# Charge sensitivity of radio frequency single-electron transistor

Alexander N. Korotkov[a)]
*Department of Physics, State University of New York, Stony Brook, New York 11794-3800*

Mikko A. Paalanen
*Low Temperature Laboratory, Helsinki University of Technology, FIN-02015 HUT, Finland*

(Received 23 November 1998; accepted for publication 4 May 1999)

A theoretical analysis of the charge sensitivity of the radio frequency single-electron transistor (rf-SET) is presented. We use the ''orthodox'' approach and consider the case when the carrier frequency is much less than $I/e$ where $I$ is the typical current through rf-SET. The optimized noise-limited sensitivity is determined by the temperature $T$, and at low $T$ it is only 1.4 times worse than the sensitivity of conventional single-electron transistor.  [S0003-6951(99)03026-0]

Single-electron devices[1] are gradually becoming useful in real applications.[2] Despite the wide variety of studied circuits, the single-electron transistor (SET)[1,3,4] remains the most important device in applied single electronics (in this letter we will discuss the new version[5] of the SET setup). At present the best reported charge sensitivity of the SET at 10 Hz is[6] $2.5\times10^{-5}\,e/\sqrt{\text{Hz}}$ (the previous record figure was[7] $7\times10^{-5}\,e/\sqrt{\text{Hz}}$). The low-frequency sensitivity of the SET is limited by 1/f noise, so it improves as the frequency increases. The best achieved figure so far[8] of $\sim10^{-5}e/\sqrt{\text{Hz}}$ was measured at 4.4 kHz. This is still an order of magnitude worse than the limit determined by the thermal/shot noise of the SET.[9–14]

The difficulty of further frequency increase is due to the relatively large output resistance $R_d$ of the SET. For the typical figure $R_d\sim10^5\ \Omega$ and wiring capacitance $C_L\sim10^{-9}$ F the corresponding $R_dC_L$ time limits the bandwidth by a few kHz (the use of filters can make it even lower). The importance of potential high-frequency applications makes urgent a significant increase of the bandwidth. This can be done in several ways.

The output resistance can be reduced in superconducting (Bloch) SET based on supercurrent modulation[1,15,16] (the use of the quasiparticle tunneling threshold does not help much because $R_d$ is limited by the quantum resistance even at the threshold[13,17]). The load capacitance $C_L$ can be decreased placing the next amplifier close to the SET.[18,19] However, while bandwidth up to 700 kHz was demonstrated[18] using this idea, the charge sensitivity was relatively poor because of extra heating and extra noise produced by the preamplifier. Finally, a bandwidth over 100 MHz has recently been demonstrated[5] in the so-called radio frequency (rf) SET in which the SET controlled the dissipation of the tank circuit which in turn affected the reflection of the carrier wave with frequency $\omega/2\pi=1.7$ GHz. A sensitivity of $1.2\times10^{-5}\,e/\sqrt{\text{Hz}}$ has been achieved[5] at 1.1 MHz. The theoretical analysis of the ultimate sensitivity of the rf-SET is the subject of the present letter.

In principle a wide bandwidth could be achieved simply by illuminating the SET with microwaves and measuring the wave reflection. The gate voltage would change the SET differential resistance $R_d$ and thus affect the reflection coefficient $\alpha=(Z-R_0)/(Z+R_0)$, where $Z^{-1}=i\omega C_s+R_d^{-1}$, $R_0\simeq50\ \Omega$ is the cable wave resistance, and $C_s$ is the stray capacitance. However, because of the large ratio $R_d/R_0\sim10^3$, the signal would be extremely small. To estimate the signal power $P\simeq A^2R_0/2R_d^2[1+(\omega C_sR_0)^2]$, let us use $R_d=10^5\ \Omega$ and the amplitude of the SET bias voltage oscillation $A=1$ mV ($A$ is limited by the Coulomb blockade threshold); then $P\sim10^{-15}$ W. This figure corresponds to the noise power of the amplifier with noise temperature of 10 K within $10^7$ Hz bandwidth and clearly makes such an experiment quite difficult.

To increase the signal, the authors of Ref. 5 inserted the SET into the tank circuit (see Fig. 1). Then at resonant frequency $\omega=(LC_s)^{-1/2}$ the circuit impedance is small, $Z\simeq L/C_sR_d\ll R_0$ (we assume $Q_{\text{SET}}\gg Q\gg1$ where $Q_{\text{SET}}=R_d/\sqrt{L/C_s}$ and $Q=\sqrt{L/C_s}/R_0$), so $\alpha\simeq-1+2L/C_sR_dR_0$. The signal power $P=[V_{\text{in}}(\alpha+1)]^2/2R_0$ ($V_{\text{in}}$ is the amplitude of the incoming wave) can be expressed via the SET bias amplitude $A\simeq2QV_{\text{in}}$ as $P=Q^2A^2R_0/2R_d^2$, indicating $Q^2$ gain in comparison with the nonresonant case.[5]

The linear analysis above can be used only as an estimate because of the considerable nonlinearity of the SET current–voltage ($I-V$) curve. For a more exact analysis let us write the differential equation (see Fig. 1) for the voltage $v(t)$ at the end of the cable (the static component $V_0$ is subtracted):

$$\ddot{v}LC_s+\dot{v}R_0C_s+v=2(1-\omega^2LC_s)V_{\text{in}}\cos\omega t-R_0I(t),$$

where $V_{\text{in}}\cos\omega t$ is the incoming wave at the end of cable and $I(t)$ is the current through the SET while the SET bias

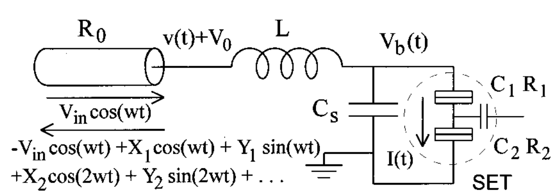


FIG. 1. The schematic of the rf-SET.

a)Electronic mail: korotkov@rsfq1.physics.sunysb.edu

 

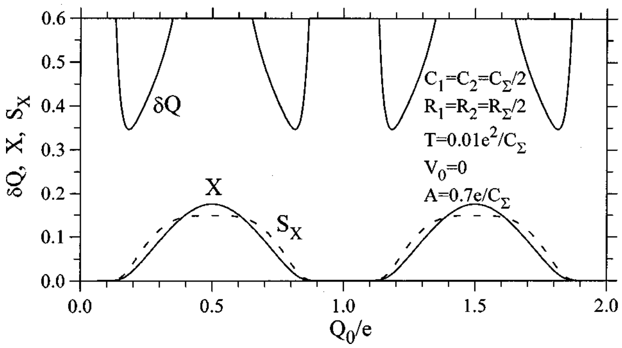


FIG. 2. The reflected wave amplitude $X_1$ [in units $2(L/C_s)^{1/2}e/R_\Sigma C_\Sigma$], its noise $S_{X1}$ (in units $4L/C_s e^2/R_\Sigma C_\Sigma$), and minimal detectable charge $\delta Q$ [in units $e(R_\Sigma C_\Sigma \Delta f)^{1/2}$] as functions of the background charge $Q_0$ for the symmetric SET at $T=0.01e^2/C_\Sigma$ for zero dc bias voltage $V_0$ and its rf amplitude $A=0.7e/C_\Sigma$.

voltage is $V_b(t)=V_0+v+(2V_{\rm in}\omega\sin\omega t+\dot{v})L/R_0$. The reflected wave can be written (at the end of cable) as $v(t)-V_{\rm in}\cos\omega t=-V_{\rm in}\cos\omega t+X_1\cos\omega t+Y_1\sin\omega t+X_2\cos 2\omega t+Y_2\sin 2\omega t+\dots$, where the coefficients $X_k$ and $Y_k$ should be calculated self-consistently [an obvious way is the iterative updating of $V_b(t)$ and $X_k, Y_k$]. While the analysis of the higher harmonics is important for the possible versions of rf-SET in which the signal is measured at the double (or triple) frequency, we will limit ourselves by the reflected wave at the basic harmonic. For simplicity we assume exact resonance, $\omega=(LC_s)^{-1/2}$, then

$$X_1=2\sqrt{L/C_s}\,\langle I(t)\sin\omega t\rangle,$$
$$Y_1=2\sqrt{L/C_s}\,\langle I(t)\cos\omega t\rangle, \tag{1}$$

where $\langle\;\rangle$ denotes averaging over time. In the first approximation (if $Q_{\rm SET}\gg Q\gg 1$) the SET bias voltage is $V_b(t)=V_0+A\sin\omega t$ where $A=2QV_{\rm in}$.

The coefficients $X_1$ and $Y_1$ (we omit index 1 below) can be measured separately using homodyne detection and both can carry information about the low frequency signal applied to the SET gate (as usual, [1] we will describe it in terms of the background charge $Q_0$ induced into the SET island). If the amplifier noise and other fluctuations are negligible, then the sensitivity of the rf-SET is determined by the intrinsic noise of the SET. The minimal detectable charge $\delta Q$ can be expressed as

$$\delta Q_X=\sqrt{S_X(f_s)\Delta f}/(dX/dQ_0),$$
$$\delta Q_Y=\sqrt{S_Y(f_s)\Delta f}/(dY/dQ_0), \tag{2}$$

while the simultaneous measurement of $X$ and $Y$ can give $\delta Q=[(1-K^2)/(\delta Q_X^{-2}+\delta Q_Y^{-2}-2K/\delta Q_X\delta Q_Y)]^{1/2}$, where $K=({\rm Re}\,S_{XY}/\sqrt{S_X S_Y})\,{\rm sign}[(dX/dQ_0)(dY/dQ_0)]$ is the correlation between two noises. Here $S_X(f_s)$ is the spectral density of $X(t)$ fluctuations at signal frequency $f_s$ (which should be within the tank circuit bandwidth, $2\pi f_s\lesssim\omega/Q$), $S_{XY}$ is the mutual spectral density, and $\Delta f$ is the measurement bandwidth (inverse ''accumulation'' time).

In this letter we consider only the case of sufficiently low carrier frequency $\omega\ll I/e$ (where $I$ is the typical current through the SET), so that the quasistationary state is reached

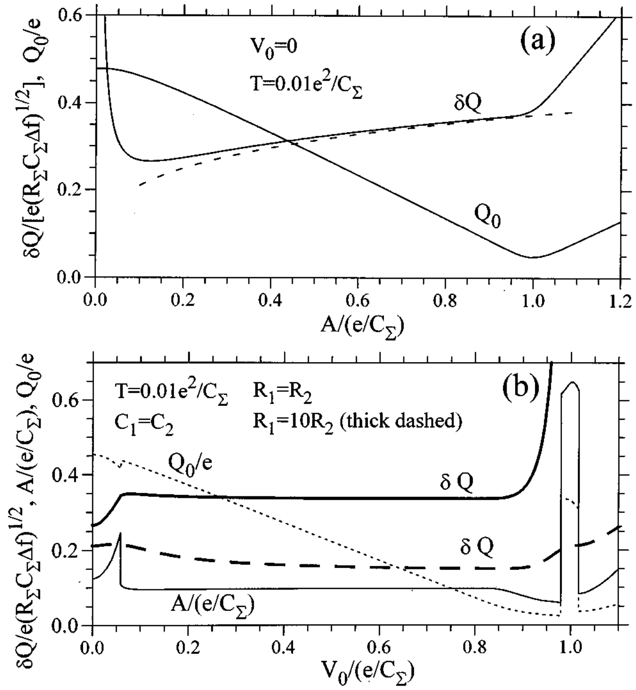


FIG. 3. (a) The sensitivity $\delta Q$ optimized over $Q_0$ and corresponding $Q_0$ as functions of rf amplitude $A$ for the symmetric SET. Dashed line shows the analytical result (see the text). (b) Dependence of $\delta Q$ minimized over $A$ and $Q_0$ and of the optimal operation point $(A,Q_0)$ on the dc bias voltage $V_0$. Dashed line is for the asymmetric SET ($R_2/R_1=10$).

at any moment during the period of oscillations. In this case the spectral density does not depend on $f_s$ (which is even lower than $\omega$) and

$$S_X=4(L/C_s)\,\langle S_I(t)\sin^2\omega t\rangle, \tag{3}$$

where $S_I(t)$ is the low frequency spectral density of the thermal/shot noise of the current through the SET, which has the time dependence because of oscillating bias voltage $V_b$. There is no need to consider $Y$ output in this case because $Y=0$ (so $\delta Q_Y=\infty$) and the noise correlation is absent, $K=0$ (nonzero $Y$ and $K$ would appear at higher $\omega$ due to delay of tunneling events).

We use the ''orthodox'' theory[1,3] for a normal SET consisting of two tunnel junctions with capacitances $C_1$ and $C_2$ and resistances $R_1$ and $R_2$ (see Fig. 1) assuming $R_j\gg R_Q=\pi\hbar/2e^2$ (as usual, the gate capacitance is distributed between $C_1$ and $C_2$ in a proper way). The effects of finite photon energy $\hbar\omega$ are neglected. We also neglect the possible rf modulation of the SET gate voltage. The low frequency thermal/shot noise of the SET current is calculated in the standard way.[9,10]

Figure 2 shows the dependence of $X, S_X$, and $\delta Q=\delta Q_X$ on the background charge $Q_0$ for a symmetric SET ($C_1=C_2, R_1=R_2$) at $T=0.01e^2/C_\Sigma$ ($C_\Sigma=C_1+C_2$), $V_0=0$, and $A=0.7e/C_\Sigma$. One can see that the minimum of $\delta Q$ is achieved near the edge of $Q_0$ range corresponding to nonzero $X$, so that the amplitude $A$ is only a little larger than the Coulomb blockade threshold $V_t$. For $V_b$ close to $V_t$ the noise of the current through the SET obeys Schottky formula, $S_I=2eI$, with a good accuracy at low temperatures,[9,10] while the current $I$ can be approximated as $I=W/eR_j[1-\exp(-W/T)]$ where $W=e(V_b-V_t)(C_1C_2/C_jC_\Sigma)=$

$(-1)^j e(Q_0 - Q_{0,t})/C_\Sigma$ ($j$th junction determines the threshold) and $|dI/dQ_0| = (dI/dV_b)C_j/C_1C_2$. (As a consequence of the Schottky formula, the dashed curve in Fig. 2 is approximately twice as high as the $X$-curve at small $X$.)

Using these equations and optimizing $Q_0$, one can find the minimum $\delta Q \simeq 1.2\, e\, (R_\Sigma C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2} \times (eA/T)^{1/4}$ for the symmetric SET at $T \ll eA < e^2/C_\Sigma$ ($R_\Sigma = R_1 + R_2$). This dependence as a function of rf amplitude $A$ is shown in Fig. 3(a) by the dashed line while the numerical result is shown by the solid line. The sensitivity gets worse ($\delta Q$ increases) at $A > e/C_\Sigma$ because of $X$ and $S_X$ increase. The sensitivity also worsens rapidly when $A$ is too small and becomes comparable to $T/e$, because of the contribution from the Nyquist noise of the SET at $V_b$ close to zero. Before optimizing the amplitude $A$, let us notice that the results shown in Fig. 3(a) correspond to relatively small $X$ that can be difficult to measure experimentally [in the approximation above $X \simeq 2\,(L/C_s)^{1/2} \times 15\,(T/eR_\Sigma)(T/eA)^{1/2}$]. However, as seen from Fig. 2, $X$ can be significantly increased for the price of a few ten per cent increase of $\delta Q$.

Figure 3(b) shows $\delta Q$ minimized over both $A$ and $Q_0$ and the corresponding optimum values of $A$ and $Q_0$ as functions of the direct current (dc) bias voltage $V_0$. One can see that for a symmetric SET the best sensitivity is achieved at $V_0 = 0$ and there is a long plateau of $\delta Q$ which ends when $V_0$ approaches $e/C_\Sigma$ leading to significant worsening of the sensitivity. For the asymmetric SET (dashed line) the best sensitivity can be achieved in the plateau range. At the plateau $\delta Q$ can be calculated analytically using the approximations above, $\delta Q \simeq 3.34\, e\, (2R_{\min} C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2}$ where $R_{\min} = \min(R_1, R_2)$. This expression can be compared with the optimized low-temperature sensitivity of the conventional SET which is given[9,10] by the same formula with the numerical factor 1.90 instead of 3.34. For the symmetric rf-SET the optimized low-temperature sensitivity (at $V_0 = 0$) is

$$\delta Q \simeq 2.65 e (R_\Sigma C_\Sigma \Delta f)^{1/2} (TC_\Sigma/e^2)^{1/2}, \tag{4}$$

only 1.4 times worse than for the conventional SET.

Figure 4 shows numerically minimized $\delta Q$ for the symmetric SET and corresponding optimal $A$ and $Q_0$ (while $V_0 = 0$) as functions of temperature. The result of Eq. (4) is shown by the dashed line. The sensitivity scales as $T^{1/2}$ at low temperatures while it significantly worsens at $T > 0.1 e^2/C_\Sigma$, similar to the result for the conventional SET (dotted line). The ''orthodox'' sensitivity improves with the decrease of tunnel resistances while the optimum value (which should be comparable to $R_Q$) could be calculated if cotunneling[1] was taken into account.

To make a comparison with experiment,[5] let us take $C_\Sigma = 0.45$ fF, $R_\Sigma = 97$ k$\Omega$, and $T = 100$ mK, then after optimization $\delta Q \simeq 2.7 \times 10^{-6}\, e/\sqrt{\mathrm{Hz}}$ in the normal case (necessity of relatively large $X$ would lead to a factor about 1.5). So, there is still an order of magnitude for possible experimental improvement. Comparison for the superconducting case is not straightforward because the sensitivity depends on the junction quality.[13]

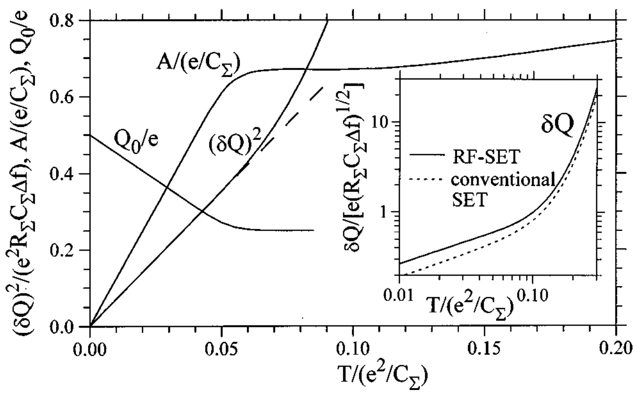


FIG. 4. The optimized $\delta Q$ (squared) and corresponding $A$ and $Q_0$ as functions of the temperature $T$ for the symmetric SET at $V_0 = 0$. Dashed line represents Eq. (4). Inset shows $\delta Q$ on the larger scale. For comparison, the result for conventional SET is shown by the dotted line.

In conclusion, we have shown that the price for the wide bandwidth of the rf-SET is only a little decrease of the noise-limited sensitivity in comparison with conventional SET.

The authors thank K. K. Likharev and H. Seppä for valuable discussions. The work was supported in part by US AFOSR, Russian Fund for Basic Research, and Finnish Academy of Sciences and Letters.


[1] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991), p. 173.

[2] A. N. Korotkov, in *Molecular Electronics*, edited by J. Jortner and M. Ratner (Blackwell, Oxford, 1997), p. 157.

[3] K. K. Likharev, IEEE Trans. Magn. **23**, 1142 (1987).

[4] T. A. Fulton and G. C. Dolan, Phys. Rev. Lett. **59**, 109 (1987).

[5] R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov, P. Delsing, and D. E. Prober, Science **280**, 1238 (1998).

[6] V. A. Krupenin, D. E. Presnov, M. N. Savvateev, H. Scherer, A. B. Zorin, and J. Niemeyer, J. Appl. Phys. **84**, 3212 (1998).

[7] E. N. Visscher, S. M. Verbrugh, J. Lindeman, P. Hadley, and J. E. Mooij, Appl. Phys. Lett. **66**, 305 (1995).

[8] B. Starmark, T. Henning, T. Claeson, P. Delsing, and A. N. Korotkov, J. Appl. Phys. (in press).

[9] A. N. Korotkov, D. V. Averin, K. K. Likharev, and S. A. Vasenko, in *Single-Electron Tunneling and Mesoscopic Devices*, edited by H. Koch and H. Lubbig (Springer, Berlin 1992), p. 45.

[10] A. N. Korotkov, Ph.D. thesis, Moscow State University (1991); Phys. Rev. B **49**, 10381 (1994).

[11] S. Hershfield, J. H. Davies, P. Hyldgaard, C. J. Stanton, and J. W. Wilkins, Phys. Rev. B **47**, 1967 (1993).

[12] U. Hanke, Yu. M. Galperin, K. A. Chao, and N. Zou, Phys. Rev. B **48**, 17209 (1993).

[13] A. N. Korotkov, Appl. Phys. Lett. **69**, 2593 (1996).

[14] A. N. Korotkov, Europhys. Lett. **43**, 343 (1998).

[15] A. B. Zorin, Phys. Rev. Lett. **76**, 4408 (1996).

[16] D. J. Flees, S. Han, and J. E. Lukens, Phys. Rev. Lett. **78**, 4817 (1997).

[17] D. V. Averin, A. N. Korotkov, A. J. Manninen, and J. P. Pekola, Phys. Rev. Lett. **78**, 4821 (1997).

[18] J. Pettersson, P. Wahlgren, P. Delsing, D. B. Haviland, T. Claeson, N. Rorsman, and H. Zirath, Phys. Rev. B **53**, 13272 (1996).

[19] E. H. Visscher, J. Lindeman, S. M. Verbrugh, P. Hadley, J. E. Mooij, and W. van der Vleuten, Appl. Phys. Lett. **68**, 2014 (1996).