Chapter VII

Nuclear Forces in Nuclei and Alpha-Clusterization

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Contents

§1. Introduction

Nuclear forces have many characteristics,[1] such as a singular short range core, strong non-central part and strong state-dependence and so on. These characteristics of realistic nuclear forces and finite nature of the nuclear

system under the Fermi statistics bring about plentiful nuclear phenomena and existence of different "phases". In the course of the development of the nuclear theory a specific aspect in the nuclear phenomena will be understood step by step as a realization of interplay of some characteristics of realistic nuclear forces and many-body correlations. Although sufficient recognition of this complicated interrelation is still difficult at the present stage in the wide region of nuclear phenomena, it seems that we are able to make a step of this way further in the two regions of nuclei, light nuclei and infinite nuclear matter. In the former region the many-body effects seem to play less important role and it is rather easy to analyze the problems starting from the realistic nuclear forces. In the latter case the nuclear structure is idealized in the "model" with some essential features of the real nuclei and the treatment of the state can be simplified.

Investigations have been performed in Japan from the veiwpoint of considering the two regions in connection.[2] The saturation problem is a typical subject which is closely related to these two regions. The clustering features or molecular aspects of the nucleus are inherently related to the saturation property of nuclei. Therefore, it is very interesting and promising to investigate the clustering phenomena on the basis of the realistic nuclear forces at the present stage.*)

In the preceding chapters it has been discussed that the states with molecule-like structures are rather widely observed near and above the threshold energies and also in some cases the clustering features are found even in the neighbourhood of the ground states. The saturation property makes two "phases" comparable in energy: the one where all nucleons are combined in one-center field and the other where the nucleons are divided into subunit nuclei. We find, as the consequence, the states with cluster structure at rather low excitation energies. Actually the alpha-clusterization is first observed, because the alpha-particle is a particularly stable unit among very light nuclei which has almost the same binding energy per particle as that in heavier nuclei. Thus, the mechanism of the alpha-clusterization of a nucleus should be closely related to the mechanism which assures the saturation overall nuclei including the alpha-particle and heavier nuclei. The saturation property of nuclei depends strongly on the characteristics of the nuclear forces. It is, therefore, particularly important to understand the alpha-clusterization in terms of the characteristics of the realistic nuclear forces.

At present the saturation problem has not yet been solved finally. There still remain some quantitative ambiguities in the treatment of the many-body problem and also uncertainties in the two-body nuclear forces, especially concerning the strength of the tensor force in the triplet-even state

*) The contents of this chapter are based on the works in Refs. 3) and 4).

and the behavior of the repulsive core.[1),5)] A potential with a large hard core and a strong tensor force gives less binding energy. For example, Hamada-Johnston (H-J) potential[6)] gives only about half of the binding energy of nuclear matter[7)] and more than 5 MeV/particle remains deficient in the present stage of the theory even if the contribution of a three-body force[8)] is taken into account. On the other hand the same potential H-J is found to give good results for the three-body problem.[3),9),10)] It is, therefore, necessary and important to investigate the three- and four-body problems (^{3}H, ^{3}He and ^{4}He) themselves, in order to develop the nuclear matter theory and also to clarify the mechanism of the variation of the binding energies overall nuclei from very light nuclei to heavier ones including nuclear matter as a limit. We call it the "overall saturation" mechanism. To understand the binding mechanism of alpha-particle is a basic subject for the problem of the alpha-clusterization. This is our first subject in this chapter and treated in §§ 2 and 3.

In spite of the above-mentioned situation many investigations[11)] up to the present have derived several important conclusions about the saturation mechanism based on the realistic nuclear forces. We summarize them briefly as follows:

1) Main contribution to the binding energy comes from the two-body correlation within nuclei and so the Independent-Pair Model[12)] is good starting basis on which nuclei are investigated with the realistic nuclear forces. This is true even for very light nuclei such as triton and alpha-particle,[3)] as is shown in the following sections.

2) Most of the contribution arises from the relative S-states containing the renormalization by the tensor force in the triplet-even state. Those from the P- and D-states are small and cancel each other.[11)]

3) The obtained binding energies depend considerably on the magnitude of the tensor-to-central (v_T/v_C) ratio in the triple-even state[11),13)] and the behavior of the repulsive core[11)] of the adopted nuclear forces.

4) The triplet-even tensor force plays a special role in realizing the "overall saturation".[3)] Because of the diminution of the tensor force effect caused by the many-body effect, the Pauli principle and the effect dependent on starting energy, the effective tow-body interactions in nuclei become less attractive from lighter to heavier nuclei and also from outer to inner region of a nucleus. Without this effect we cannot obtain the "overall saturation".[14)]

In order to see the above four points let us show in Table I the calculated results[11)] for nuclear matter. The potentials, H-J, OBEP-K[15)] and OPEG,[5)] are three typical types of the potentials, as written in the footnotes of Table I. For either potential the points 1) and 2) are seen to be justified. The results for the three potentials show the dependence on the

Table I. Two-body contributions to the binding energy per particle of nuclear matter in the cases of three types of potentials and also of the phase-shift approximation.

	H-J	OBEP-K	OPEG	phase shift
^1S	−15.9	−16.9	−17.2	−23.3
^3S	−15.8	−23.3	−19.1	−44.7
^1P	3.2	4.2	3.8	2.9
^3P	0.3	0.6	0.8	− 4.9
^1D	− 2.2	− 2.3	− 2.2	− 3.1
^3D	− 1.3	− 1.6	− 1.7	− 4.0
U	−31.7	−39.3	−35.6	−77.1
T	23.9			
E	− 7.8	−15.4	−11.7	−53.2

Characteristics of the three potentials: $r_0=1.1$ fm. (MeV)
H-J: large hard core radius and large $v_T/v_C(^3E)$ ratio,
OBEP-K: small hard core radius and small $v_T/v_C(^3E)$ ratio,
OPEG: soft core and medium $v_T/v_C(^3E)$ ratio.

v_T/v_c ratio and the behavior of the repulsive core, as mentioned in the point 3). The fourth column is the results obtained by replacing the reaction matrix elements by the phase shifts of the free nucleon-nucleon scattering. It shows far over-bound energy. Comparing the fourth column with the other ones, we find that the nuclear forces act less effectively in nuclear matter than in the free two-body scattering and the most striking reduction appears in the $^3S_1$-state contribution. The latter fact just arises from the diminution of the strong tensor force effect due to the many-body effects. The tensor force is most sentitive to the surrounding situation.

The $^3S_1$-contribution is almost twice the $^1S_0$-one in the phase shift approximation, while they are almost comparable in nuclear matter. In the alpha-particle the former is about 1.5 times the latter.[3] The effective interaction within the alpha-particle is more attractive than within larger nuclei, since the many-body effects are still not so strong because of the only four constituent nucleons.

Once the saturation property was ascribed to some exchange characters of the nuclear forces which give strong repulsion in the odd state.[16] After the nuclear forces were found to be weak in the odd state in low energies, it has been confirmed that collapse of nuclei is prevented by the short-range repulsive core. Now it should be emphasized that it is the tensor force that assures the nuclear saturation over all nuclei, from the very light nuclei to the infinite nuclear matter.

Now, our second subject of this chapter is to attempt to relate the problem of the alpha-clusterization to the characteristics of the realistic

nuclear forces, with these achievements on the saturation problem at the present stage as background. As shown in Chapter II, "externally weak interaction" is mainly brought about by the Pauli exchange between nucleons in two clusters. This weak interaction is represented effectively by a short-range repulsive core. There also exists a weak attractive interaction outside this repulsion and these two parts form a weak interaction between clusters. Thus the Pauli exchange and a characteristic of the nuclear force, which is short-range attractive in S-states and weak in P-states, are essential to realize "internally strong but externally weak" nature characterizing the alpha-cluster structure.

In addition to this we stress here an important role of the strong tensor force in the triplet-even state which is another characteristic of the nuclear forces. When a nucleus changes from a state with shell structure to a state with cluster-structure, change of the effective two-body interaction itself caused mainly by the tensor force is expected to appear.[14] It can be considerd that some additional attractive part is induced[4] in the effective two-body interaction by clusterization due to a change of the many-body effects. And this induced interaction (clustering-induced interaction) will act to enhance the clusterization itself. Such interdependence between the effective interaction of constituent nucleons and the structure of the nucleus will be a characteristic feature of the nuclear system which is self-sustained by the nuclear forces with strong state-dependence. In §§ 4 and 5 we treat this problem by applying the reaction matrix method on a cluster model.

Here we make a comment on the phenomenological effective interactions, for example, Volkov forces[17] which are often used for the model calculations. They have the repulsive potentials in the odd states which are unrealistically strong in comparison with the properties of the realistic nuclear forces. In calculations of the cluster model they are inevitably introduced in order to assure the saturation property, though only for few members of nuclei, by use of such simple interactions. According to the various calculations the strength of the odd state repulsion influences directly the degree of the clusterization of a nucleus. It should be, therefore, examined severely on the basis of the realistic nuclear forces whether the strong odd state repulsion is justified by some physical reason. In this respect the peculiar role of the tensor force is considered to be a real fact for which the strong odd state repulsion can be substituted apparently within some limited features.

§2. Application of reaction matrix theory to very light nuclei

As mentioned in §1 the clustering phenomena in nuclei are closely related to the saturation property of nuclei. However, states with arbitrary division into clusters do not appear near the ground state, but only some particular clusters are realized. In the region of light nuclei there appear

an alpha-cluster and n alpha-clusters and this fact comes from the especially stable property of the alpha-particle (^{4}He).

The ^{4}He is the lightest one of the saturating systems and its binding energy per particle is not so large in comparison with the average one. On the other hand, when the ^{4}He is compared with neighbouring nuclei, its binding energy per particle is especially large.

Thus our task in this part is to investigate the binding mechanism of the ^{4}He in relation to the saturation property over all nuclei and also to make clear the mechanism of the extra gain of its binding energy relative to neighbouring nuclei. This will surve the understanding of the "internally strong" property, which is one of characteristics of the clustering phenomena.

The saturation problem has not yet been solved completely, though it is recognized that the most part of the binding energy comes from the two-body scattering correlations, as shown in §1. It is very interesting to notice that the two-body nuclear potential, which reproduces well not only the two-body data but also the properties of the three-body bound system,[9),10)] does not necessarily bring about the binding energy of nuclear matter fully.[7)] In this context the investigation of the binding mechanism of the ^{4}He is important.

We apply the reaction matrix theory to the lightest nuclei, the ^{4}He and the ^{3}H, because we must treat properly the short range and singular core and the strong tensor component in the realistic nuclear forces and the Pauli exclusion in the scattering process, and the two-body scattering correlation is expected to play an important role in light nuclei as well as in nuclear matter. On the basis of the independent-pair model and our improved treatment,[3)] a model Hamiltonian is derived and some higher order correlations are estimated.

There are some points to be improved in application of the reaction matrix theory to light nuclei. First of all, the shell model is not necessarily good for light nuclei and the elimination of the center-of-mass (c. m.) motion of a whole nucleus becomes important. Applicability of the reaction matrix theory does not necessarily need the goodness of the independent-particle model but depends essentially on dominance of the independent-pair scattering correlation and on good convergence of the linked-cluster expansion, which are determined by a well-healing behavior of the two-body scattering wave function. In our improved treatment we start with the internal model Hamiltonian which does not contain the c. m. coordinate, and only the relative coordinate between two nucleons is treated in the reaction-matrix equation separated from the other motion. The energy denominator in the two-nucleon scattering process is, however, not reduced to a two-body operator but must be treated carefully as a difference between the total energies of the ground and intermediate states. One of the difficulties is how

to take the spectrum off the energy shell. We give up the concept of the single-particle potential for particle states and use the hole-line expansion method,[11l),11m)] where the so-called potential diagrams inserted in particle lines and A^{-1} terms are treated in each scattering diagram as a whole. It is unnatural, however, that no single-particle potential is taken into account for the particle states just above the Fermi surface. We investigate also an effect of the potential insertion in such particle states by use of the reaction matrix off the energy shell. Finally, in finite nuclei a difficulty arises in use of the self-consistent model wave function. We use a simple harmonic oscillator (h. o.) wave function, but it will be confirmed by a variational method with a superposed h. o. function that it is a rather good one.

2. 1 Formulation

We start with the internal Hamiltonian which does not contain the c. m. motion of the whole nucleus.

$$H=T+\sum_{i<j}v_{ij}, \tag{2·1}$$

where T stands for the internal kinetic energy only and contains three independent internal coordinates. v_{ij} is a two-body potential. In this part we omit the Coulomb potential.

The eigenfunction of H is given approximately by Ψ, which is connected with the model wave function Φ by the multiple scattering operator F:

$$\begin{aligned} &H\Psi\simeq E\Psi, \\ &\Psi=F\Phi, \end{aligned} \tag{2·2}$$

where F is defined by two equations

$$\begin{aligned} F&=1+\sum_{i<j}\frac{Q_{ij}}{e}G_{ij}F_{ij}, \\ F_{ij}&=1+\sum_{k<l(kl\neq ij)}\frac{Q_{kl}}{e}G_{kl}F_{kl}. \end{aligned} \tag{2·3}$$

The reaction matrix G is the solution of the equation

$$G_{ij}=v_{ij}+v_{ij}\frac{Q_{ij}}{e}G_{ij}, \tag{2·4}$$

where Q_{ij} is the Pauli projection operator.

Following the hole-line-expansion method,[11l),11m)] developed in the case of nuclear matter, we rewrite the left-hand side of Eq. (2·2) as follows:

$$\begin{aligned} H\Psi&=HF\Phi=\{T+\sum_{i<j}\bar{G}_{ij}(1-Q_{ij})+e\}F\Phi-e\Phi+(w_1+w_2)\Phi, \\ w_1&=\sum_{i<j}(1-Q_{ij})I_{ij}F_{ij}, \\ w_2&=-\sum_{i<j}Q_{ij}\bar{G}_{ij}(1-Q_{ij})F_{ij}, \end{aligned} \tag{2·5}$$

where I_{ij} is defined by

$$I_{ij}=G_{ij}-\bar{G}_{ij}(1-Q_{ij}). \tag{2·6}$$

The operator $\bar{G}_{ij}$ is defined so as to comprise the diagonal part of G_{ij} and the non-diagonal part through which the total potential operator $\sum\bar{G}(1-Q)$ comprises the non-diagonal elements between the model ground state and the states connected with it by the kinetic energy operator T.

We define a model Hamiltonian H_{M} and an energy denominator e as follows:

$$H_{\mathrm{M}}=T+\sum_{i<j}G_{ij}(1-Q_{ij}), \tag{2·7}$$

$$e=E_{\mathrm{M}}-H_{\mathrm{M}}, \qquad H_{\mathrm{M}}\Phi=E_{\mathrm{M}}\Phi. \tag{2·8}$$

The projection of Eq. (2·5) on the model wave function is obtained as

$$\langle\Phi, H\Psi\rangle=\langle\Phi, H_{\mathrm{M}}\Phi\rangle+\langle\Phi, (w_1+w_2)\Phi\rangle. \tag{2·9}$$

Using Eqs. (2·2), (2·3) and (2·8), we obtain the internal energy E as follows:

$$E\simeq E_{\mathrm{M}}+\langle\Phi, (w_1+w_2)\Phi\rangle. \tag{2·10}$$

If we can show that the last term is small, the energy E is given by E_{M} in a good approximation. The expectation value of w_2 vanishes for Φ because of Q operators. The terms in w_1 with a definite number of hole lines construct the whole scattering diagrams for a definite number of particles. They will be estimated to be small in the following sections.

Generally the true wave function and its energy are given by Goldstone's linked cluster expansion,[18] as long as the expansion series in terms of the scattering diagrams regrouped for each number of hole lines are converged, and they are connected with the unperturbed wave functions, i. e., the model wave function, and its energy adiabatically. One of our problems in §§2 and 3 is to show that this expansion with above defined H_{M} and Φ converges rapidly.

2. 2 Independent pair approximation

In the following subsections we show an application of our formalism to the case of the ^{4}He. We take a s-state function of a simple harmonic oscillator, as an approximation, for the spatial part of the model state wave function:

$$\varphi(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})=N\exp[-\frac{1}{2}\nu\sum_{i=1}^{4}(\boldsymbol{r}_i-\boldsymbol{R}_\alpha)^2]=N\exp[-\frac{1}{2}\nu(u^2+\frac{1}{2}v^2+\frac{1}{2}w^2)]$$

$$\equiv\varphi(u)\varphi(v)\varphi(w), \tag{2·11}$$

where $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$ form a set of the internal coordinate shown in Fig. 1. It is noted that the wave function and the kinetic energy of the relative motion of a pair nucleons can be factorized from the remainder.

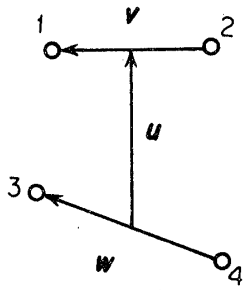

Fig. 1. Internal coordinate system in ^{4}He.

The reaction matrix equation for the pair of particles 3 and 4 (we denote that pair by α) is given as follows:

$$G_\alpha\varphi(w) = (v_\alpha + v_\alpha \frac{Q_\alpha}{e_\alpha} G_\alpha)\varphi(w),$$
$$Q_\alpha = 1 - |\varphi(w)\rangle\langle\varphi(w)|, \qquad (2\cdot12)$$

where spin and iso-spin parts are abbreviated. The energy denominator e_α is a difference between the total energy of the initial state and that of an intermediate state, in which the pair α is in an excited state $\varphi'(w)$ and the total wave function is $\varphi(u)\varphi(v)\varphi'(w)$. The expectation value of the kinetic energy in that intermediate state is given simply by $\langle T_u\rangle + \langle T_v\rangle + \langle T_w\rangle'$. On the other hand, that of the interaction energy is not so simple. The pair α and the pair $\tilde{\alpha}$ which is the pair of the particles 1 and 2 in Fig. 1, have the factorized wave functions $\varphi'(w)$ and $\varphi(v)$, respectively, but the other pairs (called β's) do not have factorized ones, even if the coordinate system is recomposed. Taking into account the Pauli projection operator Q_{ij}, the expectation values of the energy denominator in an intermediate state are obtained as follows:

$$\langle e_\alpha\rangle' = \{\langle T_w\rangle + \langle\bar{G}_\alpha\rangle + \sum_{\beta}^{4\ \text{pairs}} \langle\bar{G}_\beta\rangle\} - \langle T_\omega\rangle', \qquad (2\cdot13)$$

where the expectation values of $G_\beta(1-Q_\beta)$ in the intermediate state are approximately taken to be zero, since they are off-energy shell and also the wave function of the pair β imply partially excited components. With respect to this matter we shall investigate in another approximation of a particle state potential afterwards.

Here it is noted that, as seen from Eq. (2·13), the interaction energy of the initial state contains effectively only five pairs, not six, and so is not equal to $2V_0$, where V_0 is a single particle potential energy. This is an effect of the A^{-1} term in the energy denominator.

In compensation for the above-mentioned ambiguity in the energy denominator, which may bring only a small error to the total energy, the Pauli projection operator Q_α is related only to the relative motion of the pair and, therefore, we are free from a troublesome estimation of one particle jump excitation and from an ambiguous treatment of the c. m.

motion of the pair in the intermediate state, which one encounters in the shell model treatment.

After all, the reaction matrix equation to be solved is obtained in the following form:

$$G_\alpha = v_\alpha + v_\alpha \frac{Q_\alpha}{e_\alpha'} G_\alpha,$$
$$e_\alpha' = \langle T_w \rangle + \sum_{\text{all pairs}} \langle \bar{G} \rangle - \langle \bar{G}_{\tilde{\alpha}} \rangle - T_w. \tag{2·14}$$

The energy by the two-body scattering correlation is given by

$$E_M = \sum_{u,v,w} \langle T \rangle + \sum_{\text{all pairs}} \langle \bar{G} \rangle. \tag{2·15}$$

2. 3 Solution of reaction matrix equation

We solve the reaction matrix equation (2·14) self-consistently with respect to the energy denominator and the reaction matrix. In order to treat Pauli operator simply we divide the calculation into two steps where, first, we solve the equation

$${}^0G_\alpha = v_\alpha + v_\alpha \frac{1}{e_\alpha'} {}^0G_\alpha \tag{2·16}$$

and, afterwards, solve the equation

$$G_\alpha = {}^0G_\alpha + {}^0G_\alpha \left(\frac{Q_\alpha}{e_\alpha'} - \frac{1}{e_\alpha'} \right) G_\alpha \tag{2·17}$$

by the matrix method[19] with an approximation mentioned afterwards. The energy denominator is approximated as follows:

$$\bar{e}_\alpha' = -\frac{\hbar^2}{M} \gamma^2 - T_w, \tag{2·18}$$

where

$$-\frac{\hbar^2}{M} \gamma^2 = \langle T_w \rangle + \frac{5}{2} \{ \langle G(^3S_1) \rangle + \langle G(^1S_0) \rangle \}, \tag{2·19}$$

which is an average value for $\langle T_w \rangle + \sum_{\text{all pairs}} \langle \bar{G} \rangle - \langle \bar{G}_{\tilde{\alpha}} \rangle$. This average may not affect the final result of the total potential energy, since it is the sum of the reaction matrices of all pairs. On the other hand it will be discussed that the specific reaction matrix is greatly effected by an appreciable change of the value of $-\hbar^2/M\,\gamma^2$, which is called the "starting energy".

Since the operator $Q_\alpha - 1$ implies only one level, i. e., the initial state, after obtaining the matrix elements of ${}^0G$, the matrix elements of G for the $^3S_1$ and $^1S_0$ states can be easily obtained by solving the coupled equations algebraically

$$\langle G_\alpha \rangle = \langle {}^0G_\alpha \rangle - \frac{\langle {}^0G_\alpha \rangle}{\langle e_\alpha' \rangle} \langle G_\alpha \rangle, \tag{2·20}$$

where

$$\langle e_\alpha' \rangle = \langle \{ \langle T_w \rangle + \frac{5}{3} V_0 - T_w \} \rangle = \frac{5}{2} \{ \langle G({}^3S_1) \rangle + \langle G({}^1S_0) \rangle \}.$$

Here we treated the equation $G_\alpha = {}^0G_\alpha + {}^0G_\alpha (Q_\alpha - 1)(1/e_\alpha')(Q_\alpha - 1) G_\alpha$, which is different from Eq. (2·17), strictly speaking. The final errors may be estimated by the relation to be satisfied by the overlap integral $(\varphi \cdot \psi) = 1$. Our calculated results show these errors less than 2 %.

Thus the matrix element of G is simply expressed by

$$\langle G_\alpha \rangle = {}^\alpha C \langle {}^0G_\alpha \rangle, \tag{2·21}$$

and the scattering wave functions under this approximation are also expressed with use of the same constant as follows:

$$\psi_\alpha = {}^\alpha C \varphi_\alpha. \tag{2·22}$$

2.4 Results

We use five types of the nuclear potentials, H-J, OPEH,[5] OPEG, Reid-SC[20] and OBEP-K. In the calculations of the binding energy of nuclear matter these potentials can be divided into three groups which give results different from each other, i.e., the binding energy by H-J is about 8 MeV/particle without higher correlations, those by OPEH, OPEG and Reid-SC are abut 10~11 MeV/particle and that by OBEP-K is about 15 MeV/particle.

It has been shown that the contributions of higher correlations and a three-body force amount to about 5 MeV/particle and the binding energy of nuclear matter becomes 16 MeV/particle[21] in the case of Reid SC. On the other hand in the case of H-J the higher correlation (three-particle scattering correlation) reduces the binding energy owing to its large hard core in contrast to the case of soft core potentials. Thus it is very interesting to investigate in the lightest nuclei the effects of the repulsive core and of the tensor force in the triplet-even state, using the potentials with different strength of them which give the equivalent results in the two-body problem.

The total energies obtained self-consistently at each value of ν for the ^{4}He and the ^{3}H are shown in Figs. 2(a) and 2(b), respectively. The minimum energies of the ^{4}He (^{3}H) are -19.5 MeV (-6.3 MeV) at $\nu = 0.40$ fm^{-2} (0.32 fm^{-2}) for H-J, and about $-21.5 \sim -23.5$ MeV ($-6.5 \sim -7.0$ MeV) at $\nu = 0.45 \sim 0.48$ fm^{-2} (0.32~0.36 fm^{-2}) for other potentials. Even in the case of H-J for the ^{4}He the interaction energy of the two-body scattering correlation amounts to -56.8 MeV and is the most part of the whole interaction energy. And the lack of interaction energy by about -10 MeV is about the same, or even smaller, rate as that in nuclear matter.

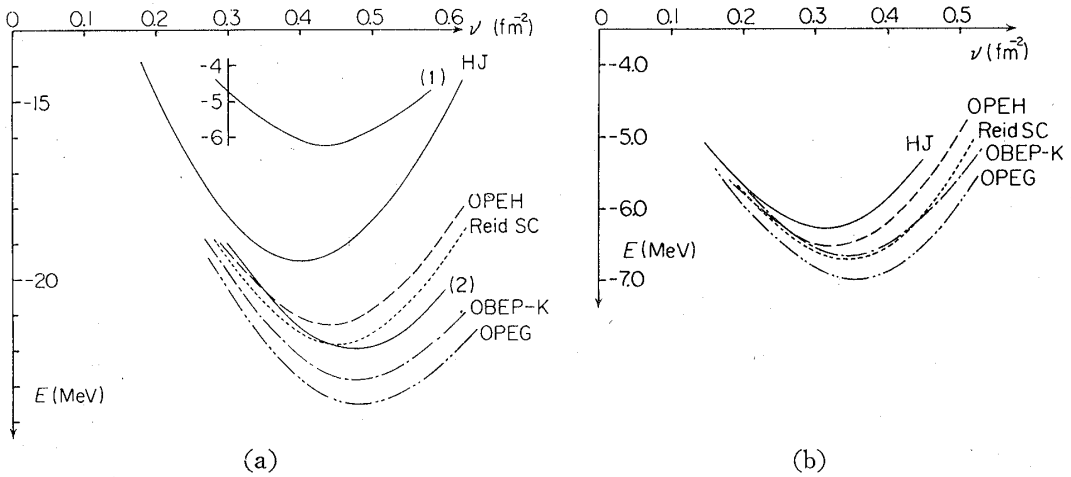

Fig. 2. Total energies for various two-body nuclear potentials versus harmonic oscillator strength ν; (a) ^{4}He: The solid line (1) shows the result by using the effective potential obtained from the nuclear matter calculation, which is discussed in §3.2. The solid line (2) shows the result of the OPEG by the method used in §4. (b) ^{3}H.

The energy of the ^{3}H is further well reproduced by the two-body scattering correlation almost irrespective of potentials used. It is known by the variational calculation[9),10)] that H-J gives a binding energy of 6.5 MeV for the ^{3}H, which is a good result if we take into account an energy gain of about 1.4 MeV by the three-body force,[22)] and OPEH may give a slightly over-

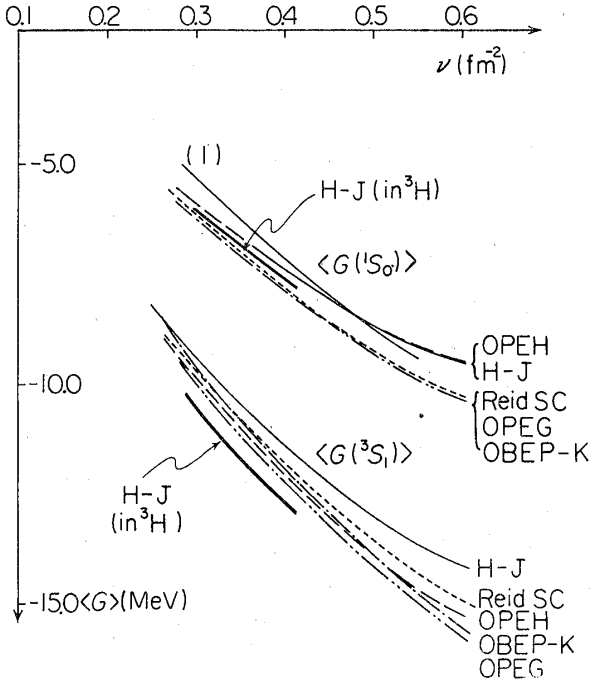

Fig. 3. Reaction matrix elements $\langle G(^1S_0)\rangle$ and $\langle G(^3S_1)\rangle$ for various two-body potentials in the case of ^{4}He and for H-J in the case of ^{3}H. The solid line (1) shows the result with the potential explained in Fig. 2, in which the same parameters are used for the $^3S_1$- and $^1S_0$-states.

bound energy.[23] If we take the results by OPEH as a criterion of giving an overbound energy or less one in a variational calculation for the lightest nuclei, we may say that OPEG and OBEP-K have a possibility of giving overbound energies and OPEH and Reid SC are near the critical region.

In Fig. 3 the reaction matrix elements $G(^3S_1)$ and $G(^1S_0)$ are shown as functions of ν. We shall give some discussions about this graph in the next section. In Fig. 4 we show the scattering wave functions of the relative motion of two particles, obtained by Eq. (2·22). The healing takes place roughly at about 2.0 fm in the relative coordinate.

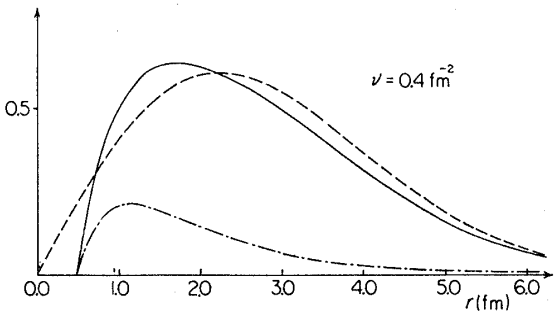

Fig. 4. Wave functions of the relative $^3S_1$-state of two particles in ^{4}He at $\nu=0.4$ fm^{-2}; the solid line shows the scattering wave function, the dashed line the model one and the dot-dashed line the tensor coupling one.

In comparison with a usual variational method it is emphasized that in the reaction matrix theory not only the short range correlation but also the D-state mixing is successfully taken into account by solving the two-particle scattering equations coupled by the tensor force in the nucleus. A large part of the binding energy comes from the tensor coupling in the scattering correlation and a large probability of D-state mixing is obtained for a strong tensor force. For H-J the probability is about 14 % (8.5 %) for the ^{4}He (^{3}H), which is estimated by use of the correlated wave function Ψ in a first order approximation for F in Eq. (2·3).

With the same approximate wave function the root mean square (r.m.s.) radius of the ^{4}He is calculated and is obtained as 1.44 fm at the minimum energy, i.e., at $\nu=0.4$ fm^{-2}. When we take into account the finite size of proton, 0.78 fm,[24] the resultant r.m.s. radius becomes 1.64 fm and is in very good agreement with the experimental value 1.63 fm.[25] If the model wave function Φ is used, the h.o. parameter ν of 0.4 fm^{-2} corresponds to the r.m.s. radius of 1.85 fm. It is noted that "shrinkage" in Ψ is large as seen easily from figures of ψ_α and φ_α in Fig. 4 and, therefore, it is necessary to use an "effective" r.m.s. radius in the model space.

In the case of ^{4}He the energy gain from about -20 MeV of H-J to -29 MeV of the experimental total energy (without Coulomb energy) should be left to higher correlations and many-body forces.

As shown in Ref. 3), the largest contribution in the various higher correlations to the energy is the hole-hole rearrangement energy of the third order, which adds the energy of about -3.6 MeV and shifts the value of ν at the minimum energy to 0.44 fm^{-2}. There also the tensor force plays an important role.

The other rearrangement correlations and the three- and the four-body scattering diagrams are not so important as regards energy. In the former case the large energy spacings of the off-energy shell propagation in the intermediate state and the small off-energy shell reaction matrix elements make their contributions smaller than that of the hole-hole rearrangement correlation, which is determined with the on-energy shell quantities. In the latter case the probability of finding the three- and four-particle clusters in the interaction range simultaneously is smaller than that of nuclear matter. This factor has a relative ratio of about 1/6 at $\nu=0.44$ to that of nuclear matter at the normal density and the energy becomes less than 1 MeV in absolute value.

In our treatment there are two important assumptions. One is about the model wave function for which we use a simple h. o. one. Since it is very difficult to obtain directly the eigenfunction of the model Hamiltonian H_{M}, we estimate the validity of a simple h. o. function by a variational calculation. Using the Slater determinant of four single-particle wave function where each of them is a superposition of h. o. functions as follows:

$$\varphi(\boldsymbol{r})=\sum_{n} C_n R_{no}(r)/r\cdot Y_{00}(\Omega), \qquad (2\cdot 23)$$

where $R_{no}(r)/r$ is the h. o. wave function. C_n's, mixing parameters of superposition, are treated as variational parameters. The results which are also shown in Ref. 3) show that the energy gain is less than 1 MeV within n up to 2. From this result it is said that a simple h. o. function of the internal coordinate is not wrong as the model wave function for the model Hamiltonian.

The other assumption is about the potential energy of the particle state in the intermediate scattering process. In above-mentioned treatment we have taken the potential energy of the particle state to be zero from the outset. This is founded on the fact that the three-particle scattering correlation reduces the potential energy off the energy shell especially at higher momenta in nuclear matter. It may not be necessarily good approximation, however, for the low-lying particle state just above the Fermi surface to give up the potential. This should be investigated by introduction of a self-consistent off-energy shell potential into the low-lying particle states.

A method is proposed,[26] where some low-lying particle states are projected out at the first step in the calculation of the reaction matrix and, afterwards, the propagation in such states is taken into account with the

Table II. Total, potential and single-particle energies obtained by the method[26] where the off-energy shell potentials are inserted into some low-lying particle states.

	$\nu=0.4$ fm^{-2}		$\nu=0.5$ fm^{-2}	
	$V_{\text{off}}=0$	$V_{\text{off}}\neq 0$	$V_{\text{off}}=0$	$V_{\text{off}}\neq 0$
Total E	−16.6(MeV)	−18.8(MeV)	−15.1(MeV)	−16.7(MeV)
Pot. E	−53.9	−56.1	−61.7	−63.4
$V(0s_{1/2})$	−27.0	−28.1	−30.9	−31.7
$V_{\text{off}}(0p_{3/2})$	0.0	− 3.1	0.0	− 2.8
$V_{\text{off}}(0p_{1/2})$	0.0	− 4.6	0.0	− 2.9
$V_{\text{off}}(0d_{5/2})$	0.0	− 0.9	0.0	− 0.9
$V_{\text{off}}(1s_{1/2})$	0.0	− 4.2	0.0	− 3.1
$V_{\text{off}}(0d_{3/2})$	0.0	− 1.2	0.0	− 0.7

off-energy shell potential. The results[3] are shown in Table II. The gain in the total energy is about 2 MeV by insertion of the off-energy shell potential into the low-lying particle states (in the case of p and sd shells). It is emphasized that the potential energies of the low-lying states are very small, while their insertion contributes appreciable energy to the total energy, and the energy spacing in virtual excitations is made very large. This fact assures that the higher correlations become less important.

§3. Binding mechanism of alpha-particle and triton

3.1 Reliability of the independent-pair model

The model Hamiltonian, Eq. (2·7), is constructed on the speculation that the two-body scattering correlation is dominant within very light nuclei as well as in nuclear matter. Adequacy of the employment of our medel Hamiltonian can be checked in the frame of the modified Bethe-Goldstone expansion, i. e., the hole-line expansion. Here we summarize the facts supporting the usefulness of our model Hamiltonian for study of bindings of light nuclei.

1) Our model Hamiltonian gives good results for energies of the ^{3}H and the ^{4}He. The values obtained with the H-J potential are −6.3 MeV for the ^{3}H and −19.5 MeV for the ^{4}He. The dominance of the independent-pair scattering correlation is clearly shown by the fact that the sum of reaction-matrix elements, i. e., the two-body scattering term, brings the most part of the interaction energies of these nuclei. In the case of the ^{4}He the interaction energy −56.8 MeV comes from the two-body scattering term. This energy is very large in absolute value compared with not only the energy −3.6 MeV from the hole-hole correlation term which gives the largest contribution among higher correlation terms investigated in this chapter, but also the additional energy needed to reproduce the experimental

one.

2) The two-body scattering wave function heals well to the model one as shown in Fig. 4. This short range healing suggests that the higher correlation energies are not so large and the independent-pair model is good. Really, a small value of the wound integral, which is determined by the degree of the healing, is the measure of smallness of higher order terms which are omitted from our model Hamiltonian.

3) Some of higher order terms are investigated quantitatively. The three- and the four-body scattering correlations give only very small contributions to the energy. It is noted that in the case of the ^{3}H and the ^{4}He this fact results not only from the smallness of the wound integral but also from smallness of the probability in which three or four nucleons encounter inside the healing distance. The largest higher order contribution comes from the hole-hole correlation which exist in the case of ^{4}He, but is small in absolute value compared with the two-body scattering contribution. These results support quantitatively the validity of our independent-pair model and the convergence of the hole-line expansion.

4) It may be somewhat unphysical that potential energies are zero for all intermediate states in the scattering process. This point is examined in the shell-model treatment. Single-particle potentials are inserted self-consistently into the particle states just above the Fermi surface. The resultant potential energies are nearly equal to zero. It is ascertained that in the case of virtual excitation in the ^{4}He there exists a large energy spacing over 30 MeV between the lowest particle state and the hole state. This fact may assure that our model wave function is a rather good one in spite of its simplicity, because the large energy spacing suppresses the mixing of other components into the model wave function. Also this fact has a role to assure the stability of the alpha-particle. It is noted that the two-body scattering correlation and therefore the reaction matrix is not so affected by potential energies of the particle states just above the Fermi surface, because the singular two-body interaction mixes mainly higher states into the scattering wave function.

Our model is a modified one from the conventional one for heavy nuclei or nuclear matter. In the case of light nuclei it is important not only to eliminate the c. m. motion of a nucleus but also to treat carefully in the scattering process the c. m. motion of a pair of nucleons. The model wave function is not a shell model one, but the internal one in which the c. m. wave function of the whole nucleus is eliminated and a relative one of any pair of nucleons can be factorized from the remainder. The Pauli operator is defined with this relatilve wave function. Also the energy denominator in the reaction-matrix equation cannot be reduced to the two-body quantity used conventionally in the shell model and must be treated as a many-body

one. Careful evaluation of energy denominators is required in our model. In fact an overcounting of pair-bonds in the shell-model results in a rather small binding energy in the case of the ^{4}He and the ^{3}H. Recently a new variational method is proposed,[10] where the two-body scattering correlation and the multiple scattering correlation are taken into account. The similar results by this method to ours confirm the results by our method and calculation in both cases of the ^{4}He and the ^{3}H.

From these facts, we can get an important conclusion with respect to the reliability of the independent-pair model in the lightest nuclei, that the model is a good starting basis on which the structure and the binding mechanism are investigated with realistic nuclear forces.

It should be mentioned that the dominance of the two-body scattering correlation is essential to the independent-pair model and may be realized not only in the shell model but also in some models, for example, a cluster one. In this case we can construct a model Hamiltonian with "effective" two-body interactions as a good starting bases. If the three- or the four-body scattering correlation remains largely, the effective three- or four-body interaction must be included into the model Hamiltonian. The independent-pair model is also applied to the cases of ^{8}Be and ^{12}C in order to investigate a mechanism of clusterization in relation to realistic nuclear forces. Results and discussions are given in §§ 4 and 5 of this chapter.

3.2 Role of the tensor force in the triplet even state for nuclear saturation and change of the effective interaction depending on the "starting energy"

Nuclear saturation has been understood historically with a well-behaved potential with some exchange characters. In this case the nuclear saturation is realized by the strong repulsive force in the odd state. It is, however, revealed in investigations of the two-body problem that the repulsive force in the odd state is not so strong as to assure the nuclear saturation, but instead, the nuclear forces have the singular repulsive core in the very small region. Brueckner et al. have shown that the short range repulsive core assures the saturation of nuclear matter and many authors have accumulated investigations for the nuclear saturation.[7),11] Our present results for the ^{3}H and the ^{4}He reveal that the tensor force in the triplet-even state plays an important role for the "overall saturation" and, therefore, may play a peculiar role for the clusterization. We begin with a simple consideration to explain the above statement.

We use tentatively the "effective" interaction obtained from the reaction matrix by the method of Ref. 27). This interaction in nuclear matter is calculated with Tamagaki's G3RS potential[5] and multiplied by a factor 1.15 so as to reproduce the binding energy of nuclear matter 16 MeV/particle.

If we use this "effective" interaction in nuclear matter as that in the alpha-particle, the energy of the ^{4}He is calculated as shown in Fig. 2(a) by a solid line (1). In this case the binding energy of the ^{4}He does not exceed 7 MeV. In comparison with the result of about 20 MeV obtained by our present model, it is obvious that the "effective" interaction in the ^{4}He is more attractive than that in nuclear matter and the increase of the binding energy caused by the increase of the attraction amounts to about 15 MeV. Matrix elements of the "effective" interaction are shown in Fig. 3. In the ^{4}He the reaction-matrix element $\langle G(^3S_1)\rangle$ is about 1.5 times as large as the $\langle G(^1S_0)\rangle$, while in nuclear matter the $\langle G(^3S_1)\rangle$ is nearly equal to the $\langle G(^1S_0)\rangle$ as seen from Table I. It is also seen from Fig. 3 that the change of the effective interaction $G(^1S_0)$ between the cases of the ^{4}He and nuclear matter is rather small. From these two facts it is deduced that the contribution of the tensor force in the triplet-even state, which is renormalized into the "effective" interaction in the $^3S_1$-state, changes largely between the cases of the ^{4}He and nuclear matter.

Mechanism of the large change of the tensor renormalization is understood as follows. The tensor force can be roughly treated by a perturbation method, because it has usually no singular core. The tensor force has no first order diagonal matrix element in the S-state and non-vanishing element starts from the second order term $v_T(Q/e)v_T$, which contributes through intermediate states. The propagation through intermediate states is affected by the level structure of nuclei. In the ^{3}H and the ^{4}He there exists only one level at the Fermi surface, while in nuclear matter many levels exist in the Fermi sea. Difference in the level structure appears as a difference of the "starting energy" $(\hbar^2/M)\,\gamma^2$ in the two-body scattering process. The starting energies obtained self-consistently with the H-J potential are about 30 MeV at $\nu=0.4$ fm^{-2} in the case of the ^{4}He and about 100 MeV in average at the normal density in the case of nuclear matter.[7] This difference changes largely the renormalization of the tensor force in the "effective" interaction in the $^3S_1$-state in which the tensor renormalization hold a large part. It is noted that the change of the tensor renormalization takes place also between the cases of the ^{3}H and the ^{4}He as seen from Fig. 3.

Now we summarize the above discussions. The tensor force in the triplet-even state plays an important role for the "overall nuclear saturation". Although the two-body scattering correlation is the main part not only in nuclear matter but also in light nuclei, the "effective" interaction in the triplet-even state is very different between the two cases. The renormalization of the tensor force depends sensitively on the starting energy. When this peculiar property of the tensor force is taken into account, it is thought that the "effective" interaction is considerably changed not only among

nuclei but also, even in a nucleus, among states with different structures, if starting energies are appreciably different among them. When a subunit-nucleus, for example, the alpha-like one is separated from the remainder, the attraction of the effective interaction in the $^3S_1$-state may increase within the subunit-nucleus. Thus it may be important to attend to the peculiar role of the tensor force in order to investigate the mechanism of clusterization on the basis of the realistic forces.

3.3 Understanding of the stability of the alpha-particle in relation to realistic nuclear forces

Among light nuclei the n alpha-like nuclei are stable compared with the neighbouring nuclei. The ^{4}He is the lightest one among the n alpha-like nuclei. In the preceding chapters it is shown and discussed that the states with the alpha-cluster structure are realized at relatively low energies. This fact comes from the especially stable property of the ^{4}He. Therefore it is important to investigate the binding mechanism of the ^{4}He in order to understand a reason of the realization of the states with the alpha-cluster structure at low energy.

Now we begin with a consideration about binding energies of the ^{3}H and the ^{4}He. In our present calculation the binding energy of the ^{4}He is, of course, much larger than that of the ^{3}H. The reason is easily understood. The number of bonds in the relative S-state increases from three in the triton to six in the alpha-particle. In the case of the saturating systems, when the nucleus expands, the matrix elements are decreased and the binding energy per particle is kept constant. On the contrary, in the case of the ^{4}He from the ^{3}H, the nucleus shrinks, the matrix elements are not decreased and the increment of the number of the S-state bonds brings directly the increase of binding energy against the increase of the kinetic energy. It should be, however, mentioned that the $^3S_1$-state effective interaction in the ^{4}He is weakened compared with that in the ^{3}H because the level in the alpha-particle becomes very deep and the renormalization of the tensor force is remarkably changed as seen from values at the same ν in Fig. 3. Thus the matrix element of the $^3S_1$-state in the ^{4}He ($\nu=0.40$ fm^{-2}) is nearly equal to that in the triton ($\nu=0.32$ fm^{-2}) for the H-J potential as seen also in Fig. 3. If the reduction of the renormalization of the tensor force is not brought about, the energy of the ^{4}He becomes overbound as large as Schmid's results[28] calculated with the "effective central" potential which is the central potential fitted to the two-body S-wave data. In the case of nuclear matter the reduction of the tensor renormalization becomes more remarkable and the "effective central" potential gives a largely overbound energy. Thus it is said that in the case of the ^{4}He the increase of the number of the S-state bonds brings the much larger binding energy than that of the ^{3}H,

and, at the same time, prevents indirectly the ^{4}He from overbinding through the reduction of the tensor renormalization.

It is also noted that a large excitation energy due to smallness of the potential energies of the low-lying particle states shown in §2 is one of assurance of the stability of the ^{4}He. It is caused by the off-energy shell property of the reaction matrix elements and the mixing of the relative higher angular momentum states than S-state into them.

The other characteristic property of the ^{4}He is that the interaction between a nucleon and the ^{4}He (N-α interaction) becomes weak. The short range part of the N-α interaction is dominated by the Pauli principle as discussed in Chapter II: The Pauli exclusion in the N-α system brings about the very short range repulsion in the S-state and "l-dependence" of the N-α interaction.[29] But the effect of the Pauli principle in the N-α system is relatively unimportant compared with that in the α-α system. The weakness of the outer part of the N-α interaction is investigated in relation to properties of realistic nuclear forces. The S-state bond, which consists of only the two-body effective interaction in the S-state, is saturated among four nucleons, and the bond between the fifth nucleon and a nucleon in the ^{4}He consists of the combination $\{9(^3O)+(^1O)+3(^3E)+3(^1E)\}/16$ of "effective" interactions in respective spin-parity states. The odd-state interaction of realistic nuclear forces makes this bond weak: The OPEP odd-state repulsion makes the net OPEP central contribution to this bond vanishing and the medium range odd-state interaction is weak, especially in the ^3O-state. It is to be noted that the $^3S_1$-state effective interaction in this bond includes the renormalization contribution from the tensor force and may be more attractive than that in the ^{4}He, because the fifth nucleon is in a very shallow level and the starting energy becomes very small. It is a further problem that the mechanism of the "externally weak" is investigated quantitatively in ^{5}He.

§4. A method of treating the alpha-clusterization on the basis of realistic nuclear forces

Based on the achievements summarized in §§ 1 and 3 we investigate in this and the next sections the mechanism of the alpha-clusterization in a nucleus, focusing its interrelation with the characteristics of realistic nuclear forces. It is first necessary to develop a method suitable to this aim. We need to combine by some way an alpha-cluster model with a method of dealing with the singular nature and the strong tensor component in the nuclear forces. We know two forms of the microscopic alpha-cluster model, the resonating-group method (RGM)[30] and the generator coordinate method (GCM).[31] For managing the nuclear forces we may choose the Brueckner theory (reaction matrix method) or the variational method by use

of such correlation function as the Jastrow type one.[32]

In taking into account the physical elements summarized and discussed in the preceding sections, the reaction matrix method is much better than the correlation function method with variational parameters, because it is very important to treat exactly the two-body correlations within the nucleus with its dependence on the structure of the nucleus and also the independent-pair model is useful enough even in such light nuclei as stressed in §3. The reaction matrix method, however, has been constructed on the model space defined always by a group of single particle orbits. This is the case also in the theory of the effective Hamiltonian for the unclosed shell nucleus.[33] It seems, therefore, to be unable to apply the reaction matrix method directly to the states with cluster structure. The cluster structure is characterized, in contrast to the shell structure, by the concept of the relative motion between the clusters, as well as by the density localization corresponding to the intrinsic configuration of the clusters. If the clusterization is the more distinct, it is the more impossible to represent it by use of the single particle orbits in rather limited model space. More appropriate model space will be the subspace connected through the change of the relative coordinate between the clusters or equivalently a "vertically truncated shell model space"[34] presented in Chapter VI. At present we have not a theory of the effective Hamiltonian which is general and also workable in practice for such a model space.

Here we adopt an approximate method which is physically reasonable and also suitable to taking into account the above-mentioned characteristics of the clusterization and the realistic nuclear forces. Let us notice that the relative oscillation between the clusters is usually slow compared with the internal motion within them and on the other hand the two-body correlations are mainly referred to the high momentum transfer processes due to the singular nature of the nuclear forces. It will be justified, therefore, that we treat in the first step the two-body correlations in some intrinsic states specified by the restricted positions of the clusters (alpha-cluster-intrinsic states) and in the next step solve the relative motion taking off the restriction of the positions. This is a kind of adiabatic approximation. The GCM is just suitable for such a two-step treatment. The alpha-cluster-intrinsic states appearing in the first step of GCM can be represented by the anti-symmetrized product of the cluster wave functions[35] which are essentially of independent particles though not so simple as the usual shell model ones. So we can apply the reaction matrix method in this stage. On the other hand it seems difficult to exploit the RGM directly on the basis of the realistic nuclear forces. It should be noted that the preference for our method is not only for the practical reason but also for the physical reason why we can see explicitly the dependence of the effective interaction on the

clusterization and also the independent particle feature[36] even in the cluster structure, by passing through the first step.

4.1 Model wave functions for the alpha-cluster-intrinsic state

The most tractable model wave functions are the many-centred harmonic oscillator's[35] corresponding to the alpha-cluster-intrinsic state of the nucleus under consideration. Such ones are used frequently in the preceding chapters for the respective purposes. Here we present our method[4] for the most simple case of ^{8}Be. Extension to the more complicated systems is straightforward, though the actual computation will be more enormous. There are shown in Appendix II the equations for ^{12}C which are necessary to the discussions in §5.

For the alpha-cluster-intrinsic state of the ground band of ^{8}Be we take two closed s-orbits around the centres separated by d;

$$\Psi(\boldsymbol{d}) \sim \mathcal{A}[\varphi_1^4 \cdot \varphi_2^4], \tag{4·1}$$

where

$$\varphi_i(\boldsymbol{r}) = (\sqrt{\pi}\, b)^{-3/2} \exp\left[-\left(\boldsymbol{r} \pm \frac{\boldsymbol{d}}{2}\right)^2 / 2b^2\right], \quad i=1,2. \tag{4·2}$$

The separation d is a measure of the density localization, an important characteristic of the alpha-clusterization, and is treated later as a generator coordinate to treat the relative motion. Note that $\Psi(\boldsymbol{d})$ in Eq. (4·1) is a wave function in model space on which the reaction matrix method is applied and so the large D-state mixture, for example, is treated not in the model space but in the reaction matrix equations. Here is not taken into account the polarization effect[37] of the alpha-clusters, which is to be treated properly in the model space as investigated in Chapter V, since our main purpose in this chapter is to relate the alpha-clusterization to the characteristic features of the realistic nuclear forces.

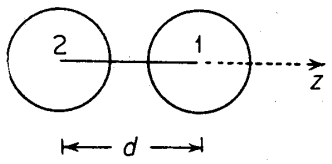

Fig. 5. Parameter-coordinate system in ^{8}Be.

In order to apply the reaction matrix method on $\Psi(\boldsymbol{d})$, we construct the orthonormal single particle wave functions from $\varphi_i(\boldsymbol{r})$ in Eq. (4·2) as follows:

$$\chi_i(\boldsymbol{r}) = \{2(1 \pm e^{-d^2/4b^2})\}^{-1/2} \{\varphi_1(\boldsymbol{r}) \pm \varphi_2(\boldsymbol{r})\}, \quad i=1,2. \tag{4·3}$$

The wave functions $\chi_i(\boldsymbol{r})$ tend to the (000) and (001) states of the usual h. o. shell model respectively when the separation d tends to zero.

Next let us transform the pair wave functions $\chi_i(\boldsymbol{r}_1)\chi_j(\boldsymbol{r}_2)$ to the relative and the centre-of-mass coordinates. The expressions are written as

$$\chi_i(\boldsymbol{r}_1)\chi_j(\boldsymbol{r}_2) = \sum_{n,N} C(i,j;n,N)\phi_n(\boldsymbol{r})\Phi_N(\boldsymbol{R}), \tag{4·4}$$

where $\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2$ and $\boldsymbol{R}=(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$. Here $\phi_n(\boldsymbol{r})$ and $\Phi_N(\boldsymbol{R})$ are the orthonormal wave functions for the relative and centre-of-mass coordinates*) (shown explicitly in Appendix I) and $C(i,j;n,N)$ are the transformation coefficients. By expanding $\phi_n(\boldsymbol{r})$ in partial waves

$$\phi_n(\boldsymbol{r})=\sqrt{4\pi}\,r^{-1}\sum_l(-)^l\sqrt{2l+1}\,\xi_l^{(n)}(r)\,Y_{l0}(\hat{\boldsymbol{dr}}), \tag{4·5}$$

we are ready for calculation of the reaction matrices.

4.2 Self-consistency between the reaction matrices and the single particle energies

The two-body correlations within the nucleus are fully taken into account by solving the reaction matrix equations

$$G=v+v\frac{Q}{\varepsilon_1+\varepsilon_2-(T_1+T_2)}G, \tag{4·6}$$

where Q is the Pauli exclusion operator and T_i is the kinetic energy operator. The use of the kinetic energy only and no single particle potential in the off-energy shell propagation has been assured in the nuclear matter theory in relation to the three-body correlations.[11e),11f)] The single particle energies ε_i are given by the G-matrix as follows:

$$\varepsilon_i\delta_{ij}=\langle\chi_i|\,T+U|\chi_j\rangle, \tag{4·7}$$

where

$$\langle\chi_i|\,U|\chi_j\rangle=\sum_k\sum_{TS}\frac{2T+1}{2}\sum_{m_s}\langle ik|G_{TSm_s}|jk\rangle. \tag{4·8}$$

Equations (4·6) ~ (4·8) are solved self-consistently determining G and ε_i for each value of d. The total energy of the intrinsic state is given by

$$E(d)=4(\varepsilon_1+\varepsilon_2)-2(U_1+U_2)-\frac{3}{4}\frac{\hbar^2}{Mb^2}, \tag{4·9}$$

where the last term expresses the subtraction of the kinetic energy of the c. m. motion of ^{8}Be.

We calculate the G-matrix in two steps, first neglecting the Pauli operator Q and then taking it into account. The equations are

$$G^0=v+v\frac{1}{\varepsilon_1+\varepsilon_2-(T_1+T_2)}G^0 \tag{4·10}$$

and

$$G=G^0+G^0\frac{Q-1}{\varepsilon_1+\varepsilon_2-(T_1+T_2)}G. \tag{4·11}$$

*) Choice of $\phi_n(\boldsymbol{r})$ and $\Phi_N(\boldsymbol{R})$ is not unique. We use the symmetry properties as far as possible and then Schmidt's orthogonalization procedure.

We solve Eq. (4·10) in the form of the differential equation. It is written as

$$(\nabla^2-\gamma^2)(\xi-\psi)=-\frac{M}{\hbar^2}v\psi, \tag{4·12}$$

where

$$\gamma^2=\frac{M}{\hbar^2}\{\langle\Phi_N|\,T_{\boldsymbol{R}}\,|\Phi_N\rangle-\varepsilon_i-\varepsilon_j\}. \tag{4·13}$$

Here we have made an approximation of replacing the c. m. kinetic energy operator by its expectation value. The quantity γ^2 is regarded as the effective starting energy and is dependent on the starting single particle states i and j and also the c. m. quantum state N. This starting-energy-dependent quantity as well as the Q-operator bring about the state dependence of the G-matrix in Eq. (4·6).

The uncorrelated (model) functions are each $\xi(r)\chi_s{}^{m_s}$ defined in Eq. (4·5), where $\chi_s{}^{m_s}$ are the spin functions of the interacting two nucleons. If the correlated functions ψ are expanded as

$$\begin{aligned}\psi_s{}^{m_s}&=\sum_{mS'}\psi_s{}^{m_s,m_s'}\chi_s{}^{m_s'}\\&=\sum_{J=0}^{\infty}\sum_{l=|J-S|}^{J+S}(2l+1)(-)^l\sqrt{\frac{4\pi}{2l+1}}\sum_{l'=|J-S|}^{J+S}\frac{1}{r}u_{ll'}^{JS}(r)\mathcal{Y}_{l'SJ}^{m_s}(loSm_s|Jm_s),\end{aligned} \tag{4·14}$$

then the equation for each partial wave is written as

$$\left\{\frac{d^2}{dr^2}-\frac{l'(l'+1)}{r^2}-\gamma^2\right\}\left\{\delta_{ll'}\xi_{l'}(r)-u_{ll'}^{JS}(r)\right\}=-\frac{M}{\hbar^2}\sum_{l''}V_{l'l''}^{JS}(r)u_{ll''}^{JS}(r), \tag{4·15}$$

where

$$V_{ll'}^{JS}(r)=\int_{\text{spin}}d\Omega_r\mathcal{Y}_{lSJ}^{m_s}v(\boldsymbol{r},S)\mathcal{Y}_{l'SJ}^{m_s}. \tag{4·16}$$

In these equations we can properly deal with the hard (or soft) core and the strong tensor component of the nuclear force. Using the solutions of Eq. (4·15) we obtain the matrix elements of G^0 by

$$\begin{aligned}\langle\xi_{l'}|G_{TSJ}^{0(ijN)}|\xi_l\rangle&=(4\pi)\sum_{l''}\int_0^{\infty}\xi_{l'}(r)\,V_{l'l''}^{JS}(r)u_{ll''}^{JS}(r)\,dr\\&=(4\pi)\frac{\hbar^2}{M}\int_0^{\infty}\{u_{ll'}^{JS}(r)-\delta_{ll'}\xi_l(r)\}\left\{\frac{d^2}{dr^2}-\frac{l'(l'+1)}{r^2}-\gamma^2\right\}\xi_{l'}(r)\,dr.\end{aligned} \tag{4·17}$$

The G^0-matrix in Eq. (4·17) depends on (ijN) through γ^2. The two-body matrix elements $\langle i'j'|G_{TSm_s}^0|ij\rangle$ are obtained by use of Eqs. (4·4), (4·5)

and (4·17).

The matrix elements of G are calculated algebraically by Eq. (4·11) or more explicitly by

$$\sum_{i,j=1}^{2}\left\{\delta_{i_1 i}\delta_{j_1 j}+\frac{\langle i_1 j_1|G^0_{TSm_S}|ij\rangle}{e(i_0 j_0, ij)}\right\}\langle ij|G_{TSm_S}|i_0 j_0\rangle=\langle i_1 j_1|G^0_{TSm_S}|i_0 j_0\rangle, \quad (4\cdot18)$$

where

$$e(i_0 j_0, ij)=\varepsilon_{i_0}+\varepsilon_{j_0}-\langle\chi_i|\,T|\chi_i\rangle-\langle\chi_j|\,T|\chi_j\rangle.$$

Thus the G-matrix is finally obtained.

4.3 Generator coordinate method

In the previous subsections we have presented a method of treating the two-body correlations in the alpha-cluster-intrinsic state with an adiabatically fixed d value. Next procedure is to improve the relative wave function between clusters, as well as to make angular momentum projection, by GCM for d. To perform it it is necessary to evaluate matrix elements of the Hamiltonian between the intrinsic states with different d values. We have shown in §4.2 that in the independent-pair approximation the energy expectation value of the intrinsic state is given by

$$\begin{aligned} &E(d)=\langle\Psi(\boldsymbol{d})|H_{\rm eff}(d)|\Psi(\boldsymbol{d})\rangle,\\ &H_{\rm eff}(d)=\sum_i T_i+\sum_{i<j}G_{ij}(d). \end{aligned} \quad (4\cdot19)$$

Now we make an approximation following the consideration mentioned in the first of this section, for the non-diagonal matrix elements as

$$\begin{aligned} \langle\Psi(\boldsymbol{d})|H_{\rm eff}|\Psi(\boldsymbol{d}')\rangle=&\frac{1}{2}\{\langle\Psi(\boldsymbol{d})|H_{\rm eff}(d')|\Psi(\boldsymbol{d}')\rangle\\ &+\langle\Psi(\boldsymbol{d}')|H_{\rm eff}(d)|\Psi(\boldsymbol{d})\rangle\}. \end{aligned} \quad (4\cdot20)$$

In the computation of Eq. (4·20) we use the G-matrix in the coordinate-representation[27]

$$G^{(ijN)}_{TS(l'l)Jm_S}(r;d)=\frac{\langle ij|G_{TSm_S}(d)|ij\rangle}{\langle ij|G^0_{TSm_S}(d)|ij\rangle}G^{0(ijN)}_{TS(l'l)Jm_S}(r;d) \quad (4\cdot21)$$

and

$$G^{0(ijN)}_{TS(l'l)Jm_S}(r;d)=\sum_{l''}V^{JS}_{l'l''}(r)u^{JS}_{ll''}(r)/\xi_l(r). \quad (4\cdot22)$$

The basic equation of GCM is given by

$$\int[H_L(d,d')-\lambda_L N_L(d,d')]f_L(d')dd'=0, \quad (4\cdot23)$$

where

$$\begin{matrix} H_L(d,d')\\ N_L(d,d')\end{matrix}=\int P_L(\cos\theta_{\boldsymbol{dd}'})\langle\Psi(\boldsymbol{d})|\begin{matrix}H_{\rm eff}\\ 1\end{matrix}|\Psi(\boldsymbol{d}')\rangle d\cos\theta_{\boldsymbol{dd}'}. \quad (4\cdot24)$$

§5. Alpha-clusterization and clustering-induced interaction

The nuclear force we adopt here is OPEG potential.[5),*)] In the treatment presented in §4 the binding energy of an alpha-particle is obtained from $E(d)$ in Eq. (4·9) by

$$E_\alpha(b)=\frac{1}{2}\left\{E(d\to\infty)-\frac{3}{4}\frac{\hbar^2}{Mb^2}\right\}, \tag{5·1}$$

where the kinetic energy of the relative oscillation between two alpha-clusters is subtracted. This corresponds to the result in §2 obtained by use of the shell model-like treatment for the energy denominator in the reaction matrix equations. The energy curve $E_\alpha(b)$ is shown in Fig. 2(a) by the solid line (2). The obtained binding energy is deficient by about 10 % of the potential energy.

Contributions to the total energies of the intrinsic states are displayed in Table III for the particular values of the size parameter. In the case of ^{8}Be we have taken the relative S, P and D waves. The P-state contributions are found small and almost cancel the D-state attractions. This is

Table III. Contributions to the total energies in the alpha-cluster-intrinsic states of ^{8}Be and ^{12}C.

	d	1.0	2.0	3.0	4.0	5.0	6.0
^{8}Be b=1.45	^1E	− 68.6	− 65.5	− 60.8	− 56.4	− 54.2	− 53.6
	^3E	− 97.8	− 93.9	− 88.1	− 82.4	− 79.8	− 79.2
	^1O	+ 3.8	+ 2.9	+ 1.8	+ 0.90	+ 0.37	+ 0.14
	^3O	− 0.77	− 0.65	− 0.40	− 0.09	+ 0.07	+ 0.01
	U	−163.0	−157.1	−147.5	−138.1	−133.6	−132.6
	T	138.5	127.1	114.8	107.0	104.2	103.6
	E	− 24.5	− 30.0	− 32.7	− 31.1	− 29.4	− 29.0
^{12}C (T) b=1.5	^1S	−114.5	−106.6	− 95.3	− 84.6	− 78.6	− 76.8
	^3S	−158.4	−149.4	−136.1	−123.3	−116.1	−114.0
	U	−272.9	−256.0	−231.3	−207.9	−194.7	−190.8
	T	219.1	201.4	179.3	162.1	154.3	152.3
	E	− 53.8	− 54.6	− 52.0	− 45.7	− 40.4	− 38.4
^{12}C (L) b=1.5	^1S	−107.9	−101.2	− 90.8	− 82.0	− 77.7	− 76.5
	^3S	−150.8	−142.9	−130.3	−119.8	−114.8	−113.5
	U	−258.7	−244.1	−221.1	−201.8	−192.5	−190.1
	T	244.9	211.1	177.9	160.0	153.6	152.2
	E	− 13.8	− 33.0	− 43.2	− 41.8	− 38.9	− 37.8

b and d are given in fm. (MeV)

*) The treatment in §4 is also applicable to the hard core potentials and main conclusions derived here are not altered for them.

the same situation as in the nuclear matter. In the case of ^{12}C only the relative S-states (with the renormalization from the tensor force) are taken into calculation. Deficiency of the total energies roughly accords with about 10 % deficiency of the potential energy in the alpha-particle. The single particle energies and the total energy surfaces are shown in Figs. 6 and 7, respectively.

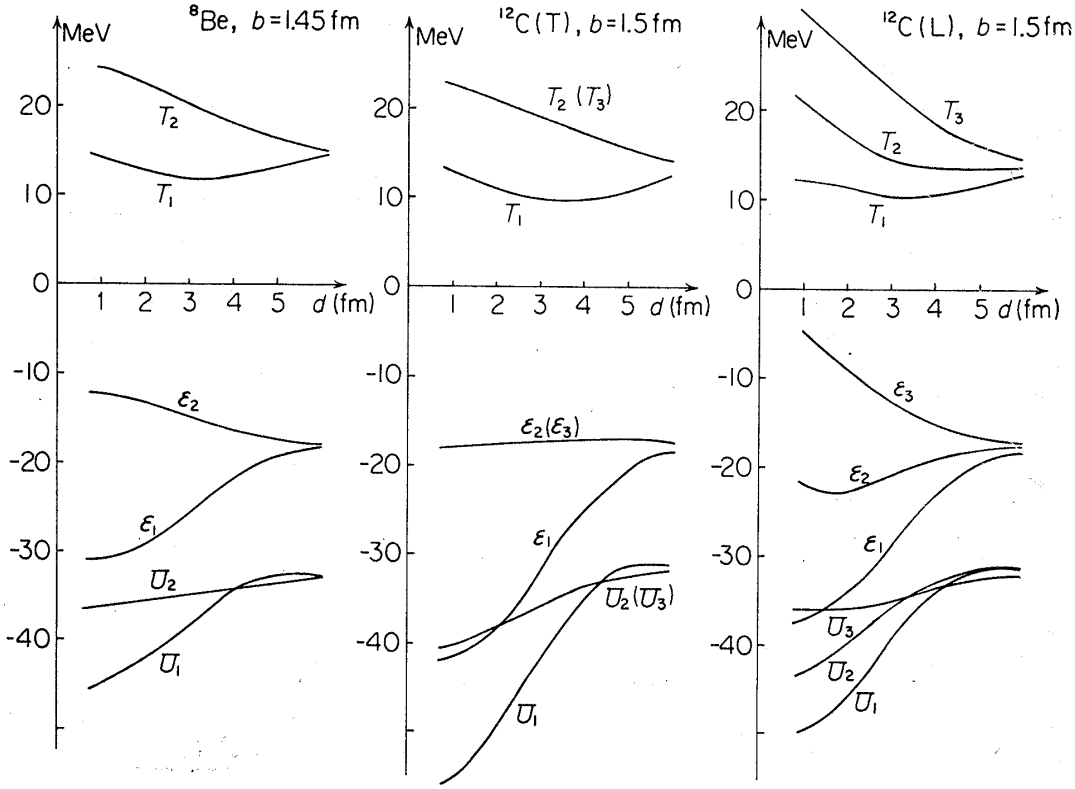

Fig. 6. Single particle energies in the alpha-cluster-intrinsic states. T_i: kinetic energies, U_i: potential energies and $\varepsilon_i = T_i + U_i$.

In ^{8}Be the energy surface has its minimum at $d \simeq 3.5$ fm, which means that the alpha-cluster structure with remarkable density localization is energetically more stable than the shell structure specified by smaller d value. This result, which is obtained repeatedly in the preceding chapters by use of the phenomenological effective interactions, is confirmed here from a realistic nuclear force, though it gives a little deficient internal energy for an alpha-particle in the present calculation. The results after angular momentum projection and solving the GCM equations are shown in Fig. 8. Here we take only the relative S-states contributions of G-matrices for simplicity and take d=1.0, 2.0, 3.0, 4.0, 5.0 as the mesh points for the generator coordinate d. The energy differences among L=0, 2, 4 states ($E_{ex}(2^+)$ = 3.3 MeV, $E_{ex}(4^+)$ = 10.7 MeV) are roughly consistent with observation (2.9 MeV, 11.4 MeV) but the absolute values are not sufficient corre-

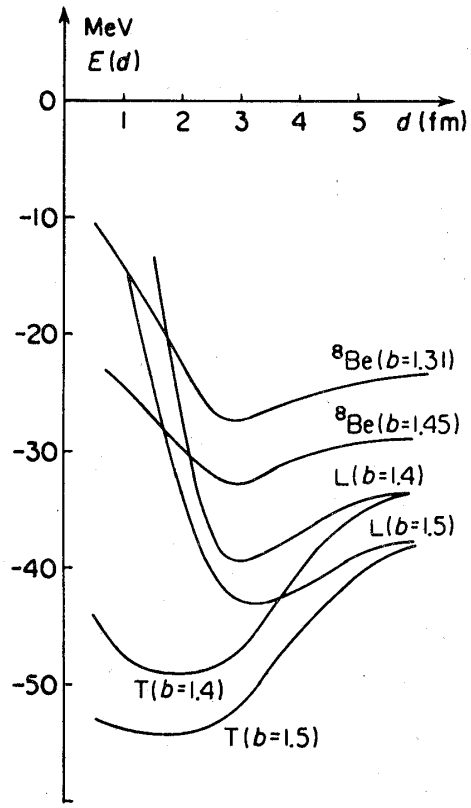

Fig. 7. Total energy surfaces for the alpha-cluster-intrinsic states.

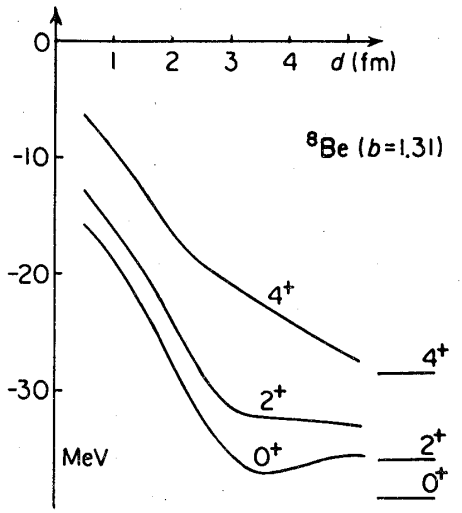

Fig. 8. Total energy surfaces of ^{8}Be after angular momentum projection and the energy levels finally obtained by GCM.

sponding to the deficiency of the internal energy of the alpha-particle.

As for the single particle energies the $(p, 2p)$ reaction experiments on ^{9}Be indicate $\varepsilon_1 \sim -26$ MeV and $\varepsilon_2 \sim -18$ MeV,[38] which means a small splitting between the s and p orbits in the single-particle field of the two alpha-clusters. Our results around $d \approx 3.5$ fm in Fig. 8 where the total energy is minimum are consistent with this observation, although the experimental data include some rearrangement effects and the field in ^{9}Be will not be quite the same as that in ^{8}Be. Anyway such small splitting is an evidence for the alpha-cluster structure.

The triangular (T)-configuration of ^{12}C has a rather shallow minimum around $d \simeq 2$ fm indicating not so distinct clusterization in this case, while in the linear-chain (L)-configuration*) the minimum is sharp around $d \simeq 3$ fm and the clusterization is more remarkable. It is noticed that such tendency to alpha-clusterization is obtained by using no odd-state repulsive

*) The L-configuration was proposed by Morinaga[39] as a possible interpretation of the excited 0^+ state at 7.66 MeV. According to the recent investigations,[40] however, this state cannot be understood as a simple linear chain structure. (See Chapter III.) We take here the L-configuration as a distinct contrast to the T-configuration, since the former is expected to have more distinct alpha-clusterization than the latter.

force, in contrast to the phenomenological effective interactions with the strong odd-state repulsions.

Let us study the relation of these results with the characteristics of the nuclear forces. As seen in Table III the repulsive contribution of the odd-state interactions has rather minor effect for the enhancement of the alpha-clusterization, although it is essential, of course, that the odd-state interactions are never strongly attractive. In order to see the roles of the repulsive core and the tensor force we investigate the d-dependence of the G-matrices. We express them in the coordinate-representation, in terms of the difference

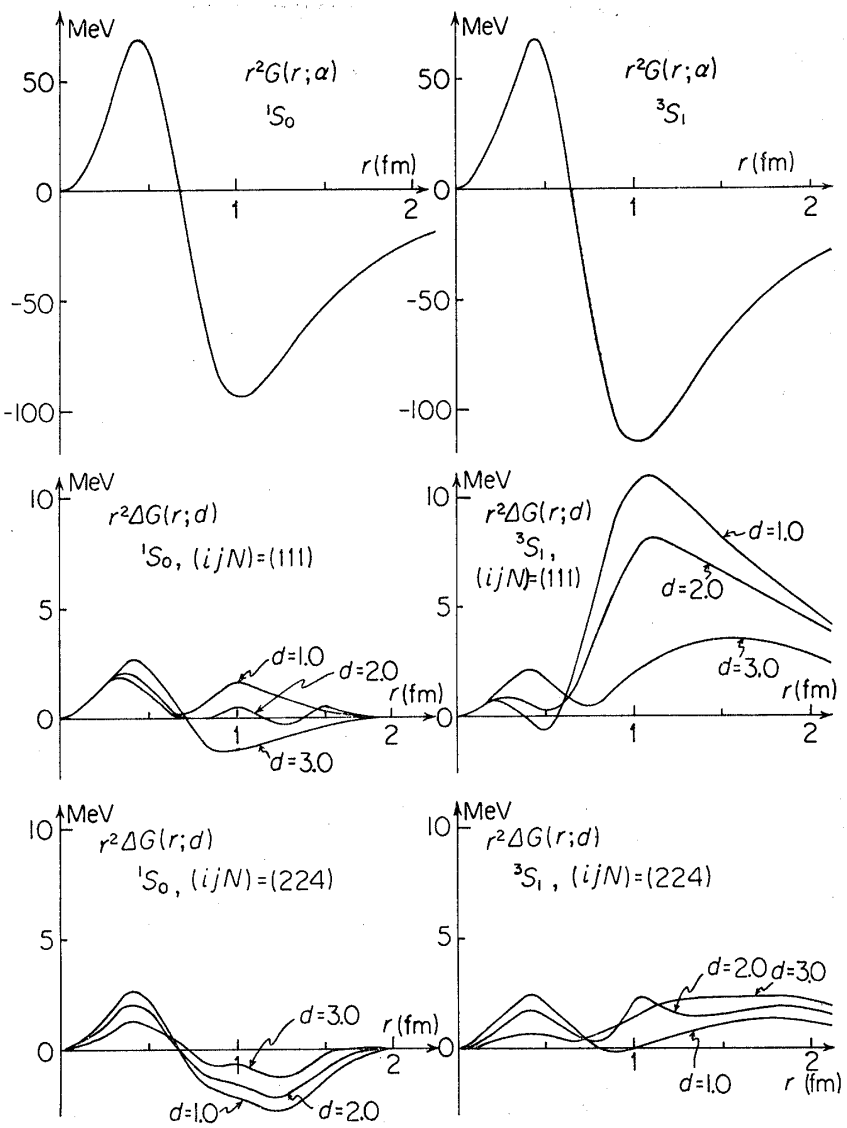

Fig. 9. Reaction matrices in the coordinate-reprsentation. The uppermost two represent the effective interations within a free alpha-particle, $r^2G(r;\alpha)$, for $^1S_0$- and $^3S_1$-states. The others are $r^2\Delta G^{(ijN)}_{TS}(r;d)=r^2\{G^{(ijN)}_{TS}(r;d)-G_{TS}(r;\alpha)\}$ for the typical cases of (ijN) in the T-configuration of ^{12}C. All of them are calculated for $b=1.4$ fm.

of $G(r;d)$ from a standard for which we adopt $G(r;d=\infty)\equiv G(r;\alpha)$, the effective interaction within an alpha-particle. In Fig. 9 are shown $\Delta G_{TS}^{(ijN)}(r;d)=G_{TS}^{(ijN)}(r;d)-G_{TS}(r;\alpha)$ multiplied by r^2 for some typical cases in the T-configuration of ^{12}C, together with $r^2\ G(r;\alpha)$. The cases $(ijn)=$ (111) and (224) are most sensitive and insensitive to d, respectively.

Repulsive (attractive) nature of $\Delta G(r;d)$ means that the effective interaction is less (more) attractive than that within the alpha-particle. By their changes with respect to d we can see how the effective interactions between nucleons vary depending on the growth of the alpha-clusterization. In the $^1S_0$-state the changes of $G(r;d)$ are small on the whole indicating the minor role of the repulsive core in the clustering-dependence of the effective interaction. On the other hand, in the $^3S_1$-state the changes of $G(r;d)$ are much more remarkable especially in the outer attractive part, which becomes more attractive with d and tends to go back to $G(r;\alpha)$ within a free alpha-particle. This originates just from the strong tensor component of the nuclear force, as explained in §3 on its mechanism. Such d-dependence of the effective interaction is responsible for the behavior of the energy surfaces for the T-configuration.

Corresponding to the growth of the alpha-clusterization from the T- to L-configuration the effective interaction is expected to become more attractive. We find this in Fig. 10 where $r^2\Delta G(r;d=2.0)$ for the T-configuration are compared with $r^2\Delta G(r;d=3.0)$ for the L-configuration, each corresponding to the respective energy minimum. Shadowed regions are the potential volumes which are induced due to the growth of the alpha-clusterization. Here again main part is in the outer attractive region of the $^3S_1$-state interaction. Physical origin of this is, as explained in §1, that the effect of

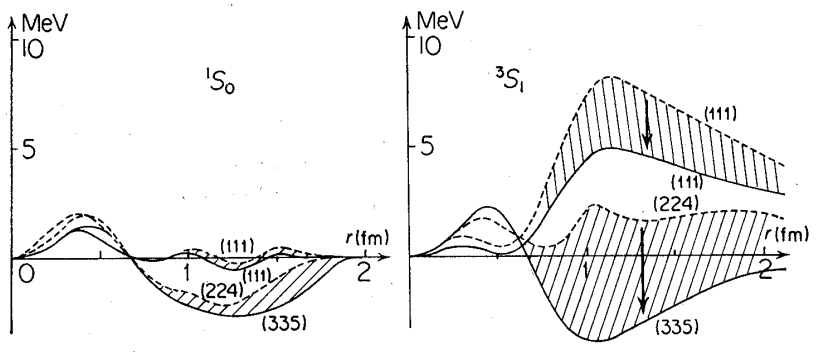

Fig. 10. $r^2\Delta G(r;d=2.0)$ for the T-configuration (dashed lines) and $r^2\Delta G(r;d=3.0)$ for the L-configuration (solid lines) of ^{12}C. $b=1.4$ fm. The suffices (ijN) are attached to each curve. The arrows indicate the inducement of the attractive interaction according to the growth of the alpha-clusterization from the T- to L-configuration.

the strong tensor force tends to revive due to the reduction of the many-body effects (the Pauli principle and the starting energy effect) according to the growth of the alpha-clusterization. This complicated mechanism can be regarded in the model space as *some attractive interaction is induced by the alpha-clusterization and that induced interaction enhances the alpha-clusterization itself.*

Form of matrix elements is more suitable for the quantitative estimate of the change of the G-matrices. We compare in Fig. 11 the matrix elements $\langle\chi_i(d)\,\chi_j(d)\,|G_{TS}(d)\,|\chi_i(d)\,\chi_j(d)\rangle$ with $\langle\chi_i(d)\,\chi_j(d)\,|G_{TS}(\alpha)\,|\chi_i(d)\,\chi_j(d)\rangle$ evaluated by use of the d-independent standard $G_{TS}(\alpha)$. We find again that the ${}^3S_1$-state G-matrices change markedly with d, while the ${}^1S_0$-state ones only slightly, as expected from the study of the G-matrices in the coordinate-representation. Thus we can conclude that the tensor force in the triplet-even state is mainly responsible for the clustering-dependence of the effective interaction.

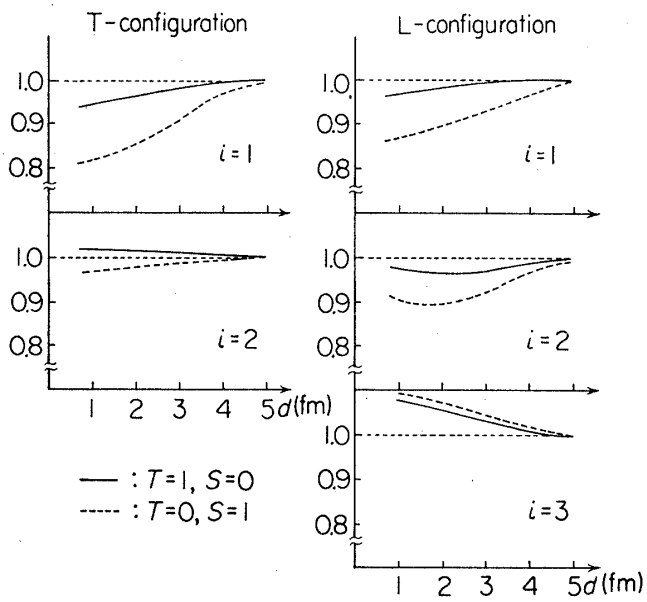

Fig. 11. Ratios $\langle\chi_i(d)\chi_i(d)\,|G_{TS}(d)\,|\chi_i(d)\chi_i(d)\rangle/\langle\chi_i(d)\chi_i(d)\,|G_{TS}(\alpha)\,|\chi_i(d)\chi_i(d)\rangle$. b=1.4 fm.

In order to see how the energy surfaces in Fig. 7 are realized due to such a clustering-dependence of the effective interection, let us compare them with those calculated by use of $G_{TS}(\alpha)$ in place of $G_{TS}(d)$. They are shown in Fig. 12. For the T-configuration the energy minimum is found at $d=0$ with no alpha-clusterization, if the clustering-dependence of the G-matrices are ignored. This situation occurs usually when we use simple phenomenological effective interactions without strong odd-state repulsions. Thus we understand the importance of the clustering-dependence of the effective interaction in causing the tendency to the alpha-clusterization. The

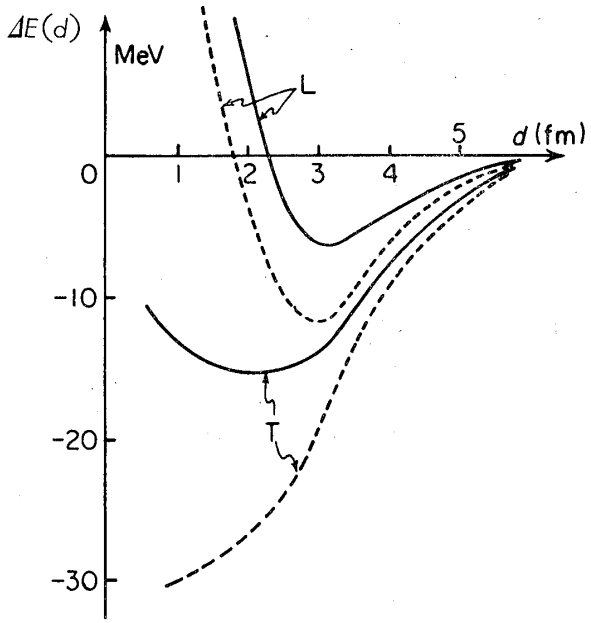

Fig. 12. Comparison of the energy surfaces obtained with (solid lines) and without (dashed lines) d-dependence of the G-matrices. Dashed lines are calculated by use of $G(\alpha)$ in place of $G(d)$ for any d-values. $\Delta E(d) \equiv E(d) - E(d=\infty)$. $b=1.4$ fm.

strong odd state repulsion is understood to be a substitute for the real mechanism, the clustering-dependence of the effective interaction or the clustering-induced interaction originating mainly from the tensor force in the triplet-even state. Its importance appears more clearly in the alpha-clusterization in ^{20}Ne.[41] In the case of the L-configuration there remains stable clusterization even if the d-dependence of the G matrices is ignored. This is mainly due to the strong effect of the exchange repulsion related to the kinetic energies.[42] The clustering-induced interaction plays a decisive role in determining the structure of the states with some transient situation, such as the ground bands of ^{12}C and ^{20}Ne. The alpha-cluster aspects in such states are realized as the result of the interdependence of the effective interaction and the structure of the nucleus.

We can estimate the net effect of the induced interactions (Fig. 12) on lowering the energy difference between the T- and the L-configurations: Energy reduction from the standard curves (dashed curves in Fig. 12 obtained from the same G-matrix $G(\alpha)$) is about 11 MeV at the energy minimum ($d=2.0$ fm) of the T-configuration, while only 5 MeV in the L-configuration ($d=3.0$ fm). This indicates that the L-state is lowered about 6 MeV due to the additional attractive interaction induced in the transition from the T-state. It should be noted that all the quantities under discussions are subject to the uncertainty related to about 10 % deficiency of the potential energy. Such effect of the clustering-induced interaction will be more noticeable in the transition from a purely shell model-like state to a purely

molecule-like state, such as the case of ^{16}O nucleus. Without taking into account this effect we are led to too high excitation energy for the molecule-like state. Here we find the overall-saturation mechanism revealing itself as the excitation mechanism of a nucleus.

§6. Conclusions and future problems

1) It is confirmed that the independent pair scattering mode plays the most important role for the nuclear saturation over all nuclei including the lightest ones. Especially the binding energy of alpha-particle is given by the two-body scattering correlations up to about 20 MeV and the other correlations add several MeV. In the alpha-particle and the triton the contributions from the tensor force are much larger than in nuclear matter.

2) The saturation property over all nuclei is founded on both of the repulsive core and the tensor component of the nuclear forces. The former prevents the nucleus from collapse and the latter modulates the values of the binding energy per particle to be nearly constant from the very light nuclei to the nuclear matter.

3) The variation of the contribution of the tensor force depends sensitively on the starting energy of the scattering particles and the Pauli exclusion principle, since the tensor force mainly contributes only in two steps through the intermediate states. This property is quite general in nuclei and possibly appears not only in the saturation problem but also in variety of the nuclear phenomena.

4) Alpha-particle has a particular binding energy in comparison with the neighbouring nuclei. A gain of the energy from triton to alpha-particle is obtained by the increment in the number of pair bonds, i. e., triton is not a saturation system. The interaction between alpha-particle and a nucleon becomes weak because the nuclear force is strongly attractive only in the S-state and alpha-particle forms the double closed shell of the s-state. Due to this exceptional binding of alpha-particle the alpha-clustering feature first reveals itself near and above the ground state.

5) The alpha-clusterizations in ^{8}Be and ^{12}C are realized with the realistic nuclear forces, which have only weak odd-state forces being different from the phenomenological effective interactions often used in the cluster model calculations.

6) Clusterization is strongly influenced by the tensor force. As the clustering aspect grows up in the nucleus, the attractive contribution by the tensor force becomes increased, as if a "clustering-induced" attraction is generated due to the clusterization. This effect turns to enhance the clusterization itself. Thus the clusterization and the induced-interaction are interdependent to each other. The clusterization of the state is finally

determined, of course, by many structural effects as well as the clustering-induced interaction.

7) The clustering-induced attraction is represented effectively in a coordinate representation of the reaction matrix, where the strength of the attractive part in the triplet-even state varies depending on a separation parameter between clusters.

8) In the case of ^{12}C the contribution of the tensor force differs in the different states, "tri-angular" and "linear" configurations, that is, the effective interaction is more attractive in the latter than the former state, corresponding to the growth of the alpha-clusterization. Therefore, this has an effect to lower the excitation energy between both configuration.

9) It can be said safely that the above-mentioned mechanism about the clustering-induced interaction will persist, even if some mechanism to complement the deficient part of the total binding energy is revealed in future. This will be also ture for the use of more improved wave functions, as far as it represents the distinct structure-change, for example, from the shell "phase" to the cluster "phase". The reason is that the contribution of the tensor force is, in any way, very sensitive to the situation of the scattering two nucleons bound within the nucleus and the situation is prescribed by the relevant structure of the nucleus.

10) We have adopted a kind of adiabatic approximation between the pair scattering correlation and the cluster correlation and have clarified the interdependence of them. It is important, however, to treat the interrelation of these correlations more dynamically. This is necessary for putting our conclusions on more reliable foundation, investigating unknown dynamical correlations and providing the method to develop the theory on the basis of realistic nuclear forces.

Acknowledgements

The authors would like to express their thanks to Mr. Y. Yamamoto for his collaboration and also to Professor R. Tamagaki and Professor H. Tanaka for their valuable discussions.

Appendix I

Case of ^{8}Be

The orthonormal relative wave functions are given by

$$\phi_1(\boldsymbol{r}) = (\sqrt{2\pi}b)^{-3/2}\exp(-r^2/4b^2),$$
$$\phi_2(\boldsymbol{r}) = \{2(1-\Delta^{1/2})\}^{-1/2}\{\phi^{(+)}(\boldsymbol{r}) - \phi^{(-)}(\boldsymbol{r})\},$$
$$\phi_3(\boldsymbol{r}) = \{\cosh(\delta/4) - 1\}^{-1/2}\left[\phi_0(\boldsymbol{r}) - \frac{1}{2}\Delta^{-1/8}\{\phi^{(+)}(\boldsymbol{r}) + \phi^{(-)}(\boldsymbol{r})\}\right],$$

where

$$\phi^{(\pm)}(\boldsymbol{r}) = (\sqrt{2\pi}\, b)^{-3/2}\exp[-(\boldsymbol{r}\pm\boldsymbol{d})^2/4b^2].$$

The orthonormal centre-of-mass wave functions are given by

$$\Phi_1(\boldsymbol{R}) = (\sqrt{\pi/2}\, b)^{-3/2}\exp(-R^2/b^2),$$
$$\Phi_2(\boldsymbol{R}) = \{2(1-\Delta^2)\}^{-1/2}\{\Phi^{(+)}(\boldsymbol{R}) - \Phi^{(-)}(\boldsymbol{R})\},$$
$$\Phi_3(\boldsymbol{R}) = \{\cosh(\delta)-1\}^{-1/2}\left[\Phi_0(\boldsymbol{R}) - \frac{1}{2}\Delta^{-1/2}\{\Phi^{(+)}(\boldsymbol{R}) + \Phi^{(-)}(\boldsymbol{R})\}\right],$$

where

$$\Phi^{(\pm)}(\boldsymbol{R}) = (\sqrt{\pi/2}\, b)^{-3/2}\exp[-(\boldsymbol{R}\pm\boldsymbol{d}/2)^2/b^2].$$

In the above $\delta = d^2/b^2$ and $\Delta = e^{-\delta}$. The functions $\phi_n(\boldsymbol{r})$ and $\Phi_N(\boldsymbol{R})$ tend to the usual harmonic oscillator wave functions with $(n-1)$ and $(N-1)$ oscillator quanta, respectively, in the limit $d\to 0$.

Appendix II

Case of ^{12}C

i) Triangular (T-) configuration of 3 alpha-clusters.

The orthonormal single particle wave functions with proper symmetries are given by

$$\chi_1(\boldsymbol{r}) = \{3(1+2\Delta^{1/4})\}^{-1/2}(\varphi_1+\varphi_2+\varphi_3),$$
$$\chi_2(\boldsymbol{r}) = \left\{\frac{3}{2}(1-\Delta^{1/4})\right\}^{-1/2}\left(\varphi_1 - \frac{1}{2}\varphi_2 - \frac{1}{2}\varphi_3\right),$$
$$\chi_3(\boldsymbol{r}) = \{2(1-\Delta^{1/4})\}^{-1/2}(\varphi_2-\varphi_3),$$

where

$$\varphi_i(\boldsymbol{r}) = (\sqrt{\pi}\, b)^{-3/2}e^{-(\boldsymbol{r}-\boldsymbol{e}_i)^2/2b^2}, \qquad i=1, 2, 3.$$

From the pair wave functions $\chi_i(\boldsymbol{r}_1)\chi_j(\boldsymbol{r}_2)$ we obtain six and seven orthonormal basic functions for the centre-of-mass $(\Phi_N(\boldsymbol{R}))$ and the relative coordinates $(\phi_n(\boldsymbol{r}))$, respectively.[4]

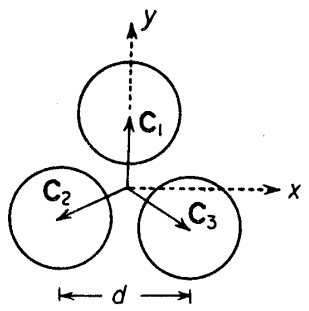

Fig. 13. Parameter-coordinate system in the T-configuration of ^{12}C.

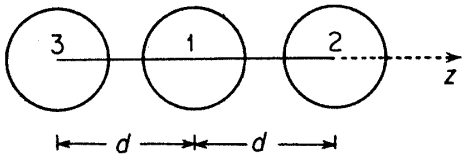

Fig. 14. Parameter-coordinate system in the L-configuration of ^{12}C.

ii) Linear chain (L-) configuration of 3 alpha-clusters.
The orthonormal single particle wave functions are taken as follows:

$$\chi_1(\boldsymbol{r}) = \{1+\sin(2\theta_1)\cosh^{-1/2}(\delta/2)\}^{-1/2} \times [\cos\theta_1\,\varphi_1+\sin\theta_1\{2(1+\Delta)\}^{-1/2}(\varphi_2+\varphi_3)],$$
$$\chi_2(\boldsymbol{r}) = \{2(1-\Delta)\}^{-1/2}(\varphi_2-\varphi_3),$$
$$\chi_3(\boldsymbol{r}) = \{1-\sin(2\theta_3)\cosh^{-1/2}(\delta/2)\}^{-1/2} \times [-\sin\theta_3\,\varphi_1+\cos\theta_3\{2(1+\Delta)\}^{-1/2}(\varphi_2+\varphi_3)],$$

with the condition

$$\sin(\theta_1-\theta_3)+\cos(\theta_1+\theta_3)\cosh^{-1/2}(\delta/2)=0,$$

assuring the orthogonality of χ_1 and χ_3. The parameter $\theta_1(\theta_3)$ is determined not by the symmetry properties but by the H-F condition $\langle\chi_1|\,T+U|\chi_3\rangle=0$. We obtain five basic functions for both $\varphi_n(\boldsymbol{r})$ and $\Phi_N(\boldsymbol{R})$.[4)]

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