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Characterization of connected Vertex Magic Total labeling graphs in Topological ideals

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Abstract

A graph with v vertices and e edges is said to be vertex magic total labeling graph if there is a one to one map taking the vertices and edges onto the integers 1,2,…v+e with the property that the sum of the label on the vertex and the labels of its incident edges is constant, independent of the choice of the vertex. An ideal space is a triplet (X,τ,I) where X is a nonempty set, τ is a topology, *I* is an ideal of subsets of X. In this paper we characterize connected vertex magic total labeling graphs through the ideals in topological spaces.

Keywords: Vertex magic total labeling graphs, ideal, topological space, Euler graph.

1.Introduction

A graph G = { V, E } has vertex set V=V(G) and the edge set E= E(G) and $e = |E|$ and $v = |V|$. Various topologies like combinatorial topology, ideal topology, mesh based topology, tree based topology were defined on the vertex set and edge set of a graph [8]. The graphs considered here are finite, simple and undirected. A general reference for graph theory notations is from West, Gary Chartrand and Ping Zhang,. A circuit in a graph is called Eulerian circuit if it contains every edge of G. A connected graph that contains an Eulerian circuit is called Eulerian graph.

A labeling for a graph is an assignment of labels, usually numbers to its vertices or its edges or sometimes to both of them. There are various attempts on labelings of graphs in the literature. Gallian completely compiled the survey of the graph labeling. MacDougall, Mirka Miller & Slamin & Wallis introduced vertex magic total labelings of graphs. Manohar and Thangavelu studied the characterization of Eulerian graphs through the concept of ideals in topological spaces for connected graphs, where as in this paper we study the characterization of connected vertex magic total labeling graphs through the ideals in topological spaces.

Let G = {V, E} be a simple graph with $v = |V|$ and $e = |E|$. A one to one and onto mapping f from V∪E to the finite subset $\{1,2,\ldots,v+e\}$ of natural numbers such that for every vertex x, $f(x) + f(xy_i) = k$, where y_i's are vertices adjacent to x is called a vertex magic total labeling graph or VMTL graph. The path P_3 is VMTL with constant k = 7.

A connected VMTL graph is both connected and VMTL in nature. It is also noted that all connected graphs are not vertex magic total labeling in nature, as a vertex magic total labeling graph may contain one isolated vertex.

A nonempty collection of subsets of a set X is said to be an ideal on X , if it satisfies (i) If A∈I and B⊆A, then B∈I and (ii) If A∈I and B∈I , then A∪B∈ I.

The idea of localization of the properties in connection with ideals in the topological spaces was treated in a general manner by Kurtowski. By a space (X, τ), we mean a topological space X with a topology τ defined on X. For a given point x in a space (X, τ), the system of open neighbourhoods of x is denoted by N(x) = {U ∈ τ / x∈U }. For a given subset A of a space (X, τ), the closure of a subset A of a topological space X with respect to the topology τ is defined as the intersection of all closed sets containing A and denoted by cl(A). An ideal space is a triplet (X, τ, *I*), where X is a nonempty set, τ is a topology on X and I is an ideal of subsets of X.

Let (X, τ, *I*) be an ideal space. Then $A^*(I,\tau) = \{ x \in X / A\cap U \notin I$ for every $U\in N(x)\}$ is called the local function of A with respect to I and τ. $A^*(I,\tau)$ can also be noted as $A^*(I)$ or simply A^*.

2.Basic results

The basic results are considered as defined by Manohar and Thangavelu .

2.1.Result

For an ideal space (X, τ, *I*), if I =P(X), then A*(I) = Ø for every A⊆X.

2.2.Result

If (X, τ) is a topological space and X is finite then A*(I) = Ø for every A⊆X if and only if I = P(X).

2.3.Result

Let (X, τ) be a topological space and I be an ideal defined on it . Then A*(I) = cl(A) for every A⊆X if I = {Ø }.

2.4.Result

Let (X,τ) be a topological space and I be an ideal defined on it . Then A*(I) = cl(A) for every A⊆X if and only if I = {Ø }.

2.5.Result

A connected graph G is Eulerian if and only if all of the vertices of G are of even degree.

3.Characterization of connected Vertex magic total labeling graphs

3.1Theorem
A connected VMTL graph G is an Euler graph if and only if all vertices of G are of even degree.
Proof:
The proof is obvious from the result 2.5, as a connected graph G is Eulerian if and only if all of the vertices of G are of even degree, it is same for a connected VMTL graph.
3.2 Theorem.
Let G be a connected VMTL graph and S be the set of all even degree vertices of G. Let τ be a topology defined on V(G) and I be the ideal of collection of all subsets of S. Then $\{v\}^* = Ø$ if and only if deg(v) is even for any v ∈ V(G).
Proof :
Let v_1∈ V(G) and $\{v_1\}^*$ = Ø. Since $\{v_1\}^*$ = Ø for every v ∈ V(G), there exists U∈ N(v) such that U ∩{ v_1} ∈ I (since I is the ideal of collection of all subsets of S). In particular, if $v = v_1$ there exists U∈ N(v_1) such that U ∩{ v_1} = {v_1}∈ I. Therefore, deg (v_1) is even. Conversely, if deg(v) is even, {v}∈ I. Then { v}* is empty.
3.3 Theorem.
Let G be a connected VMTL graph and τ be a topology defined on V(G).Let S be the set of all even degree vertices of G. Let I be the ideal of collection of all subsets of S. Then the graph is Eulerian if and only if {v}* = Ø for every v ∈ V(G).
Proof:
By theorem 3.1, A connected VMTL graph G is an Eulerian graph if and only if all vertices of G are of even degree. By theorem 3.2, for connected VMTL graph and S be the set of all even degree vertices of G. Let τ be a topology defined on V(G) and I be the ideal of collection of all subsets of S. Then {v}* = Ø if and only if deg (v) is even for any v ∈ V(G). Hence the theorem.
3.4 Theorem
Let G be a connected VMTL graph and τ be a topology defined on V(G).Let S be the set of all odd degree vertices of G. Let I be the ideal of collection of all subsets of S. Then {v}* = Ø if and only if deg (v) is odd for every v ∈ V(G).
Proof:
Let v_i ∈ V(G) and { v_i}* = Ø. Since { v_i}* = Ø for every v ∈ V(G), there exists U∈ N(v) such that U ∩{ v_i}∈ I. In particular , if $v = v_i$ there exists U∈ N(v_i) such that U ∩{ v_i}= {v_i}∈ I . Therefore,
deg (v_i) is odd. Conversely, if deg (v) is odd, { v}∈ I. Then {v}* is empty.
3.5 Theorem
Let G be a connected VMTL graph and τ be a topology defined on V(G). Let S be the set of all odd degree vertices of G. Let I be the ideal of collection of all subsets of S. Then the graph is Eulerian if and only if {v}* ≠ Ø for every v ∈ V(G).
Proof:
Assume that G is Eulerian. By theorem 3.1, all the vertices of G are of even degree.
By theorem 3.4, G is a connected VMTL graph and τ be a topology defined on V(G), S be the set of all odd degree vertices of G, I be the ideal of collection of all subsets of S. Then {v}* = Ø if and only if deg (v) is odd for every v ∈ V(G).
3.6 Theorem

Let G be a connected VMTL graph and τ be a topology defined on V(G). Let S be the set of all even degree vertices of G. Let I be the ideal of collection of all subsets of S. Then the graph is Eulerian if and only if A*(I) = Ø for every A⊆V(G).
Proof :
Assume that G is Eulerian. By theorem 3.1, all the vertices of G are of even degree. Then $\mathscr{P}$(V) = $\mathscr{P}$(S). By result 2.1, A*(I) = Ø for every A⊆V(G).Conversely assume A*(I) = Ø for every A⊆V(G). By result (ii) I = $\mathscr{P}$(S) = $\mathscr{P}$(V). Then by theorem 3.1 the graph G is Eulerian.
3.7 Theorem
Let G be a connected VMTL graph and τ be a topology defined on V(G).Let S be the set of all odd degree vertices of G. Let I be the ideal of collection of all subsets of S. Then the graph is Eulerian if and only if A*(I) = cl(A) for every A⊆V(G).
Proof:
By result 2.4, for a topological space (X, τ) and I be an ideal defined on it , then A*(I) = cl(A) for every A⊆X if I = {Ø} . Then by theorem 3.1, a connected VMTL graph G is an Euler graph if and only if all vertices of G are of even degree , here I= {Ø}.

4.Conclusion

In this paper we have seen few theorems which characterize connected vertex magic total labeling graphs through the ideals in topological spaces of vertices of graphs. A further study on connected VMTL graphs through ideals in topological spaces can also be attempted.

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Note 1. This is an example of theorem 3.1

The complete graph K_5 is VMTL [6] with degree of each vertex being 4 and k = 35 AS in figure 2, is of even degree and an Euler circuit exists in it.

Note 2. These are examples of theorem 3.2
Example.1. Let G = $K_{3,3}$-e be a connected VMTL graph as in [5] and S be the set of all even degree vertices of G. Let τ be a topology defined on V(G) and I be the ideal of collection of all subsets of S

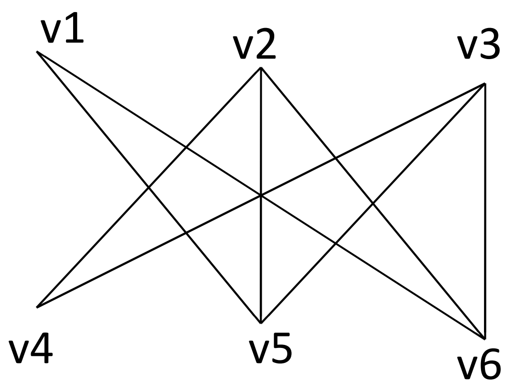

Fig 3.- $K_{3,3}$-e

Let $V(G) = \{v_1,v_2,v_3,v_4,v_5,v_6\}$, $S = \{v_1,v_4\}$, $I_1= \mathscr{P}(S) = \{ \emptyset, \{v_1\}, \{v_4\},\{v_1,v_4\}\}$, $\tau = \{ \emptyset, \{v_1\},V\}$, $U = \emptyset$ or $\{v_1\}$ or V

By the definition of the local function of A, $A^*(I, \tau) = \{x\in X / A\cap U \notin I_1$ for every $U \in N(x)\}$

By definition , here $A^*= \{v_1\}^*$, the system of open neighbourhoods is given by

$N(x) = \{U \in \tau / x\in U \}$, Here it is $N(v_1) = \{ v_1, V \}$

The local function of v_1 with respect to I_1 and τ is

$\{v_1\}^* = \{v_i\in S/ v_1\cap U \notin I_1$ for every $U \in N(v_1)\} = \emptyset$

Example. 2. Let $G = K_3$ be a connected VMTL graph and S be the set of all even degree vertices of G. Let τ be a topology defined on V(G) and I be the ideal of collection of all subsets of S

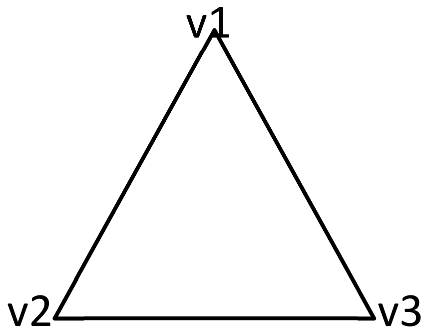

Fig 4. - K_3

Let $V(G) = \{v_1,v_2,v_3\}$, $S = \{v_1,v_2,v_3\}$,

$I_1= \mathscr{P}(S) = \{ \emptyset, \{v_1\},\{v_2\},\{v_3\},\{v_1,v_2\},\{v_2,v_3\},\{v_1,v_3\},\{v_1,v_2,v_3\}\}$, $\tau = \{ \emptyset, \{v_1\},V\}$, $U = \emptyset$ or $\{v_1\}$or V

By the definition of the local function of A, $A^*(I, \tau) = \{x\in X/A\cap U \notin I_1$ for every $U \in N(x)\}$

By definition $A^*= \{v_1\}^*$, the system of open neighbourhoods is given by

$N(x) = \{U \in \tau / x\in U \}$, Here it is $N(v_1) = \{ v_1, V\}$

The local function of v_1 with respect to I_1 and τ is

$\{v_1\}^* = \{v_i\in S/ v_1\cap U \notin I_1$ for every $U \in N(v1)\} = \emptyset$

Note 3.This is an example of theorem 3.4

Let G be a connected VMTL graph and S be the set of all odd degree vertices of G. Let τ be a topology defined on V(G) and I be the ideal of collection of all subsets of S

Let $V(G) = \{v_1, v_2,v_3 ,v_4,v_5, v_6\}$, $S = \{ v_2,v_3,v_5, v_6\}$,

$I= \mathscr{P}(S) = \{ \emptyset, \{v_2\},\{v_3\},\{v_2,v_3\}\}$,

$\tau = \{ \emptyset, \{v_2\},V\}$, $U = \emptyset$ or $\{v_2\}$or V, $N(v_2) = \{ v_2, V\}$. Then $\{v_2\}^* = \emptyset$

Fig.1-Path P_3

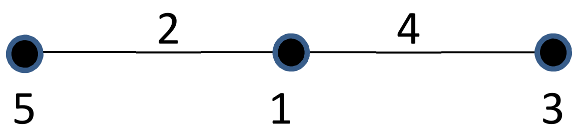

Table.1-VMTL labeling of K_5

λ(x)	a=11	b=12	c=13	d=14	e=15
a=11	-	10	2	9	3
b=12	10	-	8	1	4
c=13	2	8	-	5	7
d=14	9	1	5	-	6
e=15	3	4	7	6	-

The vertex magic labeling of K_5 is given in the table 1, where the row and column titles represent the vertices and the contents inside the table represent the edges connecting between the corresponding vertices.(i.e)The edge between a and b is labeled as 10, between a and c is 2, etc. Some Euler circuits obtained are akcmeobldna, afbgchdieja.

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