

Characteristic exponents of complex networks

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We present a novel way to characterize the structure of complex networks by studying the statistical properties of the trajectories of random walks over them. We consider time series corresponding to different properties of the nodes visited by the walkers. We show that the analysis of the fluctuations of these time series allows to define a set of characteristic exponents which capture the local and global organization of a network. This approach provides a way of solving two classical problems in network science, namely the systematic classification of networks, and the identification of the salient properties of growing networks. The results contribute to the construction of a unifying framework for the investigation of the structure and dynamics of complex systems.

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Networks are the fabric of complex systems, and network science has provided a deeper understanding of the basic mechanisms underlying the functioning and the evolution of diverse biological, technological and social systems, from the human brain to the Internet [1–6]. Recently, networks have been successfully employed for the study of dynamical systems. The basic idea consists into transforming a time series into a graph, by means of state-space proximity and recurrence [7–9], transition probabilities [10, 11] or visibility relationships [12], and then inferring information about the time series from the analysis of the corresponding network. These studies have revealed the existence of intimate connections between the statistical properties of a time series and the topology of the network constructed from it [13–16]. However, apart from a few exceptions [11, 17–19], little attention has been devoted to the dual problem, i.e. *studying the structure of complex networks by analyzing time series associated to them.*

In this Letter we aim at bridging this gap, by showing that a standard analysis of the statistical properties of time series constructed from random walks on graphs allows to characterize the topology of complex networks. In particular, the study of fluctuations in time series corresponding to different node properties, such as the degree, the average degree of nearest neighbours and the clustering coefficient, can reveal the existence of local and global correlations in the underlying graph. In this way it is possible to associate to each network a set of *characteristic exponents* which describe the scaling of fluctuations of each node property and capture the intrinsic complexity of a graph in a concise way. We show that these exponents can be employed to check the stability of the structure of growing networks and also allow to construct a taxonomy of networks, thus providing a quantitative, effective way of discriminating social from biological and technological systems by looking only at their structural properties.

I. MODEL

Let $G(V, E)$ be a connected undirected graph consisting of $N = |V|$ nodes and $K = |E|$ edges, and denote by $A = \{a_{ij}\}$ the adjacency matrix of G , whose entry $a_{ij} = 1$ if there is an edge between node i and node j , while $a_{ij} = 0$ otherwise. Let us consider a random walk on G described by a time-invariant transition matrix $\Pi \equiv \{\pi_{ji}\}$. At each time step, a walker moves from the current node i to node j with a probability π_{ji} . The probabilities π_{ji} satisfy the normalization condition $\sum_j \pi_{ji} = 1 \forall i$.

According to this definition, a walk on G corresponds to a discrete time-invariant Markov chain defined by the transition matrix Π on the state space V . Let us now consider an instance W of the walk defined by Π on G , and a real-valued property of node i , \mathcal{H}_i . If we indicate as (i_0, i_1, i_2, \dots) the sequence of nodes visited by W , we can construct the time series $(\mathcal{H}_{i_0}, \mathcal{H}_{i_1}, \mathcal{H}_{i_2}, \dots)$. For instance, if $\mathcal{H}_i \equiv k_i = \sum_j a_{ij}$, we get the time series $(k_{i_0}, k_{i_1}, k_{i_2}, \dots)$ of the degrees of the visited nodes.

We denote such time series as $T(W, \mathcal{H})$, because it depends on the node property \mathcal{H} , and on the specific order in which the nodes are visited by the walk. However, if the walk defined by the transition matrix Π on G is irreducible, then the topology of G completely determines which sequences of values can be produced by the walk and with which frequency [20]. Hence, any two time series $T'(W', \mathcal{H})$ and $T''(W'', \mathcal{H})$ constructed from two walkers W' and W'' on G corresponding to the same walk rule Π and the same node property \mathcal{H} will have, for $t \rightarrow \infty$, the same statistical properties, and will carry the same information about the structure of G . We can therefore indicate any time series produced by a transition matrix Π and by node property \mathcal{H} as $T(\Pi, \mathcal{H})$. We will now show that the analysis of the time series produced by different node properties \mathcal{H} can provide useful insights on the microscopic structure of a complex network and about its overall organization. We focus on the case of classical random walks, i.e. we set $\pi_{ji} = a_{ij}/k_i$.

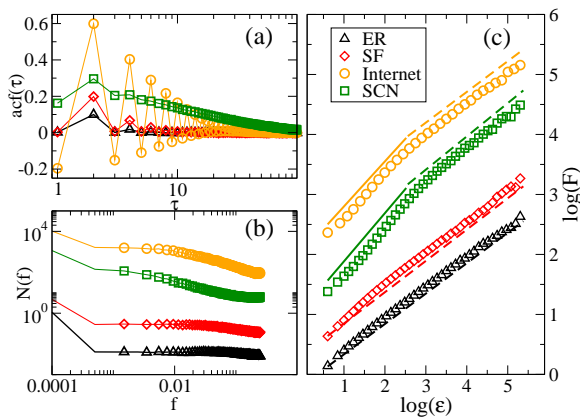


FIG. 1. (a) Autocorrelation function and (b) power spectrum of the time series constructed from the degree of nodes visited by random walks on an Erdős-Renyi random graph (ER), a scale-free graph with $\gamma = 3.0$ (SF), the Internet at the level of Autonomous Systems (Internet) and the collaboration network of scientists in condensed matter (SCN). Panel (c): the DFA of the time series reveals two distinct scaling regimes in the Internet and in SCN, while the fluctuations in ER and SF networks are compatible with Gaussian noise. The plots in (b) and (c) have been vertically displaced to improve readability.

Notice that with this rule the walkers visit each edge of a connected graph G with uniform probability, so that the time series constructed from random walks on G contain information about the distribution and correlations of the chosen node property \mathcal{H} throughout the network. We consider three possible choices of \mathcal{H} , namely the node degree, $\mathcal{H}_i \equiv k_i$, the average degree of first neighbours of a node, $\mathcal{H}_i \equiv k_i^{nn} = k_i^{-1} \sum_j a_{ij} k_j$, and the node clustering coefficient, $\mathcal{H}_i \equiv C_i$, where C_i is the number of closed triads centered on i divided by the total possible number $k_i(k_i - 1)/2$ of such triads. We decided to focus on these three node properties because broad-tailed degree distributions ($P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$), the presence of non-trivial degree correlations ($k^{nn}(i) \sim k_i^\nu$) and the abundance of triangles ($\langle C_i \rangle \gg 0$) are the basic features of most complex networks [2–4].

II. RESULTS

In fig. 1(a) and 1(b) we respectively report the autocorrelation function (ACF) and the power spectrum (PS) of the degree-based time series ($\mathcal{H}_i = k_i$) obtained in an Erdős-Renyi random graph (ER), a scale-free graph (SF) constructed by the configuration model [6], and two real-world complex networks, namely the Internet at the level of Autonomous Systems (Internet) [21] and the network of co-authorship in condensed matter (SCN) [22]. As expected, the ACF of ER and SF decays pretty fast and the corresponding PS is almost flat, indicating the absence of degree correlations. Conversely, the degree-based time series obtained from real-world networks exhibit broad

tails both in the ACF and the in the PS, a clear indication of the presence of long-range degree correlations. The peaks at even values of τ in the ACF of Internet are due to the presence of disassortative degree correlations [23]. An iterative surrogate analysis [24] has also confirmed that these time series are highly non-linear. A non-parametric statistical test [25], not depending on delay embedding reconstruction, suggested that such time series are non-linear or non-stationary with high confidence level.

In the following we report the results of the multifractal Detrended Fluctuation Analysis (DFA) [26], a standard non-linear analysis technique which allows to detect the presence of long-range correlations and to quantify the self-affinity of a time series, even if generated by a non-stationary process. Given a time series $(\mathcal{H}_{i_0}, \mathcal{H}_{i_1}, \mathcal{H}_{i_2}, \dots)$ we consider ℓ time-windows of length ε ; then, we remove the local linear trend in each time-window to obtain the detrended time series $(\overline{\mathcal{H}}_{i_0}, \overline{\mathcal{H}}_{i_1}, \overline{\mathcal{H}}_{i_2}, \dots)$ and we compute the local variance $\sigma^2(\ell, \varepsilon)$ of the detrended fluctuations. We evaluate the structure function $F(\varepsilon)$ by averaging $\sigma^2(\ell, \varepsilon)$ over all time-windows whose length is equal to ε , and we plot $F(\varepsilon)$ as a function of ε . The procedure can be generalized to build a set of structure functions depending on a parameter q [27–29], but here we focus on $q = 2$, allowing a physical interpretation of the results in term of diffusivity.

If the graph G is \mathcal{H} -uncorrelated, i.e. if the probability

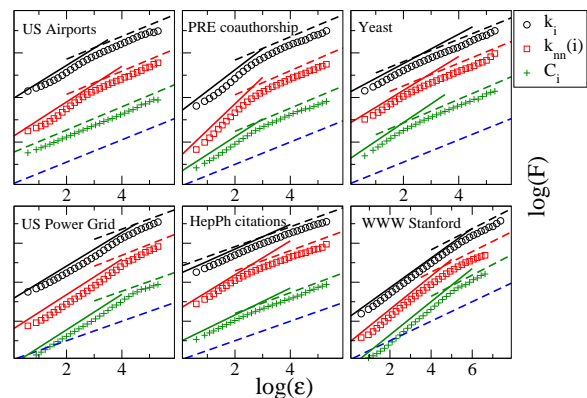


FIG. 2. DFA of different node properties. We considered time series produced by k_i , k_i^{nn} and C_i , and six networks, namely the US airports network [31], the co-authorship network from papers published in Physical Review E, the budding yeast protein interaction network [32], the US power grid [33], a citation network in high-energy physics [34] and the World Wide Web [35]. The plots have been vertically displaced to enhance readability. We observe two scaling regimes, with the actual values of the two characteristic exponents ν_1 and ν_2 varying across different networks. The dashed blue line in each panel is the DFA of the time series of the corresponding randomized networks ($F(\varepsilon) \sim \varepsilon^{1/2}$), averaged over 1000 realizations.

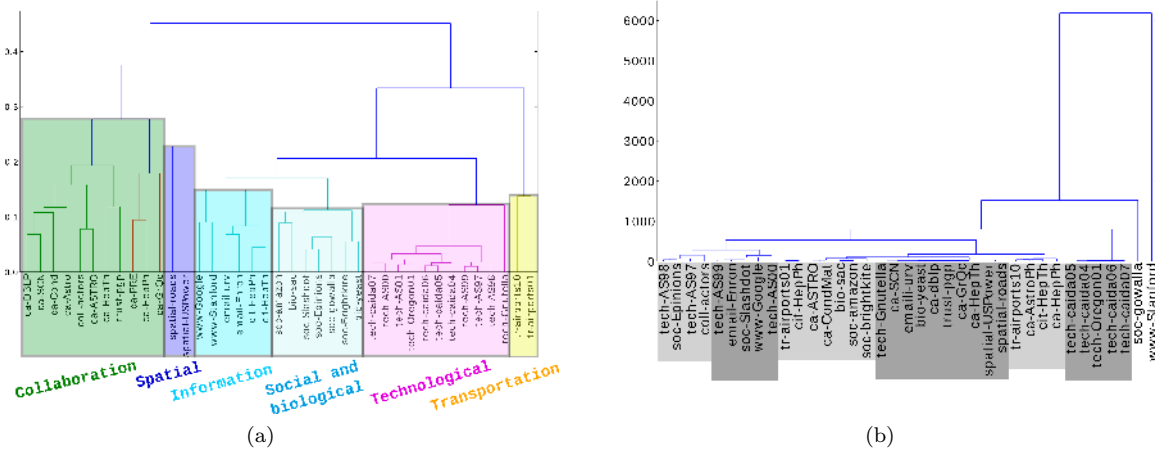


FIG. 3. (a) The dendrogram represents the hierarchical clustering of 39 real-world networks obtained by using the characteristic exponents, ν_1 and ν_2 , of the three time series (respectively based on k_i , k_i^{nn} and C_i). Notice the presence of well-defined, meaningful clusters of networks, namely collaboration and trust networks (green), spatially-embedded networks (blue), information networks (bright cyan), biological and online social networks (dark cyan), technological infrastructure networks (purple), and air transportation networks (yellow). (b) For comparison, we show the results of hierarchical clustering based on the mean and standard deviations of k_i , k_i^{nn} and C_i , in which the clusters always contain networks of different nature.

to find the edge (i, j) connecting node i to node j does not depend on the values \mathcal{H}_i and \mathcal{H}_j , then the fluctuations of the corresponding time series $T(\Pi, \mathcal{H})$ obtained from a random walk on G will be indistinguishable from an uncorrelated Gaussian noise, for which we have $F(\varepsilon) \sim \varepsilon^{1/2}$. Conversely, a scaling behaviour $F(\varepsilon) \sim \varepsilon^\alpha$ with $\alpha \neq 1/2$ is a clear signal of the existence of \mathcal{H} -correlations in the original graph G , and the value of α is a proxy for the magnitude of such correlations.

In fig. 1(c) we report the results of the DFA of $\mathcal{H}_i \equiv k_i$ for the same four networks considered in panel (a) and (b). As expected, degree fluctuations in ER and SF are compatible with Gaussian noise ($F(\varepsilon) \sim \varepsilon^{1/2}$), since the node degrees in these networks are uncorrelated. Conversely, $F(\varepsilon)$ plots corresponding to time series generated by walkers on the Internet and on the SCN appreciably deviate from Gaussian noise and are characterized by two different regimes [23]. In the first regime, corresponding to small values of ε , both time series are super-diffusive, i.e. $F(\varepsilon) \sim \varepsilon^{\nu_1}$ with $\nu_1 > 1/2$ ($\nu_1 \simeq 0.75$ for Internet and $\nu_1 \simeq 0.80$ for SCN), while for large values of ε their behaviour is almost Gaussian ($F(\varepsilon) \sim \varepsilon^{1/2}$ with $\nu_2 \simeq 0.51$ for Internet and $\nu_2 \simeq 0.52$ for SCN). In fig. 2 we report the results of the DFA of time series generated by $\mathcal{H}_i = k_i$, $\mathcal{H}_i = k_i^{nn}$ and $\mathcal{H}_i = C_i$ in six real-world networks of different nature [30]. The same two-regime behavior shown in fig. 1 for degree-based time series, is also found for the time series generated by k_i^{nn} and C_i . The two scaling regimes are a signature that the networks look different, with respect to degree, degree correlations and clustering, when observed at a local or at a global scale. On the one hand, the super-diffusive behaviour observed for small values of ε ($F(\varepsilon) \sim \varepsilon^{\nu_1}$) indicates that

a walker which explores the network for relatively short time intervals will observe correlated fluctuations in the properties of the nodes it visits, a clear signal of the presence of \mathcal{H} -correlations. On the other hand, the almost-Gaussian behaviour corresponding to large values of ε ($F(\varepsilon) \sim \varepsilon^{1/2}$) suggests that at a larger scale (i.e., if the walk continues for a sufficiently long time), the network appears uncorrelated. The transition point ε_c that separates the two regimes corresponds to the typical scale of \mathcal{H} -correlations, i.e. the typical walk length above which local heterogeneities and correlations in the values of \mathcal{H} become less important and all the walks on the network can be considered a homogeneous representation of the typical \mathcal{H} -fluctuations of the graph. We notice that in some cases the exponent ν_2 can be substantially larger than 0.5, like in the case of the US power grid [33], for which we have $\nu_2 > 0.65$ for all the three time series. In this particular case, the super-diffusive behavior for large values of ε is due to the fact that the network is embedded in a 2D space and has a strongly self-similar structure [36].

Although the presence of two scaling regimes seems to be a ubiquitous feature of different real-world networks, independently of their origin and nature, fig. 2 indicates that the actual values of the two exponents ν_1 and ν_2 may vary a lot for different node properties of the same network and, more importantly, for the same node property across different networks. In the following we show that these scaling exponents capture some key properties of a graph and can be employed to construct a taxonomy of networks [37, 38].

We considered a data set of 39 medium-to-large sized ($N \sim 10^4$ to $N \sim 10^6$) real-world networks representing

different social, biological and technological systems. We assigned to each graph G a point $p(G) \in \mathbb{R}^6$ identified by the values of the six scaling exponents obtained from the DFA of time series of degree, clustering coefficient and average degree of first neighbours. Then, we performed a hierarchical clustering on the resulting set of points, subsequently merging together at each step the two clusters whose points were separated by the smallest distance in \mathbb{R}^6 . In fig. 3(a) we report the resulting dendrogram, where the six large clusters identified (highlighted with different colors) correspond to networks with different functions. From left to right: the green cluster contains all the co-authorship ([22, 39]), trust (PGP [40]) and collaboration networks (IMDb co-starring network [33]); the blue cluster includes spatial networks (US power grid [33] and the Pennsylvania road network [35]); the bright-cyan cluster contains information networks, such as the WWW [35], citation networks [34], and email communication networks [35, 41]; the dark-cyan cluster includes online social networks [35, 42, 43] and proteomes [33, 44]; the purple cluster contains technological networks, including snapshot of the Internet sampled at different times by different institutions [21, 45, 46] and the Gnutella peer-to-peer file-sharing network [47]. Finally, the networks of US airports at two different times [31] are put together in the yellow cluster. The accuracy of characteristic exponents in classifying networks of different nature is quite remarkable [30], and becomes evident by comparing the results of fig. 3(a) with those of hierarchical clustering based on the mean and standard deviations of k_i , k_i^{nn} and C_i , reported in fig. 3(b). While in the former case clusters represent homogeneous groups of networks, in the latter case each cluster always contains networks of different nature.

The results shown in fig. 2 and fig. 3 suggest that the scaling exponents of the time series produced by random walkers visiting a complex network are indeed a key feature to characterize the network. Hence, we name them *characteristic exponents* of the network (Table I reports the characteristic exponents of all the complex networks considered in this study).

It is also interesting to investigate how the characteristic exponents of growing graphs change over time. In fig. 4(a) and 4(b) we show the temporal evolution of the characteristic exponents of k_i , k_i^{nn} and C_i respectively for the collaboration network of authors in APS Physical Review E (PRE) and for the Internet. Both networks have grown by a factor ~ 9 in the considered time intervals. However, while in PRE the characteristic exponent ν_1 for k_i and k_i^{nn} exhibits a clear decrease over time, the characteristic exponents of the Internet have remained constant in the considered 10-years interval. The different temporal behaviour of the characteristic exponents is probably due to the peculiar dynamics of edge formation in the two networks. In fact, in a co-authorship network a node continues to accumulate edges over time, even if the majority of these edges correspond to collaborations which are not active any more. Evidently,

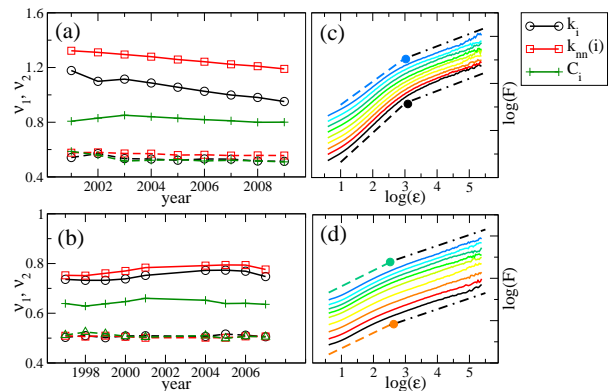


FIG. 4. (color online) Characteristic exponents of (a) a co-authorship network from Physical Review E (PRE) and (b) the Internet at the level of Autonomous Systems, over a period of ten years. In PRE the characteristic exponents for k_i and k_i^{nn} decrease over time, while the exponents for C_i remain constant. In the Internet, all the characteristic exponents are constant. Panel (c) and panel (d) report the detail of the DFA for degree time series, respectively for PRE and Internet. The plots have been vertically displaced to enhance readability, with the topmost curve corresponding to the most recent network.

the continuous addition of edges drives the network towards a homogenization of degree and clustering correlations. Conversely, the number of neighbours of a node in the Internet cannot increase indefinitely, due to technological and economical constraints. In fact, connecting to more peers usually implies handling more Internet traffic, which in turn requires more bandwidth and new hardware, and translates into an economical investment. These constraints are mostly independent from network size, thus having the same impact on the network growth at different times. This might explain why the structure of correlations has remained stable over time.

Finally, we check whether the position ε_c of the cut-off of the structure function does depend on the size of the graph, and to which extent. To this aim, we show in fig. 5 the approximate value of ε_c for the PRE and Internet networks, as a function of time. Notice that as the networks grow the corresponding values of ε_c change slightly for all the time-series, but we observe opposite trends in the two cases. In particular, ε_c usually decreases for PRE and increases in Internet. This means that ε_c is not simply determined by the size of the network (otherwise we should have observed a similar behaviour in both networks), but is instead intimately related to the local organization of the graph.

It is also worth noticing that, despite the presence of these trends, ε_c usually remains of the same order while both networks have grown by an order of magnitude in the considered time intervals. For instance, in the time-series of degrees of PRE [see fig. 5(a)] ε_c remains in the range [1250 : 1750], while for the time-series of k_i^{nn} it is in the range [300 : 600]. If we take into account the fact that

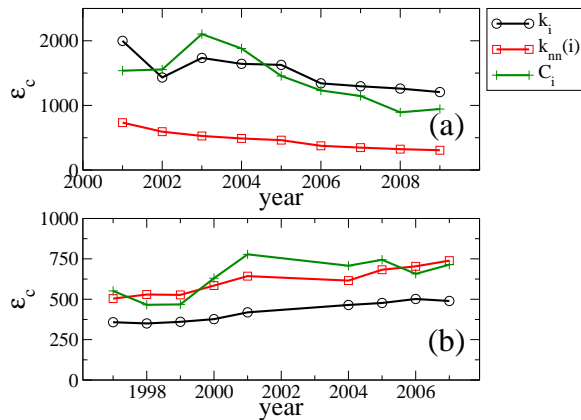


FIG. 5. (color online) The value ε_c at which the scaling of the structure function $F(\varepsilon)$ changes, respectively for (a) PRE and (b) Internet, as a function of time. Despite ε_c can slightly increase or decrease over time, its value remains of the same order of magnitude and corresponds, in both cases, to the visit of just a few hundred nodes.

a random walk on any of the snapshots of the PRE collaboration network typically requires $O(10^6)$ time-steps in order to visit all the nodes at least once, and that this network has strong communities and a high value of clustering coefficient (and both these factors contribute to keep a walker confined on a small set of nodes), then we realise that $\varepsilon_c \sim 10^3$ corresponds indeed to the exploration of a relatively small region of the graph, which usually includes no more than a few hundred nodes. Similarly, in the Internet network [fig. 5(b)] ε_c is in the interval $[350 : 500]$ for k_i and in $[450 : 700]$ for k_i^{nn} and C_i , which again correspond to visiting a relatively small portion of the graph. These results suggest that the values of the cut-off in the scaling of the structure functions tend to remain practically stable over time, even when the network undergoes substantial expansion.

Summing up, in this work we reported on the discovery of an intimate connection between the structure of a network, the properties of time-series extracted from it, and the capability of such time-series to carry useful information about the overall organization of the network. We have shown that the characteristic exponents corresponding to degree, node clustering coefficient and average degree of first neighbours can be used to cluster networks, and to distinguish social, collaboration, biological, information, transportation and spatial networks only by looking at their structure. The procedure described in this work is quite general, and can be used to extract characteristic exponents corresponding to any desired node or link property, thus allowing for a finer and more accurate classification of complex networks.

Network	ν_1			ν_2		
	k_i	k_i^{nn}	C_i	k_i	k_i^{nn}	C_i
ca-DBLP	0.838	1.092	0.769	0.519	0.523	0.516
ca-SCN	0.803	1.044	0.757	0.518	0.554	0.524
ca-Cond	0.726	1.032	0.748	0.509	0.564	0.523
ca-Astro	0.739	1.119	0.810	0.526	0.588	0.527
coll-actors	0.789	1.211	0.763	0.506	0.554	0.516
ca-ASTRO	0.838	1.215	0.780	0.531	0.606	0.506
ca-HepTh	0.811	1.034	0.831	0.535	0.601	0.615
trust-pgp	0.853	1.035	0.901	0.515	0.552	0.547
ca-PRE	0.953	1.188	0.799	0.522	0.556	0.520
ca-HepPh	0.993	1.220	0.860	0.549	0.585	0.554
ca-GrQc	1.006	1.254	0.924	0.610	0.672	0.596
spatial-roads	0.808	0.833	0.839	0.602	0.607	0.740
spatial-USPower	0.863	0.964	0.890	0.656	0.752	0.685
www-Google	0.695	0.874	0.802	0.510	0.512	0.543
www-Stanford	0.760	0.854	0.850	0.542	0.528	0.548
email-urv	0.662	0.876	0.743	0.501	0.526	0.514
email-Enron	0.681	0.813	0.735	0.511	0.516	0.522
cit-HepPh	0.629	0.871	0.696	0.506	0.524	0.518
cit-HepTh	0.593	0.847	0.700	0.515	0.518	0.521
soc-amazon	0.582	0.708	0.865	0.516	0.547	0.525
bio-sac	0.648	0.668	0.858	0.508	0.504	0.531
soc-Slashdot	0.604	0.747	0.757	0.493	0.511	0.512
soc-Epinions	0.596	0.713	0.757	0.514	0.509	0.519
soc-gowalla	0.630	0.688	0.782	0.502	0.523	0.528
soc-Brightkite	0.639	0.801	0.828	0.509	0.536	0.526
bio-yeast	0.663	0.758	0.837	0.504	0.513	0.530
tech-caida07	0.748	0.776	0.636	0.506	0.507	0.506
tech-AS00	0.738	0.770	0.647	0.509	0.505	0.508
tech-AS01	0.752	0.783	0.660	0.509	0.501	0.504
tech-Oregon01	0.748	0.781	0.655	0.503	0.510	0.502
tech-caida06	0.769	0.794	0.640	0.512	0.507	0.507
tech-caida05	0.773	0.794	0.639	0.516	0.501	0.499
tech-caida04	0.772	0.792	0.652	0.507	0.502	0.510
tech-AS99	0.732	0.761	0.638	0.502	0.510	0.517
tech-AS97	0.736	0.752	0.639	0.503	0.509	0.512
tech-AS98	0.732	0.751	0.629	0.510	0.506	0.524
tech-Gnutella	0.640	0.714	0.642	0.501	0.505	0.502
tr-airports01	0.770	0.926	0.516	0.518	0.542	0.518
tr-airports10	0.866	1.001	0.519	0.473	0.494	0.519

TABLE I. The two characteristic exponents ν_1 and ν_2 of time series constructed from node degree (k_i), average degree of first neighbours (k_i^{nn}) and node clustering coefficient (C_i) of real-world complex networks. The color correspond to the class to which a network belongs, i.e. coauthorship, collaboration and trust networks (green), spatial networks (blue), information and citation networks (bright cyan), social networks and proteomes (dark cyan), technological networks (purple) and air transportation networks (yellow).

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