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ABSTRACT

In this paper, we propose different complexity indicators for the well-known nurse scheduling problem (NSP). The NSP assigns nurses to shifts per day taking both hard and soft constraints into account. The objective is to maximize the nurses' preferences and to minimize the total penalty cost from violations of the soft constraints. The problem is known to be NP-hard.

Due to its complexity and relevance in practice, the operations research literature has been overwhelmed by different procedures to solve the problem. The complexity has resulted in the development of several (meta-)heuristic procedures, able to solve a NSP instance heuristically in an acceptable time limit. The practical relevance has resulted in a never-ending amount of different NSP versions, taking practical, case-specific constraints into account.

The contribution of this paper is threefold. First, we describe our complexity indicators to characterize a nurse scheduling problem instance. Secondly, we develop a NSP generator to generate benchmark instances to facilitate the evaluation of existing and future research techniques. Finally, we perform some preliminary tests on a simple IP model to illustrate that the proposed indicators can be used as predictors of problem complexity.

Keywords: Nurse scheduling; Benchmark instances; Problem classification

1. INTRODUCTION

The nurse scheduling problem (NSP) is a well-known combinatorial optimization problem in literature and has attracted numerous researchers to develop exact and (meta-) heuristic procedures. The NSP involves the construction of duty rosters for nursing staff and assigns the nurses to shifts per day taking both hard and soft constraints into account. The objective maximizes the preferences of the nurses and minimizes the total penalty cost from violations of the soft constraints. The problem is known to be NP-hard (Osogami and Imai, 2000).

Despite the numerous procedures for the NSP, no state-of-the art results have been presented in literature. The main reason is that comparison between procedures is very difficult, since problem descriptions and models vary drastically and depend on the need of the particular hospital. Due to the huge variety of hard and soft constraints, and the several objective function possibilities, the nurse scheduling problem has a multitude of representations, and hence, a wide variety of solution procedures has overwhelmed the optimization literature. The comparison is further hindered by the lack of benchmark problem instances and the unavailability of source code of the different procedures. Moreover, there is no general agreement on how to evaluate and compare procedures in terms of solution comparison, stop criterion, etc…. Consequently, a fair comparison between procedures seems to be an impossible idea, which undoubtedly limits the efficient development of future algorithms.

In their overview papers, Cheang et al (2003) and Burke et al (2004) express the need for a benchmark database to facilitate comparison of the various algorithms and to motivate future researchers to develop better solution procedures for the NSP. In this paper, we come towards this need of benchmarking in several ways.

The outline of the paper is as follows. In the next section, we briefly review the use of hard and soft constraints applicable to nurse scheduling problems. Section 3 presents the complexity indicators for the NSP that are used as a base for the problem instance generator (*NSPGen*). In section 4 we present our generation approach to generate problem instances under a controlled design. In section 5, we report the relevance of these indicators by computational results on a simple IP model. A decision tree has been constructed to distinguish groups of data instances based on known input parameters. Section 6 draws overall conclusions and suggestions for future research avenues.

2. THE NSP UNDER DIFFERENT ASSUMPTIONS

The basic nurse scheduling problem (NSP) can be stated as follows. A set of nurses needs to be scheduled within a pre-defined period (e.g. a week). In doing so, these nurses need to be assigned to one of a number of possible shifts in order to meet the minimal coverage constraints and other case-specific constraints and to maximize the quality of assigned working shifts. According to Warner (1976), quantifying preferences in the objective function maintains fairness in scheduling nurses over the scheduling horizon. Hence, the quality of a schedule is a subjective judgment of the nurses depending on how well the assigned schedule is conform to his/her desires to be off or on duty and to other schedule properties such as work stretch, rotation patterns, etc…. The coverage constraints determine the required nurses per shift and per day, and are inherent to each NSP instance. However, many other constraints are very case-specific, and are determined by personal time requirements, specific workplace conditions, national legislation, etc…. The majority of these extra constraints can be handled as *hard constraints*, for which no violation is possible whatsoever, or as *soft constraints*, which can be violated at a certain penalty cost. In their literature survey, Cheang et al (2003) present an overview of constraint types as appearing frequently in the literature. In the remainder of our paper, we propose different complexity indicators to describe a NSP instance. More precisely, these indicators describe a two-dimensional nurse/day preference roster and the corresponding coverage requirements, which are both inherent to any NSP instance. We assume a nurse scheduling problem where each nurse i can express its preference to work on day j in shift k as p_{ijk}. We opt for this general approach to express the preference or aversion of nurses to work on a shift/day, and hence, ignore some very case-specific preference structures, such as sequence-dependent preferences. We believe, however, that most nurse scheduling problems can be modelled by using our general preference matrix. The required number of nurses (coverage requirements) on day j for shift k can be denoted by r_{jk}.

The objective is to schedule the nurses for the complete period, such that the coverage requirements are met and the total sum of nurses' preferences and the penalty costs of soft constraints violations are minimized.

3. NSP GENERATION

In this section, we present three classes of complexity indicators in order to generate NSP instances under a controlled design. These complexity indicators should span the full range of problem complexity and should have sufficient discriminatory power to serve as predictors for the complexity of the problem under study (Elmaghraby and Herroelen, 1980). Hence, it allows the generation of instances with pre-defined values for the complexity indicators to predict the difficulty of a particular NSP instance for a particular solution procedure. Therefore, different sets with different combinations of the indicators can discriminate between easy and hard instances and these indicators can act as predictors of the computational effort of the procedures that have been developed. The CPU-time that a solution procedure needs to solve a particular problem instance to optimality can typically be used to describe the hardness of this problem instance for the particular solution procedure. Hence, the comparison of procedures and good predictions of their required CPU-time allow the a priori selection of the fastest solution procedure, based on the simple calculation of the indicators. The complexity indicators are therefore indispensable in the construction of problem sets that span the complete range of complexity of important problem characteristics.

Insert Table 1 about here

The three classes of proposed complexity indicators to generate a NSP instance measure the size of the problem instance, the preference structure of the nurses and the coverage requirements of the schedule. Table 1 serves as a guideline to following sections, where the three classes of indicators will be explained into detail.

3.1. Problem size

The size of the NSP instance under study depends on the size of the duty roster matrix. Consequently, the three complexity indicators describing the size of the problem can be defined as:

N = number of nurses
S = number of shifts
D = number of days

These three input parameters will be used to describe the two following classes of complexity indicators. The *preferences* have to be expressed by each nurse, for each shift of all days (see section 3.2). The *coverage requirements* need to be given for all shifts of all days (see section 3.3).

3.2. Preferences

The preference structure of the nurses consists of three input parameters. First, the preference distribution over all nurses (for each shift and for each day) needs to be determined by the *nurse-preference distribution*. Second, these preferences need to be distributed among all shifts of a single day, denoted by the *shift-preference distribution*. Last, the preference distribution for all days of the complete scheduling period needs to be determined, referred to as the *day-preference distribution*. In doing so, we have full control on the complete preference distribution for all nurses for each shift on each day.

3.2.1. Nurse-preference distribution (NPD)

We assume that a shift/day preference can be expressed by nurses as a ranking among shifts. More precisely, each nurse can rank each possible working shift of the day, such that the maximal number of different preference values equals the number of shifts *S*. In doing so, each nurse expresses his/her desire to work on that particular shift by assigning a number between 1 (very desirable) and *S* (very undesirable). The *NPD* measures the distribution of the preferences over all nurses for a particular shift on a particular day. In the remainder of this section, we explain the calculation of the NPD measure for a particular shift k of a particular day *j*. Although the *NPD* value can differ for each shift of each day, we assume that the nurse-preference distribution is equal for all shifts and all days.

We introduce an auxiliary variable to measure the number of times an identical preference value *l* (l = 1, …, *S*), has been selected for a particular shift and day among the nurses, as follows:

y_{il} = 1, if nurse i prefers choice *l* (for a particular shift on a particular day)
0, otherwise

and hence, $\sum_{i=1}^{N} y_{il}$ denotes the number of times a ranking value *l* has been selected by all the nurses for a particular shift/day.

NPD ∈ [0, 1] can be calculated as $NPD = \frac{\alpha_w^{NPD}}{\alpha_{\max}^{NPD}} = \frac{\sum_{l=1}^{S} \left| \sum_{i=1}^{N} y_{il} - N/S \right|}{\alpha_{\max}^{NPD}}$.

Consequently, α_w^{NPD} measures the total absolute deviation of all $\sum_{i=1}^{N} y_{il}$ -values for each preference *l* from the average number of each preference, i.e. *N / S*. Indeed, since the total number of different preferences equals S, each preference will be selected *N / S* times, on the average. Moreover, $\alpha_{\max}^{NPD}$ is used to denote the maximal possible value of α_w^{NPD}. By dividing α_w^{NPD} by $\alpha_{\max}^{NPD}$, we make sure that *NPD* lies between zero and one, inclusive. The value for $\alpha_{\max}^{NPD}$ depends on the maximal allowable value for $\sum_{i=1}^{N} y_{il}$ which is equal to *N*, and can be expressed as $2N - \frac{2N}{S}$. For more information, we refer to appendix A. In this appendix, we show that our variance measure $\frac{\alpha_w^{NPD}}{\alpha_{\max}^{NPD}}$ is a general measure for the distribution of any parameter which will also be used in this paper to describe other complexity indicators. It has been proposed by Vanhoucke et al (2004) for the generation of project networks and has been adapted by Labro and Vanhoucke (2005) to design general costing systems for management accounting.

The *NPD* measures whether the preference structure is distributed equally over the nurses (there is no clear preference among the nurses for this particular shift, or *NPD* = 0) or shows a clear pattern for one preference (all nurses have the same ranking value for that shift, or *NPD* = 1). In table 2, we display three different *NPD* values for three shifts (k = 1, 2 or 3) and 15 nurses. As an example, NPD_1 = (|13 – 5| + |1 – 5| + |1 – 5|) / (2 * 15 – (2 * 15 / 3)) = 0.80, denoting that the majority of nurses (13) have shift 1 as their first choice (and 1 nurse as the second choice and 1 nurse as the last choice). The *NPD* only measures the distribution of preferences among nurses, but does not assign the individual preferences of the generated distribution to particular shifts of a particular day. The shift-preference distribution indicator of section 3.2.2 and the day-preference distribution indicator of section 3.2.3 assigns the individual preferences among shifts and days, respectively. As an example, table 3 displays a preference matrix for which the *NPD*-values equal 0.80 for the first shift of each day, 0.50 for the second shift of each day and 0.20 for the third shift of each day.

Insert Table 2 about here

3.2.2. Shift-preference distribution (SPD)

The *NPD* measures the distribution of the preferences l (expressed as a ranking value between 1 and *S*) among nurses, but does not assign the individual preference values to individual nurses to express his/her desire to work on that shift of that day. The *SPD* assigns these preferences to nurses and measures the distribution of these preferences over all shifts of a single day. Although the *SPD* value can differ for each day, we assume that the shift-preference distribution is equal for all days over the complete scheduling horizon.

$SPD \in [0, 1]$ can be calculated as $SPD = \dfrac{\sum_{i=1}^{N} \Delta_i^{SPD} - N}{(S-1)N}$.

Δ_i^{SPD} measures the number of different preference values for nurse *i* over all shifts *k* of a particular day. Using P as a temporary set for preference ranking values, Δ_i^{SPD} can be calculated for each nurse *i* as follows:

$\Delta_i^{SPD} = 0$

P = ∅

for k = 1, …, S

if (pijk ∉ P) then

$\Delta_i^{SPD} = \Delta_i^{SPD} + 1$

P = P ∪ pijk

where p_{ijk}–values can vary between 1 and *S*.

A minimal value for Δ_i^{SPD} equals 1 when nurse *i* expresses no clear preference among the shifts (and hence, assigns a similar preference value to each shift of the day to express indifference among shifts). The maximal value for Δ_i^{SPD} equals *S* and means that nurse *i* has a clear ranking between each shift on a day. Consequently, the maximal value for $\sum_{i=1}^{N} \Delta_i^{SPD}$ equals *SN* and minimal value equals *N* and the *SPD* always lies between zero and one, inclusive. The *SPD* measures the preference structure over all shifts within a day and equals 0 if all nurses express indifference between the shifts and equals 1 if each nurse expresses a preference ranking among the individual shifts. As an example, table 3 displays the four-days preference matrix with a *SPD*-value equal to 0.2 for day 1, 0.4 for day 2, 0.6 for day 3 and 0.8 for the last day. Note that the *NPD* equals 0.8, 0.5 or 0.2 for the first, second or third shift, respectively, for each day.

3.2.3. Day-preference distribution (DPD)

The *SPD* of previous section can be applied to each day of the complete scheduling period in order to control the preference structure over all shifts of each day. In order to control the preference structure over all days of the complete scheduling period, an indicator to measure the day-preference distribution is necessary.

The *DPD* indicator is similar to the *SPD*, but measures the distribution of the preferences over all days, instead of a single-day distribution over all shifts. In analogy with Δ_i^{SPD}, Δ_{ik}^{DPD} measures the number of different preference values for nurse *i* on shift *k* over all days.

$DPD \in [0, 1]$ can be calculated as $DPD = \dfrac{\sum_{i=1}^{N}\sum_{k=1}^{S}\Delta_{ik}^{DPD} - NS}{(S-1)NS}$.

Since the maximal value for $\sum_{i=1}^{N}\sum_{k=1}^{S}\Delta_{ik}^{DPD}$ equals *SSN* and minimal value equals *SN*, the *DPD* always lies between zero and one, inclusive. When *DPD* equals 0, then all nurses have expressed a similar preference or aversion for similar shifts over all days. On the other hand, *DPD* equals 1 when each nurse has assigned a different preference value for similar shifts over the days (i.e. the nurses have clearly a day-dependent preference for each shift). The *DPD*-value for table 3 equals 0.70.

In order to clarify the different indicators, we have displayed some extreme preference structures measured by the three preference distribution measures in appendix B.

3.3. Coverage constraints

The coverage requirements, expressed as the required number of nurses on day *j* for shift *k*, will be expressed as r_{jk}. Furthermore, we use $\overline{r_j} = \dfrac{\sum_{k=1}^{S} r_{jk}}{S}$ to denote the average number of nurses required per shift on day j and $\bar{r} = \dfrac{\sum_{j=1}^{D}\sum_{k=1}^{S} r_{jk}}{D}$ to denote the average number of nurses required per day. The coverage requirements of the nurse scheduling problem can be generated by means of three input parameters as follows. In a first step, the total number of required nurses will be generated which has a major influence on the constrainedness and hence on the feasibility of the NSP instance. In a second step, the total number of required nurses will be distributed among the days (day-coverage) and the shifts per day (shift-coverage).

3.3.1. Total-coverage constrainedness (TCC)

The *TCC* serves as an indicator to generate the total number of nurses required for the complete scheduling period (e.g. a week). The required number of nurses (i.e. the coverage requirements) as well as the different case-specific constraints have a major influence on the feasibility of the NSP instance under study. An NSP instance with only the coverage requirement constraints has a feasible schedule when the total daily coverage is lower than or equal to the number of nurses. Hence, the maximal allowable daily coverage is equal to the number of nurses in the instance, and higher coverage values will lead to infeasible solutions. However, the feasibility can no longer be guaranteed when the NSP instance is subject to additional constraints. For these instances, the total coverage needs to be decreased (lower than the number of nurses) and, therefore, can only be a fraction of the maximal coverage. Note that Koop (1988) has discussed lower bounds for the workforce size on the multiple shift manpower scheduling problem by taking both the minimal number of required working shifts (i.e. the coverage constraints) as well as other case-specific constraints into account.

The *TCC* $\in [0, 1]$ can be calculated as $TCC = \frac{\sum_{j=1}^{D}\sum_{k=1}^{S} r_{jk}}{ND} = \frac{\bar{r}}{N}$.

The total-coverage constrainedness (*TCC*) is measured as the average number of nurses required per day divided by the number of nurses. The *TCC* is measured as a fraction of the maximal coverage requirements (when the *TCC* = 1, the total daily coverage equals the number of nurses). When the NSP instance is the subject to additional constraints, the

TCC needs to be lower than or equal to 1 to guarantee feasibility. The *TCC* value of table 3 equals (3 + 3 + 3 + 1 + 2 + 3 + 4 + 1 + 1 + 0 + 9 + 0) / (15 * 4) = 0.5.

3.3.2. Day-coverage distribution (DCD)

The *TCC* determines the total coverage requirement for the complete scheduling period but does not assign requirements to individual days or shifts. The *DCD* divides the total coverage requirement (obtained by the *TCC* measure) among days in a controlled way as follows:

$$DCD = \frac{\alpha_w^{DCD}}{\alpha_{\max}^{DCD}} = \frac{\sum_{j=1}^{D} \left| \sum_{k=1}^{S} r_{jk} - \bar{r} \right|}{\alpha_{\max}^{DCD}}$$

The *DCD* is similar to the *SPD* indicator as explained in appendix A. α_w^{DCD} measures the total absolute deviation of a one-day coverage $\sum_{k=1}^{S} r_{jk}$ from the total average coverage requirement over all days. Moreover, $\alpha_{\max}^{DCD}$ is used to denote the maximal possible value of α_w^{DCD}. Similar to the *SPD* indicator of section 3.2.1, we divide α_w^{DCD} by $\alpha_{\max}^{DCD}$ to make sure that *DCD* lies between zero and one, inclusive. The value for $\alpha_{\max}^{DCD}$ depends on the maximal allowable value for $\sum_{k=1}^{S} r_{jk}$ (which is equal to N) and is equal to $\left(N - 2\bar{r}\right) \cdot \left\lfloor \frac{D\bar{r}}{N} \right\rfloor - \bar{r}(1 - D) + \left| \left(D\bar{r}\right) \operatorname{mod}\left(N\right) - \bar{r} \right|$ (see appendix A).

DCD measures whether the daily coverage is distributed equally over all days, and does not measure the intra-day coverage requirement over the shifts. When *DCD* is equal to 0, the coverage requirements are equally distributed among all days. When *DCD* equals 1, the coverage requirements are maximal for one or several days (depending on the *TCC* value), and zero for all remaining days. The *DCD* value of table 3 equals *DCD* = (|9 – 7.5| +

|6 – 7.5| + |6 – 7.5| + |9 – 7.5|) / ((15 – 15) * $\left\lfloor \frac{4*7.5}{15} \right\rfloor$– 7.5 * (1-4) + |(4 * 7.5)mod(15) – 7.5|) = 6 / 30 = 0.2.

3.3.3. Shift-coverage distribution (SCD)

While the *DCD* measures the daily coverage distribution over the complete scheduling problem, the *SCD* does the same, on a shift level. More precisely, the *SCD* measures the distribution of the coverage requirements over all shifts for a particular day *j*. Although the *SCD* indicator can differ per day, we assume that the shift-coverage distribution is equal for all days.

The *SCD* $\in$ [0, 1] can be calculated as $SCD = \frac{\alpha_w^{SCD}}{\alpha_{\max}^{SCD}} = \frac{\sum_{k=1}^{S} \left| r_{jk} - \overline{r_j} \right|}{\alpha_{\max}^{SCD}}$.

Similar to *SPD* and *DCD*, α_w^{SCD} measures, for a given day *j*, the total absolute deviation of all shift coverage requirements r_{jk} from the total average coverage requirement of that day (which is a result of the *DCD* calculations). $\alpha_{\max}^{SCD}$ is used to denote the maximal possible value of α_w^{SCD} and depends on the maximal allowable value for r_{jk}. This maximal value equals *N*, and can never be exceeded since the *DCD* calculations guaranteed that $\sum_{k=1}^{S} r_{jk} \le N$. Therefore, no explicit upper value needs to be taken into consideration, and $\alpha_{\max}^{SCD}$ can be – according to appendix A – calculated as $2S\overline{r_j} - 2\overline{r_j}$. When *SCD* equals 0, the coverage requirements for a single day are equally distributed. When *SCD* equals 1, there is a single shift with a given coverage requirement (determined by the *DCD* indicator), while all other shifts on that day do not need nurses. The *SCD* values of table 3 equal 0, 0.25, 0.5 and 1 for day 1, 2, 3 and 4.

4. THE GENERATION PROCESS OF THE NSP INSTANCE GENERATOR *NSPGEN*

In this section, we describe a simple but efficient way to generate NSP instances with given values for the 9 indicators of section 3. In the remainder of this section, we refer to a 'shift vector' to denote one column of a preference matrix as shown in table 3. Moreover, we refer to a day matrix (week matrix) to denote a combination of shift vectors for a complete day (week). We describe our generation approach for weekly preference matrices, but it can easily be extended, without loss of generality, to larger scheduling periods.

The generation process boils down to the combination of randomly generated shift vectors with a pre-specified *NPD* value in order to obtain a preference structure with known values for all the indicators. This process is followed by an improvement step, until a pre-specified value for *SPD* and *DPD* is obtained. The pseudo-code to generate periodically (e.g. weekly) preference matrices with a given value for *NPD*, *SPD* and *DPD* (denoted as *NPD'*, *SPD'* and *DPD'*) is given below.

```
Procedure Generate instance (NPD', SPD', DPD')
      Initialize d1 = 0
      Step 1. Construct NPD' shift vectors
      Randomly generate C1 shift vectors with NPD'-value
      Step 2. Construct D1 SPD' day matrices
      Select randomly one shift vector c1 from the C1 vectors
      Construct a day matrix with SPD = 0 (i.e. all shifts equal c1)
      Set SPDold = 0
      For k = 2 to S
            For c2 = 1 to C1
                  Replace shift vector c1 with vector c2 for shift k
                  and calculate SPDnew
                  If |SPD' - SPDnew| < |SPD' - SPDold| then
                        Replace vector c1 with vector c2
                        SPDold = SPDnew
      Save the day matrix and set d1 = d1 + 1
      If d1 < D1 repeat step 2
      Step 3. Construct W1 DPD' week matrices
      Select randomly one day matrix d1 from the D1 matrices
      Construct a week matrix with DPD = 0 (i.e. all days equal d1)
      Set DPDold = 0
      For j = 2 to D
            For d2 = 1 to D1
                  Replace day matrix d1 with matrix d2 for day j
                  and calculate DPDnew
                  If |DPD' - DPDnew| < |DPD' - DPDold| then
```

```
                    Replace matrix d1 with matrix d2
                    DPDold = DPDnew
      Save the week matrix and set w1 = w1 + 1
      If w1 < W1 repeat step 3
      Step 4. Improvement SPD' and DPD'
      Select the best found week matrix with known SPD and DPD value
      For j = 1 to D
            For k = 1 to S
                  For i1 = 1 to N
                        For i2 = i1 to N
                              Swap(pi1,j,k, pi2,j,k)
                              If SPD or DPD improves, save new week
      matrix
Return
```

The first step randomly generates C_1 shift vectors with a *NPD*-value as close as possible to *NPD'* (a shift vector is one column of a week preference matrix as given in table 3). In step 2, the procedure combines these vectors to generate a day preference matrix with a given value for *SPD'*. To that purpose, the procedure randomly selects one shift vector and creates a day matrix with a *SPD*-value of 0. This day matrix contains the selected shift vector for all shifts of the day. The procedure aims at improving the *SPD*-value by replacing the shift vectors one at a time with the other generated shift vectors of step 1. Each time an improvement has been made (i.e. the newly found *SPD* value lies closer to the pre-specified *SPD'*-value), the new shift vector replaces the old one in the day matrix. The best found day matrix will be saved and this process will be repeated until D_1 day matrices are obtained. In step 3, these day matrices, on their turn, are used to combine them to week matrices with a given *DPD'*-value in a similar way as the construction of the day matrices. Instead of scanning all the shifts (step 2), the procedure scans all the days to find good combinations of day matrices to result in week matrices with a *DPD*-value close to *DPD'*. This process is repeated until W_1 week schedules are obtained. Step 4 selects the best week matrix found and aims at improving the *DPD* and *SPD* value. More precisely, the procedure swaps individual nurse preferences for each day and each shift in order to look for improvement for *SPD* and *DPD*. The *NPD* value remains unchanged during this step, since swaps are made within one single shift. Computational tests have revealed that a stop criterion $C_1 = D_1 = W_1 = 100$ for each step results in a fast and efficient procedure with excellent performance for all indicators. All preference matrices are extended with coverage requirements by the controlled random generation of numbers with known values for the *TCC*, *SCD* and *DCD* indicators.

In their overview papers, Cheang et al (2003) and Burke et al (2004) call the importance of benchmark instances to test the exact and/or (meta-)heuristic procedures for the nurse scheduling problem. In doing so, a benchmark dataset that is shared among the research community facilitates the systematic evaluation and comparison of the performance of the different procedures. The proposed instance generator *NSPGen* is a useful tool to generate these instances based on the set of complexity indicators proposed in section 3. Hence, a library of NSP instances (NSPLib) has been presented by Vanhoucke and Maenhout (2005) that is accessible by the research community. The benchmark instances have been grouped in six different sets and are characterized by systematically varied levels of all the complexity indicators. In total, NSPLib contains 7,290 problem instances with a one-week scheduling period and 1,920 instances with a one-month scheduling period. Each instance of the dataset can be extended by a particular set of case-specific constraints. More precisely, the user can choose among 16 sets of possible constraints, where each set consists of a combination of constraints identified by Cheang et al (2003) as appearing frequently in literature. For more details about the proposed benchmark set, we refer the reader to Vanhoucke and Maenhout (2005). The problem instances and the corresponding case constraint files can be downloaded from www.projectmanagement.ugent.be/nsp.php.

5. COMPUTATIONAL EXPERIMENTS

5.1. Preliminary test results

Small sized nurse scheduling problems can be solved using branch and bound procedures typically provided with commercially available software. We have programmed a simple IP model to solve small instances of the NSP procedures in Visual C++ version 6.0 and run it on a Toshiba personal computer with a Pentium IV 2.4 GHz processor under Windows XP. The model has been linked with the industrial LINDO optimization library version 5.3 (Schrage, 1995). In this paper, we do not have the intention to present an efficient model to solve large-sized realistic nurse scheduling problem instances. Instead, we aim at detecting the influence of preference structures and coverage requirements with given values of the pre-defined indicators on the problem complexity. To that purpose, we have generated small-sized NSP instances with the complexity indicators as given in table 4. Using 5 instances for each problem class, we obtain 46,875 data instances.

Insert Table 4 about here

The simple IP model contains the following case-specific constraints:

- Number of assignments per nurse equals 5
- minimum 2 consecutive working days per nurse
- minimum 2 identical consecutive working shifts per nurse

The hardness of a problem instance is typically measured by the amount of CPU-time that a solution procedure needs to find an exact solution for the problem at hand. It is therefore of great importance to possess a set of problem characteristics that discriminates between easy and hard instances and that acts as a predictor of the computational effort of the procedures. If good predictions of the required CPU-time for different solution procedures were available, it would be possible to a priori select the fastest solution procedure, based on the simple calculation of these problem characteristics. The following tables display the average CPU time (Avg.) required to solve the problem instances to optimality and the number of instances for which a feasible solution exists (#Sol). Each table contains the required CPU time for the complexity indicators (either preference related or coverage related) and the number of instances that could be solved to optimality within a pre-determined time of 180 seconds.

Table 5 displays the one-dimensional effect of the six indicators on the required CPU-time to solve the problem instances to optimality. Tables 5, 6, 7 and 8 clarify the effects of the different indicators on problem complexity (measured by the CPU-time).

Insert Table 5 about here

The effect of *NPD* shows an increasing pattern on the CPU-time. Indeed, the more nurses express an identical preference for a particular shift, the more conflicts between nurses exist, resulting in an increasing problem complexity.

The *SPD* shows a hard-easy-hard transition effect, and has been further explained in combination with *NPD*, as shown in table 6. For low *NPD* values, the table shows a decreasing effect for increasing *SPD*-values. In general, low *NPD* values results in a few conflicts between nurses, and hence, in a rather easy schedule. Combined with high *SPD* values results in a clear nurse preference for each shift of the day and hence, there are not much conflicts, neither between the different nurses nor between their shift preferences on each day. However, a low *SPD* value means that nurses are indifferent between shifts, and hence it is not a priori clear which shift assignment is best for each nurse. The table shows an opposite behaviour for high *NPD* values (which result in a higher complexity anyway). Both low and high *SPD* values results in a conflict between nurses. In the former, all nurses have an identical preference for all shifts, and hence, a switch for a nurse to another shift does not influence the total preference cost but might resolve some case-specific constraint violations. High *SPD* values results in a clear conflict between the shifts, since all nurses express an identical preference for each shift. As a result, the (preference) cost of switching a particular nurse to his/her second or third choice results in an increase of the preference cost. The effect between *NPD* and *DPD* shows a similar behaviour as table 6, although somewhat less outspoken, as shown in table 7.

Insert Table 6 and Table 7 about here

The *TCC* measures the constrainedness of the problem instances, and has a positive correlation with problem complexity. If more nurses are required by the hospital, then the freedom to schedule a subset of nurses on a particular shift/day without violating case-specific constraints is dramatically reduced. Increasing values for *DCD* result in an increasing complexity. Table 8 further clarifies the effect of *DCD* on the CPU-time, in combination with the different settings for the *TCC* complexity indicator.

Insert Table 8 about here

This table reveals that the *DCD* has a negative impact on the CPU-time for low *TCC*-values, and an opposite behaviour for medium or high values for *TCC*. Low *TCC*-values and high *DCD* values result in tight coverage requirements for a small number of days while all other days do not require any nurses. Hence, only a small number of days are constrained, which results in much freedom to schedule the nurses over the complete time horizon. On the other hand, high *DCD* values with high *TCC* values result in tight coverage requirements for almost all days. Hence, a careful trade-off needs to be made to schedule the nurses without violating many case-specific constraints.

The effect of *SCD* shows, in general, an easy-hard-easy transition, i.e. an increasing (from low to medium values) followed by a decreasing (from medium to high values) effect, on the CPU-time. Low *SCD* values mean that the coverage requirements are almost equally distributed among the shifts, and hence, the probability of violating case-specific constraints (like the consecutiveness constraints) is rather low. As the *SCD* value goes up, violating these constraints is more likely, resulting in a higher problem complexity. However, large values for *SCD* either results in easy schedules or infeasible schedules, which explains the decreasing trend of the CPU-time. Indeed, all daily nurse requirements occur on one single shift, which results in either an almost unconstrained problem instance (there is no much choice than assigning nurses to this shift) or infeasible instances (due to the limited choice and the consecutiveness and succession constraints, no feasible assignment can be found). In appendix C, we tested the significance of the mean differences by means of a one-way ANOVA test. Moreover, we extended the table by a post hoc analysis to detect which mean values are different. The appropriate post hoc test was selected based on the homogeneity of variances indicated by Levene's test for homogeneity of variances.

5.2. A CHAID regression tree

In order to gain further insights on the influence of the proposed indicators on problem complexity, we clustered our data instances based on the required CPU-time in a way that reduces variation. This categorization is performed using a decision tree based on the CHAID algorithm (Chi-square automatic interaction detection (Kass, 1980)) of which the goal is to create a concise model which a priori predicts the hardness of new problem instances based on their characteristics. CHAID investigates the effects of independent variables on a dependent variable. Starting with all observations in a single group and a set of independent variables, the observations are subsequently split into two or more groups by the same or an alternative independent variable until further splitting will not reduce the variation of the dependent variable. The splitting is performed by the independent variable that is judged to be most important in reducing the total variation in the dependent variable. The criterion for evaluating a splitting rule is based on a statistical significance test, namely an F-test with a p-value of 0.05 as a stopping rule. Furthermore, the split is performed subject to a limit of 5 branches (which is equal to the (maximal) number of different values for each indicator of table 4) and a limit of minimal 10 observations assigned to each branch. For these criteria, the best split is the one with the smallest p-value. The decision tree, presented in appendix D, is created using a training data set (records sampled from the entire dataset of table 4) and validated on a testing dataset (remaining records). The dataset was randomly split into a training and a testing dataset, 70 – 30 respectively. For each split the splitting variable and the resulting branches with their corresponding split values are indicated. The tree counts 29 splitting nodes and 49 end nodes (leaves) which are designated by a number. The descriptive statistics (average; standard deviation) for the leaves are indicated below the tree.

In order to visualize the discriminative power and to give an indication of the predictive power of the decision tree, we generated new data instances by NSPGen based on the values for the complexity indicators for 10 different end nodes. Furthermore, we generated data instances with neighbouring values for some of these classes. The data instances of classes 11, 12, 13 and 14 are created in the vicinity of classes 2, 3, 9 and 10, respectively (and hence, the “end node” column values correspond to each other). The values for the complexity indicators for newly generated problem instances for the designated end nodes are indicated in table 9.

Insert Table 9 about here

The required CPU-time to solve these data instances to optimality are presented in figure 1. This figure reveals that instances within a class are rather homogeneous with respect to the computational performance, while the CPU-time varies between different classes. Furthermore, instances generated with neighbouring values for the complexity indicators have more or less the same behaviour in computational complexity. Comparing the required CPU-time to solve the newly generated data instances to optimality with the computational results upon which the decision tree in appendix D is built exposes the predictive power of the decision tree and the proposed complexity indicators.

Insert Figure 1 about here

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6. CONCLUSIONS

In this paper, we presented three classes of indicators to characterise nurse scheduling problem instances. The first class describes the size of the problem instances, measured by the number of nurses, number of shifts and number of days of the roster matrix. The second class consists of three indicators to characterise the preference structure of the roster matrix. The last class represents the coverage constraints of the roster.

We have presented a simple, yet efficient generation approach to generate NSP instances with given values for the aforementioned indicators. This generator allows researchers to generate instances that can be used to test existing and newly developed state-of-the-art procedures. Moreover, the generator has been used by Vanhoucke and Maenhout (2005) to create a benchmark dataset in order to facilitate future research comparison of newly developed procedures. Both the generator and the benchmark dataset can be downloaded from www.projectmanagement.ugent.be/nsp.php.

Finally, we have used a straightforward IP model to test the influence of the proposed complexity indicators on the complexity of the nurse scheduling problem instances. The results show promising effects of the indicator settings on the required CPU-time to solve the problem instances, both for the preference related and the coverage related indicators. Inspired by these results, we would like to call for using the proposed dataset to test newly developed procedures to facilitate comparison with current state-of-the-art procedures.

Our future intensions are threefold. First, we will investigate the explanatory and predictive power of the proposed indicators in depth. Based on the preliminary results of section 5, we believe that the indicators can predict problem complexity into more detail (e.g. by investigating three-or-more dimensional effects or a more detailed investigation of the classification trees). Classification and regression trees have been successfully used in other areas of health care (Smith et al, 1992; Harper and Shahani, 2002; Garbe et al, 1995; Ridley et al, 1998; Harper and Winslett, 2006). These trees classify individual observations in groups based on simple splitting rules and allow the prediction of the outcome of interest of new observations based on known parameter values of the associated class. Secondly, we want to develop new approaches to solve the NSP and compare existing state-of-the-art procedures on our dataset. In doing so, we will investigate the occurrence of phase transitions in nurse scheduling problems that give an indication of dramatic changes in problem complexity. In doing so, we can a-priori select the fastest and best solution procedures based on some simple calculations of the indicators. Last, we will investigate the influence of different case-specific constraints on the performance of an algorithm and the influence of the constructed schedule. The relation between the proposed indicators and the specific constraints might reveal some interesting results.

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APPENDIX A

The complexity indicators

In this appendix, we define a general measure of variance that will be used to describe three complexity indicators in the paper. The *measure of variance* is defined as $\upsilon = \frac{\alpha_w}{\alpha_{\max}} = \frac{\sum_{t=1}^{m} \left| x_t - \bar{x} \right|}{\alpha_{\max}}$ with $\bar{x} = \frac{\sum_{t=1}^{m} x_t}{m}$ the average value of all x_t's. Consequently, it measures the distribution of all x_t values (t = 1, …, m) by calculating the total absolute deviations α_w and α_{max}. α_w measures the total absolute deviation of all x_t values (i.e. (x_1, x_2,…x_m)) from the average $\bar{x} = \frac{\sum_{t=1}^{m} x_t}{m}$ as follows: $\alpha_w = \sum_{t=1}^{m} \left| x_t - \bar{x} \right|$. α_{max} is used to denote the maximal possible value of α_w. By dividing α_w by α_{max}, we make sure that our measure of variance lies between zero and one inclusive. The maximal deviation α_{max} depends on the maximal allowable value (u) of each variable x_t. α_{max} can be shown to be equal to

$$\alpha_{\max} = \left(u - \bar{x}\right) \cdot \left\lfloor \frac{\sum_{t=1}^{m} x_t}{u} \right\rfloor + \left| \left(\sum_{t=1}^{m} x_t \right) \operatorname{mod}(u) - \bar{x} \right| + \left(\bar{x}\right) \cdot \left(n - 1 - \left\lfloor \frac{\sum_{t=1}^{m} x_t}{u} \right\rfloor \right) \qquad [1]$$

If no constraining maximal x_t values (i.e. $u = \sum_{t=1}^{m} x_t$) are imposed, this formula collapses to $\alpha_{\max} = (\sum_{t=1}^{m} x_t - \bar{x}) + (m-1) * \bar{x}$. [2]

This occurs in a situation where one of the x_t's is at its maximum value of $u = \sum_{t=1}^{m} x_t$ (first term) and all the other (m - 1) terms are equal to zero. The intuition behind the general formula for α_{max} [1] is as follows. The first term measures the deviation for all x_t's that can be put at their maximum value of u. The second term measures the variance for the x_t, if any, with a value between $\bar{x}$ and u. The third term sets the remainder of the x_t's to zero.

The measure of variance is used for three complexity indicators, i.e. the *NPD*, the *DCD* and the *SCD*.

A.1 The NPD

The *NPD* distributes the different preferences l (from 1 to S) among nurses, and therefore, $x_t = \sum_{i=1}^{N} y_{il}$ (i.e. $x_1 = \sum_{i=1}^{N} y_{i1}$, $x_2 = \sum_{i=1}^{N} y_{i2}$, …, $x_D = \sum_{i=1}^{N} y_{iS}$), $\bar{x} = \frac{N}{S}$ and $m = S$. The total sum of all nurses for the different preferences, i.e. $\sum_{l=1}^{S}\sum_{i=1}^{N} y_{il}$ equals N and therefore, no explicit upper value needs to be imposed (the upper value u of $\sum_{i=1}^{N} y_{il}$ equals N, which is always guaranteed since $\sum_{l=1}^{S}\sum_{i=1}^{N} y_{il} = N$). Consequently, equation [1] collapses to equation [2] and is equal to $2N - \frac{2N}{S}$

A.2 The DCD

The *DCD* distributes the total coverage requirement for a complete scheduling period to the individual days, and therefore, $x_t = \sum_{k=1}^{S} r_{jk}$ (i.e. $x_1 = \sum_{k=1}^{S} r_{1k}$, $x_2 = \sum_{k=1}^{S} r_{2k}$, …, $x_D = \sum_{k=1}^{S} r_{2k}$), $\bar{x} = \bar{r} = \frac{\sum_{j=1}^{D}\sum_{k=1}^{S} r_{jk}}{D}$ and $m = D$. The maximal coverage per day $\sum_{k=1}^{S} r_{jk}$ equals N ($u = N$) and hence equation [1] collapses to

$$\alpha_{\max} = \left(N - \bar{r}\right)\cdot\left\lfloor \frac{D\bar{r}}{N} \right\rfloor + \left|\left(D\bar{r}\right)\operatorname{mod}\left(N\right) - \bar{r}\right| + \left(\bar{r}\right)\cdot\left(D - 1 - \left\lfloor \frac{D\bar{r}}{N} \right\rfloor\right)$$

$$= \left(N - 2\bar{r}\right)\cdot\left\lfloor \frac{D\bar{r}}{N} \right\rfloor - \bar{r}(1 - D) + \left|\left(D\bar{r}\right)\operatorname{mod}\left(N\right) - \bar{r}\right|$$

A.3 The SCD

The *SCD* distributes the daily coverage requirement to the individual shift, and therefore, x_t $= r_{jk}$ (i.e. $x_1 = r_{j1}$, $x_2 = r_{j2}$, …, $x_S = r_{jS}$), $\overline{x} = \overline{r_j} = \frac{\sum_{k=1}^{S} r_{jk}}{S}$ and $m = S$. The maximal value *u* for r_{jk} but is always guaranteed since $\sum_{k=1}^{S} r_{jk} \leq N$. Therefore, no explicit upper value needs to be taken into consideration, and equation [2] collapses to $2S\overline{r_j} - 2\overline{r_j}$.

APPENDIX B

Example preference matrices with extreme settings for *NPD/SPD/DPD*

0/0/0	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2
3	2	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1
7	3	3	3	3	3	3	3	3	3
8	3	3	3	3	3	3	3	3	3
9	3	3	3	3	3	3	3	3	3

0/0/1	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	2	2	2	1	1	1	3	3	3
2	2	2	2	1	1	1	3	3	3
3	2	2	2	1	1	1	3	3	3
4	1	1	1	3	3	3	2	2	2
5	1	1	1	3	3	3	2	2	2
6	1	1	1	3	3	3	2	2	2
7	3	3	3	2	2	2	1	1	1
8	3	3	3	2	2	2	1	1	1
9	3	3	3	2	2	2	1	1	1

0/1/0	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	2	1	3	2	1	3	2	1	3
2	2	1	3	2	1	3	2	1	3
3	2	1	3	2	1	3	2	1	3
4	1	3	2	1	3	2	1	3	2
5	1	3	2	1	3	2	1	3	2
6	1	3	2	1	3	2	1	3	2
7	3	2	1	3	2	1	3	2	1
8	3	2	1	3	2	1	3	2	1
9	3	2	1	3	2	1	3	2	1

0/1/1	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	2	3	1	1	2	3	3	1	2
2	2	3	1	1	2	3	3	1	2
3	2	3	1	1	2	3	3	1	2
4	1	2	3	3	1	2	2	3	1
5	1	2	3	3	1	2	2	3	1
6	1	2	3	3	1	2	2	3	1
7	3	1	2	2	3	1	1	2	3
8	3	1	2	2	3	1	1	2	3
9	3	1	2	2	3	1	1	2	3

1/1/0	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	1	2	3	1	2	3	1	2	3
2	1	2	3	1	2	3	1	2	3
3	1	2	3	1	2	3	1	2	3
4	1	2	3	1	2	3	1	2	3
5	1	2	3	1	2	3	1	2	3
6	1	2	3	1	2	3	1	2	3
7	1	2	3	1	2	3	1	2	3
8	1	2	3	1	2	3	1	2	3
9	1	2	3	1	2	3	1	2	3

1/0/1	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	2	2	2	3	3	3	1	1	1
2	2	2	2	3	3	3	1	1	1
3	2	2	2	3	3	3	1	1	1
4	2	2	2	3	3	3	1	1	1
5	2	2	2	3	3	3	1	1	1
6	2	2	2	3	3	3	1	1	1
7	2	2	2	3	3	3	1	1	1
8	2	2	2	3	3	3	1	1	1
9	2	2	2	3	3	3	1	1	1

1/1/0	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	1	2	3	1	2	3	1	2	3
2	1	2	3	1	2	3	1	2	3
3	1	2	3	1	2	3	1	2	3
4	1	2	3	1	2	3	1	2	3
5	1	2	3	1	2	3	1	2	3
6	1	2	3	1	2	3	1	2	3
7	1	2	3	1	2	3	1	2	3
8	1	2	3	1	2	3	1	2	3
9	1	2	3	1	2	3	1	2	3

1/1/1	Day 1			Day 2			Day 3		
Nurse	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**	**S1**	**S2**	**S3**
1	1	2	3	3	1	2	2	3	1
2	1	2	3	3	1	2	2	3	1
3	1	2	3	3	1	2	2	3	1
4	1	2	3	3	1	2	2	3	1
5	1	2	3	3	1	2	2	3	1
6	1	2	3	3	1	2	2	3	1
7	1	2	3	3	1	2	2	3	1
8	1	2	3	3	1	2	2	3	1
9	1	2	3	3	1	2	2	3	1

APPENDIX C

ANOVA table for the test results of sectio 4.2 (the significance of the meand differences by means of one-way ANOVA-test)

	NPD	*SPD*	*DPD*	*SCD*	*DCD*
ANOVA	< 0.001**	< 0.001**	< 0.001**	< 0.001**	< 0.001**
Levene's Test	< 0.001**	< 0.001**	< 0.001**	< 0.001**	< 0.001**
Post Hoc					
0 vs 0.25	0.992	< 0.001**	0.057	0.524	1.000
0 vs 0.5	0.903	0.865	0.147	< 0.001**	0.013*
0 vs 0.75	< 0.001**	0.628	< 0.001**	< 0.001**	< 0.001**
0 vs 1	< 0.001**	< 0.001**	< 0.001**	1.000	< 0.001**
0.25 vs 0.5	1.000	< 0.001**	< 0.001**	< 0.001**	0.070
0.25 vs 0.75	< 0.001**	< 0.001**	< 0.001**	0.116	< 0.001**
0.25 vs 1	< 0.001**	< 0.001**	< 0.001**	0.877	< 0.001**
0.5 vs 0.75	< 0.001**	1.000	0.201	0.797	0.018*
0.5 vs 1	< 0.001**	< 0.001**	0.070	< 0.001**	< 0.001**
0.75 vs 1	< 0.001**	< 0.001**	1.000	< 0.001**	0.268

	TCC
ANOVA	< 0.001**
Levene's Test	< 0.001**
Post Hoc	
0.2 vs 0.35	0.006*
0.2 vs 0.5	< 0.001**
0.35 vs 0.5	< 0.001**

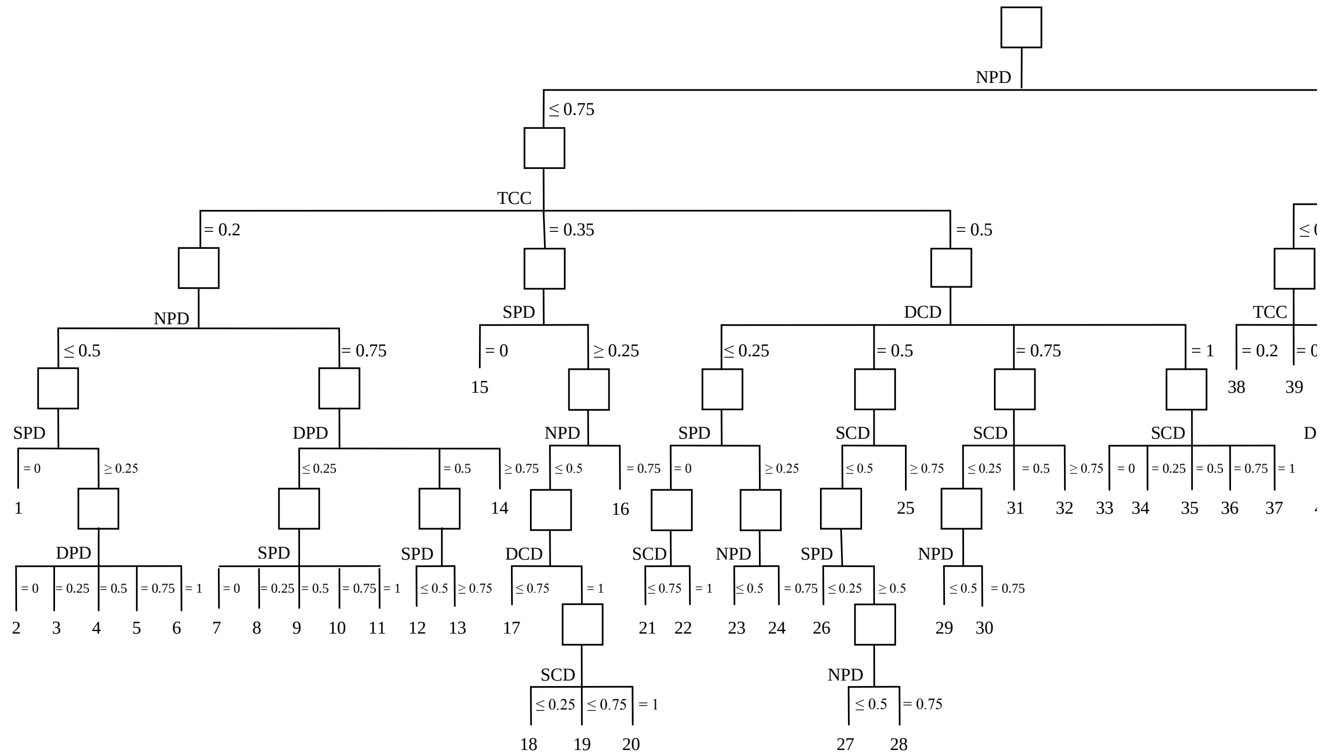
* The p-value is smaller than 0.05

** The p-value is smaller than 0.01

(a) An LSD test or Dunnett T3 test is used as a Post Hoc Test whether the H0 hypothesis of the Levene's test for homogenity of variances is respectively accepted or rejected

APPENDIX D

Decision tree: The resulting CHAID regression tree



1: (2.21; 4.59)
2: (0.50; 3.17)
3: (0.29; 0.82)
4: (1.60; 9.58)
5: (0.94; 5.26)
6: (0.71; 3.17)
7: (2.17; 1.95)
8: (0.73; 1.43)
9: (1.40; 4.83)
10: (0.96; 1.51)
11: (1.27; 2.71)
12: (1.61; 5.99)
13: (13.73; 41.71)
14: (3.37; 15.81)
15: (4.95; 7.94)
16: (4.51; 19.72)
17: (1.32; 7.23)
18: (1.04; 3.39)
19: (8.46; 35.50)
20: (0.50; 1.13)
21: (7.24; 2.51)
22: (4.15; 3.55)
23: (2.49; 10.59)
24: (5.62; 16.46)
25: (11.13;34.96)
26: (8.00; 20.86)
27: (2.27; 10.58)
28: (7.92; 22.69)
29: (3.53; 10.15)
30: (9.44; 25.22)
31: (15.11; 43.90)
32: (24.18; 54.15)
33: (9.84; 35.34)
34: (16.96; 47.76)
35: (40.32; 72.99)
36: (12.19; 41.79)
37: (0.73; 2.18)
38: (1.70; 2.84)
39: (6.45; 21.77)
40: (9.04; 23.57)
41: (23.94; 55.73)
42: (4.69; 14.91)
43: (9.43; 26.08)
44: (14.38; 35.91)
45: (18.90; 44.73)
46: (9.11; 21.68)
47: (18.93; 48.88)
48: (36.28; 66.40)
49: (47.26;71.89)

Node number: (average CPU; standard deviation)

TABLE 1

The three classes of indicators measuring the size, preferences and coverage requirements of a NSP isntance

Size	**Preferences**	**Coverage**
Number of nurses	Preference distribution among nurses (*NPD*)	Total number of nurses required (*TCC*)
Number of shifts in a day	Preference distribution over all shifts (*SPD*) (for each day)	Distribution of required number of nurses over all shifts (*SCD*) (for each day)
Number of days in a complete scheduling period	Preference distribution over all days (*DPD*) (for the scheduling period)	Distribution of required number of nurses over all days (*DCD*) (for the scheduling period)

TABLE 2

The different *NPD* scenarios for 15 nurses and three shifts

k	***NPD***	***l* = 1**	***l* = 2**	***l* = 3**
1	0.80	13	1	1
2	0.50	3	2	10
3	0.20	5	3	7

TABLE 3

The four-days preference matrix (top) and coverage requirements (bottom) with known values for the different indicators

Day 1			Day 2			Day 3			Day 4		
S1	S2	S3	S1	S2	S3	S1	S2	S3	S1	S2	S3
3	3	3	2	3	3	1	3	2	2	1	3
1	1	1	2	2	3	1	1	3	2	3	1
3	3	3	1	1	1	1	3	3	2	3	1
3	2	2	2	2	2	1	1	2	2	1	3
3	3	3	2	1	1	3	3	1	3	2	2
3	1	1	2	1	1	1	1	2	2	1	3
3	3	3	2	2	3	1	3	2	2	1	3
3	3	2	2	1	1	1	1	2	2	1	3
3	3	1	2	1	1	1	3	3	2	1	3
3	3	1	2	1	2	1	1	3	2	3	2
3	3	3	3	3	3	1	3	2	2	1	1
3	3	3	2	1	2	1	1	3	1	2	2
3	1	1	2	1	1	1	1	1	2	1	1
3	3	3	2	1	1	2	3	1	2	1	2
2	2	2	2	1	2	1	1	2	2	1	3

3	3	3	1	2	3	4	1	1	0	9	0

TABLE 4

Test settings used for our computational tests

Problem size	
N	10
S	3 (including the free shift)
D	7
Preference distribution	
NPD	0, 0.25, 0.50, 0.75 or 1
SPD	0, 0.25, 0.50, 0.75 or 1
DPD	0, 0.25, 0.50, 0.75 or 1
Coverage constraints	
TCC	0.20, 0.35 or 0.50
DCD	0, 0.25, 0.50, 0.75 or 1
SCD	0, 0.25, 0.50, 0.75 or 1

TABLE 5

The effect of the indicators on the required CPU-time

	NPD		*SPD*		*DPD*		*SCD*		*DCD*	
	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol
0	3.101	8,380	5.231	8,365	5.007	8,418	4.885	9,367	4.569	9,375
0.25	3.321	8,405	4.090	8,400	4.182	8,425	5.436	9,108	4.690	9,120
0.5	3.416	8,382	5.630	8,389	5.822	8,366	6.942	8,380	5.504	8,471
0.75	5.406	8,379	5.750	8,404	6.696	8,346	6.347	7,908	6.663	7,761
1	13.296	8,387	7.842	8,375	6.848	8,378	4.980	7,170	7.689	7,206

	TCC	
	Avg.	#Sol
0.2	3.348	15,416
0.35	4.943	14,080
0.5	9.499	12,437

TABLE 6

The two-dimensional effect of *NPD* and *SPD* on the required CPU-time

NPD	**0**		**0.25**		**0.5**		**0.75**		**1**	
SPD	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol
0	4.625	1,680	5.557	1,683	5.199	1,665	5.141	1,659	5.630	1,678
0.25	3.785	1,668	3.494	1,674	3.266	1,688	3.259	1,686	6.640	1,684
0.5	2.372	1,688	2.935	1,678	2.506	1,682	5.230	1,664	15.136	1,677
0.75	2.303	1,684	2.199	1,695	3.236	1,671	6.163	1,681	14.914	1,673
1	2.425	1,660	2.422	1,675	2.887	1,676	7.228	1,689	24.208	1,675

TABLE 7

The two-dimensional effect of *NPD* and *DPD* on the required CPU-time

NPD	**0**		**0.25**		**0.5**		**0.75**		**1**	
DPD	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol
0	4.252	1,693	3.149	1,688	3.792	1,675	5.301	1,676	8.538	1,686
0.25	2.485	1,667	2.882	1,688	3.147	1,697	4.660	1,704	7.754	1,669
0.5	2.992	1,669	3.150	1,663	3.614	1,683	6.703	1,672	12.617	1,679
0.75	2.936	1,685	3.364	1,677	3.303	1,649	6.082	1,665	17.799	1,670
1	2.827	1,666	4.055	1,689	3.223	1,678	4.294	1,662	19.768	1,683

TABLE 8

The two-dimensional effect of *TCC* and *DCD* on the required CP-time

DCD	**0**		**0.25**		**0.5**		**0.75**		**1**	
TCC	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol	Avg.	#Sol
0.2	3.684	3,125	3.490	3,125	3.793	3,125	2.891	3,102	2.847	2,939
0.35	4.690	3,125	5.163	3,125	4.513	2,927	4.539	2,522	5.943	2,381
0.5	5.333	3,125	5.480	2,870	8.914	2,419	14.644	2,137	17.438	1,886

TABLE 9

10 classes of data instances with different indicator values

Class	NPD	SPD	DPD	TCC	SCD	DCD	End node
1	≤ 0.5	0	0 - 1	0.2	0 - 1	0 - 1	1
2	0.75	0.25	≤ 0.25	0.2	0 - 1	0 - 1	8
3	0.75	0 - 1	≥ 0.75	0.2	0 - 1	0 - 1	14
4	≤ 0.75	0	0 - 1	0.35	0 - 1	0 - 1	15
5	≤ 0.75	≤ 0.25	0 - 1	0.5	≤ 0.5	0.5	26
6	≤ 0.75	0 - 1	0 - 1	0.5	0	1	33
7	≤ 0.75	0 - 1	0 - 1	0.5	0.25	1	34
8	1	≤ 0.25	0 - 1	0.2	0 - 1	0 - 1	38
9	1	≤ 0.25	0 - 1	0.35	0 - 1	0 - 1	39
10	1	0.5 - 0.75	≥ 0.5	0 - 1	0 - 1	0 - 1	45
11	0.75 - 0.8	0.2 - 0.25	0.2 - 0.25	0.2	0 - 1	0 - 1	8
12	0.75 - 0.8	0 - 1	0.75	0.15 - 0.2	0 - 1	0 - 1	14
13	1	0.2 - 0.25	0 - 1	0.3 - 0.35	0 - 1	0 - 1	39
14	1	0.6	≥ 0.5	0 - 1	0 - 1	0 - 1	45

FIGURE 1

The required CPU-time for each class for the NSP