

## CHARACTERIZATION OF THE RESTRICTED TYPE SPACES $R(X)$

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*Abstract.* We study functorial properties of the spaces  $R(X)$ , which have been recently introduced as a central tool in the analysis of the Hardy operator minus the identity on decreasing functions. In particular, we provide conditions on a minimal Lorentz space  $\Lambda_\phi$  so that the equation  $R(X) = \Lambda_\phi$  has a solution within the category of rearrangement invariant (r.i.) spaces. Moreover, we show that if  $R(X) = \Lambda_\phi$ , then we can always take  $X$  to be the minimal r.i. Banach range space for the Hardy operator defined in  $\Lambda_\phi$ .

*Mathematics subject classification (2010):* 26D10, 46E30.

*Keywords and phrases:* Rearrangement invariant spaces, Lorentz spaces, restricted type, Hardy operator, optimal range.

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