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Charged Particle Behavior in Low-Frequency Geomagnetic Pulsations
1. Transverse Waves

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The behavior of charged particles in low-frequency geomagnetic pulsations is examined with particular emphasis on what a spacecraft-borne detector would observe. We concentrate on the effects of purely transverse electromagnetic signals. The time scale of a particle's motion relative to the wave period is shown to determine the nature of its response. For low-energy particles, the acceleration in the last gyroperiod before detection is what matters. At higher energies, what has occurred over recent bounce and drift motions becomes increasingly important and convection of gradients by the wave E × B drift must be considered. Distinguishing features such as phase differences between signals in back-to-back detectors or between channels of different energy are catalogued. In particular, we assess the detectability of resonance effects in the light of detector characteristics and finite signal bandwidth. Recent observations are used to illustrate the ideas developed.

INTRODUCTION

This paper sets out the theory of particle behavior in a purely transverse low-frequency wave in the magnetosphere. We specifically ask what a spacecraft particle detector would see in the presence of a wave. This turns out to be a strong function of the energy and pitch angle of the particles to which the detector responds. Because the story is quite complex, we have limited this paper to the study of purely transverse signals. In a separate work we intend extending the analysis to waves of arbitrary polarization.

The problem is worth studying just as an exercise in our understanding of collisionless plasma behavior, but there are potential applications to specific problems of magnetospheric interest: The waves we are interested in can be fairly large scale phenomena, occupying an entire flux tube length and displacing plasma by as much as a fraction of an earth radius perpendicular to the field near the equator. Such wave amplitudes are given in or can be deduced from papers like *Kivelson* [1976], *Cummings et al.* [1978], and *Hughes et al.* [1979].

In the presence of these large scale waves, one may be able to probe plasma conditions away from the spacecraft locations. *Hughes et al.* [1979] have already provided such an analysis of steep spatial gradients in electrons of about 10 keV energy near synchronous orbit. Their results have implications for our understanding of the overall convection pattern in the magnetosphere [*Kivelson et al.*, 1979; *Southwood and Kaye*, 1979]. One may also be able to examine particles actually in resonance with a plasma wave. There is mounting evidence that resonance phenomena are one source of low-frequency wave noise in the magnetosphere (most recently in *Hughes et al.* [1978, 1979]). Also, suggestions have been made that resonant effects can cause major spatial scattering of particles [*Hasegawa and Mima*, 1978].

A succession of papers using data from a variety of spacecraft has examined variations in particle flux in magnetospheric pulsation events [*Brown et al.*, 1968; *Sonnerup et al.*, 1969; *Barfield et al.*, 1971; *Baxter and LaQuey*, 1973; *Kivelson*, 1976; *Kokubun et al.*, 1977; *Lin et al.*, 1976; *Su et al.*, 1977, 1979; *Cummings et al.*, 1978; *Hughes et al.*, 1978, 1979]. Up to this point, no paper has fully codified particle flux behavior expected in low-frequency waves (0.001 ot 0.1 Hz), and this limits interpretation of many of the above observational data. We attempt a full codification for transverse signals here. Many of the above papers report signals which are not purely transverse; compressional magnetic oscillations are commonly present in this frequency band. Purely compressional signals are, however, rare [*Barfield et al.*, 1971] as are reports of flux oscillations in the absence of magnetic perturbations [*Baxter and LaQuey*, 1973]. Purely transverse magnetic oscillations are commonly seen. Figure 1 (which is taken from *Kokubun et al.* [1977]) shows a purely transverse magnetic oscillation producing flux oscillations in both very low energy plasma (ions) detected by the Lockheed ion mass spectrometer on OGO 5 and in high-energy protons and electrons detected by UCLA and LLL instruments on OGO 5. It is not surprising in itself that both high- and low-energy particles respond to the wave. As we shall show, a wave both accelerates particles and displaces them in space.

We aim to explain what a particle detector on a spacecraft should see in the presence of a particular wave. We are led to point up several effects that have not been emphasized in earlier theoretical work. For example, we assess the detectability of particles actually in resonance with a wave. In this instance, the energy bandwidth and pitch angle response of the detector control what can be seen. Detector characteristics are also important in our analysis of acceleration within a gyroperiod. The effect we describe as gyration acceleration is ignored in many theoretical works but is important because of the directional nature of most detectors.

TRANSVERSE MAGNETIC SIGNALS

Dungey [1954] first suggested that the magnetic signals in the ULF band seen on the earth's surface were due to hydromagnetic waves in the magnetosphere. It is now firmly established that this is so. An important feature of the theory is that the lower-frequency pc 3–5 pulsations have a standing structure along the earth's field. Phase measurements at magnetically conjugate points first confirmed this (see, e.g., *Sugiura*

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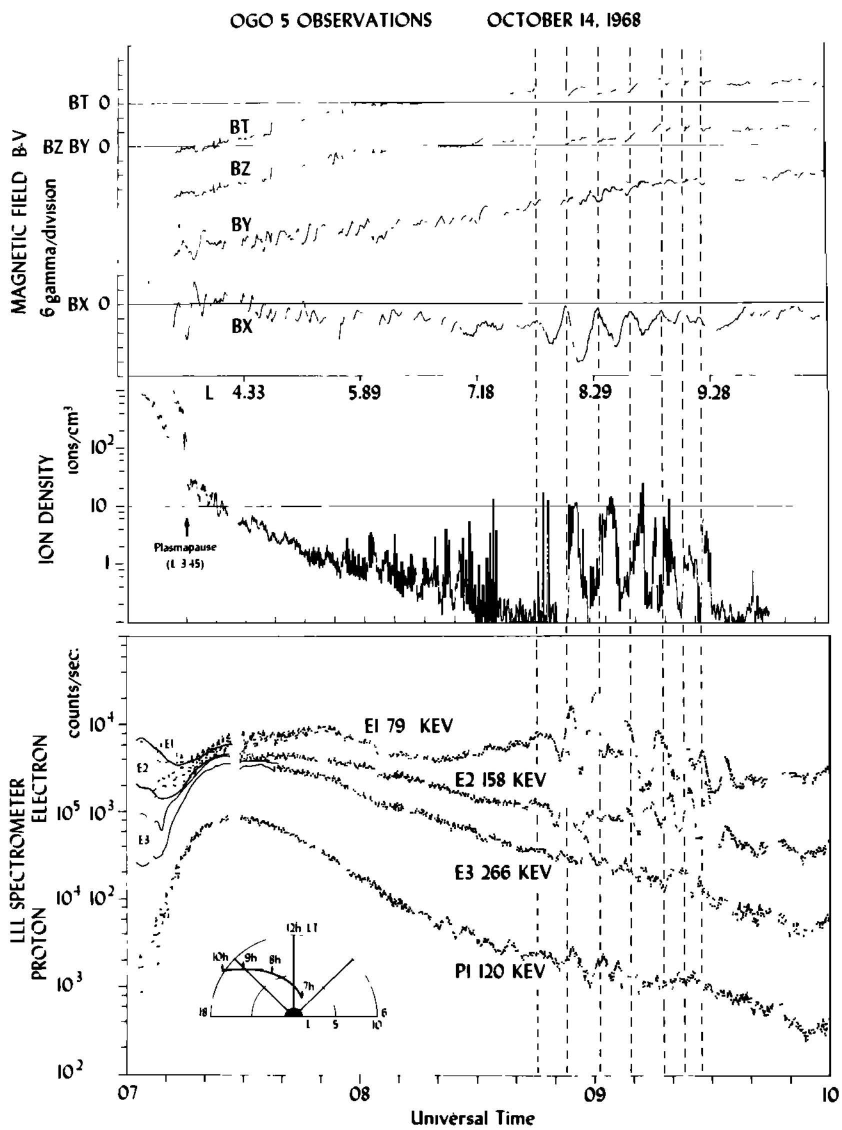

Fig. 1. Particle flux oscillations in a transverse pulsation event [from *Kokubun et al.,* 1977]. From top to bottom, the traces show the total field and its components, the low-energy ion flux (not density as labelled), the fluxes of energetic electrons (79, 158, and 266 keV), and the 120 keV proton flux. Note that the low-energy fluxes are modulated in quadrature with **b**. The energetic particle fluxes show irregular modulations whose phases are energy dependent.

and Wilson [1964]). The wave Poynting flux along **B** is zero in a standing wave, and so in space the wave electric field component perpendicular to the magnetic field **B** is in quadrature with the wave magnetic perturbation across **B**. Particle measurements in space have confirmed this now [*Kivelson,* 1976; *Kokubun et al.,* 1977; *Cummings et al.,* 1978]. The standing structure also means that the wave field should exhibit some symmetry about the field line equator. If we assume north-south symmetry in the background field, the fundamental mode should have the electric field, field displacement and plasma velocity perturbation symmetric about the equator while the transverse magnetic field perturbation is antisymmetric. In contrast, the next higher harmonic has a transverse magnetic perturbation that is symmetric about the equator and **E** field, field displacement and velocity perturbation that are antisymmetric. Figure 2 shows the field line configuration at extremes of the oscillations and illustrates this point. Inspection of Figure 2 shows that such statements are dependent on the field displacement being small in the ionosphere (fixed field line end condition). The ionospheric boundary condition has received some attention (see, e.g., *Hughes and Southwood* [1976*a*], *Newton et al.* [1978], and *Allan and Knox* [1979]) and at times the conductivity may be so low that a free end condition may be more appropriate. Such would seem to be the exception rather than the rule and we shall ignore this possibility throughout the remainder of the paper.

Transverse magnetic signals, Alfvén mode signals, have a transverse electric field associated with them, but unless the wavelength is as short as the mean proton Larmor radius we do not expect a significant parallel electric field signal (see, e.g., *Coroniti and Kennel* [1970]). We can be sure there is a large class of signals with wavelengths across the field much larger than the proton Larmor radius because of direct measurements [e.g., *Green,* 1976; *Hughes et al.,* 1978; *Mier-Jedrzejowicz and Southwood,* 1979]. In fact, if all pulsation signals varied across the Earth's field on scales comparable with the thermal Larmor radius (~300 km for a ring current proton at geostationary orbit), signal amplitudes on the ground would probably be very small. This is because such signals, when mapped to the ionosphere, would have a horizontal variation

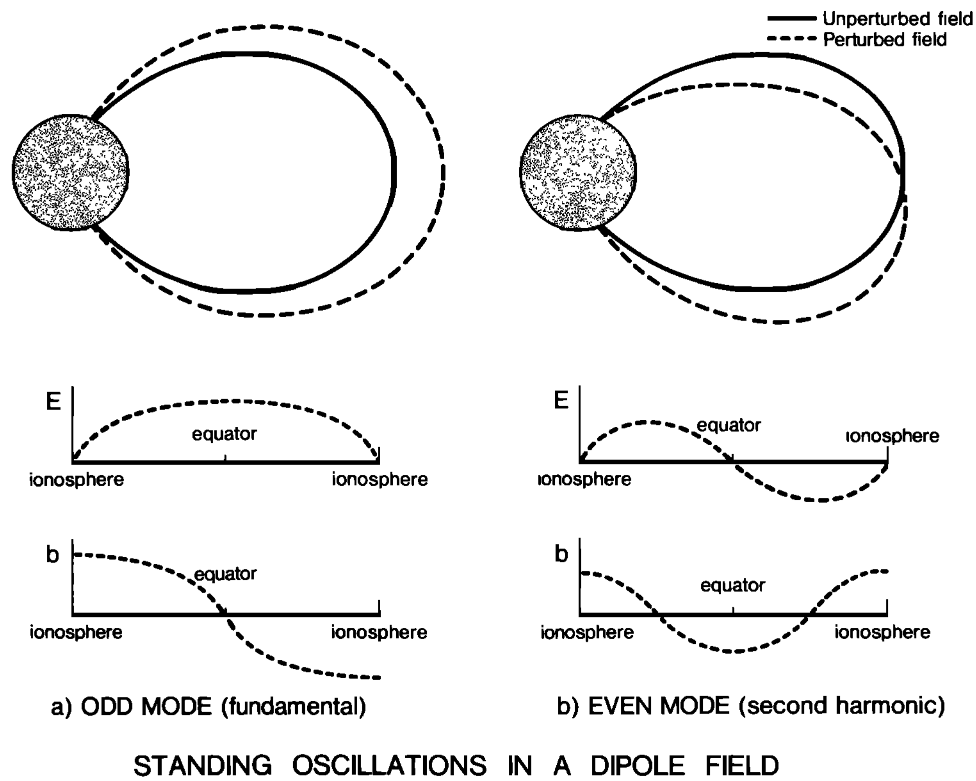

Fig. 2. Schematic representations of standing mode waves in a dipole geometry. Note that the electric field is symmetric about the equator for the fundamental mode and antisymmetric about the equator for the second harmonic.

shorter than the E region height, and these signals are strongly shielded from the ground [*Hughes and Southwood,* 1976*a*, *b*]. It thus seems admissible initially to ignore parallel electric fields in wave signals. Other authors [e.g., *Hasegawa,* 1979; *Su et al.,* 1979] have chosen to emphasize the parallel E field's role. As we make clear here, it is not necessary for a wave to have a parallel E component to modulate particle flux.

Time Scales of Particle Motion

The relative time scales of wave and particle motion are crucial to ordering our problem. The shortest time scales involved are the gyration times of electrons and protons. Both are much less than the period of the waves, and we shall also assume the corresponding spatial scales, the Larmor radii, are much smaller than the wavelength. This does not mean we need not consider effects which act on a particle on the scale of its gyration. We shall see that we do. This is because we are orienting our work to what is seen in a particle detector and a detector is much smaller than a particle Larmor radius.

The next smallest scale we need to consider is that of particle motion back and forth along the Earth's field. A particle whose bounce period greatly exceeds the wave period responds only to the wave fields in the spacecraft vicinity. In contrast, a fast particle with a bounce period much less than the wave period responds to the overall wave field distribution it sees along its entire bounce orbit. Medium energy protons (1–10 keV) have bounce periods of about a minute at geosynchronous orbit which is also a typical pulsation period. An electron of the same energy bounces 40 times faster. The spread of energies measured on a spacecraft and the fact that both protons and electrons may be measured mean that commonly information is available on particles responding to both local and bounce orbit averaged fields. Note, however, that near the equator a detector looking at 90° pitch angle particles is seeing particles which are sampling only a small (local) fraction of the wave field whatever the energy being measured because of the small amplitude of the bounce motion of such particles. In contrast, a detector looking at small pitch angle, wherever it is, sees particles which sample the wave over a vast fraction of the flux tube if their energy is high enough.

There is one more time scale in particle adiabatic motion in the magnetosphere. This is the drift period, the time taken to move around the earth under ∇B and curvature drifts. It is inversely proportional to energy and much exceeds the period of a pulsation for any reasonable energy. However, pulsations have a finite scale length in the east-west direction and thus the important time scale is the time to drift through a wavelength. As in many theoretical works, we shall describe the east-west wave variation by exp $(im\phi)$ (ϕ, longitude), so that the east-west angular wavelength is $2\pi/m$; m need not be an integer. We write the mean east-west angular drift rate as $\bar{\omega}_d$, and so the time scale to drift through a wavelength is $2\pi/m\bar{\omega}_d$; once this is comparable to or less than the wave period, drift motion east-west is important.

When the wave period is comparable to $2\pi/m\bar{\omega}_d$, a resonance is possible; a particle drifts at the rate the wave moves east-west and the particle can thus see a steady component in the wave signal. Resonances are also possible when wave period and bounce period are comparable. The generalized condition for resonance is

$$\omega - m\bar{\omega}_d = N\omega_b$$

where $\bar{\omega}_b$ is the bounce frequency and N is integer or zero. As *Dungey* [1965] first pointed out, the symmetry of the wave field about the equator determines which resonances are important (see, e.g., *Southwood* [1976]).

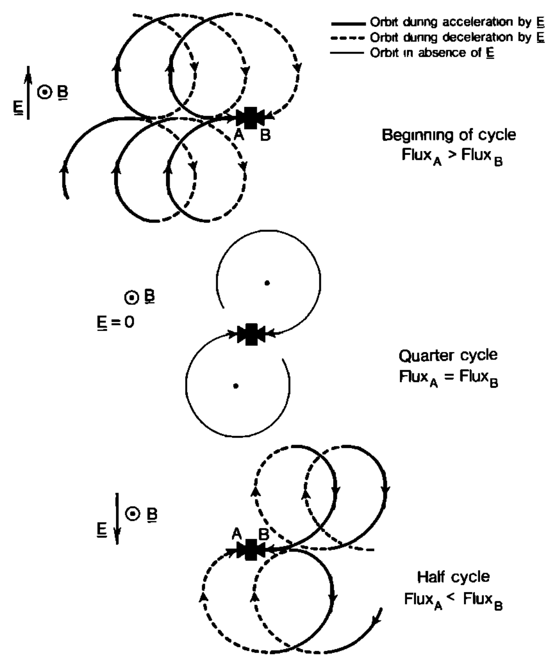

Fig. 3. Schematic illustration of the gyroorbit acceleration of particles entering back-to-back detectors in the presence of a low-frequency wave.

ACCELERATION OF A PARTICLE IN ITS GYROORBIT

In a transverse hydromagnetic wave any change in particle energy must be due to acceleration by the wave electric field **E**. Over many gyrations the mean rate of change of energy ($\dot{W}$) for a particle of charge q is given by the well-known expression [*Northrop*, 1963]

$$\dot{W} = q\mathbf{E} \cdot \mathbf{v}_d \tag{1a}$$

where $\mathbf{v}_d$ is the magnetic gradient and curvature drift. Although this represents the mean rate, it is not the instantaneous acceleration rate. The gyration motion through the wave electric field means there is a continued acceleration and deceleration on the scale of a Larmor orbit. As the wave period much exceeds the proton (and of course the electron) gyroperiod, on a gyration scale the wave appears static to the particle and the particle's net excursion in energy in a gyration is

$$2q|E|v_\perp/\Omega \tag{2}$$

where $v_\perp$ and Ω are the particle gyration velocity and gyration frequency, respectively; $|E|$ is the wave electric field amplitude.

What a particle detector sees depends on its look direction relative to the wave **E** field direction. Figure 3 illustrates this. A detector whose look direction has a component antiparallel to $\mathbf{E} \times \mathbf{B}$ sees particles that have accelerated over their last half gyration before detection. It should be clear that the acceleration can be described just as that due to each particle's velocity being changed by $\mathbf{E} \times \mathbf{B}/B^2$ due to the presence of the wave. The net excursion in energy is given by (2). The energy oscillation, δW_g, for particles seen in a detector with look direction, $\hat{l}$, is

$$\delta W_g = q\,\frac{\mathbf{E}}{B} \cdot (\mathbf{B} \times \hat{l})\,\frac{\mathbf{v}}{\Omega} \cdot \hat{l} = -M\left(\frac{\mathbf{E} \times \mathbf{B}}{B^2}\right) \cdot \hat{l}(\mathbf{v} \cdot \hat{l}) \tag{3}$$

where M is the particle mass and **v**, particle velocity. The subscript g is to emphasize that the acceleration takes place on the scale of a gyration.

The effect that we have described in this section has been recognized in several experimental analyses [*Kivelson*, 1976; *Cummings et al.*, 1978; *Kokubun et al.*, 1977; *Hughes et al.*, 1979] but has been ignored in theoretical developments (see, e.g., *Southwood* [1973, 1976], *Su et al.* [1977], *Tamao* [1978], and *Hasegawa* [1979]). In low-frequency plasma wave theory one is normally interested in the mean distribution averaged over gyrophase. A detector with fixed look direction makes no such average. Only if the detector were omnidirectional would it sample particles at all gyrophases at once and in doing so would smear out the effect we have described here.

We shall refer to the acceleration process looked at in this section as 'gyration acceleration' in the remainder of the paper and continue to denote it by δW_g.

ACCELERATION OVER MANY GYRATIONS: EQUATORIAL PARTICLES

In this section we proceed to consider the acceleration a particle experiences over many gyrations. Equation (1a) gives the rate of change of energy. Henceforward we shall add a subscript A to signify this rate is averaged over many gyrations:

$$\dot{W}_A = q\mathbf{E} \cdot \mathbf{v}_d \tag{1b}$$

The acceleration represented by (1b) is a slower process than the gyration acceleration described above. As a result we need to consider particle motion over time scales much longer than the gyration time, but by the same token, the amount of acceleration is not gyrophase dependent. An omnidirectional detector would detect a change in flux just as well as a directional detector would.

To compute the change in energy a particular particle has experienced, we need to integrate the right-hand side of (1b) back over its past orbit and inevitably over many gyrations. We restrict ourselves to particles with Larmor radii small compared to the scale on which the wave varies. The orbit integration can then be done back along the path of the particle guiding center. This motion consists of an east-west motion due to ∇B and curvature drift and bounce motion along **B**. We clearly may need to deal with wave amplitude variation both along **B** and in longitude across **B**. To order the latter, we take the signal to vary as exp $i(m\phi - \omega t)$ and ω is the wave frequency.

We defer a discussion of the structure of the wave field along **B** by first focusing our attention on particles with near 90° equatorial pitch angles. Such particles will not sample much of the field variation along **B** because their bounce amplitudes are small. Despite the fact that they respond solely to the equatorial field amplitude, the 90° particles reveal many of the complexities of the wave-particle interaction problem.

Equatorially mirroring particles of velocity v in a dipole magnetic field drift azimuthally with a velocity

$$v_d = LR_E\omega_d = 3v^2/2\Omega LR_E$$

where Ω is the gyration frequency of the particle. From (1b) with the assumed wave electric field,

$$\dot{W}_A = qE_\phi(3v^2/2\Omega LR_E)\exp i(m\phi - \omega t) \tag{4}$$

where ϕ increases westward.

The particles' unperturbed orbit is given by

$$\phi = \omega_d t + \phi_0$$

where ϕ_0 is an initial longitude. On the face of it, any integration back in time will leave a result dependent on initial conditions. In practice, after a finite time, a particle's behavior should not be strongly dependent on how it initially interacted with the signal. In a real signal a particle will see a sinusoid only for a finite interaction time. This time is inversely proportional to the bandwidth of the signal or proportional to the time a particle takes to travel through a coherence length. The smoothing of the particle 'memory' of a Fourier component of the wave signal in the long term can be usefully simulated mathematically by assuming a weakly growing signal and taking the integration far back in time to where the amplitude is negligibly small. Doing this, we find that the first order (in the wave amplitude) change in the particle energy produced by the wave over many gyrations is

$$\delta W_A = i\,\frac{qE_\phi}{\Omega}\,\frac{3v^2}{2LR_E}\,\frac{\exp i\,(m\phi - \omega t)}{\omega - m\omega_d} \tag{5}$$

Now (5) is obviously unsatisfactory when

$$\omega = m\omega_d \tag{6}$$

When (6) holds the particle's grad B drift is just such as to keep the particle moving on a phase front of the wave. A resonance, drift resonance, is occurring. A particle in resonance sees a constant disturbance and a secular change in energy results. The interaction time mentioned above is again a useful concept. The secular gain for a particular particle in resonance lasts only for an interaction time, the length of time that the particle sees a sinusoid. Recomputing the energy change for a particle strictly in resonance for a time T yields

$$\delta W_A = T\,\frac{qE_\phi}{\Omega}\,\frac{3v^2}{2LR_E}\exp i(m\phi - \omega t) \tag{7}$$

It is immediately apparent that, as $T > \omega^{-1}$, by definition, the resonant response is large and the energy of strictly resonant particles oscillates in phase with $E_\phi(\mathbf{r}, t)$. In contrast, as (5) shows, at lower and higher energy the oscillation in particle energy is in quadrature with E_ϕ. To compute systematically how this phase variation with energy behaves, we return to our mathematical formulation with a small wave growth rate, γ, which represents in many respects the effect of a finite bandwidth. Near resonance one finds

$$\delta W_A = \frac{qE_\phi}{\Omega}\,\frac{3v^2}{2LR_E}\exp i(m\phi - \omega t)\left[\frac{\gamma - i(\omega_r - m\omega_d)}{\gamma^2 + (\omega_r - m\omega_d)^2}\right] \tag{8}$$

where ω_r is the real part of the frequency. Equation (8) shows the feature we remarked above, namely, that the response of resonant particles is in quadrature with the response at lower and higher energies.

At this stage it is important to consider another feature of particle detectors. The form of (8) shows that for resonance, one needs a particle to have a drift such that

$$|\omega_r - m\omega_d| < \gamma \tag{9}$$

An actual detector is very unlikely to be able to detect just those particles in resonance. It is likely to respond to a range of particle energies much wider than the energy range of particles close enough to resonance that (9) holds. For the sake of argument, let us assume the detector measures in a limited range of pitch angle over a range of velocities

$$v_D - \Delta v_D < v < v_D + \Delta v_D$$

and that the detector response is flat over this range of velocity. The mean energy perturbation in the channel is then proportional to an integral of the form

$$2\Delta v_D\,\overline{\delta W}_{\text{res}} = \int_{v_D-\Delta v_D}^{v_D+\Delta v_D} dv\,\dot{W}\,\frac{\gamma - i(\omega_r - m\omega_d)}{\gamma^2 + (\omega_r - m\omega_d)^2}$$

$$\simeq \frac{\dot{W}}{m(\partial\omega_d/\partial v)}\left[\pi - i\ln\left|\frac{v_D - v_{\text{res}} + \Delta v_D}{v_D - v_{\text{res}} - \Delta v_D}\right|\text{sign}\,(v_D - v_{\text{res}})\right] \tag{10}$$

where v_{res} is the velocity of a particle actually at resonance, i.e.:

$$3v_{\text{res}}^2/2\Omega(LR_E)^2 = \omega/m$$

Now $\partial\omega_d/\partial v = 2\omega_d/v$, so provided $[v_D - v_{\text{res}} + \Delta v_D]/[v_D - v_{\text{res}} - \Delta v_D] \ll ln^{-1}\,\pi \simeq 20$,

$$\overline{\delta W}_{\text{res}} = \frac{\dot{W}\pi}{\omega_r}\,\frac{v_{\text{res}}}{\Delta v_D} \tag{11}$$

To get the result (11) we assumed

$$\frac{\Delta v_D}{v_{\text{res}}} \gg \frac{\gamma}{\omega_r} \tag{12}$$

Equation (11) shows that as long as this requirement, (12), that the detector energy 'bandwidth' exceed the signal bandwidth holds, the resonant response measured by the detector is independent of the signal bandwidth. The sharpness of the resonant response is a function of the detector characteristics, the range of energies measured, the range of pitch angles around 90° actually measured, and any weighting over those ranges. In the opposite instance, when the detector energy range is relatively narrow compared with signal bandwidth, the response in a particular detector is limited by the signal bandwidth, $\Delta\omega \sim T^{-1}$ (see (7)). So $\overline{\delta W}_{\text{res}} = \dot{W}/\Delta\omega$. The former case is most commonly applicable to instrumentation already flown in space. Typically Δv_D is less than an order of magnitude smaller than v_D and they can be of exactly the same order.

Before we can combine the results derived above to discuss the actual particle flux variations which a particular wave should produce, we need to recognize that flux changes produced by a wave in a spacecraft-borne detector need not be due to particle acceleration alone. The wave also convects particles back and forth across the ambient field, and if there is a pre-existing gradient in the spatial distribution in the direction of wave motion, flux oscillations can result. To keep matters simple, we shall assume the major gradients are in the meridian. This is not a very binding assumption, and results are readily adaptable when it is not good. If the magnetic field is not strongly varying east-west, we can use the magnetic shell parameter L as a space coordinate, where L is the radial distance (in Earth radii, R_E) of a field line's equatorial crossing point. We shall adopt a convention that L is a space coordinate, and so labels field lines only when the wave is absent.

An equatorial particle moves in L at a rate

$$\dot{L} = \mathbf{E} \times \mathbf{B}/B^2R_E$$

and so integrating just as before, for nonresonant particles

$$\delta L = \frac{E_\phi}{BR_E}\,\frac{i\exp i\,(m\phi - \omega t)}{\omega - m\omega_d} \tag{13}$$

Resonant particles can also be treated just as before. The mean change in L over a finite energy range containing resonant particles is

$$\overline{\delta L}_{\text{res}} = -\frac{E_\phi\,(\mathbf{r}, t)}{B\omega R_E}\,\frac{v_{\text{res}}}{\Delta v_D} \tag{14}$$

Comparison of (13) and (14) with (5) and (11) shows that δL for 90° pitch angle particles is directly proportional to δW_A, the adiabatic change in energy. As a result, the relative importance of gradients in space and energy can be assessed easily. One finds using (5) and (13) or (10) and (11) that $\delta W_A/\delta L = -3W/L$ for 90° equatorial pitch angle. Let us write the distribution function as f. The gyration averaged change in f produced by the wave is given by

$$\delta f_A = -\delta W_A \frac{\partial f}{\partial W} - \delta L \frac{\partial f}{\partial L}$$

$$= -\delta W_A \left[\frac{\partial f}{\partial W} - \frac{L}{3W}\frac{\partial f}{\partial L}\right]$$

Whether the adiabatic response is dominated by gradients in energy or space depends on whether

$$\frac{\partial f}{\partial W} \gtrless \frac{L}{3W}\frac{\partial f}{\partial L}$$

The critical gradient

$$\frac{\partial f}{\partial L} = \frac{3W}{L}\frac{\partial f}{\partial W} \equiv \left.\frac{\partial f}{\partial L}\right]_{\text{ad}} \tag{15}$$

is precisely the gradient one expects for a distribution which has been convected in from large distances conserving μ, the first invariant (so that $W \propto L^{-3}$). Hence we refer to it as the adiabatic gradient, and label it $(\partial f/\partial L)_{\text{ad}}$. We can write

$$\delta f_A = -\delta L \left\{\frac{\partial f}{\partial L} - \left.\frac{\partial f}{\partial L}\right]_{\text{ad}}\right\} \tag{16}$$

The most generally accepted means of injection of the ring current particles into the dipolar regions of the magnetosphere is adiabatic convection. Because sources are at high L and sinks (e.g., large atmospheric loss cone) are at low L, one expects $\partial f/\partial L > (\partial f/\partial L)_{\text{ad}}$. At the inner edge of a distribution formed by any mechanism whatever, self-evidently $\partial f/\partial L > 0$, and also one expects $\partial f/\partial L \gg (\partial f/\partial L)_{\text{ad}}$ in this case, but elsewhere gradients are typically Earthward and close to the adiabatic value (first tested by *Nakada et al.* [1965]).

As well as the gyrophase independent part of δf, δf_A, there will be a part which is due to the local gyration acceleration energy change, δW_G, discussed previously. The total 90° equatorial pitch angle particle flux change brought about by the wave then is proportional to the distribution function change

$$\delta f = -\delta W_g \frac{\partial f}{\partial W} - \delta L \left\{\frac{\partial f}{\partial L} - \left.\frac{\partial f}{\partial L}\right]_{\text{ad}}\right\} \tag{17}$$

Equation (17) is ostensibly simple, but it is important to note that it predicts possibly very different responses at different energies. To compare orders of magnitude, one can use (15) to note that

$$\frac{\delta L(\partial f/\partial L)_{\text{ad}}}{\delta W_g(\partial f/\partial W)} = \frac{\delta W_A}{\delta W_g}$$

For all particles with $\omega \gg m\omega_d$, we can estimate

$$\delta W_A \sim |qEv^2/2LR_E\Omega\omega|$$

and

$$\delta W_g \sim |qEv/\Omega|$$

So one expects the gyration energy change to be dominant for any particle with $v < \omega LR_E$, unless the spatial gradient is very much steeper than adiabatic. At higher energies the second term is likely to be dominant unless the spatial distribution is very close to adiabatic.

Say we consider a detector looking radially outward. At low energies we expect to see particle flux increase in the detector in phase with E_ϕ. As one moves to higher energy, the adiabatic acceleration becomes more important and for nonresonant particles, this produces a flux change in quadrature with E_ϕ (see (5)). Because the adiabatic term becomes increasingly important, the flux oscillation expected as one moves to higher energies has its phase shifted with respect to E_ϕ and with respect to lower energy channels. Such phase shifting with energy has been observed [e.g., *Su et al.*, 1977], and it is important to note that an effect as simple as that described here may provide the explanation. Once the energy is high enough that $\omega_d \sim \omega/m$, there is again a phase shift with energy. As (7), (8), and (14) show at resonance the response is in phase (or strict antiphase, depending on the sign of the distribution gradients) with E_ϕ. But note here that drift resonance is only possible for one species in any particular wave.

ACCELERATION OVER MANY GYRATIONS: BOUNCING PARTICLES

Particles which mirror off the equator sample how the wave fields vary along **B**. Let us write s, measured from the equator, as the coordinate parallel to **B**. Particle motion is periodic in s. The periodicity can be described with a bounce phase, θ, which can be defined by

$$\theta = \int_{\text{eq}}^{s} \frac{ds\,\omega_b}{v_\parallel}$$

θ is also measured from the equator; s is an odd function of θ in a magnetic field that is symmetric about the equator.

As has been done in previous theoretical works [e.g., *Southwood*, 1973], we can use the periodic motion in s to express the rate of increase of particle energy as a Fourier series in bounce phase, θ:

$$\begin{aligned}\dot{W}_A &= qE_\phi(s)v_d(s)\exp i(m\phi - \omega t)\\ &= \sum_{N=-\infty}^{\infty} \dot{W}_N e^{iN\theta}\exp i(m\tilde{\omega}_d - \omega)t\end{aligned} \tag{18}$$

where $\tilde{\omega}_d$ represents the particle's mean (bounce averaged) drift rate in longitude. The coefficients $\dot{W}_N$ are Fourier coefficients and are independent of bounce phase. They depend on how the wave electric field varies along the particle bounce orbit and thus how the electric field varies along **B**.

Now we expect the electric field to be either an odd or even function of s or θ depending on whether the field line oscillation is an even or odd harmonic respectively, as we described earlier. It is apparent that in either case, coefficients in the summation (18) are interrelated and some terms will cancel in either case. From now on, we separate cases according to symmetry.

First, let us consider an odd harmonic such as the fundamental. We must in principle be able to rearrange the Fourier series (18) into the form

$$\dot{W}_A(\theta, \phi, t) = \sum_{l=0}^{\infty} \dot{W}_{2l}\cos 2l\theta\exp i(m\tilde{\omega}_d - \omega)t \tag{19a}$$

The series is a cosine series because of the requirement that the electric field be an even function of s (and so also of θ). Also because of the symmetry of E_ϕ about the equator, the particle sees E_ϕ oscillate at twice the particle's bounce period and so only even integers appear in the cosine argument. Integrating (19a) backwards in time using the procedure described in the previous section, we find

$$\delta W_A = \sum_{l=0}^{\infty} \dot{W}_{2l} \exp i(m\bar{\omega}_d - \omega)t \cdot \frac{i(\omega - m\bar{\omega}_d)\cos 2l\omega_b t - 2l\omega_b \sin 2l\omega_b t}{(\omega - m\bar{\omega}_d)^2 - (2l\omega_b)^2}$$

This expression is the energy perturbation experienced by a particle at bounce phase $\theta = \omega_b t$ at time t. Provided the integration is taken over many bounces the particle's longitude at time t is approximately $\phi = \bar{\omega}_d t$. If we make this assumption, we can express δW_A at time t as a function of the particle's longitude and bounce phase.

$$\delta W_A = \sum_{l=0}^{\infty} \dot{W}_{2l} \exp i(m\phi - \omega t) \cdot \frac{i \cos 2l\theta(\omega - m\bar{\omega}_d) - 2l\omega_b \sin 2l\theta}{(\omega - m\bar{\omega}_d)^2 - (2l\omega_b)^2} \qquad (20a)$$

The formula (20a) is superficially complicated but is readily interpretable if underlying particle behavior is remembered. We will illustrate this but before doing so, we close this section by deriving an equivalent expression for even harmonic standing structures. In this case the electric field and acceleration are antisymmetric about the equator and an odd function of s or θ. Equivalent arguments to those used above for odd harmonics show δW_A must be expressible in the form

$$\dot{W}_A(s, \phi, t) = \sum_{l=0}^{\infty} \dot{W}_{2l+1} \sin (2l+1)\omega_b t \exp i(m\bar{\omega}_d - \omega)t \qquad (19b)$$

for an even mode. Under the same rules as above, the integration yields

$$\delta W_A = \sum_{l=0}^{\infty} \dot{W}_{2l+1} \exp i(m\phi - \omega t) \cdot \frac{i(\omega - m\bar{\omega}_d)\sin(2l+1)\theta - (2l+1)\omega_b \cos(2l+1)\theta}{(\omega - m\bar{\omega}_d)^2 - (2l+1)^2 \omega_b^2} \qquad (20b)$$

The denominators of the expressions on the right-hand sides of (20a) and (20b) show resonant behavior wherever

$$\omega - m\omega_d = N\omega_b$$

and in these circumstances the Nth term in the Fourier series (19a) and (19b) will dominate the rest. Inspection shows even N resonances or pure drift resonance can occur in response to a symmetric disturbance (see (20a)). Odd N resonances appear in response to an antisymmetric disturbance (as in (20b)). The time integrations need to be redone for particles near resonance. In a signal with real frequency ω_r and small growth rate, γ, one finds for N odd, even or zero,

$$\delta W_A = \dot{W}_N \exp i(m\phi - \omega t)e^{iN\theta} \cdot \frac{\gamma - i(\omega_r - m\bar{\omega}_d - N\omega_b)}{(\omega_r - m\bar{\omega}_d - N\omega_b)^2 + \gamma^2} \qquad (20c)$$

Just as we argued earlier, in obtaining (11) the dependence on bandwidth or growth rate, γ, is eliminated if we work with the mean response over a range of velocities, Δv_D, near resonance. Much as before in the manipulation of (10), we obtain

$$\overline{\delta W}_{A\text{res}} = \frac{\dot{W}_N \pi}{2m\bar{\omega}_d + N\omega_b} \frac{v_{\text{res}}}{\Delta v_D} \qquad (21)$$

Motion in L must also be allowed for both resonant and nonresonant particles. For a particle with a finite parallel velocity, $v_\parallel$, the rate of change of L is (see, e.g., *Southwood* [1973])

$$\dot{L} = \left(\frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{v_\parallel \mathbf{b}}{B}\right) \cdot \nabla L \qquad (22)$$

The new term introduced here is the drift in the L direction due to the tilt in magnetic field produced by $\mathbf{b}$, the wave magnetic field. $\mathbf{E}$ and $\mathbf{b}$ are related by Faraday's law, and this fact can be used to rewrite $\dot{L}$ as [*Southwood*, 1973, 1976]

$$\dot{L} = \frac{\dot{W}_A m}{qB_e L\omega R_E^2} - \frac{d}{dt}\left[\frac{irE_\phi(\mathbf{r}, t)}{\omega B_e L R_E^2}\right] \qquad (23)$$

where B_e is the equatorial field on the field line, r is the local radial distance to the symmetry axis, and d/dt is the derivative along the unperturbed particle orbit (L = const). Integrating (23), we find

$$\delta L = \frac{\delta W_A m}{qB_e L\omega R_E^2} - \frac{irE_\phi(\mathbf{r}, t)}{\omega B_e L R_E^2} \qquad (24)$$

The change in L has a part proportional to the adiabatic chance in energy δW_A and a part which depends only on the local electric field. This latter past is independent of particle energy and can be thought of as the local field line displacement in the L coordinate.

The flux into a detector is proportional to the change in distribution function and as before, taking $f = f(\mu, W, L)$,

$$\delta f = -\delta W \frac{\partial f}{\partial W} - \delta L \frac{\partial f}{\partial L}$$

so

$$\delta f = -\delta W_g \frac{\partial f}{\partial W} - \delta W_A \left(\frac{\partial f}{\partial W} + \frac{m}{q\omega B_e L R_E^2}\frac{\partial f}{\partial L}\right) + \frac{irE_\phi(\mathbf{r}, t)}{\omega B_e L R_E^2}\frac{\partial f}{\partial L} \qquad (25)$$

The partial derivatives in equation (25) are taken at constant μ. In the discussion of 90° pitch angle particles (e.g., in (15) and (17)) the derivatives are taken at a fixed pitched angle (α = 90°). If $\partial f/\partial\alpha = 0$ at α = 90°, and the initial distribution is symmetric about 90°, the derivatives in (25) and those used in the earlier section are the same at 90°. Equation (25) thus effectively subsumes the results of the previous section.

BOUNCING PARTICLES: CHARACTERISTICS OF THE PARTICLE RESPONSE

Inclusion of particle bounce motion certainly complicates matters. The general expression (25) for δf and (20a) and (20b) for δW_A are far more complex than the corresponding equations (17) and (5) for equatorially mirroring particles.

The complications arise because of the variation of wave field along the bounce orbit. However, because we are interested in the lowest frequency standing signals, the variation along **B** should be relatively slow and thus the dominant terms in the Fourier series (19*a*) and (19*b*) should be the leading terms. Further simplification can be made by directing our attention to what is seen near the equatorial plane. This is a useful special case; many observations are made in such circumstances. Lastly, simplification can be achieved by noting that there are clear instances where one or more terms in the equation we have derived are dominant. In this section we shall use what simplifying arguments we can to reduce the complexity of the general results.

At low enough energy, both $m\omega_d$ and low multiples of ω_b are small compared with ω. For such particles the bounce is unimportant. Their response is determined predominantly by the local electric field. Mathematically, this means the term in ω dominates the numerator and denominator of each Fourier component in (20*a*) or (20*b*). One finds (by reference to (18), (19*a*), and (19*b*))

$$\delta W_A(\mathbf{r}, t) \simeq \frac{-iqE_\phi(\mathbf{r}, t)v_d(\mathbf{r})}{\omega} \tag{26}$$

Now note that

$$\omega_b \sim v/LR_E$$

and

$$v_d \sim \frac{v^2}{\Omega LR_E}$$

so that if $\omega \gg \omega_b$, we also have

$$\delta W_A \ll \frac{qE_\phi v_d}{\omega_b} \ll \frac{qE_\phi v}{\Omega} \sim \delta W_G$$

and thus we approximately have (see (25))

$$\delta f = -\delta W_G \frac{\partial f}{\partial W} - \frac{iE_\phi}{\omega BR_E}\frac{\partial f}{\partial L}$$

It is straightforward to check also that the former term will dominate unless the spatial gradient scale, l, is short enough that

$$l > v/\omega$$

i.e., very steep gradients would be required. We thus generally expect gyration acceleration to dominate for $\omega \gg \omega_b$, $m\bar{\omega}_d$.

Once particle energy is high enough that $\omega \sim \omega_b$ (but still $\omega > m\bar{\omega}_d$), none of the above arguments hold. Gyration scale energy changes, adiabatic energy changes and changes in L due to convection can all have comparable effects. For comparison, let us introduce an adiabatic gradient, defined by analogy with our earlier definition (15) for 90° equatorial pitch angle particles (see also, e.g., *Southwood and Kivelson* [1975]):

$$(\partial f/\partial L)_{\text{ad}} = (\partial W/\partial L)_{\mu,J}\,(\partial f/\partial W)$$
$$= qB_e\bar{\omega}_d R_E{}^2 L\,\frac{\partial f}{\partial W} = \frac{\nu W}{L}\frac{\partial f}{\partial W}$$

where ν depends on pitch angle and lies between 2 and 3 [*Southwood and Kivelson*, 1975].

If $|\partial f/\partial L| \gg |(\partial f/\partial L)_{\text{ad}}|$, the gradient effect will dominate the distribution change, but in many practical circumstances all effects are of similar magnitude. However, each effect does have distinguishable features. Gyration acceleration has the unique feature that it provides signals in antiphase in detectors looking in opposite directions when projected into a plane perpendicular to **B**. Gyration acceleration is proportional to the local value of the wave electric field and is largest for 90° pitch angle particles (see (3)). Gyration acceleration is negligible wherever the local E field is small. In particular, it is negligible near the equator in an antisymmetric wave, as there is an E field node at the equator (see Figure 2).

The gradient effect is proportional to the local field line displacement in the meridian. In a standing wave, this displacement is in quadrature with the local east-west electric field. This effect is omnidirectional; it provides the same flux whatever the detector look direction. The effect, as well, is small at any point along **B** where the local electric field is small.

The adiabatic energy change may be large even where the local electric field is small, as will become clear. The adiabatic energy change's unique feature is its dependence on bounce phase (see (20*a*) or (20*b*)) when $\omega \sim \omega_b$. A bounce phase dependence can be detected by two detectors back to back. If one detector is seeing particles with pitch angle α, the other sees pitch angle $\pi - \alpha$. In other words, it sees particles of the same energy and pitch angle coming in the opposite sense with respect to **B**, i.e., at a different phase of their bounce motion.

The bounce phase dependence of δW_A is a strong function of the wave symmetry with respect to the equator. This is shown up particularly if one considers what happens near the equator where $\theta = 0, \pi$. Inspection of (20*a*) and (20*b*) shows

$$\delta W_A(\theta = 0) = \delta W_A(\theta = \pi)$$

for a symmetric disturbance. In other words, the bounce dependence would be negligible near the equator. In sharp contrast, (20*b*) shows that

$$\delta W_A(\theta = 0) = -\delta W_A(\theta = \pi)$$

for an antisymmetric disturbance. In this case, back-to-back detectors should see flux oscillations in antiphase. At other points of the field line, there should be some phase difference between back-to-back detectors in both cases, but the effect will always be strongest for an antisymmetric wave because the symmetric wave provides a bounce independent contribution to δW_A, the term proportional to $\dot{W}_0$ in the Fourier series (20*a*).

There is also a pitch angle dependence in δW_A. Again consider particles seen near the equator. Ninety degree pitch angle particles have experienced the wave field only near the equator. If the wave is symmetric about the equator, the E field has a local maximum there, and so 90° pitch angle particles are accelerated more than particles of smaller pitch angle, which spend part of their bounce in weaker E fields. An antisymmetric disturbance has the reverse effect. Ninety degree particles see no electric field and receive no acceleration. Particles of smaller pitch angle receive more acceleration, as wave E field amplitudes are large far from the equator.

There is one other feature of the adiabatic acceleration term in (25), namely, that resonances occur when $\omega \sim \omega_b$, $\omega \sim 2\omega_b$, etc. As long as we are concerned with the lowest frequency signals standing along **B**, the lowest harmonic resonances are likely to be the only significant ones. The $\omega \sim \omega_b$ resonance is possible only with an antisymmetric disturbance; the second bounce harmonic resonance can occur only with a symmetric signal. Many of the comments of the preceding two paragraphs carry over to the resonance case. Near the equator, the antisymmetric mode will produce the most obvious discrepancies in back-to-back detectors looking up and down with respect to **B**; signals should be close to 180° out of phase between detectors, and the most effectively accelerated particles mirror some way off the equator. The particles most accelerated in the $\omega = \omega_b$ resonance will also mirror some way off the equator; for the resonant Fourier coefficient $\dot{W}_2$ to be important in the series (19*a*) and (20*a*), the particle must see substantial variation along its bounce orbit, i.e., must travel far off the equator.

What distinguishes an energy channel containing resonant particles from neighboring channels is the phase of the oscillation and also possibly the amplitude. The amplitude should stand out above neighbors if the detector energy bandwidth is much less than the central energy, a condition not fulfilled by many detectors. Inspection of the acceleration expression (20*c*) for particles near resonance shows that the phase of δW_A with respect to $\dot{W}_N$ is a function of energy. Particles just above and below resonant energy oscillate in quadrature with those in exact resonance, and there is a net phase difference of ~180° between the lower and higher energy particles. This latter feature could be an unreliable diagnostic if taken too literally as resonance may affect a narrow range of energies. If one is making measurements at a set of fixed energies, the channels adjacent to resonance may still be far enough away in energy to have their response dominated by one of the alternative effects. It is appropriate to note here that in a standing wave structure, the wave electric field at most changes sign as one moves along **B**; in no other way will the signal phase vary along **B**. The Fourier components, $\dot{W}_N$, as defined here, are thus all in phase or exact antiphase with E_ϕ at any point along *B*. Equation (20*c*) shows the resonant response is thus in phase or exact antiphase with the local east-west component of the electric field.

Let us now consider particles with energies such that $\omega_b \gg \omega$. We still require $\omega_b \gg m\omega_d$. Naturally, the symmetry about the equator is very important in classifying the possible behavior of these particles, but because they make many bounces in a wave period, their net adiabatic acceleration is an average of the acceleration they see over their bounce. If the wave is antisymmetric, that average is close to zero, and one can conclude that the adiabatic contribution to the flux variation is likely to be small. Gyration acceleration is also likely to be insignificant for these particles. The gradient term should dominate in an antisymmetric disturbance if

$$\partial f/\partial L \gtrsim (\partial f/\partial L)_{ad}\,(\omega L/v) \tag{27}$$

Condition (27) is not very stringent. When it holds and the gradient is dominant, the flux should oscillate in quadrature with the local value of E_ϕ and show no dependence on detector look direction with respect to **B** at fixed pitch angle.

The situation is more complicated for symmetric waves. The adiabatic acceleration need not be negligible; the first term in the Fourier series for δW_A in (20*a*) becomes dominant once $\omega_b \gg \omega$. It is straightforward to check that gyration acceleration should be a small effect in general; thus the adiabatic term competes with the gradient term. For comparison, it is useful to introduce a mean electric field $\bar{E}_\phi$ defined so that

$$W_0 = q\bar{E}_\phi L R_E \bar{\omega}_d$$

Once more introducing $(\partial f/\partial L)_{ad}$, we can rearrange the gradient and adiabatic contributions to (25) to give

$$\delta f = \frac{i\bar{E}_\phi}{R_E B_e(\omega - m\bar{\omega}_d)}\left\{\left[\frac{\partial f}{\partial L}\right]_{ad} - \frac{\partial f}{\partial L}\right\} + \frac{(1-\varepsilon)i\bar{E}_\phi}{B_e\omega R_E}\frac{\partial f}{\partial L} \tag{28}$$

where

$$\varepsilon = rE_\phi/(LR_E\bar{E}_\phi)$$

Equation (28) is superficially complicated. It does reduce to the much simpler form (17), previously derived for equatorially mirroring particles, when $\varepsilon = 1$, $\bar{E}_\phi = E_\phi$ and $\omega_d = \bar{\omega}_d$. One can note that ε will depart far from one only for particles mirroring far off the equator, and unless ε is much less than unity, the size of the flux variation is controlled by how far the local gradient departs from the adiabatic slope $(\partial f/\partial L)_{ad}$. Note that the resonant response when $\omega = m\bar{\omega}_d$ goes to zero if the distribution is adiabatic, a point of some significance in the theory of wave generation by resonance (see, e.g., *Southwood* [1976]).

At high energies, one can have $m\bar{\omega}_d \sim \omega_b \gg \omega$. This condition marks the border of our stated interest as the condition is precisely equivalent to the east-west wavelength being comparable to the particle Larmor radius. The condition is of no great significance for particles interacting with a symmetric disturbance, but it gives a further resonance possibility in an antisymmetric signal. Once again, bounce phase dependence becomes significant, and overall behavior in the vicinity of resonance is like that described above for the $\omega \sim \omega_b$ circumstance.

COMMENTS ON DETECTORS

In the preceding, we have delineated the varying ways that particles of different energies can interact with a transverse hydromagnetic wave, and have particularly concentrated on what controls the flux oscillations a detector looking in a particular direction should see. Table 1 summarizes the key features of the particle responses we have noted. We have pointed out specifically the effect of the finite energy bandwidth of any detector in limiting the resolution of resonant particles (originally briefly discussed by *Dungey and Southwood* [1971]). One needs also to recognize that any detector responds to a finite range of pitch angle, and because both bounce and drift frequencies are functions of pitch angle, a detector's pitch angle range may limit resolution.

We have pointed out the virtue of back-to-back detectors. Such a configuration can potentially distinguish gyrophase and bounce phase dependence, both of which are important diagnostic tests of competing effects. On many spacecraft, the same capability can be achieved by a single detector mounted at right angles to the spacecraft spin axis if the spin rate is rapid compared to the wave frequency.

Commonly, detectors scan in pitch angle. When this is done mechanically it can be very limiting for wave studies, as a full mechanical scan tends to take times comparable to wave periods. For instance, this limitation applies to West's detector [*West et al.*, 1973], energetic proton and electron data from which are displayed in Figure 1. Only waves with frequencies as low as the Pc 5 period range signal can be studied. Scanning of pitch angles is also a feature of the University of California at San Diego (UCSD) ion and electron electrostatic analyzer on ATS 6. The most effective work on low-frequency waves done using this detector [e.g., *Hughes et al.*, 1979] has

TABLE 1. Classification of Particle Responses

Wave Mode	Dominant Mechanism	Phase Relation to $\mathbf{b}(t)$	Relative Phase in Back-to-Back Detectors Near Equator	Pitch Angle Dependence Near Equator
		$\omega \gg \omega_b$ and $m\omega_d$		
Symmetric and antisymmetric disturbances	g	quadrature with $\mathbf{b} \cdot \hat{l}$	antiphase in detectors perpendicular to **B**	local 90° particles most strongly affected
		Nonresonant Particles: $\omega_b \sim \omega \gg m\omega_d$		
Symmetric disturbance (fundamental)	g, a, and c comparable	(g) in quadrature with $\hat{l} \cdot \mathbf{b}$ (a) phase relative to $\mathbf{b} \cdot \hat{n}$ varies with energy (c) at 0° or 180° relative to $\mathbf{b} \cdot \hat{n}$	(g) antiphase for detectors perpendicular to **B** (a) phase difference small in detectors parallel to **B** (c) no phase difference	(g) local 90° particles most strongly affected (a) strongest modulation for 90° particles (c) independent unless spatial gradients are pitch angle dependent
Antisymmetric disturbance (second harmonic)	g, a, and c comparable	(g) as above (a) as above (c) as above	(g) as above (a) phase difference near 180° in detectors parallel to **B** (c) no phase difference	(g) as above (a) strongest modulation for off-equatorially mirroring particles (c) as above
		Nonresonant Particles: $\omega_b \gg \omega$, $m\omega_d$		
Symmetric disturbance (fundamental)	a and c	in phase or antiphase	no phase difference	(a) strongest modulation for 90° particles (c) independent unless spatial gradients are pitch angle dependent
Antisymmetric disturbance (second harmonic)	c	in phase or antiphase	no phase difference	(c) as above
		Resonant Particles: $\omega \simeq \omega_b$		
Antisymmetric disturbance	a	in quadrature with $\hat{n} \cdot \mathbf{b}$	antiphase in detectors parallel to **B**	strongest modulation for off-equatorially mirroring particles
		Resonant Particles: $\omega \simeq m\omega_d$		
Symmetric disturbance	a	in quadrature with $\hat{n} \cdot \mathbf{b}$	no phase difference	strongest modulation for 90° particles

Mechanisms g, a, and c denote, respectively: gyration acceleration (equation (3)), adiabatic acceleration (for example, equations (5), (8), etc.), and convection of a gradient (for example, equations (13), (24), etc.). 'Antiphase' denotes 180° phase difference. The unit vector $\hat{n}$ is in the meridian plane, and $\hat{l}$ is the look direction of the detector. 'Parallel to **B**' for back-to-back detectors denotes that $\hat{l} \cdot \mathbf{B}$ has opposite signs in back-to-back detectors. 'Perpendicular to **B**' denotes that $\hat{l} \times \mathbf{B}$ has opposite signs in back-to-back detectors.

been during periods when no scanning was being done and measurements were being obtained at fixed pitch angle.

The UCSD ATS 6 instrument also scans in energy and this raises another timing problem. The instrument measures ions and electrons in 64 energy steps between 0 and 77 keV. In a typical cycle of measurements, it will dwell at a particular energy for 16 s and then step over the entire range through each channel in the remaining 32 s of the cycle. Once again, scanning is being done on the time scale of a hydromagnetic wave, and this is a limitation. *Hughes et al.* [1979] solved this problem in an ingenious way by ordering measurements by wave phase and putting together many measurements over many cycles to determine the phase relationship between the wave and particles in a particular energy channel.

The UCSD ATS 6 detector is an example of a detector with a very wide energy range and relatively good energy resolution. The same spacecraft carried an array of telescopes from NOAA's Space Environment Laboratory, which measured energetic ions above 25 keV. Data from these instruments have given rise to a series of papers on hydromagnetic wave and particle interactions [*Su et al.*, 1977, 1980]. The three telescopes are arranged in a rectangular triad so that one looks at a pitch angle in excess of 90°, while the other two look at pitch angles less than 90°. One detector has a look direction component eastward, another westward. The overall arrangement is appropriate for looking at gyrophase or bounce phase dependence. In the next section we shall argue that some of Su et al.'s conclusions in earlier papers may need revising in the light of our analysis here.

OBSERVATIONS

We have shown a variety of different particle responses is possible to a wave as simple as a transverse Alfvén wave. The crucial demarcation energies, where the character of the response changes, are when the bounce frequency, ω_b, and the Doppler shift due to east-west motion, $m\omega_d$, are comparable to the wave frequency.

At synchronous orbit a typical wave period observed is of the order of a minute. An electron with a 1-min bounce period in the synchronous orbit region has an energy of between 10 and 40 eV depending on pitch angle but protons with the same bounce period have energies between 20 to 80 keV. Heavier ions would in turn have proportionately higher energies. It is apparent that electron and ion behavior can well be controlled by very different effects over the range 10 eV to 80 keV, an energy range covered by many detectors flown up to now. Detection of electrons below ~10 eV is in fact a very difficult problem because of the spacecraft sheath of photoelectrons that typically exists and this means that ion measurements seem the most reliable way of looking at particles with $\omega \gg \omega_b$. Such measurements provide a diagnostic of the wave electric field because we have shown one can generally be sure that gyration acceleration is dominant. In the sample wave event illustrated in Figure 1, the response of the Lockheed light ion mass spectrometer is dominated by gyration acceleration. The detector look direction is along the spacecraft orbital direction. The vertical dashed lines on the figure show that B_x is exactly in quadrature with the low-energy ion flux (plotted as an equivalent density, the density the ions would have if the plasma were stationary). The coordinate system is chosen so that B_x aligns with the component of the look direction of the detector perpendicular to **B**. The electric field one infers from the ion flux oscillation must be at right angles to B_x. The fact that the flux oscillation is in quadrature with B_x indicates the corresponding electric field is in quadrature just as one would expect for a standing mode along **B**. As is implied by our expression (17) one needs to know $\partial f/\partial W$ to deduce the actual E field amplitude. Assuming the ions are cold, i.e., that the $E \times B$ drift much exceeds their thermal speed, *Kokubun et al.* [1977] find that wave electric fields of a few mV/m associated with transverse magnetic amplitudes of a few nanotesla. *Cummings et al.* [1978] actually measured the low-energy ion distribution during a wave event on ATS 6, derived electric amplitudes of a few mV/m and also found derived electric oscillations were in quadrature with the transverse magnetic perturbation, **b**. Knowledge of whether **E** leads or lags **b** can indicate whether a mode is symmetric or antisymmetric. *Cummings et al.* [1978] and *Kokubun et al.* [1977] deduce that the events they studied are symmetric about the equator (like the fundamental, see Figure 2).

Low-energy (<1 keV) ions certainly have their behavior dominated by gyration acceleration. The high-energy particles shown in Figure 1 certainly do not. The 79 keV electron channel shows oscillations roughly in phase with the x magnetic perturbation. The 158 keV electrons appear to switch phase relative to the 79 keV channel, somewhere near 0910 UT from being in phase to being in antiphase. Protons in the 120 keV channel show far less modulation as do the most energetic electrons shown (266 keV). The x direction, the look direction of the Lockheed ion detector, is along the spacecraft orbital direction (the ram direction), and the insert showing the orbit in Figure 1 shows that this has a substantial radial component. Correspondingly, the wave electric field must have a substantial east-west component. Adiabatic acceleration and convection associated with the east-west electric field should be what gives rise to the energetic particle flux oscillations. We can probably discount the importance of the convection effect because the count rate is varying little with radial distance. We thus predict from (25)

$$\delta f = -\delta L(\partial f/\partial L)_{\rm ad}$$
$$= \frac{iq\bar{E}_\phi \tilde{\omega}_d L R_E}{(\omega - m\tilde{\omega}_d)} \frac{\partial f}{\partial W}$$

We can see by inspection of Figure 1 that for electrons, $\partial f/\partial W < 0$. If $\omega > m\tilde{\omega}_d$, particle energy rises when the field is displaced earthward and so one expects the flux to rise when the field is displaced earthward. The spacecraft is some 20°–30° above the equator which means one expects B_x to be maximum when the field line is displaced inwards if the wave is symmetric about the equator (cf. Figure 2). This prediction fits with the 79 keV behavior. If we now assume the wave is moving eastward, the switch in phase of the 158 keV channel oscillations could be due to the $\omega \sim m\tilde{\omega}_d$ resonance lying within the channel at some time. Note that because the spacecraft is moving outwards the drift velocity of the particles seen in a particular channel increases with time (as $\tilde{\omega}_d \propto L$). We can thus suggest that near 0908 UT where the phase switch occurs the particles dominating the detector response were close to resonance. Prior to that point one has the response for $\omega > m\omega_d$, i.e., the signal is in phase with the lower energy channel, afterwards the signal is closer to being in antiphase and $\omega < m\omega_d$. These ideas fit well with the observed lower response in the 120 keV proton channel. If the wave is moving eastward, drift resonance does not occur for protons and in the energy range where $|\omega| \sim |m\tilde{\omega}_d|$ the response should be much smaller than for electrons of comparable energy. No very clear actual resonant response is seen in the 158 keV channel; the amplitude does not become extreme. The nominal energy of the channel is 158 ± 36 keV. The drift velocity of a 150 keV particle at $L \sim 7$ is of the order of 60 km/s. If we take this as the wave phase velocity and map it to the expected magnetopause in the mid-afternoon one arrives at ~100 km/s. This speed and the eastward direction deduced for the phase velocity are consistent with wave energy being fed by the Kelvin-Helmholtz instability at the magnetopause [*Southwood*, 1974; *Chen and Hasegawa*, 1974] as *Kokubun et al.* [1977] remark.

The *Kokubun et al.* [1977] event appears to represent a symmetric mode. The best evidence for observation of particle modulation in an antisymmetric transverse mode comes in a paper by *Sú et al.* [1980], who report energetic ion flux oscillations which are in antiphase in detectors looking up and down with respect to the magnetic field direction. The amplitude of the oscillation is larger in the detector looking at the smallest pitch angle. The events occur in conjunction with transverse magnetic signals. *Su et al.* [1980] discuss in detail an event seen with the NOAA low-energy positive ion detector on the ATS 6 spacecraft on October 23, 1974. The wave is close to linearly polarized. The perturbation magnetic field makes an angle of 80° with respect to the background field and has a substantial component in the meridian. The phase difference between the oscillations in flux and the magnetic oscillations is a function of energy. It shifts from about 180° for the 47.8–70.8 keV channel to about 120° for the 100.2–150.5 keV channel. The wave has a 96 s period and it is not unreasonable to consider that the >100 keV ions were bounce resonant. This idea fits in two significant ways. The 100.2–150.5 keV channel shows the biggest oscillations and the oscillations in this channel are nearly in quadrature with the magnetic perturbation. If the wave is a standing wave the 100.2–150.5 keV ions would thus be oscillating in phase with the local electric field as one expects for resonance (cf. (21)).

The above facts argue strongly for the October 23, 1974, event's being an example of a transverse mode with antisymmetric form about the equator which is setting up bounce modulation of ion fluxes. This may not however be the full story as a preliminary study of low-energy ion behavior using the UCSD detector on the same spacecraft reveals an electric field in quadrature with the magnetic perturbation but with the wrong phase relationship for an even mode (cf. Figure 2) (J. Quinn, personal communication, 1979). More study should be fruitful.

CONCLUDING REMARKS

We have attempted in this paper to delimit particle behavior in a hydromagnetic wave with a very simple polarization, namely, a purely transverse wave. Our emphasis has been on what a spacecraft-borne detector would see during a wave event. As Table 1 indicates, there are a large number of responses possible at different energy, different pitch angle, in detectors with different look directions and in disturbances with different amplitude distributions.

At low energies, acceleration of a particle over its most recent gyration is the important effect. We called this gyration acceleration. It is an effect which produces changes in flux which depend on the look direction of the detector used. For gyration acceleration the important parameter is the angle between the look direction and the wave-induced $E \times B$ drift. At higher energies another direction-dependent effect can be important; in particular, waves with electric field antisymmetrically distributed about the equator can produce strong bounce phase dependence, i.e., detectors looking in opposite directions with respect to the magnetic field see different responses in this case. Other effects, e.g., flux changes due to convection of a spatial gradient back and forth by the wave, produce a nondirectional effect.

The phase of the oscillatory response seen in a particular detector is a strong function of the energy measured and variation between energy channels of the phase of flux oscillations is expected. The dynamically significant circumstance under which this is found is when a group of energy channels contains particles capable of resonating with the wave, but this is far from the only circumstance where it is expected.

Some authors [*Hasegawa*, 1979; *Su et al.*, 1979] have appeared to suggest that a parallel electric field component is necessary to produce particular effects. *Hasegawa* [1979] suggests it is necessary to produce any interaction at all. *Su et al.* [1979] feel it should be introduced to explain flux differences in back-to-back detectors. By example here we have shown parallel electric fields need not be invoked in either case but we would not wish to suggest that parallel electric field components are necessarily unimportant. In further work we aim to include them. Important questions of wave dynamics hinge on the existence of such components, and it would be useful to establish a diagnostic test for them.

We have also ignored the effect of a compressional magnetic component. In this instance we know that signals in the low-frequency pulsation band commonly have such polarization [e.g., *Sonnerup et al.*, 1969; *Barfield and McPherron*, 1978; *Hedgecock*, 1976]. Future work should thus also be directed at including the compressional component.

We have only briefly discussed observations here, but other wave-particle events of various types have already been described in the literature and merit further analysis. Because particle flux oscillations can contain unique information about the wave characteristics locally and also far from the point of measurement, studies of oscillations are worthwhile and should be an integral part of hydromagnetic wave analysis in the magnetosphere.

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