

Characterization of Clocks and Oscillators

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This publication covers portions of NBS Monograph 140 published in 1974. The purpose of this document is to replace this outdated monograph in the areas of methods for characterizing clocks and oscillators and definitions and standards relating to such characterization.



U.S. DEPARTMENT OF COMMERCE, Robert A. Mosbacher, Secretary
NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY, John W. Lyons, Director

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PREFACE

For many years following its publication in 1974, "TIME AND FREQUENCY: Theory and Fundamentals," a volume edited by Byron E. Blair and published as NBS Monograph 140, served as a common reference for those engaged in the characterization of very stable clocks and oscillators. Monograph 140 has gradually become outdated, and with the recent issuance of a new military specification, MIL-0-55310B, which covers general specifications for crystal oscillators, it has become especially clear that Monograph 140 no longer meets the needs it so ably served in earlier years. During development of the new military specification, a process involving discussion and input from many quarters, a key author of the specification, John Vig of the US Army Electronics Technology and Devices, urged the National Bureau of Standards (now the National Institute of Standards and Technology, NIST) to issue a revised publication to serve as reference for the characterization of clocks and oscillators. With NIST having agreed to this task, the framers of the military specification used the nomenclature "NBS Monograph 140R" in their document, anticipating a revised (R) volume which had not yet been prepared.

Considering the availability of a number of newer books in the time and frequency field, the rewriting of a major volume like Monograph 140 seemed inappropriate. The real need has not been for rework of everything in Monograph 140, but only for those parts which provide reference to definitions and methods for measurement and characterization of clocks and oscillators, subjects which are fully covered in a number of papers distributed through a variety of conference proceedings, books, and journals. For the near term, we concluded that the most effective procedure would be to collect a representative set of these papers into one reference source with introductory comments which permit the reader to quickly access material required to meet particular needs. Thus, we arrived at this particular collection. The editors' challenge has been to select representative papers, to organize them in a convenient manner, and to deal with errata and notation inconsistencies in a reasonable manner. In the longer term, the material in this volume needs to be more completely integrated. This task would profitably await further developments in the area of phase noise measurements.

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<p>This is a collection of published papers assembled as a reference for those involved in characterizing and specifying high-performance clocks and oscillators. It is an interim replacement for NBS Monograph 140, <i>Time and Frequency: Theory and Fundamentals</i>, an older volume of papers edited by Byron E. Blair. This current volume includes tutorial papers, papers on standards and definitions, and a collection of papers detailing specific measurement and analysis techniques. The discussion in the introduction to the volume provides a guide to the content of the papers, and tables and graphs provide further help in organizing methods described in the papers.</p>				
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PREFACE	iii
TOPICAL INDEX TO ALL PAPERS	x
ABSTRACT	TN-1
A. INTRODUCTION	
A.1 OVERVIEW	TN-1
A.2 COMMENTS ON INTRODUCTORY AND TUTORIAL PAPERS	TN-2
A.3 COMMENTS ON PAPERS ON STANDARDS AND DEFINITIONS	TN-3
A.4 COMMENTS ON SUPPORTING PAPERS	TN-3
A.5 GUIDE TO SELECTION OF MEASUREMENT METHODS	TN-5
A.6 RELATIONSHIP OF THE MODIFIED ALLAN VARIANCE TO THE ALLAN VARIANCE	TN-9
A.7 SUPPLEMENTARY READING LIST	TN-13
B. INTRODUCTORY AND TUTORIAL PAPERS	
B.1 Properties of Signal Sources and Measurement Methods	
D.A. Howe, D.W. Allan, and J.A. Barnes	
<i>35th Annual Symposium on Frequency Control, 1981 Proceedings</i>	TN-14
I. The Sine Wave and Stability	TN-14
II. Measurement Methods Comparison	TN-20
III. Characterization	TN-22
IV. Analysis of Time Domain Data	TN-23
V. Confidence of the Estimate and Overlapping Samples	TN-26
VI. Maximal Use of the Data and Determination of the Degrees of Freedom	TN-28
VII. Example of Time Domain Signal Processing and Analysis	TN-30
VIII. Spectrum Analysis	TN-32
IX. Power-Law Noise Processes	TN-38
X. Pitfalls in Digitizing the Data	TN-39
XI. Translation From Frequency Domain Stability Measurement to Time Domain Stability Measurement and Vice-Versa	TN-48
XII. Causes of Noise Properties in a Signal Source	TN-50
XIII. Conclusion	TN-55
B.2 Frequency and Time - Their Measurement and Characterization	
Samuel R. Stein	
<i>Precision Frequency Control, Volume 2, Chapter 12, 1985</i>	TN-61
12.1 Concepts, Definitions, and Measures of Stability	TN-63
12.1.1 Relationship Between the Power Spectrum and the Phase Spectrum	TN-65
12.1.2 The IEEE Recommended Measures of Frequency Stability	TN-65
12.1.3 The Concepts of the Frequency Domain and the Time Domain	TN-73
12.1.4 Translation Between the Spectral Density of Frequency and the Allan Variance	TN-73
12.1.5 The Modified Allan Variance	TN-75
12.1.6 Determination of the Mean Frequency and Frequency Drift of an Oscillator	TN-77
12.1.7 Confidence of the Estimate and Overlapping Samples	TN-80
12.1.8 Efficient Use of the Data and Determination of the Degrees of Freedom	TN-83
12.1.9 Separating the Variances of an Oscillator and the Reference	TN-86

	Page	
12.2	Direct Digital Measurement	TN-87
12.2.1	Time-Interval Measurements	TN-87
12.2.2	Frequency Measurements	TN-89
12.2.3	Period Measurements	TN-89
12.3	Sensitivity-Enhancement Methods	TN-90
12.3.1	Heterodyne Techniques	TN-90
12.3.2	Homodyne Techniques	TN-91
12.3.3	Multiple Conversion Methods	TN-97
12.4	Conclusion	TN-101
B.3 Time and Frequency (Time-Domain) Characterization, Estimation, and Prediction of Precision		
Clocks and Oscillators		
David W. Allan		
	<i>IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, 1987</i>	TN-121
1.	Introduction	TN-121
2.	Systematic Models for Clocks and Oscillators	TN-121
3.	Random Models for Clocks and Oscillators	TN-123
4.	Time-Domain Signal Characterization	TN-123
5.	Time and Frequency Estimation and Prediction	TN-126
6.	Conclusions	TN-127
B.4 Extending the Range and Accuracy of Phase Noise Measurements		
F.L. Walls, A.J.D. Clements, C.M. Felton, M.A. Lombardi and M.D. Vanek		
	<i>42nd Annual Symposium on Frequency Control, 1988 Proceedings</i>	TN-129
I.	Introduction	TN-129
II.	Model of a Noisy Signal	TN-129
III.	Two Oscillator Method	TN-130
IV.	The New NBS Phase Noise Measurement Systems	TN-135
V.	Conclusion	TN-137
C. PAPERS ON STANDARDS AND DEFINITIONS		
C.1 Standard Terminology for Fundamental Frequency and Time Metrology		
David Allan, Helmut Hellwig, Peter Kartaschoff, Jacques Vanier, John Vig, Gernot M.R. Winkler, and Nicholas Yannoni		
	<i>42nd Annual Symposium on Frequency Control, 1988 Proceedings</i>	TN-139
1.	Introduction	TN-139
2.	Measures of Frequency and Phase Instability	TN-139
3.	Characterization of Frequency and Phase Instabilities	TN-139
4.	Confidence Limits of Measurements	TN-140
5.	Recommendations for Characterizing or Reporting Measurements of Frequency and Phase Instabilities	TN-141

C.2 Characterization of Frequency Stability

James A. Barnes, Andrew R. Chi, Leonard S. Cutler, Daniel J. Healey, David B. Leeson,
 Thomas E. McGunigal, James A. Mullen, Jr., Warren L. Smith, Richard L. Sydnor,
 Robert F.C. Vessot, and Gernot M.R. Winkler

IEEE Transactions on Instrumentation and Measurement, 1971 TN-146

I.	Introduction	TN-147
II.	Statement of the Problem	TN-148
III.	Background and Definitions	TN-149
IV.	Definition of Measures of Frequency Stability	TN-149
V.	Translations Among Frequency Stability Measures	TN-151
VI.	Applications of Stability Measures	TN-153
VII.	Measurement Techniques for Frequency Stability	TN-154
VIII.	Conclusions	TN-157

C.3 Characterization of Frequency and Phase Noise

Comité Consultatif International Des Radiocommunications (CCIR)

Report 580 of the CCIR, 1986 TN-162

1.	Introduction	TN-162
2.	Fourier Frequency Domain	TN-163
3.	Time Domain	TN-163
4.	Conversion Between Frequency and Time Domains	TN-164
5.	Measurement Techniques	TN-166
6.	Confidence Limits of Time Domain Measurements	TN-166
7.	Conclusion	TN-167

D. SUPPORTING PAPERS**D.1 Characterization and Measurement of Time and Frequency Stability**

P. Lesage and C. Audoin

Radio Science, 1979 TN-171

1.	Introduction	TN-171
2.	Definitions: Model of Frequency Fluctuations	TN-171
3.	Noise Processes in Frequency Generators	TN-173
4.	Measurements in the Frequency Domain	TN-174
5.	Measurements in the Time Domain	TN-177
6.	Characterization of Frequency Stability in the Time Domain	TN-178
7.	Characterization of Frequency Stability via Filtering or Frequency Noise	TN-182
8.	Spectral Analysis Inferred From Time Domain Measurements	TN-183
9.	Structure Functions of Oscillator Fractional Phase and Frequency Fluctuations	TN-184
10.	Power Spectral Density of Stable Frequency Sources	TN-185
11.	Conclusion	TN-187

D.2 Phase Noise and AM Noise Measurements in the Frequency Domain

Algie L. Lance, Wendell D. Seal, and Frederik Labaar

Infrared and Millimeter Waves, Vol. 11, 1984 TN-190

I.	Introduction	TN-190
II.	Fundamental Concepts	TN-191
	A. Noise Sidebands	TN-194
	B. Spectral Density	TN-194
	C. Spectral Densities of Phase Fluctuations in the Frequency Domain	TN-197
	D. Modulation Theory and Spectral Density Relationships	TN-199
	E. Noise Processes	TN-202
	F. Integrated Phase Noise	TN-203
	G. AM Noise in the Frequency Domain	TN-205
III.	Phase-Noise Measurements Using the Two-Oscillator Technique	TN-206
	A. Two Noisy Oscillators	TN-208
	B. Automated Phase-Noise Measurements Using the Two-Oscillator Technique	TN-209
	C. Calibration and Measurements Using the Two-Oscillator System	TN-210
IV.	Single-Oscillator Phase-Noise Measurement Systems and Techniques	TN-218
	A. The Delay Line as an FM Discriminator	TN-219
	B. Calibration and Measurements Using the Delay Line as an FM Discriminator	TN-225
	C. Dual Delay-Line Discriminator	TN-232
	D. Millimeter-Wave Phase-Noise Measurements	TN-234
V.	Conclusion	TN-238

D.3 Performance of an Automated High Accuracy Phase Measurement System

S. Stein, D. Glaze, J. Levine, J. Gray, D. Hilliard, D. Howe, and L. Erb

36th Annual Symposium on Frequency Control, 1982 Proceedings TN-241

1.	Review of the Dual Mixer Time Difference Technique	TN-241
2.	Extended Dual Mixer Time Difference Measurement Technique	TN-242
3.	Hardware Implementation	TN-242
4.	Conclusions	TN-243

D.4 Biases and Variances of Several FFT Spectral Estimators as a Function of Noise Type and Number of Samples

F.L. Walls, D.B. Percival and W.R. Ireland

43rd Annual Symposium on Frequency Control, 1989 Proceedings TN-248

I.	Introduction	TN-248
II.	Spectrum Analyzer Basics	TN-248
III.	Expected Value and Bias of Spectral Estimates	TN-249
IV.	Variances of Spectral Estimates	TN-251
V.	Conclusions	TN-252

D.5 A Modified "Allan Variance" with Increased Oscillator Characterization Ability

David W. Allan and James A. Barnes

35th Annual Symposium on Frequency Control, 1981 Proceedings TN-254

1. Introduction TN-254
2. Definition of "Allan Variance" and Related Concepts TN-254
3. Development of the Modified Allan Variance TN-255
4. Comparisons, Tests, and Examples of Usage of the Modified Allan Variance TN-256
5. Conclusion TN-257

D.6 Characterization of Frequency Stability: Analysis of the Modified Allan Variance and Properties of Its Estimate

Paul Lesage and Théophile Ayi

IEEE Transactions on Instrumentation and Measurement, 1984 TN-259

- I. Introduction TN-260
- II. Background and Definitions TN-260
- III. The Modified Allan Variance TN-260
- IV. Uncertainty on the Estimate of the Modified Allan Variance TN-262
- V. Conclusion TN-263

D.7 The Measurement of Linear Frequency Drift in Oscillators

James A. Barnes

15th Annual PTTI Meeting, 1983 Proceedings TN-264

- I. Introduction TN-264
- II. Least Squares Regression of Phase on a Quadratic TN-265
- III. Example TN-266
- IV. Drift and Random Walk FM TN-270
- V. Summary of Tests TN-270
- VI. Discussion TN-276
- VII. Conclusions TN-276

D.8 Variances Based on Data With Dead Time Between the Measurements

James A. Barnes and David W. Allan

NIST Technical Note 1318, 1990 TN-296

1. Introduction TN-297
2. The Allan Variance TN-298
3. The Bias Function $B_1(N, r, \mu)$ TN-299
4. The Bias Function $B_2(r, \mu)$ TN-300
5. The Bias Function $B_3(N, M, r, \mu)$ TN-300
6. The Bias Functions TN-302
7. Examples of the Use of the Bias Functions TN-303
8. Conclusion TN-304
- Appendix TN-312

E. APPENDIX - Notes and Errata TN-336

TOPICAL INDEX TO ALL PAPERS

The following index is designed for quick location of substantial discussion of a particular topic. The page numbers are those where the topic is first cited in the reference.

Aliasing	TN-40
Allan Variance	TN-24, TN-71, TN-123, TN-140, TN-150, TN-163, TN-179, TN-254, TN-260, TN-298
Amplitude Modulation Noise	TN-194, TN-205, TN-235
Bias Functions	TN-162, TN-164, TN-299
Clock-Time Prediction	TN-126, TN-140
Common Measurement Problems	TN-156
Components Other than Oscillators (Methods)	TN-37, TN-131, TN-176
Confidence of the Estimate	TN-26, TN-80, TN-140, TN-156, TN-166, TN-180, TN-262
Conversion Between Time and Frequency Domain	TN-48, TN-73, TN-142, TN-151, TN-164
Correlation Method	TN-132, TN-176, TN-216
Dead Time	TN-140, TN-164, TN-180, TN-296
Degrees of Freedom	TN-28, TN-83
Digitization of Data	TN-39
Direct Measurements	
o Measurements at the Fundamental Frequency	TN-15, TN-87
o Measurements after Multiplication/Division	TN-101, TN-135
Drift	TN-30, TN-77, TN-122, TN-151, TN-184, TN-264
Efficient Use of Data	TN-28, TN-83
Filter Method	TN-182
Frequency Synthesis	TN-97
Heterodyne or Beat-Frequency Methods	TN-17, TN-90, TN-154, TN-177
o Single-Conversion Methods	TN-17, TN-90, TN-154, TN-177
o Multiple-Conversion Methods	TN-97
o Time-Difference (Time-Interval-Counter)	TN-20, TN-87, TN-177
- Dual-Mixer, Time-Difference Method	TN-17, TN-99, TN-177, TN-241
Homodyne Measurements	
o Phase-Lock-Loop Methods	TN-18 TN-34, TN-93, TN-130, TN-176, TN-209
o Discriminator Methods	TN-92, TN-133, TN-174, TN-218
- Cavity-Discriminator Method	TN-133, TN-174, TN-219
- Delay-Line Method	TN-92, TN-134, TN-176, TN-219
Least Squares (Regression)	TN-31, TN-77, TN-265
Modified Allan Variance	TN-75, TN-143, TN-164, TN-255, TN-260
Overlapping Samples	TN-26, TN-80, TN-140
Pickett-Fence Effect	TN-46
Power Law Spectra (Random Noise)	TN-38, TN-73, TN-123, TN-141, TN-167, TN-173, TN-200
Quantization Uncertainty	TN-40, TN-89
Reference-Phase-Modulation Technique	TN-133
Separating Oscillator and Reference Variances	TN-86
Spectral Density	TN-32, TN-43, TN-65, TN-130, TN-139, TN-150, TN-163, TN-172, TN-194
Systematic Fluctuations (non-Random-Noise)	TN-22, TN-121, TN-150
Window Function (Leakage)	TN-44