CHAPTER 16

CHARACTERISTIC WAVE PERIOD

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ABSTRACT : The wave period estimates obtained from different procedures are not consistant unlike statistical distribution analysis of wave heights. Thus not one difinition of wave period is satisfactory for engineering analysis of coastal processes.

There are at least 10 different measures of wave periods including the zero up-crossing period, the average wave period, significant height period and peak of the energy density spectrum period.

For the analysis of periods, 20 min. records were obtained from offshore pressure recorders. Summer and winter records were analysed separately.

In the analysis, zero up-crossing period and average period were taken as reference periods. There were significant differences between the wave periods and they were found to depend also on the spectral width parameter.

Finally comparison was made between the energy flux obtained under the spectral diagrams and energy flux obtained using various wave periods and heights.

Study shows that if the total energy flux is desired, then the most appropriate values to be used are the root mean square wave height and period corresponding to that wave height. Use of significant wave height, along with zero up-crossing period gives higher values.

INTRODUCTION : Ocean waves are extremely complex in character. The period, celerity, wave length and wave height are irregular. Though individual wave heights vary considerably, they are easier to define statistically and the expected wave heights can be predicted reasonably. Such consistancy does not, however, occur with wave periods and therefore, not one definition of wave period is sufficient in wave analysis.

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There are, at least, 10 measures of wave periods, at present, in use namely (i) average period of all waves called period of wave crests, (ii) zero up-crossing period, (iii) average period of one-third the highest waves, (iv) average period of one-tenth the highest waves, (v) period of root mean square wave height, (vi) average period contained in a single wave group, (vii) period of the highest wave, (viii) period most prominent in the record, (ix) period corresponding to the peak of the energy density spectrum and (x) period of the maximum energy density.

THEORY : At a given point on the ocean surface, the water surface elevation y is a function of time and can be assumed as the sum of an infinite number of sinusoidal waves, each with a frequency $\omega_i = \frac{2\pi}{T_i}$ and phase angle ϕ_i where ω_i = angular frequency of the i th component of the wave spectrum. Thus

$$y(t) = \sum_{i}^{N} a_i \cos(\omega_i t + \phi_i) \tag{1}$$

where y (t) is a stationary random function. The probability density of y (t) is a Gaussian distribution

$$p(y) = \frac{1}{\sqrt{2 m_o}} \; e^{\frac{-y^2}{2 m_o}} \tag{2}$$

where $\sqrt{m_o}$ = square root of the zero moment or area of the energy density spectrum and

$$H_{rms} = 2\sqrt{2 m_o} \tag{3}$$

where H_{rms} = root mean square wave height.

Not going into mathematical details of the Raleigh Distribution, it can be stated that consideration of the energy of the component waves leads to the energy density spectrum. The energy of each component

$$e_i = \frac{1}{2} \rho g a_i^2$$

where e_i varies with angular frequency ω depending upon wave characteristics. The energy density of the spectrum E (ω) is the energy contained in the frequency limits ω, and (ω + d ω). Thus :

$$E(\omega)\, d\omega = \frac{1}{2} \sum_{\omega}^{\omega + d\omega} a_i^2$$

and the total energy in the whole spectrum is

$$m_o = \int_o^\infty \varepsilon(\omega)\, d\omega = \sum_1^N \frac{1}{2} a_i^2 \qquad (3)$$

Statistical analysis (3, 4) also shows that average zero up-crossing period T_z of the sea surface and the average crest period T_c can be determined from the wave spectrum. Thus :

$$T_z = 2.\pi \sqrt{\frac{m_o}{m_2}} \qquad (4)$$

$$T_c = 2.\pi \sqrt{\frac{m_2}{m_4}} \qquad (5)$$

The spectral width parameter ε is given by :

$$\varepsilon = \sqrt{1 - \frac{m_2^{\ 2}}{m_o m_4}} \qquad (6)$$

where m_2 and m_4 are spectral moments of 2nd and 4rth order respectively.

If the spectrum has a constant shape, then it may be characterised by a single wave height directly related to the wave energy described by m_o and a single wave period. The characteristic period (T_o) adopted is often corresponding to the frequency ω_o of the spectral peak and can be obtained from the energy spectrum (with $\varepsilon(\omega)$ on the y axis and $\omega = \frac{2\pi}{T}$ on the x axis). The above period of the maximum energy density obtained from spectral analysis is also useful in finding the average period of the waves composing the dominant wave groups in a swell (7, 8).

ANALYSIS : In the study, waves were recorded by offshore pressure operated recorders (OSPOS) anchored at the bottom in 6 m to 7 m depths at three stations off the Nile Delta Coast.

Draper/Tucker (3,9) method of manual analysis was made of the pressure charts, each record of which was of 20 min. duration recorded at intervals of 4 hours. Analysis was made separately for summer and winter seasons since the former usually experienced swells from wind waves more or less continuously and the latter generated storms separated by lull periods.

To obtain a more objective analysis and since it represents an integration of waves present in the 20 min. record, the period of zero up-crossing was taken as the characteristic period and other periods were compared to that period. However, since zero up-crossing method depends on a practical method for determining the zero line, the wave crest period was also used as reference period in some cases.

There were significant differences between the various wave periods. Further the differences depended on the type of waves namely swells or storm waves or whether the waves occurred with one or more peaked spectra. Comparing the zero up crossing period with significant period, the former was found to be normally more than the latter, the ratio for summer and winter waves being 1.2 and 1.13 respectively (Fig. 1). The reason for this is probably because a large wave results when several of the spectral components are in phase at a given point and time. The period of the resulting wave will be influenced most by the higher frequencies present and therefore will be less than the zero up-crossing period T_z. Also larger values of ε occur for storm waves since wide range of frequencies are present. Comparison of period of maximum wave height with zero up-crossing period (fig.2) shows similar trend (for the same reasons) but to a lesser extent. Their ratios (T_{Hmax} to T_z) are also dependent upon on the spectral width parameters which vary from o.4 to o.8 with summer swells (ε small) having smaller values than storm waves (ε large). Analysis of the period of the peak of the energy spectrum with zero up-crossing period gave a relationship $T_o = 1.12\ T_z$ (fig.3) which corresponds to Bretschneider spectrum $^{(1)}$ for wind generated waves such as those of Pierson and Moskowitz$^{(6)}$. The above relation was found to be mostly true for swells and for fully risen storm waves. This relationship will, however, differ if a spectrum of different form or of more than one peak occur. Comparison of significant wave height periods with those of maximum wave heights showed periods to be more or less equal though slightly less for swells. Considering spectral peak period in relation to the significant wave period, the former was higher than the latter varying from 1.22 to 1.32 (fig.4), the higher value representing summer waves.

Using wave crest period as reference, the significant wave height period, the spectral peak period, maximum wave height period, period corresponding to the root mean square wave height period were then analysed (figs.5 to 8). The spectral peak period showed the largest variation with the root mean square wave height period showing the least variation and the period of the maximum wave height equalling the period of the wave crests for swells (ε small). The fact that the spectral peak period showed the largest variation is not surprising since it represents a single peaked spectra of higher frequencies.

Studies were also made between the period of maximum energy density and the period corresponding to the peak of energy density spectrum. There seemed to be no correlation between them and therefore, no further study was made of the former.

Groups of waves (not swell trains) present in wind waves during storms were then examined and their periods determined. Similarly periods of prominent waves during storms were also analysed. Fig. 9 shows that they are related to T_z in some way or other. The analysis of these is not complete since swell trains have yet to be studied. However, the present indications are that for wind waves, zero up crossing period is fair representation of wave group period.

Unlike the earlier study of Harris$^{(5)}$ which showed very little correlation of different wave period definitions, this study seems to indicate the existance of relations between some of them and that fact has been made use of to evaluate the total energy flux correctly using various wave height and period definitions.

Fig.10 shows the comparison between the energy flux obtained under the spectral diagrams and energy flux obtained using the various wave periods and heights. It shows that if the correct total energy flux is desired, then the most appropriate values to be used are the root mean square wave height and the period corresponding to that wave height. Use of significant wave height along with the zero up crossing period or spectral peak period results in considerably higher values representing higher energies present in the wave train. Though they may be useful in the study of wave dynamics, they will certainly give higher sediment transport rates than the actual. Lines representing the various energy fluxes using different wave periods have been drawn for the purpose of comparison.

CONCLUSIONS : An extensive study of the various definitions (ten) of wave periods was made. It showed that for wind waves and for swells associated with such wind waves, certain relations exist between some wave period definitions. It also showed that use of zero up-crossing period to represent the period of a wave recording is not to be recommended in all cases. Comparison of the actual energy flux from wave spectrum with the various wave periods and wave heights showed that the most appropriate combinations are those involving root mean square wave height and period corresponding to that height and that combinations involving significant wave height and zero up crossing period gave considerably higher values.

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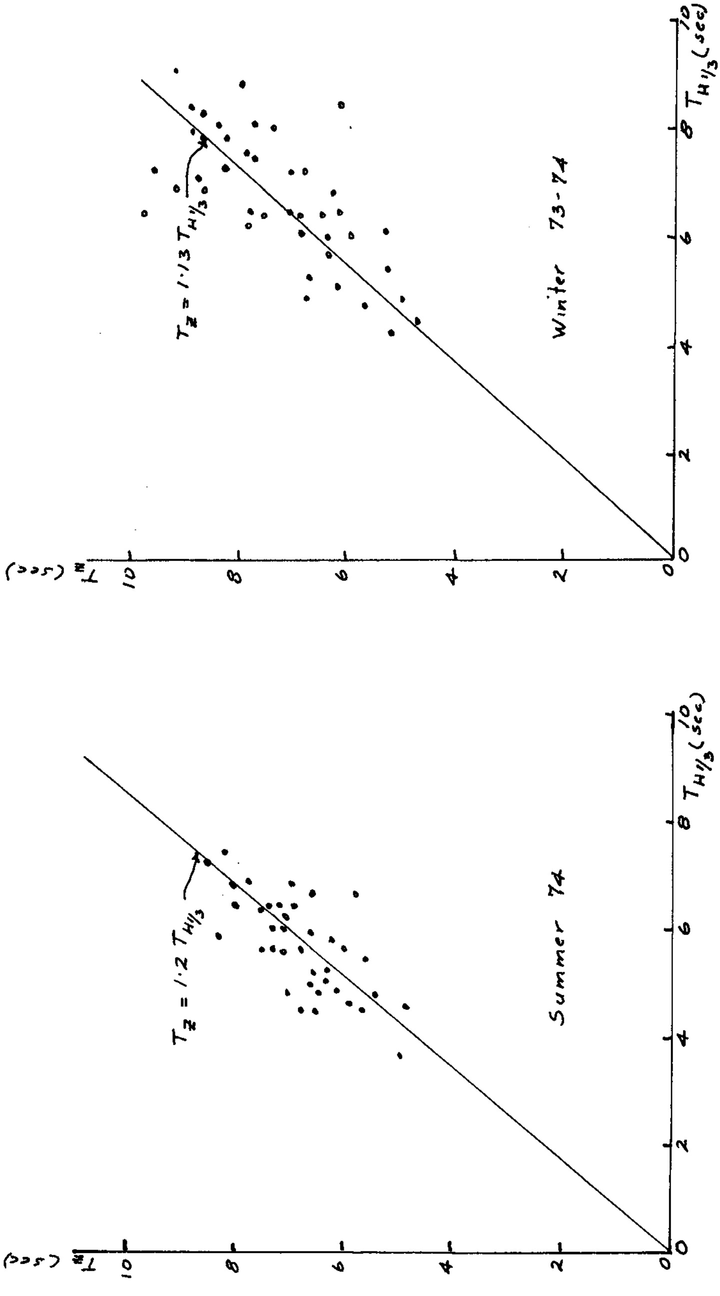

FIG. 1: NILE DELTA COAST : T_z vs $T_{H1/3}$

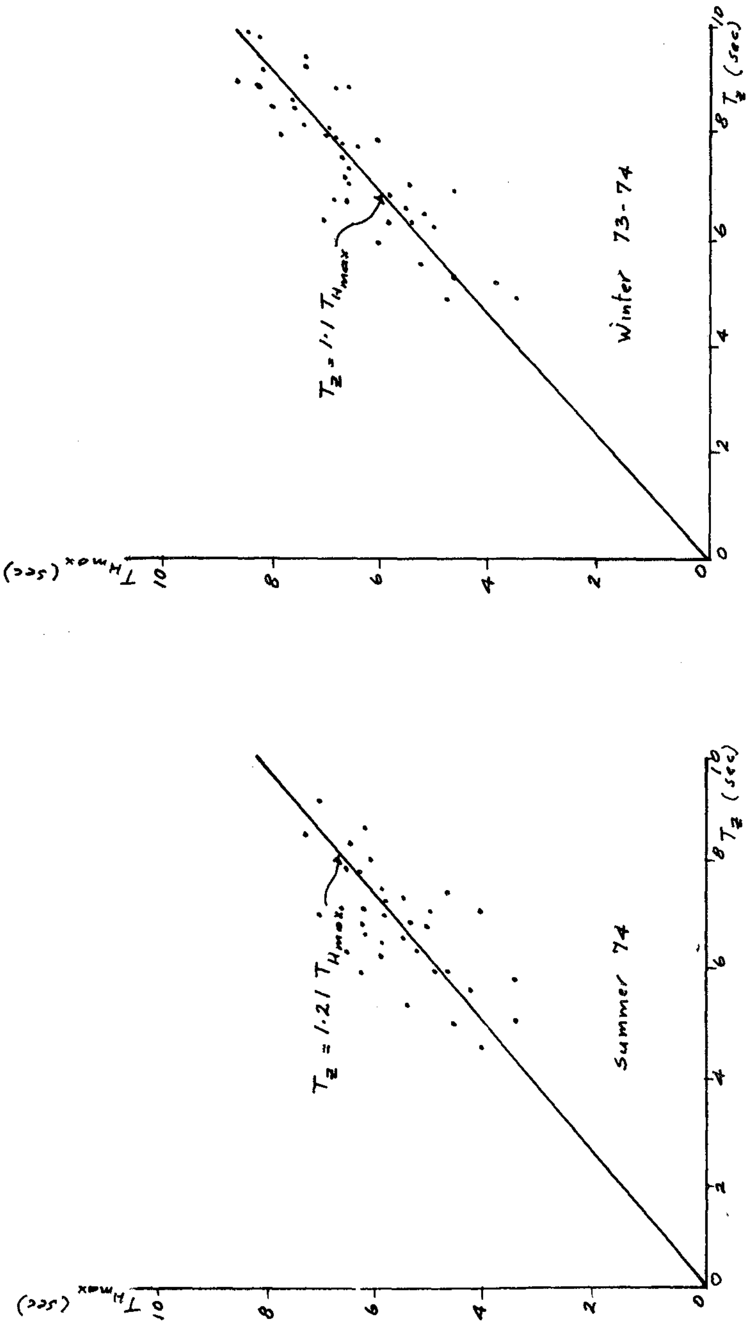

FIG. 2: NILE DELTA COAST : T_z vs $T_{H_{max}}$

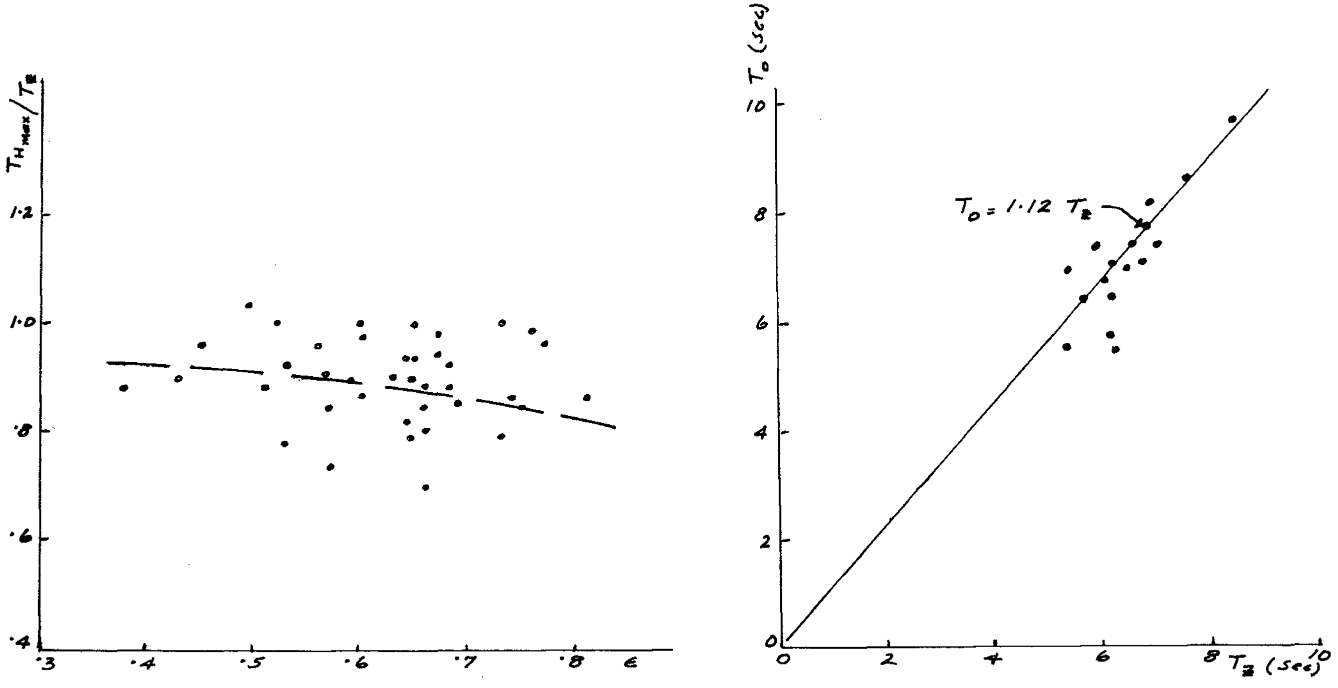

FIG.3: NILE DELTA COAST: $T_{H_{max}}/T_z$ vs ε AND T_o vs T_z – WINTER 73-74

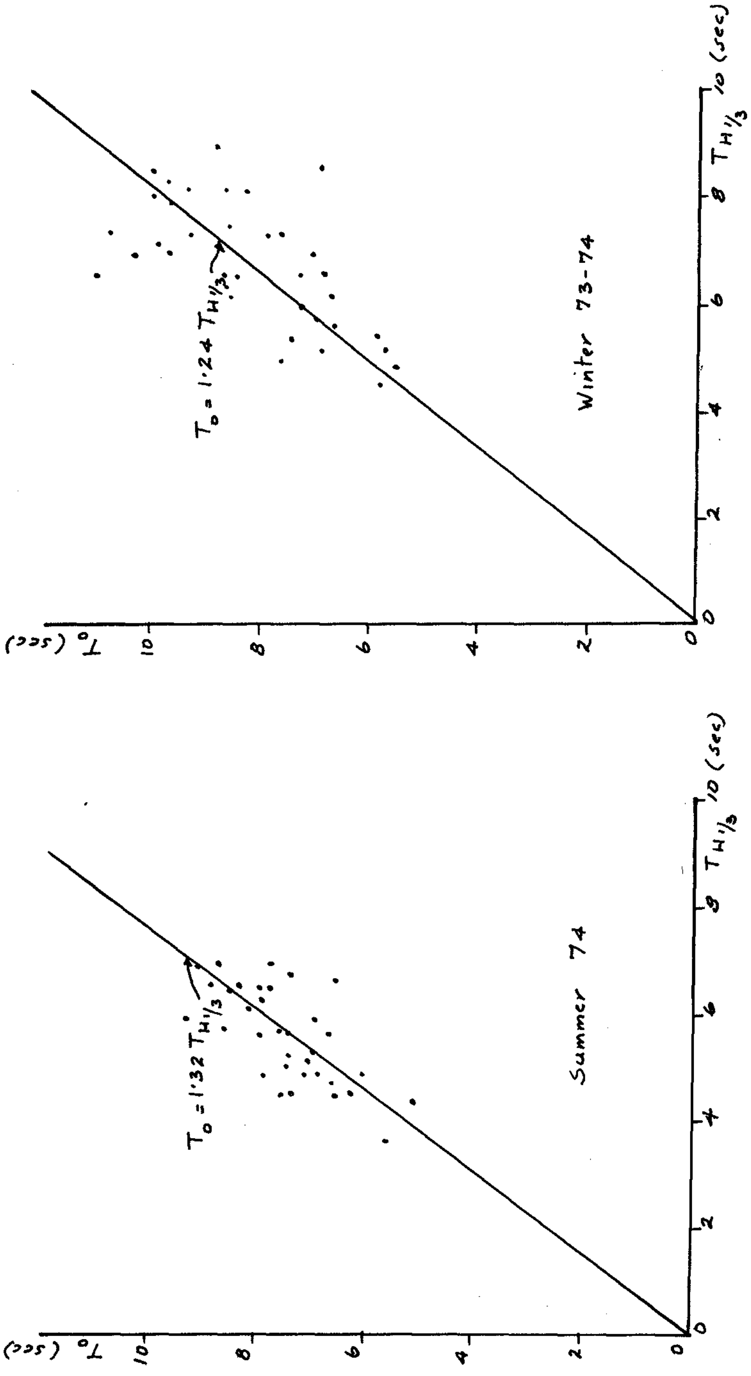

FIG. 4 : NILE DELTA COAST : T_o vs $T_{H1/3}$

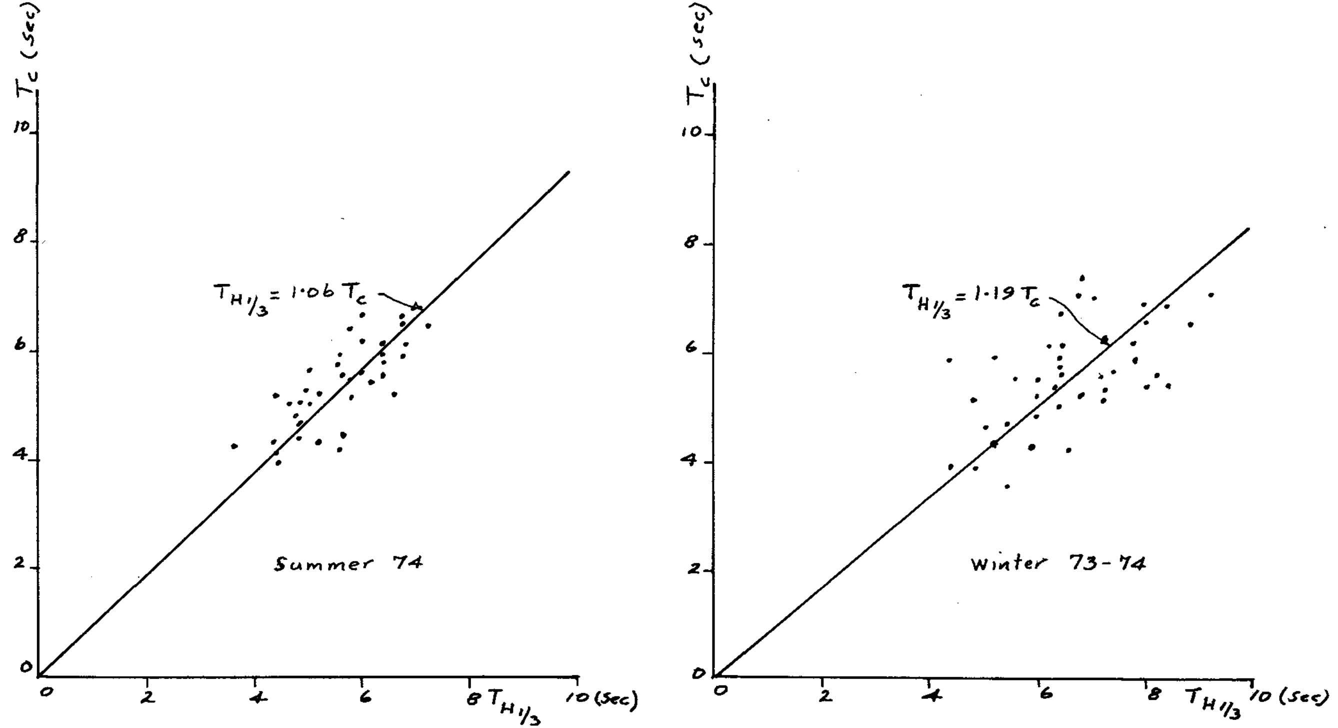

FIG. 5 : NILE DELTA COAST : $T_{H1/3}$ vs T_c

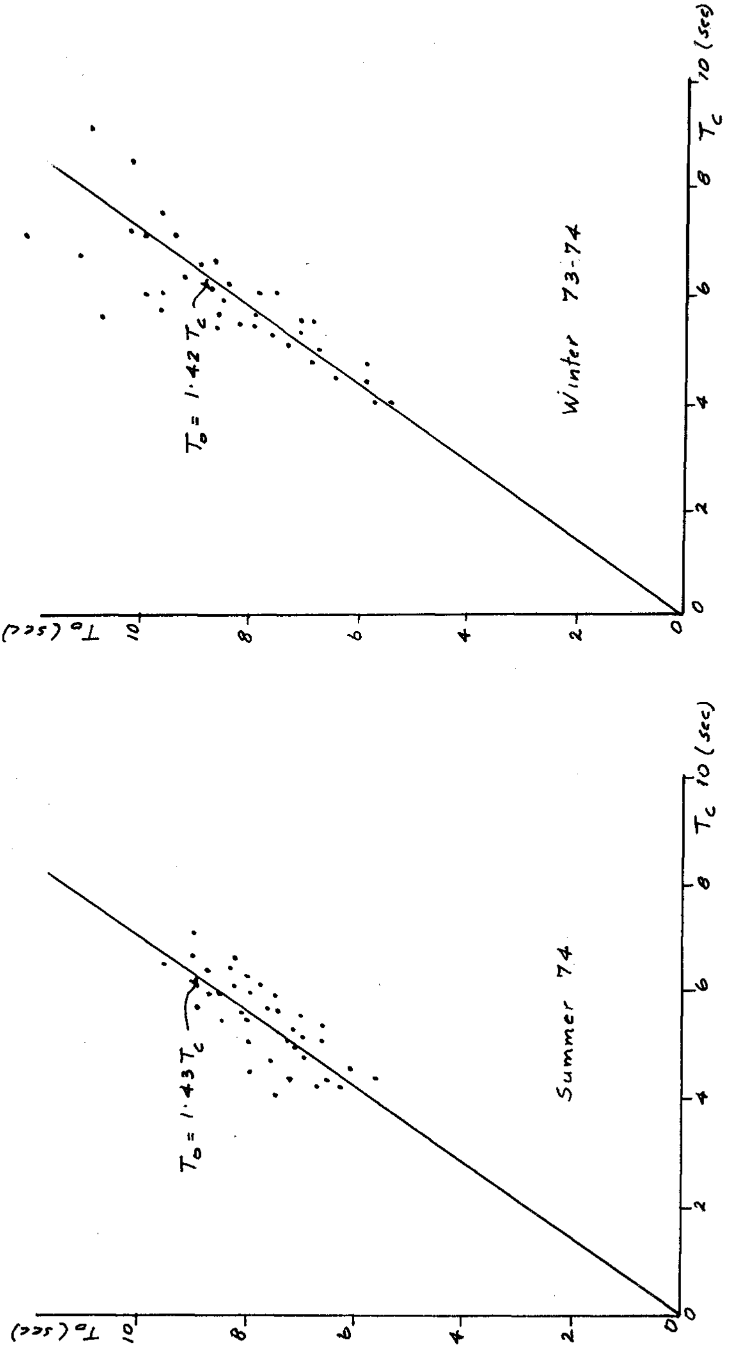

FIG. 6: NILE DELTA COAST: T_o vs T_c

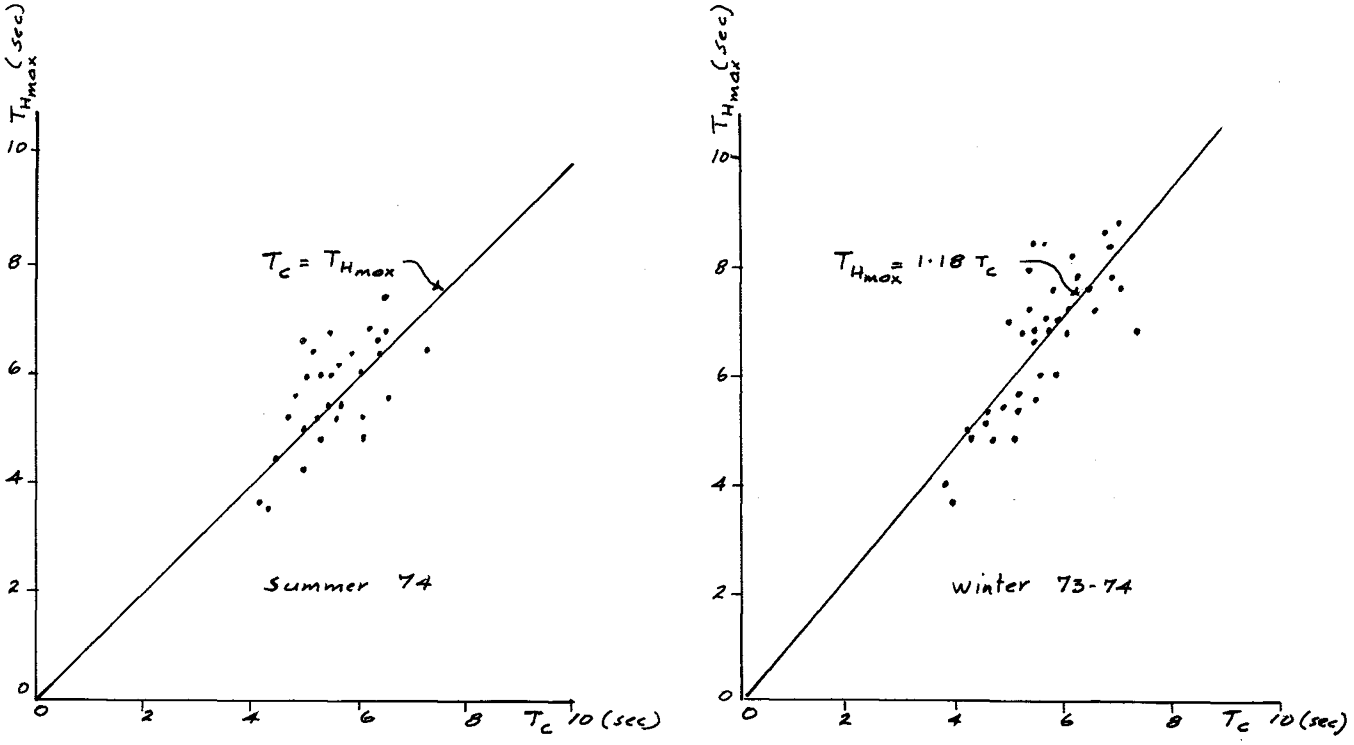

FIG. 7 : NILE DELTA COAST : $T_{H_{max}}$ vs T_c

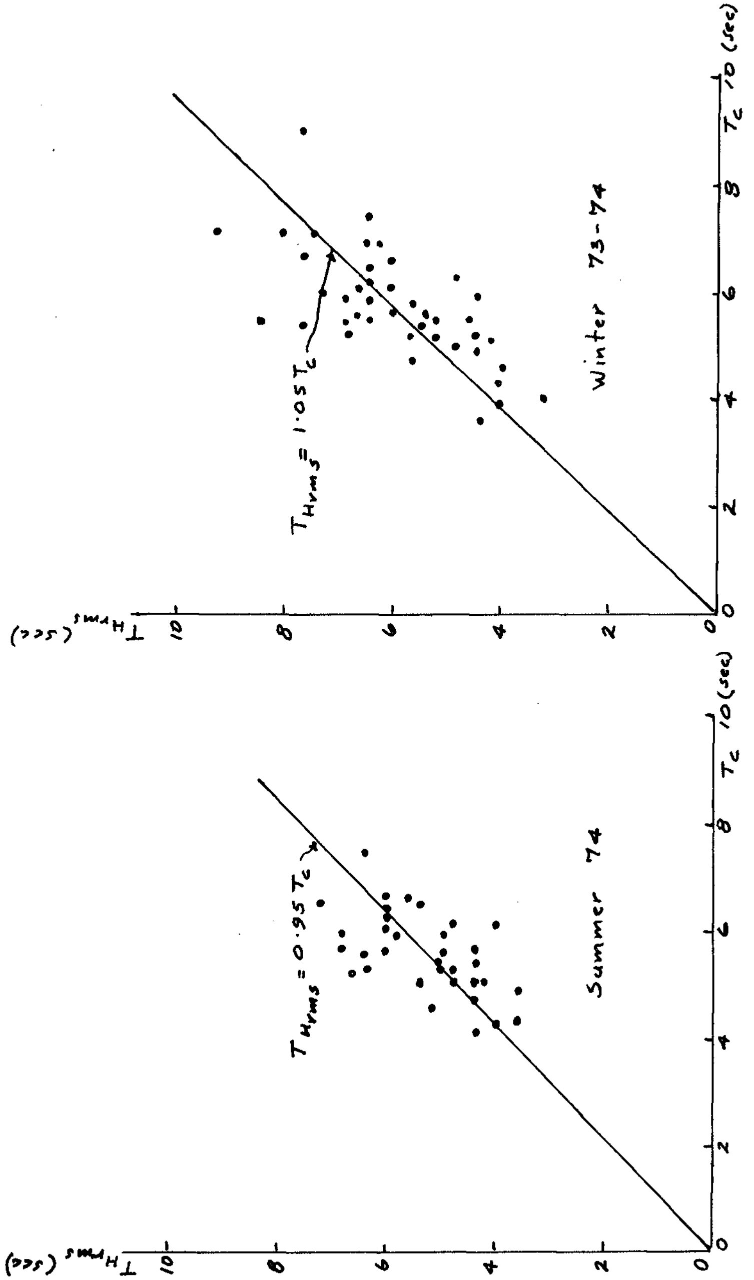

FIG 8: NILE DELTA COAST : $T_{H_{rms}}$ vs T_c

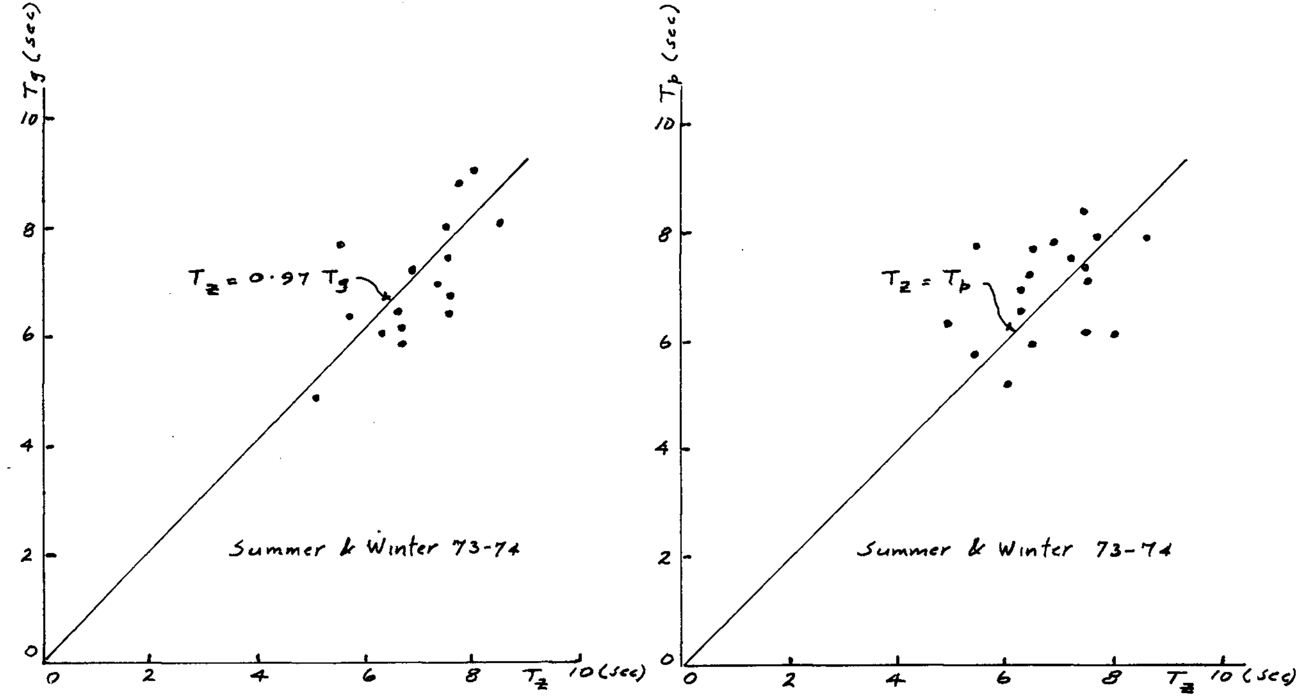

FIG. 9: NILE DELTA COAST: T_z vs T_g & T_p

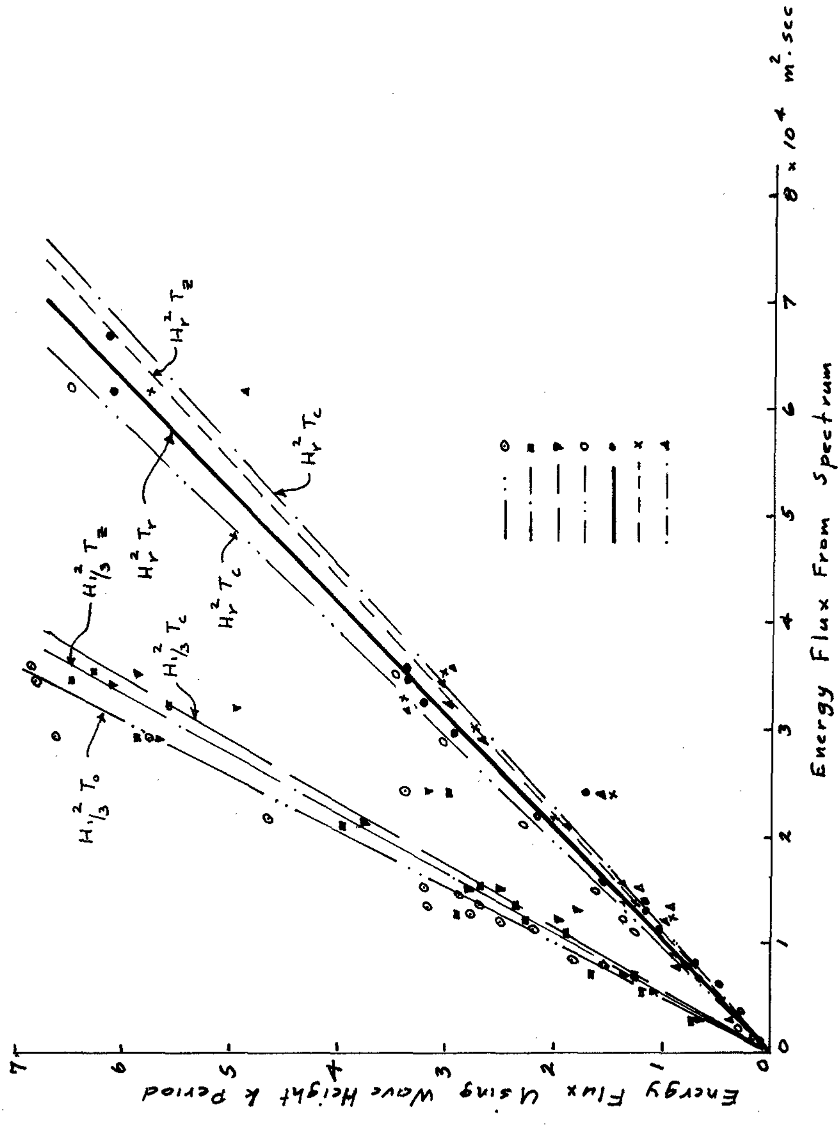

FIG.10: NILE DELTA COAST : ENERGY FLUX