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Characteristics of laser-driven electron acceleration in vacuum

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ABSTRACT

The interaction of free electrons with intense laser beams in vacuum is studied using a 3D test particle simulation model that solves the relativistic Newton-Lorentz equations of motion in analytically specified laser fields. Recently, a group of solutions was found for very intense laser fields that show interesting and unusual characteristics. In particular, it was found that an electron can be captured within the high-intensity laser region, rather than expelled from it, and the captured electron can be accelerated to GeV energies with acceleration gradients on the order of tens of GeV/cm. This phenomenon is termed the capture and acceleration scenario (CAS) and is studied in detail in this paper. The maximum net energy exchange by the CAS mechanism is found to be approximately proportional to a_0^2, in the regime where $a_0 \gtrsim 100$, where $a_0 = eE_0/m_e\omega c$ is a dimensionless parameter specifying the magnitude of the laser field. The accelerated GeV electron bunch is a macro-pulse, with duration equal or less than that of the laser pulse, which is composed of many micro-pulses that are periodic at the laser frequency. The energy spectrum of the CAS

electron bunch is presented. The dependence of the energy exchange in the CAS on various parameters, e.g., a_0^2 (laser intensity), w_0 (laser radius at focus), τ (laser pulse duration), b_0 (the impact parameter), and θ_i (the injection angle with respect to the laser propagation direction), are explored in detail. A comparison with diverse theoretical models is also presented, including a classical model based on phase velocities and a quantum model based on nonlinear Compton scattering.

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I. INTRODUCTION

Recent advances in laser technology have yielded light intensities as high as $I\lambda^2 = 10^{20}$ W/cm$^2 \cdot \mu$m^2, where I and λ are the laser intensity and wavelength in units of W/cm^2 and μm, respectively. Consequently, there have emerged many new frontier research areas in both applied and fundamental physics [1]. Among these, the development of laser-driven electron acceleration mechanisms is a fast advancing area of scientific research [2]. Compared with the 20 MV/m acceleration gradient provided by contemporary linear accelerators, the 10^7 MV/m electric field gradients of the laser field have made laser acceleration a very promising candidate for the development of compact high-energy accelerators. But laser acceleration has several technological difficulties. For instance, most of the reported acceleration mechanisms have involved plasma [3], [4]. To avoid the problems inherent in laser-plasma interaction such as plasma instabilities, the far-field laser acceleration of free electrons in vacuum has received new attention. In this research area, there is a long-standing question of whether or not an electron can get a net energy gain, assuming an unlimited interaction length, from a laser beam in free space. According to the Lawson-Woodward Theorem, the electron can get no net energy gain through the entire interaction [5], [6], [7], [8], [9]. But this conclusion is only confined to low intensity laser fields, i.e., energy gains that are linearly proportional to the laser field. Malka et al. [10] reported the observation of electrons accelerated to MeV energy in vacuum by intense lasers with $a_0 = 3$, where $a_0 \equiv eE_0/m_e\omega c$ is a dimensionless parameter specifying the magnitude of the laser field, $-e$ and m_e are the electron charge and mass, respectively, c the speed of light in vacuum, and ω the angular frequency of the electromagnetic wave. In terms of the peak laser intensity and wavelength, $a_0 = 0.85 \times 10^{-9}\lambda[\mu\text{m}](I[\text{W/cm}^2])^{1/2}$. Earlier, electrons accelerated to a fraction of eV [11] or a few keV [12] at low intensity and 100 KeV [13] at higher intensity had been observed.

To give a more exact answer to the above question, we devised a model to study the interaction of electrons with a laser field based upon a 3D computer simulation code to solve the relativistic Newton-Lorentz equations of motion [14]. The results show that a large net

energy gain is possible [14]. In this model, electrons were injected at a specified angle into a continuous laser beam. For $a_0 \leqslant 0.1$, there is no noticeable energy transfer between the electron and the laser beam. As a_0 increases from 0.1 to more than 10, the electron begins to obtain more and more net energy, which is of several MeV magnitude when a_0 nears 10. The most surprising and meaningful result is that as a_0 approaches or exceeds 100 ($a_0 \gtrsim 100$), the electron can be captured and violently accelerated to GeV energy by either continuous or sufficiently long-pulsed laser beams with acceleration gradients on the order of tens of GeV/cm. We refer to electron acceleration in this regime as the capture and acceleration scenario (CAS).

The main purpose of this paper is to study the characteristics of electron scattering by intense pulsed laser beams and to determine the dependence of the net energy exchange on various parameters such as the laser intensity. Special attention has been paid to exploring the physics of the CAS, such as determining the conditions under which a capture trajectory emerges, and finding the scaling of the maximum energy gain of the accelerated electrons with respect to laser intensity. The numerical results are compared to various theoretical models. Some of these results have been recently and breifly presented in Ref. [15], and in this paper these results, as well as additional aspects of CAS, are studied in detail. This study has significance in determining parameters for experimentally testing laser-driven electron acceleration in vacuum.

Theoretical models based upon classical physics that we consider are the ponderomotive potential model (PPM) and a phase velocity synchronization model. The PPM is a classical description in which the time-averaged electron motion is modeled by assuming that the electron moves in an effective ponderomotive potential, which is obtained by averaging the Newton-Lorentz equations of motion over the fast quiver oscillation of the electron in the laser field [3], [16], [17], [18], [19], [20], [21]. Such quivering motion is argued [11] to be analogous to a kind of stimulated scattering process. At low laser field intensities ($a_0 \leqslant 0.1$) PPM stands well in describing the electron averaged motion in the electromagnetic field [22]. In this paper we will extend the PPM to the high laser intensity region and compare

its results with that obtained from the full Newton-Lorentz equations of motion.

Phase velocity synchronization plays an important role in the CAS. For an electron moving near the speed of light c in a straight line along the axis, the phase velocity of the laser field is greater than c. In this case phase synchronism and, hence, a significant energy gain does not occur. However, for an electron moving in a curved trajectory, as is the case in the CAS, the effective phase velocity can be $\lesssim c$ over a sufficiently long distance so as to result in a large energy gain.

Although the numerical model discussed in this paper is entirely classical, it is insightful to make some comparisons with quantum electro-dynamics (QED). According to QED, there are three fundamentally different energy-exchange mechanisms between free electrons and lasers in vacuum: normal Compton scattering (NCS), stimulated Compton scattering (SCS), and nonlinear Compton scattering (NLCS), which is a multi-photon exchange process in which an electron absorbs simultaneously many photons with emission of one high-frequency photon. These mechanisms play different roles in the laser acceleration of electrons at different laser intensities [18]. Furthermore, we will examine the connection between the NLCS effect and the validity of the PPM, as well as to explain the violent acceleration law based on the NLCS effect.

In Section II, we discuss the analytical expressions for the laser fields used in the simulations. In Section III, we present various theoretical models and results, both classical and quantum in nature. Our numerical simulation results are presented and discussed in Section IV. A brief summary will be given at the end.

II. FIELD EQUATIONS

In our consideration of relativistic electrons interacting with intense laser fields, the following inequalities are assumed to be satisfied [23],

$$\hbar\omega << m_e c^2 \text{ and } \sqrt{\frac{E_0}{E_c}} = \sqrt{\frac{a_0 \hbar\omega}{m_e c^2}} << \gamma << \frac{E_c}{E_0} = \frac{m_e c^2}{a_0 \hbar\omega}, \tag{1}$$

where $E_c = m_e^2 c^3/(e\hbar) \simeq 1.3 \times 10^{16}$ V/cm is the so-called critical field strength for production of $e^+ e^-$ pairs, E_0 is the peak amplitude of the laser field, and γ is the Lorentz factor representing the electron energy. The maximum field strength $E_0[\text{V/cm}] = 3.21 \times 10^{10} a_0/\lambda[\mu m]$ used in the examples given below is $E_0 = 9.63 \times 10^{12}$ V/cm $<< E_c$ for $a_0 = 300$ and $\lambda = 1\mu m$. Hence, a classical description of the radiation field and electron is adequate.

Numerical simulation methods used here are similar to those we used previously [14]. The configuration of the laser-electron interaction is shown in Fig. 1. The laser beam we adopted is the lowest-order Hermite-Gaussian (0,0) mode and it is polarized in the x-direction and propagating along the z-axis. The transverse component of the vacuum wave equation describing the evolution of the slowly varying amplitude of the laser field $\hat{E}_x(r, \zeta, z)$, where $E_x = (\hat{E}_x/2)\exp(ik\zeta)+$ complex conjugate, can be written as [24],

$$\left[\nabla_\perp^2 - 2\left(ik + \frac{\partial}{\partial\zeta} \right)\frac{\partial}{\partial z} + \frac{\partial^2}{\partial z^2} \right] \hat{E}_x = 0, \tag{2}$$

where $\zeta = ct - z$ and $k = 2\pi c/\lambda$. Typically, the operators in the above equation scale as $\nabla_\perp \sim 1/w_0$, $\partial/\partial\zeta \sim 1/L$, and $\partial/\partial z \sim 1/Z_R$, where w_0 is the laser radius at focus, L is the laser pulse length, and $Z_R = kw_0^2/2$ is the Rayleigh length [24]. The well-known paraxial approximation to the wave equation involves neglecting the terms $\partial^2/\partial\zeta\partial z$ and $\partial^2/\partial z^2$ in Eq. (2). For a continuous laser beam ($L = \infty$), $\hat{E}_x(r, z)$ is independent of ζ and $\partial^2/\partial\zeta\partial z$ is set to zero in Eq. (2). For a continuous laser beam, analytical expressions for the fields beyond the paraxial approximation have been derived by Davis [25] by retaining the term $\partial^2/\partial z^2$ in Eq. (2) and expanding the solutions in terms of the small parameter ϵ, where $\epsilon^2 = 1/(2kZ_R) = 1/(kw_0)^2$. Later, Barton [26] extended this procedure to obtain symmetric fifth-order corrected formulae for the electromagnetic field components. The non-paraxial solutions for a continuous laser beam can be expressed as follows [26],

$$E_x = E_0\{1 + \epsilon^2(-\rho^2\Theta^2 + i\rho^4\Theta^3 - 2\Theta^2\xi^2)$$
$$+\epsilon^4[2\rho^4\Theta^4 - 3i\rho^6\Theta^5 - 0.5\rho^8\Theta^6 + (8\rho^2\Theta^4 - 2i\rho^4\Theta^5)]\xi^2\}\psi_0\exp\left(-i\alpha/\epsilon^2\right), \tag{3}$$

$$E_y = E_0\{\epsilon^2(-2\Theta^2) + \epsilon^4(8\rho^2\Theta^4 - 2i\rho^4\Theta^5)\}\xi\eta\psi_0\exp\left(-i\alpha/\epsilon^2\right), \tag{4}$$

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$$E_z = E_0\{\epsilon(-2\Theta) + \epsilon^3(6\rho^2\Theta^3 - 2i\rho^4\Theta^4)$$

$$+\epsilon^5(-20\rho^4\Theta^5 + 10i\rho^6\Theta^6 + \rho^8\Theta^7)\}\xi\psi_0 \exp{(-i\alpha/\epsilon^2)}, \tag{5}$$

$$cB_x = E_0\{\epsilon^2(-2\Theta^2) + \epsilon^4(8\rho^2\Theta^4 - 2i\rho^4\Theta^5)\}\xi\eta\psi_0 \exp{(-i\alpha/\epsilon^2)}, \tag{6}$$

$$cB_y = E_0\{1 + \epsilon^2(-\rho^2\Theta^2 + i\rho^4\Theta^3 - 2\Theta^2\eta^2)$$

$$+\epsilon^4[2\rho^4\Theta^4 - 3i\rho^6\Theta^5 - 0.5\rho^8\Theta^6 + (8\rho^2\Theta^4 - 2i\rho^4\Theta^5)\eta^2]\}\psi_0 \exp{(-i\alpha/\epsilon^2)}, \tag{7}$$

$$cB_z = E_0\{\epsilon(-2\Theta) + \epsilon^3(6\rho^2\Theta^3 - 2i\rho^4\Theta^4)$$

$$+\epsilon^5(-20\rho^4\Theta^5 + 10i\rho^6\Theta^6 + \rho^8\Theta^7)\}\eta\psi_0 \exp{(-i\alpha/\epsilon^2)}, \tag{8}$$

$$\xi = \frac{x}{w_0}, \quad \eta = \frac{y}{w_0}, \quad \alpha = \frac{z}{kw_0^2}, \quad \rho = \sqrt{\xi^2 + \eta^2}, \tag{9}$$

$$\Theta = \frac{1}{i + 2\alpha}, \quad \psi_0 = i\Theta \exp(-i\rho^2\Theta + i\omega t + i\phi_0). \tag{10}$$

where E_0 is the reference electric field strength and ϕ_0 the initial phase. In the usual paraxial solutions, electromagnetic field components of the laser are given by

$$E_x = E_0\psi_0 \exp{(-i\alpha/\epsilon^2)}$$

$$= \frac{E_0 w_0}{w} \exp\left[i(\omega t - kz + \phi_0) - \left(1 + i\frac{z}{Z_R}\right)\frac{r^2}{w^2} + i\tan^{-1}\frac{z}{Z_R}\right], \tag{11}$$

$$E_z = -\frac{i}{k}\frac{\partial E_x}{\partial x}, \tag{12}$$

$$\mathbf{B} = \frac{i}{\omega}\nabla \times \mathbf{E}, \tag{13}$$

where $w = w_0(1 + z^2/Z_R^2)^{1/2}$. According to the discussions of Barton [26], the fifth-order corrected field equations are of high accuracy. Our studies indicate that when $kw_0 \gtrsim 60$, as in most of the cases of interest, the paraxial expressions can be readily regarded to be very good approximations to the actual fields.

For the case of laser pulses with a finite pulse length L, the term $\partial^2/\partial\zeta\partial z$ can be important in Eq. (2). Solutions to Eq. (2) describing ultra-short pulses with $L \ll 2Z_R$ have been derived by Esarey et al. [24] by retaining the term $\partial^2/\partial\zeta\partial z$ while neglecting the term $\partial^2/\partial z^2$. However, for sufficiently long pulses, $L \gg 2Z_R$, the term $\partial^2/\partial\zeta\partial z$ can be neglected compared to $\partial^2/\partial z^2$. Hence, for long pulses with $L \gg 2Z_R$, the field components can be

approximated by multiplying the continuous pulse solutions, Eqs. (3)-(8), by a time envelope function $f(\zeta)$, which is assumed to be Gaussian,

$$f(\zeta) = \exp\left[-\frac{(t - z/c)^2}{\tau^2}\right],\qquad(14)$$

where $\tau = L/c$ is the pulse duration ($\tau \to \infty$ corresponds to a continuous beam). Recall that since the fields must satisfy $\nabla \cdot E = \nabla \cdot B = 0$, simply multiplying Eqs. (3)-(8) by $f(\zeta)$ implies that terms of order $1/kL$ have been neglected to these expressions. Furthermore, Esarey et al. [24] point out that the envelope of a finite duration laser pulse travels at a group velocity $v_g \leq c$, i.e., the axial profile is of the form $f(v_g t - z)$, where $v_g/c = 1 - 2/(k^2 w_0^2)$ near the laser focus. This effect can be neglected, however, provided that the envelope slippage length $\Delta L = z(1 - v_g/c)$ is small compared to the pulse length $L = c\tau$ over the interaction distance z. This implies $kL \gg |z|/Z_R$. Note that this inequality is typically not as constraining as the inequality that must be satisfied to be in the long pulse regime, $L \gg 2Z_R$, which implies $kL \gg k^2 w_0^2$ or $kL \gg 1/\epsilon^2$.

We use a four-dimensional energy-momentum configuration to specify the electron state (γ, P_x, P_y, P_z), where the Lorentz factor γ, the momentum P are normalized in the units of $m_e c^2$ and $m_e c$, respectively. Besides, for simplicity, throughout the paper, time and length are normalized by $1/\omega$ and $1/k$. The electron dynamics are governed by the following relativistic Newton-Lorentz equations.

$$\frac{d\mathbf{P}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}),\qquad(15)$$

$$\mathbf{P} = \gamma\mathbf{v}, \quad \gamma = \frac{1}{\sqrt{1 - \mathbf{v}^2}},\qquad(16)$$

where $\mathbf{v}$ is the electron velocity normalized to c.

Without losing generality, we assume that the pulsed beam center reaches the point $x = y = z = 0$ at $t = 0$, and that the electron is incident in the $x - z$ plane ($b_0 = 0$) with the initial time chosen such that the electron arrives at $x = y = z = 0$ at time $t = -\Delta t_d$ under the condition of free motion, i.e., without the influence of the laser fields. Thus, Δt_d specifies the relative delay between the laser pulse and the electron. Here, we take the sign of Δt_d

such that when $\Delta t_d < 0$, the laser pulse propagates ahead of the electron and the electron may mainly interact with the trailing temporary edge of the pulse, while for $\Delta t_d > 0$, the electron may interact with the leading edge of the pulse.

Except specification, all the results in this paper were obtained by numerical integration of the full Newton-Lorentz force equations, Eqs. (15)-(16), with the fields given by Eqs. (3)-(10) and (14).

III. THEORETICAL MODELS

A. One-Dimensional Theory

Assuming that the laser field is a one-dimensional (1D) plane wave of the form $a = a(z - ct)$, the electron orbits can be calculated exactly [27], [28], [29]. For example, the normalized energy γ and axial momentum P_z are given by

$$\gamma = \frac{(1 + \beta_0)\gamma_0}{2} \left[(1 + a^2) + \frac{1}{(1 + \beta_0)^2 \gamma_0^2} \right], \tag{17}$$

$$P_z = \frac{(1 + \beta_0)\gamma_0}{2} \left[(1 + a^2) - \frac{1}{(1 + \beta_0)^2 \gamma_0^2} \right], \tag{18}$$

and the transverse momentum is $P_x = a_x$, where β_0 is the initial normalized velocity in the z-direction and $\gamma_0 = (1 - \beta_0^2)^{-1/2}$. For an initially stationary electron, $P_z = a^2/2$ and $\gamma = 1 + a^2/2$. Notice that the electron only gains energy while it is inside the laser pulse. Physically, as the laser pulse impinges upon the electron, the nonlinear ponderomotive force associated with the front (rise) of the laser pulse accelerates the electron. Eventually, the laser pulse outruns the electron and the electron is decelerated by ponderomotive force on the back of the pulse. Once the electron exits the back of the pulse, there is no net energy gain. A finite energy gain can result, however, if the electron leaves the vicinity of the laser pulse before it has a chance to be decelerated by the back of the pulse. In 3D, this can occur by transverse scattering of the electrons, as discussed in Refs. [10], [21], or by the pulse diffracting, as is discussed in the following.

Kaw and Kulsrud [28] analyzed electron motion in a 1D model of a laser pulse with a slowly varying amplitude of the form $a = \hat{a}(z)\Phi(z - ct)$, where Φ is a function of only $z - ct$ and includes the fast varying phase function and $\hat{a}(z)$ is the slowly varying envelope. The scale length for variations in $\hat{a}$ is assumed to be on the order of the Rayleigh length, $|d\hat{a}/dz| \sim |\hat{a}|/Z_R$, and thus approximately account for the effects of diffraction within a 1D model. In the adiabatic limit, in which the quiver oscillation time is short compared to the diffraction time, the final electron energy is given by [28]

$$\gamma \simeq a_0^2(f_k - 1)/f_k^2 \simeq (1 - Z_R/a_0^2 L)Z_R/L, \tag{19}$$

where $f_k \simeq a_0^2 L/Z_R$ with L the laser pulse length, along with the initial conditions of a particle at rest at the focal position, and the assumptions of $f_k > 1$, $(f_k - 1)$ not too small, and $a_0 \gg 1$. For a fixed laser pulse energy, the energy gain is optimized for $f_k \simeq 3$, which physically states that the time it takes the electron to slip relative to the laser pulse by the pulse length, $L/(c - v_z) \simeq a_0^2 L/2$, is approximately equal to the diffraction time Z_R/c. As an example, a laser pulse with $a_0 = 4.2$, $kL = 1000$, and $kZ_R = 5800$ can accelerate an electron from rest to $\gamma \simeq 10$.

The above results assumed 1D and an adiabatic approximation (many quiver oscillations per Rayleigh length). For a laser pulse of the form $a = a_0(z - ct)\cos k(z - ct)$, where $k = 2\pi/\lambda$, the amplitude of the transverse quiver oscillation x_q and the time required to complete this quiver oscillation t_q are given by

$$x_q = (1 + \beta_0)\gamma_0 a_0(\lambda/2\pi), \tag{20}$$

$$ct_q = \left[1 + (1 + \beta_0)^2\gamma_0^2(1 + a_0^2/2)\right]\lambda/2. \tag{21}$$

The higher the initial energy, the larger the quiver orbit and period, since the electron is moving closer to synchronism with the laser field. One would expect that these 1D orbits would be an approximately valid description of the dynamics near the focus of a 3D laser field provided that (i) the quiver amplitude remain small compared to the laser spot size, $x_q \ll w_0$, and (ii) the quiver period remain small compared to the diffraction time, $ct_q \ll Z_R$. These two conditions imply, respectively,

$$w_0/\lambda \gg (1 + \beta_0)\gamma_0 a_0/2\pi, \tag{22}$$

$$w_0^2/\lambda^2 \gg \left[1 + (1 + \beta_0)^2\gamma_0^2(1 + a_0^2/2)\right]/2\pi, \tag{23}$$

These conditions become more difficult to satisfy at high values of a_0 and γ_0. For the parameters of the CAS regime, these two conditions are generally violated, and the energy gain characteristic of the CAS regime cannot be described by 1D or adiabatic (i.e., time-averaged over the quiver motion) theories.

Nevertheless, it is interesting to note that if one were to terminate the interaction when the electron has slipped to the center of laser pulse, such that the electron resides at the peak of the laser intensity, the exact 1D orbits given by Eqs. (17) and (18) predict an energy gain that scales as $\gamma \sim a^2$. In the CAS mechanism, the laser-electron interaction is terminated, in effect, by diffraction. The scaling $\gamma \sim a^2$ is in approximate agreement with that observed in the CAS simulations discuss in the following sections.

B. Ponderomotive Potential Model

It is of interest to compare the solutions of the full Newton-Lorentz equations of motion with that of a simplified equation of motion, the so-called ponderomotive potential model (PPM), which is often used to describe the interaction of intense lasers fields with electrons. The PPM is typically valid in cases in which an electron experiences many quiver oscillations in the laser field such that a time-averaging over the fast quiver motion can be justified. Furthermore, in the PPM, the canonical momentum is approximately conserved, $i.e.$, $p_\perp - A_\perp \simeq constant$. These assumptions are generally not valid for the capture and acceleration scenario (CAS). Nevertheless, it is of interest to explore the differences between the numerical solutions of the full Lorentz equations of motion and the PPM. In the PPM, the time-averaged equation of motion is given by

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{F}_{\text{pond}}(t) = -\nabla V_{\text{pond}}(r, z, t), \tag{24}$$

where V_{pond} is the ponderomotive potential given by [3], [16], [17], [18], [19], [20], [21],

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$$V_{\text{pond}}(r, z, t) = \left(\sqrt{1 + a^2(r, z, t)/2} - 1 \right) m_e c^2. \tag{25}$$

Here, $a^2/2$ is the normalized time-averaged laser intensity profile, which in the paraxial approximation is given by

$$a^2(r, z, t) = a_0^2 \frac{w_0^2}{w^2(z)} f^2(ct - z) \exp\left(-\frac{2r^2}{w^2(z)} \right). \tag{26}$$

C. Phase Velocity Synchronization

To explain the mechanism leading to the large electron energy gains in the CAS, it is instructive to observe the phase variation experienced by the electron in the laser field. As we know, the phase slippage velocity of an electron (relative the laser field phase fronts) in a vacuum electromagnetic plane wave can be approximately estimated by $c/(2\gamma_\parallel^2)$, where $\gamma_\parallel = (1 - v_\parallel^2/c^2)^{1/2}$ and $v_\parallel$ is the electron velocity along the wave propagation direction. Thus it would be expected that when $\gamma_\parallel$ is not large, as in the early acceleration stage, there should be noticeable phase slippage. To study the physical reason of this phenomenon, we note that the laser field concerned is not a plane wave, but a Gaussian beam in which the radius of the curvature varies due to the diffraction effect of the optical beam. The phase of a Gaussian beam is given by [30]

$$\varphi = kz - \omega t - \phi(z) - \phi_0 + \frac{kr^2}{2R(z)}, \tag{27}$$

where $\phi(z) = \tan^{-1}(z/Z_R)$ is the Gouy phase shift and $R(z) = z(1 + Z_R^2/z^2)$ is the radius of the curvature. Note that $R(z)$ first decreases from $z = 0$ to Z_R, the Rayleigh range, and then increases from Z_R to the infinity.

The phase velocity of the wave along a particle trajectory can be calculated by the equation

$$\partial \varphi / \partial t + (V_\varphi)_J (\nabla \varphi)_J = 0, \tag{28}$$

where $(V_\varphi)_J$ is the phase velocity of the wave along the trajectory and $(\nabla \varphi)_J$ is the gradient of the phase along the trajectory. In particular, denoting the unit vector along the electron

trajectory as $\mathbf{e}_e = (v_r/v_0)\mathbf{e}_r + (v_z/v_0)\mathbf{e}_z$, where $v_0 = (v_r^2 + v_z^2)^{1/2}$ is the magnitude of the electron velocity, the magnitude of the phase velocity along the electron trajectory is $V_{\varphi e} = ck/(\mathbf{e}_e \cdot \nabla\varphi)$, which can be written as

$$V_{\varphi e} = ck \left(\frac{v_z}{v_0}\frac{\partial\varphi}{\partial z} + \frac{v_r}{v_0}\frac{\partial\varphi}{\partial r} \right)^{-1}, \tag{29}$$

where

$$\frac{\partial\varphi}{\partial z} = k \left[1 - \frac{(1 - f_\varphi)}{kZ_R(1 + z^2/Z_R^2)} \right], \tag{30}$$

$$\frac{\partial\varphi}{\partial r} = \frac{krz}{Z_R^2(1 + z^2/Z_R^2)}, \tag{31}$$

with

$$f_\varphi = \frac{r^2(1 - z^2/Z_R^2)}{w_0^2(1 + z^2/Z_R^2)}. \tag{32}$$

The above expressions give magnitude of the phase velociy along the electron trajectory $V_{\varphi e}$ as a function of the electron velocity (v_r, v_z) and position (r, z). Note that the minimum value of the phase velocity occurs for an electron trajectory angle of

$$\tan\theta_{min} = \frac{v_r}{v_z} = \frac{\partial\varphi/\partial r}{\partial\varphi/\partial z} \tag{33}$$

and is given by $V_{\varphi,min} = ck/|\nabla\varphi|$.

Consider an electron propagating at a small angle θ_e with respect to the z-axis, where $v_r = v_0 \sin\theta_e$ and $v_z = v_0 \cos\theta_e$. The phase velocity along the trajectory is given by

$$V_{\varphi e} \simeq c \left[1 + \frac{(1 - f_\varphi)}{kZ_R(1 + z^2/Z_R^2)} - \frac{rz\theta_e}{Z_R^2(1 + z^2/Z_R^2)} + \frac{\theta_e^2}{2} \right] \tag{34}$$

assuming $\theta_e^2 \ll 1$, $(1 - k^{-1}\partial\varphi/\partial z)^2 \ll 1$ and $(\theta_e k^{-1}\partial\varphi/\partial r)^2 \ll 1$, i.e., the last three terms on the right side of the above equation are assumed to be small compared to unity. For an electron moving parallel to the z-axis ($\theta_e = 0$), subluminous phase velocities $V_{\varphi e} < c$ require $f_\varphi > 1$, which can only occur in the region $|z| < Z_R$ and at $z = 0$ only for $r > w_0$. The phase velocity is minimum at the angle

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$$\theta_{min} \simeq \frac{rz}{Z_R^2(1 + z^2/Z_R^2)} \tag{35}$$

and is given by

$$V_{\varphi,min} \simeq c\left[1 + \frac{(1 + z^2/Z_R^2 - r^2/w_0^2)}{kZ_R(1 + z^2/Z_R^2)^2}\right]. \tag{36}$$

The condition $V_{\varphi,min} < c$ requires $r^2/w_0^2 > 1 + z^2/Z_R^2$.

For example, an electron moving along the z-axis ($r = 0$) with a velocity $v_z \simeq c$ has the phase velocity $V_{\varphi 0}$ approximately given by

$$V_{\varphi 0} \simeq c\left[1 + \frac{1}{kZ_R(1 + z^2/Z_R^2)}\right]. \tag{37}$$

Consequently, the distance it takes for this electron to phase slip with respect to the laser field by an amount $\lambda/2$ is $z \simeq Z_R$. This is a result of the Gouy phase factor $\phi(z) = \tan^{-1}(z/Z_R)$. Furthermore, this forms the basis of the Lawson-Woodward theorem, as applied to an electron moving with $v_z \simeq c$ in a straight line that experiences only a linear acceleration force (proportional to the electric field of the laser). Integrating this force along the straight line trajectory from $-\infty < z < \infty$ yields zero net energy gain. This is a consequence of $v_{p0} > c$ and phase slippage.

This is not the case, however, for an electron undergoing a general nonlinear, curved trajectory for which the velocity is not constant. As the laser field acts upon the electron and alters its velocity, the term $\mathbf{v} \times \mathbf{B}$ in the Lorentz force can become important. Furthermore, the effective phase velocity along the nonlinear electron trajectory can be less than the speed of light in vacuum. It then becomes possible for the electron to be phase synchronous with the laser field over a significant distance, which can lead to a substantial net energy gain. We find this to be the case in the simulations of the CAS trajectories, and the numerical simulation results will be presented in the following.

D. Quantum Estimation of Maximum Energy Gain

Our simulations indicate, that in the CAS regime ($a_0 \gtrsim 100$), with maximum net electron energy gain scales as a_0^2. One possible explanation for this is the following. An upper limit on

the electron acceleration can be estimated by counting the total momentum of the optical field shining on the effective area of an electron area per unit time. Here, we assume that the characteristic area of an electron in the electron-photon interaction is $\pi\lambda_c^2$, where $\lambda_c = \hbar/(m_e c)$ is the electron Compton radius, and that the characteristic interaction length of the electron with highly focused laser beam is the Rayleigh length $Z_R = kw_0^2/2$. This implies

$$[\Delta E_{\max}]_{up} = I \times \pi\lambda_c^2 \times Z_R/c \tag{38}$$

where $I[\text{W/cm}^2] = 1.37 \times 10^{18} a_0^2/(\lambda[\mu\text{m}])^2$. Also, the maximum acceleration gradient is

$$[\partial(\Delta E)/\partial s]_{up} = I \times \pi\lambda_c^2 \times c^{-1} \tag{39}$$

For $kw_0 = 200$, $\lambda = 1\mu m$, and $a_0 = 100$, the above expressions give

$$[\Delta E_{\max}]_{up} \sim a_0^2/2 \tag{40}$$

and 14.3 GeV/cm for the acceleration gradient, which is in approximate agreement with the simulations presented below. These simple estimates imply that the upper limit to the energy gain $\Delta E_{\max}$ in the CAS is proportional to a_0^2, and that the relevant acceleration gradient can reach tens of GeV/cm. It should be emphasized that since the net energy exchange is influenced by numerous factors, the above-mentioned law is intended only an approximate scaling relationship. This statement is also consistent with the theoretical analysis of NLCS in a plane-wave field. Because the required laser intensity is very high, experimental research on NLCS in the regime of CAS is beyond current technology. Nevertheless, recently C. Bula *et al.* have succeeded in observing the absorption of up to four photons simultaneously by an electron interacting with a laser field with $a_0 = 0.6$ [31], which demonstrates that experimental investigation of physics relevant to these processes is now possible.

E. NLCS and PPM

In the numerical simulations presented below, we find that in the regime in which the energy gain is small, $a_0 < 10$, PPM can provide an adequate approximation to the electron

dynamics. However, in the regime of large energy gain of CAS, $a_0 \gtrsim 100$, PPM is a very poor approximation, and the energy gain mechanism can be described by NLCS. We propose the following explanations to the suggested connection between NLCS and PPM. First, if we neglect the contribution of NCS (because the energy of a photon concerned is in the order of an eV), the net electron energy gain from the mono-frequency continuous laser fields by the mechanism of SCS is zero since the continuous laser beam is composed of plane waves with the same frequency. Thus the net energy exchange between charged particle and continuous laser beam comes chiefly from NLCS. On the other hand, we note the fact that since the ponderomotive potential of a continuous beam is conservative, the electron scattering by such a potential is bound to be elastic with no net energy exchange. Thus PPM cannot describe the process dominated by NLCS. In other words, we can say that the invalidity of PPM can be regarded as a judgment that NLCS plays a noticeable role. This statement can also essentially be applied to the case of pulsed lasers. M.V.Fedorov *et al*. [32] verified that as $a_0 < 1$, one can use PPM to describe SCS. As for $a_0 > 1$, however, up to now there is no definite connection that has been identified between PPM and SCS.

Regarding a pulsed laser beam, it is possible for an electron to exchange both energy and momentum with a pulsed field by the mechanism of SCS since the pulsed laser beam can be *Fourier* decomposed into plane waves not only of different traveling directions but also of different frequency. Likewise, V_{pond} of a pulsed laser beam is no longer conservative and, hence, it is possible for an electron to gain net energy in the PPM with a pulsed laser beam. The problem of invalidity of PPM has also been studied by Quesnel and Mora recently [21]. From the simulation results presented below, we find that when $10 < a_0 \leqslant 30$, NLCS begins to play a noticeable role, and for $a_0 \gtrsim 30$, NLCS becomes dominant. When the field strength is sufficiently strong ($a_0 \gtrsim 100$) the electron can be captured and violently accelerated by the laser field, and this effect comes chiefly from the NLCS mechanism.

IV. SIMULATION RESULTS AND DISCUSSION

A. Characteristics of Output Electron Bunches

Our study shows that the electron dynamic regime, CAS, emerges only when the laser intensity is strong enough $a_0 \gtrsim 100$ and when the electron injection angle is sufficiently small. The phase space of the electron incoming momenta required by CAS is not small and readily achievable in experiments. Especially, the optimum incident momentum is not very sensitive to the laser intensity, and can be in the range $10 - 20 \; MeV$.

As found in our previous studies [14], the electron final energy γ_f is sensitive to the laser wave's initial phase ϕ_0 as well as the delay time Δt_d. Figure 2 shows examples that the electron final energy γ_f as a function of the initial phase ϕ_0 when (a) $a_0 = 30$ and (b) $a_0 = 100$, where $\gamma_{f\,max}$ is the maximum γ_f in the whole phase range $\phi_0 \in [0, 2\pi]$. The other parameters used in Fig. 2 are $\Delta t_d = 0$, $\tau = 1000$, $w_0 = 200$, $P_{xi} = 4$, $P_{yi} = 0$, $P_{zi} = 40$, and $b_0 = 0$. In the electron capture case, we have terminated the calculation at $t = 3 \times 10^5$. From Fig. 2(b), it can be seen that about 20% of electrons can be accelerated to GeV which display typical CAS trajectories [14], if the electrons are uniformly distributed in all phase $\phi_0 \in [0, 2\pi]$.

Fig. 3 shows two typical cases of electron dynamics, *i.e.*, the CAS trajectory and the electron inelastic scattering (IS). They correspond to the points A and B shown in Fig. 2(b), respectively. Other parameters are the same as those in Fig. 2(b). Figure 3(a) shows electron trajectories in the x, z plane. It clearly indicates whether the electron is captured or reflected. In Fig. 3(b), we present the electron energy γ as a function of time. Fig.3(c) shows the variation of the laser phase φ experienced by an electron during the interaction. The most prominent feature of Fig. 3(c) is that the phase experienced by the CAS electron varies extremely slowly even in the early acceleration stage. For the CAS, the electrons can be captured into the intense field region rather than expelled from it and the captured electrons can be accelerated to GeV energies with acceleration gradients of tens of GeV/cm.

As for a real laser beam, its initial phase ϕ_0 is fixed, but electrons with different delay time Δt_d in a bunch will feel different phases. Figure 4 shows that the electron outgoing energy γ_f as a function of the relative delay time Δt_d. Without losing generality, we chose $\phi_0 = 0$ and other parameters are the same as those in Fig. 2(b). Figure 4 corresponds to the case of an incident electron bunch. Obviously, the output of the acceleration mechanism is a GeV electron macro-pulse which consists of many micro-pulses. The macro-pulse corresponds to the duration of the laser pulse and the micro-pulse to the periodicity of the laser wave. Each of the micro-pulses has the same shape-factor as that of Fig. 2(b). This output feature is analogous to that of conventional linacs. By combining Fig. 2(b) and Fig. 4, we can find that the total capture and accelerating fraction for an incident electron bunch with a length comparable or less than that of the laser pulse is not small, in contrast to the "bucket" phenomenon for the laser-plasma acceleration schemes [2].

The energy spread and angular spread of the accelerated electron bunches are important research subjects for practical applications of the CAS mechanism. Fig. 5 and Fig. 6 present an example where the incoming electron bunch is assumed to be a prolate ellipsoid with the same size as that of the laser pulse: the major axis is $L_0 = c\tau = 500$ and the minor axis equals to $w_0 = 150$. The momentum of all the electrons uniformly distributed in the ellipsoid are the same with $P_{0i} = 19.544$ and incident angle $\theta_i = \tan^{-1}(0.1)$, corresponding to an incoming energy $\gamma_i = 10$ MeV. The electron bunch arrives synchronously and interacts with the laser pulse.

Fig. 5 shows that the outgoing electrons can generally be divided into two groups. The IS electrons correspond to the peak at larger scattering angle. This, along with the feature of low outgoing energy, causes the IS electrons to spread greatly in space. The left peak in Fig. 5 corresponds to the CAS electrons, which consists of more than 30% of the total incident electrons. Due to the features of high outgoing energies and small angle spread, the outgoing CAS electrons compose a high-energy bunch with a limited spread in space. Fig. 6 presents the energy spectrum of the outgoing CAS electrons. It can be seen that the energies of the CAS electrons spread widely from 0.5 to 3.5 GeV. The poor energy spread of

the output CAS electrons can be improved by using an electron spectrometer to tailor the electron beam.

B. Effective Phase Velocity

Figure 7(a) shows the phase velocity distribution along the z-axis. Figure 7(b) and Figure 7(c) compare the wave phase velocity along the electron trajectory $(V_\varphi)_J$ with the electron velocity for CAS and IS, respectively. From Fig.7(c) we can see the wave phase velocity (solid line) for the IS trajectory is much faster than the electron dynamic velocity (dotted line). Thus the electron phase slippage in the wave will be very fast as shown in Fig. 3(c) (dotted line). As a consequence, the electron cannot get considerable net energy gain from the laser field. In contrast to that, from Fig. 7(b), we see that in the path between 0 and Z_R, the wave phase velocity (solid line) of the CAS trajectory is even less than the electron velocity (dotted), and in the following path, the effective phase velocity is kept very close to the electron velocity. This is the reason that the phase slippage of the electron in the wave field remains extremely low. Consequently, the electron can be trapped in the acceleration phase for long times to gain considerable energy from the laser field.

When an electron is captured, due to the diffraction effect of the optical beam near the focused region, the effective wave phase velocity along the dynamic trajectory of the captured particle is found to be less than c, the speed of light in vacuum, or even less than the speed of the particle. Thus the captured electron can be kept in the acceleration phase of the wave for long times, and gain considerable energy from the laser field. It is also found that the emergence of CAS trajectories is sensitive to the laser wave phase experienced by the incident electron when it reaches the laser intense region.

C. Comparisons to Theory

Figure 8 shows the envelopes of the maximum outgoing energy of the electron $\gamma_{f\,\text{max}}$ as a function of the relative delay time Δt_d for the three typical cases at $a_0 = 10$, 30 and

100, respectively. It can be found that there are two kinds of peaks as the delay time is varied over the range of interest. The first type of peak occurs when the electron interacts with the leading ($\Delta t_d > 0$) and trailing ($\Delta t_d < 0$) temporal edges of the pulse, the other type occurs near the point $\Delta t_d = 0$. The width of the former is very narrow (referred to as a narrow peak), whereas the width of the latter is relatively wide (referred to as a wide peak). The narrow peaks always appear near the so-called turning points where $P_{xi} = a/\sqrt{2}$, which represents a transition between penetration and reflection [22]. At the turning point, the time interval in which the electron moves in the strong field region is longer than the nearby trajectories. Thus, the electron may have more net energy exchange with the laser fields. Detailed study shows that the wide peak begins to appear only when $a_0 \gtrsim 10$, as shown in Fig. 8(a). The value of the wide peak will be greater than that of the narrow peak when $a_0 \gtrsim 30$, as shown in Fig. 8(b). Furthermore, the narrow peaks almost can be neglected compared with the wide peak when a_0 approaches 100, as shown in Fig. 8(c). For the pulsed laser beam, we can only obtain the narrow peaks but not the wide peak by using the PPM, *e. g.*, the numerical solution to Eq. (24). However, for the continuous laser beam, the narrow peaks are not observed for either the Lorentz force model or the PPM. The wide peak becomes prominent only when $a_0 \gtrsim 30$, which stems chiefly from the NLCS mechanism, *i.e.*, numerical solution to the Lorentz equations, Eq. (15).

To explore the scaling law for the net energy gain of the electrons from the laser field in vacuum, we use $E_m = m_e c^2 \gamma_{fm}$ to represent the outgoing electrons' maximum energy as ϕ_0 and Δt_d vary over the whole range of interest. Figure 9 shows the variation of γ_{fm} versus a_0. This figure is chiefly concerned with the laser intensity range of $a_0 \leqslant 100$. For a comparison, the results obtained using three models, namely (i) the continuous laser with numerical integration of the Lorentz equations, (ii) the pulsed laser with numerical integration of the Lorentz equations, and (iii) the pulse laser with numerical integration of the PPM, are presented by the dot-dashed line, solid line and dotted line, respectively in the figure. From Fig. 9, we can see that when $a_0 \leqslant 10$, the PPM describes very well the electron motion, and when $10 < a_0 \leqslant 30$, the PPM is still approximately valid. But when $a_0 \gtrsim 30$, the PPM is

totally invalid. Furthermore, the electron energy gain increases sharply after $a_0 > 70$ as a consequence of the electron dynamics entering the CAS.

From Fig. 9 and many other calculation results, it seem to us that when NLCS begins to emerge, then PPM becomes invalid. As discussed above, for continuous laser beams, energy gain from SCS is zero, since the laser field is monochromatic. Likewise, the energy gain from PPM is zero, since the potential is conservative. Hence, the net energy exchange between charged particle and continuous laser beam comes chiefly from NLCS. However, for a pulsed laser beam, it is possible for an electron to exchange both energy and momentum with a pulsed field by the mechanism of SCS, since the pulsed field is no longer monochromatic, and by the mechanism of PPM, since the potential is no longer conservative. In view of the above-mentioned arguments, as well as from Fig. 8 and Fig. 9, we can say that when $10 < a_0 \leqslant 30$ the NLCS begins to play a noticeable role, and for $a_0 \gtrsim 30$ the NLCS becomes dominant. When the field strength is sufficiently strong ($a_0 \gtrsim 100$) the electron can be captured and violently accelerated by the laser field, and this effect comes chiefly from the NLCS mechanism.

D. Scaling Laws for the CAS Mechanism

The dependence of the electron final energy γ_{fm} on the incoming energy γ_i is shown in Fig. 10. From Fig. 10, we can find that the general trend for CAS is that as the incoming energy γ_i is increased, the electron final energy γ_{fm} will first increase rapidly and then decrease at a much slower rate after reaching the maximum. Also, the values of the incoming energy γ_i which correspond to the maximum electron final energy γ_{fm} are not sensitive to the laser intensity, $viz.$ $25 \lesssim \gamma_i \lesssim 35$ for a_0 varying in the range $[70, 300]$. This will be very useful for the design of laser acceleration experiments as well as future laser-driven accelerators because we need only a relatively low incoming energy $\gamma_i \approx 30$ (~ 15 MeV) to get the maximum final energy γ_{fm}.

Figure 11 shows the variation of the maximum net energy exchange $\Delta E_{\max} =$

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$m_e c^2 c (\gamma_{fm} - \gamma_i)_m$, which is defined as the maximum value of $\gamma_{fm} - \gamma_i$, as three parameters (the initial laser phase, the delay time and the incoming electron momenta) are varied over the range of interest, as a function of the laser intensity a_0^2 for $a_0 \gtrsim 100$, which corresponds to the regime of CAS. The prominent feature of Fig. 11 is that the maximum net energy gain in CAS, $\Delta E_{\max}$, is approximately proportional to a_0^2.

E. Energy Exchange versus w_0, τ, b_0, and θ_i

The dependences of the net energy exchange on the focal spot size w_0, the pulse length τ, the impact parameter b_0, and the injection angle $\theta_i = \tan^{-1}(P_{xi}/P_{zi})$ were also studied, since these parameters are important in obtaining ultra-high laser intensities and high energy exchange.

Figure 12 shows the variation of γ_{fm} versus the beam width at fixed laser field strength. The solid line, the crosses ($\times$) and the circles ($\bigcirc$) in Fig. 12 correspond to $\tau = 1000$, 300, and the case of continuous laser beams, respectively. These results demonstrate that γ_{fm} decreases as w_0 increases, and that γ_{fm} varies rapidly around $w_0 = 200$. It is of interest to see from Fig. 12 that the results are not sensitive to the pulse duration τ if $\tau > 300$. Detailed study shows that the electron cannot be captured even if the laser field strength is sufficiently strong ($a_0 = 100$) when $w_0 \geqslant 250$ and that the smaller the beam width is, the easier the electron trends to be captured and violently accelerated. Also, detailed study shows that as w_0 decreases, the threshold of the laser intensity for CAS will decrease, whereas the fraction of the initial phase ϕ_0 range for CAS will sharply increase. Obviously, this will be useful for the design of future laser-accelerators.

This feature can be understood from the following classical explanation. For sufficiently large values of w_0, the phase velocity synchronization effect will become less prominent. This results in a reduced energy gain, as discussed in the previous sections.

Figure 13 shows the dependence of γ_{fm} on the laser field strength for different pulse durations τ. We find that γ_{fm} increases as τ decreases when $a_0 < 30$, whereas γ_{fm} is

not sensitive to τ when $a_0 > 30$. The former is easy to understand from the quantum-mechanical viewpoint. We know a pulse with a Gaussian envelope will have a minimum-duration-bandwidth product $\Delta\nu \cdot \tau = 0.4$ [1], where $\Delta\nu$ is the pulse bandwidth. Hence, the contribution to the net energy exchange by SCS will increase as τ decreases. Since SCS is important for $a_0 < 30$, thus γ_{fm} will increase as τ decreases for $a_0 < 30$. On the other hand, the contribution to the net energy exchange in a strong laser field ($a_0 > 30$) mainly stems from NLCS, which shows different feature compared with that of SCS.

Figure 14 shows that the electron maximum outgoing energy $\gamma_{f\,max}$ as a function of the impact parameter b_0, where $\gamma_{f\,max}$ is the maximum γ_f when ϕ_0 varies in the whole phase range $\phi_0 \in [0, 2\pi]$. A pulsed laser with $\tau = 500$ and $w_0 = 150$ is used. Three typical cases, namely, $\Delta t_d = -250$, 0, $+250$, are given by the dash-dotted line, the solid line and dotted line respectively in the figure. The electron incident momentum chosen are $P_{xi} = 2$, $P_{yi} = 0$, $P_{zi} = 20$, corresponding to the incoming energy $\gamma_i \approx 10.28$ MeV and crossing angle $\theta_i = \tan^{-1}(0.1)$. Other parameters are the same as those in Fig. 2(b). One may find from Fig. 14 and Fig. 4 that electrons inside the internal region of the bunch, $b_0 < \frac{1}{2}w_0$ and $\Delta t_d < \frac{1}{2}\tau$, can be captured and accelerated to GeV energy under the conditions given above. It means the output GeV electron bunch can have comparable sizes as that of the laser pulse, provided the incident electron bunch is large enough.

Figure 15 shows the variation of $\gamma_{f\,max}$ versus $\theta_i = \tan^{-1}(P_{xi}/P_{zi})$, the electron injection angle in the x-z plane, for the parameters $P_{zi} = 20$, $P_{xi} = P_{zi}\tan\theta_i$, and $P_{yi} = 0$. The solid line is for $w_0 = 150$, the dotted line for $w_0 = 200$ and the dot-dashed line for $w_0 = 100$. A prominent feature of Fig. 15 is that the electron dynamic regime, CAS, emerges only when the electron injection angle is sufficiently small. Furthermore, the CAS angle range is strongly dependent on the laser beam width. Generally, the smaller w_0, the wider the CAS angle range. Still, when w_0 is large enough, the lowest-order Hermite-Gaussion(0,0) mode E-M wave tends to become a plane-wave and there would be no any CAS phenomenon.

V. SUMMARY

Using test particle simulations of electron trajectories in analytically prescribed laser fields, the physics of the CAS mechanism has been explored, which shows the following characteristics.

(1) Electrons can be captured into the intense field region, rather than expelled from it; and the captured electrons can be accelerated to GeV energies with acceleration gradients on the order of tens of GeV/cm.

(2) The required laser intensity for CAS to emerge is extremely high. As shown in Fig. 9, the electron energy gain increases sharply after $a_0 > 70$, and only as a_0 approaches 100 will there appear typical CAS trajectories.

(3) The CAS is distinct from that predicted by PPM. We find that whenever the NLCS effect appears to be prominent, PPM becomes invalid.

(4) For a capture electron in the CAS regime, the effective phase velocity of the laser field can be less than c, allowing for phase synchronism with the laser field and large energy gain.

(5) From Figs. 3 and 4, one can see that corresponding to each incident electron bunch, the output of the laser-electron interactions in CAS is a GeV electron macro-pulse composed of many micro-pulses.. Each of the micro-pulses corresponds to the periodicity of the laser wave. The features in the structure of the accelerated electron bunches are analogous to that of the conventional linacs, but with much high acceleration gradients. Furthermore, despite the electron energy gain being sensitive to the laser wave phase, the total fraction of electrons captured and accelerated is not small, provided the incident electron bunch length is comparable or less than that of the laser pulse. This is in contrast to the "bucket" phenomenon that characterizes laser-plasma acceleration schemes [2].

(6) The phase space of the incident electron momenta required by CAS is not small and is readily achievable in experiments. Furthermore, the optimum incident momentum is not very sensitive to the laser intensity, which is around 10-20 MeV with small crossing angle,

as shown in Fig. 8.

(7) Within the CAS regime ($a_0 \gtrsim 100$), the maximum electron energy gain is approximately proportional to a_0^2, as shown in Fig. 11.

(8) From Fig. 12, CAS is sensitive to the lateral field gradient of Gaussian beams. The larger w_0, the weaker the capture effect, resulting in less electron energy gain.

(9) The electron energy gain increases as τ decreases when $a_0 < 30$, but is not sensitive to τ when $a_0 > 30$ as long as $\tau > 300$. The contribution to the net energy exchange in a strong laser field ($a_0 > 30$) mainly stems from NLCS, which shows different features than that of SCS.

(10) A possible practical application of the CAS is the acceleration to high energy of an initially relativistic, high quality electron bunch that is injected, at a finite angle, into the focal region of an intense laser pulse. As shown in Fig. 5 and Fig. 6, The CAS can really generate bunches with large amount of GeV electrons and modest angle emittance. However, the energy of the outgoing CAS electron bunches spreads in a wide (more than GeV) energy rangy. This poor energy emittance may be improved by using electron spectrometer to tailor the output electron beams.

VI. ACKNOWLEDGMENTS

References

[1] G. A. Mourou, C. P. Barty and M. D. Perry, *Phys. Today*, 22**,** Jan. (1998); M. D. Perry and G. Mourou, *Science* **264**, 917 (1994).

[2] 2. Modena, Z. Najmudln, A. E. Dangor, C. E. Clayton, K. A. Marsh, C. Joshl, V. Malka, C. B. Darrow, C. Danson, D. Neely & F. N. Walsh, *Nature* **377**, 606 (1995); P. Sprangle, E. Esarey, J. Krall, *Phys.Plasmas* **3**, 2183 (1996); F. V. Hartemann, E. C. Landahl, A. L. Troha, Jr., J. R. Van Meter, H. A. Baldis, R. R. Freeman, N. C. Luhmann, Jr., L. Song, A. K. Kerman, D. U. L. Yu, *Phys. Plasmas.* **6**, 4104 (1999).

[3] For a review, see E. Esarey, P. Sprangle, J. Krall, and A. Ting, *IEEE Trans. Plasma Sci.* **24**, 252 (1996).

[4] P. Sprangle, E. Esarey, A. Ting, G. Joyce, *Appl. Phys. Lett.* **53** 2146 (1988); D. Umstadter, S.-Y. Chen, A. Maksimchuk, G. Mourou, R. Wagner, *Science* **273**, 472 (1996); C. I. Moore, A. Ting, S. J. McNaught, J. Qiu, H. R. Burris, and P. Sprangle, *Phys. Rev. Lett.* **82**, 1688 (1999); A. Ting, C. I. Moore, K. Krushelnick, C. Manka, E. Esarey, P. Sprangle, R. Hubbard, H. R. Burris, R. Fischer, and M. Baine, *Phys. Plasmas* **4**, 1889 (1997); F. Amiranoff, S. Baton, D. Bernard, B. Cros, D. Descamps, F. Dorchies, F. Jacquet, V. Malka, J. R. Marquè, G. Matthieussent, P. Miné, A. Modena, P. Mora, J. Morillo, and Z. Najmudin , *Phys. Rev. Lett.* **81**, 995 (1998); K. Krushelnick, E. L. Clark, Z. Najmudin, M. Salvati, M. I. K. Santala, M. Tatarakis, and A. E. Dangor, *Phys. Rev. Lett.* **83**, 737 (1999); K. Nakajima, D. Fisher, T. Kawakubo, H. Nakanishi, A. Ogata, Y. Kato, Y. Kitagawa, R. Kodama, K. Mima, H. Shiraga, K. Suzuki, K. Yamakawa, T. Zhang, Y. Sakawa, T. Shoji, Y. Nishida, N. Yugami, M. Downer, and T. Tajima, *Phys. Rev. Lett.* **74**, 4428 (1995).

[5] P.M. Woodward, *J. Inst. Electr. Eng.* **93**, 1554 (1947).

[6] J.D. Lawson, *IEEE Trans. Nucl. Sci.* **NS-26**, 4217 (1979).

[7] R.B. Palmer, *Part. Accel.* **11**, 81 (1980).

[8] E. Esarey, P. Sprangle, and J. Krall, *Phys. Rev. E*, **52**, 5443 (1995).

[9] K. T. McDonald, *Phys. Rev. Lett.* **80**, 1350 (1998).

[10] G. Malka, E. Lefebvre and J. L. Miquel, *Phys. Rev. Lett.* **78**, 3314 (1997); *Phys. Rev. Lett.* **80**, 1352 (1998).

[11] P.H. Bucksbaum, M. Bashkansky and T.J. McIlrath, *Phys. Rev. Lett.* **58**, 349 (1987).

[12] P. Monot, T. Auguste, L. A. Lompré, G. Mainfrary, and C. Manus, *Phys. Rev. Lett.* **70**, 1232 (1993).

[13] C. I. Moore, J. P. Knauer and D. D. Meyerhofer, *Phys. Rev. Lett.* **74**, 2439 (1995).

[14] L. J. Zhu, Y. K. Ho, J. X. Wang, and Q. Kong, *Phys. Lett. A* **248**, 319 (1998); J. X. Wang, Y. K. Ho, Q. Kong, L. J. Zhu, L. Feng, S. Scheid, and H. Hora, *Phys. Rev. E*, **58**, 6575 (1998).

[15] P.X. Wang, Y.K. Ho, X.Q. Yuan, Q. Kong, A.M. Sessler, and E. Esarey, Appl. Phys. Lett. **78**, 2253-5 (2001).

[16] A.V. Gaponov and M.A. Miller, Sov. Phys. J. Exp. Theor. Phys. **25**, 168 (1958).

[17] T. W. B. Kibble, *Phys. Rev. Lett.* **23**, 1054 (1966); *Phys. Rev.* **150**, 1060 (1966).

[18] A.Gover and A. Yariv, *Appl. Phys.* **16**, 121 (1979).

[19] P. Mora and T.M. Antonsen, *Phys. Rev. E* **53**, 2068 (1996); *Phys. Plasmas* **4**, 217 (1997).

[20] E.A. Startev and C.J. McKinstrie, *Phys. Rev. E*, **55**, 7527 (1997).

[21] Brice Quesnel and Patrick Mora, *Phys. Rev. E* **58**, 3719 (1998).

[22] L. J. Zhu, Y. K. Ho, J. X. Wang, and L. Feng, *J. Phys.* **B** : *At. Mol. Opt. Phys.* **32**, 939 (1999).

[23] K. T. McDonald, in Laser *Acceleration of Particles*, Edited by Chan Joshi and Thomas Katsouleas (AIP, New York, 1985), No.130, P.23

[24] E. Esarey, P. Sprangle, M. Pilloff, and J. Krall, *J. Opt. Soc. Am. B* **12**, 1695 (1995); E. Esarey and W.P. Leemans, *Phys. Rev. E* **59**, 1082 (1999).

[25] M. Lax, *Phys. Rev. A* **11**, 1365 (1975); L. W. Davis, *Phys. Rev. A* **19**, 1177 (1979).

[26] J.P.Barton and D.R.Alexander, *J. Appl. Phys.* **66(7)**, 2800 (1989).

[27] E.S. Sarachik and G.T. Schappert, *Phys. Rev. D* **1**, 2738 (1970).

[28] P.K Kaw and R.M. Kulsrud, *Phys. Fluids* **16**, 321 (1973).

[29] E. Esarey, S.K. Ride, and P. Sprangle, *Phys. Rev. E* **48**, 3003 (1993).

[30] P. W. Milonni and J. H. Eberly, *Lasers*, (John Wily & Sons, New York, 1988).

[31] C. Bula, K.T.McDonald, and E. J. Prebys, *Phys. Rev. Lett.* **76**, 3116 (1996); F.V.Hartemann and A.K.Kerman, *Phys. Rev. Lett.* **76**, 624 (1996);

[32] M.V.Fedorov, S.P.Goreslavsky, and V.S.Letokhov *Phys. Rev. E* **55**, 1015 (1997).

[33] G. Stupakov and M. Zolotorev have examined this process for the acceleration of electrons from rest to several MeV (submitted for publication).

Figure Captions

FIG. 1. Schematic geometry of electron scattering by laser beam. The laser propagates along the z-axis, w_0 is the beam width at the waist. Without losing generality, we assume the electrons are coming in from the negative-x side parallel to the x-z plane. (γ_i, P_{xi}, $P_{yi} = 0$, P_{zi}) denote the incoming energy and momentum of the electron and (γ_f, P_{xf}, P_{yf}, P_{zf}) that of outgoing state. γ is the Lorentz factor and b_0 the impact parameter. $\theta_i = \tan^{-1}(P_{xi}/P_{zi})$ is the electron injection angle in the x-z plane.

FIG. 2. The electron final energy γ_f as a function of initial phase ϕ_0 when $\Delta t_d = 0$. The results of Fig. 2(a) are for $a_0 = 30$ and Fig. 2(b) for $a_0 = 100$. Other parameters chosen are $\tau = 1000$, $w_0 = 200$, $P_{xi} = 4$, $P_{yi} = 0$, $P_{zi} = 40$, and $b_0 = 0$. In the electron capture case, we have terminated the calculation at $t = 3 \times 10^5$.

FIG. 3. Two typical cases of electron dynamics: Capture and acceleration scenario (CAS) and inelastic scattering (IS). Other parameters are the same as those in Fig. 2(b). (a) Electron trajectories in the x-z plane. The dot-dot-dashed lines show the spatial profile of the focused laser beam. (b) Electron energy γ as a function of time. (c) The laser wave phase experienced by the electron as a function of time. The solid line is for the case of electron capture with $\phi_0 = 210^0$ and the dotted line for the electron inelastic scattering with $\phi_0 = 0^0$. They are respectively corresponding to the points A and B shown in Fig. 2(b).

FIG. 4. (a) Dependence of the electron final energy γ_f on the relative delay time Δt_d when $\phi_0 = 0$. Other parameters are the same as those in Fig. 2(b). (b) An enlargement of the part denoted by the arrow in Fig. 4(a).

FIG. 5. Angular distribution $dn/(Nd\Omega) \sim \theta_f$ of the outgoing electrons. The parameters used in the calculations are $\tau = 500$, $w_0 = 150$, $P_{0i} = 19.544$, $\theta_i = \tan^{-1}(0.1)$. We have terminated the calculation at $t = 3 \times 10^5$.

FIG. 6. Energy spectrum of the outgoing CAS electrons. The parameters used in the calculations are the same as those in Fig. 5.

FIG. 7. (a) The effective phase velocity of Gaussian laser beam along z-axis. (b) Variation of effective phase velocities of Gaussian laser waves along electron trajectories (solid line) of CAS compared with the electron's velocity (dotted line) (c) Same as Fig. 7(b) but for IS. The parameters chosen are the same as those in Fig. 3.

FIG 8. Dependence of the electron final energy $\gamma_{f\max}$ on the relative delay time Δt_d, where the results of Fig. 8(a) are for $a_0 = 10$, Fig. 8(b) for $a_0 = 30$, and Fig. 8(c) for $a_0 = 100$. Other parameters are the same as those in Fig. 2.

FIG. 9. Dependence of the electron final maximum energy γ_{fm} on the laser field intensity a_0. The dot-dashed line is for a continuous laser beam, the solid line for a pulsed laser beam, and the dotted line for that of a pulsed laser beam with the PPM. The maximum electron final energy is obtained by varying ϕ_0 and Δt_d over a wide range of values. Other parameters are the same as those in Fig. 2. The inset is an enlargement of the part denoted by the arrow.

FIG. 10. The electron final energy γ_{fm} as a function of the incoming energy γ_i. The four curves in the figure correspond to $a_0 = 70, 100, 200$, and 300. Other parameters are the same as those in Fig. 2.

FIG. 11. Dependence of the maximum net energy exchange $\Delta E_{max} = (\gamma_{fm} - \gamma_i)_m m_e c^2$ on the parameter a_0^2. The solid line is for the case of a pulsed laser and the dot-dashed line for a continuous laser. The maximum net energy exchange is obtained as ϕ_0, Δt_d and the incoming energy γ_i are varied over a wide range. Other parameters are the same as those in Fig. 2.

FIG. 12. Dependence of the electron final energy γ_{fm} on the beam width at the waist w_0. The solid line is for $\tau = 1000$, the cross ($\times$) for $\tau = 300$, and the circle ($\bigcirc$) for the case of a continuous laser beam. Other parameters are the same as those in Fig. 2(b).

FIG. 13. The electron final energy γ_{fm} as a function of the laser field intensity a_0. The solid line is for the case when $\tau = 1000$, the doted line is for $\tau = 300$, and the dot-dashed line is for the case of a continuous laser. Other parameters are the same as those in Fig. 2.

FIG. 14. Dependence of the electron maximum outgoing energy $\gamma_{f\,\mathrm{max}}$ on the impact parameter b_0. The dash-dotted line, the solid line and dotted line correspond to the cases $\Delta t_d = -250, 0, +250$ respectively. Other parameters are $\tau = 500$, $w_0 = 150$, $P_{xi} = 2$, $P_{yi} = 0$, $P_{zi} = 20$. In the electron capture case, we have terminated the calculation at $t = 3 \times 10^5$.

FIG. 15. The variation of $\gamma_{f\,\mathrm{max}}$ versus the electron injection angle θ_i in the x-z plane when $\Delta t_d = 0$. The solid line is for $w_0 = 150$, the dotted line for $w_0 = 200$, and the dot-dashed line for $w_0 = 100$. Other parameters are the same as those in Fig. 14.