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CHARTS FOR PREDICTING THE SUBSONIC VORTEX-LIFT CHARACTERISTICS OF ARROW, DELTA, AND DIAMOND WINGS

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16. Abstract

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CHARTS FOR PREDICTING THE SUBSONIC VORTEX-LIFT CHARACTERISTICS OF ARROW, DELTA, AND DIAMOND WINGS

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SUMMARY

The leading-edge-suction analogy method of predicting the aerodynamic characteristics of slender delta wings has been extended to cover arrow- and diamond-wing planforms. Charts for use in calculating the potential- and vortex-flow terms for the lift and drag are presented, and a subsonic compressibility correction procedure based on the Prandtl-Glauert transformation is outlined.

INTRODUCTION

The leading-edge vortex lift associated with the leading-edge-separation vortex which occurs on slender sharp-edge wings has, during the past decade, become more than an aerodynamic curiosity with airplanes such as the Concorde supersonic transport and the Viggen fighter utilizing this flow phenomenon as a means of eliminating the need for flow control devices and high-lift flaps. (See refs. 1 to 3.) Although many analytical methods of predicting the aerodynamic characteristics associated with leading-edge vortex flow have been developed (some of which are reported in refs. 4 to 8), they have been limited primarily to delta planform wings or wings with unswept trailing edges. Because of the increased use of slender wings exhibiting leading-edge vortex flow, at least in the many off-design conditions if not at the design condition, analytical methods applicable to arbitrary planforms are needed. The leading-edge-suction analogy, described in references 8 and 9 appears to provide an accurate method of predicting the vortex-lift characteristics which, at least in concept, is not limited to delta planforms and has been shown in reference 10 to provide accurate estimates for a fairly wide range of fully tapered wings. Although the subsonic analysis was limited to incompressible flow, an appropriate application of the Prandtl-Glauert transformation should provide a subsonic compressibility correction. The purpose of this paper is to present, in chart form, the potential-flow and vortex-flow constants, including subsonic compressibility effects, for a wide series of arrow-, delta-, and diamond-wing planforms.

SYMBOLS

A	wing aspect ratio, b^2/S
a	longitudinal distance from root trailing edge to wing tip station, positive rearward (see fig. 1)
a/l	wing notch ratio, positive for arrow wings and negative for diamond wings
b	wing span
C_D	drag coefficient
$C_{D,o}$	drag coefficient at zero lift
ΔC_D	drag-due-to-lift coefficient, $C_D - C_{D,o}$
C_L	lift coefficient
C_p	pressure coefficient
e	leading-edge length of wing (see fig. 1)
e'	leading-edge length of transformed wing
f_M	compressibility factor (see eq. (5))
K_p	constant in potential-flow-lift term
K_v	constant in vortex-lift term
l	longitudinal distance from apex to wing tip station (see fig. 1)
M	Mach number
S	wing area
α	angle of attack

$\beta = \sqrt{1 - M^2}$

Λ_{le} leading-edge sweep of actual wing (see fig. 1)

Λ'_{le} leading-edge sweep of transformed wing, $\tan \Lambda'_{le} = \frac{\tan \Lambda_{le}}{\beta}$

All primes refer to the transformed wing.

ANALYTICAL METHODS

In references 8 and 9 it has been shown that excellent predictions of lift and drag due to lift of sharp-edge delta wings over a wide range of angles of attack and aspect ratios can be obtained by combining the potential-flow lift and the vortex lift as predicted by the leading-edge-suction analogy. The resulting equations are

$$C_L = K_p \sin \alpha \cos^2\alpha + K_v \sin^2\alpha \cos \alpha \tag{1}$$

and

$$\Delta C_D = K_p \sin^2\alpha \cos \alpha + K_v \sin^3\alpha \tag{2}$$

or

$$\Delta C_D = C_L \tan \alpha \tag{3}$$

where, in equations (1) and (2), the first term represents the potential-flow contribution and the second term represents the vortex-lift contribution.

In reference 10 it was shown that equation (1) is applicable for wings of arbitrary planform providing, of course, that the constants K_p and K_v are calculated for the desired planform. The analogy method makes it possible to use potential-flow theory to predict both the potential-flow term and the vortex-flow term. For the arrow and diamond planforms of interest in this paper, any accurate potential-flow lifting-surface method, such as the methods of references 11 and 12, can be used. Since the method of reference 12 appears to offer some advantages with regard to more general planforms involving broken leading edges, it has been programed at Langley for use in certain lifting-surface studies and was used for the present calculations of the potential- and vortex-lift constants. The constant K_p is simply the potential-flow lift-curve slope and the constant K_v is related to the potential-flow leading-edge thrust parameter. (See eq. (3) of ref. 10.)

The subsonic effects of compressibility can be accounted for by use of the Prandtl-Glauert transformation and the Goethert rule form (see ref. 13) will be used herein. This rule relates the pressure coefficient at a given nondimensionalized point on the real wing at a given Mach number to a pressure coefficient at the same nondimensionalized point on a transformed wing (stretched in longitudinal direction by $1/\beta$) in incompressible flow. For a wing of zero thickness, the rule can be stated as follows:

$$\left(C_p\right)_{M,\alpha,A,\Lambda_{le}} = \frac{1}{\beta^2}\left(C_p\right)_{M=0,\alpha\beta,A\beta,\Lambda'_{le}}$$

Application to the potential-flow-lift constant K_p is well known, and the effect of compressibility can be accounted for simply by determining the incompressible value for a transformed wing having a reduced aspect ratio equal to $A\beta$ and an increased leading-edge sweep angle whose tangent is greater by $1/\beta$, and then increasing the resulting value of K_p by the factor $1/\beta$. The $1/\beta$ correction to K_p results from combining the $1/\beta^2$ correction to the pressure and the effect of the reduced angle of attack $\alpha\beta$. Therefore, if K'_p is the incompressible value for the transformed wing, then K_p for the real wing at its Mach number is given by

$$K_p = \frac{K'_p}{\beta}$$

With regard to the effect of compressibility on the vortex-lift constant K_v, it was assumed that the leading-edge-suction analogy can also be applied in compressible flow; therefore, the problem can be reduced to that of determining the effect of compressibility on the leading-edge suction. Although the same transformed wing is used for the leading-edge suction and the resulting vortex-lift constant K'_v, the compressibility factor that must be applied differs from the $1/\beta$ that is used for K_p. This is due to two factors. First, since the leading-edge suction increases with the square of the angle of attack, the angle-of-attack reduction associated with the transformed wing completely cancels the $1/\beta^2$ term that is applied to the pressures on the transformed wing. Second, since the method used must be equivalent to applying the transformed wing pressures along the real-wing leading edge (rather than the reference area as in the potential-flow lift case), and since the transformed leading-edge length e' does not increase as rapidly as the transformed-wing area S', the value of K'_v must be corrected to the real wing ratio of the leading-edge length to the area. In other words

$$K_v = K'_v \frac{S'}{S} \frac{e}{e'}$$

and since $\frac{S'}{S} = \frac{1}{\beta}$ and $\frac{e}{e'} = \sqrt{\frac{1 + \tan^2\Lambda}{1 + \frac{\tan^2\Lambda}{\beta^2}}}$,

$$K_v = K_v' \sqrt{\frac{1 + \tan^2\Lambda}{\beta^2 + \tan^2\Lambda}} = K_v' f_M \tag{4}$$

PRESENTATION OF RESULTS

Lifting-surface solution values for the potential-flow-lift constant $K_p\beta$ as a function of $A\beta$ and Λ_{le}' are presented in figure 2 by the solid lines. Also presented as an aid in locating a particular wing are dashed lines which represent constant values of notch ratio a/l. These constant notch-ratio lines are also convenient for applying the Prandtl-Glauert transformation since the notch ratio is unaffected by the transformation. Following a constant notch-ratio line removes the need for determining the sweep angles Λ_{le}' of the various transformed wings.

Figure 3 presents values of the vortex-lift constant in the form K_v/f_M as a function of $A\beta$ and Λ_{le}'. Again, lines of constant notch ratio are presented for convenience. Values of f_M as determined from equation (4) are presented in figure 4 as a function of leading-edge sweep angle and Mach number.

For convenience in using the equations, table I presents values of the various combinations of trigonometric functions needed.

With regard to the expected accuracy of the method, reference 10 presents correlations with experimental results for the incompressible case.

CONCLUDING REMARKS

The leading-edge-suction analogy method of predicting the aerodynamic characteristics of slender delta wings has been extended to cover arrow- and diamond-wing planforms. The method of applying compressibility corrections to the leading-edge suction has been examined, and the resulting procedure applied to the vortex-lift constant. Charts for use in calculating the potential- and vortex-flow terms for the lift and drag in subsonic compressible flow are presented for a wide range of planform parameters.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 26, 1971.

REFERENCES

1. Maltby, R. L.: The Development of the Slender Delta Concept. Aircraft Eng., vol. XL, no. 3, Mar. 1968, pp. 12-17.
2. Küchemann, D.; and Weber, J.: An Analysis of Some Performance Aspects of Various Types of Aircraft Designed to Fly Over Different Ranges at Different Speeds. Progress in Aeronautical Sciences, Vol. 9, D. Küchemann, ed., Pergamon Press, Inc., c.1968, pp. 329-456.
3. Behrbohm, Hermann: Basic Low Speed Aerodynamics of the Short-Coupled Canard Configuration of Small Aspect Ratio. SAAB TN 60, Saab Aircraft Co. (Linköping, Sweden), July 1965.
4. Brown, Clinton E.; and Michael, William H., Jr.: On Slender Delta Wings With Leading-Edge Separation. NACA TN 3430, 1955.
5. Mangler, K. W.; and Smith, J. H. B.: A Theory of the Flow Past a Slender Delta Wing With Leading Edge Separation. Proc. Roy. Soc. (London), ser. A., vol. 251, no. 1265, May 26, 1959, pp. 200-217.
6. Sacks, Alvin H.; Lundberg, Raymond E.; and Hanson, Charles W.: A Theoretical Investigation of the Aerodynamics of Slender Wing-Body Combinations Exhibiting Leading-Edge Separation. NASA CR-719, 1967.
7. Smith, J. H. B.: Improved Calculations of Leading-Edge Separation From Slender Delta Wings. Tech. Rep. No. 66070, Brit. R.A.E., Mar. 1966.
8. Polhamus, Edward C.: A Concept of the Vortex Lift of Sharp-Edge Delta Wings Based on a Leading-Edge-Suction Analogy. NASA TN D-3767, 1966.
9. Polhamus, Edward C.: Application of the Leading-Edge-Suction Analogy of Vortex Lift to the Drag Due to Lift of Sharp-Edge Delta Wings. NASA TN D-4739, 1968.
10. Polhamus, Edward C.: Prediction of Vortex-Lift Characteristics Based on a Leading-Edge Suction Analogy. AIAA Paper No. 69-1133, Oct. 1969.
11. Lamar, John E.: A Modified Multhopp Approach for Predicting Lifting Pressures and Camber Shape for Composite Planforms in Subsonic Flow. NASA TN D-4427, 1968.
12. Wagner, Siegfried: On the Singularity Method of Subsonic Lifting-Surface Theory. AIAA Paper No. 69-37, Jan. 1969.
13. Shapiro, Ascher H.: The Dynamics and Thermodynamics of Compressible Fluid Flow. Vol. I. Ronald Press Co., c.1953.

TABLE I.- VALUES OF TRIGONOMETRIC FUNCTIONS

α, deg	$\sin \alpha \cos^2\alpha$	$\sin^2\alpha \cos \alpha$	$\sin^3\alpha$
2	0.0349	0.0012	0.0000
4	.0694	.0049	.0003
6	.1034	.0109	.0011
8	.1365	.0192	.0027
10	.1684	.0297	.0052
12	.1990	.0423	.0090
14	.2278	.0568	.0142
16	.2547	.0730	.0209
18	.2795	.0908	.0295
20	.3020	.1099	.0400
22	.3220	.1301	.0256
24	.3395	.1511	.0673
26	.3541	.1727	.0842
28	.3660	.1946	.1035
30	.3750	.2165	.1250

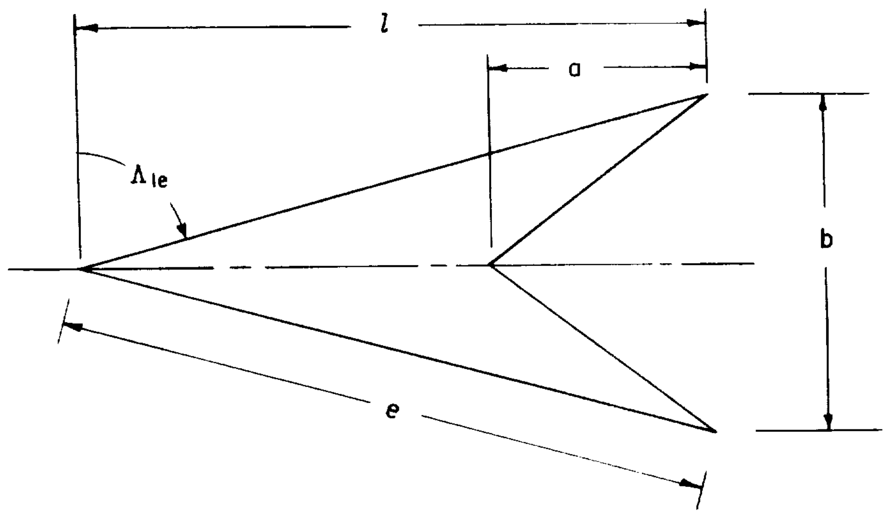

Figure 1.- Sketch defining wing geometry nomenclature.

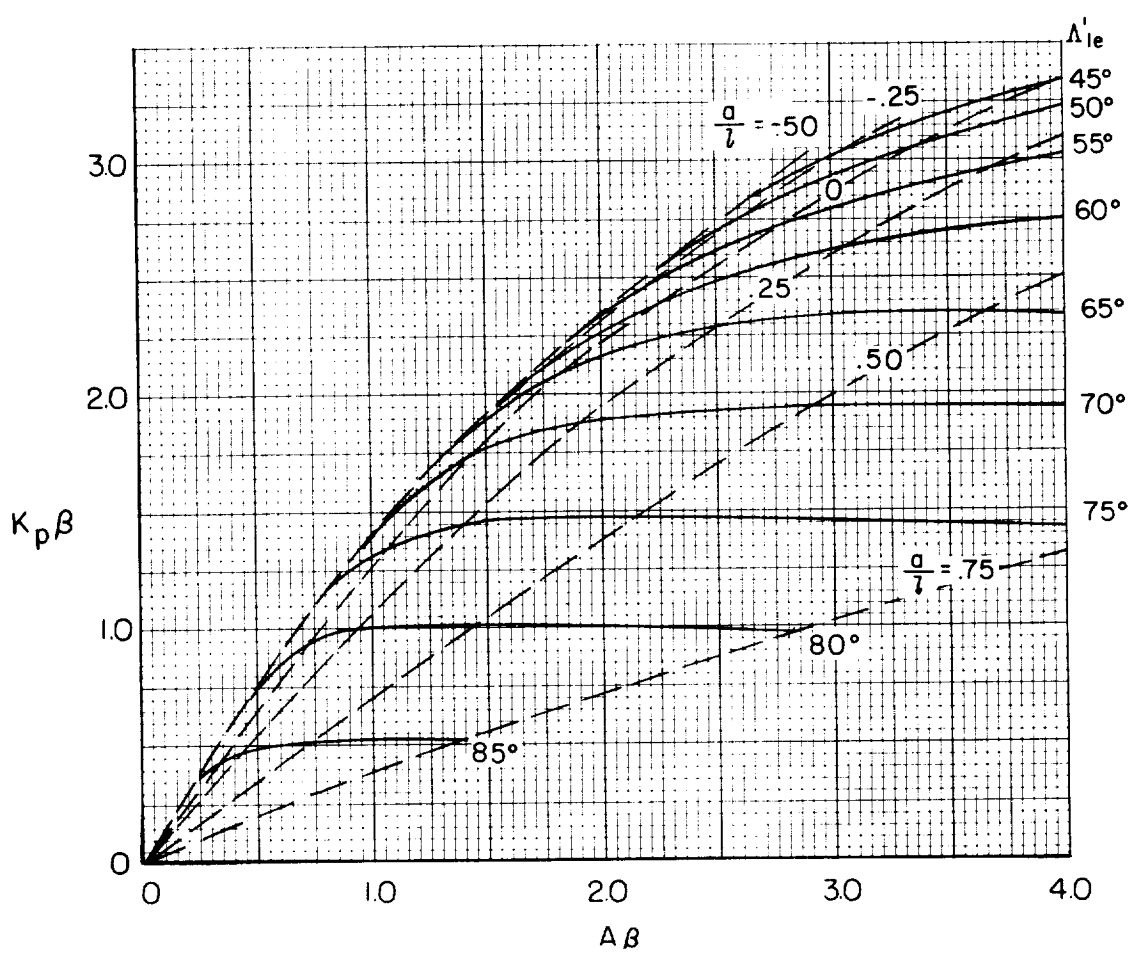

Figure 2.- Variation of potential-flow lift constant with planform parameters.

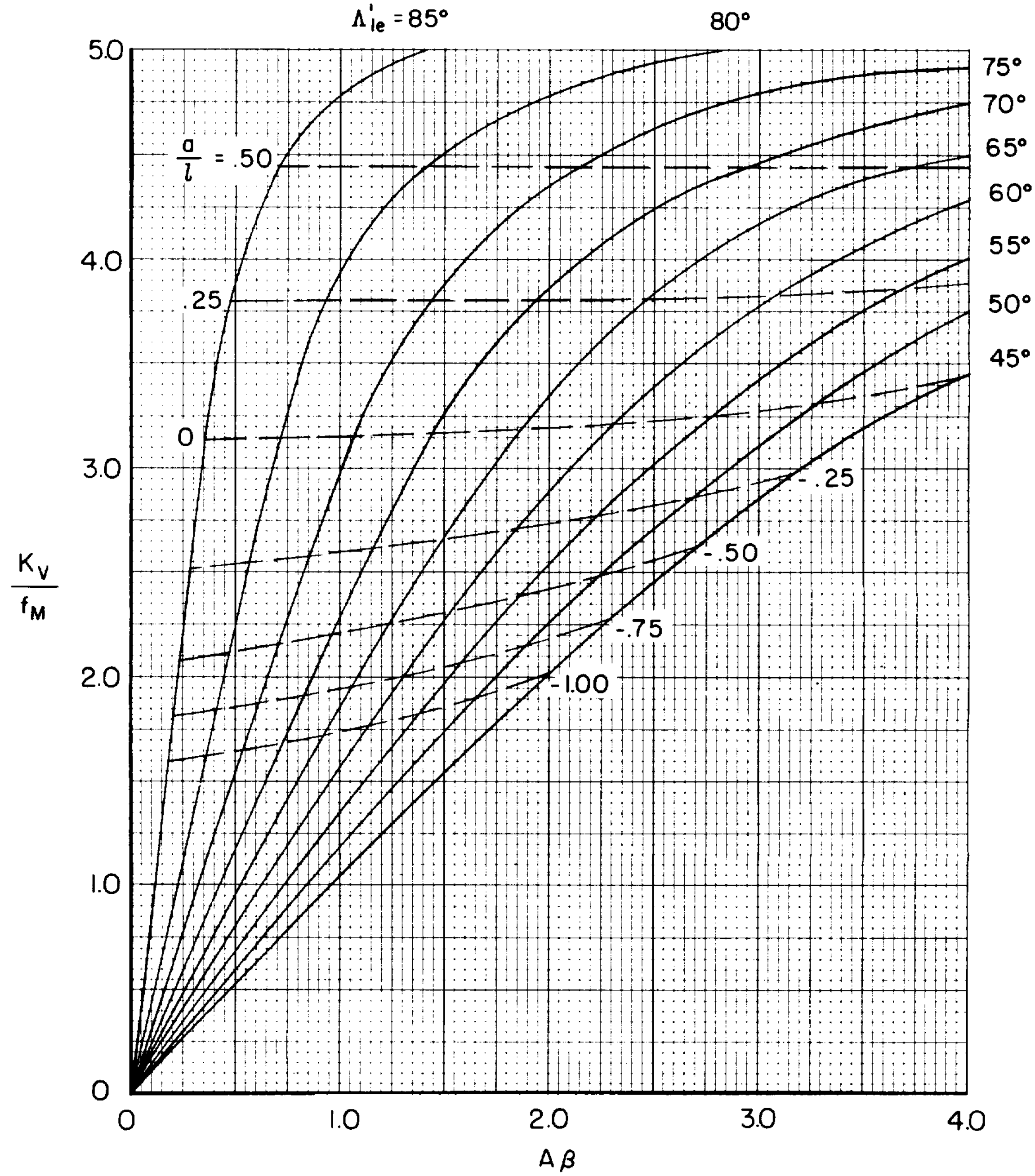

Figure 3.- Variation of vortex-lift constant with planform parameters.

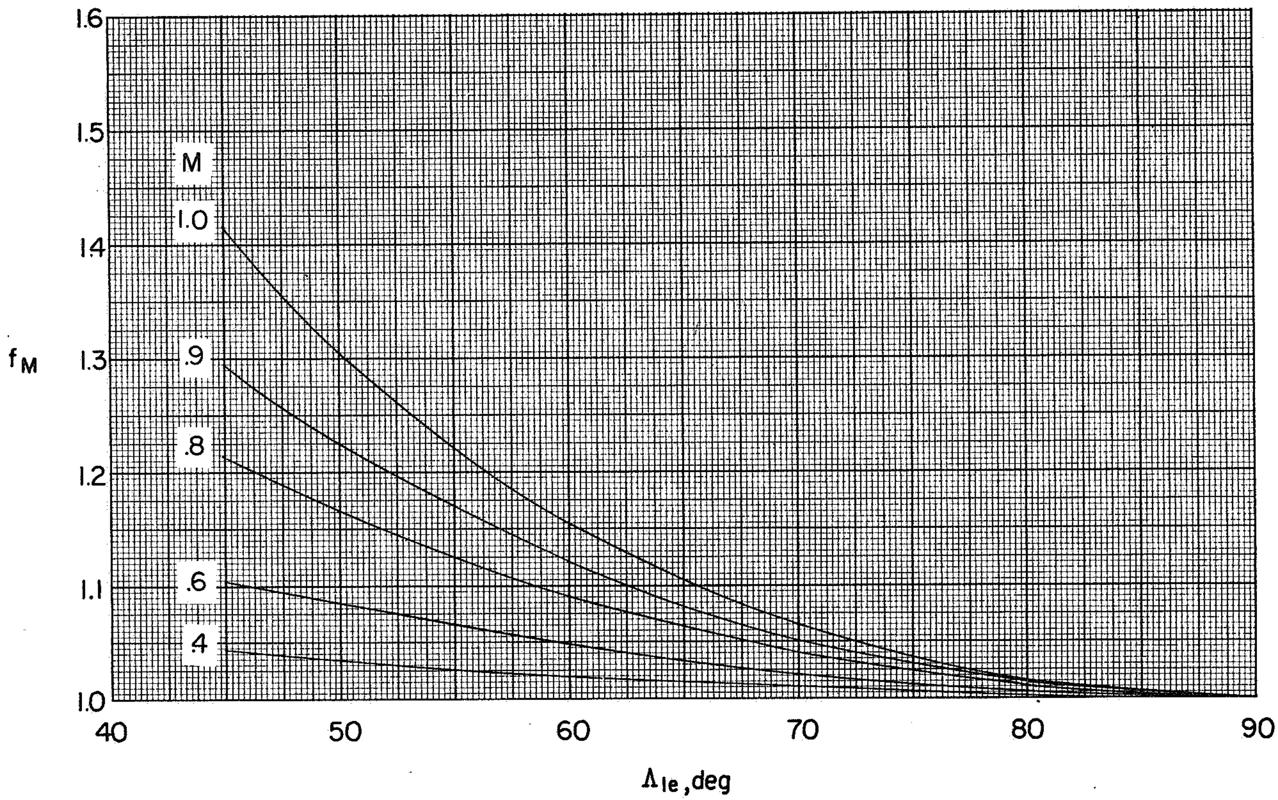

Figure 4.- Variation of compressibility factor with sweep and Mach number.