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CHARACTERIZATIONS OF REAL HYPERSURFACES IN COMPLEX SPACE FORMS IN TERMS OF CURVATURE TENSORS

By

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§1. Introduction.

A complex n-dimensional Kähler manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. A complete and simply connected complex space form consists of a complex projective space P_nC, a complex Euclidean space C^n or a complex hyperbolic space H_nC, according as $c>0$, $c=0$ or $c<0$.

In this study of real hypersurfaces of P_nC, Takagi [8] classified all homogeneous real hypersurfaces and Cecil and Ryan [2] showed also that they are realized as the tubes of constant radius over Kähler submanifolds if the structure vector field ξ is principal. And Berndt [1] classified all homogeneous real hypersurfaces of H_nC and showed that they are realized as the tubes of constant radius over certain submanifolds. According to Takagi's classification theorem and Berndt's one, the principal curvatures and their multiplicities of homogeneous real hypersurfaces of $M_n(c)$ are given.

Now, let M be a real hypersurface of $M_n(c)$, $c\neq 0$. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the Kähler metric and the almost complex structure of $M_n(c)$. We denote by A the shape operator in the direction of the unit normal on M. Then Okumura [7] and Montiel and Romero [6] proved the following

THEOREM A. *Let M be a real hypersurface of P_nC, $n\geqq 2$. If it satisfies*

$$A\phi-\phi A=0\,, \tag{1.1}$$

then M is locally a tube of radius r over one of the following Kähler submanifolds:

(A_1) *a hyperplane $P_{n-1}C$, where $0<r<\pi/2$,*

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(A_2) *a totally geodesic* P_kC ($1 \leqq k \leqq n-2$), *where* $0<r<\pi/2$.

THEOREM B. *Let* M *be a real hypersurface of* H_nC, $n \geqq 2$. *If it satisfies* (1.1), *then* M *is locally one of the following hypersurfaces*:

(A_0) *a horosphere in* H_nC, *i.e., a Montiel tube,*

(A_1) *a tube of a totally geodesic hyperplane* $H_{n-1}C$,

(A_2) *a tube of a totally geodesic* H_kC ($1 \leqq k \leqq n-2$).

Such real hypersurfaces in Theorems A and B are said to be of *type A*. On the other hand, Kimura and Maeda [4] gave the following

THEOREM C. *Let* M *be a real hypersurface of* P_nC, $n \geqq 2$. *If the structure vector field* ξ *is principal and if it satisfies*

$$\nabla_\xi R = 0\,, \qquad g(A\xi, \xi) \neq 0\,, \tag{1.2}$$

then M *is of type A, where* ∇ *denotes the Riemannian connection and* R *denotes the Riemannian curvature tensor on* M.

The purpose of this paper is to give some characterizations of real hypersurfaces in $M_n(c)$, $c \neq 0$, in terms of the Riemannian curvature tensor R. Firstly, as generalizations of Theorem C, we obtain the following

THEOREM 1. *Let* M *be a real hypersurface of* $M_n(c)$, $c \neq 0$, $n \geqq 3$. *If it satisfies* (1.2), *then* M *is of type A*.

THEOREM 2. *Let* M *be a real hypersurface of* P_nC, $n \geqq 3$. *If* $\nabla_\xi R=0$, *then* M *is locally congruent to one of the following*:

(a) *a non-homogeneous real hypersurface which lies on a tube of radius* $\pi/4$ *over a certain Kähler submanifold in* P_nC,

(b) *a real hypersurface of type A*.

Next, we also have a complete classification of real hypersurfaces in $M_n(c)$ satisfying $\mathcal{L}_\xi R=0$, where $\mathcal{L}_\xi$ denotes the Lie derivative in the direction of the structure vector field ξ. Namely, we prove the following

THEOREM 3. *Let* M *be a real hypersurface of* $M_n(c)$, $c \neq 0$, $n \geqq 2$. *If* $\mathcal{L}_\xi R = 0$, *then* M *is of type A*.

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§ 2. Preliminaries.

First of all, we recall fundamental properties about real hypersurfaces of a complex space form. Let M be a real hypersurface of a complex n-dimensional complex space form $M_n(c)$ of constant holomorphic sectional curvature c, and let C be a unit normal vector field on a neighborhood in M. We denote by J the almost complex structure of $M_n(c)$. For a local vector field X on the neighborhood in M, the images of X and C under the linear transformation J can be represented as

$$JX=\phi X+\eta(X)C\,, \qquad JC=-\xi\,,$$

where ϕ defines a skew-symmetric transformation on the tangent bundle TM of M, while η and ξ denote a 1-form and a vector field on the neighborhood in M, respectively. Then it is seen that $g(\xi, X)=\eta(X)$, where g denotes the Riemannian metric tensor on M induced from the metric tensor on $M_n(c)$. The set of tensors (ϕ, ξ, η, g) is called an *almost contact metric structure* on M. They satisfy the following properties:

$$\phi^2=-I+\eta\otimes\xi, \qquad \phi\xi=0, \qquad \eta(\xi)=1\,,$$

where I denotes the identity transformation. Furthermore, the covariant derivatives of the structure tensors are given by

$$\nabla_X\xi=\phi AX\,, \qquad \nabla_X\phi(Y)=\eta(Y)AX-g(AX, Y)\xi \tag{2.1}$$

for any vector fields X and Y on M, where ∇ is the Riemannian connection on M and A is the shape operator of M in the direction of C.

Since the ambient space is of constant holomorphic sectional curvature c, the equations of Gauss and Codazzi are respectively obtained:

$$\begin{aligned} R(X, Y)Z=\frac{c}{4}\{&g(Y, Z)X-g(X, Z)Y \\ &+g(\phi Y, Z)\phi X-g(\phi X, Z)\phi Y-2g(\phi X, Y)\phi Z\} \\ &+g(AY, Z)AX-g(AX, Z)AY\,, \end{aligned} \tag{2.2}$$

$$\nabla_X A(Y)-\nabla_Y A(X)=\frac{c}{4}\{\eta(X)\phi Y-\eta(Y)\phi X-2g(\phi X, Y)\xi\}, \tag{2.3}$$

where R denotes the Riemannian curvature tensor of M and $\nabla_X A$ denotes the covariant derivative of the shape operator A with respect to X.

Next, we suppose that the structure vector field ξ is principal with corresponding principal curvature α. Then it is seen in [3] and [5] that α is constant on M and it satisfies

$$A\phi A=\frac{c}{4}\phi+\frac{1}{2}\alpha(A\phi+\phi A) \tag{2.4}$$

and hence, by (2.1) and (2.3), we get

$$\nabla_\xi A=-\frac{1}{2}\alpha(A\phi-\phi A). \tag{2.5}$$

§ 3. Proof of Theorems 1 and 2.

We consider about the covariant derivative of the Riemannian curvature tensor R. The covariant derivative $\nabla_\xi R$ of R with respect to the structure vector field ξ is defined by

$$\nabla_\xi R(X, Y, Z)=\nabla_\xi(R(X, Y)Z)-R(\nabla_\xi X, Y)Z-R(X, \nabla_\xi Y)Z-R(X, Y)\nabla_\xi Z$$

for any vector fields X, Y and Z.

Now, we shall prove the following proposition.

PROPOSITION. *Let M be a real hypersurface of $M_n(c)$, $c\neq 0$, $n\geqq 3$. If $\nabla_\xi R=0$, then $\nabla_\xi A=0$.*

PROOF. By the definition of the covariant derivative $\nabla_\xi R$ and (2.2), our assumption is equivalent to

$$\begin{aligned}\frac{c}{4}[&\{\eta(Y)g(A\xi, Z)-\eta(Z)g(A\xi, Y)\}\phi X\\ &-\{\eta(X)g(A\xi, Z)-\eta(Z)g(A\xi, X)\}\phi Y\\ &-2\{\eta(X)g(A\xi, Y)-\eta(Y)g(A\xi, X)\}\phi Z\\ &+g(\phi Y, Z)\{\eta(X)A\xi-g(A\xi, X)\xi\}\\ &-g(\phi X, Z)\{\eta(Y)A\xi-g(A\xi, Y)\xi\}\\ &-2g(\phi X, Y)\{\eta(Z)A\xi-g(A\xi, Z)\xi\}]\\ &+g(\nabla_\xi A(Y), Z)AX-g(\nabla_\xi A(X), Z)AY\\ &+g(AY, Z)\nabla_\xi A(X)-g(AX, Z)\nabla_\xi A(Y)\\ =0&\end{aligned} \tag{3.1}$$

for any vector fields X, Y and Z.

Let T_0 be a distribution defined by the subspace $T_0(x)=\{u\in T_xM: g(u, \xi(x))=0\}$ of the tangent space T_xM of M at any point x, which is called the *holo-*

morphic distribution. Suppose that the structure vector field ξ is not necessarily principal. Then we can put $A\xi=\alpha\xi+\beta U$, where U is a unit vector field in the holomorphic distribution T_0, and α and β are smooth functions on M. Let M_0 be the non-empty open subset of M consisting of points x at which $\beta(x)\neq 0$. Hereafter unless otherwise stated, we shall discuss on the subset M_0 of M. By the form $A\xi=\alpha\xi+\beta U$, (3.1) is reformed as

$$
\begin{aligned}
(3.2)\qquad & \frac{c}{4}\beta[\{\eta(Y)g(Z, U)-\eta(Z)g(Y, U)\}\phi X-\{\eta(X)g(Z, U)-\eta(Z)g(X, U)\}\phi Y \\
& -2\{\eta(X)g(Y, U)-\eta(Y)g(X, U)\}\phi Z \\
& +g(\phi Y, Z)\{\eta(X)U-g(X, U)\xi\}-g(\phi X, Z)\{\eta(Y)U-g(Y, U)\xi\} \\
& -2g(\phi X, Y)\{\eta(Z)U-g(Z, U)\xi\}] \\
& +g(\nabla_\xi A(Y), Z)AX-g(\nabla_\xi A(X), Z)AY \\
& +g(AY, Z)\nabla_\xi A(X)-g(AX, Z)\nabla_\xi A(Y) \\
& =0
\end{aligned}
$$

for any vector fields X, Y and Z. Putting $Z=\xi$ and taking X and Y in T_0 in the above equation, we get

$$
\begin{aligned}
(3.3)\qquad & \frac{c}{4}\beta(-g(Y, U)\phi X+g(X, U)\phi Y-2g(\phi X, Y)U\} \\
& +g(\nabla_\xi A(\xi), Y)AX-g(\nabla_\xi A(\xi), X)AY \\
& +\beta\{g(Y, U)\nabla_\xi A(X)-g(X, U)\nabla_\xi A(Y)\} \\
& =0\,.
\end{aligned}
$$

Next, putting $Y=Z=\xi$ and taking X in T_0 in (3.2) again, and calculating directly, we have

$$
(3.4)\qquad \alpha\nabla_\xi A(X)=g(\nabla_\xi A(\xi), X)A\xi+\beta g(X, U)\nabla_\xi A(\xi)-d\alpha(\xi)AX\,.
$$

Combining the above two equations, we get

$$
\begin{aligned}
(3.5)\qquad & \frac{c}{4}\alpha\beta\{-g(Y, U)\phi X+g(X, U)\phi Y-2g(\phi X, Y)U\} \\
& +\beta\{g(Y, U)g(\nabla_\xi A(\xi), X)-g(X, U)g(\nabla_\xi A(\xi), Y)\}A\xi \\
& +\{\alpha g(\nabla_\xi A(\xi), Y)-\beta d\alpha(\xi)g(Y, U)\}AX \\
& -\{\alpha g(\nabla_\xi A(\xi), X)-\beta d\alpha(\xi)g(X, U)\}AY \\
& =0
\end{aligned}
$$

for any vector fields X and Y in T_0.

Let $L(\xi, U, \phi U)$ be a distribution defined by the subspace $L_x(\xi, U, \phi U)$ in the tangent space T_xM spanned by the vectors $\xi(x)$, $U(x)$ and $\phi U(x)$ at any point x, and let T_1 be the orthogonal complement in the tangent bundle TM of the distribution $L(\xi, U, \phi U)$. Then T_1 is not empty, because $n \geqq 3$. For any unit vector field X in T_1, putting $Y=\phi X$ in (3.5), we have

$$\frac{c}{2}\beta U = g(\nabla_\xi A(\xi), \phi X)AX - g(\nabla_\xi A(\xi), X)A\phi X \tag{3.6}$$

if we assume that $\alpha \neq 0$. Suppose that there is a unit vector field X_0 in T_1 at which $g(\nabla_\xi A(\xi), X_0)=0$. Then we get by (3.6)

$$\frac{c}{2}\beta U = g(\nabla_\xi A(\xi), \phi X_0)AX_0 \neq 0\,.$$

Accordingly we can put $AX_0=\omega(X_0)U$, where ω is a 1-form on M_0. Putting $X=X_0$ and $Y=U$ in (3.5), we have

$$\frac{c}{4}\alpha\beta\phi X_0 - \omega(X_0)\{\alpha g(\nabla_\xi A(\xi), U) - \beta d\alpha(\xi)\}U = 0\,.$$

Since ϕX_0 and U are orthonormal vector fields, this equation implies $\beta=0$, a contradiction. Accordingly we get

$$g(\nabla_\xi A(\xi), X) \neq 0$$

for any non-zero vector field X in T_1.

On the other hand, putting $Y=\phi U$ in (3.5) again, we have

$$g(\nabla_\xi A(\xi), \phi U)AX - g(\nabla_\xi A(\xi), X)A\phi U = 0 \tag{3.7}$$

for any vector field X in T_1 under the assumption $\alpha \neq 0$. If $g(\nabla_\xi A(\xi), \phi U)=0$, then, by the above equation, $A\phi U=0$. Now, we suppose that $g(\nabla_\xi A(\xi), \phi U) \neq 0$. From (3.7), we get

$$AX = \theta(X)A\phi U\,, \qquad \theta(X) \neq 0 \tag{3.8}$$

for any non-zero vector field X in T_1, where θ is a 1-form on M_0. Putting $X=Y$ in (3.8) and substituting the second one from the first one, we obtain

$$A(\theta(Y)X - \theta(X)Y) = 0, \quad \theta(X) \neq 0, \quad \theta(Y) \neq 0 \tag{3.9}$$

for any non-zero vector fields X and Y in T_1. If we put $Z_1=\theta(Y_1)X_1-\theta(X_1)Y_1$ for given linearly independent vector fields X_1 and Y_1 in T_1, then $AZ_1=0$ by (3.9) and hence $A\phi U=0$ by (3.8).

Next, putting $X=U$ and $Y=\phi U$ in (3.5), we have

$$\frac{3c}{4}\alpha\beta U - g(\nabla_\xi A(\xi),\ \phi U)(\alpha AU - \beta A\xi) = 0\,,$$

where we have used that $A\phi U=0$. Consequently, we get $g(\nabla_\xi A(\xi),\ \phi U)\neq 0$ and hence $AX=0$ for any vector field X in T_1 by (3.7). Lastly, putting $X=\xi$ and taking Y and Z in T_1 in (3.2), we obtain

$$\alpha g(\nabla_\xi A(Y),\ Z)\xi + \beta\left\{\frac{c}{4}g(\phi Y,\ Z) + g(\nabla_\xi A(Y),\ Z)\right\}U = 0\,.$$

Accordingly it turns out to be $\beta=0$ on M_0 under the assumption $\alpha\neq 0$, a contradiction. This means that ξ is principal on M', where M' denotes the open subset of M consisting of points x at which $\alpha(x)\neq 0$. Thus, putting $Y=Z=\xi$ in (3.2), we get $\nabla_\xi A=0$ on M', where we have used that $\nabla_\xi A(\xi)=0$.

Now, let us denote by $\mathrm{Int}\,(M-M')$ the interior of the subset $M-M'$. Then $\alpha=0$ on $\mathrm{Int}\,(M-M')$. Suppose that ξ is not principal on $\mathrm{Int}\,(M-M')$. Then the subset M_1 of $\mathrm{Int}\,(M-M')$ consisting of points x at which $\beta(x)\neq 0$ is non-empty open set. Hence we have by (3.4)

$$g(\nabla_\xi A(\xi),\ X)U + g(X,\ U)\nabla_\xi A(\xi) = 0 \tag{3.10}$$

on M_1 for any vector field X in T_0. Accordingly $g(\nabla_\xi A(\xi),\ Y)=0$ for any vector field Y in T_0 othogonal to U. Since $g(\nabla_\xi A(\xi),\ X)g(X,\ U)=0$ by (3.10), we get $g(\nabla_\xi A(\xi),\ U)=0$ and hence $\nabla_\xi A(\xi)=0$ on M_1. Taking X and Y in T_0 orthogonal to U in (3.3), we obtain $g(\phi X,\ Y)=0$ on M_1, a contradiction. This means that ξ is principal with corresponding principal curvature $\alpha=0$. Accordingly we have $\nabla_\xi A=0$ on $\mathrm{Int}\,(M-M')$ by (2.5). This completes the proof by the continuity of $\nabla_\xi A$. □

REMARK. If $\nabla_\xi A=0$, then ξ is principal and hence it satisfies the condition $\nabla_\xi R=0$ by (3.2).

PROOF OF THEOREMS 1 AND 2. Suppose that $\alpha\neq 0$. Since ξ is principal by Proposition, we have $A\phi-\phi A=0$ by (2.5). It completes the proof of Theorem 1 by Theorems A and B.

Theorem 2 is also verified by a theorem due to Kimura and Maeda [4]. □

§4. Proof of Theorem 3.

In this section, we are concerned with the proof of Theorem 3. Let M be a real hypersurface of $M_n(c)$, $c\neq 0$, $n\geqq 2$. We consider $A\xi=\alpha\xi+\beta U$, where U is a unit vector field in the holomorphic distribution T_0, and α and β are smooth functions on M. And the Lie derivative $\mathcal{L}_\xi R$ of R with respect to ξ is defined by

$$\mathcal{L}_\xi R(X, Y, Z)=\mathcal{L}_\xi(R(X, Y)Z)-R(\mathcal{L}_\xi X, Y)Z-R(X, \mathcal{L}_\xi Y)Z-R(X, Y)\mathcal{L}_\xi Z$$

for any vector fields X, Y and Z. Hence, by the assumption, we have

$$\begin{aligned}(4.1)\quad & \frac{c}{4}\beta[\{\eta(Y)g(Z, U)-\eta(Z)g(Y, U)\}\phi X-\{\eta(X)g(Z, U)-\eta(Z)g(X, U)\}\phi Y \\ &\quad -2\{\eta(X)g(Y, U)-\eta(Y)g(X, U)\}\phi Z \\ &\quad +g(\phi Y, Z)\{\eta(X)U-g(X, U)\xi\}-g(\phi X, Z)\{\eta(Y)U-g(Y, U)\xi\} \\ &\quad -2g(\phi X, Y)\{\eta(Z)U-g(Z, U)\xi\}] \\ &\quad -\frac{c}{4}\{g(\phi Y, Z)\phi(A\phi-\phi A)X-g(\phi X, Z)\phi(A\phi-\phi A)Y \\ &\quad -2g(\phi X, Y)\phi(A\phi-\phi A)Z \\ &\quad +g((A\phi-\phi A)Y, Z)X-g((A\phi-\phi A)X, Z)Y \\ &\quad +g((A\phi^2-\phi^2 A)Y, Z)\phi X-g((A\phi^2-\phi^2 A)X, Z)\phi Y \\ &\quad -2g((A\phi^2-\phi^2 A)X, Y)\phi Z\} \\ &\quad +g(\nabla_\xi A(Y), Z)AX-g(\nabla_\xi A(X), Z)AY \\ &\quad +g(AY, Z)\{\nabla_\xi A(X)+(A\phi-\phi A)AX\} \\ &\quad -g(AX, Z)\{\nabla_\xi A(Y)+(A\phi-\phi A)AY\} \\ & =0\end{aligned}$$

for any vector fields X, Y and Z. Putting $Z=\xi$ and taking X and Y in the holomorphic distribution T_0 in this equation, we have

$$\begin{aligned}(4.2)\quad & \frac{c}{4}\beta\{g(Y, \phi U)X-g(X, \phi U)Y\} \\ &\quad +g(\nabla_\xi A(\xi), Y)AX-g(\nabla_\xi A(\xi), X)AY \\ &\quad +\beta[g(Y, U)\{\nabla_\xi A(X)+(A\phi-\phi A)AX\} \\ &\quad -g(X, U)\{\nabla_\xi A(Y)+(A\phi-\phi A)AY\}] \\ & =0.\end{aligned}$$

Again, putting $Y=Z=\xi$ and taking X in T_0 in (4.1), we get

$$\begin{aligned}(4.3)\quad \alpha\nabla_\xi A(X)= & \beta g(X, U)\nabla_\xi A(\xi)-d\alpha(\xi)AX+g(\nabla_\xi A(\xi), X)A\xi \\ & +\frac{c}{4}\beta g(X, \phi U)\xi+\beta^2 g(X, U)(A\phi-\phi A)U \\ & -\alpha\beta^2 g(X, U)\phi U-\alpha(A\phi-\phi A)AX.\end{aligned}$$

Eliminating $\nabla_\xi A(X)$ and $\nabla_\xi A(Y)$ in (4.2) and (4.3), we obtain

$$\begin{aligned}(4.4)\qquad &\frac{c}{4}\beta[\alpha\{g(Y, \phi U)X-g(X, \phi U)Y\}\\ &\quad+\beta\{g(X, \phi U)g(Y, U)-g(X, U)g(Y, \phi U)\}\xi]\\ &\quad+\alpha\{g(\nabla_\xi A(\xi), Y)AX-g(\nabla_\xi A(\xi), X)AY\}\\ &\quad+\beta[g(Y, U)\{g(\nabla_\xi A(\xi), X)A\xi-d\alpha(\xi)AX\}\\ &\quad-g(X, U)\{g(\nabla_\xi A(\xi), Y)A\xi-d\alpha(\xi)AY\}]\\ &=0\end{aligned}$$

for any vector fields X and Y in T_0.

Now, putting $X=U$ and $Y=\phi U$ in (4.4), we get

$$\begin{aligned}&\frac{c}{4}\beta(\alpha U-\beta\xi)+\alpha\{g(\nabla_\xi A(\xi), \phi U)AU-g(\nabla_\xi A(\xi), U)A\phi U\}\\ &\quad-\beta\{g(\nabla_\xi A(\xi), \phi U)A\xi-d\alpha(\xi)A\phi U\}\\ &=0\,.\end{aligned}$$

Taking the inner product of this equation with ξ, we obtain $\beta=0$. Thus the structure vector field ξ is principal. If $\alpha=0$, then, putting $X=\xi$ in (4.1), we get $A\phi-\phi A=0$. Next, suppose that $\alpha\neq 0$. Then we have

$$(4.5)\qquad \nabla_\xi A(X)+A\phi AX-\phi A^2X=0$$

for any vector field X in T_0 by (4.3), where we have used that $\nabla_\xi A(\xi)=0$. Furthermore, (4.5) holds for any vector field X. This implies that

$$\phi\Big(A^2-\alpha A-\frac{c}{4}I\Big)X=0$$

for any vector field X, where I denotes the identity transformation and we have used (2.4) and (3.3). This is equivalent to

$$A^2-\alpha A-\frac{c}{4}(I-\eta\otimes\xi)=0\,,$$

from which it follows that the shape operator A satisfies

$$(A\phi-\phi A)^2=0\,,$$

where we have used that (2.4) and $A\phi^2=\phi^2A=-A+\alpha\eta\otimes\xi$. Accordingly $A\phi-\phi A=0$, because $A\phi-\phi A$ is symmetric. It completes the proof by Theorems A and B. □

REMARK. If M is of type A, then $A\phi-\phi A=0$ and hence $\nabla_{\xi}A=0$ by (2.5). Accordingly, by (4.1), it satisfies the condition $\mathcal{L}_{\xi}R=0$.

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