NASA TM-86292

NASA Technical Memorandum 86292

CHARACTERISTIC BOUNDARY CONDITIONS FOR THREE-DIMENSIONAL TRANSONIC UNSTEADY AERODYNAMICS

WOODROW WHITLOW, JR.

OCTOBER 1984

NASA
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

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       85N10915*#  ISSUE 2  PAGE 134  CATEGORY 2  RPT#: NASA-TM-86292 NAS
       1.15:86292  84/10/00  12 PAGES  UNCLASSIFIED DOCUMENT
UTTL: Characteristic boundary conditions for three-dimensional transonic
      unsteady aerodynamics
AUTH: A/WHITLOW, W., JR.
CORP: National Aeronautics and Space Administration. Langley Research Center,
      Hampton, Va.    AVAIL.NTIS   SAP: HC A02/MF A01
MAJS: /*AERODYNAMICS/*BOUNDARY VALUE PROBLEMS/*FAR FIELDS/*THREE DIMENSIONAL
      FLOW
MINS: / FINITE DIFFERENCE THEORY/ KERNEL FUNCTIONS/ MACH NUMBER/ TRANSONIC FLOW/
      WAVE EQUATIONS
ABA:  Author
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CHARACTERISTIC BOUNDARY CONDITIONS FOR THREE-DIMENSIONAL TRANSONIC UNSTEADY AERODYNAMICS

Woodrow Whitlow, Jr.
NASA Langley Research Center
Hampton, VA 23665

Introduction

The primary tool for three-dimensional transonic unsteady aerodynamic analysis is the XTRAN3S code.[1] In that code, finite difference methods are used to numerically solve the transonic small disturbance potential equation. Steady state far-field conditions are implemented on the boundaries of the computational region.

Using steady far-field conditions causes disturbances that are incident on the boundaries to be reflected back into the computational domain. The reflected disturbances can cause significant errors in the calculated aerodynamic loading. This work presents characteristic boundary conditions that reduce the reflected disturbances. The boundary conditions are extensions of those developed for two-dimensional flows by Engquist and Majda[2,3] and Whitlow.[4] Some representative results are presented to show the effectiveness of the characteristic conditions.

Symbols

c_{ℓ_α}	unsteady lift curve slope
c_r	reference chord
k	reduced frequency based on reference chord ($\omega c_r/U_\infty$)
M_∞	free stream Mach number
t	time
U_∞	free stream speed
x,y,z	streamwise, spanwise, and lateral coordinate directions
α	wing root angle of attack

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γ	ratio of specific heats
ϕ	transonic small disturbance velocity potential
ω	circular frequency

Unsteady Transonic Small Disturbance Potential Equation

The XTRAN3S code is used to solve the unsteady transonic small disturbance (TSD) potential equation

$$(C\phi_t + A\phi_x)_t = (E\phi_x + F\phi_x^2 + G\phi_y^2)_x + (\phi_y + H\phi_x\phi_y)_y + (\phi_z)_z \qquad (1)$$

where ϕ is a disturbance velocity potential normalized by $U_\infty c_r$, c_r is a wing reference chord, and U_∞ is free stream speed. The spatial coordinates, x, y, and z, are normalized by c_r, and time, t, is normalized by ω^{-1}, where ω is the circular frequency of unsteady motion. In (1),

$$C = k^2M_\infty^2$$

$$A = 2kM_\infty^2$$

$$E = 1 - M_\infty^2$$

The constants F, G, and H may be defined either as proposed in ref. 5

$$F = -\frac{1}{2}(\gamma+1)M_\infty^2$$

$$G = \frac{1}{2}(\gamma-3)M_\infty^2$$

$$H = -(\gamma-1)M_\infty^2$$

or as proprosed in ref. 6

$$F = -\frac{1}{2}\left[3-(2-\gamma)M_\infty^2\right]M_\infty^2$$

$$G = -\frac{1}{2}M_\infty^2$$

$$H = -M_\infty^2$$

In the original version of XTRAN3S, the boundary conditions on the outer edges of the computational domain are

$\phi = 0$, upstream

$\phi_x = 0$, downstream

$\phi_y = 0$, wing root

$\phi_y = 0$, side

$\phi_z = 0$, top and bottom

Formulation of the Characteristic Boundary Conditions

To formulate the characteristic far-field boundary conditions, the cross flow terms are dropped from the TSD potential equation. That is, G = H = 0, and (1) reduces to

$$C\phi_{tt} + A\phi_{xt} = B\phi_{xx} + \phi_{yy} + \phi_{zz} \tag{2}$$

where $B = E + 2F\phi_x$. Assuming B to be locally constant, the transformations

$$\xi = \frac{x}{\sqrt{B}}$$

$$\tau = \frac{A}{BD}x + \frac{2}{D}t$$

where $D^2 = 4C + \frac{A^2}{B}$, are used to transform (2) into the wave equation

$$\phi_{\tau\tau} = \phi_{\xi\xi} + \phi_{yy} + \phi_{zz} \tag{3}$$

A characteristic far-field boundary condition that satisfies (3) is[7]

$$\phi_\tau + \phi_r + \frac{\phi}{r} = 0 \tag{4}$$

where $r^2 = \xi^2 + y^2 + z^2$. In untransformed coordinates, (4) becomes

$$\frac{1}{2}\left(D - \frac{A}{B}\frac{x}{r}\right)\phi_t + \frac{x}{r}\phi_x + \frac{y}{r}\phi_y + \frac{z}{r}\phi_z + \frac{\phi}{r} = 0 \tag{5}$$

Allowing x to approach - ∞ with y and z finite, the following first order plane wave condition at the upstream boundary is obtained

$$\frac{1}{2}\left(\frac{A}{B} + \frac{D}{\sqrt{B}}\right)\phi_t - \phi_x = 0 \tag{6}$$

Letting $x \rightarrow +\infty$ with y and z finite results in the downstream condition

$$\frac{1}{2}\left(-\frac{A}{B} + \frac{D}{\sqrt{B}}\right)\phi_t + \phi_x = 0 \tag{7}$$

At the top and bottom boundaries, $z \rightarrow \pm\infty$ with x and y finite, and the boundary conditions are

$$\frac{D}{2}\phi_t \pm \phi_z = 0 \tag{8}$$

where + and - represent the top and bottom boundaries, respectively. Outboard of the wing tip, $y \rightarrow +\infty$ with x and z finite, resulting in the condition

$$\frac{D}{2}\phi_t + \phi_y = 0 \tag{9}$$

A symmetry condition, as implemented in the original XTRAN3S code, is imposed at the wing root. The boundary conditions are summarized in fig. 1.

Results

The boundary conditions were implemented in XTRAN3S and tested by calculating the unsteady force response for a flat plate rectangular wing with a pulse in wing root angle of attack α. All computations were made using the CDC CYBER 203 vector processor at the NASA Langley Research Center. The calculations were made for $M_\infty = 0.85$ on a 60 x 40 x 20 grid in x,y,z that extended $\pm 20c_r$ in x, $6.66c_r$ in y, and $\pm 10c_r$ in z. The extent of the grid in the z direction is less than that of the default grid, which extends $\pm 25c_r$ in z.[8] The wing tip was located at $3.33c_r$ in the y direction.

Using the pulse-transfer function technique of ref. 8, the frequency response function for the unsteady lift curve slope c_{ℓ_α} was calculated with and without the characteristic boundary conditions. Fig. 2a shows a comparison of the lift response calculated using steady-state far-field conditions with that obtained using a kernel function method.[9] Below $k \approx 0.6$, the XTRAN3S results have spurious oscillations due to disturbances reflected from the boundaries. When the characteristic conditions, (6)-(9), were implemented, the reflected disturbances were small. Fig. 2b shows that the oscillations in the unsteady lift response have been eliminated by using (6)-(9) on the boundaries.

The cause of the oscillations in the lift response obtained using steady state conditions can be seen in fig. 3. In the pulse-transfer function method, after α is increased to a maximum and returned to its starting value, calculation of the unsteady forces is continued until they return to their starting values. Fig. 3 shows that with steady state conditions implemented on the boundaries, the unsteady lift begins to decline toward zero but shows an increase beginning near a time when a disturbance has traveled approximately $20c_r$. This is the time required for a disturbance emanating from the wing to travel to the z boundaries and return. Thus, the increase in the lift at $U_\infty t/c_r \approx 20$ and the spurious oscillations in the XTRAN3S lift response in fig. 2a are attributed to disturbances reflected from the z boundaries. When the characteristic conditions are used, the lift history shows a smooth decline toward zero after the initial transient. As a result, the lift response in fig. 2b has no spurious oscillations.

For this study, no calculations were made for flows past wings with thickness. However previous calculations of two-dimensional transonic flows[4] indicate that the present characteristic boundary conditions would be effective in reducing reflected disturbances in those cases.

Concluding Remarks

Characteristic boundary conditions for the three-dimensional transonic unsteady small disturbance potential equation have been developed. They were implemented in the NASA Langley version of XTRAN3S and tested for a flat plate rectangular wing with a pulse in angle of attack. The unsteady lift response for calculations with and without the characteristic conditions shows that use of those conditions eliminates most of the disturbances reflected from the computational boundaries.

Acknowledgements

The author wishes to thank Drs. Samuel R. Bland, Robert M. Bennett, and John W. Edwards of NASA Langley for many useful discussions and Mr. David A. Seidel of NASA Langley for generating the figures used in this report.

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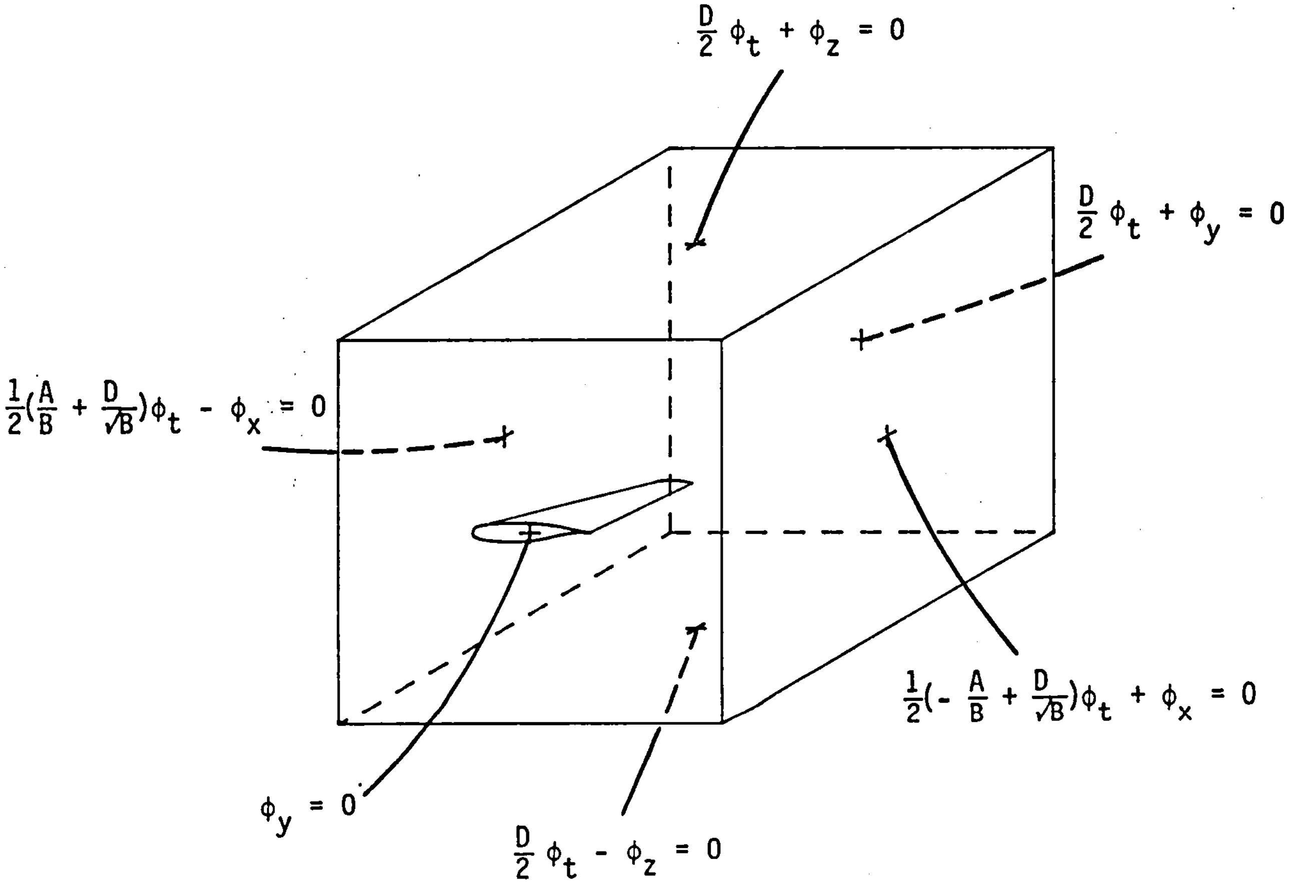
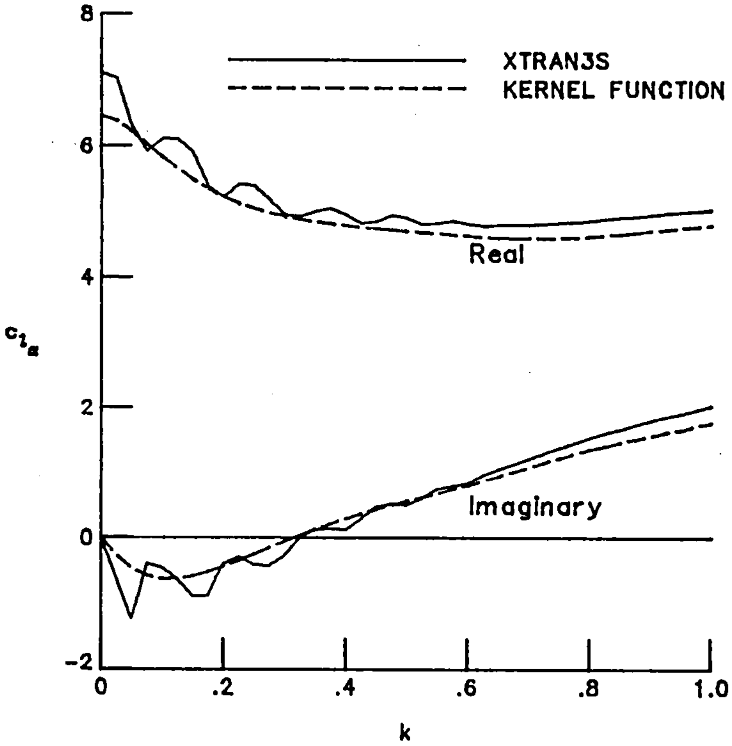
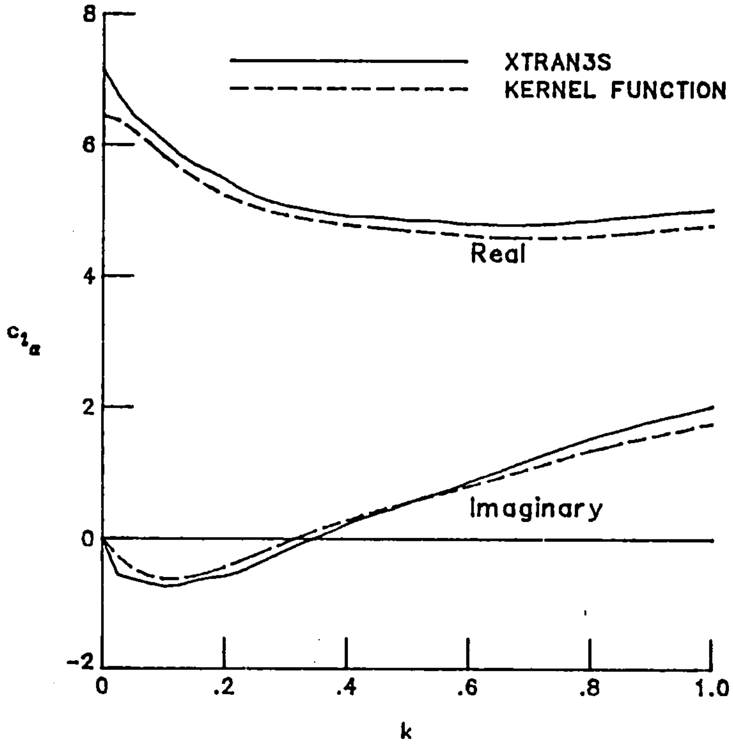


Figure 1. Characteristic boundary conditions.



(a) Steady state boundary conditions



(b) Characteristic boundary conditions

Figure 2. Force response for a flat plate rectangular wing, $M_\infty = 0.85$.

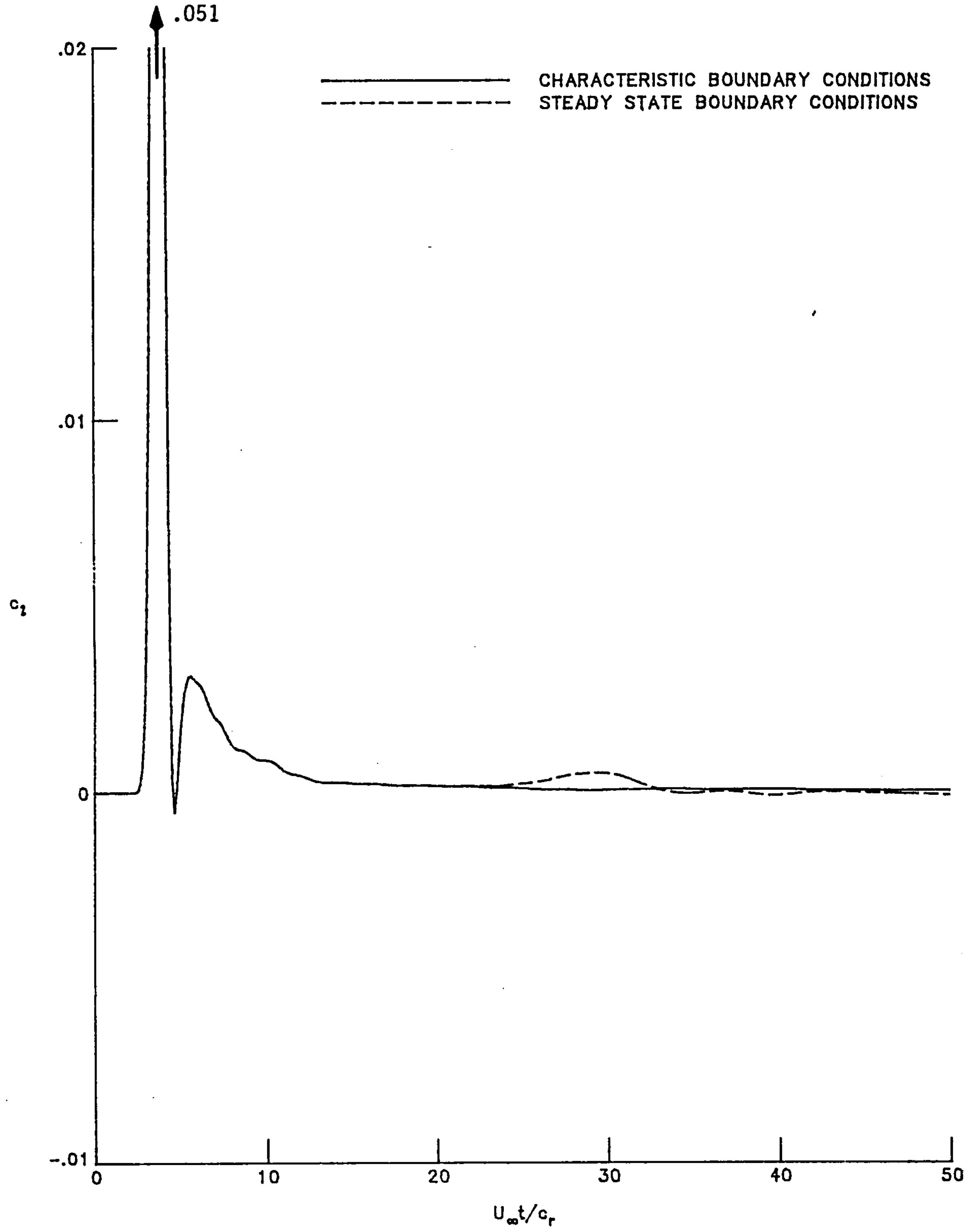


Figure 3. Time history of calculated lift, $M_\infty = 0.85$.

1. Report No. NASA TM-86292	2. Government Accession No.	3. Recipient's Catalog No.

4. Title and Subtitle Characteristic Boundary Conditions for Three-Dimensional Transonic Unsteady Aerodynamics	5. Report Date October 1984
	6. Performing Organization Code 505-33-43-09
7. Author(s) Woodrow Whitlow, Jr.	8. Performing Organization Report No.
	10. Work Unit No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

Characteristic far-field boundary conditions for the three-dimensional unsteady transonic small disturbance potential equation have been developed. The boundary conditions were implemented in the XTRAN3S finite difference code and tested for a flat plate rectangular wing with a pulse in angle of attack; the freestream Mach number was 0.85. The calculated force response shows that the characteristic boundary conditions reduce disturbances that are reflected from the computational boundaries.

17. Key Words (Suggested by Author(s)) Characteristic Boundary Conditions Transonic Unsteady Aerodynamics Transonic Small Disturbance Potential	18. Distribution Statement Unclassified - Unlimited Subject Category - 02

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 11	22. Price A02

N-305

For sale by the National Technical Information Service, Springfield, Virginia 22161