{00}|^2-|\eta_{00}|^2+|\varphi_{00}|^2=-N,\end{aligned} \tag{4·26}$$

in accordance with Eq. (2·45) or (2·46).

Here, it should be noticed that, if the strength of the effective two-body interaction responsible for the four-body correlations, especially G_V, increases and passes through a critical value which leads ω_{0+}, ω_{00} and ω_{0-} to have the same value $2(\varepsilon_a+\varepsilon_b)$, the eigenvalues of Eq. (4·20) become complex and the corresponding eigenmodes have no physical meaning. Moreover, we should notice that, as seen from Eq. (4·26), the norm corresponding to the physical eigenmode $\alpha_0^\dagger$ or β_0 is positive but that of the unphysical mode $\gamma_0^\dagger$ is negative. This fact seems to be a general character of the four-body eigenmode under consideration. Then we can safely put for the normalization constant N_n appeared in Eq. (2·45) or (2·65) (and also in Eq. (4·9))

$$N_n=\begin{cases}1 & \text{for the physical eigenmodes } \alpha_n^\dagger \text{ and } \beta_n,\\ -1 & \text{for the unphysical eigenmode } \gamma_n^\dagger.\end{cases} \tag{4·27}$$

As discussed in the last subsection, the ground state of the "core" $|A_0;\,0\rangle\equiv|\Phi_0\rangle$, which satisfies

$$\alpha_0|\Phi_0\rangle=0,\quad \beta_0|\Phi_0\rangle=0,\quad \gamma_0^\dagger|\Phi_0\rangle=\gamma_0|\Phi_0\rangle=0, \tag{4·28}$$

has the structure

$$|\Phi_0\rangle = C_0|\phi_0\rangle + \sum_{JT} C_1(JT) \sum_{MZ} \langle JJM-M|00\rangle \langle TTZ-Z|00\rangle \times A^{\dagger}_{JM,TZ}(-)^{J+T-M-Z} B^{\dagger}_{J-M,T-Z}|\phi_0\rangle + \sum_{JT}\sum_{J'T'} C_2(JT, J'T') \sum_{MZ}\sum_{M'Z'} \langle JJM-M|00\rangle \langle TTZ-Z|00\rangle \times \langle J'J'M'-M'|00\rangle \langle T'T'Z'-Z'|00\rangle \times A^{\dagger}_{JM,TZ} A^{\dagger}_{J-M,T-Z} B^{\dagger}_{J'M',T'Z'} B^{\dagger}_{J'-M',T'-Z'}|\phi_0\rangle + \cdots\cdots. \quad (4\cdot29)$$

Conditions (4·28) determine the coefficients C. For example, the equation corresponding to Eqs. (4·5) lead us to

$$C_1(JT) = C_0\left(\frac{2J_m+1}{2}\right)^{1/2} K^{-3/2} \frac{\sqrt{2}\,G_V}{\{2(\varepsilon_a-\varepsilon_b)-B_4\}+\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V{}^2}} \times \delta_{JJ_m}\delta_{T0}, \quad (4\cdot30a)$$

$$C_2(JT, J'T') = C_0 \frac{1}{4!} \frac{\{2(\varepsilon_a-\varepsilon_b)-B_4\}-\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V{}^2}}{\{2(\varepsilon_a-\varepsilon_b)-B_4\}+\sqrt{\{2(\varepsilon_a-\varepsilon_b)-B_4\}^2-2G_V{}^2}} \times \delta_{JJ_m}\delta_{J'J_m}\delta_{T0}\delta_{T'0}. \quad (4\cdot30b)$$

Here it should be emphasized that condition $\gamma_0^{\dagger}|\Phi_0\rangle = \gamma_0|\Phi_0\rangle = 0$ is indispensable for determining the structure of the ground state $|\Phi_0\rangle$, in spite of "unphysical property" of $\gamma_0^{\dagger}$ in the sense of the eigenmode.

4.3 Stability of the "normal" Hartree-Fock ground state and bound alpha-like cluster

So far we have assumed that the Hartree-Fock field forthe "core" under consideration is still stable, i.e., that the free ground state of the "core" $|\phi_0\rangle$ is normal and well described by the normal shell-model ground state. However, in order to investigate the structure of the four-body correlations in detail, we should see what happens when the strength of the effective two-body interaction responsible for the four-body correlations increases and passes through a critical value. In this situation it may happen that the ground-state wave function changes suddenly and cannot any longer be obtained by the ordinary perturbation theory. Hereafter we call this a "phase transition", which means an instability of the normal shell-model ground state $|\phi_0\rangle$. Typical examples of such a "phase transition" in the theory of nuclear structure are the transition of the ground state from the spherical shape to the deformed shape and the transition from the normal state to the superconducting state. As is well known, in the former case the eigenvalue of the $1p$-$1h$ collective excitation mode with $J=2$ becomes zero and in the latter case that of the two-body scattering eigenmode with $J=0$ does.

Following the original general theory proposed by Sawada and Fukuda,[14]

we can formulate a theorem concerning our new "phase transition" due to the four-body correlations. Let $\alpha_0^\dagger$ be the creation operator of the alpha-like four-body mode with the *lowest* eigenvalue ω_{0+} (defined by Eq. (2·63)). The operator $\alpha_0^\dagger$ satisfies the following equation of motion:

$$[\alpha_0^\dagger, H] = -\omega_{0+}\alpha_0^\dagger + Z_0, \tag{4·31}$$

where

$$H = \sum_\alpha \varepsilon_\alpha c_\alpha^\dagger c_\alpha + \frac{1}{2}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|v|\gamma\delta\rangle : c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma : \tag{4·32}$$

and the "interaction" Z_0 is composed of the terms with sixth-order normal product with respect to $a^\dagger(a)$ and $b^\dagger(b)$.

Now, we consider a family of variational functions defined by

$$|\Psi(\mu)\rangle = \exp\{i\mu(\alpha_0^\dagger + \alpha_0)\}\,|\phi_0\rangle, \quad |\Psi(0)\rangle = |\phi_0\rangle, \tag{4·33}$$

where μ is a variational parameter and $|\phi_0\rangle$ is the normal Hartree-Fock (shell-model) ground state. Then the variational energy is given by

$$E(\mu) \equiv \langle\Psi(\mu)|H|\Psi(\mu)\rangle = \langle\phi_0|e^{-i\mu(\alpha_0^\dagger+\alpha_0)}He^{i\mu(\alpha_0^\dagger+\alpha_0)}|\phi_0\rangle, \tag{4·34}$$

which is, of course, equal to the Hartree-Fock energy for $\mu=0$. With the aid of Eq. (4·31), we can easily see

$$\frac{\partial}{\partial\mu}E(\mu)\,|_{\mu=0} = (-i)\langle\phi_0|\,[\alpha_0^\dagger + \alpha_0, H]\,|\phi_0\rangle = 0. \tag{4·35}$$

Equation (4·35) tells us that the normal shell-model ground state $|\phi_0\rangle$ is not only the ordinary variational one with respect to the single-particle modes but also a variational one with respect to the family of variational functions (4·33).

In order to check whether the normal Hartree-Fock ground state $|\phi_0\rangle$ is really stable, we have to see the sign of the second derivative of $E(\mu)$ at $\mu=0$, which is given by

$$\begin{aligned}\frac{\partial^2}{\partial\mu^2}E(\mu)\,|_{\mu=0} &= (-i)^2\langle\phi_0|\,[\alpha_0^\dagger+\alpha_0,\,[\alpha_0^\dagger+\alpha_0, H]]\,|\phi_0\rangle \\ &= (\omega_{0+}+\omega_{0+}^*)\langle\phi_0|\,[\alpha_0, \alpha_0^\dagger]\,|\phi_0\rangle.\end{aligned} \tag{4·36}$$

Now, let ω_c be the eigenvalue at the critical point where the eigenvalue ω_{0+} changes from real to complex one. Without loss of generality, we can choose the single-particle energies ε_α as the critical eigenvalue ω_c becomes zero. Then, for the physical eigenmode $\alpha_0^\dagger$, the eigenvalue ω_{0+} is real and positive and

$$\langle\phi_0|\,[\alpha_0, \alpha_0^\dagger]\,|\phi_0\rangle = \sum_{\alpha\beta\gamma\delta}\{\psi_{0+}^2(\alpha\beta\gamma\delta) - \varphi_{0+}^2(\alpha\beta\gamma\delta)\} > 0. \tag{4·37}$$

We obtain therefore

$$\frac{\partial^2}{\partial\mu^2}E(\mu)\,|_{\mu=0}>0 \qquad \text{for} \quad \omega_{0+}>0. \tag{4·38}$$

Thus the Hartree-Fock ground state $|\phi_0\rangle$ satisfies the minimum condition with respect to the family of variational functions. This is consistent with our original assumption that the ground state of the "core" is normal for the four-body correlations. However, if the strength of the effective interaction increases and passes through a critical value which leads ω_{0+} to become zero, then we have

$$\frac{\partial^2}{\partial\mu^2}E(\mu)\,|_{\mu=0}=0 \qquad \text{for} \quad \omega_{0+}=\text{imaginary}. \tag{4·39}$$

In this case, the minimum condition is usually violated and there appears a new phase transition. Sawada and Fukuda[14] have shown that there exists in this case another ground state with lower energy which is to be obtained by the variational principle by making use of trial functions similar to Eq. (4·33):

$$|\Psi(\theta)\rangle=\exp\{i(\alpha^\dagger(\theta)+\alpha(\theta))\}\,|\phi_0\rangle, \tag{4·40}$$

where the operator $\alpha(\theta)$ has the same structure as $\alpha_0^\dagger$ with respect to operators $a^\dagger(a)$ and $b^\dagger(b)$ but the coefficients contain some variational parameters.

In order to investigate the structure of the new ground state $|\Psi(\theta)\rangle$, we now use the schematic model developed in the last section. In this model, the alpha-like mode is written as Eq. (4·18). Therefore, the new ground state after the phase transition is considered to be

$$|\Psi(\theta)\rangle=\exp\{iS(\theta)\}\,|\phi_0\rangle=U(\theta)\,|\phi_0\rangle, \tag{4·41}$$

$$U^\dagger(\theta)=U^{-1}(\theta), \tag{4·42}$$

where

$$S(\theta)=\sum_M\langle J_mJ_mM-M|00\rangle\{\psi(\theta)C^\dagger_{J_mM,00}C^\dagger_{J_m-M,00}+\psi^*(\theta)C_{J_m-M,00}C_{J_mM,00}\} \tag{4·43}$$

with

$$C^\dagger_{JM,TZ}\equiv A^\dagger_{JM,TZ}+B_{JM,TZ}. \tag{4·44}$$

The pair operator $A^\dagger$ and B have been defined by Eq. (4·11). Now, we define new operators with the unitary transformations of the annihilation operators a_α and b_α as

$$A_\alpha\equiv U(\theta)a_\alpha U^{-1}(\theta), \quad B_\alpha\equiv U(\theta)b_\alpha U^{-1}(\theta). \tag{4·45}$$

These new operators are expanded as follows:

$$\left.\begin{aligned} A_\alpha &= a_\alpha + i[S, a_\alpha] + \frac{i^2}{2!}[S, [S, a_\alpha]] + \cdots, \\ B_\alpha &= b_\alpha + i[S, b_\alpha] + \frac{i^2}{2!}[S, [S, b_\alpha]] + \cdots. \end{aligned}\right\} \tag{4·46}$$

If we adopt an approximation in which only the lowest order terms with respect to the original operators $a^\dagger(a)$ and $b^\dagger(b)$ in each commutators appearing in the right-hand side of Eq. (4·46) are taken into account, we can write the operators A_α and B_α as

$$\left.\begin{aligned} A_\alpha &\cong \cos\theta\, a_\alpha - s_\alpha \sin\theta\, (a^\dagger C^\dagger)_{-\alpha}, \\ B_\alpha &\cong \cos\theta\, b_\alpha - s_\alpha \sin\theta\, (b^\dagger C)_{-\alpha}, \end{aligned}\right\} \tag{4·47}$$

where $s_\alpha = (-1)^{j-m_\alpha+1/2-\tau_\alpha}$ and

$$\left.\begin{matrix} (a^\dagger C^\dagger)_{-\alpha} \\ (b^\dagger C)_{-\alpha} \end{matrix}\right\} = \frac{1}{\sqrt{K}} \sum_m \langle jJ_m m - m_\alpha - m \,|\, j - m_\alpha\rangle \left\langle \frac{1}{2} 0 - \tau_\alpha 0 \,\middle|\, \frac{1}{2} - \tau_\alpha \right\rangle \times \left\{ \begin{matrix} a^\dagger_{m,-\tau_\alpha} C^\dagger_{J_m - m_\alpha - m, 00} \\ b^\dagger_{m,-\tau_\alpha} C_{J_m - m_\alpha - m, 00} \end{matrix} \right. \tag{4·48}$$

with K defined by Eq. (4·17). Here we have assumed

$$\theta = \sqrt{\frac{8K}{2j+1}}\, i\psi. \tag{4·49}$$

From Eq. (4·47), we can define a new quasi-particle operator α_α as

$$\alpha_\alpha = A_\alpha + B^\dagger_\alpha = u_a c_\alpha - s_\alpha v_a (c^\dagger C^\dagger)_{-\alpha}, \tag{4·50}$$

which just corresponds to the Bogoliubov transformation in the theory of superconductor. From a different standpoint, Flowers and Vujičič have already introduced a transformation similar to Eq. (4·50).[15] This operator α_α satisfies the following anticommutation relation:

$$\{\alpha_\alpha, \alpha_\beta^\dagger\}_+ = \delta_{\alpha\beta} + [\text{higher order terms with respect to } a^\dagger(a) \text{ and } b^\dagger(b)]. \tag{4·51}$$

If we neglect the higher order terms in the right-hand side of Eq. (4·51), we can consider the new quasi-particle operator as a fermion operator.

From definition, the new ground state $|\Psi(\theta)\rangle$ is determined by

$$A_\alpha |\Psi(\theta)\rangle = B_\beta |\Psi(\theta)\rangle = 0. \tag{4·52}$$

By considering the pseudo-fermion property of A_α and B_β, the ground state is, therefore, written as

$$|\Psi(\theta)\rangle = \prod_{\alpha,\beta>0} \{u_a + s_\alpha v_a a_\alpha^\dagger (a^\dagger C^\dagger)_{-\alpha}\} \{u_b + s_\beta v_b b_\beta^\dagger (b^\dagger C)_{-\beta}\} |\phi_0\rangle$$
$$\Rightarrow \left(\sqrt{\frac{2j+1}{K}} \frac{V_a}{U_a} \sum_M \langle J_m J_m M - M | 00\rangle C^\dagger_{J_m M,00} C^\dagger_{J_m - M,00}\right)^{A/4} |0\rangle, \quad (4\cdot53)$$

where $|0\rangle$ is the vacuum defined by $c_\alpha|0\rangle = 0$ and

$$\begin{pmatrix} U_\alpha \\ V_\alpha \end{pmatrix} = \begin{pmatrix} 1-\theta_a & \theta_a \\ \theta_a & 1-\theta_a \end{pmatrix} \begin{pmatrix} u_a \\ v_a \end{pmatrix} \quad (4\cdot54)$$

with

$$\theta_a = \begin{cases} 1 & \text{for states occupied in } |\phi_0\rangle, \\ 0 & \text{for states unoccupied in } |\phi_0\rangle. \end{cases} \quad (4\cdot55)$$

The right-hand side of Eq. (4·53) shows that the new ground state after phase transition is of the bound alpha-like cluster with $J=0$ and $T=0$, which has been discussed by Baranger[16] and by Flowers and Vujičič[15] and many authors in connection with the four-body correlations in light nuclei.

4.4 New Tamm-Dancoff calculations for ^{20}Ne within the vertically truncated subspace

In the previous section, we solved the eigenvalue equation for the four-body modes in ^{20}Ne within the framework of the TD approximation so as to investigate the structure of the spatial correlations. We have then introduced the vertically truncated subspace $\mathcal{H}_\Phi$ which is suitable for that aim. In the present subsection, in order to take account of the ground-state correlations together with the spatial correlations, we will treat the four-body modes in ^{20}Ne under the NTD approximation within the vertically truncated subspace.

The eigenvalue equation determining the four-body modes in the NTD approximation is a three-component equation (2·50). Since it seems to be reasonable that each component of the three-component eigenvector $\boldsymbol{\Psi}_n$ appearing in Eq. (2·50) has the [4] symmetry property, we assume

$$\boldsymbol{\Psi}_n(x_1x_2x_3x_4) \equiv \boldsymbol{\psi}_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\chi_{S=0,T=0}(1234), \quad (4\cdot56)$$

where

$$\boldsymbol{\Psi}_n = \begin{pmatrix} \Psi_n^{(1)} \\ \Psi_n^{(3)} \\ \Psi_n^{(5)} \end{pmatrix}, \qquad \boldsymbol{\psi}_{LM} = \begin{pmatrix} \psi_{LM}^{(1)} \\ \psi_{LM}^{(3)} \\ \psi_{LM}^{(5)} \end{pmatrix}. \quad (4\cdot57)^{*)}$$

The correlation amplitude $\boldsymbol{\psi}_{LM}$ is assumed to be a vector in the vertically truncated subspace $\mathcal{H}_\Phi$ so that it is written in the similar form as Eqs. (3·7):

*) Hereafter, Gothic Greek letters such as $\boldsymbol{\Psi}$, $\boldsymbol{\psi}$, $\boldsymbol{\Phi}$, etc. denotes three-component vectors.

$$\psi_{LM}(\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3\boldsymbol{r}_4)\equiv\int\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R})^2\right\}\langle\boldsymbol{R}\,|\,\boldsymbol{\Phi}_{LM}\rangle d\boldsymbol{R}, \tag{4·58a}$$

$$\langle\boldsymbol{R}\,|\,\boldsymbol{\Phi}_{LM}\rangle\equiv\boldsymbol{\Phi}_L(R)\,Y_{LM}(\Omega_R). \tag{4·58b}$$

Substituting the correlation function (4·56) with Eqs. (4·58) into the eigenvalue equation (2·50), we obtain the following integral equation in R-coordinate:

$$\int[\boldsymbol{H}_L(R,R')-\omega_L\boldsymbol{N}_L(R,R')]\,\boldsymbol{\Phi}_L(R')R'^2dR'=0, \tag{4·59}$$

where both of the overlap kernel $\boldsymbol{N}_L$ and the energy kernel $\boldsymbol{H}_L$ are (3×3)-component matrix and are defined by

$$\begin{aligned}\begin{pmatrix}\boldsymbol{N}_L(R,R')\\ \boldsymbol{H}_L(R,R')\end{pmatrix}=&\int\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R})^2\right\}Y^*_{LM}(\Omega_R)\chi^*_{S=0,T=0}(1234)\\ &\times\begin{pmatrix}\langle x_1x_2x_3x_4\,|\,\boldsymbol{P}\,|\,x_1'x_2'x_3'x_4'\rangle\\ \langle x_1x_2x_3x_4\,|\,\boldsymbol{H}\,|\,x_1'x_2'x_3'x_4'\rangle\end{pmatrix}\exp\left\{-\frac{\alpha^2}{2}\sum_{i=1}^{4}(\boldsymbol{r}_i'-\boldsymbol{R}')^2\right\}Y_{LM}(\Omega_{R'})\\ &\times\chi_{S=0,T=0}(1'2'3'4')\,d\Omega_Rd\Omega_{R'}\Big(\prod_{i=1}^{4}dx_idx_{i'}\Big).\end{aligned} \tag{4·60}$$

The matrices $\boldsymbol{P}$ and $\boldsymbol{H}$ are given by Eqs. (2·48) and (2·52) respectively. From relation (2·54), the normalization condition of $\boldsymbol{\Phi}_L(R)$ becomes

$$\begin{aligned}&\int\boldsymbol{\Phi}_L^\dagger(R)\,\boldsymbol{N}_L(R,R')\boldsymbol{\Phi}_L(R')R^2R'^2dRdR'\\ &\qquad=\begin{cases}1 & \text{for physical eigenmodes,}\\ -1 & \text{for unphysical eigenmodes.}\end{cases}\end{aligned} \tag{4·61}$$

Similarly to the case of the TD calculation represented in the previous section, we assume the harmonic oscillator potential and a Gaussian interaction for the single-particle potential and the residual interaction, respectively, whose form are given by Eqs. (3·19) and (3·20). Then the explicit forms of the overlap kernel $\boldsymbol{N}_L(R,R')$ and the energy kernel $\boldsymbol{H}_L(R,R')$ could be written in terms of well-known functions but, as they are very complicated, we do not write them here.

Now, we expand the eigenvector $\boldsymbol{\Phi}_L(R)$ by the orthonormal set of the basic functions defined by Eq. (3·28a):

$$\boldsymbol{\Phi}_L(R)=\sum_N\boldsymbol{C}_{NL}\boldsymbol{\phi}^{(0)}_{NL}(R), \tag{4·62}$$

where the coefficients $\boldsymbol{C}_{NL}$ are three-component vectors

$$\boldsymbol{C}_{NL}=\begin{pmatrix}C^{(1)}_{NL}\\ C^{(3)}_{NL}\\ C^{(5)}_{NL}\end{pmatrix}. \tag{4·63}$$

With the use of expansion (4·62), the eigenvalue equation (4·59) can be rewritten in the form

$$\sum_{N'}(\widetilde{\boldsymbol{H}}^{L}_{NN'}-\omega_L\widetilde{\boldsymbol{N}}^{L}_{NN'})\boldsymbol{C}_{N'L}=0, \tag{4·64}$$

where the matrix elements $\widetilde{\boldsymbol{H}}^{L}_{NN'}$ and $\widetilde{\boldsymbol{N}}^{L}_{NN'}$ are defined by

$$\begin{pmatrix}\widetilde{\boldsymbol{N}}^{L}_{NN'}\\ \widetilde{\boldsymbol{H}}^{L}_{NN'}\end{pmatrix}=\int\varphi^{(0)}_{NL}{}^{*}(R)\begin{pmatrix}\boldsymbol{N}_L(R,R')\\ \boldsymbol{H}_L(R,R')\end{pmatrix}\varphi^{(0)}_{N'L}(R')R^2R'^2dRdR'. \tag{4·65}$$

The low-lying even-parity states in ^{20}Ne are studied by solving Eq. (4·64) for each angular momentum L. The size parameter β^2 of the ^{16}O-core is assumed to be $\beta^2=m\omega_0/\hbar=0.410$ fm^{-2} which corresponds to $\hbar\omega_0=17.0$ MeV. The strength of the interaction defined by Eq. (3·43) and the range parameter $\lambda=\beta/(\sqrt{2}\,\mu)$ are treated as adjustable parameters in order to reproduce the energy spectrum and the binding energy of ^{20}Ne. As an example, in Fig. 11, calculated eigenvalues ω_n of the eigenvalue equation (4·64) with $L=0^+$ are shown in unit of $\hbar\omega_0$ for the case of the "size" parameter $\alpha^2/\beta^2=1.2$ and the interaction-range parameter $\lambda=\beta/(\sqrt{2}\,\mu)=0.79$ for various values of the interaction strength. In this figure, the solid line with the least energy corresponds to the most strongly correlated alpha-like eigenmode in ^{20}Ne and then to the ground state of ^{20}Ne. Here, it should be noticed that, if the absolute value of the interaction strength, $|V_0|$, becomes greater than the critical value for which the eigenvalue of the ground state of ^{20}Ne (represented by the solid line mentioned above) is equal to the eigenvalue of the unphysical modes (represented by the dashed lines in Fig. 11), the eigenvalue of the alpha-like four-body mode becomes complex and, as discussed in § 4.3, an instability of the Hartree-

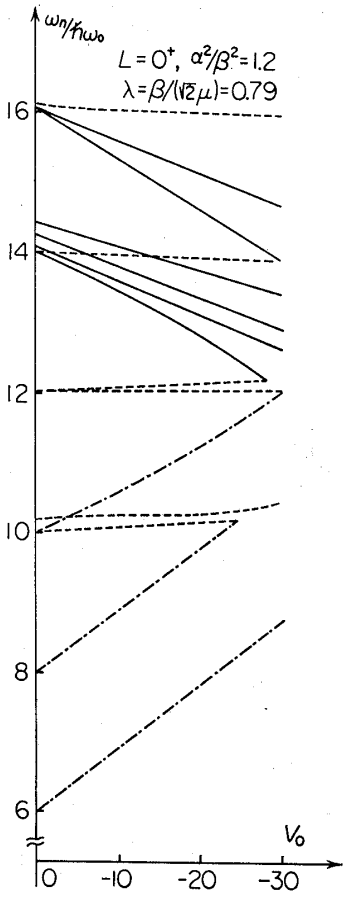

Fig. 11. Calculated eigenvalues ω_n of the eigenvalue equation (4·64) with $L=0^+$ are shown in unit of $\hbar\omega_0$(=17.0 MeV) for the case of $\alpha^2/\beta^2=1.2$ and $\lambda=\beta/(\sqrt{2}\,\mu)=0.79$. The eigenvalues of the alpha-like four-body modes are given by solid lines and those of the correlated four-hole modes by dotted-dashed lines. Dashed lines represent the eigenvalues of the unphysical modes.

Fock field appears.

Similarly to §3.4, we hereafter measure the energy of the ground and excited states of ^{20}Ne, subtracting the total single-particle energy of $(sd)^4$ configuration. The energies W_n thus defined are

$$W_n=\omega_n-4\left(\frac{7}{2}\hbar\omega_0\right), \tag{4·66}$$

where ω_n are obtained by solving the eigenvalue equation (4·64). In order that we bring the calculated value W_n for $L=0^+$ close to the observed binding energy 31.38 MeV of the ground state of ^{20}Ne with appropriate choice of the parameters V_0 and λ, we must assume a larger value of $\hbar\omega_0$ than 15.0 MeV chosen in the TD calculation in § 3.4; otherwise the calculated value W_n becomes complex beforce gaining the satisfactory binding energy. This is the reason why we assumed $\hbar\omega_0=17.0$ MeV in the present NTD calculation.

Calculated energies for various values of size parameter α^2 are shown in

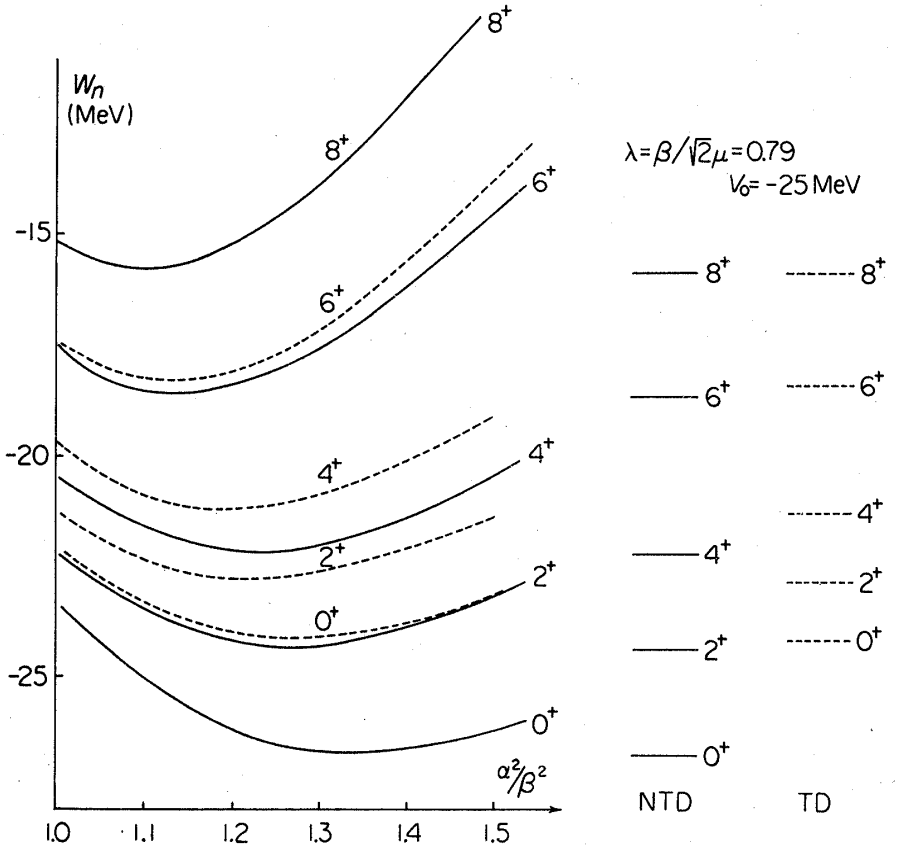

Fig. 12. Level structure of ^{20}Ne calculated in the NTD approximation (solid lines). Eigenenergies W_n are plotted against the "size" parameter α^2/β^2. The energy spectrum consisting of the lowest value for each curve is shown on the right. Dashed lines represent the TD calculations for the same interaction parameters as those of the NTD calculations.

Fig. 12 assuming $\lambda=0.79$ and $V_0=-25$MeV. An energy spectrum consisting of the lowest value of each curve is put on the right of the curves together with that of the TD calculation. Comparing these two spectra with each other, we can see that the NTD calculation gives about 3 MeV greater binding energy than the TD calculation for the same interaction parameters. It can also be seen in Fig. 12 that, the lower the state comes from 8^+ to 0^+, the more ground-state correlations are involved in it. The states 6^+ and 8^+ are hardly affected by the ground-state correlations, but the 0^+, 2^+ and 4^+ states are considerably influenced. Therefore, it is supposed that there is a wide difference in property of the state vectors between these two groups of states. In addition, we can find the following characteristic feature in Fig. 12: The values of the size parameter α^2 at the lowest point of each energy curve are greater for the NTD calculation than for the TD calculation. This fact implies that the ground-state correlations enhances the spatial correlations.

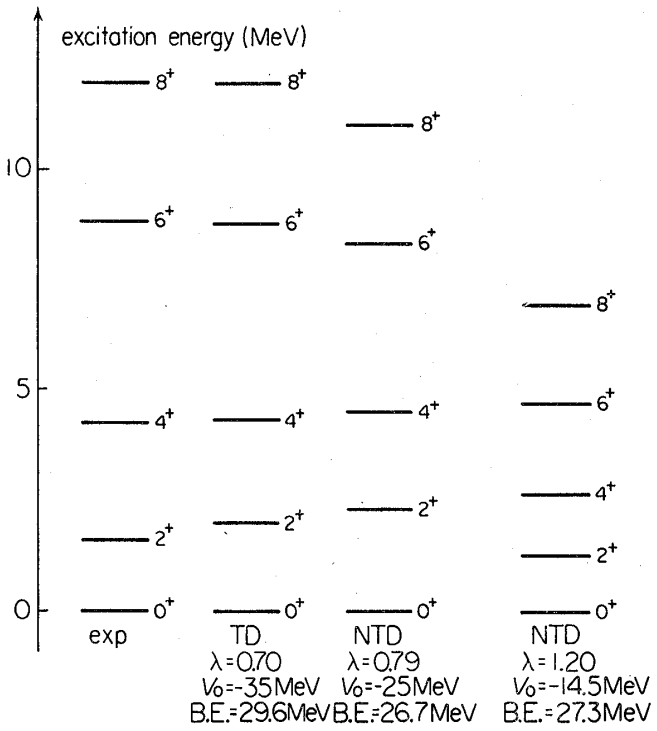

Fig. 13. Energy spectra obtained by the TD and the NTD calculations are compared with the experimental one.

In Fig. 13, energy spectra for different values of the interaction-range parameter are shown together with the observed spectrum of the ground band in ^{20}Ne. We obtained a better fit of spectrum with $\lambda=0.79$ for the NTD calculation.

It is more interesting to investigate how the ground-state correlations affect the other physical values than the energy spectrum or the binding

energy; the other physical values mean, for example, the $E2$-transition probability and the effective charge. It is considered that, in such physical value as $B(E2)$, the difference in the property of the state vectors between the group of the 0^+, 2^+ and 4^+ states and the group of the 6^+ and 8^+ states appears explicitly. This is just a direct reflection of the ground-state correlations due to the alpha-like four-body correlations.

4.5 First excited 0^+ states in ^{16}O and ^{40}Ca and discussion on the mode-mode coupling

As is well-known, one of the puzzling properties of light closed-shell nuclei such as ^{16}O and ^{40}Ca is that they display particularly low-lying first excited 0^+ states; their excitation energies are 6.05 MeV for ^{16}O and 3.35 MeV for ^{40}Ca. According to the standpoint of our picture, the appearance of these mysterious 0^+ states may be understood as follows: Let us assume that the ground state of the closed-shell nucleus under consideration is "normal", but suppose that the alpha-like four-body correlations force the normal state to be very near to a corresponding "phase transition", (to the bound alpha-like cluster), discussed in §4.3. Then we may expect a corresponding collective mode of the closed-shell nucleus to have very low excitation energy.

The corresponding collective excited state of the closed-shell nucleus may, as the first-order approximation, be considered to be of the form

$$\alpha_0^\dagger \beta_0^\dagger |\Phi_0\rangle \text{ (the correlated } 4p\text{-}4h \text{ state)}, \tag{4·67}$$

where $\alpha_0^\dagger$ is the creation operator of the alpha-like four-body mode with the lowest eigenvalue ω_{0+} and $\beta_0^\dagger$ is the creation operator of the correlated four-hole mode with the highest eigenvalue ω_{0-}. Here the state $|\Phi_0\rangle$ is the true ground state of the closed-shell nucleus under consideration, i.e., $|\Phi_0\rangle = |A_0 = 16; 0\rangle$ for ^{16}O or $|\Phi_0\rangle = |A_0 = 40; 0\rangle$ for ^{40}Ca. From Eq. (2·64), the following relations must be satisfied;

$$\alpha_0 |\Phi_0\rangle = \beta_0 |\Phi_0\rangle = 0. \tag{4·68}$$

In the first-order approximation, the excitation energy of the state (4·67) is given by

$$W_0 = \omega_{0+} - \omega_{0-}, \tag{4·69}$$

which becomes very small in the neighbourhood of the corresponding "phase transition". We can estimate the value W_0 from the empirical binding energies of neighbouring nuclei of closed-shell one in the following way. For the $4p$-$4h$ states of ^{16}O and ^{40}Ca, we have

$$W_0=\omega_{0+}-\omega_{0-}$$
$$\sim\begin{cases}2B(^{16}\mathrm{O})-\{B(^{20}\mathrm{Ne})+B(^{12}\mathrm{C})\}=2.431\ \mathrm{MeV} & \text{for } ^{16}\mathrm{O},\\ 2B(^{40}\mathrm{Ca})-\{B(^{44}\mathrm{Ti})+B(^{36}\mathrm{Ar})\}=1.806\ \mathrm{MeV} & \text{for } ^{40}\mathrm{Ca},\end{cases} \tag{4·70}$$

where, for instance, $B(^{16}\mathrm{O})$ means the binding energy of $^{16}\mathrm{O}$. Similarly, we can estimate the excitation energies of the other np-nh states of $^{16}\mathrm{O}$ and $^{40}\mathrm{Ca}$. From the special smallness of the excitation energy of the $4p$-$4h$ state, one of the authors (T. M.) and Suzuki[2] concluded that the four-body correlations is of essential characteristic for the light, closed-shell nuclei and the "phase transition" from the normal ground state to the bound alpha-like cluster is going to come into existence.

In order to investigate the detailed structure of the mysterious 0^+ state or the other $4p$-$4h$ states, we should take account of the particle-hole interaction H_{ph} which has been neglected in process of construction of the four-body modes $\alpha_n^\dagger$ and $\beta_n^\dagger$. This is done by diagonalizing the particle-hole interaction H_{ph} with the use of the linearized relation

$$[H,\ \alpha_m^\dagger\beta_n^\dagger]=\sum_{kl}(M_{mnkl}\alpha_k^\dagger\beta_l^\dagger+N_{mnkl}\alpha_k\beta_l). \tag{4·71}$$

Here, matrices M and N mean the couplings between the four-body eigenmodes and are determined by taking the appropriate matrix elements of Eq. (4·71) with respect to the ground state $|\Phi_0\rangle$ and $4p$-$4h$ states $\alpha_m^\dagger\beta_n^\dagger|\Phi_0\rangle$. It seems to be not so easy to evaluate the matrices M and N, because it is rather difficult to determine the true ground state $|\Phi_0\rangle$. A thinkable way to determine the matrices without using the detailed structure of $|\Phi_0\rangle$ is to expand the Hamiltonian H by means of the four-body modes $\alpha_n^\dagger(\alpha_n)$ and $\beta_n^\dagger(\beta_n)$ assuming them as boson-operators on the basis of their quasi-boson character (4·9). Then we can define the creation operators of eigenmodes $Y_\lambda^\dagger$ (i.e., phonons composed of correlated $4p$-$4h$) for positive-parity excited states in $^{16}\mathrm{O}$ and $^{40}\mathrm{Ca}$ as solutions of the equation

$$[H,\ Y_\lambda^\dagger]=W_\lambda Y_\lambda^\dagger, \tag{4·72}$$

where

$$Y_\lambda^\dagger=\sum_{mn}\{\xi_\lambda(mn)\alpha_m^\dagger\beta_n^\dagger+\eta_\lambda(mn)\alpha_m\beta_n\} \tag{4·73}$$

with $\lambda\equiv\{L, IM, TT_z\}$. Here λ denotes the set of quantum numbers composed of the angular momentum I and its projection M, the isospin T and its projection T_z and a set of additional quantum numbers L. The new ground state $|\Psi_0\rangle$ of $^{16}\mathrm{O}$ or $^{40}\mathrm{Ca}$ defined by $Y_\lambda|\Psi_0\rangle=0$ contain the ground-state correlations not only due to the four-body correlations (coming from H_{pp}, H_{hh} and H_V in Fig. 10) but also due to the particle-hole interaction H_{ph}.

Thus the first excited 0^+ (correlated $4p$-$4h$) states in $^{16}\mathrm{O}$ and $^{40}\mathrm{Ca}$ are given by

$$Y^{\dagger}_{L_0,I=0,T=0}|\Psi_0\rangle, \tag{4·74}$$

which clearly has an intrinsic deformation produced by the four-body correlations and the particle-hole interaction. The correlated $4p$-$4h$ states

$$Y^{\dagger}_{L_0,I,T=0}|\Psi_0\rangle\ , \qquad I^{\pi}=0^+, 2^+, 4^+, \cdots \tag{4·75}$$

form a "rotational" band built on the first excited 0^+ state (4·74).

In the scheme mentioned above, we have employed the two-step method; the first step is the construction of the alpha-like four-body eigenmodes and the second step is the diagonalization of the mode-mode coupling due to the particle-hole interaction within the subspace defined by the eigenmodes. The philosophy of this two-step method is very similar to the weak coupling model proposed by Arima, Horiuchi and Sebe.[4] Along our scheme, we are now in process of obtaining and diagonalizing the mode-mode coupling matrices M and N in Eq. (4·71).

§5. Concluding remarks

In this chapter, we aimed at construction of a unified microscopic theory in which the shell-model aspect and the alpha-cluster aspect in the light nuclei are combined. For this purpose, we adopted the picture: Although there exist very strong alpha-like four-body correlations which are against the stability of the Hartree-Fock field, the field is still stable in the neighbourhood of the ground states to realize the shell structure. In addition, we discussed that, if the strength of the four-body correlations increases and passes through a critical value, the ground-state wave function changes suddenly and there happens a "phase transition" from the shell-model "phase" to the alpha-cluster "phase".

According to the various investigations developed in the present chapter, it has become clear that our picture stated above is considerably reasonable and plausible. However, our work on such microscopic description based on the picture has just been started and there are many open problems being left:

(1) Importance of the alpha-like spatial correlations and the ground-state correlations has been clarified through the analyses in §§3 and 4. Relation between these two kinds of correlations is discussed a little in §4, but not enough. This is an important future problem.

(2) In §4.3, we discussed a new "phase transition" from the shell-model "phase" to the alpha-cluster "phase" and studied the property of the new ground-state wave function after the "phase transition". On the other hand, the Bloch-Brink type of model wave function is often used in phenomenological analyses. Then it seems to be very important to investigate the relation

between the two types of wave functions.

(3) Qualitative discussion on the first excited 0^+ states in ^{16}O and ^{40}Ca, which has been a motive of our picture, was developed in §4.5, but quantitative analysis has not yet been done. Particularly serious problem in this analysis exists in the interaction between the various alpha-like four-body modes. As stated in §4.5, we adopted the two-step method. The serious problem mentioned above is in the second step; that is, the problem is how we express the residual interaction (the particle-hole and the Y-type interactions) not included in construction of the alpha-like eigenmodes in the form of mode-mode coupling.

(4) Our theory in the present chapter has been developed only for the ground and low-excited states. However, as discussed in the previous chapters, new "phases" in atomic nucleus, (for example the molecular-structure "phase"), appear in the higher-excited states. Therefore, we must generalize our microscopic theory to be applicable to such states. One of the typical example of such generalization is the scattering problem of α-particle by doubly closed-shell nucleus. Our theory of the alpha-like four-body mode seems to be applicable to this problem as it is. An example is the calculation of the 6^+ and 8^+ resonance states in ^{20}Ne represented in §3.6. However, in order to generalize our theory to such problems strictly, we should solve the so-called problem of centre-of-mass motion.

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