Characterizing the effects of disorder in metamaterial structures

Jonah Gollub[a]
Department of Physics, University of California, San Diego, California, 92037, USA

Thomas Hand, Soji Sajuyigbe, Shawn Mendonca, Steve Cummer, and David R. Smith[b]
Department of Electrical and Computer Engineering, Duke University, Durham, North Corolina, 27708, USA

(Received 12 September 2007; accepted 1 October 2007; published online 19 October 2007)

We investigate the effects of disorder on metamaterial samples composed of split ring resonators with randomly introduced variation in their geometrical dimensions. We demonstrate that disorder broadens the negative permeability band and introduces effective losses into the system. Transmission measurements on samples with varying degrees of disorder are found to be in excellent agreement with predictions based on standard homogenization theories. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2801391]

A class of artificial electromagnetic media, called metamaterials, has provided access to electromagnetic properties beyond those of natural materials.[1–3] These materials derive their unique properties from the collective response of many periodic subwavelength resonant structures. Metamaterial designs generally assume that these structures are identical, but in reality, fabrication error will introduce geometrical perturbations and cause some degree of deviation in their resonant properties. As has been shown in several prior studies,[4,5] a variation in the geometrical parameters of resonant metamaterial elements results in a spreading of the oscillator strengths across a finite bandwidth, effectively reducing the strength of the collective resonance, increasing the apparent loss, and introducing other artifacts into the effective constitutive parameters. We demonstrate these effects experimentally in the microwave regime with a magnetic metamaterial consisting of split-ring resonators (SRRs) fabricated with varying amounts of disorder. We also explore a predictive model to determine the effective electromagnetic parameters of these structures from numerical simulations. Numerical calculations furthermore suggest that normal geometrical variations cannot vary more than 5.5% to maintain a negative permeability regime. This is easily satisfied with photolithographic methods in the microwave regime, but fabrication accuracy declines as metamaterial geometry is scaled down, and this becomes a potential limitation for samples being considered at terahertz and optical frequencies.[6]

To investigate the effect of the variation in metamaterials, we intentionally designed structural variations into SRR unit cells resonant at microwave frequencies and distributed these structures randomly throughout the media. Numerical simulations of unit cells, carried out using HFSS in a standard manner (Ansoft, a commercial finite-element solver),[7] demonstrated that varying the width of the outer and inner copper rings (while holding the unit cell size, gap size, and ring trace width constant), as shown in Fig. 1, excited a large range of resonant frequencies. Using these simulations as a guide, the structural parameters of a set of SRR structures were chosen to give an approximately linear distribution of resonant frequencies over 0%, ±4% (with 9 distinct cells), ±14% (with 21 distinct cells), ±19% (with 21 distinct cells), and ±25% (with 21 distinct cells) with a mean value of 12.5 GHz. The SRRs were designed so that their resonance occurred over a band of microwave frequencies consistent with our measurement equipment.

To determine effective constituent parameters for the fabricated samples, it was necessary to apply a mixing model. A common model is Maxwell-Garnett,[8] in which a sufficiently dilute material is assumed such that a hierarchy of macroscopic and microscopic fields can be defined. However, the underlying assumption of diluteness becomes suspect when investigating metamaterials which are composed of closely packed and strongly resonant structures. An alternative model, which we apply here, is the Bruggeman mixing model.[8,9] The Bruggeman model applies to mixtures of two or more media and makes no distinction between species, such as host or inclusion. The Bruggeman model is thus conceptually closer to our experimental configuration. To determine the effective permeability of the metamaterial structure, we assign each unit cell its associated bulk metamaterial permeability. These "macroscopic" parameters are straightforwardly obtained with present numerical methods.[7]

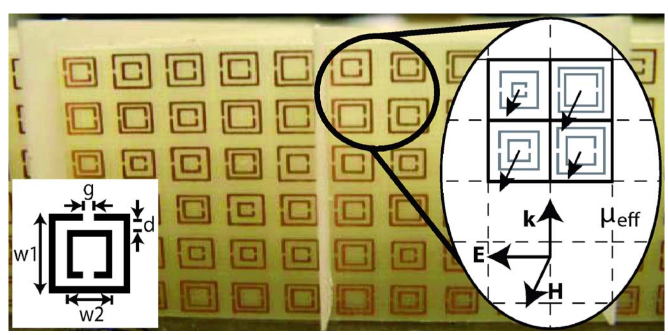

FIG. 1. (Color online) The characteristic dimensions of the SRR structure are shown. The parameters $w1$ and $w2$ were varied in the samples to induce various amounts of disorder as pictured. The SRR used in the 0% SRR structure had dimensions $g=0.22$ mm, $c=0.08$ mm, $w1=2.25$ mm, $w2=0.45$ mm, and a unit cell size of 3.33 mm. Also shown is an illustration of the Bruggeman model for our structure in which the magnetic dipole moment of SRR unit cells are determined with respect to an average background permeabilty.

[a]Electronic mail: jng6@duke.edu
[b]Also with Department of Physics, University of California, San Diego, CA 92037.

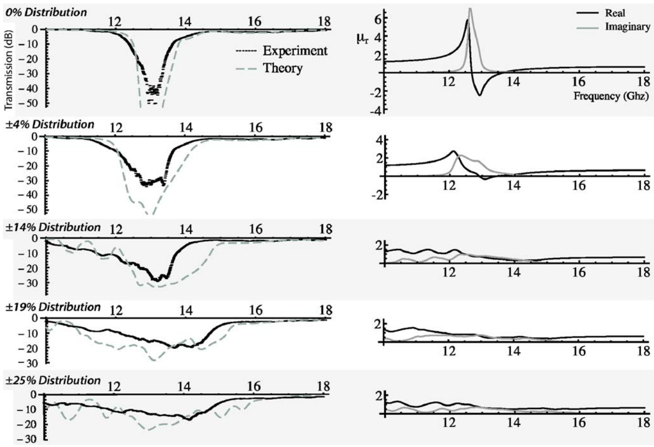

FIG. 2. (Color online) The experimental data shown for transmission through disordered metamaterials of various linear distributions. Theoretical transmission for the sample is calculated using the Bruggeman mixing model to determine the material's effective permeability (and permittivity—not shown).

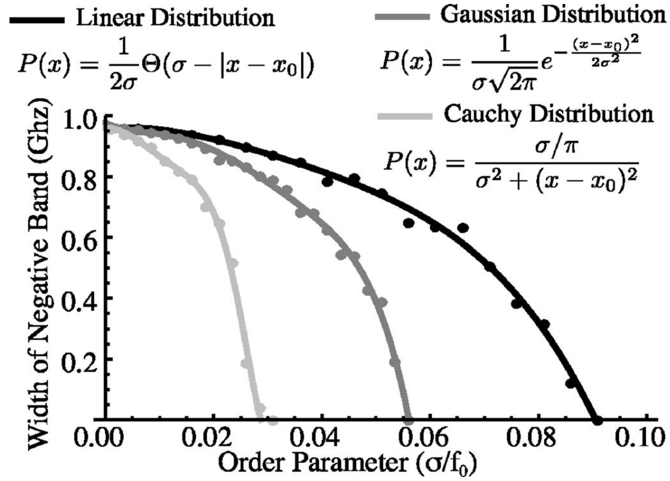

FIG. 3. For a characteristic SRR structure in our experiment (f_0=12.5 GHz, Q=78, F=0.14), a numerical calculation of the width of negative region as a function of disorder introduced is shown for linear, Gaussian, and Cauchy distributions.

The magnetic dipole moment of each unit cell is then found with respect to a yet to be determined homogenous "background" effective permeability, as illustrated in Fig. 1. The Bruggeman model assumes spheroid constituents, but in the quasistatic limit, where the unit cell is much smaller than the operating wavelength (as we are interested in here), the polarization of a cube is equivalent to that of a spheroid.[10] By self-consistency, the dipole moments (with respect to the background) must sum to 0 and the material's effective permeability can be determined. The general Bruggeman equation for the permeability and similarly the permittivity are then

$$\sum_{i=1}^{n} f_i \frac{\mu_i - \mu_{\text{eff}}}{\mu_i + 2\mu_{\text{eff}}} = 0,$$

$$\sum_{i=1}^{n} f_i \frac{\epsilon_i - \epsilon_{\text{eff}}}{\epsilon_i + 2\epsilon_{\text{eff}}} = 0, \qquad (1)$$

where μ_i and ϵ_i are the parameters of the constituent materials and f_i is the filling fraction of each element such that $\Sigma_i^n f_i = 1$.

To apply the Bruggeman model, it is necessary to extract the effective bulk parameters of each distinct SRR structure. The commercially available electromagnetic simulator Ansoft HFSS was used to determine the effective permeability and permittivity[7] for each unique SRR structure. These "homogenous" material parameters were applied to Eq. (1) and a numerical root finding algorithm was implemented in MATHEMATICA to deduce the effective Bruggeman material parameters μ_{eff} and ϵ_{eff}. The retrieved μ_{eff} and ϵ_{eff} were then used in the Fresnel equations[7] to determine the theoretical transmission coefficient to which measured transmission was compared.

The SRR metamaterial structures were fabricated on FR4 sheets (250 μm thick with 17 μm copper thickness) using a standard optical lithographic method. A mask was designed consisting of strips of 3.33 mm SRR unit cells extending six cells wide and 45 cells long. MATHEMATICA was used to generate the layout and to randomize the positions of the SRR unit cells on the mask. Upon completion of the photolithographic process, the strips of SRRs were cut out to dimensions 2 cm in width (6 unit cells) and 15 cm in height (45 unit cells). Supporting cross strips, constructed from bare FR4 substrate material, were used to assemble the strips into three-dimensional slabs, as shown in Fig. 1. In total five samples were constructed with various amounts of disorder in the SRR structures.

A two horn antenna system was used to measure the transmission through each disordered metamaterial sample. Rexolite planoconvex lenses were placed in front of each antenna so that the exiting beam from the first horn was collimated and focused at 30 cm to a 7 cm diameter beam. The horn antennas were connected to a network analyzer (Agilent N5230A PNA-L) and the transmission between both horns was calibrated. The random media samples were placed at the focal point of the lens and measured. The significant size of the sample (15×15 cm^2) ensured that the focused 7 cm microwave beam was fully incident on the sample. The focused beam illuminated approximately 1700 unit cells along its path through the sample, providing a well distributed measurement.

In Fig. 2 the results of the transmission experiment for the disordered metamaterials samples are plotted versus the theoretical transmission calculated using the Bruggeman effective permeability and permittivity parameters. Good correlation is seen for each of the disordered samples considered. As the distribution in the samples is increased, the stop band is widened and the response decreased, consistent with effective additional loss in the permeability of the material. The numerically determined Bruggeman effective permeability is also shown for each sample and predicts that the negative region should only exist for 0% and ±4% distributions. It is useful to note that a metamaterial can have a strong stop band without its parameters necessarily taking on negative values—simply due to impedance mismatch of its positive parameters with free space.

We further performed a numerical investigation to determine the effect of disorder on the negative permeability band of our SRR structures for several distributions. For simplification, we assumed a Lorentz form, $\mu_i = 1 - F\omega^2/[\omega_{0j}^2 - \omega^2 + i(\omega\omega_0/Q)]$, for our constituent material permeabilities. The general parameters, F=0.14 and Q=78, were determined by fitting the Lorentz function to the numerically extracted permeability of our uniform SRR metamaterial with geometry listed in Fig. 1 (ω_0=12.5 GHz). A 500 element material was considered with linear (as in the experiment), Gaussian, and Cauchy distributions in the resonant frequencies ω_0 and the Bruggeman formula was solved with a numerical root finding method. The width of the negative band of the extracted permeability is shown in Fig. 3 as a function of the order parameter σ/f_0, where σ is a parameter of the probability functions, as defined in the figure. For the linear case, the numerical results are consistent with the extracted Bruggeman parameters for our samples. That is, the negative band does not exist for variation above ±9%. One might also consider the Gaussian distribution that one would expect from normal fabrication error of a metamaterial. As is shown, the standard deviation cannot be greater than ±5.5%. This cannot be translated into an exact constraint on geometrical variation since it depends on the specific parameter that is varied, but it can be noted that simulations show that geometrical variation translates into a slightly greater resonant frequency variation, so 5.5% can be used as conservative estimate on the allowable standard deviation of our geometrical structure in order to maintain a negative band. This is slightly larger than the theoretical predictions by Shadrivov *et al.*[4] who used a Cauchy distribution in their theory (we plot the Bruggeman parameters for a Cauchy distribution here as a comparison).

In summary, transmission through metamaterials composed of disordered SRR structures were measured and their spectra matched very closely with that predicted by a Bruggeman mixing model. Numerical modeling also suggest that tolerances for maintaining a negative band are about $\sigma=\pm 6\%$ for normal fabrication. This study of disorder also has implications for nongeometrically induced variability that might be found in tunable[11] and active[12] metamaterials due to integrated electrical components.

This research was supported by U.S. Army Research Office DOA under Grant No. W911NF-04-1-0247. We also acknowledge support from the Air Force Office of Scientific Research through a Multiply University Research Initiative under Contract No. FA9550-06-1-0279.

[1] J. Pendry, IEEE Trans. Microwave Theory Tech. **47**, 2075 (1999).
[2] D. Smith, Phys. Rev. Lett. **85**, 2933 (2000).
[3] H. Chen, L. Ran, D. Wang, J. Huangfu, Q. Jiang, and J. A. Kong, Appl. Phys. Lett. **88**, 031908 (2006).
[4] I. Shadrivov, Appl. Phys. Lett. **90**, 201919 (2007).
[5] M. Gorkunov, Phys. Rev. E **73**, 056605 (2006).
[6] T. J. Yen, W. J. Padilla, N. Fang, D. C. Vier, D. R. Smith, J. B. Pendry, D. N. Basov, and X. Zhang, Science **303**, 1494 (2006).
[7] D. Smith, Phys. Rev. B **65**, 195104 (2002).
[8] A. Sihvola, *Electromagnetic Mixing Formulas and Applications* (IEE, London 1999).
[9] R. Landauer, J. Appl. Phys. **23**, 779 (1952).
[10] A. D. Yaghjian, Proc. IEEE **68**, 248 (1980).
[11] Shadrivov, Opt. Express **14**, 9344 (2006).
[12] B. Popa, Microwave Opt. Technol. Lett. **49**, 2574 (2007).