

# Characterization of Unruh channel in the context of open quantum systems

Subhashish Banerjee,<sup>a</sup> Ashutosh Kumar Alok,<sup>a</sup> S. Omkar<sup>b</sup> and R. Srikanth<sup>c</sup>

<sup>a</sup>Indian Institute of Technology Jodhpur,  
Jodhpur 342011, India

<sup>b</sup>Seoul National University,  
Seoul, S. Korea

<sup>c</sup>Poornaprajna Institute of Scientific Research,  
Sadashivnagar, Bengaluru- 560080, India

E-mail: [subhashish@iitj.ac.in](mailto:subhashish@iitj.ac.in), [akalok@iitj.ac.in](mailto:akalok@iitj.ac.in),  
[omkar.shrm@gmail.com](mailto:omkar.shrm@gmail.com), [srik@poornaprajna.org](mailto:srik@poornaprajna.org)

**ABSTRACT:** In this work, we study an important facet of field theories in curved space-time, viz. the Unruh effect, by making use of ideas of statistical mechanics and quantum foundations. Aspects of decoherence and dissipation, natural artifacts of open quantum systems, along with foundational issues such as the trade-off between coherence and mixing as well as various aspects of quantum correlations are investigated in detail for the Unruh effect. We show how the Unruh effect can be quantified mathematically by the Choi matrix approach. We study how environmentally induced decoherence modifies the effect of the Unruh channel. The differing effects of a dissipative or non-dissipative environment are noted. Further, useful parameters characterizing channel performance such as gate and channel fidelity are applied here to the Unruh channel, both with and without external influences. Squeezing, which is known to play an important role in the context of particle creation, is shown to be a useful resource in a number of scenarios.

**KEYWORDS:** Quantum Dissipative Systems, Stochastic Processes

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## 1 Introduction

In recent times the tools borrowed from the foundations of quantum mechanics and quantum information have been widely used to probe intriguing topics related to the understanding of the fundamental aspects of nature, such as the physics of the early universe. This includes particle creation in an expanding universe [1, 2], quantum correlations in curved spacetime such as deSitter space time [3–5] and scalar field models of an expanding universe using the Robertson-Walker spacetime, conformally invariant to the Minkowski spacetime [6–8]. A common theme in all these is two sets of vacua connected by a Bogoliubov transformation. The study of quantum correlations can shed light on these issues.

The ideas of quantum statistical mechanics such as decoherence and dissipation have been applied, fruitfully to study phenomenon such as vacuum fluctuations and particle creation. These enable a deeper understanding as well as provide a statistical interpretation [9] for the well known effects such as the Unruh [10, 11], Hawking [12], and Gibbons-Hawking [13] effects. Interestingly, in recent times, finite time aspects of the Unruh effect have revealed an inverse relation between the acceleration and the temperature, a phenomenon known as the anti-Unruh effect [14, 15]. This sets the scene for the present work where the ideas of open quantum systems and quantum foundations are used to provide a common platform to address the above problems. This would invoke a detailed understanding of various aspects of decoherence and dissipation, natural artifacts of application

of open system ideas to the above scenarios. Here, in particular, we study the Unruh effect [10, 11, 16] which predicts thermal like effects from observing uniform acceleration of the Minkowski vacuum. This has attracted intense interest [17–27] and has emerged as a natural quest in the direction of relativistic quantum information [28–41].

In brief, open quantum system is the study of the dynamics of the system of interest taking into account the effect of the system’s environment. In general, open system dynamics can be broadly classified into (a) quantum non-demolition (QND), involving decoherence without any dissipation and (b) dissipative, where decoherence is accompanied with dissipation. QND evolution has its roots in gravitational wave detection [42, 43] and is the precursor for Laser Interferometer Gravitational-Wave Observatory (LIGO) [44]. The squeezed generalized amplitude damping (SGAD) [45, 46] is a very general dissipative channel of the Lindblad class and incorporates the well known amplitude damping and generalized amplitude damping channels as limiting cases. The role of the squeezed thermal bath in the present context is very pertinent as squeezing is connected to parametric amplification which is known to play an important role in the context of particle creation [1, 2, 9] and at the same time plays a constructive role in preserving quantum coherence in the presence of decoherence [45, 47, 48].

It is important to realize here that a monochromatic Minkowski mode is associated to a Rindler mode which corresponds to a (highly) non-monochromatic field excitation, as first observed in [49]. As noted there, this is essentially because the relevant Bogoliubov transformation linking the Minkowski to Rindler modes are such that a plane-wave (monochromatic) Minkowski creation operator cannot be written, in general, as a combination of monochromatic Rindler modes. In fact, the assumption that it can be so written is applicable to a class of Minkowski wave packets that are appropriately peaked to satisfy constraints from a suitable Fourier transform. In this work, for simplicity we shall adopt this assumption.

Technically, this means that the entangled state analyzed here corresponds to an entanglement between a Minkowski mode and a specific Unruh mode, namely one for which  $q_R = 1$  and  $q_L = 0$  in eq. (16) of [49]. Fortunately, as noted already there, the exact, polychromatic treatment reproduces qualitatively all the degrading effects of noise associated with the single-mode assumption.

Another justification for this assumption is that it suffices to exhibit in a simple fashion two features not often studied in the literature:

- to distinguish between the influence of a dissipative or damping environment and non-dissipative or nondemolition environment;
- to highlight the role of squeezing in the external environment, as against a purely thermal effect.

Here, we study various facets of quantum correlations, such as nonlocality, entanglement, teleportation fidelity, coherence and quantum measurement-induced disturbance (a discord-like measure) for the Unruh channel of a Dirac field mode. We highlight the distinction between dephasing and dissipative environmental interactions, by considering the

actions of QND and SGAD channels, respectively. The Unruh effect is conventionally seen as a thermal radiation by a uniformly accelerated detector coupled to an appropriate quantized field. The detector is typically a localized and controllable quantum system that is locally coupled to the ambient field. The Unruh channel for the Dirac qubit, i.e., the qubit acted upon by the Unruh channel of a Dirac field mode, under the influence of possible interactions of the QND and SGAD type, is characterized by constructing the corresponding Kraus operators. The trade-off between quantum coherence, a fundamental characteristic of quantumness in the system and mixing is studied. Useful parameters characterizing channel performance are the gate fidelity [50] as well as the average gate fidelity [51]. They represent how well a (noisy) gate performs the operation it is supposed to implement. How well a gate preserves the distinguishability of states is captured by another channel performance parameter, the channel fidelity, introduced in [46]. These channel parameters are applied here to the Unruh channel, both with and without external influences.

Experimental progress in this direction is now attracting considerable attention from the community. Circuit quantum electrodynamics, using Superconducting Quantum Interferometric Devices, is a promising effort in this direction. Here tuneable boundary conditions are possible, corresponding to mirrors moving at speeds close to the speed of light in the medium. This was used to experimentally simulate the scenario of dynamical Casimir effect [52]. This paved the way for investigations into various facets of relativistic quantum information.

The plan of the work is as follows. In section II, we briefly discuss the Unruh channel for the Dirac qubit. This is followed by studying how external influences, characterized by parameters like temperature, squeezing and evolution time, effect various aspects of quantum correlations, such as nonlocality, entanglement, teleportation fidelity and measurement-induced disturbance. Quantum coherence is a characteristic of a quantum operation. Mixing, which is inevitable with evolution, will result in the degradation of coherence. In order to have an operational estimation of the utility of a quantum task, it is imperative to have an understanding of the trade-off between the two [53]. This is done for the Unruh channel, pure as well as in the presence of ambient effects. We then study the average gate and channel fidelities in order to gain insight into the nature of the Unruh channel. In the penultimate section, we discuss how one can address the problems treated here by going beyond the single mode approximation. We then make our conclusions.

## 2 Invitation to Unruh effect

We consider two observers, Alice (A) and Rob (R) sharing a maximally entangled initial state of two qubits at a point in flat Minkowski spacetime. Then Rob moves with a uniform acceleration and Alice stays stationary. Moreover, we assume that the observers are equipped with detectors that are sensitive only to their respective modes and share the following maximally entangled initial state:

$$|\psi\rangle_{A,R} = \frac{|00\rangle_{A,R} + |11\rangle_{A,R}}{\sqrt{2}}. \quad (2.1)$$

Suppose R gets uniformly accelerated with acceleration  $a$ . Under the caveat about the scope and limitations of the monochromaticity assumption in light of [49], as discussed above, from R's frame the Minkowski vacuum state transforms to the Unruh mode giving a two mode squeezed state

$$|0\rangle_R = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II},$$

and the excited state is

$$|1\rangle_R = \cos r |1\rangle_I |0\rangle_{II},$$

where  $\cos r = \frac{1}{\sqrt{1+e^{-\frac{2\pi\omega c}{a}}}}$  and  $\omega$  is Dirac particle frequency with acceleration  $a$ ,  $c$  being the speed of light in vacuum. In the above equations, the regions I and II are causally disconnected regions, in Rindler's spacetime. The state in eq. (2.1) has contribution from the regions I and II. Since these regions are disconnected, the mode corresponding to II can be traced out to obtain the following density matrix

$$\rho_{A,I} = \frac{1}{2} [\cos^2 r |00\rangle\langle 00| + \cos r (|00\rangle\langle 11| + |11\rangle\langle 00|) + \sin^2 r |01\rangle\langle 01| + |11\rangle\langle 11|]. \quad (2.2)$$

It would be pertinent, here, to have a discussion related to the choice of tensor product decomposition in a fermionic system. This arises from the issues related to superselection rules, restricting the superpositions of bosons and fermions [54]. A careful use of the superselection rules is important for determining the class of states permissible for describing a composite system of, for e.g., two fermions [55–58]. Thus, for e.g., given a fermionic vacuum state  $|0, 0\rangle$ , one could generate the states  $|1, 0\rangle = a_1^\dagger |0, 0\rangle$ ,  $|0, 1\rangle = a_2^\dagger |0, 0\rangle$ ,  $|1, 1\rangle = a_2^\dagger a_1^\dagger |0, 0\rangle$ . Here the operators acting in the space  $\mathcal{C}^2 \otimes \mathcal{C}^2$  have the form  $a_1 = a \otimes \mathcal{I}_2$ ,  $a_2 = \mathcal{I}_2 \otimes a$ , with analogous creation operators. However, the tensor product  $\otimes$ , used here, is a modified tensor product defined by the graded multiplication rule [59]

$$(O \otimes m)(n \otimes P) = (-1)^{F(n)F(m)} O n \otimes m P, \quad (2.3)$$

where  $m, n$  are monomials in the annihilation and creation operators and  $F(n)$  is the fermion number, equal to the excess of the number of creation operators to the number of annihilation operators required to build  $n$ .

Consider the maximally entangled two mode state in which the second mode is Unruh accelerated. The state is represented by

$$\rho_u = \frac{1}{2} \begin{pmatrix} \cos^2 r & 0 & 0 & \cos r \\ 0 & \sin^2 r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cos r & 0 & 0 & 1 \end{pmatrix}. \quad (2.4)$$

Leaving out the factor  $1/2$  in  $\rho_u$ , the corresponding Choi matrix [60, 61] is  $|i\rangle\langle i| \mathcal{E}(|j\rangle\langle j|)$ , from which the Kraus representation of the Unruh channel can be obtained. Following the procedure for constructing Kraus operators we have

$$K_1^u = \begin{pmatrix} \cos r & 0 \\ 0 & 1 \end{pmatrix}, \quad K_2^u = \begin{pmatrix} 0 & 0 \\ \sin r & 0 \end{pmatrix}. \quad (2.5)$$

The above Kraus operators are similar to

$$K_1 = \begin{pmatrix} \sqrt{1-\gamma} & 0 \\ 0 & 1 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 0 & 0 \\ \sqrt{\gamma} & 0 \end{pmatrix}, \quad (2.6)$$

which represents the dissipative interaction of a qubit with the vacuum bath, i.e., an amplitude damping channel [45, 46]. Here  $\gamma = \frac{1}{e^{\frac{\omega}{k_B T}}}$ . However, this identification is not strict in the sense that the Unruh channel, though resembling the effect of a vacuum bath is not exactly synonymous with it, a point that was elucidated in [62]. This also brings into focus a point raised recently in [15] by the question whether the influence of the Unruh effect on the system of interest, there a topological qubit, was same as a thermal one or not. From the above, we have provided an answer to this.

Once we have the Kraus operators of the Unruh channel, we can calculate its effect on a qubit in pure state given by

$$\rho = \begin{pmatrix} \cos^2(\theta/2) & e^{i\phi} \cos(\theta/2) \sin(\theta/2) \\ e^{-i\phi} \cos(\theta/2) \sin(\theta/2) & \sin^2(\theta/2) \end{pmatrix}. \quad (2.7)$$

The action of the Unruh channel on the state  $\rho$  is

$$\mathcal{E}_u(\rho) = \begin{pmatrix} \cos^2 r \cos^2(\theta/2) & \cos r e^{i\phi} \cos(\theta/2) \sin(\theta/2) \\ \cos r e^{-i\phi} \cos(\theta/2) \sin(\theta/2) & \sin^2 r \cos^2(\theta/2) + \sin^2(\theta/2) \end{pmatrix}. \quad (2.8)$$

### 3 Degradation of quantum information under Unruh channel

The nonclassicality of quantum correlations can be characterized in terms of nonlocality  $B$  (for e.g., Bell inequality violation [63, 64]), entanglement, characterized here by concurrence  $C$  [65], teleportation fidelity  $F_{\max}$  [66] or weaker nonclassicality measures like measurement induced disturbance  $M$  [67, 68]. In the accelerated reference frame, the Unruh effect degrades the quantumness of the state (2.2) [40, 62]. To achieve our goal, we consider the scenario wherein only Rob's qubit is interacting with a noisy environment, for e.g., a scalar field. The other case in which both the qubits of the two observers interact with a noisy environment is not seen here to produce any qualitatively useful insight and hence is not considered in what follows.

#### 3.1 Effect of QND noise

QND is a purely dephasing noise channel whose action on a qubit, characterized by frequency  $\omega_0$ , can be studied using the following Kraus operators [69–71]

$$K_1 = \sqrt{\frac{1 - e^{-\omega_0^2 \gamma(t)}}{2}} \begin{pmatrix} e^{i\omega_0 t} & 0 \\ 0 & -1 \end{pmatrix}; \quad K_2 = \sqrt{\frac{1 + e^{-\omega_0^2 \gamma(t)}}{2}} \begin{pmatrix} e^{i\omega_0 t} & 0 \\ 0 & 1 \end{pmatrix}. \quad (3.1)$$

Assuming an Ohmic bath spectral density with an upper cut-off frequency  $\omega_c$ , it can be shown that

$$\begin{aligned} \gamma(t) = & \left( \frac{\gamma_0 k_B T}{\pi \hbar \omega_c} \right) \cosh(2s) \left( 2\omega_c t \tan^{-1}(\omega_c t) + \ln \left[ \frac{1}{1 + \omega_c^2 t^2} \right] \right) \\ & - \left( \frac{\gamma_0 k_B T}{2\pi \hbar \omega_c} \right) \sinh(2s) \left( 4\omega_c(t-a) \tan^{-1}[2\omega_c(t-a)] - 4\omega_c(t-2a) \tan^{-1}[\omega_c(t-2a)] \right. \\ & \left. + 4a\omega_c \tan^{-1}(2a\omega_c) + \ln \left[ \frac{(1 + \omega_c^2(t-2a)^2)^2}{1 + 4\omega_c^2(t-a)^2} \right] + \ln \left[ \frac{1}{1 + 4a^2\omega_c^2} \right] \right). \end{aligned} \quad (3.2)$$

Here  $T$  is the reservoir temperature, while  $a$  and  $s$  are bath squeezing parameters. Now, the corresponding Choi matrix have the form

$$\begin{pmatrix} \cos^2 r & 0 & 0 & e^{i\omega_0 t} e^{-\omega_0^2 \gamma(t)} \cos r \\ 0 & \sin^2 r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e^{-i\omega_0 t} e^{-\omega_0^2 \gamma(t)} \cos r & 0 & 0 & 1 \end{pmatrix}, \quad (3.3)$$

The new Kraus operators are

$$\begin{aligned} K_1 &= \frac{\sqrt{\cos 2r + 3 - 2e^{-\frac{1}{4}(\omega_0^2 \gamma(t))} \sqrt{\mathcal{A}}}}{2\sqrt{\frac{1}{4} \sec^2 r \left( \sqrt{\mathcal{A}} + e^{\frac{1}{4}\omega_0^2 \gamma(t)} \sin^2 r \right)^2 + 1}} \times \begin{pmatrix} -\frac{1}{2} e^{i\omega_0 t} \sec r \left( \sqrt{\mathcal{A}} + e^{\frac{1}{4}\omega_0^2 \gamma(t)} \sin^2 r \right) & 0 \\ 0 & 1 \end{pmatrix} \\ K_2 &= \frac{\sqrt{\cos 2r + 3 + 2e^{-\frac{1}{4}(\omega_0^2 \gamma(t))} \sqrt{\mathcal{A}}}}{2\sqrt{\frac{1}{4} \sec^2 r \left( \sqrt{\mathcal{A}} - e^{\frac{1}{4}\omega_0^2 \gamma(t)} \sin^2 r \right)^2 + 1}} \times \begin{pmatrix} \frac{1}{2} e^{i\omega_0 t} \sec r \left( \sqrt{\mathcal{A}} - e^{\frac{1}{4}\omega_0^2 \gamma(t)} \sin^2 r \right) & 0 \\ 0 & 1 \end{pmatrix} \\ K_3 &= \begin{pmatrix} 0 & 0 \\ \sin r & 0 \end{pmatrix}, \end{aligned} \quad (3.4)$$

where  $\mathcal{A} = e^{\frac{1}{2}\omega_0^2 \gamma(t)} \sin^4 r + 2 \cos 2r + 2$  and the Kraus operators satisfy the completeness  $\sum_i^3 K_i^\dagger K_i = \mathbb{I}$ .

For the initial time  $t = 0$ , when the QND interaction has not begun,  $e^{-\omega_0^2 \gamma(t)} = 1$  and the above Kraus operators reduce to that in eq. (2.5). The composition of the dephasing channel with the Unruh channel has 3 Kraus operators, essentially because only three of the resulting four operators obtained under composition, are linearly independent. To see this, let the two Kraus operators of the amplitude damping channel be denoted

$$\mathcal{A}_1 \equiv \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, \quad \mathcal{A}_2 \equiv \begin{pmatrix} 0 & 0 \\ \sqrt{\lambda} & 0 \end{pmatrix},$$

and those for the dephasing channel by

$$\mathcal{D}_1 \equiv \sqrt{p} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathcal{D}_2 \equiv \sqrt{1-p} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The composition of these two channels has the Kraus operators  $\mathcal{D}_1\mathcal{A}_1$ ,  $\mathcal{D}_2\mathcal{A}_1$ ,  $\mathcal{D}_1\mathcal{A}_2$  and  $\mathcal{D}_2\mathcal{A}_2$ , where the last two terms, namely,

$$\mathcal{D}_1\mathcal{A}_2 = \sqrt{p\lambda} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathcal{D}_2\mathcal{A}_2 = \sqrt{p\lambda} \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}, \quad (3.5)$$

are equivalent in that they produce the same noise effect. Thus, the composed noise channel has a rank of three corresponding to the independent Kraus operators  $\mathcal{D}_1\mathcal{A}_1$ ,  $\mathcal{D}_2\mathcal{A}_1$ , and  $\mathcal{D}_1\mathcal{A}_2$  or  $\mathcal{D}_2\mathcal{A}_2$ .

The QND channel acting on the Unruh qubit effects its quantum characteristics and can be studied by the behavior of the different facets of quantum correlations. For the case of QND noise acting on Rob, analytical expressions can be obtained for the corresponding measures of quantum correlations which are as follows,

$$\begin{aligned} M = \frac{1}{8} & \left[ 4 + \frac{3 + \cos 2r - 2\sqrt{4e^{-\omega_0^4\gamma^2(t)} \cos^2 r + \sin^4 r}}{8} \right. \\ & \times \log \left( \frac{3 + \cos 2r - 2\sqrt{4e^{-\omega_0^4\gamma^2(t)} \cos^2 r + \sin^4 r}}{8} \right) \\ & + \frac{3 + \cos 2r + 2\sqrt{4e^{-\omega_0^4\gamma^2(t)} \cos^2 r + \sin^4 r}}{8} \\ & \times \log \left( \frac{3 + \cos 2r + 2\sqrt{4e^{-\omega_0^4\gamma^2(t)} \cos^2 r + \sin^4 r}}{8} \right) \\ & \left. - 4 \cos^2 r \log \left( \frac{\cos^2 r}{2} \right) \right], \quad (3.6) \end{aligned}$$

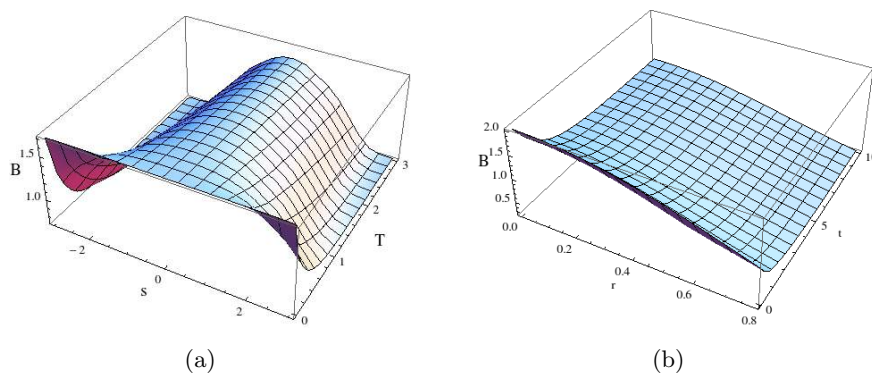
$$F_{\max} = \frac{1}{2} \left[ 1 + \frac{\cos r}{3} \left( 2e^{-\omega_0^2\gamma(t)} + \cos r \right) \right], \quad (3.7)$$

$$B = 2e^{-\omega_0^4\gamma^2(t)} \cos^2 r. \quad (3.8)$$

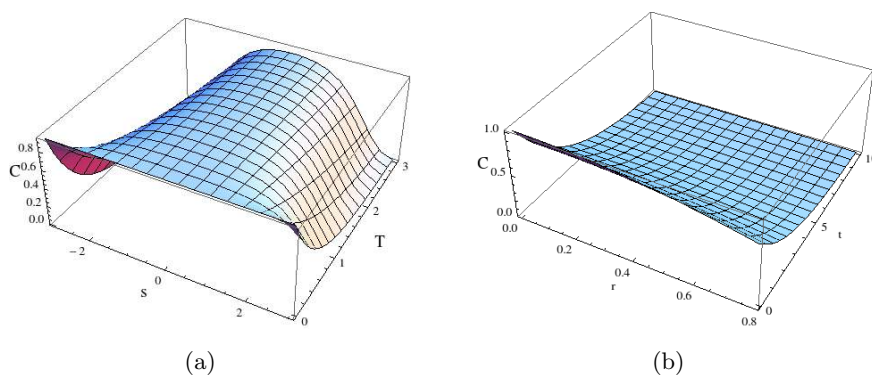
The analytical expression for entanglement  $C$  turns out to be very involved. Hence we only provide a numerical analysis. For the initial time  $t = 0$  when QND interaction has not begun,  $e^{-\omega_0^2\gamma(t)} = 1$  and the expressions  $F_{\max}$ ,  $B$  and  $M$  in the above equations reduce to the pure Unruh-effect case.

The figures 1 to 4 correspond to the behavior of various aspects of quantum correlations of the Unruh channel under the influence of QND noise. It can be seen from figure 1(a) that as reservoir squeezing  $s$  increases, the channel becomes local even for a small external temperature  $T$ . Also, from figure 1(b) the channel is seen to become local with increase in the Unruh acceleration depicted, here, by  $r$ . Entanglement is seen, in figure 2(a), to decrease with increase in  $s$ . This feature is more prominent for  $T > 1$ . From figure 2(b), for a given value of  $r$ , entanglement is seen to decrease with time. For  $|s| < 2$ , figure 3(a) shows that  $F_{\max} > \frac{2}{3}$ ,  $\frac{2}{3}$  being the classical threshold, for the given temperature range. Also,  $F_{\max}$  decreases with increase in  $r$  and time of evolution  $t$ , figure 3(b).  $M$ , figure 4, is seen to decrease with increase in the parameters  $t$ ,  $s$ ,  $T$  and  $r$ .

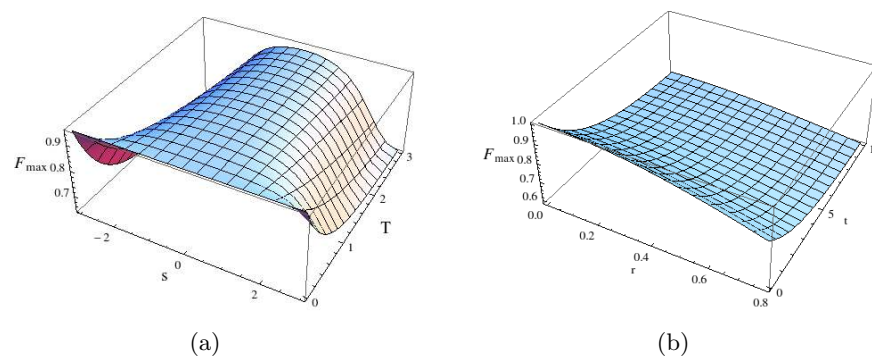




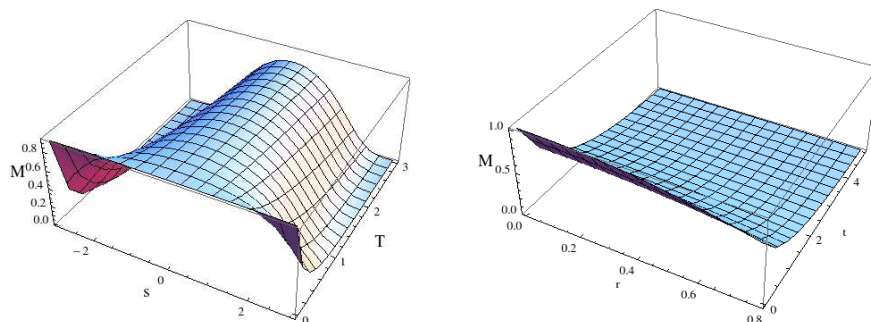
**Figure 1.** (a) Plot of Bell's inequality  $B$  against bath temperature  $T$  and squeezing  $s$  due to action of QND channel on Rob's qubit accelerated at  $r = \pi/8$  and the qubit-bath interaction time  $t = 0.5$ . (b) Plot of  $B$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 1$ ,  $a = 0$  and  $\gamma_0 = 0.1$ .



**Figure 2.** (a) Plot of concurrence  $C$  against bath temperature  $T$  and squeezing  $s$  due to action of QND channel on Rob's qubit accelerated at  $r = \pi/8$  and the qubit-bath interaction time  $t = 0.5$ . (b) Plot of  $C$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 1$ ,  $a = 0$  and  $\gamma_0 = 0.1$ .



**Figure 3.** (a) Plot of teleportation fidelity  $F_{\max}$  against bath temperature  $T$  and squeezing  $s$  due to action of QND channel on Rob's qubit accelerated at  $r = \pi/8$  and the qubit-bath interaction time  $t = 0.5$ . (b) Plot of  $F_{\max}$  against  $t$  and  $r$  while  $T = 1.5$  and  $s = 1.5$ . The other parameters are  $\omega_0 = 1$ ,  $a = 0$  and  $\gamma_0 = 0.1$ .



**Figure 4.** (a) Plot of  $M$  against bath temperature  $T$  and squeezing  $s$  due to action of QND channel on Rob's qubit accelerated at  $r = \pi/8$  and the qubit-bath interaction time  $t = 0.5$ . (b) Plot of  $M$  against  $t$  and  $r$  while  $T = 1.5$  and  $s = 1.5$ . The other parameters are  $\omega_0 = 1$ ,  $a = 0$  and  $\gamma_0 = 0.1$ .

### 3.2 Effect of SGAD noise

The Kraus corresponding to the SGAD channel are

$$\begin{aligned}
 K_1 &\equiv \sqrt{p_1} \begin{bmatrix} \sqrt{1-\alpha} & 0 \\ 0 & 1 \end{bmatrix}, & K_2 &\equiv \sqrt{p_1} \begin{bmatrix} 0 & 0 \\ \sqrt{\alpha} & 0 \end{bmatrix}, \\
 K_3 &\equiv \sqrt{p_2} \begin{bmatrix} \sqrt{1-\mu} & 0 \\ 0 & \sqrt{1-\nu} \end{bmatrix}, & K_4 &\equiv \sqrt{p_2} \begin{bmatrix} 0 & \sqrt{\nu} \\ \sqrt{\mu}e^{-i\phi_s} & 0 \end{bmatrix},
 \end{aligned} \tag{3.9}$$

where  $p_1 + p_2 = 1$  [45]. Here

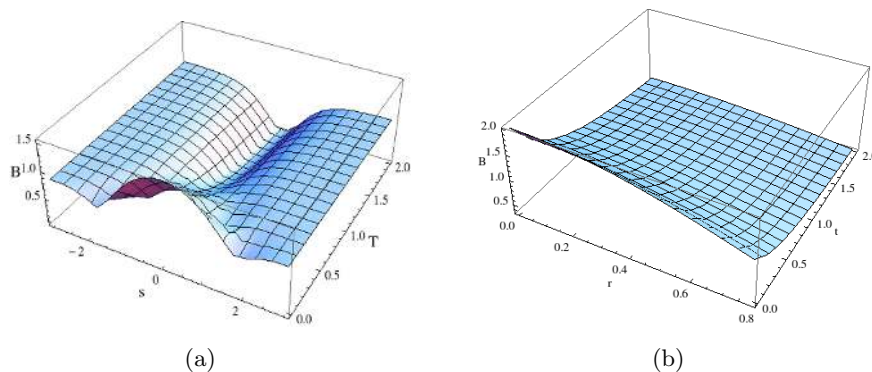
$$\begin{aligned}
 p_2 = \frac{1}{(A+B-C-1)^2 - 4D} \times [ &A^2B + C^2 + A(B^2 - C - B(1+C) - D) - (1+B)D \\
 &- C(B+D-1) \pm 2\sqrt{D(B-AB+(A-1)C+D)(A-AB+(B-1)C+D)} ],
 \end{aligned} \tag{3.10}$$

where

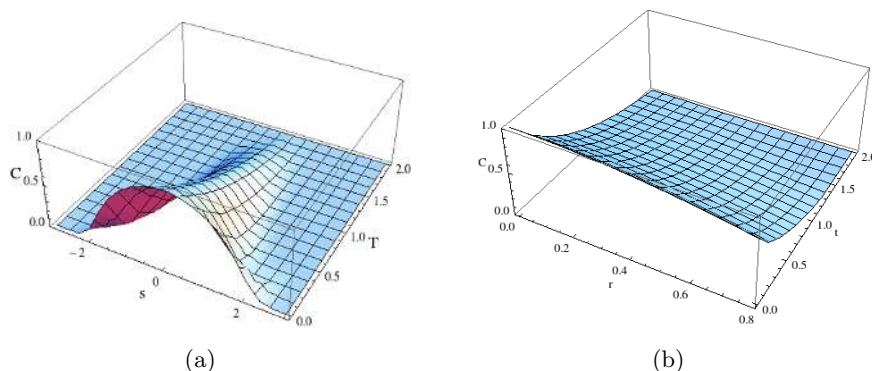
$$\begin{aligned}
 A &= \frac{2N+1}{2N} \frac{\sinh^2(\gamma_0 at/2)}{\sinh(\gamma_0(2N+1)t/2)} \exp(-\gamma_0(2N+1)t/2), \\
 B &= \frac{N}{2N+1} (1 - \exp(-\gamma_0(2N+1)t)), \\
 C &= A + B + \exp(-\gamma_0(2N+1)t), \\
 D &= \cosh^2(\gamma_0 at/2) \exp(-\gamma_0(2N+1)t).
 \end{aligned} \tag{3.11}$$

Also,

$$\begin{aligned}
 \nu &= \frac{N}{(p_2)(2N+1)} (1 - e^{-\gamma_0(2N+1)t}), \\
 \mu &= \frac{2N+1}{2(p_2)N} \frac{\sinh^2(\gamma_0 at/2)}{\sinh(\gamma_0(2N+1)t/2)} \exp\left(-\frac{\gamma_0}{2}(2N+1)t\right), \\
 \alpha &= \frac{1}{p_1} \left(1 - p_2[\mu(t) + \nu(t)] - e^{-\gamma_0(2N+1)t}\right).
 \end{aligned} \tag{3.12}$$



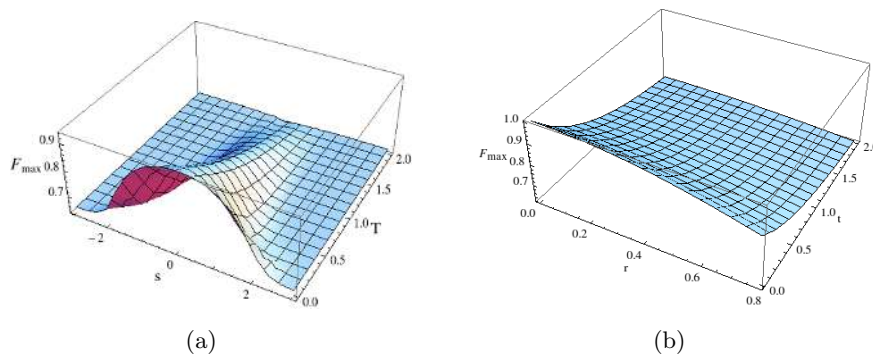
**Figure 5.** (a) Plot of Bell's inequality  $B$  against bath temperature  $T$  and squeezing  $s$  due to action of SGAD channel on Rob's qubit accelerated at  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $B$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ .



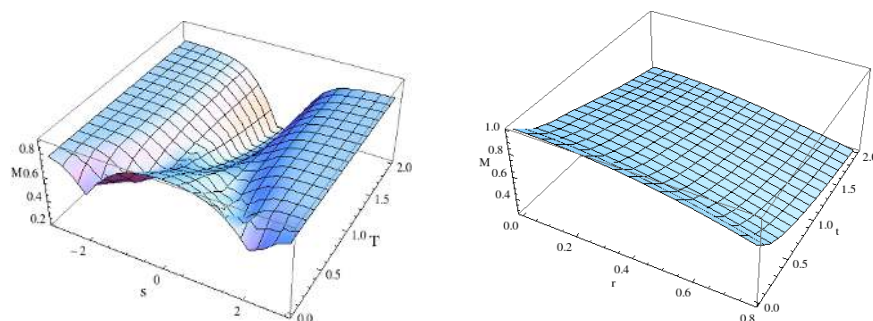
**Figure 6.** (a) Plot of concurrence  $C$  against bath temperature  $T$  and squeezing  $s$  due to action of SGAD channel on Rob's qubit accelerated at  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $C$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ .

Also  $N = N_{\text{th}}[\cosh^2(s) + \sinh^2(s)] + \sinh^2(s)$ ,  $a = \sinh(2s)(2N_{\text{th}} + 1)$  where  $N_{\text{th}} = 1/(e^{\hbar\omega_0/k_B T} - 1)$  is the Planck distribution giving the number of thermal photons at the frequency  $\omega_0$ ;  $s$  and  $\phi_s$  are bath squeezing parameters.

The analytical expressions are complicated and hence we resort to numerical discussions. The figures 5–8 correspond to the behavior of various aspects of quantum correlations of the Unruh channel under the influence of SGAD noise. From figure 5(a) it can be seen that for certain range of  $T$ , squeezing enhances  $B$ . However, for the values of the parameters indicated, it never crosses the threshold of nonlocality ( $B > 1$ ). From figure 5(b) it can be seen that with increase in  $r$  and  $t$ ,  $B$  decreases and the channel becomes local. From figure 6(a), concurrence is seen to drop drastically to zero with increase in  $T$ . Also for large values of  $s$ , concurrence is seen to fall to zero, irrespective of  $T$ . Figure 6(b) depicts the decrease of concurrence with respect to time for any give value of  $r$ . From figure 7,  $F_{\text{max}}$  is seen to decrease with  $T$ ,  $s$ ,  $r$  and  $t$ . From figure 8,  $M$  is seen to decrease with increase in the parameters  $r$  and  $t$ . However for certain range of  $T$ ,  $M$  is seen to increase with bath squeezing  $s$ , reiterating the usefulness of squeezing.



**Figure 7.** (a) Plot of teleportation fidelity  $F_{\max}$  against bath temperature  $T$  and squeezing  $s$  due to action of SGAD channel on Rob's qubit accelerated at  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $F_{\max}$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ .



**Figure 8.** (a) Plot of measurement induced disturbance  $M$  against bath temperature  $T$  and squeezing  $s$  due to action of SGAD channel on Rob's qubit accelerated at  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $M$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ .

#### 4 Coherence and mixedness

Coherence plays a central role in quantum mechanics [37] enabling operations or tasks which are impossible within the regime of classical mechanics. As a resource in quantum operations, it has attracted much attention in recent times [53].

For a quantum state represented by density matrix  $\rho$  in basis  $\{|i\rangle\}$ , the  $l_1$  norm of coherence is given by

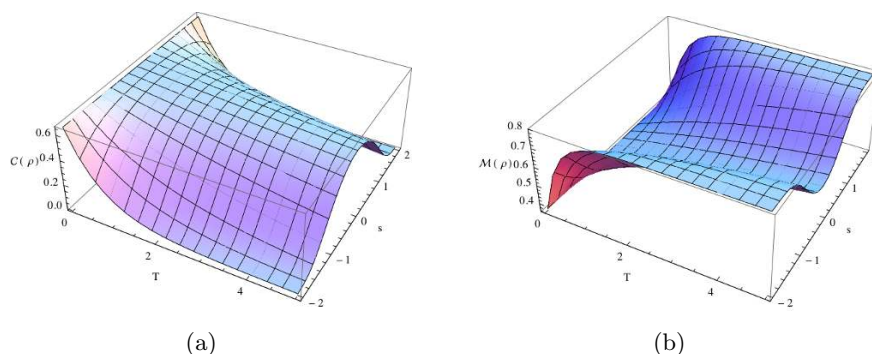
$$\mathcal{C}(\rho) = \sum_{i \neq j} |\rho_{i,j}|. \quad (4.1)$$

The mixedness, which is basically normalized linear entropy, of a  $d$  dimensional quantum state  $\rho$  is given by [72]

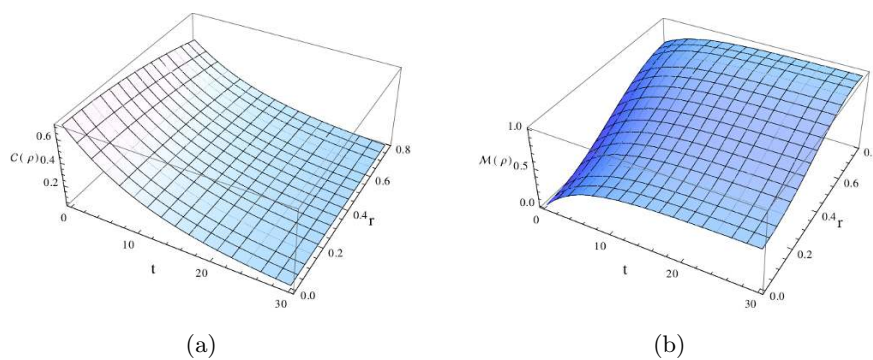
$$\mathcal{M}(\rho) = \frac{d}{d-1} (1 - \text{Tr} \rho^2). \quad (4.2)$$

The inequality relation between the  $\mathcal{C}(\rho)$  and  $\mathcal{M}(\rho)$  is [53]

$$\frac{\mathcal{C}(\rho)^2}{(d-1)^2} + \mathcal{M}(\rho) \leq 1. \quad (4.3)$$



**Figure 9.** Plot of (a) coherence  $\mathcal{C}(\rho)$  and (b) mixedness  $\mathcal{M}(\rho)$  due to action of QND channel on Rob's qubit parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$  w.r.t. bath squeezing  $s$  and temperature  $T$ , while qubit-bath interaction time  $t = 2$  and Rob's acceleration  $r = \pi/8$ . The other parameters are  $a = 0$ ,  $\omega_0 = 0.1$  and  $\gamma_0 = 0.1$ .



**Figure 10.** Variation of (a) coherence  $\mathcal{C}(\rho)$  and (b) mixedness  $\mathcal{M}(\rho)$  due to action of QND channel on Rob's qubit parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$  w.r.t. qubit-bath interaction time  $t$  and Rob's acceleration  $r$ , while bath temperature  $T = 0.5$  and bath squeezing  $s = 0.5$ . The other parameters are  $a = 0$ ,  $\omega_0 = 0.1$  and  $\gamma_0 = 0.1$ .

When this inequality is saturated for certain values of  $\mathcal{C}(\rho)$  and  $\mathcal{M}(\rho)$ , the situation corresponds to states which have maximum coherence for a given mixedness in the states. Such states are known as Maximally Coherent Mixed States (MCMS).

#### 4.1 QND channel

The analytical expressions for coherence and mixedness of the Unruh channel under the influence of the QND noise are

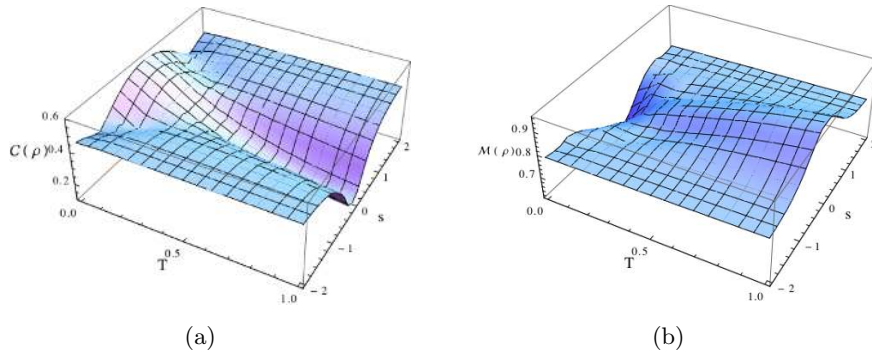
$$\mathcal{C}(\rho) = \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos^r e^{-\frac{\omega_0^2 \gamma(t)}{4}}, \quad (4.4)$$

$$\mathcal{M}(\rho) = \cos^2 r \left( \cos^2 \frac{\theta}{2} (3 - \cos 2r - 2 \cos^2 r \cos \theta) - e^{-\frac{\omega_0^2 \gamma(t)}{2}} \sin^2 \theta \right), \quad (4.5)$$

respectively.

From figure 9, coherence  $\mathcal{C}$  is seen to decrease with increase in temperature  $T$  as well as reservoir squeezing  $s$ . Mixing  $\mathcal{M}$  increases with both  $T$  and  $s$ . From figure 10, it can





**Figure 11.** Plot of (a) coherence  $\mathcal{C}(\rho)$  and (b) mixedness  $\mathcal{M}(\rho)$  due to action of SGAD channel on Rob's qubit parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$  w.r.t. bath squeezing  $s$  and temperature  $T$ , while qubit-bath interaction time  $t = 2$  and Rob's acceleration  $r = \pi/8$ . The other parameters are  $\phi_s = 0$ ,  $\omega_0 = 0.1$  and  $\gamma_0 = 0.1$ .

be seen that coherence decreases with  $t$  whereas mixedness increases with  $t$  as well as  $r$ , a feature which is consistent with common intuition.

## 4.2 SGAD channel

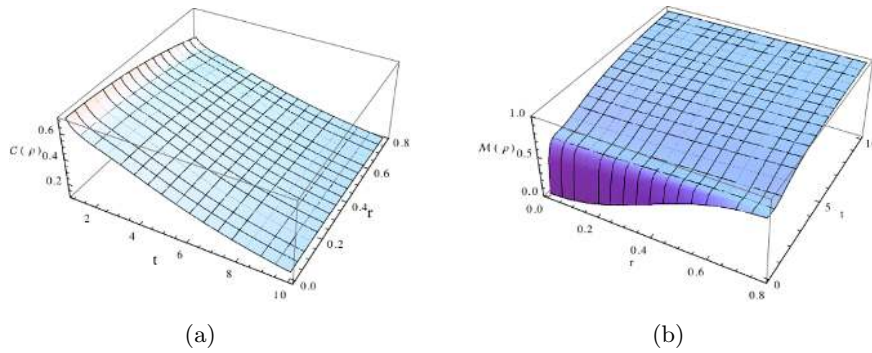
The analytical expressions for coherence and mixedness of the Unruh channel under the influence of the SGAD noise is

$$\mathcal{C}(\rho) = \cos r \sin \theta \sqrt{(p_1 \sqrt{1-\alpha} + p_2 \sqrt{(1-\mu)(1-\nu)})^2 + p_2^2 \mu \nu + 2 \cos(2\phi - \phi_s) \sqrt{\mu \nu} (p_1 \sqrt{1-\alpha} + p_2 \sqrt{(1-\mu)(1-\nu)})}, \quad (4.6)$$

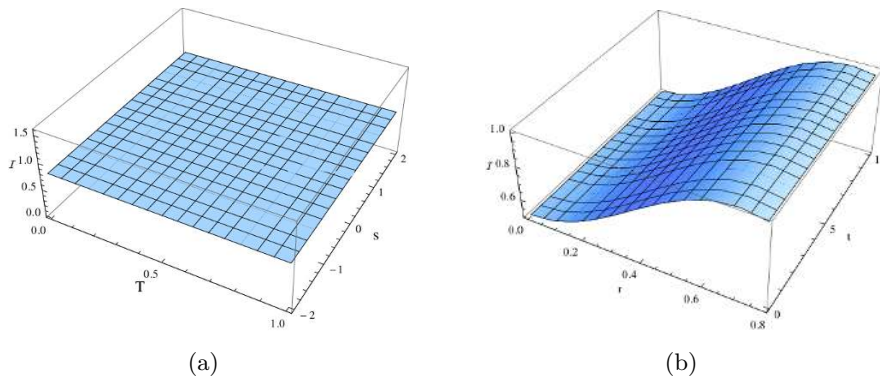
$$\begin{aligned} \mathcal{M}(\rho) = & 2 \left( 1 - \left( \cos \frac{\theta^2}{2} ((p_1 + p_2 - p_1 \alpha - p_2 \mu) \cos^2 r + p_2 \nu \sin^2 r) + p_2 \nu \sin^2 \frac{\theta}{2} \right)^2 \right. \\ & - \left( (p_1 \alpha + p_2 \mu) \cos^2 r \cos^2 \frac{\theta}{2} + (p_1 + p_2 - p_2 \nu) \left( \cos^2 \frac{\theta}{2} \sin^2 r + \sin^2 \frac{\theta}{2} \right) \right)^2 \\ & - \frac{1}{2} e^{-i\phi - i(\phi + \phi_s)} \left( e^{i\phi_s} (p_1 \sqrt{1-\alpha} + p_2 \sqrt{1-\mu} \sqrt{1-\nu}) + e^{2i\phi} p_2 \sqrt{\mu} \sqrt{\nu} \right) \\ & \left. \times \left( e^{2i\phi} (p_1 \sqrt{1-\alpha} + p_2 \sqrt{1-\mu} \sqrt{1-\nu}) + e^{i\phi_s} p_2 \sqrt{\mu} \sqrt{\nu} \right) \cos^2 r \sin^2 \theta \right), \quad (4.7) \end{aligned}$$

respectively. From figure 11(a), it is observed that for a certain range of temperature  $T$ , coherence  $\mathcal{C}$  increases with squeezing  $s$  while in another range, it decreases with  $s$ , in consistence with the quadrature nature of squeezing. Also, finite  $s$  brings a notion of stability in the behavior of coherence  $\mathcal{C}$  as a function of external temperature  $T$ . Thus, squeezing  $s$  once again emerges as a useful resource. The expected increase in mixing  $\mathcal{M}$  with increase in  $T$  is observed in figure 11(b). From figure 12, it can be seen that coherence decreases with increase in  $t$  and  $r$  whereas mixedness increases rapidly with  $t$ .

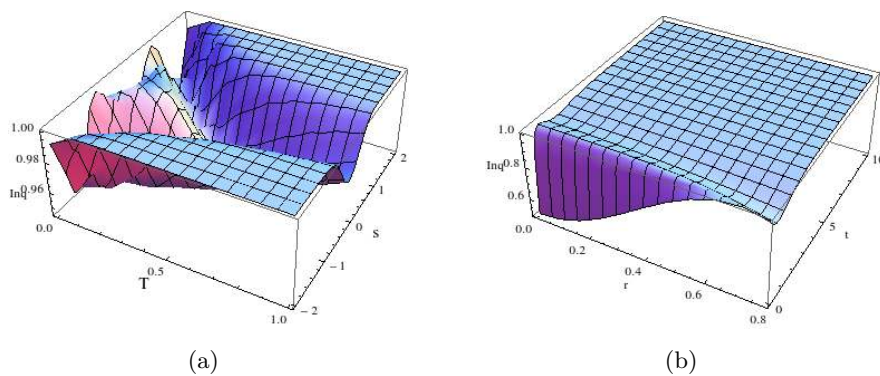
From figure 13 and 14, it is clear that the inequality eq. (4.3) is respected for both QND and SGAD noises, respectively. In particular, from figure 13(a), due to the action of QND noise, it can be seen that for  $t = 2$  and  $r = \pi/8$ , by varying parameters  $T$  and  $s$  we cannot get any MCMS as the inequality is not saturated. However, fixing  $T$  and  $s$  to 0.5



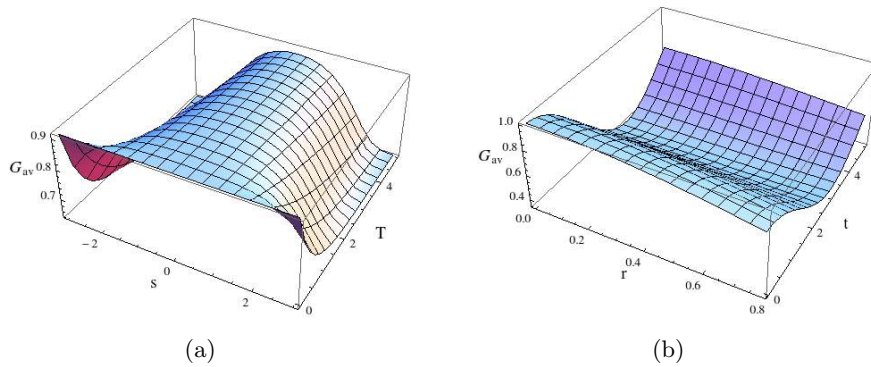
**Figure 12.** Variation of (a) coherence  $\mathcal{C}(\rho)$  and (b) mixedness  $\mathcal{M}(\rho)$  due to action of SGAD channel on Rob's qubit parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$  w.r.t. qubit-bath interaction time  $t$  and Rob's acceleration  $r$ , while bath temperature  $T = 0.5$  and bath squeezing  $s = 0.5$ . The other parameters are  $\phi_s = 0$ ,  $\omega_0 = 0.1$  and  $\gamma_0 = 0.1$ .



**Figure 13.** (a) Plot of inequality  $\mathcal{I}$  in eq. (4.3) due to action of QND channel on Rob's qubit w.r.t. bath temperature  $T$  and bath squeezing  $s$  while qubit-bath interaction time  $t = 2$  and Rob's acceleration  $r = \pi/8$ . (b)  $\mathcal{I}$  plotted against  $t$  and  $r$  while  $T = 0.5$ ,  $s = 0.5$ . The other parameters are  $a = 0$ ,  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and the input state is parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$ .



**Figure 14.** (a) Plot of inequality  $\mathcal{I}$  in eq. (4.3) due to action of SGAD channel on Rob's qubit w.r.t. bath temperature  $T$  and bath squeezing  $s$  while qubit-bath interaction time  $t = 2$  and Rob's acceleration  $r = \pi/8$ . (b)  $\mathcal{I}$  plotted against  $t$  and  $r$  while  $T = 0.5$ ,  $s = 0.5$ . The other parameters are  $\phi_s = 0$ ,  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and the input state is parameterized by  $\theta = \pi/4$ ,  $\phi = \pi/4$ .



**Figure 15.** (a) Plot of average gate fidelity  $G_{\text{av}}$  against bath temperature  $T$  and squeezing  $s$  when Rob's accelerated qubit is subjected to a QND channel, while Rob's acceleration  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Plot of  $G_{\text{av}}$  against  $t$  and  $r$ , while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 1$ ,  $\gamma_0 = 0.1$  and  $a = 0$ .

and varying  $r$ , MCMS can be achieved, as can be seen from figure 13(b). For the case of SGAD noise, figure 14(a) depicts that for  $t = 2$  and  $r = \pi/8$ , by varying the parameters  $T$  and  $s$  MCMS states can be attained. Also, by fixing  $T$  and  $s$  to 0.5 and varying  $r$  and  $t$ , MCMS can be achieved as can be seen from figure 14(b).

## 5 Average gate and channel fidelity

In this work we are trying to understand the Unruh channel under the influence of external noisy effects. A useful way to understand this is to analyze the average gate and channel fidelities [46] of the Unruh channel under the ambient environmental conditions.

The average gate fidelity [45, 50, 51] has a closed expression

$$G_{\text{av}} = \frac{d + \sum_i |\text{Tr}(E_i)|^2}{d(d+1)}. \quad (5.1)$$

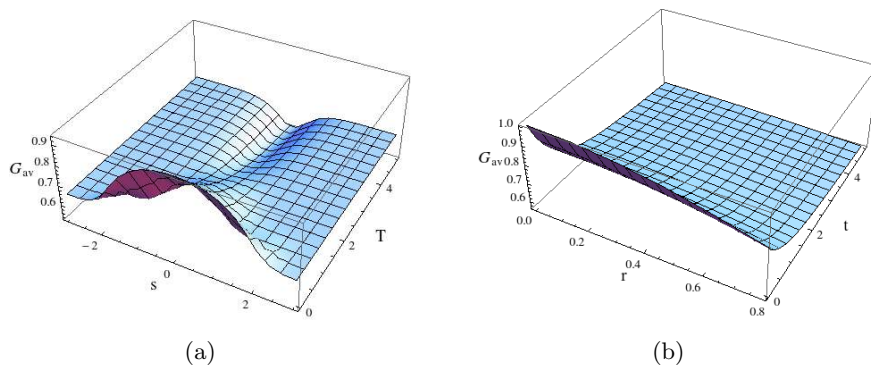
For the Unruh channel  $G_{\text{av}} = \frac{1}{6} (2 + (1 + \cos r)^2)$ , where  $d$  is the dimensionality of the system on which channel  $\mathcal{E}$  acts with operator sum representation elements  $E_i$ .

For the QND-Unruh channel

$$\begin{aligned} |\text{Tr}(E_i)|^2 = & \frac{(2\sqrt{\mathcal{A}}e^{-\frac{1}{4}\gamma\omega_0^2} + \cos 2r + 3) \left( \sec r \cos \omega_0 t \left( \sqrt{\mathcal{A}} - e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right) + \frac{1}{4} \sec^2 r \left( \sqrt{\mathcal{A}} - e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right)^2 + 1 \right)}{\sec^2 r \left( \sqrt{\mathcal{A}} - e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right)^2 + 4} \\ & + \frac{(-2\sqrt{\mathcal{A}}e^{-\frac{1}{4}\gamma\omega_0^2} + \cos r + 3) \left( -\sec r \cos \omega_0 t \left( \sqrt{\mathcal{A}} + e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right) + \frac{1}{4} \sec^2 r \left( \sqrt{\mathcal{A}} + e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right)^2 + 1 \right)}{\sec^2 r \left( \sqrt{\mathcal{A}} + e^{\frac{\gamma\omega_0^2}{4}} \sin^2 r \right)^2 + 4}, \end{aligned} \quad (5.2)$$

using which  $G_{\text{av}}$  can be calculated. In the limit  $t \rightarrow 0$  this reduces to the Unruh case. Unlike the QND case, the analytical expression for  $G_{\text{av}}$  for the SGAD channel is very involved, and hence we discuss this case numerically.





**Figure 16.** (a) Plot of average gate fidelity  $G_{\text{av}}$  against bath temperature  $T$  and squeezing  $s$  when Rob’s accelerated qubit is subjected to a SGAD channel, while Rob’s acceleration  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $G_{\text{av}}$  against  $t$  and  $r$  while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ .

From figure 15(a), it can be seen that  $G_{\text{av}}$  is stable for a certain range of squeezing, after which it falls.  $G_{\text{av}}$  is also seen to decrease with  $r$ . However with time,  $G_{\text{av}}$  first decreases and then is then seen to increase. Since the expression for  $G_{\text{av}}$  has the oscillatory term  $\cos \omega_0 t$ , oscillation is seen in figure 15(b) with time  $t$ .

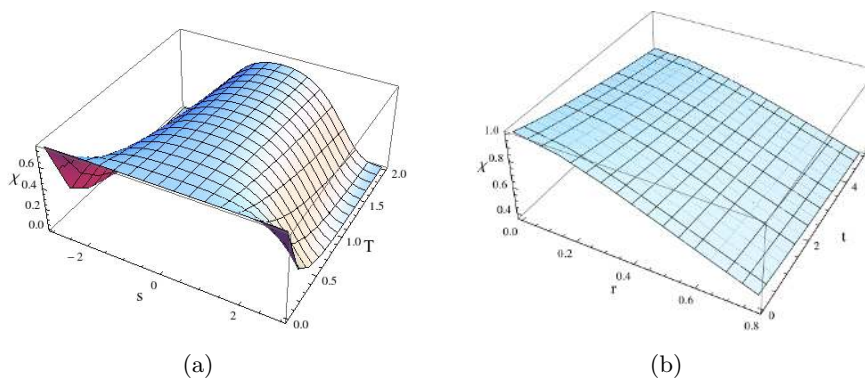
From figures 16, a general trend is observed of average gate fidelity  $G_{\text{av}}$ , under the influence of the SGAD channel, decreasing with increase in  $T$  as well as evolution time  $t$ , for a given  $r$ . However, as can be observed from figure 16(a), for a certain range of  $T$ ,  $G_{\text{av}}$  is seen to increase with reservoir squeezing  $s$ . This indicates that squeezing can be a useful quantum resource in this scenario.

Another quantity frequently used to access the channel’s performance is the channel fidelity [45, 46]

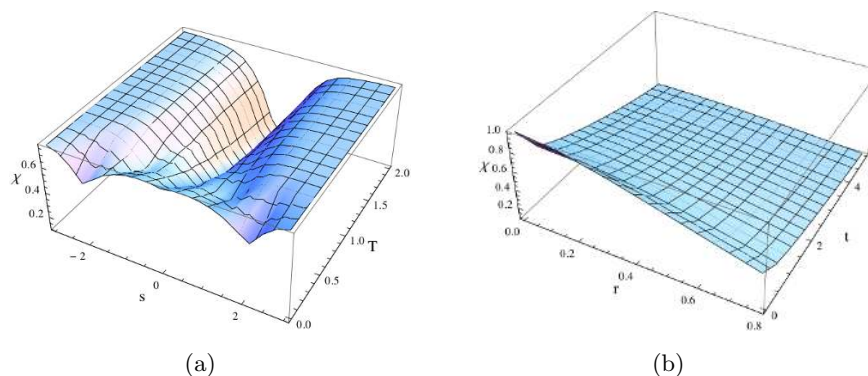
$$\chi = \max_{\mathcal{B}} \kappa(\mathcal{B}, \mathcal{E}), \tag{5.3}$$

where  $\kappa(\mathcal{B}, \mathcal{E})$  is the Holevo bound for states prepared in basis  $\mathcal{B}$  and passed through the channel  $\mathcal{E}$ . By numerical techniques it was learned that the maximum is achieved for the states prepared in the basis states  $\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$ .  $\kappa$  signifies how well the quantum input states are distinguishable after the action of the channel  $\mathcal{E}$ . When classical information is encoded in to the quantum states, it can also be interpreted as the amount of extractable classical information.

From figure 17(a) it can be seen that  $\chi$ , for the Unruh-QND channel, decreases with both  $s$  and  $T$ . This behavior is due to the dephasing caused by QND channel and alteration of the diagonal and off-diagonal terms by Unruh channel of the input states. The  $\chi$  also decreases with the  $r$  and  $t$  as seen from the figure 17(b). The Unruh-SGAD channel drives the input states towards an asymptotic state determined by the channel parameters reducing the distinguishability of the states, i.e., reducing the  $\chi$ . By increasing the bath squeezing, the coherence in the input states increases [46] leading to a rise in  $\chi$  for a given range of temperature, as seen in the figure 18(a). Figure 18(b) shows that as the Unruh-SGAD channel acts  $\chi$  decreases with both  $r$  and  $t$  as both the parameters drive the input states towards an asymptotic state.



**Figure 17.** (a) Plot of channel fidelity  $\chi$  against bath temperature  $T$  and squeezing  $s$  when Rob's accelerated qubit is subjected to QND channel, while Rob's acceleration corresponds to  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $\chi$  against  $t$  for and  $r$ , while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $a = 0$ . In the figures input states used are  $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ .



**Figure 18.** (a) Plot of channel fidelity  $\chi$  against bath temperature  $T$  and squeezing  $s$  when Rob's accelerated qubit is subjected to SGAD channel, while Rob's acceleration corresponds to  $r = \pi/8$  and qubit-bath interaction time  $t = 0.5$ . (b) Variation of  $\chi$  against  $t$  for and  $r$ , while  $T = 0.5$  and  $s = 0.5$ . The other parameters are  $\omega_0 = 0.1$ ,  $\gamma_0 = 0.1$  and  $\phi_s = 0$ . In the figures input states used are  $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ .

## 6 Limitations and future directions

In this article, we have worked with the combinations of Minkowski modes [49, 73–75], using the single mode approximation [49] which attempts to relate a single frequency Minkowski mode (observed by an inertial observer) with a single frequency Rindler mode (observed by a uniformly accelerated observer). A monochromatic Minkowski mode is associated to a Rindler mode which corresponds to a non-monochromatic field excitation. The relevant Bogoliubov transformations are such that a plane-wave Minkowski creation operator cannot be written, in general, as a combination of monochromatic Rindler modes [49].

The present analysis can be extended beyond the single mode approximation. As an example, one can consider a general Unruh mode of the form

$$d_{k,U}^\dagger = q_R \left( D_{k,R}^\dagger \otimes \mathcal{I}_L \right) + q_L \left( \mathcal{I}_R \otimes D_{k,L}^\dagger \right), \quad (6.1)$$

where  $D_{k,R}^\dagger$ ,  $D_{k,L}^\dagger$  are Unruh creation operators and  $q_L$ ,  $q_R$  are complex numbers, the sum of whose modulus square is equal to one. In our analysis  $q_L = 0$  and  $q_R = 1$ . Inclusion of both  $q_L$  and  $q_R$  in the analysis would enable the study of particle and antiparticle modes of Rob and anti-Rob in the study of fermionic entanglement. This results in entanglement redistribution between the particle and anti-particle sectors, a feature not possible in the bosonic sector. As a result, there is non-vanishing minimum value of fermionic entanglement in the limit of infinite acceleration, a result that is also seen in the case of a single-mode approximation scheme [62]. However, keeping both  $q_L$  and  $q_R$  enables one to reach a physical understanding of this residual entanglement [73, 74]. This is independent of the choice of the Unruh mode. This was made explicit in [76].

Here we have investigated in detail, in terms of various aspects of quantum correlations, how environmentally induced decoherence and dissipation, natural artifacts of open quantum systems, modify the effect of the Unruh channel. The single-mode treatment, such as that adopted here, can still be useful for exhibiting in a simple way important facets of external noise, such as the differing effects of a dissipative or non-dissipative environment, the role of squeezing, which is known to play an important role in the context of particle creation, and finally, how external noise can enhance the Unruh effect's degrading effect on quantum information. Keeping in mind the bigger picture that emerges if one were to go beyond the single mode approximation, it would be worth extending the present analysis in this direction. This would be possible by a straightforward application of the tools developed in this work and would enable a deeper insight into the effect of external noise channels on the tradeoff between the particle anti-particle sector.

Another key progress that has been made in the field of relativistic quantum information in recent times, is the introduction and use of localized modes for Alice and the accelerated observer, here the detector, Rob [77, 78]. The principle behind the degradation of quantum correlations emerged to be due to the mode mismatch between what was received and what could be observed by the accelerated detector. It would be interesting, albeit an algebraically involved procedure, to apply the tools of quantum statistical mechanics, as used here, to the localized mode formulation of the problem.

## 7 Conclusions

In this work, we make use of ideas of statistical mechanics and quantum foundations on an important facet of field theories in curved space time, viz. the Unruh effect. We study how environmentally induced decoherence modifies the effect of the Unruh channel, essentially by investigating the degradation of quantum correlations, as quantified by measures such as nonlocality, mixed-state entanglement, teleportation fidelity, coherence and a discord-like quantity. The differing effects of an environment that interacts dissipatively or non-dissipatively are noted. In particular, the latter is shown to lead to a non-Pauli channel of rank 3. Further, useful parameters characterizing channel performance such as gate and channel fidelity are applied here to the Unruh channel, both with and without external influences. Squeezing, which is known to play an important role in the context of particle

creation, is shown to be a useful resource in a number of scenarios. We hope this work is a contribution towards the understanding of the Unruh effect.

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