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Charged current cross section for massive cosmological neutrinos impinging on radioactive nuclei

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Abstract

We discuss the cross section formula for both massless and massive neutrinos on stable and radioactive nuclei. The latter could be of interest for the detection of cosmological neutrinos whose observation is one of the main challenges of modern cosmology. We analyse the signal-to-background ratio as a function of the ratio m_ν/Δ , i.e. the neutrino mass over the detector resolution and show that an energy resolution $\Delta \leq 0.5$ eV would be required for sub-eV neutrino masses, independently of the gravitational neutrino clustering. Finally, we mention the non-resonant character of neutrino capture on radioactive nuclei.

1. Introduction

Modern big-bang cosmology firmly predicts the existence of a relic neutrino background, and relates its temperature to the temperature of the background microwave radiation

$$T_\nu/T_\gamma = (4/11)^{1/3}, \quad (1)$$

see, e.g., the basic texts [1, 2]³. Verifying the existence of the relic neutrino sea represents one of the main challenges of modern cosmology.

Clearly, in contrast to the study of the background microwave radiation that has a long history and has reached an unprecedented accuracy (see, e.g., the latest results in [4]), detection of relic neutrinos remains an unfulfilled dream. Various strategies have been proposed so far, based on laboratories searches [5–11] and astrophysical observations [12–16], such as absorption dips in the flux of ultra high energy neutrinos (for a review see, e.g., [17]). As far as their detection in laboratory experiments is concerned, one needs to overcome two main obstacles: the low cross section characteristic of weak interactions and the low energy of relic neutrinos. The second obstacle can be overcome if the corresponding detection reactions

³ Corrections to the above ratio caused by the incomplete neutrino decoupling are only at a few per cent level [3].

have vanishing thresholds. Therefore, we discuss here the possibility of detecting the relic neutrinos by the charged current reactions using radioactive unstable nuclei as targets.

The paper is organized as follows. In the following section, we derive expressions for the charged current neutrino induced reaction cross sections involving nonrelativistic neutrinos. We show that such cross sections, when the corresponding reaction has a vanishing threshold, scale with c/v_ν so that the number of events converges to a constant for $v_\nu \rightarrow 0$. In the following section, we discuss the possibility to use a tritium target to detect the cosmological ν_e . We show that the main challenge is the separation of the produced electrons with energies just above the endpoint of the β spectrum from the overwhelming flux of the electrons from the tritium β decay that extends just below the 18.6 keV endpoint. We also discuss the possibility of a resonance enhancement of reactions involving cosmological neutrinos. We show that the charged current reactions, included the radiative ones, do not have a resonance character. In the conclusion, we summarize our findings and stress the need for an extreme energy resolution if sensitivity to detect sub-eV mass relic neutrinos should be reached.

2. Cross sections for massive neutrinos

Let us first recapitulate briefly the cross section for massless neutrinos. We use the reaction

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad (2)$$

as an example. This can be easily modified for reactions without threshold such as

$$\nu_e + n \rightarrow e^- + p. \quad (3)$$

Since we are interested in very low energy neutrinos, we can treat nucleons nonrelativistically, and keep only the lowest order terms in E_ν/M and E_e/M . The standard expression is then ($\hbar = c = 1$, see, e.g., [18])

$$\frac{d\sigma}{dq^2} = \frac{G_F^2 \cos^2 \theta_C}{\pi} \frac{|\mathcal{M}|^2}{(s - M_p^2)^2}, \quad (4)$$

where q^2 is the momentum transferred squared and s is the square of the centre-of-mass (CM) energy.

Starting with the usual current \times current weak interaction

$$[\bar{u}_n(\gamma_\mu f - \gamma_\mu \gamma_5 g)u_p][\bar{\nu}_e \gamma^\mu (1 - \gamma_5)v_e], \quad (5)$$

where f, g are the vector and axial-vector form factors, respectively, we arrive at the squared matrix element (see, e.g., [19])

$$|\mathcal{M}|^2 = (f + g)^2(p_p \cdot p_e)(p_n \cdot p_\nu) + (f - g)^2(p_n \cdot p_e)(p_p \cdot p_\nu) + (g^2 - f^2)M_n M_p(p_e \cdot p_\nu). \quad (6)$$

Evaluating it in the laboratory frame where the proton is at rest, and keeping only the leading terms one gets

$$|\mathcal{M}|^2 = M_n M_p E_\nu E_e [(f^2 + 3g^2) + (f^2 - g^2)v_e v_\nu \cos \theta], \quad (7)$$

with M_n, M_p the neutron and proton masses. Furthermore, using $s = (p_\nu + p_p)^2 = M_p^2 + 2M_p E_\nu$ in the laboratory frame, and using the Jacobian

$$\frac{dq^2}{d \cos \theta} = 2E_\nu p_e, \quad (8)$$

we obtain the usual lowest order expression [18]

$$\frac{d\sigma}{d \cos \theta} = \bar{G}^2 E_e p_e [(f^2 + 3g^2) + (f^2 - g^2)v_e v_\nu \cos \theta], \quad (9)$$

with $\bar{G} = G_F \cos \theta_C / \sqrt{2\pi}$.

Let us now consider the case of massive neutrinos. In equation (4) one should then substitute [20]

$$(s - M_p^2)^2 \rightarrow [s - (M_p + m_\nu)^2][s - (M_p - m_\nu)^2] = 4M_p^2 p_\nu^2, \quad (10)$$

where the last expression is again in the laboratory frame. The Jacobian in equation (8) becomes instead

$$\frac{dq^2}{d \cos \theta} = 2p_\nu p_e. \quad (11)$$

The cross section is then given by (using $M_n \sim M_p$ as in the equation (8))

$$\frac{d\sigma}{d \cos \theta} = \frac{\bar{G}^2}{v_\nu} E_e p_e [(f^2 + 3g^2) + (f^2 - g^2)v_e v_\nu \cos \theta]. \quad (12)$$

presenting a $1/v_\nu$ dependence⁴. Such a behaviour is in agreement with the general form for the cross section associated with exothermic reactions of nonrelativistic particles [22]. We see that this form is a ‘natural one’ that in most cases of practical importance, when $v_\nu \rightarrow c$, acquires the standard form equation (9).

The interaction cross section of very low energy neutrinos, equation (12) was implicitly used long time ago by Weinberg [23]. Very recently, this process has attracted particular interest thanks to the work by Cocco *et al* [24], where the authors have considered the possibility of detecting cosmological neutrinos through their capture on radioactive beta decaying (and hence with no threshold) nuclei.

In the case of nuclear (stable or unstable) targets, i.e. reactions

$$\nu_e + A_Z \rightarrow e^- + A_{Z+1}^* \quad \text{or} \quad \bar{\nu}_e + A_Z \rightarrow e^+ + A_{Z-1}^* \quad (13)$$

when possible, one would use the usually known ft value of the inverse radioactive decay to eliminate the fundamental constants and the nuclear matrix element (see, e.g., [25])

$$\sigma = \sigma_0 \times \left\langle \frac{c}{v_\nu} E_e p_e F(Z, E_e) \right\rangle \frac{2I' + 1}{2I + 1} \quad (14)$$

with

$$\sigma_0 = \frac{G_F^2 \cos^2 \theta_C m_e^2}{\pi} |M_{\text{nucl}}|^2 = \frac{2.64 \times 10^{-41}}{f t_{1/2}} \quad (15)$$

in units of cm^2 . Here the averaging is done over the incoming flux, $t_{1/2}$ is in seconds, the statistical function f , the electron energy E_e and momentum p_e are evaluated with m_e as a unit of energy, and the nuclear matrix element is excluded using the relation $|M_{\text{nucl}}|^2 \simeq 6300/f t_{1/2}$. For the neutrino capture on stable targets (i.e. with a threshold) the electron energy is simply (neglecting recoil) $E_e = E_\nu - E_{\text{thres}} + m_e$. So, for the $\bar{\nu}_e$ induced reactions $E_{e^+} = E_\nu - Q_\beta - m_e$ and for the ν_e induced reactions $E_{e^-} = E_\nu + Q_{EC} + m_e$. For the capture on a radioactive target $E_{e^-} = E_\nu + Q_\beta + m_e$ and $E_{e^+} = E_\nu + Q_{EC} + m_e$.

3. Applications

Let us now consider the possibility to use the neutrino capture by radioactive beta-decaying nuclei to detect cosmological neutrinos. For this aim the relevant quantity is the number of events, i.e. the cross section times the flux. For the latter, the dependence on v_ν in equations (12) and (14) is cancelled out and one obtains the number of events that converges

⁴ Note that in [21], where the cross section for charged current neutrino capture on a free neutron, equation (3), was evaluated, these modifications were not made and are not reflected in figure 1 of that paper.

to a constant when $v_\nu \rightarrow 0$, in agreement with [23, 24]. As an example of the possible application of the above finding let us consider the ν_e capture on tritium, as done in [24]. While our conclusions are qualitatively similar to the conclusions reached by Cocco *et al* [24], they differ in several significant details.

Tritium decays into ${}^3\text{He}$ with the half-life of 12.3 years. The decay Q_β value is 18.6 keV, and $ft_{1/2} = 1143$. From equations (14)–(15) we deduce the cross section for $T = 1.9$ K nonrelativistic neutrinos

$$\sigma = 1.5 \times 10^{-41} \left(\frac{m_\nu}{\text{eV}} \right) \text{cm}^2, \quad \text{or} \quad \sigma \frac{v_\nu}{c} \simeq 7.6 \times 10^{-45} \text{cm}^2. \quad (16)$$

Here, in the first equation we used that $v_\nu/c \sim 3T/m_\nu$. In making that estimate we neglected the $v_\nu \sim 10^{-3}c$ virial motion of massive neutrinos in the galactic halo, and the motion of Earth and solar system with respect to the random motion of the neutrinos.

In order to evaluate possible count rate, we have to know the number density of the background ν_e sea. Its average value, for neutrinos evenly distributed throughout the whole universe, corresponds to $T_\nu \sim 1.9$ K. For neutrinos (or antineutrinos) of one flavour only $\langle n_\nu \rangle$ is $\sim 55 \text{cm}^{-3}$. Massive neutrinos will be gravitationally clustered on the scale of $\sim \text{Mpc}$ for neutrinos with $m_\nu \sim 1$ eV, that is on the scale of galaxy clusters (probably the clustering scale is even larger). Assuming that in that case the ratio of the dimensionless neutrino and baryon densities $\Omega_\nu/\Omega_b \sim 0.5 \frac{m_\nu}{\text{eV}}$ remains the same as in the universe as a whole, we obtain

$$\frac{n_\nu}{\langle n_\nu \rangle} \sim 9 \times 10^6 n_b \left(\frac{m_\nu}{\text{eV}} \right) \sim 10^3 - 10^4, \quad (17)$$

for $m_\nu = 1$ eV and $n_b = (10^{-3} - 10^{-4}) \text{cm}^{-3}$ for a cluster of galaxies. In the following, we do not use the last estimate and treat this ratio as an unknown m_ν dependent parameter. A more elaborated study of neutrino clustering is made, for example, in [26], giving smaller but non-negligible clustering for $m_\nu = 1$ eV.

Note, however, that much larger neutrino clustering was considered in [27, 28]. The authors of these papers speculate that features in the cosmic-ray spectra, in particular the ‘knee’ at ~ 3 PeV, are associated with the threshold of the $p + \bar{\nu}_e \rightarrow n + e^+$ reaction on ~ 0.5 eV mass neutrinos. The physics basis for the required clustering of $\frac{n_\nu}{\langle n_\nu \rangle} \sim 10^{13}$ is not provided in those papers.

The capture rate per tritium atom is

$$R = \sigma \times v_\nu \times n_\nu \simeq 10^{-32} \times \frac{n_\nu}{\langle n_\nu \rangle} \text{s}^{-1}. \quad (18)$$

Let us assume, probably much too optimistically, that one can use a Megacurie source of tritium (1 Mcu = 3.7×10^{16} decays s^{-1} , i.e. 2.1×10^{25} tritium atoms ~ 100 g of tritium). The number of events is then

$$N_{\nu \text{capt}} \simeq 6.5 \times \frac{n_\nu}{\langle n_\nu \rangle} \text{yr}^{-1} \text{Mcu}^{-1}. \quad (19)$$

Thus, if our assumption about the gravitational clustering is at least nearly correct, the capture rate would be reasonably large. However, the main issue would be whether the primordial background neutrino capture signal would be detectable given the overwhelming rate of the radioactive decay.

Electrons from the ordinary β decay are distributed over the kinetic energy interval $(0 - (Q_\beta - m_\nu))$, smeared by the resolution of the detection apparatus. On the other hand, electrons from the background neutrino captures are monoenergetic with the kinetic energy $Q_\beta + m_\nu$ again smeared by resolution. Thus, the signal-to-noise ratio will critically depend on

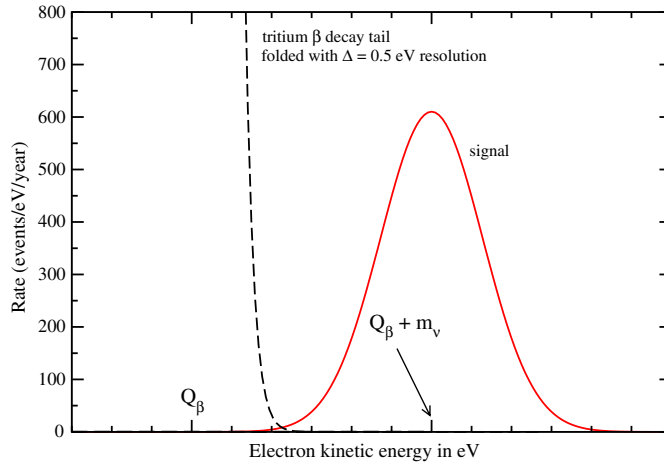


Figure 1. An illustration of the spectrum of detected electrons. The 1 eV mass of the neutrinos is assumed, and resolution (full width at half maximum) $\Delta = 0.5$ eV. The tail of the tritium β -decay spectrum, folded with that resolution is depicted with the dashed curve. The signal, also folded with the Gaussian resolution function is shown by the full line for $\frac{n_\nu}{\langle n_\nu \rangle} = 50$.

the neutrino mass m_ν and on the energy resolution Δ . Note that the fraction of electrons in an energy interval of width Δ just below the endpoint is $\sim (\Delta/Q_\beta)^3$.

To appreciate the problem, we show in figure 1, the tail of the spectrum of tritium β decay folded with a Gaussian resolution function and the signal of the cosmological ν_e capture electrons evaluated for $\frac{n_\nu}{\langle n_\nu \rangle} = 50$ and clearly separated in this idealized situation from the background.

Remarkably, the ratio of the background neutrino capture rate and the competing β decay with final electron within the resolution interval Δ just below the endpoint, does not depend on the corresponding Q_β value (see [24], their equation (23)) and, naturally, on the nuclear matrix element. For $m_\nu < \Delta$ the corresponding ratio is

$$\frac{\lambda_\nu}{\lambda_\beta} \simeq 6\pi^2 \frac{n_\nu}{\Delta^3} \simeq 2.5 \times 10^{-11} \times \frac{n_\nu}{\langle n_\nu \rangle} \times \frac{1}{(\Delta(\text{eV}))^3}. \quad (20)$$

This appears to be a hopelessly small number.

Before discussing the issues further, let us point out that other sources of background, for example the capture of solar pp neutrinos, are not dangerous. The total solar pp neutrino flux is $\sim 6 \times 10^{10} \nu_e \text{ cm}^{-2} \text{ s}^{-1}$ distributed over 420 keV [25]. Thus, the flux in the lowest 10 eV is only about $10^6 \nu_e \text{ cm}^{-2} \text{ s}^{-1}$, which is less than 1% of the effective flux of the primordial ν_e even for $m_\nu \geq 1$ eV.

One should note that the discussed method is interesting only as long as neutrinos are non-relativistic with $v \ll c$. For higher energy neutrinos (like thermal solar neutrinos that have an estimated flux of $10^8 - 10^9 \text{ cm}^{-2} \text{ s}^{-1} \text{ MeV}^{-1}$ and energies ~ 1 keV [29]) signal becomes well separated from radioactive ion decay background; however, the number of expected events will be very small, since one is obliged to work with very small amount of active material. The \sim keV mass sterile neutrinos, considered in the literature (see, e.g., [30]), are unobservable due to their extremely small mixing with the active neutrinos.

If one could achieve a resolution Δ that is less than the neutrino mass m_ν , the signal-to-background ratio would increase since the β spectrum ends at $Q - m_\nu$ while the electrons

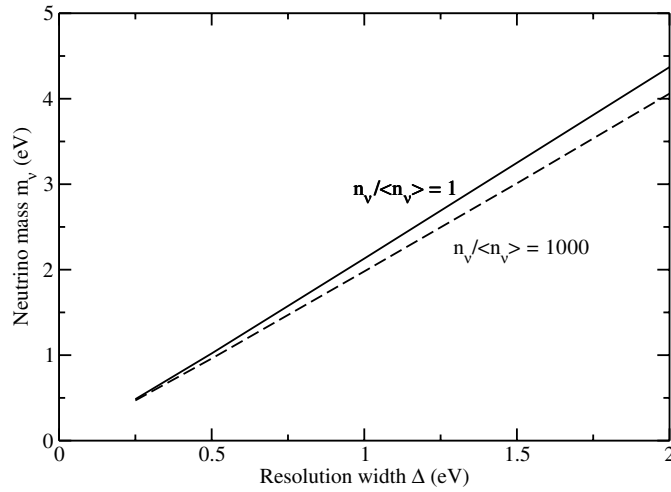


Figure 2. The values of neutrino mass m_ν for which the signal-to-noise ratio $\lambda_\nu/\lambda_\beta = 1$ as a function of the resolution width Δ . The lines are labelled by the assumed neutrino clustering values. For details see the text.

from neutrino capture have energy $Q + m_\nu$. The corresponding gain, i.e. the suppression of the tail of the β decay spectrum, is estimated in [24]. For $m_\nu \geq \Delta$ it is

$$\rho \simeq \frac{1}{\sqrt{2\pi}} e^{-2(m_\nu/\Delta)^2}, \tag{21}$$

i.e., it is an extremely steep function of m_ν/Δ . According to such estimate a signal-to-background ratio of the order of unity could be reached if the ratio $m_\nu/\Delta \sim 3$. Since m_ν remains unknown, an experiment with a fixed resolution Δ would be able to observe the background neutrino sea only if m_ν is large enough.

A numerical calculation suggests that in fact $m_\nu/\Delta \sim 2$ is enough to achieve a signal-to-background ratio of order of unity. This is illustrated in figure 2, where Δ is the full width at half maximum of the assumed Gaussian resolution function and the signal as well as the background are centred at $Q + m_\nu$ and integrated over an interval of width Δ . The figure also shows that this ratio is such a steep function of m_ν/Δ that it is essentially independent of the enhancement of λ_ν due to the gravitational clustering. (Note, however, that the signal itself, i.e. the number of events, is proportional to the clustering ratio $n_\nu/\langle n_\nu \rangle$.) Figure 2 shows that in order to achieve sensitivity to sub eV neutrino masses a resolution width below ~ 0.5 eV would be necessary. Obviously, that is a very challenging requirement.

We wish now to discuss the possible resonance character of the neutrino capture on nuclei at threshold. The interest in the possible resonance effects was whetted by the attempts of Raghavan [31], that even so they are unsuccessful so far, stimulated lively discussion. For the primordial background neutrinos the de Broglie wavelength is extremely large,

$$\bar{\lambda}_\nu = \frac{\hbar}{p_\nu} \sim 0.04 \text{ cm} \tag{22}$$

for $T_\nu = 1.9$ K. That estimate is no longer valid for the gravitationally clustered massive neutrinos, which acquire the corresponding virial velocity. Nevertheless, their de Broglie wavelength remains macroscopically large.

If a resonance reaction could occur, and if it can be somehow described by a Breit–Wigner-type formula where the integrated reaction cross section is

$$\int \sigma_{\text{reaction}} dE = 2\pi^2 \bar{\lambda}^2 \frac{\Gamma_r \Gamma_e}{\Gamma}, \quad (23)$$

a very large cross section could be obtained. Here Γ_e is the partial width for the elastic scattering, Γ_r for the capture and Γ is the total width. In the case of neutrino induced reactions, obviously, there is only one channel, and all Γ values should be identical.

Mikaleyan *et al* [32] considered a resonance scenario for the endothermic reaction (not observed as yet)



where A_Z is stable, A_{Z-1} is radioactive with an endpoint E_0 and the captured electron is an orbital one in A_Z . The reaction occurs when $E_\nu = E_0 + E_b$, where E_b is the binding energy of the captured electron, so its threshold is $\sim 2m_e c^2$ below the threshold for the inverse β decay. It is shown in [32] that for the process (22) the factor $\bar{\lambda}_\nu^2$ in the cross section formula equation (23) is compensated by the E_ν^2 dependence of the total width Γ ($\Gamma_r = \Gamma_e = \Gamma$ in this case). Moreover, the resonance electron capture cannot occur for a radioactive (hence exothermic) target A_Z .

While the reaction (24) is indeed of resonance character and occurs only when the incoming $\bar{\nu}_e$ has a fixed energy, the zero threshold reactions (with radioactive A_Z)



are not of resonance character. They can proceed for any energy of the incoming $\bar{\nu}_e$, including nonrelativistic energies, but they cannot be described by the resonance formula and the cross section is never close to the value of $2\pi^2 \bar{\lambda}^2$ as in equation (23).

4. Conclusions

We have shown that the charged current cross section of nonrelativistic neutrinos scales like $1/v_\nu$ in agreement with the generic behaviour of the cross sections for slow particles. That means, in particular, that the charged current reactions with vanishing threshold of nonrelativistic neutrinos have a rate that converges to a finite value as $v_\nu \rightarrow 0$. With that in mind we evaluate the cross section and reaction rate of the relic $\bar{\nu}_e$ on tritium nuclei. If a modest gravitational clustering enhancement of the relic neutrino number density is present, the number of events is large enough that it might be potentially observable. The more important issue is the elimination of the overwhelming background of the electrons from tritium β decay. We show that signal/background ratio of the order of unity can be achieved only if the neutrino mass m_ν exceeds the characteristic experimental resolution width Δ by a factor of two or more. Thus an energy resolution $\Delta \leq 0.5$ eV would be required in order to be possible, even in an ideal experiment, to detect relic neutrinos with sub-eV neutrino masses, independently of the gravitational neutrino clustering.

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References

- [1] Peebles P J E 1971 *Physical Cosmology* (Princeton, NJ: Princeton University Press)
- [2] Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)
- [3] Mangano G, Miele G, Pastor S and Peloso M 2002 *Phys. Lett. B* **534** 8
- [4] Spergel D N *et al* 2007 *Astrophys. J. Suppl.* **170** 377 (Preprint [astro-ph/0603449](#))
- [5] Orpher R 1974 *Astron. Astrophys.* **37** 135
- [6] Lewis R R 1980 *Phys. Rev. D* **21** 663
- [7] Cabibbo N and Maiani L 1982 *Phys. Lett. B* **114** 115
- [8] Langacker P, Leveille J P and Sheiman J 1983 *Phys. Rev. D* **27** 1228
- [9] Stodolsky L 1975 *Phys. Rev. Lett.* **34** 110
- [10] Stodolsky L 1975 *Phys. Rev. Lett.* **34** 508 (erratum)
- [11] Hagmann C 1999 Preprint [astro-ph/9902102](#)
- [12] Smith P F and Lewin J D 1984 *Acta Phys. Pol. B* **15** 1201
- [13] Smith P F 2002 *Prepared for 4th Int. Workshop on the Identification of Dark Matter (IDM 2002) (York, England)* pp 2–6
- [14] Weiler T J 1982 *Phys. Rev. Lett.* **49** 234
- [15] Weiler T J 1984 *Astrophys. J.* **285** 495
- [16] Eberle B, Ringwald A, Song L and Weiler T J 2004 *Phys. Rev. D* **70** 023007 (Preprint [hep-ph/0401203](#))
- [17] Weiler T J 1999 *Astropart. Phys.* **11** 303 (Preprint [hep-ph/9710431](#))
- [18] Fargion D, Mele B and Salis A 1999 *Astrophys. J.* **517** 725
- [19] Gelmini G B 2005 *Phys. Scripta T* **121** 131 (Preprint [hep-ph/0412305](#)) and references therein
- [20] Vogel P and Beacom J F 1999 *Phys. Rev. D* **60** 053003 (Preprint [hep-ph/9903554](#))
- [21] Azimov Yu I and Shekhter V M 1962 *Sov. Phys.—JETP* **14** 424
- [22] Beresteckii V B, Lifshitz E M and Pitaevskii L P 1971 *Relativistic Quantum Theory, Part 1* (Oxford: Pergamon) (see equation (65.15a))
- [23] Strumia A and Vissani F 2003 *Phys. Lett. B* **564** 42 (Preprint [astro-ph/0302055](#))
- [24] Landau L D and Lifshitz E M 1977 *Quantum Mechanics: Non-Relativistic Theory* 3rd edn (Oxford: Pergamon)
- [25] Weinberg S 1962 *Phys. Rev.* **128** 1457
- [26] Cocco A G, Mangano G and Messina M 2007 *J. Cosmol. Astropart. Phys.* **JCAP06(2007)015** (Preprint [hep-ph/0703075](#))
- [27] Bahcall J N 1989 *Neutrino Astrophysics* (Cambridge: Cambridge University Press)
- [28] Ringwald A and Wong Y Y Y 2004 *J. Cosmol. Astropart. Phys.* **JCAP12(2004)005** (Preprint [hep-ph/0408241](#)) and references therein
- [29] Wigmans R 2003 *Astropart. Phys.* **19** 379
- [30] Hwang W-Y P and Bo-Qiang Ma 2005 *New J. Phys.* **7** 41
- [31] Haxton W C and Lin Wei 2000 *Phys. Lett. B* **486** 263
- [32] Kusenko A 2006 Preprint [hep-ph/0609158](#) and references therein
- [33] Raghavan R S 2007 Private communication (Preprint [hep-ph/0703028](#)) (withdrawn)
- [34] Mikaleyan L A, Tsinoev B G and Borovoi A A 1968 *Sov. J. Nucl. Phys.* **6** 254