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Charged Vector Particles Tunneling From Black Ring and 5D Black Hole

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Abstract

In this paper, we have investigated the Hawking radiations process as a semiclassical quantum tunneling phenomenon from black ring and $5D$ Myers-Perry black holes. Using Lagrangian of Glashow-Weinberg-Salam model with background electromagnetic field (for charged W-bosons) and the WKB approximation, we have evaluated the tunneling rate/probability of charged vector particles at through the horizons by taking into account the electromagnetic vector potential. Moreover, we have calculated the corresponding Hawking temperature via Boltzmann factor for both type of the considered background and analyzed the whole spectrum generally.

Keywords: Black rings; Myers-Perry black holes; Charged vector particles; Semiclassical quantum tunneling phenomenon; Hawking radiation.
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1 Introduction

In the early universe, by assuming the quantum back ground, Hawking observed that black hole (BH) emits particles and the spectrum of the passed

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off particles is purely thermal. Hawking expressed the emission spectrum for all kinds of particles (neutrinos, photons, gravitons, electrons, positrons etc.) and found the emission rate for these particles. The vector particles or vector bosons (with spin equals to 1), i.e., $W^{\pm}$ and Z are important components of the standard model for electroweak interactions, therefore the emission of such particles should also be of more significance in the analysis of Hawking radiations. Different methods have been suggested for the analysis of Hawking radiations as a tunneling spectrum across the horizons of BHs. The tunneling method is based on the fundamental physical particles action which allows BH radiations.

Existence of the positive and negative energy pair of virtual particles is similar to the existence of particle and anti-particle pair creation, respectively. Instantaneously, the negative and positive energy virtual particles are created and annihilated in form of pairs. The positive energy virtual particle disappears by tunnel through the horizon and it is emitted as a depart of the Hawking radiation, whereas, the negative energy particle goes inside the BH or absorbed by the BH. The rule of conservation of energy is adapted in the procedure. The specific value of temperature at which particles are emitted from the BH is known as Hawking temperature.

Emparan and Reall [1] discussed a rotating 5D black ring. Lim and Teo [2] studied the particles motion in the spacetime of the rotating black ring. Chen and Teo [3] discussed rotating black rings on Taub-NUT space by applying the inverse-scattering procedure. Matsumoto et al.[4] pointed out the loss in BH mass and angular momenta through Hawking radiations. Cvetic and Guber [5] discussed regular charged BHs (particular BHs in gauged supergravity in $D = 5$ and $D = 7$). Aliev [6] considered that a BH may posses a small electric charged and constructed a 5-vector potential for electromagnetic field in the background of Myers-Perry metric. Sharif and Wajiha [7] have computed the Hawking radiation spectrum for charged fermions tunneling from charged accelerating and rotating BHs with NUT parameter. Jan and Gohar [8] applied the Hamilton-Jacobi method to the Klein-Gordon equation by using WKB approximation. They determined the tunneling probability of outgoing charged scalar particles from the event horizon of BH and also detect the corresponding Hawking temperature. Kruglov [9] studied the Hawking radiation of spin-1 particles from BHs in (1+1)-dimensions by using Proca equation. The procedure has been used as quantum tunneling of bosons through the event horizon. Also, he has established the result that the emission temperature with the Schwarzschild background geometry is similar to the Hawking temperature of scalar particles emission. Li and Zu [10] have studied the tunneling rate and Hawking temperature of scalar particles for Gibbons-Maeda-Dilation BH.

Feng et al.[11] have analyzed the Hawking radiation of vector particles from $4D$ and $5D$ BHs. Saleh et al. [12] obtained the Hawking radiations from a $5D$ Lovelock BH by applying the Hamilton-Jacobi procedure. Lin et al. [13] studied the outgoing and ingoing probabilities and temperature of the spin-$\frac{3}{2}$ particles. Chen and Huang [14] studied the Hawking radiation of vector particles for Vaidya BHs. Li and Chen [15] looked into vector particles tunneling (uncharged and charged bosons) from the Kerr and Kerr-Newman BHs. Singh et al. [16] have used the WKB approximation and Hamiliton-Jacobi ansatz to the Proca equation and calculated the tunneling rate and Hawking temperature of vector bosons for Kerr-Newman BH. In this paper, the Hawking temperature is discussed for three coordinate systems. Gursel and Sakalli [17] studied the Hawking radiation of massive vector particles from rotating warped anti-de Sitter BH in 3D and showed that the radial mapping gives the tunneling rate for outgoing particles. Different authors [18, 19] have found the Hawking temperature for various types of particles from BHs. Li and Zhao [20] analyzed the massive vector particles tunneling and found the Hawking temperature from the neutral rotating anti-de Sitter BHs in conformal gravity by applying tunneling procedure. Jusufi and Övgün [21] studied the quantum gravity effects on the Hawking radiation of charged spin-1 particles for noncommutative charged BHs, RN BHs and charged BHs.

Sakalli and Övgün [22] have investigated the Hawking radiations phenomenon as a tunneling spectrum of vector particles for Lorentzian wormholes in (3+1)-dimensions. Moreover, they investigated radiation process of spin-2 massive particles for a generic (3+1)-dimensional static BH. They also studied Parikh-Wilczek tunneling method for resolving the information loss problem during the process of Hawking radiation. Also, they have investigated the quantum gravitational effects on Hawking radiation of massive scalar particles for rotating acoustic BHs. The same authors [23] have analyzed the radiations spectrum of vector particles for $5D$ Kerr-Gödel spacetime. Qian-Li [24] has investigated the tunneling of scalar, fermions and massive bosons for $5D$ Schwarzschild-like BH. All the above mentioned studies predict that scalars, fermions and vector particles impart the same effective Hawking temperature.

In order to study Hawking temperature by using Kerner and Mann's formulation, Sharif and Javed [25] have studied the fermions tunneling phenomenon through the horizons for charged anti-de Sitter BHs, charged torus-like BHs, *Plebański − Demiański* family of BHs, regular BHs and and traversable wormholes. The same authors [26, 27] have discussed the Hawking radiations as fermions tunneling for a pair of charged accelerating and rotating BHs with NUT parameter. They have calculated the corresponding Hawking temperature. Javed et al.[28] have discussed the charged vector particle tunneling from a pair of accelerating and rotating and 5D gauged super-gravity BHs.

Jusufi and Övgün [29] have investigated Hawking radiation phenomenon through quantum tunneling effects of scalar and vector particles for rotating 5-dimensional Myres-Perry BH. They recovered the same Hawking temperature for both vector and scalar particles and discussed the quantum tunneling in Painleve coordinates system and co-rotating coordinates system. They showed that the coordinates systems do not affect the Hawking temperature. Hawking temperature is independent of the selection of the coordinate system. Sakalli and Övgün [30] have studied the Hawking radiation of massive vector (spin-1) particles for Schwarzschild BH in Kruskal-Szekeres and dynamic Lemaitre coordinates systems. They obtained the original Hawking temperature of Schwarzschild BH in both coordinates systems. Thus, the Hawking radiation is independent of the coordinates systems.

It is to be observed that the kinetics of charged vector particles are ruled along the Proca field equations by using WKB approximation and the tunneling rate and Hawking temperature for the passed off particles can be deduced. It is observed that the tunneling effects are not associated to the mass of BHs but depends on the mass/energy of the outgoing particles. In this paper, we have explored the charged vector particles tunneling for 5D spacetimes with electromagnetic background. This paper is organized as follows: We discuss in the section **2**, the tunneling rate and Hawking temperature of charged vector $W^{\pm}$-bosons for black rings. Section **3** is based on the analysis for Myers-Perry metric. Section **4** deals the summary of the work.

2 Rotating Charged Black Ring

In this section, we have analyzed the Hawking radiation process as tunneling of charged vector particles from 5D charged black rings. The discussion of tunneling phenomenon for Dirac particles from 5D charged black rings is studied in Ref.[31]. The black rings can go around in the azimuthal focus of the S^2. The result is identifying a revolving black ring, which takes essential conic singularity, because at that place no centrifugal force exists to produce equilibrium due to self-gravity of black rings. The black ring reduces to the electrically charged BH, as the NUT parameter reduces to the similar Kaluza-Klein monopole [3].

The study of BH thermodynamics has significance in gravitational physics. Laws of BH thermodynamics have suggested that BHs have finite temperature (known as Hawking temperature) which is proportional to their surface gravity and BH entropy is proportional to the horizon area, which corresponds to the first law of thermodynamics. The thermodynamical relationships for 5D physical objects at horizons are more complicated as compare to BHs, in particular for spinning black rings and black saturns. Matsmoto *et al.*[4] have studied the time evolution of thin black ring via Hawking radiation. They observed the energy and angular momentum of emitted scalar particles. The minimum (or zero) velocity is deduced for black ring which allows a limit for jumping the particles to be bounded inside the black ring. The analysis is based on geodesics to analyze the complicated mathematical structure for particles to go across through the ring [32].

In this section, we focus on studying Hawking radiation of charged vector particles via tunneling from $5D$ rotating charged black ring, which is special solution of the Einstein-Maxwell-Dilaton gravity model (EMD) in $5D$. The line element of black rings in a unit electric charged can be written in the following form [31]

$$\begin{aligned} ds^2 &= -\frac{-F(y)}{F(x)K^2(x,y)}\left(dt - C(\nu,\lambda)R\frac{1+y}{F(y)}\cosh^2\alpha d\phi\right)^2 \\ &+\frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\phi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\varphi^2\right], \quad (2.1) \end{aligned}$$

where

$$\begin{aligned} C(\nu,\lambda) &= \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}} \quad K(x,y) = 1+\lambda(x-y)\sinh^2\alpha F(x) \\ F(\zeta) &= 1+\lambda\zeta, \quad G(\zeta) = (1-\zeta^2)(1+\nu\zeta) \end{aligned}$$

and parameters λ and ν are dimensionless and takes values in the range of $0 < \nu \leq \lambda < 1$ and does not take the conical singularity at $x = 1$, λ and ν associated to each other say $\lambda = 2\nu/(1+\nu^2)$ and α is the parameter acting as the electric charge. The coordinate φ and ϕ are two cycles of the black ring and x and y the values rang $-1 \leq x \leq 1$ and $-\infty \leq y \leq -1$ and R has the dimensional length. The explicit calculation of the electric charged is $Q = 2M \sinh 2\alpha \diagup (3(1 + \frac{4}{3}\sinh^2\alpha))$. The mass of the black ring is $M = 3\pi R^2\lambda/[4(1-\nu)]$, and its angular momentum takes the form $J = \pi R^3 p\sqrt{\lambda(\lambda-\nu)(1+\lambda)}/[2(1-\nu)^2]$.

The line element given by Eq.(2.1) can be rewritten as

$$ds^2 = Adt^2 + Bd\phi^2 + Cdy^2 + Ddx^2 + Ed\varphi^2 + 2Fdtd\phi. \tag{2.2}$$

The charged bosons $(W^{\pm})$ act differently as from the uncharged bosons (Z) in the presence of electromagnetic field. The W^{+} bosons behave similarly as W^{-} bosons and their tunneling processes are similar too. In order to calculate the tunneling rate for charged vector particles through the BH's event horizon r_+, we will consider the Proca equation with electromagnetic effects. The motion of massive spin-1 charged vector field is traced out by the given Proca equation in the background of electromagnetic field, i.e.,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\psi^{\nu\mu}) + \frac{m^2}{h^2}\psi^\nu + \frac{i}{h}eA_\mu\psi^{\nu\mu} + \frac{i}{h}eF^{\nu\mu}\psi_\mu = 0, \tag{2.3}$$

where the anti-symmetric tensor $\psi^{\mu\nu}$ is defined by

$$\psi_{\nu\mu} = \partial_\nu\psi_\mu - \partial_\mu\psi_\nu + \frac{i}{h}eA_\nu\psi_\mu - \frac{i}{h}eA_\mu\psi_\nu \ \text{ and } \ F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\nu A^\mu.$$

The values of the components of ψ^μ and $\psi^{\nu\mu}$ are given as follows

$$\psi^0 = \frac{B}{AB-F^2}\psi_0 - \frac{F}{AB-F^2}\psi_1, \quad \psi^1 = \frac{-F}{AB-F^2}\psi_0 + \frac{A}{AB-F^2}\psi_1,$$
$$\psi^2 = \frac{1}{C}\psi_2, \quad \psi^3 = \frac{1}{D}\psi_3, \quad \psi^4 = \frac{1}{E}\psi_4, \quad \psi^{01} = \frac{F^2\psi_{10} + AB\psi_{01}}{(AB-F^2)^2},$$
$$\psi^{02} = \frac{B\psi_{02} - F\psi_{12}}{C(AB-F^2)}, \quad \psi^{03} = \frac{B\psi_{03} - F\psi_{13}}{D(AB-F^2)}, \quad \psi^{04} = \frac{B\psi_{04} - F\psi_{14}}{E(AB-F^2)},$$
$$\psi^{12} = \frac{-F\psi_{02} + A\psi_{12}}{C(AB-F^2)}, \quad \psi^{13} = \frac{-F\psi_{03} + A\psi_{13}}{D(AB-F^2)}, \quad \psi^{14} = \frac{-F\psi_{04} + A\psi_{14}}{E(AB-F^2)},$$
$$\psi^{23} = \frac{\psi_{23}}{DC}, \quad \psi^{24} = \frac{\psi_{24}}{EC}, \quad \psi^{34} = \frac{\psi_{34}}{DE}.$$

The electromagnetic vector potential is given by

$$A_\mu = A_t dt + A_\phi d\phi, \tag{2.4}$$

where

$$A_t = \frac{\lambda(x-y)\sinh\alpha\cosh\alpha}{F(x)K(x,y)}, \quad A_\phi = \frac{C(\nu,\lambda)R(1+y)\sinh\alpha\cosh\alpha}{F(x)K(x,y)}.$$

Using, WKB approximation [32], i.e.,

$$\psi_\nu = c_\nu \exp[\frac{i}{\hbar}S_0(t,r,\theta,\phi) + \Sigma\hbar^n S_n(t,r,\theta,\phi)], \tag{2.5}$$

to the Lagrangian (2.3), where c_ν are arbitrary constants, S_0 and S_n are arbitrary functions. By ignoring the terms for $n = 1, 2, 3, 4, ...$, we obtain the following set of equations

$$\frac{F^2}{AB-F^2}[c_0(\partial_1 S_0)^2 - c_1(\partial_1 S_0)(\partial_0 S_0) + c_0(\partial_1 S_0)eA_1 - c_1(\partial_1 S_0)eA_0]$$
$$+\frac{AB}{AB-F^2}[c_1(\partial_1 S_0)(\partial_0 S_0) - c_0(\partial_1 S_0) + c_1(\partial_1 S_0)eA_0 - c_0(\partial_1 S_0)eA_1]$$
$$+\frac{B}{C}[c_2(\partial_0 S_0)(\partial_2 S_0) - c_0(\partial_2 S_0)^2 + c_2(\partial_2 S_0)eA_0] - \frac{F}{C}[c_2(\partial_1 S_0)(\partial_2 S_0)$$
$$-c_1(\partial_2 S_0)^2 + c_2(\partial_2 S_0)eA_1] + \frac{B}{D}[c_3(\partial_0 S_0)(\partial_3 S_0) - c_0(\partial_3 S_0)^2 + c_3(\partial_3 S_0)$$

$$eA_0] - \frac{F}{D}[c_3(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_3 S_0)^2 + c_3(\partial_3 S_0)eA_1] + \frac{B}{E}[c_4(\partial_0 S_0)$$
$$(\partial_4 S_0) - c_0(\partial_4 S_0)^2 + c_4(\partial_4 S_0)eA_0] - \frac{F}{E}[c_4(\partial_1 S_0)(\partial_4 S_0) - c_1(\partial_4 S_0)^2$$
$$+c_4(\partial_4 S_0)eA_1] - m^2 Bc_0 + m^2 Fc_1 + \frac{eA_1 F^2}{AB - F^2}[c_0(\partial_1 S_0) - c_1(\partial_0 S_0)$$
$$+eA_1 c_0 - eA_0 c_1] + \frac{eA_1 AB}{AB - F^2}[c_1(\partial_0 S_0) - c_0(\partial_1 S_0) + eA_0 c_1 - eA_1 c_0]$$
$$= 0, \tag{2.6}$$
$$\frac{-F^2}{AB - F^2}[c_0(\partial_1 S_0)(\partial_0 S_0) - c_1(\partial_0 S_0)^2 + c_0(\partial_0 S_0)eA_1 - c_1(\partial_0 S_0)eA_0]$$
$$-\frac{AB}{AB - F^2}[c_1(\partial_0 S_0)^2 - c_0(\partial_1 S_0)(\partial_0 S_0) + c_1(\partial_0 S_0)eA_0 - c_0(\partial_0 S_0)eA_1]$$
$$-\frac{F}{C}[c_2(\partial_0 S_0)(\partial_2 S_0) - c_0(\partial_2 S_0)^2 + c_2(\partial_2 S_0)eA_0] + \frac{A}{C}[c_2(\partial_1 S_0)(\partial_2 S_0)$$
$$-c_1(\partial_2 S_0)^2 + c_2(\partial_2 S_0)eA_1] - \frac{F}{D}[c_3(\partial_0 S_0)(\partial_3 S_0) - c_0(\partial_3 S_0)^2 + c_3(\partial_3 S_0)$$
$$eA_0] + \frac{A}{D}[c_3(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_3 S_0)^2 + c_3(\partial_3 S_0)eA_1] - \frac{F}{E}[c_4(\partial_0 S_0)$$
$$(\partial_4 S_0) - c_0(\partial_4 S_0)^2 + c_4(\partial_4 S_0)eA_0] + \frac{A}{E}[c_4(\partial_1 S_0)(\partial_4 S_0) - c_1(\partial_4 S_0)^2$$
$$+c_4(\partial_4 S_0)eA_1] + m^2 Fc_0 - m^2 Ac_1 - \frac{eA_0 F^2}{AB - F^2}[c_0(\partial_1 S_0) - c_1(\partial_0 S_0)$$
$$-eA_1 c_0 + eA_0 c_1] + \frac{eA_0 AB}{AB - F^2}[c_3(\partial_1 S_0) - c_1(\partial_3 S_0) + eA_1 c_3] = 0, \tag{2.7}$$
$$\frac{F}{AB - F^2}[c_2(\partial_1 S_0)(\partial_0 S_0) - c_1(\partial_0 S_0)(\partial_1 S_0) + c_2(\partial_0 S_0)eA_1]$$
$$-\frac{B}{AB - F^2}[c_2(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_2 S_0) + c_2(\partial_0 S_0)eA_0]$$
$$+\frac{F}{AB - F^2}[c_2(\partial_0 S_1)(\partial_0 S_0) - c_0(\partial_2 S_0)(\partial_1 S_0) + c_2(\partial_1 S_0)eA_0]$$
$$-\frac{A}{AB - F^2}[c_2(\partial_1 S_0)^2 - c_1(\partial_2 S_0)(\partial_1 S_0) + c_2(\partial_1 S_0)eA_1] - \frac{1}{D}[c_3(\partial_2 S_0)$$
$$(\partial_3 S_0) - c_2(\partial_3 S_0)^2] + \frac{1}{E}[c_4(\partial_4 S_0)(\partial_2 S_0) - c_2(\partial_4 S_0)^2] - m^2 c_2 +$$
$$\frac{eA_0 F}{AB - F^2}[c_2(\partial_1 S_0) - c_1(\partial_2 S_0) + c_2 eA_1] - \frac{B}{AB - F^2}[c_2(\partial_0 S_0)$$

$$-c_0(\partial_2 S_0) + c_2 eA_0] + \frac{eA_1 F}{AB - F^2}[c_2(\partial_0 S_0) - c_0(\partial_2 S_0) + eA_0 c_2]$$
$$-\frac{eA_1 A}{AB - F^2}[c_2(\partial_1 S_0) - c_1(\partial_2 S_0) + eA_1 c_2] = 0, \quad (2.8)$$
$$-B[c_3(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_3 S_0) + c_3(\partial_0 S_0)eA_0] + F[c_3(\partial_0 S_0)(\partial_1 S_0)$$
$$-c_1(\partial_0 S_0)(\partial_3 S_0) + c_3(\partial_0 S_0)eA_1] + F[c_3(\partial_0 S_0)(\partial_1 S_0) - c_0(\partial_1 S_0)(\partial_3 S_0)$$
$$+c_3(\partial_1 S_0)eA_0] - A[c_3(\partial_1 S_0)^2 - c_1(\partial_1 S_0)(\partial_3 S_0) + c_3(\partial_1 S_0)eA_1]$$
$$-\frac{AB - F^2}{C}[c_3(\partial_2 S_0)^2 - c_2(\partial_2 S_0)(\partial_3 S_0)] + \frac{AB - F^2}{E}[c_4(\partial_3 S_0)(\partial_4 S_0) -$$
$$c_3(\partial_4 S_0)^2] - m^2 c_3 \frac{AB - F^2}{D} + eA_0 B[c_3(\partial_0 S_0) - c_0(\partial_3 S_0) + c_3 eA_0]$$
$$+eA_0 F[c_3(\partial_1 S_0) - c_1(\partial_3 S_0) + c_3 eA_1] + eA_1 F[c_3(\partial_0 S_0) - c_0(\partial_3 S_0) +$$
$$eA_0 c_3] - eA_1 A[c_3(\partial_1 S_0) - c_1(\partial_3 S_0) + eA_1 c_3] = 0, \quad (2.9)$$
$$F[c_4(\partial_0 S_0)(\partial_1 S_0) - c_1(\partial_0 S_0)(\partial_4 S_0) + c_4(\partial_0 S_0)eA_1] - B[c_4(\partial_0 S_0)^2$$
$$-c_0(\partial_0 S_0)(\partial_4 S_0) + c_4(\partial_0 S_0)eA_0] + F[c_4(\partial_0 S_0)(\partial_1 S_0) - c_0(\partial_1 S_0)(\partial_4 S_0)$$
$$+c_4(\partial_1 S_0)eA_0] - A[c_4(\partial_1 S_0)^2 - c_1(\partial_1 S_0)(\partial_4 S_0) + c_4(\partial_1 S_0)eA_1]$$
$$-\frac{AB - F^2}{C}[c_4(\partial_2 S_0)^2 - c_2(\partial_2 S_0)(\partial_4 S_0)] - \frac{AB - F^2}{D}[c_4(\partial_3 S_0)^2$$
$$-c_3(\partial_4 S_0)(\partial_3 S_0)] - m^2 c_4 AB - F^2 + eA_0 F[c_4(\partial_1 S_0) - c_1(\partial_4 S_0)$$
$$+c_4 eA_1] - eA_0 B[c_4(\partial_0 S_0) - c_0(\partial_4 S_0) + c_4 eA_0] + eA_1 F[c_4(\partial_0 S_0)$$
$$-c_0(\partial_4 S_0) + eA_0 c_4] - eA_1 A[c_4(\partial_1 S_0) - c_1(\partial_4 S_0) + eA_1 c_4] = 0. \quad (2.10)$$

Using the following rule for separation of variables [33], i.e.,

$$S_0 = -(E - \sum_{i=1}^{2} j_i \check{\Omega}_i)t + J\phi + W(x, y) + L\varphi, \quad (2.11)$$

where E is the energy of the particle, J and L represent the particle's angular momentums corresponding to the angles ϕ and φ, respectively, while K is the complex constant. From Eqs.(2.6)-(2.10), we can obtain a 5×5 matrix equation

$$G(c_0, c_1, c_2, c_3, c_4)^T = 0,$$

where the matrix is labeled as "**G**", whose components are given as follows:

$$
\begin{aligned}
G_{00} &= \frac{1}{AB-F^2}[F^2J^2+eA_1]-AB[J-eA_1J]-\frac{1}{C}(\dot{W}^2-\frac{1}{D}(\partial_3 W)^2 \\
&- \frac{L^2}{E}-m^2B+\frac{eA_1}{AB-F^2}[F^2J-eA_1]-AB[J+eA_1] \\
G_{01} &= \frac{1}{AB-F^2}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)J-eA_0J]-AB[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)J-eA_0J] \\
&+ F[(\partial_2W)^2+eA_2(\partial_2W)]-F[(\partial_3W)^2-L^2]+\frac{eA_1}{AB-F^2}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) \\
&- eA_0]-AB[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)-eA_0] \\
G_{02} &= \frac{B}{C}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_2W)+eA_0(\partial_2W)]-F[J(\partial_3W)+eA_1(\partial_2W)] \\
G_{03} &= \frac{B}{D}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_3W)+eA_0(\partial_3W)]-F[J(\partial_3W)+eA_1(\partial_3W)] \\
G_{04} &= \frac{B}{E}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)L+eA_0L]+F[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)L-eA_1L] \\
G_{10} &= \frac{F^2}{AB-F^2}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)J+eA_1(E-j\check{\Omega})]+AB[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)J \\
&-eA_1(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)]+\frac{F}{C}(\partial_2W)^2+\frac{F}{D}(\partial_3W)^2+\frac{F}{E}L^2+m^2F \\
&-\frac{eA_0F^2}{AB-F^2}[J-eA_0] \\
G_{11} &= \frac{1}{AB-F^2}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)^2-eA_0(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)]+AB[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)^2 \\
&-eA_0(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)]-\frac{A}{C}(\partial_2W)+\frac{1}{D}(\partial_3W)^2A-\frac{1}{E}AL^2-m^2A \\
&-\frac{eA_0F^2}{AB-F^2}[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)-eA_0-(\partial_3W)AB]
\end{aligned}
$$

$$
\begin{aligned}
G_{12} &= \frac{F}{C}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)+eA_0(\partial_2 W)]+\frac{A}{C}[J(\partial_2 W)+eA_1(\partial_2 W)] \\
G_{13} &= \frac{F}{D}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_3 W)-eA_0(\partial_3 W)]+\frac{A}{D}[J(\partial_3 W)+eA_1(\partial_3 W)] \\
&+ \frac{eA_0}{AB-F^2}[ABJ+ABeA_1] \\
G_{14} &= \frac{F}{E}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)L-eA_0L]+\frac{A}{E}[JL+eA_1L] \\
G_{20} &= \frac{-B}{AB-F^2}(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_2 W)-\frac{1}{AB-F^2}(\partial_2 W)+\frac{eA_0B}{AB-F^2}(\partial_2 W) \\
&- \frac{eA_1}{AB-F^2}(\partial_2 W) \\
G_{21} &= \frac{F}{AB-F^2}(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_2 W)+\frac{A}{AB-F^2}J(\partial_2 W)-\frac{eA_0}{AB-F^2}(\partial_2 W) \\
&+ \frac{eA_1A}{AB-F^2}(\partial_2 W) \\
G_{22} &= \frac{-F}{AB-F^2}[J(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)+eA_1(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)]-\frac{B}{AB-F^2}[(E \\
&\quad -\sum_{i=1}^{2} j_i\check{\Omega}_i)^2-eA_0(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)]-\frac{F}{AB-F^2}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)J-eA_0J] \\
&\quad -\frac{A}{AB-F^2}[J^2+eA_1J]-\frac{1}{D}(\partial_3 W)^2-\frac{L^2}{E}-m^2+\frac{eA_0F}{AB-F^2}[J+eA_1] \\
&+ \frac{B}{AB-F^2}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)+eA_0]-\frac{eA_1F}{AB-F^2}[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)+eA_0] \\
&- \frac{eA_1A}{AB-F^2}[J+eA_1], \qquad G_{23}=\frac{1}{D}(\partial_2 W)(\partial_3 W) \qquad G_{24}=\frac{1}{E}(\partial_2 W)L \\
G_{30} &= -B(\partial_3 W)(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)-FJ(\partial_3 W)-eA_0(\partial_3 W)+eA_1F(\partial_3 W) \\
G_{31} &= F(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_3 W)+AJ-eA_0F(\partial_3 W)+eA_1A(\partial_3 W) \\
G_{32} &= \frac{AB-F^2}{C}(\partial_2 W)(\partial_3 W)
\end{aligned}
$$

$$
\begin{aligned}
G_{33} &= -B[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)^2 - eA_0(E-j\check{\Omega})] - F[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)J \\
&\quad -eA_1(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)] - F[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)J - eA_0J] - \\
&\quad A[J^2+eA_1J] - \frac{AB-F^2}{C}(\partial_2 W)^2 - \frac{AB-F^2}{E}L - \\
&\quad \frac{AB-F^2}{D}m^2 - eA_0B[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] + eA_0F[J+eA_1] \\
&\quad -eA_1F[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - AeA_1[J+eA_1] \\
G_{34} &= \frac{AB-F^2}{E}(\partial_3 W)L \\
G_{40} &= -B(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)L - FJL + eA_0BL - eA_1FL \\
G_{41} &= F(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)L + AJL - eA_0FL + eA_1A \\
G_{42} &= \frac{AB-F^2}{C}(\partial_2 W)L, \quad G_{43} = \frac{AB-F^2}{D}(\partial_3 W)L \\
G_{44} &= -F(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)J - FeA_1(E-j\check{\Omega}) - B[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)^2 - \\
&\quad eA_0(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)] - F[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0J] - A[J^2+eA_1J] \\
&\quad -\frac{AB-F^2}{C}(\partial_2 W)^2 - \frac{AB-F^2}{D}(\partial_3 W)^2 - m^2(AB-F^2) + eA_0F \\
&\quad [J+eA_1] + eA_0B[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - eA_1F[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) \\
&\quad +eA_0] - eA_1A[J+eA_1].
\end{aligned}
$$

For the non-trivial solution, the absolute value $|\mathbf{G}|$ is equal to *zero*, so that one can yield

$$\begin{aligned} ImW^{\pm} &= \pm \int \sqrt{\frac{(E - \sum_{i=1}^{2} j_i \check{\Omega}_i - eA_0 - \Omega_h eA_1)^2 + X}{\frac{F^2 - AB}{BD}}}, \\ &= \pm i\pi \frac{E - eA_0 - \sum_{i=1}^{2} j_i \check{\Omega}_i - \Omega_h eA_1}{2\kappa(r_+)}, \end{aligned} \tag{2.12}$$

where $-\Omega_h = \frac{F^2}{B^2}$, while + and − represent the outgoing and incoming particles, respectively. The function X can be defined as

$$\begin{aligned} X &= \frac{FJ}{B}(E - j\check{\Omega}) - e^2 A_1^2 \frac{F}{B}[\frac{F}{B} - 1] + \frac{FJ}{B}[(E - j\check{\Omega}) - eA_0] + \frac{AJ}{B} \\ & [J + eA_1] + m^2 \frac{(AB - F^2)}{B} - eA_0 \frac{F}{B} J + eA_1 \frac{A}{B}(J + eA_1) \end{aligned} \tag{2.13}$$

and the surface gravity is

$$\kappa(r_+) = \frac{1}{2}\sqrt{K_y(x,y) H_y(x,y)},$$

where we have choosed $-C_y(x,y) = K_y(x,y)$ and $D_y^{-1}(x,y) = H_y(x,y)$. The tunneling probability for charged vector particles is given by

$$\Gamma = \frac{\Gamma_{(emission)}}{\Gamma_{(absorption)}} = \exp\left[-4\pi \frac{(E - \sum_{i=1}^{2} j_i \check{\Omega}_i - eA_0 - \Omega_h eA_1)}{\sqrt{K_y(x,y) H_y(x,y)}}\right]. \tag{2.14}$$

By comparing the above expression with the Boltzmann factor, $\exp[-\beta E]$, one can derive the Hawking temperature, which is $T_H = \frac{1}{\beta}$ at the outer horizon r_+. For this case, we can obtain the following Hawking temperature as

$$\begin{aligned} T_H &= \frac{\sqrt{K_y(x,y) H_y(x,y)}}{4\pi}, \\ &= \frac{\sqrt{(x-y)^2(1-x^2)(1+\upsilon x)[2 + \upsilon y + \upsilon y^3 + x\upsilon - 2xy - 3xy^2\upsilon]}}{4\pi\sqrt{(1-y^2)(1+y\upsilon)}}. \end{aligned} \tag{2.15}$$

The Hawking temperature of electrically charged black ring is depending on x, y and υ. The resulting Hawking temperature at which vector particles tunnel through the horizon is similar to the Hawking temperature for scalar and Dirac particles at which they tunnel through the horizon of black ring [31].

3 Myers-Perrry Black Hole

Black holes are most valuable astrophysical objects predicted in general relativity [29]. Kaluza's theory (1921) along with Klein's version (1929) known as Kaluza-Klein theory which provides the proposal to unify the theory of general relativity and electromagnetic theory in 5D vacuum spacetime. The Einstein field equations for 5D spacetime are equivalent to 4D Einstein's equation on with matter comprising of scalar and electromagnetic fields. Abdolrahimi et al. [34] have studied distorted Myers-Perry BH in an external gravitational field with a single angular momentum as exact solution of the 5D Einstein equations in vacuum.

The Myers-Perry BH [35] is a solution of vacuum Einstein field equations in an arbitrary dimension. Here, we consider the 5D case which represents a regular rotating BH (with two rotation parameters) which is a generalization of the Kerr solution. The corresponding line element in the Boyer-Lindquist coordinates $(t, r, \theta, \phi, \varphi)$ is defined as follows [36]

$$\begin{aligned} ds^2 &= \frac{\rho^2 r^2}{\Delta}dr^2 + \rho^2 d\theta^2 - dt^2 + (r^2+a^2)\sin^2\theta d\phi^2 + (r^2+b^2)\cos^2\theta d\varphi^2 \\ &+ \frac{r_0^2}{\rho^2}[dt + a\sin^2\theta d\phi + b\cos^2\theta d\varphi]^2. \end{aligned} \quad (3.1)$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \quad \Delta = (r^2+a^2)(r^2+b^2) - r_0^2 r^2,$$

and the angle ϕ and φ assume measures from the interval $[0, 2\pi]$ and θ assume measures in $[0, \frac{\pi}{2}]$; a and b are the two angular momenta, r_0 associated to the BH mass. The BH horizons are turned as

$$r_\pm^2 = \frac{1}{2}[r_0^2 - a^2 - b^2 \pm \sqrt{(r_0^2 - a^2 - b^2)^2 - 4a^2b^2}] \quad \text{for} \quad (r_0^2 - a^2 - b^2)^2 > 4a^2b^2.$$

The Myers-Perry BH is regular except if $a = b = 0$ that corresponds to 5D Schwarzschild-Tangherlini BH solution [37] and only in this case it has a singularity at $r = 0$.

The line element given by Eq.(3.1) can be rewritten as

$$ds^2 = \tilde{A}dt^2 + \tilde{B}dr^2 + \tilde{C}d\theta^2 + \tilde{D}d\phi^2 + \tilde{E}d\varphi^2 + 2\tilde{F}dtd\phi + 2\tilde{G}dtd\varphi + 2\tilde{H}d\phi d\varphi. \quad (3.2)$$

The electromagnetic vector potential is defined as $A_\mu = (A_1, 0, 0, A_4, 0)$, where

$$A_1 = \frac{b}{\kappa\tilde{\phi}^2} b\cos^2\theta \frac{r_0^2}{\rho^2} \quad \text{and} \quad A_4 = \frac{r_0^2}{\rho^2\kappa\tilde{\phi}^2} ab\sin^2\theta\cos^2\theta.$$

In Lagrangian Eq.(2.3) the components of ψ^ν and $\psi^{\mu\nu}$ are given by

$$\begin{aligned}
\psi^0 &= J\psi_0 + M\psi_3 + N\psi_4, \quad \psi^1 = T\psi_1, \quad \psi^2 = U\psi_2,\\
\psi^3 &= M\psi_0 + P\psi_3 + R\psi_4, \quad \psi^4 = \tilde{E}^{-1}\psi_4,\\
\psi^{01} &= JT\psi_{01} + MT\psi_{31} + NT\psi_{41}, \quad \psi^{02} = JU\psi_{02} + MU\psi_{32} + NU\psi_{42},\\
\psi^{03} &= (JP - M^2)\psi_{03} + (JR - MN)\psi_{04} + (MR - NP)\psi_{34},\\
\psi^{04} &= MN\psi_{30} + (M^2 - JS)\psi_{40} + JR\psi_{03} + NR\psi_{43}, \quad \psi^{12} = TU\psi_{12},\\
\psi^{13} &= TM\psi_{10} + TP\psi_{13} + TR\psi_{14}, \quad \psi^{14} = TN\psi_{10} + TR\psi_{13} + TS\psi_{14},\\
\psi^{23} &= MU\psi_{20} + UP\psi_{23} + UR\psi_{24}, \quad \psi^{24} = MU\psi_{20} + UP\psi_{23} + UR\psi_{24},\\
\psi^{34} &= (MR - PN)\psi_{03} + PS\psi_{34},
\end{aligned}$$

where

$$\begin{aligned}
J &= \frac{DE - H^2}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
M &= \frac{FE - GH}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
N &= \frac{FH - GD}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
P &= \frac{EA - G^2}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
R &= \frac{-HA - FG}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
S &= \frac{-DE + F^2}{-ADE + AH^2 + F^2E - 2HFG + DG^2},\\
T &= \frac{1}{B}, \quad U = \frac{1}{C}.
\end{aligned}$$

Using Eq.(2.3) and WKB approximation, by neglecting the terms of order

$n = 1, 2, 3, 4...$ we obtain the following set of equations

$$JT[c_1(\partial_0 S_0)(\partial_1 S_0) - c_0(\partial_1 S_0)^2 + eA_0c_1(\partial_1 S_0)] + MT[c_1(\partial_1 S_0)(\partial_3 S_0) -c_3(\partial_1 S_0)^2 + eA_3c_1(\partial_1 S_0)] + NT[c_1(\partial_1 S_0)(\partial_4 S_0) - c_4(\partial_1 S_0)^2] + JU[c_2 (\partial_0 S_0)(\partial_2 S_0) - c_0(\partial_2 S_0)^2 + eA_0c_2(\partial_2 S_0)] + MU[c_2(\partial_2 S_0)(\partial_3 S_0) - c_3 (\partial_2 S_0)^2 + eA_3c_2(\partial_2 S_0)] + NU[c_2(\partial_2 S_0)(\partial_4 S_0) - c_4(\partial_2 S_0)^2] + [JP - M^2] [c_3(\partial_0 S_0)(\partial_3 S_0) - c_0(\partial_3 S_0)^2 + eA_3c_2(\partial_2 S_0)] + [JR - MN][c_4(\partial_0 S_0)(\partial_3 S_0) -c_0(\partial_4 S_0)(\partial_3 S_0) + eA_0c_4(\partial_3 S_0)] + [MR - NP][c_4(\partial_3 S_0)^2 - c_3(\partial_4 S_0)(\partial_3 S_0) +eA_3c_4(\partial_3 S_0)] + [JR - MN][c_3(\partial_0 S_0)(\partial_4 S_0) - c_0(\partial_3 S_0)(\partial_4 S_0) + c_3(\partial_4 S_0) -eA_3c_0(\partial_4 S_0)] + [JS - M^2][c_4(\partial_0 S_0)(\partial_4 S_0) - c_0(\partial_4 S_0)^2 + eA_0c_4(\partial_4 S_0)] +NR[c_3(\partial_4 S_0)^2 - c_4(\partial_3 S_0)(\partial_4 S_0) - eA_3c_4(\partial_4 S_0)] - m^2[c_0J + C_3M + c_4N] + eA_3[c_3(\partial_0 S_0) - c_0(\partial_3 S_0) + eA_0c_3 - eA_3c_0] + eA_3[JR - MN] [c_4(\partial_0 S_0) - c_0(\partial_4 S_0) + eA_0c_4] + eA_3[MR - NP][c_4(\partial_3 S_0) - c_3(\partial_4 S_0) +eA_3c_4] = 0 \tag{3.3}$$

$$-JT[c_1(\partial_0 S_0)^2 - c_0(\partial_0 S_0)(\partial_1 S_0) + eA_0c_1(\partial_0 S_0)] - MT[c_1(\partial_0 S_0)(\partial_3 S_0) -c_3(\partial_0 S_0)(\partial_1 S_0) + eA_3c_1(\partial_0 S_0)] - NT[c_1(\partial_0 S_0)(\partial_4 S_0) - c_4(\partial_0 S_0)(\partial_1 S_0)] +TU[c_2(\partial_1 S_0)(\partial_2 S_0) - c_1(\partial_2 S_0)^2] + TM[c_0(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_0 S_0)(\partial_3 S_0) -eA_0c_1(\partial_3 S_0)] + TP[c_3(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_3 S_0)^2 - eA_3c_1(\partial_3 S_0)] + TR [c_4(\partial_1 S_0)(\partial_3 S_0) - c_1(\partial_3 S_0)(\partial_4 S_0)] + TN[c_0(\partial_1 S_0)(\partial_4 S_0) - c_1(\partial_0 S_0)(\partial_4 S_0) -eA_0c_1(\partial_4 S_0)] + TR[c_3(\partial_1 S_0)(\partial_4 S_0) - c_1(\partial_3 S_0)(\partial_4 S_0) - eA_3c_1(\partial_4 S_0)] +TS[c_4(\partial_1 S_0)(\partial_4 S_0) - c_1(\partial_4 S_0)^2] - m^2Tc_1 - eA_0JT[c_0(\partial_1 S_0) - c_1(\partial_0 S_0)^2 -eA_0c_1] - MTeA_0[c_1(\partial_3 S_0) - c_3(\partial_1 S_0) - eA_3c_1] - eA_0NT[c_1(\partial_4 S_0) - c_4 (\partial_1 S_0)] + TMeA_3[c_0(\partial_1 S_0) - c_1(\partial_0 S_0) - eA_0c_1] + eA_3TP[c_3(\partial_1 S_0) - c_1 (\partial_3 S_0) - eA_3c_1] + TReA_3[c_4(\partial_1 S_0) - c_1(\partial_4 S_0)] = 0, \tag{3.4}$$

$$JU[c_0(\partial_0 S_0)(\partial_2 S_0) - c_2(\partial_0 S_0)^2 - eA_0c_2(\partial_0 S_0)] - MU[c_2(\partial_0 S_0)(\partial_3 S_0) -c_3(\partial_0 S_0)(\partial_2 S_0) + eA_3c_2(\partial_0 S_0)] - NU[c_2(\partial_0 S_0)(\partial_4 S_0) - c_4(\partial_0 S_0)(\partial_2 S_0)] -TU[c_2(\partial_1 S_0)^2 - c_1(\partial_2 S_0)(\partial_1 S_0)] + MU[c_0(\partial_2 S_0)(\partial_3 S_0) - c_2(\partial_0 S_0)(\partial_3 S_0) -eA_0c_2(\partial_3 S_0)] + UP[c_3(\partial_2 S_0)(\partial_3 S_0) - c_2(\partial_3 S_0)^2 - eA_3c_2(\partial_3 S_0)] + UR [c_4(\partial_2 S_0)(\partial_3 S_0) - c_2(\partial_3 S_0)(\partial_4 S_0)] + MN[c_0(\partial_2 S_0)(\partial_4 S_0) - c_2(\partial_0 S_0)(\partial_4 S_0) -eA_0c_2(\partial_4 S_0)] + UP[c_3(\partial_4 S_0)(\partial_2 S_0) - c_2(\partial_3 S_0)(\partial_4 S_0)] + UR[c_4(\partial_4 S_0) (\partial_2 S_0) - c_2(\partial_4 S_0)^2] - m^2Uc_2 - eA_0JU[c_0(\partial_2 S_0) - c_2(\partial_0 S_0) - eA_0c_2] -MUeA_0[c_2(\partial_3 S_0) - c_3(\partial_2 S_0)] - eA_0NU[c_2(\partial_4 S_0) - c_4(\partial_2 S_0)] + UM eA_3[c_0(\partial_2 S_0) - c_2(\partial_0 S_0) - eA_0c_2] + eA_3UP[c_3(\partial_2 S_0) - c_2(\partial_3 S_0)]$$

$$+UReA_3[c_4(\partial_2S_0)-c_2(\partial_4S_0)]=0, \tag{3.5}$$
$$-(JP-M^2)[c_3(\partial_0S_0)^2-c_0(\partial_0S_0)(\partial_3S_0)+eA_0c_3(\partial_0S_0)-eA_3c_0(\partial_0S_0)]$$
$$-(JR-MN)[c_4(\partial_0S_0)^2-c_0(\partial_0S_0)(\partial_4S_0)+eA_0c_4(\partial_0S_0)]-NU[c_2(\partial_0S_0)$$
$$(\partial_4S_0)-c_4(\partial_0S_0)(\partial_2S_0)]-(MR-NP)[c_4(\partial_0S_0)(\partial_3S_0)-c_3(\partial_0S_0)(\partial_4S_0)]$$
$$-TM[c_0(\partial_1S_0)^2-c_1(\partial_0S_0)(\partial_1S_0)-eA_0c_1(\partial_1S_0)]-TP[c_3(\partial_1S_0)^2-c_1$$
$$(\partial_3S_0)(\partial_1S_0)-eA_3c_1(\partial_1S_0)]-TR[c_4(\partial_1S_0)^2-c_1(\partial_1S_0)(\partial_4S_0)]-MU$$
$$[c_0(\partial_2S_0)^2-c_2(\partial_0S_0)(\partial_2S_0)-eA_0c_2(\partial_2S_0)]-UP[c_3(\partial_2S_0)^2-c_2(\partial_2S_0)$$
$$(\partial_3S_0)-eA_3c_2(\partial_2S_0)]-UR[c_4(\partial_2S_0)^2-c_2(\partial_2S_0)(\partial_4S_0)]+(MR-PN)$$
$$[c_3(\partial_0S_0)(\partial_4S_0)-c_0(\partial_3S_0)(\partial_4S_0)+eA_0c_3(\partial_4S_0)-eA_3c_0(\partial_4S_0)]+PS$$
$$[c_4(\partial_4S_0)(\partial_3S_0)-c_3(\partial_4S_0)+eA_3c_4(\partial_4S_0)]-eA_0(JP-M^2)[c_3(\partial_0S_0)$$
$$-c_0(\partial_3S_0)+eA_0c_3-eA_3c_0]-eA_3(JR-MN)[c_4(\partial_0S_0)-c_0(\partial_4S_0)$$
$$+eA_0c_4]-eA_0(MR-NP)[c_4(\partial_3S_0)-c_3(\partial_4S_0)+eA_3c_4]=0, \tag{3.6}$$
$$(MN-JR)[c_0(\partial_0S_0)(\partial_3S_0)-c_3(\partial_0S_0)^2+eA_3c_0(\partial_0S_0)-eA_0c_3(\partial_0S_0)]$$
$$+(M^2-JS)[c_0(\partial_0S_0)(\partial_4S_0)-c_4(\partial_0S_0)^2-eA_0c_4(\partial_0S_0)]+NR[c_3$$
$$(\partial_0S_0)(\partial_4S_0)-c_4(\partial_3S_0)(\partial_0S_0)-eA_3c_4(\partial_0S_0)]+TN[c_0(\partial_1S_0)^2-c_1$$
$$(\partial_0S_0)(\partial_1S_0)-eA_0c_1(\partial_1S_0)]+TR[C_3(\partial_1S_0)^2-c_1(\partial_1S_0)(\partial_3S_0)-eA_3$$
$$c_1(\partial_1S_0)]+TS[c_4(\partial_1S_0)^2-c_1(\partial_4S_0)(\partial_1S_0)]-MN[c_0(\partial_2S_0)^2-c_2(\partial_0S_0)$$
$$(\partial_2S_0)-eA_0c_2(\partial_2S_0)]-UP[c_3(\partial_2S_0)^2-c_2(\partial_2S_0)(\partial_3S_0)-eA_3c_2(\partial_2S_0)]$$
$$-UR[c_4(\partial_2S_0)^2-c_2(\partial_2S_0)(\partial_4S_0)]-(MR-PN)[c_3(\partial_0S_0)(\partial_3S_0)-c_0$$
$$(\partial_3S_0)^2+eA_0c_3(\partial_3S_0)-eA_3c_0(\partial_3S_0)]-PS[c_4(\partial_3S_0)^2-c_3(\partial_3S_0)(\partial_4S_0)$$
$$+eA_3c_4(\partial_3S_0)]+eA_0(MN-JR)[c_0(\partial_3S_0)-c_3(\partial_0S_0)+eA_3c_0-eA_0c_3]$$
$$+eA_0(M^2-JS)[c_0(\partial_4S_0)-c_4(\partial_0S_0)-eA_0c_4]+eA_0NR[c_3(\partial_4S_0)-c_4$$
$$(\partial_3S_0)-eA_3c_4]-eA_3(MR-NP)[c_3(\partial_0S_0)-c_0(\partial_3S_0)+eA_0c_3-eA_3$$
$$c_0]-eA_3PS[c_4(\partial_3S_0)-c_3(\partial_4S_0)+eA_3c_4]=0. \tag{3.7}$$

Using separation of variables technique, we can define the particles action for this BH as (also defined in Eq.(2.11))

$$S_0=-(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)t+W(r,\theta)+L\phi+K(\varphi). \tag{3.8}$$

For the above S_0 the preceding set of Eqs.(3.3)-(3.7) can be written in terms of matrix equation, i.e., $\Lambda(c_0,c_1,c_2,c_3,c_4)^T=0$, the elements of the required matrix provide in the following form

$$\begin{aligned}
\Lambda_{00} &= -JT(\partial_1 W)^2 - JU(\partial_2 W) - (JP - M^2)L^2 - (JP - M^2)eA_3L - (JR \\
&\quad -MN)L(\partial_3 K) - (JR - MN)L(\partial_3 K) - (JR - MN)[L(\partial_3 K) + eA_3 \\
&\quad (\partial_4 K)] - (JS - M^2)(\partial_4 K) - m^2J - eA_3L - e^2A_3^2 - eA_3(JR - MN) \\
&\quad (\partial_4 K) \\
\Lambda_{01} &= -(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)JT(\partial_1 W) + JTeA_0(\partial_1 W) + JTeA_0(\partial_1 W) + MT(\partial_1 W)L \\
&\quad +eA_3(\partial_1 W) + NT(\partial_1 W)(\partial_4 K) \\
\Lambda_{02} &= -(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)JU(\partial_2 W) + JUeA_0(\partial_2 W) + MU(\partial_1 W)L + MUeA_3(\partial_2 W) \\
&\quad +NU(\partial_2 W)(\partial_4 K) \\
\Lambda_{03} &= -MT(\partial_1 W)^2 - MU(\partial_2 W)^2 - (JP - M^2)(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)L + (JP - M^2)eA_0 \\
&\quad L - (MR - NP)L(\partial_4 K) - (JR - MN)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K) - eA_0(\partial_4 K)] \\
&\quad +NR(\partial_4 K)^2 - m^2M - (JP - M^2)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)eA_3 - e^2A_0A_3] - eA_3(M \\
&\quad R - NP)(\partial_4 K) \\
\Lambda_{04} &= -NT(\partial_1 W) - NU(\partial_2 W) - (JR - MN)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0L] + (MR \\
&\quad -NP)[L^2 - eA_3L] - (JS - M^2)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K) - eA_0(\partial_4 K)] - NR \\
&\quad [(\partial_4 K) + eA_3(\partial_4 K)] - m^2N - eA_3(JR - MN)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] + eA_3 \\
&\quad (MR - NP)[L + eA_3] \\
\Lambda_{10} &= -(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_1 W)JT + TM(\partial_1 W)L + TN(\partial_1 W)(\partial_4 K) - eA_0JT(\partial_1 W) \\
&\quad +TMeA_3(\partial_1 W)
\end{aligned}$$

$$\begin{aligned}
\Lambda_{11} &= -JT[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)^2 - eA_0(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)] - MTL[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) + eA_3] \\
&+NT(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_4 K) - TU(\partial_2 W) + TML[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) - eA_0] \\
&-TPL[L+eA_3] - TRL(\partial_4 K) + TN[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_4 K) + eA_0(\partial_4 K)] \\
&-TR[(\partial_4 K)L + eA_3(\partial_4 K)] - TS(\partial_4 K) - m^2 T - eA_0 JT[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) \\
&-eA_0] - MTeA_0[L+eA_3] - eA_0 NT(\partial_4 K) + TMeA_3[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) \\
&-eA_0] - TP[L+eA_3] - eA_3 TR(\partial_4 K) \\
\Lambda_{12} &= TU(\partial_1 W)(\partial_2 W) \\
\Lambda_{13} &= -(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_1 W)MT + TP(\partial_1 W)L + TR(\partial_1 W)(\partial_4 K) + TMeA_0 \\
&(\partial_1 W) + TPeA_3(\partial_1 W) \\
\Lambda_{14} &= -(E-j\check{\Omega})(\partial_1 W)NT + TR(\partial_1 W)L + TS(\partial_1 W)(\partial_4 K) + eA_0 NT(\partial_1 W) \\
&+eA_3 TR(\partial_1 W) \\
\Lambda_{20} &= -UJ(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_2 W) + MUL(\partial_2 W) + MU(\partial_2 W)(\partial_4 K) - JUeA_0 \\
&(\partial_2 W) + MUeA_3(\partial_2 W) \\
\Lambda_{21} &= TU(\partial_2 W)(\partial_1 W) \\
\Lambda_{22} &= -UJ((E-\sum_{i=1}^{2}j_i\check{\Omega}_i)[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) - eA_0] + MU(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)[L+eA_3] \\
&+NU(E-\sum_{i=1}^{2}j_i\check{\Omega}_i)(\partial_4 K) - TU(\partial_1 W) + MUL[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) - eA_0] \\
&-UPL[L+eA_3] - URL(\partial_4 K) + MU(\partial_4 K)[(E-\sum_{i=1}^{2}j_i\check{\Omega}_i) - eA_0] - UPL \\
&(\partial_4 K) - UR(\partial_4 K) - m^2 U + JUeA_0[eA_0 - (E-\sum_{i=1}^{2}j_i\check{\Omega}_i)] - MULeA_0
\end{aligned}$$

$$-NUeA_0(\partial_4 K) + MUeA_3[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - UPLeA_3 - UReA_3$$
$$(\partial_4 K)$$

$$\Lambda_{23} = -(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_2 W)MU + UPL(\partial_2 W) + UP(\partial_2 W)(\partial_4 K) + MUeA_0$$
$$(\partial_2 W) + eA_3 UP(\partial_2 W)$$

$$\Lambda_{24} = -NU(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_2 W) + UR(\partial_2 W)L + UR(\partial_2 W)(\partial_4 K)NUeA_0(\partial_2 W)$$
$$+UReA_3(\partial_2 W)$$

$$\Lambda_{30} = -(JP - M^2)(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)[L + eA_3] - (JR - MN)(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K)$$
$$-TM(\partial_1 W) - MU(\partial_2 W)^2 - (MR - PN)(\partial_4 K)[L + eA_3] + eA_0(JP -$$
$$M^2)[L + eA_3] + eA_0(JR - MN)(\partial_4 K)$$

$$\Lambda_{31} = -TM(\partial_1 W)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] + TP(\partial_1 W)[L + eA_3] + TR(\partial_1 W)$$
$$(\partial_4 K)$$

$$\Lambda_{32} = -MU(\partial_2 W)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] + UP(\partial_2 W)[L + eA_3] + UR(\partial_2 W)$$
$$(\partial_4 K)$$

$$\Lambda_{33} = -(JP - M^2)(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - (MR - NP)(E -$$
$$\sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K) - TP(\partial_1 W) + UP(\partial_2 W) - (MR - PN)(\partial_4 K)[(E -$$
$$\sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - PS(\partial_4 K) + eA_0(JP - M^2)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0]$$
$$+eA_0(\partial_4 K)$$

$$\Lambda_{34} = -(JR - MN)(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] + (MR - NP)(E -$$
$$\sum_{i=1}^{2} j_i\check{\Omega}_i)L - TR(\partial_1 W) - UR(\partial_2 W) + PS(\partial_4 K)[L + eA_3] + eA_0(JR$$

$$
\begin{aligned}
& -MN)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)-eA_0]-eA_0(MR-NP)[L+eA_3] \\
\Lambda_{40} &= -(MN-JR)(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)[L+eA_3]-(M^2-JS)(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) \\
& (\partial_4 K)+TN(\partial_1 W)^2-MN(\partial_2 W)^2+(MR-PN)L[L+eA_3]+eA_0 \\
& (MN-JR)[L+eA_3]+eA_0(M^2-JS)(\partial_4 K)+eA_3(MR-PN) \\
& [L+eA_3] \\
\Lambda_{41} &= TN(\partial_1 W)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)+eA_1]-TR(\partial_1 W)[L+eA_3]+TS(E- \\
& \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_1 W) \\
\Lambda_{42} &= -MN(\partial_2 W)[(E-j\check{\Omega})-eA_0]+UP(\partial_2 W)[L+eA_3]+UR(\partial_2 W) \\
& (\partial_4 K) \\
\Lambda_{43} &= -(MN-JR)(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)-eA_0]-NR(E- \\
& \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K)+TR(\partial_1 W)^2-UP(\partial_2 W)^2+(MR-PN)L[(E- \\
& \sum_{i=1}^{2} j_i\check{\Omega}_i)-eA_0]+PSL(\partial_4 K)+eA_0(MN-JR)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i) \\
& -eA_0]+eA_0NR(\partial_4 K)+eA_3(MR-PN)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)-eA_0] \\
\Lambda_{44} &= (JS-M^2)(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)[(E-\sum_{i=1}^{2} j_i\check{\Omega}_i)-eA_0]+NR(E-\sum_{i=1}^{2} \\
& j_i\check{\Omega}_i)[L+eA_3]+TS(\partial_1 W)-UR(\partial_2 W)^2-PSL[L+eA_3] \\
& -e^2A_0^2(M^2-JS)-eA_0NR[L+eA_3]
\end{aligned}
$$

For the non-trivial solution, the determinant Λ is equal to zero provides the following expression

$$ImW^{\pm} = \pm \int \sqrt{\frac{(E - eA_0 - \Omega_1 eA_3 - \sum_{i=1}^{2} j_i\check{\Omega}_i)^2 + \tilde{X}}{\frac{TR}{MN-JR}}},$$
$$= \pm \iota\pi \frac{(E - eA_0 - \Omega_1 eA_3 - \sum_{i=1}^{2} j_i\check{\Omega}_i)}{2\kappa(r_+)},$$

where

$$X = \frac{NR}{MN - JR}(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)(\partial_4 K) + \frac{UP}{MN - JR}(\partial_2 W)^2 - \frac{(MR - PN)L}{MN - JR}[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - \frac{PSL}{MN - JR}(\partial_4 K) - eA_0 \frac{NR}{MN - JR}(\partial_4 K) - eA_3 \frac{(MR - PN)}{MN - JR}[(E - \sum_{i=1}^{2} j_i\check{\Omega}_i) - eA_0] - e^2 A_3^2 + 2(E - \sum_{i=1}^{2} j_i\check{\Omega}_i)eA_3 - 2e^2 A_3 A_0$$

and the surface gravity is

$$\kappa(r_+) = \frac{1}{2}\sqrt{\tilde{M}_r(r,\theta)\tilde{N}_r(r,\theta)},$$

where $-\tilde{B}_r(r,\theta) = \tilde{M}_r(r,\theta)$ and $\tilde{C_r^{-1}}(r,\theta) = \tilde{N}_r(r,\theta)$. The required tunneling probability is

$$\tilde{\Gamma} = \frac{\tilde{\Gamma}_{emission}}{\tilde{\Gamma}_{absorption}} = \exp[-4ImW^+] = \exp\left[-4\pi \frac{(E - eA_0 - \Omega_1 eA_3 - \sum_{i=1}^{2} j_i\check{\Omega}_i)}{\sqrt{\tilde{M}_r(r,\theta)\tilde{N}_r(r,\theta)}}\right]. \tag{3.9}$$

In order to calculate the corresponding Hawking temperature by using the above tunneling probability, we use the Boltzmann factor, $\exp[-\beta E]$, where $\beta = \frac{1}{T_H}$ at the outer horizon r_+. The corresponding Hawking temperature can be deduced as

$$T_H = \frac{\sqrt{\tilde{M}_r(r,\theta)\tilde{N}_r(r,\theta)}}{4\pi} = \frac{\sqrt{2r^2[r_0^2 r^3 - b^2 - rb^2 \sin^2\theta[b^2 + 2a^2 - 2r^2 - 1]}}{4\pi(r^2 + a^2\cos^2\theta + b^2\sin^2\theta)(r^4 + b^2r^2 + a^2r^2 - r_0^2 r^2)}. \tag{3.10}$$

The tunneling probability is related to E, A_0, A_3, $\check{\Omega}$ angular momentum and surface gravity of a BH. The Hawking temperature depends on parameters r_0, a and b.

4 Outlook

In this paper, we have used the Lagrangian equation to investigate the tunneling of charged particles from electrically charged black ring and Myers-Perry BHs. In 5D, black rings have many unusual properties not shared by Myers-Perry BHs with spherical topology, e.g., their event horizon topology is not spherical for the cases of neutral, dipole and charged black rings.

For black rings, the tunneling spectrum of scalar particles has already been discussed by using the Hamilton-Jacobi method [38] and the Dirac particles tunneling phenomenon for black ring has also been discussed [31]. Recently, the anomalous derivation of Hawking radiation has attempted to recover the Hawking temperature of black rings via gauge and gravitational anomalies at the horizon [39]. As far as we know, till now, there is no references to report Hawking radiation of charged vector particles across single electrically charged black ring. So it is interesting to see if charged bosons tunneling process is still applicable in such exotic spacetime. In our analysis, we have found that the tunneling probability given by Eq.(2.14) depends on vector potential components, i.e., A_0 and A_1, energy, angular momentum, particle's charge and surface gravity of black ring. While, the Hawking temperature (2.15) depends on parameter α, i.e., charge of a black ring. We have found that the recovered Hawking temperature is same as already obtained in the literature for various particles.

For Myers-Perry BH, the tunneling spectrum of massive scalar and vector particles tunneling have been discussed in different coordinate systems [29]. They investigated the Hawking temperature in the Painlevé coordinates and in the corotating frames and showed that the coordinate system do not affect the Hawking temperature. Here, we have evaluated charged vector particles tunneling from Myers-Perry BH by solving Proca equations by applying the WKB approximation to the Hamilton-Jacobi method. For this 5D BH, the tunneling probability given by Eq.(3.9) is associated to the energy, vector potential, angular momentum and surface gravity of BH. While, the Hawking temperature (3.10) depends on parameters r_0, a and b, which is similar to the temperature as given in [29]. It is to be noted that the effect of electromagnetism appear only on the tunneling probability of these vector particles which tunnel through the horizon but not on the Hawking temperature.

In this paper, we have found that the Hawking temperature related to the emission rate is similar for every type of particles, i.e., scalars, fermions, vectors (bosons either charged or uncharged). The Hawking temperature calculated recovered by various methods is same and agrees with the temperature generally calculated in the literature [40]. However, the tunneling probably is different for different cases.

Conflict of Interest

We hereby declare that there is no conflict of interest regarding the publication of this article.

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