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CHARTS FOR THE ANALYSIS OF ONE-DIMENSIONAL STEADY COMPRESSIBLE FLOW

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SUMMARY

Charts are presented in terms of dimensionless parameters to facilitate the computation involved in the solution of equations of one-dimensional steady flow of a compressible fluid. These charts can be effectively used in analyses involving constant or variable specific heats in the subsonic and supersonic ranges.

Examples are given in which the charts are applied to the one-dimensional analysis of ideal adiabatic steady flow in ducts of varying area and of nonisentropic frictionless steady flows. A discussion is also given of the application of the charts in the analysis of adiabatic steady flow with external forces and of flows simultaneously involving friction, changes in area, and addition of heat.

INTRODUCTION

Many of the problems involving the steady flow of a compressible fluid in a duct can be solved with sufficient accuracy for engineering purposes by treating the flow as "one-dimensional"; that is, a flow in which the velocity, the pressure, and the temperature of the fluid are assumed to be constant over any given cross section of the duct and are therefore functions only of the distance along the duct.

In many cases the algebraic solution of these one-dimensional compressible flow problems is difficult, particularly for flows with velocities near the velocity of sound and flows with supersonic velocities for which variations in specific heat and gas constant are important. Numerous charts have been published for use in the solution of specific problems involved in the steady flow of a compressible fluid. This report, however, presents charts of a general nature that facilitate the solution of a large number of problems concerning flow processes, which involve variable or constant specific heats, friction, heat transfer, and combustion including a change in mass flow and the accompanying change in momentum.

The application of the charts to the analysis of ideal adiabatic steady flow in ducts of varying area is illustrated by sample computations of the flow in a convergent-divergent nozzle and in an under-expanding jet and by calculation of the total temperature of a fluid from its measured jet thrust. The use of the charts in the analysis of nonisentropic frictionless flows is illustrated by sample computations of theoretical normal compressibility shock, combustion in a tube of constant cross section, flow in a duct with an abrupt change in cross-sectional area, and flow in a jet pump. A discussion is also given of the application of the charts in the analysis of adiabatic steady flow with external forces and of flows simultaneously involving friction, changes in area, and addition of heat.

ANALYSIS AND DESCRIPTION OF CHARTS

The steady flow of a compressible fluid in a duct can be considered one dimensional if velocity, pressure, and temperature of the fluid are assumed to be constant over any cross section of the duct. Each flow parameter at any cross section can be uniquely determined from the principle of conservation of energy, the principle of conservation of mass, the equation of state of the fluid, and the specific heat of the fluid, when any four independent quantities such as total energy, mass flow, static pressure, and cross-sectional area are specified.

Equations of one-dimensional flow of perfect gas with variable specific heats. - If the fluid is a perfect gas, its total energy is a function only of its total temperature. A specification of the total temperature is therefore equivalent to a specification of the total energy.

In accordance with the principle of the conservation of mass as applied to a one-dimensional flow, the total mass flow W through any cross-sectional area A is a constant

$$W = \rho u A = \text{constant} \qquad (1)$$

where u and ρ are the velocity and the density, respectively. For convenience, all symbols are defined in the appendix.

The equation of state of a perfect gas is

$$p = \rho gRT \tag{2}$$

where p, R, and T are static pressure, gas constant, and static temperature, respectively, and g is the ratio of gravitational to absolute unit of mass.

The important flow variables defined by the conservation laws and the equation of state may be combined into a set of related dimensionless quantities $u/\sqrt{gRT_t}$, $p_tA/W\sqrt{gRT_t}$, and $pA/W\sqrt{gRT_t}$, called herein the velocity parameter, the total-pressure parameter, and the static-pressure parameter, respectively. This set of dimensionless quantities is convenient because it explicitly contains the mass flow W, the total temperature T_t, and the total pressure p_t, which remain constant in many flow processes, and because their interdependence can be represented in a simple graphical form.

Reference 1 shows that for combustion gases within the range of temperatures from about 700° to 2700° R the variation of the ratio of specific heats γ during an isentropic expansion can be represented, in the symbols of this report, by the equation

$$\frac{\gamma}{\gamma_t} = \left(\frac{p}{p_t}\right)^{-0.014} \tag{3}$$

where γ_t is the ratio of specific heats at the total temperature T_t of the fluid. The static temperature T corresponding to each value of the pressure ratio p/p_t is given by

$$\frac{T}{T_t} = \left(\frac{p}{p_t}\right)^{\frac{\gamma_T - 1}{\gamma_T}} \tag{4}$$

where γ_T is an effective value of γ defined by the relation

$$\frac{\gamma_T}{\gamma_t} = \left(\frac{p}{p_t}\right)^{\frac{-0.014}{2}} \tag{5}$$

Because, for an ideal fluid, the ratio of specific heats γ is a function of only temperature, equations (3), (4), and (5) provide a relation between the instantaneous ratio of specific heats γ and the static temperature T. It is further shown in reference 1 that the velocity u is related to the static pressure p, total pressure p_t, gas constant R, and total temperature T_t by the relation

$$\frac{u}{\sqrt{gRT_t}} = \sqrt{\frac{2\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \qquad (6)$$

where γ_h is an effective value of γ defined by the relation

$$\frac{\gamma_h}{\gamma_t} = \left(\frac{p}{p_t}\right)^{\frac{-0.014}{3}} \qquad (7)$$

The density is related to the pressure and temperature by the relation

$$\rho = \frac{p}{gRT} = \frac{p_t}{gRT_t}\left(\frac{p}{p_t}\right)^{\frac{1}{\gamma_T}} \qquad (8)$$

Equations (4), (6), and (8) can be replaced by similar equations in which $\gamma_h = \gamma_T = \gamma_t = 1.40$ when the fluid is air at low temperatures for which the specific heat may be assumed to be constant.

In terms of the velocity and the density as described by equations (6) and (8), the mass flow W through a duct of area A is given by

$$W = \rho A u = \frac{Ap_t}{\sqrt{gRT_t}}\left(\frac{p}{p_t}\right)^{\frac{1}{\gamma_T}}\sqrt{\frac{2\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]} \qquad (9)$$

which may be rearranged in two ways to define the total-pressure and static-pressure parameters:

$$\frac{p_t A}{W\sqrt{gRT_t}} = \frac{\left(\frac{p_t}{p}\right)^{\frac{1}{\gamma_T}}}{\sqrt{\frac{2\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]}} \qquad (10)$$

and

$$\frac{pA}{W\sqrt{gRT_t}} = \frac{\left(\frac{p}{p_t}\right)^{\frac{\gamma_T - 1}{\gamma_T}}}{\sqrt{\frac{2\gamma_h}{\gamma_h - 1}\left[1 - \left(\frac{p}{p_t}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right]}} \tag{11}$$

As shown in reference 2 (equation (52)), the equation of motion of a one-dimensional steady flow may be written (in the notation of this paper) as

$$\frac{d}{dx}(Wu + pA) = \phi \tag{12}$$

where ϕ is the sum of all external forces acting on the fluid per unit length including the force of friction at the walls of a tube or a duct.

The total momentum $(Wu + pA)$ has a special significance in many problems involving the flow of a compressible fluid. The total-momentum parameter, which is the sum of the velocity and the static-pressure parameters, is obtained by reducing the total momentum to dimensionless form by dividing by the term $W\sqrt{gRT_t}$

$$\frac{Wu + pA}{W\sqrt{gRT_t}} = \frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}} \tag{13}$$

Equations of motion for underexpanding nozzle. - When a compressible fluid is discharged from a nozzle at a higher static pressure than the pressure of the fluid into which the jet is flowing, the fluid of the jet will expand beyond the nozzle. In figure 1, a jet of gas is considered to discharge through a convergent nozzle from a large closed reservoir. A boundary is passed through the jet of gas just at the end of the nozzle and is drawn entirely around the reservoir. The thrust F produced by the jet is equal to the momentum of the jet Wu_n plus the pressure forces $p_nA_n - p_0A_n$; that is,

$$F = Wu_n + p_nA_n - p_0A_n \tag{14}$$

where 0 is the subscript referring to the discharge environment and n is the subscript indicating conditions within the gas at the nozzle exit; that is u_n, p_n, and A_n are the velocity, the pressure, and the area at the mouth of the nozzle, respectively. The thrust F may be so used to define an effective velocity u_e that $F = Wu_e$.

The effective velocity parameter $u_e/\sqrt{gRT_t}$ is therefore given by

$$\frac{u_e}{\sqrt{gRT_t}} = \frac{u_n}{\sqrt{gRT_t}} + \frac{p_n A_n}{W\sqrt{gRT_t}} - \frac{p_0 A_n}{W\sqrt{gRT_t}} \tag{15}$$

In general, when p_0 is less than p_n, the magnitudes of p_n and u_n are unchanged by changes in p_0. In this case, a simple relation exists between the velocity u_e and the pressure of the discharge environment p_0, namely

$$\frac{\partial(u_e/\sqrt{gRT_t})}{\partial(p_0 A_n/W\sqrt{gRT_t})} = -1 \tag{16}$$

Equations (15) and (16) are also valid for the flow of a compressible fluid from a supersonic (convergent-divergent) nozzle into a region in which the static pressure is less than the discharge pressure as determined at the mouth of the nozzle. In this case, the subscript n in equations (15) and (16) designates the cross section at which free expansion begins; that is, the exit cross section of the divergent section of the nozzle.

Equations (5), (6), (7), (8), (10), (11), and (13) indicate that the dimensionless parameters $u/\sqrt{gRT_t}$, $p_tA/W\sqrt{gRT_t}$, $pA/W\sqrt{gRT_t}$, and $u/\sqrt{gRT_t} + pA/W\sqrt{gRT_t}$ and the temperature ratio T/T_t are functions only of the pressure ratio p/p_t and the ratio of specific heats at the total temperature γ_t. For fixed values of γ_t, the values of all the dimensionless flow parameters at one cross section of a duct can be calculated if the value of the pressure ratio, the value of any one of the dimensionless flow parameters, or the temperature ratio is known. If the dimensionless flow parameters at one cross section of a duct are known and if the variation of mass flow, of area, and of total temperature along the duct is known, the dimensionless flow parameters at any other cross section of the duct can be computed with the aid of one additional relation, for example, constancy of total pressure, constancy of total momentum, or, in the case of the presence of an external force, the use of equation (12).

Description of the charts. - The one-dimensional flow charts consist of plots of the velocity parameter $u/\sqrt{gRT_t}$, the total-pressure parameter $p_tA/W\sqrt{gRT_t}$, and the total-momentum parameter $u/\sqrt{gRT_t} + pA/W\sqrt{gRT_t}$ against the static-pressure parameter $pA/W\sqrt{gRT_t}$. The reciprocal of the velocity parameter $\sqrt{gRT_t}/u$ has also been plotted to improve the accuracy at high values of the independent variable $pA/W\sqrt{gRT_t}$. Figure 2(a) covers the range of static-pressure parameter from 0 to 2.2 for cold air, that is, a fluid with a constant ratio of specific heats of 1.40. A curve of Mach number M plotted as a function of the static-pressure parameter is included in figure 2(a) for use when the initial conditions are stated in terms of Mach number. Figure 2(b) covers the range of static-pressure parameter from 0 to 2.2 for hot gases with variable specific heats. Three values of the instantaneous ratios of the specific heats at the total temperature of the fluid γ_t of 1.26, 1.30, and 1.34 were chosen for calculations with variable specific heats. A linear vertical interpolation may be used to obtain the values of the dependent functions for intermediate values of γ_t.

Figure 2(c) covers the range of static-pressure parameter from 2.2 to 4.4. Included in figure 2(c) is a curve of the reciprocal of the Mach number plotted as a function of the static-pressure parameter for cold air with a constant ratio of specific heats of 1.40. The use of Mach number as the independent variable has been purposely avoided in obtaining curves of the flow parameters for variable specific heats in order to obtain curves (fig. 2(c)) that coincide for high values of the static-pressure parameter for all values of the ratio of specific heats at the total temperature. Within the range of figure 2(c), the effects of the variation of γ are negligible on the scale used and one plot is therefore sufficient for all values of γ from 1.26 to 1.40.

Beyond the range of the abscissa of figure 2(c), that is, for values of static-pressure parameter greater than 4.4, the following approximations to the compressible flow equations (equations (6), (10), (11), and (13)) may be used to calculate the various functions:

$$\frac{pA}{W\sqrt{gRT_t}} = \sqrt{\frac{p}{2(p_t - p)}} \qquad (17)$$

$$\frac{p_t A}{W\sqrt{gRT_t}} = \frac{pA}{W\sqrt{gRT_t}} + \frac{1}{2}\,\frac{W\sqrt{gRT_t}}{pA} \qquad (18)$$

$$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}} = \frac{pA}{W\sqrt{gRT_t}} + \frac{W\sqrt{gRT_t}}{pA} \qquad (19)$$

$$\frac{\sqrt{gRT_t}}{u} = \frac{pA}{W\sqrt{gRT_t}} + \frac{1}{\gamma}\,\frac{W\sqrt{gRT_t}}{pA} \qquad (20)$$

Equations (17), (18), and (19) are equations of incompressible flow arranged in dimensionless form. If a constant value of the ratio of specific heats γ of 1.40 is used in place of the effective values of γ in equations (6) and (10), an expression for the reciprocal of the velocity parameter can be obtained in terms of the quantity $(p_t/p)^{\frac{\gamma-1}{\gamma}}$ and the static-pressure parameter. The quantity $(p_t/p)^{\frac{\gamma-1}{\gamma}}$ can be expanded in a Taylor's series in powers of $(p_t - p)/p$ in which all terms after the second are found to be negligibly small. From equation (17) and the expression for the reciprocal of the velocity parameter containing the first two terms of the Taylor's series expansion of $(p_t/p)^{\frac{\gamma-1}{\gamma}}$, equation (20) is determined.

In figure 2(a) for which $\gamma = 1.40$, a vertical dot-dash line labeled $u = a$ has been drawn through the minimum values of the total-pressure parameter and total-momentum parameter, which occur at a local Mach number of 1. At this point the static-pressure parameter is 0.7715. Subsonic flows are characterized by values of static-pressure parameter greater than 0.7715; supersonic flows have values of static-pressure parameter less than 0.7715.

Subsonic and supersonic flows in fluids with variable specific heats are characterized by values of the static-pressure parameter that are greater or less, respectively, than the critical values of the static-pressure parameter indicated by the slant lines labeled $u = a$ on figure 2(b), which intersect the curves of total-pressure parameter and total-momentum parameter at their respective minimums.

The existence of minimum values of the total-pressure parameter and total-momentum parameter automatically imposes the condition that no flow is possible for which the calculated value of either of the functions is less than their respective minimums.

The dotted lines with the slope of -1 in figures 2(a) and 2(b) have been drawn tangent to the velocity-parameter curve for the free expansion of a compressible fluid through a convergent nozzle to a pressure below the critical discharge pressure as indicated by equation (16). The total-momentum parameter associated with the jet discharged from an underexpanding nozzle is a constant equal to that at the critical discharge pressure (equation (15)); the horizontal dotted line tangent to the curve of total-momentum parameter at the minimum point thus corresponds to the flow from a convergent nozzle to a region in which the pressure is below the critical discharge pressure of the nozzle.

Similarly dotted lines with a slope of -1 could be drawn from the curve of velocity parameter on figures 2(a) and 2(b) to represent the free expansion of a compressible fluid from supersonic nozzles. The point on the curve of velocity parameter from which the dotted line with slope of -1 is to be drawn would correspond to the value of velocity parameter existing at the nozzle-exit cross section. The total-momentum parameter associated with free expansion from the supersonic nozzle is a constant equal to the total-momentum parameter at the nozzle exit.

For all cases of free expansion, the lines representing the effective velocity parameter and the total-momentum parameter intersect at a value of static-pressure parameter equal to zero.

For free expansion, the area A used in the flow parameters is the nozzle-exit area and the downstream static pressure p is the pressure of the environment into which the fluid is flowing p_0.

The values of $pA/W\sqrt{gRT_t}$, $u/\sqrt{gRT_t}$, and T/T_t as a function of p/p_t are shown on figure 3(a) for constant γ of 1.40 and in figure 3(b) for variable specific heats with γ_t of 1.26, 1.30, and 1.34. A curve of Mach number plotted as a function of the pressure ratio is included in figure 3(a) as an aid to calculations in which the given conditions are stated in the form of Mach number. Curves of Mach number, however, are not included in figure 3(b) because the use of Mach number is not desirable when the one-dimensional flow charts are used for analyses involving fluids with variable specific heats.

The value of the ratio of specific heats of the fluid at its total temperature is shown in figure 4 for two hydrogen-carbon ratios of the combustion gas. The value of the gas constant R for the same fluids is shown in figure 5. Equations for γ_t and R are given in reference 1.

The static-pressure and total-pressure parameters are made dimensionless by multiplying the parameter computed from the given numerical data by the factor K given in table I. In order to facilitate plotting special curves where greater accuracy is desired than can be obtained using figures 2 and 3, the computed data used in plotting these figures are given in tables II and III. The values of the nondimensional parameters at the critical pressure ratio for four values of the ratio of specific heats γ are given in table IV.

STEADY-FLOW PROBLEMS ILLUSTRATING USE OF ONE-DIMENSIONAL FLOW CHARTS

The following examples illustrate the use of the one-dimensional flow charts in the analysis of ideal adiabatic steady flow in ducts of varying area and of nonisentropic frictionless flows. The values of the flow parameters used in the following examples were taken from large-scale plots of figures 2(a), 2(b), 3(a), and 3(c). A discussion of the application of the charts to the analysis of flows with external forces acting is also included.

Ideal Adiabatic Flow in Ducts of Varying Area

In a reversible-adiabatic or isentropic one-dimensional steady-flow process, the flow is characterized by constant values of total temperature, total pressure, and mass flow. The total-pressure parameter $p_tA/W\sqrt{gRT_t}$ is therefore proportional to the flow area. The use of the charts to solve steady-flow problems in ducts of varying area is illustrated by the following examples:

Convergent-divergent nozzle. - Consider the case of a steady-flow process in a tube with a convergent-divergent nozzle at the exit in which upstream velocity, throat area, nozzle-exit area, and discharge velocity will be computed with the following conditions given:

Upstream passage area, A_1, sq in.	10
Upstream static pressure, p_1, lb/sq in. abs.	100
Total temperature, T_t, °R	710
Mass flow, W, lb/sec	15.0
Nozzle-exit static pressure, p_3, lb/sq in. abs.	14.7
Ratio of specific heats, γ, (constant)	1.40

The upstream static-pressure parameter calculated from these conditions is

$$\frac{p_1A_1}{W\sqrt{gRT_t}} = \frac{100 \times 10 \times K}{15.0\sqrt{32.174 \times 53.35 \times 710}}$$

$$= 1.943$$

where K is found from table I (See footnote 2, table I) to be 32.174 for the particular set of units used.

From figure 2(a) at a static-pressure parameter of 1.943

$$\frac{u_1}{\sqrt{gRT_t}} = 0.498$$

and

$$\frac{p_tA_1}{W\sqrt{gRT_t}} = 2.210$$

The upstream velocity is

$$u_1 = 0.498\sqrt{32.174 \times 53.35 \times 710} = 550 \text{ feet per second}$$

The total pressure is found by use of the value of the total-pressure parameter corresponding to the upstream static-pressure parameter.

$$p_t = 2.210 \times \frac{15 \times \sqrt{32.174 \times 53.35 \times 710}}{32.174 \times 10}$$

$$= 114 \text{ pounds per square inch absolute}$$

The total pressure p_t can also be found from figure 3(a) because the upstream static pressure and static-pressure parameter are known.

The total-pressure parameter for critical velocity at the throat of the nozzle is its minimum value. From figure 2(a)

$$\frac{p_tA_2}{W\sqrt{gRT_t}} = 1.461$$

The throat area of the nozzle A_2 is therefore

$$A_2 = 10 \times \frac{1.461}{2.210} = 6.61 \text{ square inches}$$

The static-pressure parameter and velocity parameter at the nozzle exit are found by first computing the ratio of static pressure to total pressure at the nozzle exit.

$$\frac{p_3}{p_t} = \frac{14.7}{114} = 0.129$$

Then from figure 3(a) at $p_3/p_t = 0.129$

$$\frac{p_3A_3}{W\sqrt{gRT_t}} = 0.312$$

and

$$\frac{u_3}{\sqrt{gRT_t}} = 1.77$$

The nozzle-exit area A_3 and discharge velocity u_3 are therefore

$$A_3 = 0.312 \times \frac{15 \times \sqrt{32.174 \times 53.35 \times 710}}{14.7 \times 32.174}$$

$$= 10.92 \text{ square inches}$$

$$u_3 = 1.77 \times \sqrt{32.174 \times 53.35 \times 710} = 1954 \text{ feet per second}$$

<u>Underexpanding jet.</u> - If the diverging section of the nozzle in the previous example is removed, the fluid will expand freely after the nozzle to the discharge static pressure p_3 of 14.7 pounds per square inch absolute. The effective velocity u_e, which can be found for this type of flow from equation (15), is

$$\frac{u_e}{\sqrt{gRT_t}} = \frac{u_2}{\sqrt{gRT_t}} + \frac{p_2A_2}{W\sqrt{gRT_t}} - \frac{p_3A_2}{W\sqrt{gRT_t}}$$

where the subscript 2 refers to the nozzle-exit section. A simpler procedure, however, is to use the dashed extension of the curve of velocity parameter in figure 2(a). The term $\frac{p_3A_2}{W\sqrt{gRT_t}}$ is

$$\frac{p_3A_2}{W\sqrt{gRT_t}} = \frac{14.7 \times 6.61 \times 32.174}{15\sqrt{32.174 \times 53.35 \times 710}} = 0.1888$$

From the dashed extension of the curve of velocity parameter of figure 2(a), the effective velocity parameter $u_e/\sqrt{gRT_t}$, which corresponds to an abscissa of $\frac{p_3A_2}{W\sqrt{gRT_t}}$ of 0.1888, is 1.663.

The effective velocity is

$$u_e = 1.663\sqrt{32.174 \times 53.35 \times 710}$$

$$= 1836 \text{ feet per second}$$

Calculation of total temperature of a fluid from jet thrust. - The total temperature of a jet of fluid discharging from a convergent nozzle can be determined if the mass flow W, the jet thrust F, the discharge area A_n, and the static pressure of the environment into which the jet is discharging p_0 are known. A relation between the total temperature T_t and the known quantities can be derived as follows: The product of the effective velocity parameter $u_e/\sqrt{gRT_t}$ and the static-pressure parameter $p_0A_n/W\sqrt{gRT_t}$ is

$$\left(\frac{u_e}{\sqrt{gRT_t}}\right)\left(\frac{p_0A_n}{W\sqrt{gRT_t}}\right) = \frac{u_ep_0A_n}{WgRT_t}$$

The total temperature can therefore be expressed as

$$T_t = \frac{(Wu_e)\ p_0A_n}{gRW^2\left(\frac{u_e}{\sqrt{gRT_t}}\right)\left(\frac{p_0A_n}{W\sqrt{gRT_t}}\right)}$$

$$T_t = \frac{1}{gR} \times \frac{Fp_0(A_n/W^2)}{\left(\frac{u_e}{\sqrt{gRT_t}}\right)\left(\frac{p_0A_n}{W\sqrt{gRT_t}}\right)} \tag{21}$$

The ratio of the unknown parameters $u_e/\sqrt{gRT_t}$ and $p_0A_n/W\sqrt{gRT_t}$ is

$$\frac{\frac{u_e}{\sqrt{gRT_t}}}{\frac{p_0A_n}{W\sqrt{gRT_t}}} = \frac{Wu_e}{p_0A_n} = \frac{F}{p_0A_n} \tag{22}$$

Equation (22) could be represented in figure 2 by a straight line through the origin of coordinates with a slope of F/p_0A_n. The values of $u_e/\sqrt{gRT_t}$ and $p_0A_n/W\sqrt{gRT_t}$ to be used in equation (21) are found at the intersection of the straight line representing equation (22) and the curve of velocity parameter or if the nozzle is underexpanding, the effective velocity parameter. If the specific heat of the fluid is variable, the method of successive approximations should be used to make the value of γ_t compatible with the total temperature. For example, the total temperature T_t of the fluid is computed when an underexpanding jet of air flows from a convergent nozzle under the following conditions:

Mass flow, W, lb/sec	60
Discharge area, A_n, sq ft	2
Thrust, F, lb	3800
Discharge pressure, p_0, lb/sq in. abs.	6

The term F/p_0A_n is

$$\frac{F}{p_0A_n} = \frac{3800}{6 \times 144 \times 2} = 2.199$$

As a first approximation, γ_t is assumed to be 1.34. If a straight line is drawn through the origin of coordinates on figure 2(b) with a slope of 2.199, this line would intersect the curves of velocity parameter to the left of the curves marked $u = a$ and thus indicate that the jet is underexpanding. At the intersection between the line with slope F/p_0A_n and the dashed extension (slope = -1) of the velocity-parameter curve for $\gamma_t = 1.34$, the values of the effective velocity parameter and the static-pressure parameter would be

$$\frac{u_e}{\sqrt{gRT_t}} = 1.282$$

and

$$\frac{p_0 A}{W\sqrt{gRT_t}} = 0.582$$

From equation (21) the total temperature therefore is

$$T_t = \frac{3800 \times 6 \times 144 \times 2 \times 32.174}{60^2 \times 53.35 \times 1.282 \times 0.582}$$

$$= 1474^\circ \text{ R}$$

In figure 4, for a fuel-air ratio of 0 and a total temperature of 1474° R, the second approximation to the correct value of γ_t is 1.352. If the procedure, as described, is repeated for $\gamma_t = 1.352$, the total temperature T_t is 1480° R. In figure 4, the value of γ_t of 1.352 corresponds closely to the temperature 1480° R; hence, the second approximation of the value of γ_t is sufficiently accurate for the calculation of the total temperature.

Where the charts are applied to the problem of determining the total temperature of a jet, the discharge area A_n should actually be multiplied by a discharge coefficient C_W based on the ratio of total pressure to static pressure at the discharge cross section of the nozzle. In practice, discharge coefficients are based on the ratio of upstream total pressure to discharge static pressure. Use of the usual discharge coefficients, however, will not introduce any appreciable error in the calculated temperature.

The total temperature of the fluid in an underexpanding jet discharging from a convergent-divergent nozzle can also be determined if the static pressure at the discharge cross section of the nozzle is known in addition to the thrust, the discharge area, the mass flow, and the static pressure in the discharge environment. The procedure in determining the temperature is the same as that used for the convergent nozzle except that the straight line drawn through the origin of coordinates on figure 2(a) or figure 2(b) must intersect the line representing free expansion from the convergent-divergent nozzle. This line, as previously explained, has a slope of -1 and intersects the curve of velocity parameter at a value of static-pressure parameter equal to that computed from the static pressure and the total temperature at the discharge cross section. Because, in this case, the total temperature is unknown, a series of values of the total temperature is assumed and then by trial and error the position of the free expansion line is found such that the computed total temperature is equal to the assumed value.

This trial-and-error technique can be considerably shortened by making a simple geometrical construction on figure 2(a) or figure 2(b). For example, in figure 6, which is a replot of part of figure 2(b), a line has been drawn with slope F/p_0A_n. Another straight line is drawn through the origin of coordinates with a slope of F/p_nA_n where p_n is the static pressure at the discharge cross section of the nozzle. From a point A chosen on the line labeled F/p_0A_n, a horizontal line is drawn intersecting the line labeled F/p_nA_n at point B. From point B, a vertical line is drawn intersecting the curve labeled $u/\sqrt{gRT_t}$ at point C. Through point C a line is constructed having a slope of -1 intersecting the line labeled F/p_0A_n at point D. Point A is then moved along the line labeled F/p_0A_n and the construction repeated until point D, determined by the construction, coincides with point A. The final slant line with slope -1 is the curve representing the free expansion from the convergent-divergent nozzle considered. The total temperature can now be obtained by applying the procedure illustrated by the example given for the convergent nozzle.

Nonisentropic Frictionless Flows

The principal features of several nonisentropic steady flows in ducts may be described approximately without considering the effects of wall friction. Examples of such flow processes are normal compressibility shock, heat addition or combustion, flow in a tube with an abrupt change in area, and flow in the jet pump with a constant-area mixing length. Each of these flows is characterized by the constancy of the total momentum and the mass flow.

Two states of flow corresponding to a single value of the total-momentum parameter theoretically are always possible: one subsonic and the other supersonic. It has been shown (reference 3) that the subsonic regime corresponds to a state of higher entropy than the supersonic regime. In nearly all cases the flow, when disturbed, will be expected to adjust itself to the state of higher entropy, namely the subsonic regime.

The use of the charts in the solution of problems involving constant total momentum is illustrated by the following examples:

Theoretical normal compressibility shock. - When a compressible fluid flows through a tube at a velocity greater than the local speed of sound, the flow may revert more or less discontinuously to a regime in which the velocity is less than the local speed of sound. This phenomenon is called compressibility shock. The conditions at some distance on either side of the discontinuity are such that total energy, mass flow, and total momentum are conserved.

If the following conditions of a gas involved in a normal shock are given, the final velocity, the final static pressure, the initial and final total pressures, and the ratio of densities before and after the shock can be determined:

Ratio of specific heats, γ 1.40
Total temperature, T_t, ^oR 550
Initial velocity, u_1, ft/sec. 1690
Initial static pressure, p_1, lb/sq in. abs. 20

The velocity parameter before shock is

$$u_1/\sqrt{gRT_t} = \frac{1690}{\sqrt{32.174 \times 53.35 \times 550}} = 1.739$$

From figure 2(a), the value of the static-pressure parameter $p_1A/W\sqrt{gRT_t}$ is 0.327 and the corresponding value of the total-momentum parameter $u_1/\sqrt{gRT_t} + p_1A/W\sqrt{gRT_t}$ is equal to 2.067. After the shock to subsonic flow, the total-momentum parameter remains unchanged at a value of 2.067 and this value locates the point on the curve of total-momentum parameter in the subsonic region. The static-pressure parameter and other parameters in the subsonic region are then determined. The following quantities obtained from figure 2(a) correspond to the values of the total-momentum parameter in the supersonic and the subsonic regions:

	Supersonic, 1	Subsonic, 2
$\frac{u}{\sqrt{gRT_t}}$	1.739	0.671
$\frac{pA}{W\sqrt{gRT_t}}$	.327	1.399
$\frac{p_tA}{W\sqrt{gRT_t}}$	2.362	1.765

The final velocity is found from the value of $u_2/\sqrt{gRT_t}$.

$$u_2 = 0.671\sqrt{gRT_t}$$

$$= .671\sqrt{32.174 \times 53.35 \times 550}$$

$$= 652.2 \text{ feet per second}$$

The final static pressure and the initial and final total pressures are found as follows:

$$p_2 = p_1 \frac{\dfrac{p_2 A}{W\sqrt{gRT_t}}}{\dfrac{p_1 A}{W\sqrt{gRT_t}}} = 20 \times \frac{1.399}{0.327} = 85.6 \text{ pounds per square inch absolute}$$

$$p_{t,1} = p_1 \frac{\dfrac{p_{t,1} A}{W\sqrt{gRT_t}}}{\dfrac{p_1 A}{W\sqrt{gRT_t}}} = 20 \times \frac{2.362}{0.327} = 144.5 \text{ pounds per square inch absolute}$$

$$p_{t,2} = p_1 \frac{\dfrac{p_{t,2} A}{W\sqrt{gRT_t}}}{\dfrac{p_1 A}{W\sqrt{gRT_t}}} = 20 \times \frac{1.765}{0.327} = 108.0 \text{ pounds per square inch absolute}$$

From the equation of continuity, the ratio of the density after shock to the density before shock ρ_2/ρ_1 is equal to the inverse ratio of the velocities. That is,

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{1690}{652.2} = 2.591$$

In this case, for which γ is a constant, the product of the velocity parameters before and after shock is equal to the square of the critical velocity parameter (reference 4). For air

$$\frac{a_{cr}}{\sqrt{gRT_t}} = \sqrt{\frac{2\gamma}{\gamma + 1}} = \sqrt{\frac{7}{6}}$$

that is

$$1.739 \times 0.671 = 1.167 \approx \frac{7}{6}$$

where a_{cr} is the critical velocity of sound at which the velocity of the fluid equals the velocity of sound. This relation is only approximately valid for shocks in fluids having variable specific heats.

Combustion in tube of constant area. - In an ideal one-dimensional steady-flow combustion process in a constant-area duct, the total momentum before and after combustion is constant.

$$(Wu + pA)_1 = (Wu + pA)_2$$

If, for example, the fuel for combustion is assumed to be added with no increase in the momentum of the gas stream, the total-momentum parameter after combustion

$$(pA + Wu)_2/W_2\sqrt{gR_2T_{t,2}}$$

is

$$\frac{u_2}{\sqrt{gR_2T_{t,2}}} + \frac{p_2A}{W_2\sqrt{gR_2T_{t,2}}} = \frac{W_1\sqrt{gR_1T_{t,1}}}{W_2\sqrt{gR_2T_{t,2}}}\left(\frac{u_1}{\sqrt{gR_1T_{t,1}}} + \frac{p_1A}{W_1\sqrt{gR_1T_{t,1}}}\right) \qquad (23)$$

The change in total temperature during combustion can be determined by use of the constant-pressure combustion charts described in reference 5 or by other similar methods.

The initial and final velocities and the decrease in static pressure and total pressure during combustion can therefore be determined from the following conditions:

Mass rate of air flow, W, lb/sec	2.0
Fuel-air ratio, f	0.01
Lower enthalpy of combustion of fuel, h_c, Btu/lb	-18,900
Hydrogen-carbon ratio of fuel, m	0.175
Area of combustion chamber, A, sq in.	10.0
Total temperature of inlet air, $T_{t,1}$, $^{\circ}R$	600.0
Initial total pressure, $p_{t,1}$, lb/sq in. abs.	18.0

With the aid of figure 1 of reference 5, the total temperature after combustion $T_{t,2}$ is found to be 1348° R. The gas constants of the air and the burned mixture, as obtained from figure 5, are 53.35 and 53.39 foot-pounds per pound °R, respectively. The reduction factors are

$$\sqrt{gR_1T_{t,1}} = \sqrt{32.174 \times 53.35 \times 600} = 1014.8 \text{ feet per second}$$

and

$$\sqrt{gR_2T_{t,2}} = \sqrt{32.174 \times 53.39 \times 1348} = 1521.6 \text{ feet per second}$$

The static-pressure, total-momentum, and velocity parameters before combustion are found from figure 2(c) corresponding to the value of the initial total-pressure parameter, which is

$$\frac{p_{t,1}A}{W_1\sqrt{gR_1T_{t,1}}} = \frac{18 \times 10 \times 32.174}{2.0 \times 1014.8} = 2.853$$

Hence, from figure 2(c)

$$\frac{p_1A}{W_1\sqrt{gR_1T_{t,1}}} = 2.667$$

$$\frac{u_1}{\sqrt{gR_1T_{t,1}}} + \frac{p_1A}{W_1\sqrt{gR_1T_{t,1}}} = 3.035$$

$$\frac{\sqrt{gR_1T_{t,1}}}{u_1} = 2.718$$

from which

$$\frac{u_1}{\sqrt{gR_1T_{t,1}}} = 0.368$$

The initial static pressure p_1 and the initial velocity u_1 are then calculated to be 16.83 pounds per square inch absolute and 373.4 feet per second, respectively.

From equation (23), the total-momentum parameter after combustion is

$$\frac{u_2}{\sqrt{gR_2T_{t,2}}} + \frac{p_2A}{W_2\sqrt{gR_2T_{t,2}}} = \frac{1014.8}{1.01 \times 1521.6} \times 3.035$$

$$= 2.004$$

The instantaneous value of the ratio of specific heats after combustion γ_t is 1.352 as determined at the total temperature 1348° R for the fuel-air ratio of 0.01 in figure 4.

At a value of γ_t of 1.352 and at a total-momentum parameter of 2.004, the values of the flow parameters on figure 2(b) corresponding to the regime of higher entropy (the subsonic regime) are

$$u_2/\sqrt{gR_2T_{t,2}} = 0.729$$

$$p_2A/W_2\sqrt{gR_2T_{t,2}} = 1.275$$

$$p_{t,2}A/W_2\sqrt{gR_2T_{t,2}} = 1.672$$

hence,

$$u_2 = 0.729 \times 1521.6$$

$$= 1109 \text{ feet per second}$$

$$p_2 = \frac{1.275 \times 2.0 \times 1.01 \times 1521.6}{10 \times 32.174}$$

$$= 12.18 \text{ pounds per square inch absolute}$$

$$p_{t,2} = \frac{1.672 \times 2.0 \times 1.01 \times 1521.6}{10 \times 32.174}$$

$$= 15.97 \text{ pounds per square inch absolute}$$

The decreases during combustion in static pressure and total pressure, respectively, are

$$p_1 - p_2 = 16.81 - 12.18$$

$$= 4.63 \text{ pounds per square inch}$$

$$p_{t,1} - p_{t,2} = 18.00 - 15.97$$

$$= 2.03 \text{ pounds per square inch}$$

The amount of heat that can be added to a fluid for the given initial conditions is a maximum when the velocity of flow after combustion is equal to the local speed of sound. Thermal choking, which is evidenced by a decrease in the inlet velocity of flow, occurs when this limit of heat addition is exceeded. The minimum fuel-air ratio required to produce choking for the example cited is approximately 0.013 pound of fuel per pound of air. This value was determined by adjusting the fuel-air ratio to the value that produced the minimum possible total-momentum parameter on figure 2(b).

Flow in tube with sudden change in cross section. - When a compressible fluid flows in a tube having a cross-sectional area that enlarges suddenly from area A_1 to an area A_2 (fig. 7), the flow can be defined by the following equation relating the forces acting on the fluid:

$$Wu_3 + p_3A_3 = Wu_1 + p_1A_1 + p_2(A_2 - A_1) \tag{24}$$

where $A_3 = A_2$ and the subscripts 1, 2, and 3 correspond to an upstream section, the section just downstream of the point of enlargement, and a section located a sufficient distance downstream that uniform flow again exists, respectively. Because the mass flow W and the total temperature of the fluid T_t are constants, equation (24) can be written in terms of the dimensionless parameters as follows:

$$\frac{u_3}{\sqrt{gRT_t}} + \frac{p_3A_2}{W\sqrt{gRT_t}} = \frac{u_1}{\sqrt{gRT_t}} + \frac{p_1A_1}{W\sqrt{gRT_t}} - \frac{p_2A_1}{W\sqrt{gRT_t}} + \frac{p_2A_2}{W\sqrt{gRT_t}} \tag{25}$$

The calculations that use equation (25) can be divided into two classes: (a) critical or supersonic velocity at section 1 such that the mass flow W, the velocity u_1, and the pressure p_1 would be unchanged by a reduction of the pressure p_2; and (b) subsonic flow at section 1 such that the mass flow W depends on the pressure p_2.

In class (a), equation (25) can be used directly to solve for either of the pressures p_2 or p_3 if the other pressure or the velocity u_3 is known together with the mass flow W, or the total pressure $p_{t,1}$, the total temperature T_t, and the areas A_1 and A_2. If the pressure p_2 is of such a value that a shock occurs within a nozzle terminating at section 1, equation (25) is no longer applicable because the flow cannot be treated as one dimensional.

As an example of a problem of the first class, the mass flow and the static pressure of hot gases flowing through a convergent nozzle into an enlarged tube can be determined at the point of enlargement for the following conditions:

Downstream area, A_2, sq in.	7
Downstream static pressure, p_3, lb/sq in. abs.	25
Total temperature, T_t, ^{o}R	2000
Upstream area, A_1, sq in.	3
Upstream static pressure, p_1, lb/sq in. abs.	20

It is tentatively assumed that the flow in section 1 is critical. From figure 4 for a total temperature of 2000° R, the value of γ_t for air is 1.329. From the slant line marked $u = a$ and the curve of total-momentum parameter in figure 2(b), the total-momentum parameter and static-pressure parameter at cross section 1 are found by interpolation to be

$$\frac{u_1}{\sqrt{gRT_t}} + \frac{p_1A_1}{W\sqrt{gRT_t}} = 1.870$$

and

$$\frac{p_1A_1}{W\sqrt{gRT_t}} = 0.805$$

The mass flow W is found from the static-pressure parameter $p_1A_1/W\sqrt{gRT_t}$ to be

$$W = \frac{20 \times 3 \times 32.174}{0.805\sqrt{32.174 \times 53.35 \times 2000}}$$

$$= 1.294 \text{ pounds per second}$$

The downstream static-pressure parameter is

$$\frac{p_3A_2}{W\sqrt{gRT_t}} = \frac{25 \times 7 \times 32.174}{1.294\sqrt{32.174 \times 53.35 \times 2000}}$$

$$= 2.348$$

From figure 2(c), the total-momentum parameter corresponding to the downstream static-pressure parameter of 2.348 is

$$\frac{u_3}{\sqrt{gRT_t}} + \frac{p_3A_2}{W\sqrt{gRT_t}} = 2.760$$

The quantity $p_2A_2/W\sqrt{gRT_t} - p_2A_1/W\sqrt{gRT_t}$ in equation (25) can therefore be computed.

$$\frac{p_2A_2}{W\sqrt{gRT_t}} - \frac{p_2A_1}{W\sqrt{gRT_t}} = \left(\frac{u_3}{\sqrt{gRT_t}} + \frac{p_3A_2}{W\sqrt{gRT_t}}\right) - \left(\frac{u_1}{\sqrt{gRT_t}} + \frac{p_1A_1}{W\sqrt{gRT_t}}\right)$$

$$= 2.760 - 1.870$$

$$= 0.890$$

The static pressure near the point of enlargement therefore is

$$p_2 = \frac{0.890\ W\sqrt{gRT_t}}{A_2 - A_1}$$

$$= \frac{0.890 \times 1.294\sqrt{32.174 \times 53.35 \times 2000}}{32.174(7-3)}$$

$$= 16.57 \text{ pounds per square inch absolute}$$

Because the pressure p_2 proved to be less than the given static pressure p_1, the flow in section 1 is actually critical as assumed in the calculation.

An example of the type of calculation involved in the solution of problems of class (b) is given implicitly in the following example on the jet pump.

Jet pump. - A jet pump is a device that uses the momentum of a high-velocity jet to pump a fluid from a region of low pressure to a region of high pressure. If the mixing process takes place in a duct of constant area, the sum of the total momentums of the two fluids at the entrance section is equal to the total momentum of the mixture of fluids at the exit section.

In order to illustrate the use of the charts in the solution of problems of this type, a jet pump having a constant-area mixing length and an underexpanding convergent-divergent nozzle for the primary air as shown in figure 8 is assumed to operate under the following conditions:

Total pressure of primary air, $p_{t,a}$, lb/sq in. abs.	40
Total temperature of primary air, $T_{t,a}$, ^{o}R	2000
Throat area of nozzle, A_{min}, sq in.	2.00
Discharge area of nozzle, $A_{n,a}$, sq in.	2.15
Total pressure of secondary air, $p_{t,b}$, lb/sq in. abs.	14.7
Total temperature of secondary air, $T_{t,b}$, ^{o}R	540
Mixing area, A_3, sq in.	10
Discharge static pressure, p_3, lb/sq in. abs.	18.25

The mass flows of primary and secondary air can then be computed in the following manner: The instantaneous value of the ratio of specific heats of the primary air at 2000° R is found from figure 4 for a fuel-air ratio of 0 to be 1.329. Because the nozzle is a convergent-divergent type operating with an over-all pressure ratio p/p_t less than the critical pressure ratio, the total-pressure parameter at the throat of the nozzle is equal to the critical value of 1.482 as obtained from figure 2(b) corresponding to a value of γ_t of 1.329. The mass flow of primary air is found from the total-pressure parameter to be

$$W_a = \frac{p_{t,a} A_{min}}{1.482 \sqrt{gRT_{t,a}}}$$

$$= \frac{40 \times 2 \times 32.174}{1.482 \sqrt{32.174 \times 53.35 \times 2000}}$$

$$= 0.9373 \text{ pound per second}$$

Because the total pressure of the fluid in the nozzle is a constant, the total-pressure parameters along the nozzle are proportional to the cross-sectional areas along the nozzle. The total-pressure parameter at the discharge cross section is therefore

$$\frac{p_{t,a} A_{n,a}}{W_a \sqrt{gR_aT_{t,a}}} = \left(\frac{p_{t,a} A_{min}}{W_a \sqrt{gR_aT_{t,a}}}\right)\left(\frac{A_{n,a}}{A_{min}}\right)$$

$$= 1.482 \times \frac{2.15}{2.00}$$

$$= 1.593$$

As previously explained, the total momentum associated with the jet from an underexpanding nozzle is a constant equal to the total momentum of the fluid at the discharge cross section of the nozzle. The total-momentum parameter corresponding to the total-pressure parameter of 1.593 in the supersonic region at a value of γ_t of 1.329 (fig. 2(b)) is

$$\frac{u_{n,a}}{\sqrt{gR_aT_{t,a}}} + \frac{p_{n,a}\,A_{n,a}}{W_a\sqrt{gR_aT_{t,a}}} = 1.92$$

The total momentum of the primary air is therefore

$$W_au_{n,a} + p_{n,a}A_{n,a} = \frac{1.92 \times 0.9373}{32.174} \times \sqrt{32.174 \times 53.35 \times 2000}$$

$$= 103.6 \text{ pounds}$$

The mass flow of secondary air is found by determining a static pressure of the secondary air at cross section 2, which is consistent with the specified discharge static pressure of 18.25 pounds per square inch absolute. The required static pressure p_2 can be determined and, hence, the mass flow of secondary air, by assigning a series of static pressures p_2, computing the corresponding discharge static pressures p_3, and graphically finding the value of p_2 corresponding to the specified value of p_3.

For example, a static pressure p_2 of 14 pounds per square inch absolute is chosen. The mass flow of secondary air is found from the static-pressure parameter at cross section 2. The static-pressure parameter corresponding to a pressure ratio $p_2/p_{t,b}$ of 0.9523 is determined to be 3.185 from figure 3(a).

The mass flow of secondary air W_b is therefore

$$W_b = \frac{p_2A_2}{3.185\sqrt{gRT_{t,b}}} = \frac{14 \times (10-2.15) \times 32.174}{3.185\sqrt{32.174 \times 53.35 \times 540}}$$

$$= 1.153 \text{ pounds per second}$$

With the aid of figure 2(c) corresponding to a value of $p_2A_2/W_b\sqrt{gRT_{t,b}}$ of 3.185, the total-momentum parameter of the secondary air at cross section 2 is found to be

$$\frac{u_2}{\sqrt{gRT_{t,b}}} + \frac{p_2A_2}{W_b\sqrt{gRT_{t,b}}} = 3.492$$

from which the total momentum is

$$W_b u_2 + p_2A_2 = 3.492 \times \frac{1.153}{32.174}\sqrt{32.174 \times 53.35 \times 540}$$

$$= 120.5 \text{ pounds}$$

The total momentum at the discharge cross section 2, which is equal to the sum of the momentums of the fluids at cross section 2, is

$$(W_a + W_b)\, u_3 + p_3A_3 = 103.6 + 120.5$$

$$= 224.1 \text{ pounds}$$

The total temperature of the mixture at the discharge cross section $T_{t,3}$ is found to be 1220° R from a heat balance between the primary and secondary air and the mixture. With these values of the total momentum, of the mass flows of primary and secondary air, and of the mixture temperature, the total-momentum parameter is computed to be

$$\frac{u_3}{\sqrt{gRT_{t,3}}} + \frac{p_3A_3}{(W_a + W_b)\sqrt{gRT_{t,3}}} = \frac{224.1 \times 32.174}{2.090\sqrt{32.174 \times 53.35 \times 1220}}$$

$$= 2.384$$

From figure 2(a), the static-pressure parameter corresponding to the total-momentum parameter of 2.384 is

$$\frac{p_3A_3}{(W_a + W_b)\sqrt{gRT_{t,3}}} = 1.874$$

The static pressure p_3 is therefore

$$p_3 = 1.874 \times \frac{2.090\sqrt{32.174 \times 53.35 \times 1220}}{32.174 \times 10}$$

$$= 17.61 \text{ pounds per square inch absolute}$$

Similar calculations are made for a series of other discharge pressures. Figure 9 presents a plot of the static pressure at cross section 2 p_2 and the mass flow of secondary air W_b as functions of the discharge static pressure p_3. The required static pressure p_2 and the mass flow of secondary air W_b are 14.266 pounds per square inch absolute and 0.928 pound per second, respectively.

Steady flows with external forces. - The principal external forces acting on a fluid flowing in a duct are frictional forces and pressure forces exerted on the fluid by the walls of the duct. The differential equation expressing the relations between the forces acting on the fluid is not, in general, readily solved. It is therefore convenient to apply approximate numerical methods of solution. The use of the charts in applying numerical methods to the solution of the equations of motion permits a considerable reduction in the amount of labor involved.

The general case of nonadiabatic steady flow of a compressible fluid with friction and continuous changes in area can be described by the differential equation relating the forces acting on the fluid and by the equation of conservation of energy. The equation of motion presented in reference 2 (equation (59)) can be expressed in the notation of this report in the following form:

$$d(Wu + pA) - pdA + \frac{C_D}{2} \rho u^2 P dx = 0 \qquad (26)$$

where C_D is the surface drag coefficient or friction factor defined as the frictional force per unit wetted area divided by the dynamic pressure ($1/2\ \rho u^2$) and P is the wetted perimeter.

Equation (26) can be transformed into an integral equation of the form

$$(Wu + pA)_x = (Wu + pA)_{x=0} + \int_0^x \left(p \frac{dA}{dx} - \frac{C_D}{2} \frac{WPu}{A}\right) dx \qquad (27)$$

A solution of equation (27) can be obtained by the method of successive approximations (reference 6), if the area A, the wetted perimeter P, the total temperature T_t, and the gas constant R can be expressed as functions of the distance x along the duct. In the application of this method, the solution is obtained by assuming an arbitrary relation between the static pressure and the distance x as a first approximation. The static-pressure parameter at each distance x can be computed from the values of area A_x, mass flow W, total temperature $T_{t,x}$, and assumed pressure p_x. The velocity parameter corresponding to the static-pressure parameter can be obtained from figure 2; the velocity u_x can then be calculated. From the value of the Reynold's number $\frac{4W}{P\mu}$ for a duct, where μ is the viscosity of the fluid, the friction factor $C_{D,x}$ can be computed at any distance along the duct. The integrand in equation (26) can therefore be plotted as a function of the distance along the duct, and the area under the curve between the initial distance (when x is 0) and increasing values of distance x can be determined with a planimeter or by some method of approximate integration such as Simpson's rule. A relation between the total momentum and the distance along the duct can then be found. By dividing the total momentum at a series of points along the duct by the factor $W\sqrt{gR_xT_{t,x}}$, approximate values of the total-momentum parameter are determined. These values of total-momentum parameter are used to obtain values of static-pressure parameter from figure 2. From the values of static-pressure parameter, a second approximation to the variation of static pressure with distance along the duct is determined. The process is repeated using the second approximation to the variation of static pressure with distance along the duct. Successive applications of the foregoing procedure lead to a relation between static pressure and distance along the duct that is consistent with the equation of motion and the equation of conservation of energy.

Flight Propulsion Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, May 20, 1947.

APPENDIX

SYMBOLS

The following symbols are used in this report:

A	area
a	velocity of sound
a_{cr}	critical velocity of sound
C_D	friction factor or surface drag coefficient (based on wetted area)
C_W	discharge coefficient of nozzle
f	fuel-air ratio
F	jet thrust
g	ratio of gravitational to absolute unit of mass (32.174 lb/slug)
h_c	lower enthalpy of combustion of fuel, Btu/lb
M	Mach number
m	hydrogen-carbon ratio of fuel
P	wetted perimeter
p	static pressure
p_t	total pressure
R	gas constant, ft-lb/lb $^{\circ}F$
T	static temperature, $^{\circ}R$
T_t	total temperature, $^{\circ}R$
u	velocity, ft/sec
u_e	effective velocity, ft/sec
W	rate of mass flow

x position coordinate

γ ratio of specific heats

γ_h effective value of γ defined by equation (7)

γ_T effective value of γ defined by equation (5)

γ_t ratio of specific heats at total temperature of fluid

μ absolute viscosity

ρ density, slugs/cu ft

ϕ sum of external forces acting on fluid per unit length of duct

Numerical subscripts are used to indicate successive stations within a flow system irrespective of the assignments of other subscripts for geometrical concepts.

The subscripts a and b are used to differentiate two different fluids.

Parameters:

$\frac{u}{\sqrt{gRT_t}}$ velocity parameter

$\frac{pA}{W\sqrt{gRT_t}}$ static-pressure parameter

$\frac{p_tA}{W\sqrt{gRT_t}}$ total-pressure parameter

$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$ total-momentum parameter

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TABLE I - CONSISTENT UNITS FOR THE DIMENSIONLESS PARAMETERS[1]

Pressure, p	Area, A	Mass flow, W	Constant, K (2)
lb/sq in. absolute	sq in.	slug/sec	1
		lb/sec	32.174
	sq ft	slug/sec	144
		lb/sec	4633.1
lb/sq ft absolute	sq in.	slug/sec	0.0069444
		lb/sec	0.22343
	sq ft	slug/sec	1
		lb/sec	32.174
in. Hg absolute[3]	sq in.	slug/sec	0.49115
		lb/sec	15.802
	sq ft	slug/sec	70.727
		lb/sec	2275.6
ft water absolute[4]	sq in.	slug/sec	0.43314
		lb/sec	13.936
	sq ft	slug/sec	62.372
		lb/sec	2006.74

[1]The conversion factor g, the gas constant R, and the total temperature T_t are to be taken with the following units in order to make the units of the term $\sqrt{gRT_t}$ feet per second:

g	32.174 lb/slug
R	ft-lb/(lb)(°R)
T_t	°R

The velocity u is to be expressed in feet per second.

[2]Static-pressure and total-pressure parameters are made dimensionless by multiplying the parameter computed from the given numerical data by the factor K.

[3]Based on density of mercury at 32° F.

[4]Based on density of water at 59° F.

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TABLE II

VALUES OF THE DIMENSIONLESS PARAMETERS FOR CONSTANT SPECIFIC HEAT

[Ratio of specific heats γ of 1.40]

Pressure ratio $\frac{p}{p_t}$	Velocity parameter $\frac{u}{\sqrt{gRT_t}}$	Reciprocal of velocity parameter $\frac{\sqrt{gRT_t}}{u}$	Static-pressure parameter $\frac{pA}{W\sqrt{gRT_t}}$	Total-pressure parameter $\frac{p_tA}{W\sqrt{gRT_t}}$	Total-momentum parameter $\frac{Wu + pA}{W\sqrt{gRT_t}}$	Temperature ratio $\frac{T}{T_t}$	Mach number M
0.99	0.14168	7.05841	7.03817	7.1093	7.1798	0.99713	0.11991
.98	.20072	4.98206	4.95339	5.0545	5.1541	.99424	.17013
.97	.24628	4.06042	4.02524	4.1497	4.2715	.99134	.20905
.96	.28490	3.50997	3.46927	3.6138	3.7542	.98840	.24220
.95	.31912	3.13360	3.08801	3.2505	3.4071	.98545	.27169
.94	.35023	2.85525	2.80522	2.9843	3.1554	.98248	.29863
.93	.37901	2.63846	2.58432	2.7788	2.9633	.97948	.32366
.92	.40595	2.46338	2.40538	2.6145	2.8113	.97646	.34720
.91	.43140	2.31805	2.25642	2.4796	2.6878	.97341	.36954
.90	.45561	2.19486	2.12977	2.3664	2.5854	.97034	.39090
.89	.47878	2.08865	2.02025	2.2699	2.4990	.96725	.41144
.88	.50105	1.99580	1.92422	2.1866	2.4253	.96414	.43127
.87	.52255	1.91370	1.83905	2.1138	2.3616	.96099	.45051
.86	.54336	1.84041	1.76278	2.0498	2.3061	.95782	.46922
.85	.56357	1.77442	1.69391	1.9928	2.2575	.95463	.48749
.84	.58324	1.71458	1.63126	1.9420	2.2145	.95140	.50536
.82	.62118	1.60985	1.52111	1.8550	2.1423	.94488	.54009
.80	.65754	1.52082	1.42689	1.7836	2.0844	.93823	.57372
.78	.69260	1.44384	1.34489	1.7242	2.0375	.93147	.60650
.76	.72657	1.37633	1.27253	1.6744	1.9991	.92458	.63862
.74	.75963	1.31644	1.20792	1.6323	1.9675	.91757	.67022
.72	.79191	1.26278	1.14964	1.5967	1.9416	.91041	.70144
.70	.82353	1.21428	1.09663	1.5666	1.9202	.90311	.73240
.68	.85460	1.17013	1.04805	1.5412	1.9026	.89566	.76318
.66	.88521	1.12968	1.00322	1.5200	1.8884	.88806	.79389
.64	.91543	1.09238	.96161	1.5025	1.8770	.88028	.82461
.62	.94533	1.05783	.92278	1.4884	1.8681	.87234	.85542
.60	.97498	1.02566	.88637	1.4773	1.8614	.86420	.88639
.58	1.00444	.99558	.85208	1.4691	1.8565	.85587	.91761
.56	1.03377	.96734	.81966	1.4637	1.8534	.84733	.94914
.54	1.06301	.94073	.78887	1.4609	1.8519	.83857	.98107
.52	1.09222	.91557	.75954	1.4606	1.8518	.82958	1.01348
.50	1.12145	.89170	.73150	1.4630	1.8529	.82034	1.04646
.48	1.15076	.86899	.70460	1.4679	1.8554	.81082	1.08008
.46	1.18019	.84732	.67873	1.4755	1.8589	.80102	1.11446
.44	1.20979	.82659	.65376	1.4858	1.8636	.79091	1.14970
.42	1.23964	.80669	.62960	1.4990	1.8692	.78047	1.18591
.40	1.26978	.78754	.60614	1.5154	1.8759	.76967	1.22324
.38	1.30027	.76907	.58332	1.5350	1.8836	.75847	1.26183
.36	1.33120	.75120	.56103	1.5584	1.8922	.74684	1.30186
.34	1.36264	.73387	.53921	1.5859	1.9018	.73474	1.34353
.32	1.39467	.71702	.51778	1.6180	1.9124	.72213	1.38707
.30	1.42740	.70058	.49666	1.6555	1.9240	.70893	1.43278
.28	1.46093	.68449	.47579	1.6992	1.9367	.69510	1.48096
.26	1.49542	.66871	.45508	1.7503	1.9505	.68053	1.53205
.24	1.53100	.65317	.43445	1.8102	1.9654	.66515	1.58655
.22	1.56790	.63780	.41381	1.8810	1.9817	.64881	1.64510
.20	1.60633	.62254	.39306	1.9653	1.9994	.63138	1.70854
.18	1.64662	.60730	.37207	2.0671	2.0187	.61266	1.77951
.16	1.68917	.59201	.35070	2.1919	2.0399	.59239	1.85484
.14	1.73451	.57653	.32875	2.3482	2.0632	.57021	1.94130
.12	1.78340	.56073	.30596	2.5496	2.0894	.54564	2.04046
.10	1.83694	.54438	.28196	2.8106	2.1189	.51795	2.15719
.08	1.89692	.52717	.25618	3.2023	2.1531	.48596	2.29978
.06	1.96640	.50854	.22763	3.7938	2.1940	.44761	2.48403
.04	2.05170	.48740	.19430	4.8575	2.2460	.39865	2.74635
.02	2.17044	.46074	.15087	7.5336	2.3211	.32702	3.20771
.01	2.26321	.44185	.11854	11.8535	2.3817	.26827	3.69296

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TABLE III

VALUES OF THE DIMENSIONLESS PARAMETERS FOR VARIABLE SPECIFIC HEATS

[Ratio of specific heats at the total temperature of the fluid γ_t of 1.34]

Pressure ratio $\frac{p}{p_t}$	Velocity parameter $\frac{u}{\sqrt{gRT_t}}$	Reciprocal of velocity parameter $\frac{\sqrt{gRT_t}}{u}$	Static-pressure parameter $\frac{pA}{W\sqrt{gRT_t}}$	Total-pressure parameter $\frac{p_tA}{W\sqrt{gRT_t}}$	Total-momentum parameter $\frac{Wu + pA}{W\sqrt{gRT_t}}$	Temperature ratio $\frac{T}{T_t}$
0.99	0.14169	7.05761	7.03964	7.1108	7.1813	0.99745
.98	.20075	4.98127	4.95580	5.0569	5.1566	.99489
.97	.24634	4.05951	4.02824	4.1528	4.2746	.99230
.96	.28499	3.50885	3.47266	3.6174	3.7576	.98969
.95	.31925	3.13230	3.09176	3.2545	3.4110	.98706
.94	.35041	2.85382	2.80931	2.9886	3.1597	.98441
.93	.37923	2.63694	2.58876	2.7836	2.9679	.98173
.92	.40621	2.46176	2.41014	2.6197	2.8164	.97903
.91	.43172	2.31634	2.26146	2.4851	2.6932	.97631
.90	.45599	2.19304	2.13508	2.3723	2.5911	.97357
.89	.47922	2.08672	2.02577	2.2762	2.5050	.97080
.88	.50156	1.99380	1.93000	2.1932	2.4316	.96800
.87	.52311	1.91163	1.84508	2.1208	2.3682	.96518
.86	.54400	1.83824	1.76900	2.0570	2.3130	.96233
.85	.56428	1.77217	1.70035	2.0004	2.2646	.95947
.84	.58403	1.71224	1.63789	1.9499	2.2219	.95658
.82	.62214	1.60737	1.52811	1.8636	2.1502	.95069
.80	.65868	1.51820	1.43426	1.7928	2.0929	.94471
.78	.69392	1.44108	1.35260	1.7341	2.0465	.93860
.76	.72810	1.37344	1.28056	1.6850	2.0087	.93238
.74	.76137	1.31341	1.21624	1.6436	1.9776	.92601
.72	.79389	1.25962	1.15823	1.6086	1.9521	.91951
.70	.82576	1.21101	1.10548	1.5793	1.9312	.91286
.68	.85709	1.16674	1.05716	1.5546	1.9142	.90608
.66	.88797	1.12616	1.01256	1.5342	1.9005	.89913
.64	.91848	1.08876	.97118	1.5175	1.8897	.89201
.62	.94869	1.05408	.93254	1.5041	1.8812	.88469
.60	.97867	1.02180	.89636	1.4939	1.8750	.87724
.58	1.00847	.99160	.86227	1.4867	1.8707	.86957
.56	1.03815	.96325	.83000	1.4822	1.8682	.86167
.54	1.06776	.93654	.79940	1.4804	1.8672	.85357
.52	1.09737	.91127	.77022	1.4812	1.8676	.84521
.50	1.12704	.88728	.74230	1.4846	1.8693	.83660
.48	1.15678	.86447	.71554	1.4907	1.8723	.82772
.46	1.18669	.84268	.68978	1.4995	1.8765	.81856
.44	1.21678	.82184	.66494	1.5112	1.8817	.80908
.42	1.24713	.80184	.64089	1.5259	1.8880	.79928
.40	1.27782	.78258	.61754	1.5439	1.8954	.78911
.38	1.30892	.76399	.59477	1.5652	1.9037	.77851
.36	1.34046	.71602	.57255	1.5904	1.9130	.76748
.34	1.37254	.72858	.55075	1.6199	1.9233	.75593
.32	1.40528	.71160	.52936	1.6543	1.9346	.74390
.30	1.43875	.69505	.50822	1.6941	1.9470	.73120
.28	1.47306	.67886	.48732	1.7404	1.9604	.71785
.26	1.50840	.66295	.46660	1.7946	1.9750	.70382
.24	1.54486	.64731	.44593	1.8581	1.9908	.68891
.22	1.58274	.63182	.42517	1.9326	2.0079	.67293
.20	1.62223	.61644	.40432	2.0216	2.0266	.65590
.18	1.66369	.60108	.38312	2.1284	2.0468	.63738
.16	1.70746	.58567	.36153	2.2595	2.0690	.61729
.14	1.75418	.57006	.33927	2.4234	2.0935	.59514
.12	1.80453	.55416	.31611	2.6343	2.1206	.57043
.10	1.85981	.53769	.29168	2.9168	2.1515	.54247
.08	1.92172	.52037	.26524	3.3155	2.1870	.50972
.06	1.99344	.50165	.23581	3.9302	2.2292	.47007
.04	2.08139	.48045	.20121	5.0303	2.2826	.41880
.02	2.20297	.45393	.15544	7.7720	2.3584	.34243

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TABLE III - Continued

VALUES OF THE DIMENSIONLESS PARAMETERS FOR VARIABLE SPECIFIC HEATS - Continued

[Ratio of specific heats at the total temperature of the fluid γ_t of 1.30]

Pressure ratio $\frac{p}{p_t}$	Velocity parameter $\frac{u}{\sqrt{gRT_t}}$	Reciprocal of velocity parameter $\frac{\sqrt{gRT_t}}{u}$	Static-pressure parameter $\frac{pA}{W\sqrt{gRT_t}}$	Total-pressure parameter $\frac{p_tA}{W\sqrt{gRT_t}}$	Total-momentum parameter $\frac{Wu + pA}{W\sqrt{gRT_t}}$	Temperature ratio $\frac{T}{T_t}$
0.99	0.14171	7.05686	7.04051	7.1116	7.1822	0.99768
.98	.20077	4.98092	4.95775	5.0589	5.1585	.99535
.97	.24638	4.05877	4.03032	4.1550	4.2767	.99299
.96	.28506	3.50808	3.47516	3.6200	3.7602	.99061
.95	.31934	3.13143	3.09453	3.2574	3.4139	.98822
.94	.35053	2.85282	2.81233	2.9918	3.1629	.98580
.93	.37938	2.63590	2.59205	2.7872	2.9714	.98336
.92	.40641	2.46059	2.41360	2.6235	2.8200	.98090
.91	.43195	2.31509	2.26513	2.4892	2.6971	.97842
.90	.45626	2.19173	2.13897	2.3766	2.5952	.97592
.89	.47954	2.08534	2.02987	2.2808	2.5094	.97340
.88	.50192	1.99234	1.93425	2.1980	2.4362	.97084
.87	.52353	1.91011	1.84950	2.1259	2.3730	.96827
.86	.54447	1.83666	1.77361	2.0623	2.3181	.96567
.85	.56480	1.77054	1.70513	2.0060	2.2699	.96306
.84	.58461	1.71054	1.64281	1.9557	2.2274	.96040
.82	.62284	1.60556	1.53337	1.8700	2.1562	.95504
.80	.65951	1.51628	1.43977	1.7997	2.0993	.94955
.78	.69490	1.43905	1.35843	1.7416	2.0533	.94397
.76	.72923	1.37131	1.28663	1.6929	2.0159	.93825
.74	.76267	1.31118	1.22258	1.6521	1.9852	.93243
.72	.79536	1.25729	1.16482	1.6178	1.9602	.92645
.70	.82742	1.20857	1.11231	1.5890	1.9397	.92035
.68	.85896	1.16420	1.06422	1.5650	1.9232	.91412
.66	.89005	1.12353	1.01985	1.5452	1.9099	.90772
.64	.92079	1.08603	.97871	1.5292	1.8995	.90119
.62	.95124	1.05126	.94029	1.5166	1.8915	.89444
.60	.98147	1.01888	.90430	1.5072	1.8858	.88754
.58	1.01155	.98859	.87044	1.5008	1.8820	.88049
.56	1.04152	.96013	.83837	1.4971	1.8799	.87318
.54	1.07144	.93332	.80797	1.4962	1.8794	.86569
.52	1.10138	.90795	.77897	1.4980	1.8804	.85794
.50	1.13139	.88387	.75125	1.5025	1.8826	.84996
.48	1.16151	.86095	.72467	1.5097	1.8862	.84171
.46	1.19179	.83907	.69911	1.5198	1.8909	.83319
.44	1.22232	.81811	.67447	1.5329	1.8968	.82442
.42	1.25312	.79801	.65060	1.5490	1.9037	.81528
.40	1.28428	.77864	.62743	1.5686	1.9117	.80580
.38	1.31588	.75995	.60484	1.5917	1.9207	.79589
.36	1.34798	.74185	.58274	1.6187	1.9307	.78552
.34	1.38064	.72430	.56114	1.6504	1.9418	.77474
.32	1.41402	.70720	.53992	1.6872	1.9539	.76345
.30	1.44817	.69053	.51893	1.7298	1.9671	.75150
.28	1.48322	.67421	.49822	1.7794	1.9814	.73897
.26	1.51937	.65817	.47762	1.8370	1.9970	.72568
.24	1.55675	.64236	.45712	1.9047	2.0139	.71162
.22	1.59561	.62672	.43649	1.9840	2.0321	.69647
.20	1.63618	.61118	.41577	2.0789	2.0520	.68028
.18	1.67884	.59565	.39472	2.1929	2.0736	.66268
.16	1.72398	.58005	.37324	2.3328	2.0972	.64346
.14	1.77227	.56425	.35108	2.5077	2.1234	.62220
.12	1.82448	.54810	.32802	2.7335	2.1525	.59846
.10	1.88198	.53136	.30362	3.0362	2.1856	.57141
.08	1.94648	.51375	.27726	3.4657	2.2237	.53968
.06	2.02167	.49464	.24772	4.1287	2.2694	.50081
.04	2.11444	.47294	.21294	5.3236	2.3274	.45026
.02	2.24402	.44563	.16657	8.3284	2.4106	.37378

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TABLE III - Concluded

VALUES OF THE DIMENSIONLESS PARAMETERS FOR VARIABLE SPECIFIC HEATS - Concluded

[Ratio of the specific heats at the total temperature of the fluid γ_t of 1.26]

Pressure ratio $\frac{p}{p_t}$	Velocity parameter $\frac{u}{\sqrt{gRT_t}}$	Reciprocal of velocity parameter $\frac{\sqrt{gRT_t}}{u}$	Static-pressure parameter $\frac{pA}{W\sqrt{gRT_t}}$	Total-pressure parameter $\frac{p_tA}{W\sqrt{gRT_t}}$	Total-momentum parameter $\frac{Wu + pA}{W\sqrt{gRT_t}}$	Temperature ratio $\frac{T}{T_t}$
0.99	0.14164	7.05990	7.04529	7.1164	7.1869	0.99793
.98	.20077	4.98087	4.96015	5.0614	5.1609	.99584
.97	.24648	4.05712	4.03169	4.1564	4.2782	.99373
.96	.28520	3.50627	3.47682	3.6217	3.7620	.99160
.95	.31947	3.13015	3.09716	3.2602	3.4166	.98946
.94	.35063	2.85200	2.81575	2.9955	3.1664	.98729
.93	.37953	2.63481	2.59558	2.7910	2.9751	.98511
.92	.40656	2.45965	2.41759	2.6278	2.8242	.98290
.91	.43218	2.31387	2.26914	2.4936	2.7013	.98067
.90	.45660	2.19008	2.14286	2.3810	2.5995	.97844
.89	.47985	2.08399	2.03433	2.2858	2.5143	.97617
.88	.50235	1.99066	1.93866	2.2030	2.4410	.97388
.87	.52394	1.90863	1.85435	2.1314	2.3783	.97156
.86	.54500	1.83485	1.77839	2.0679	2.3234	.96923
.85	.56537	1.76874	1.71016	2.0120	2.2755	.96688
.84	.58519	1.70884	1.64817	1.9621	2.2334	.96450
.82	.62355	1.60373	1.53903	1.8769	2.1626	.95966
.80	.66043	1.51416	1.44562	1.8070	2.1060	.95473
.78	.69594	1.43690	1.36463	1.7495	2.0606	.94971
.76	.73046	1.36899	1.29308	1.7014	2.0235	.94455
.74	.76404	1.30883	1.22938	1.6613	1.9934	.93930
.72	.79696	1.25477	1.17183	1.6275	1.9688	.93390
.70	.82918	1.20602	1.11964	1.5995	1.9488	.92838
.68	.86094	1.16153	1.07179	1.5762	1.9327	.92274
.66	.89226	1.12075	1.02768	1.5571	1.9199	.91695
.64	.92324	1.08314	.98677	1.5418	1.9100	.91103
.62	.95396	1.04826	.94857	1.5300	1.9025	.90490
.60	.98448	1.01577	.91283	1.5214	1.8973	.89866
.58	1.01486	.98535	.87915	1.5158	1.8940	.89222
.56	1.04511	.95683	.84733	1.5131	1.8924	.88556
.54	1.07538	.92990	.81717	1.5133	1.8926	.87877
.52	1.10568	.90442	.78839	1.5161	1.8941	.87170
.50	1.13606	.88024	.76088	1.5218	1.8969	.86440
.48	1.16655	.85723	.73452	1.5302	1.9011	.85685
.46	1.19727	.83524	.70919	1.5417	1.9065	.84909
.44	1.22825	.81417	.68472	1.5562	1.9130	.84100
.42	1.25951	.79396	.66106	1.5740	1.9206	.83261
.40	1.29122	.77446	.63807	1.5952	1.9293	.82389
.38	1.32337	.75565	.61567	1.6202	1.9390	.81476
.36	1.35604	.73744	.59381	1.6495	1.9498	.80523
.34	1.38936	.71976	.57240	1.6835	1.9618	.79526
.32	1.42343	.70253	.55135	1.7230	1.9748	.78481
.30	1.45832	.68572	.53059	1.7686	1.9889	.77377
.28	1.49416	.66927	.51007	1.8217	2.0042	.76212
.26	1.53124	.65307	.48962	1.8831	2.0209	.74972
.24	1.56957	.63712	.46931	1.9554	2.0389	.73661
.22	1.60949	.62131	.44884	2.0402	2.0583	.72240
.20	1.65124	.60561	.42833	2.1416	2.0796	.70727
.18	1.69522	.58989	.40743	2.2635	2.1027	.69069
.16	1.74191	.57408	.38608	2.4130	2.1280	.67252
.14	1.79191	.55806	.36409	2.6006	2.1560	.65241
.12	1.84613	.54167	.34114	2.8428	2.1873	.62978
.10	1.90602	.52465	.31687	3.1687	2.2229	.60396
.08	1.97351	.50671	.29054	3.6318	2.2640	.57339
.06	2.05257	.48719	.26099	4.3499	2.3136	.53571
.04	2.15067	.46497	.22607	5.6518	2.3768	.48621
.02	2.28916	.43684	.17918	8.9592	2.4684	.41018

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TABLE IV

VALUES OF THE DIMENSIONLESS PARAMETERS AT THE CRITICAL PRESSURE RATIO

Ratio of specific heats γ	Critical pressure ratio $\frac{p}{p_t}$	Velocity parameter $\frac{u}{\sqrt{gRT_t}}$	Reciprocal of velocity parameter $\frac{\sqrt{gRT_t}}{u}$	Static-pressure parameter $\frac{pA}{W\sqrt{gRT_t}}$	Total-pressure parameter $\frac{p_tA}{W\sqrt{gRT_t}}$	Total-momentum parameter $\frac{Wu + pA}{W\sqrt{gRT_t}}$	Temperature ratio $\frac{T}{T_t}$
[a]1.40	0.52828	1.0801	0.92582	0.77152	1.4604	1.8516	0.83333
[b]1.34	.53625	1.0733	.93169	.79382	1.4803	1.8671	.85203
[b]1.30	.54340	1.0664	.93777	.81302	1.4962	1.8794	.86697
[b]1.26	.55090	1.0589	.94438	.83342	1.5128	1.8923	.88250

[a]Constant ratio of specific heats.

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[b]Variable ratio of specific heats; value tabulated is ratio of specific heats at total temperature of the fluid.

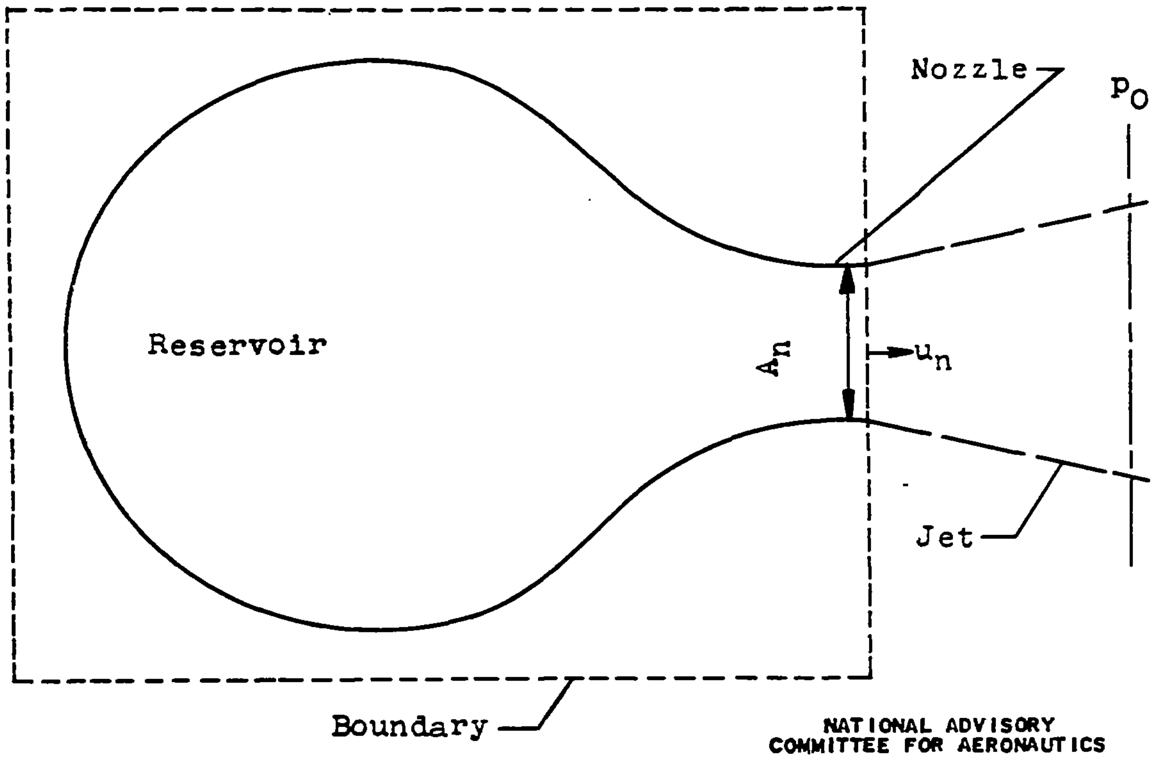
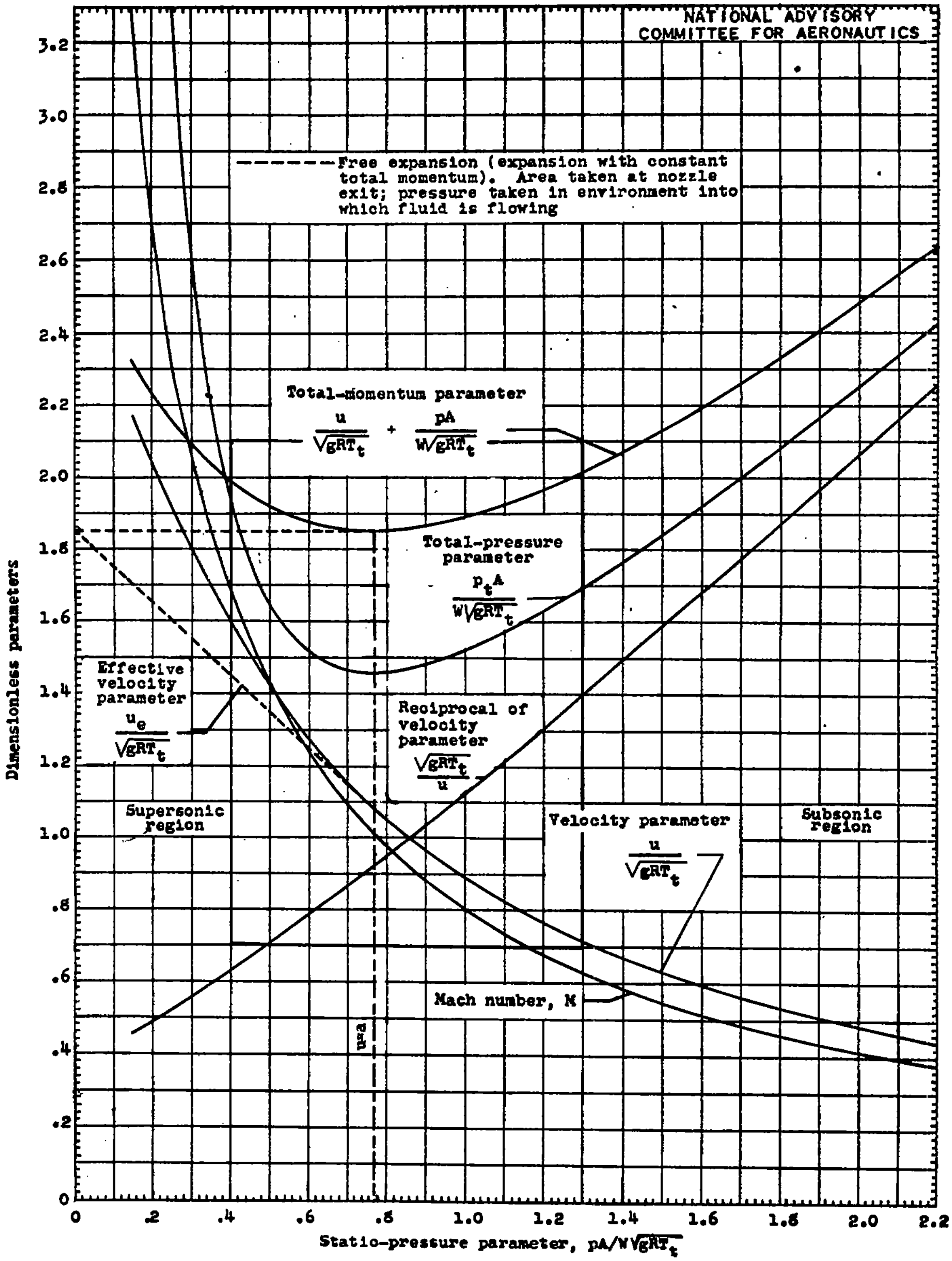


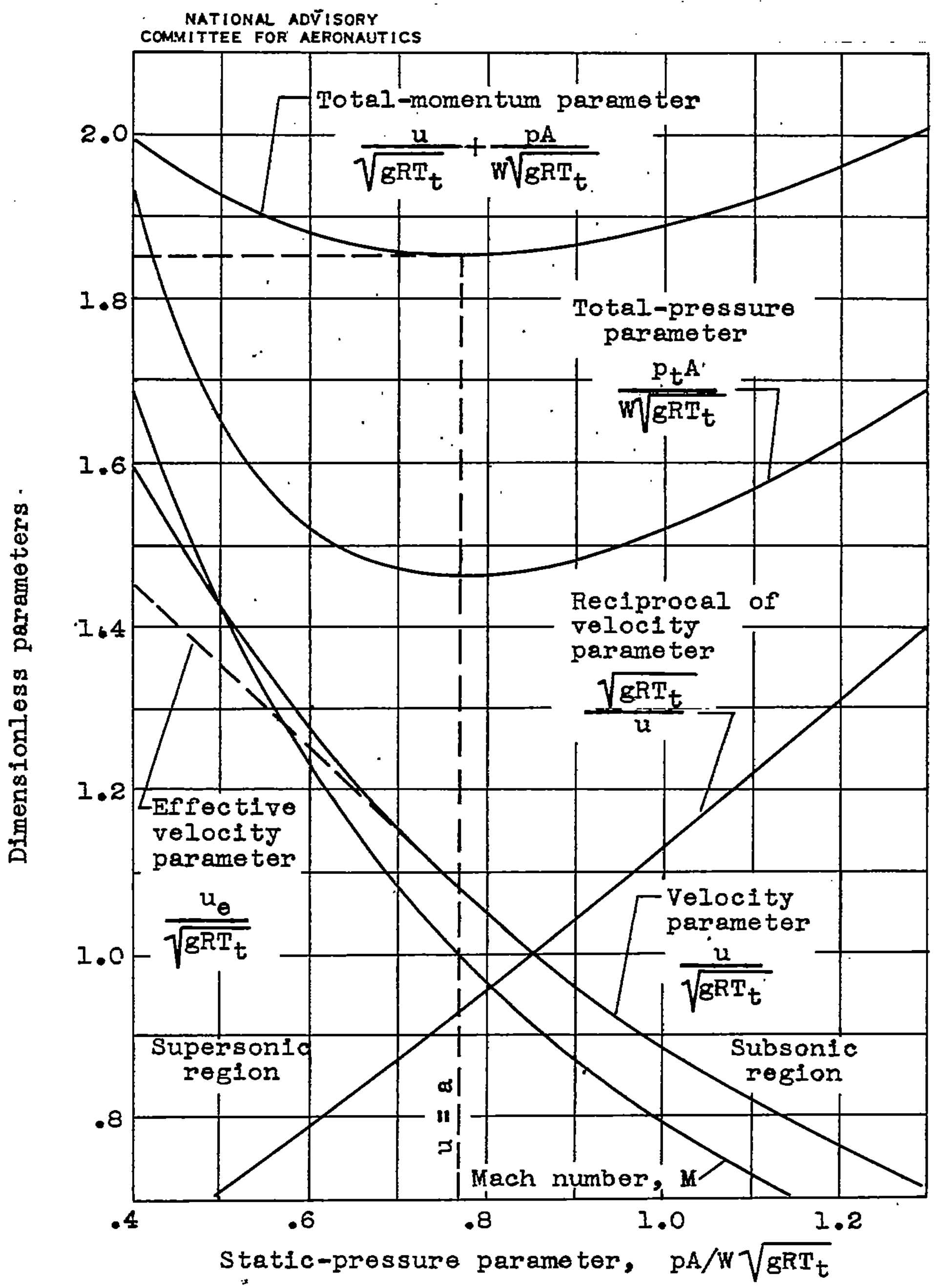
Figure 1. - Free expansion of compressible fluid from nozzle.



(a) Constant ratio of specific heats of 1.40 for range of static-pressure parameter from 0 to 2.2. Over-all plot; enclosed area enlarged in figure 2(a) concluded.

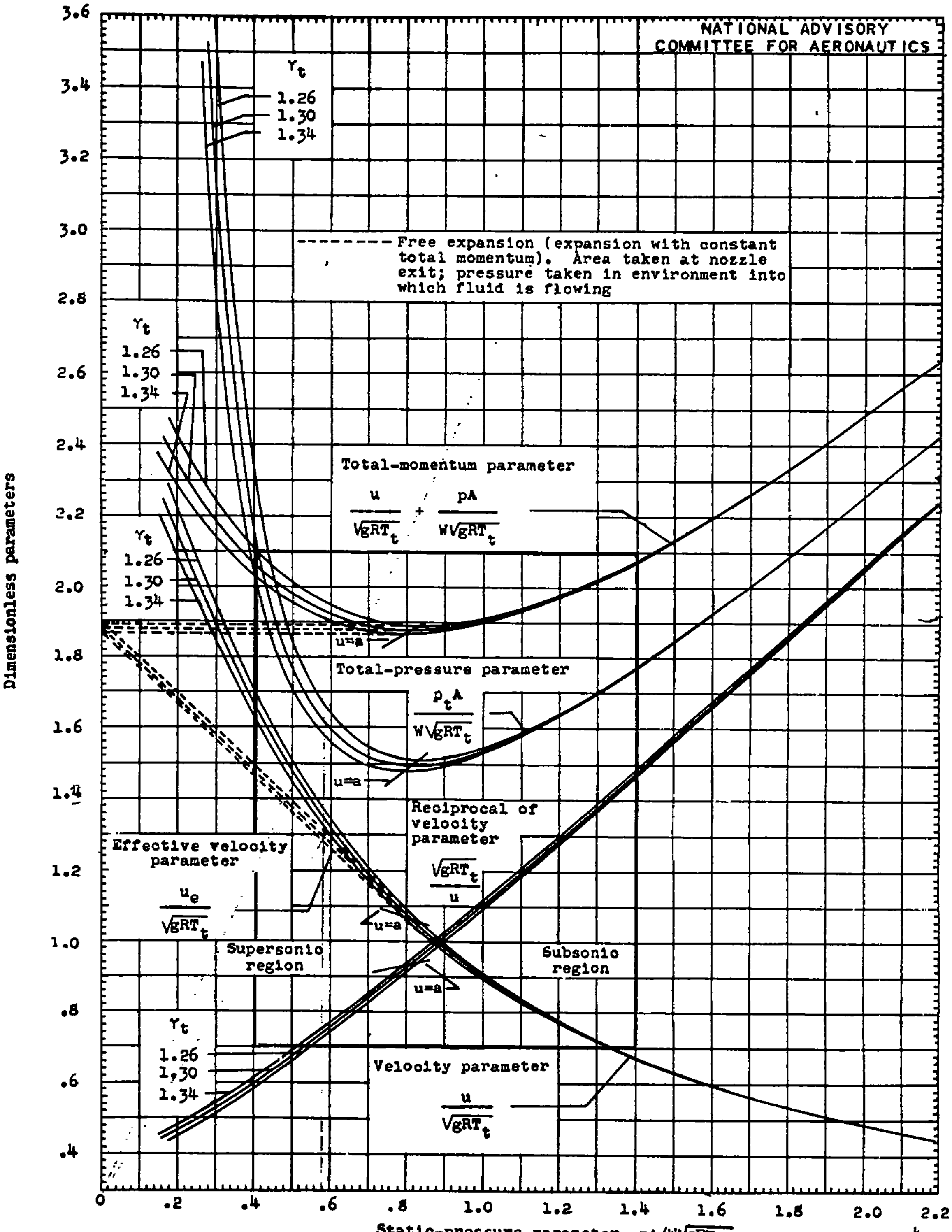
Figure 2. - Relations between dimensionless parameters for compressible fluid. (A 17 in. by 22 in. print of this figure is enclosed.)

757



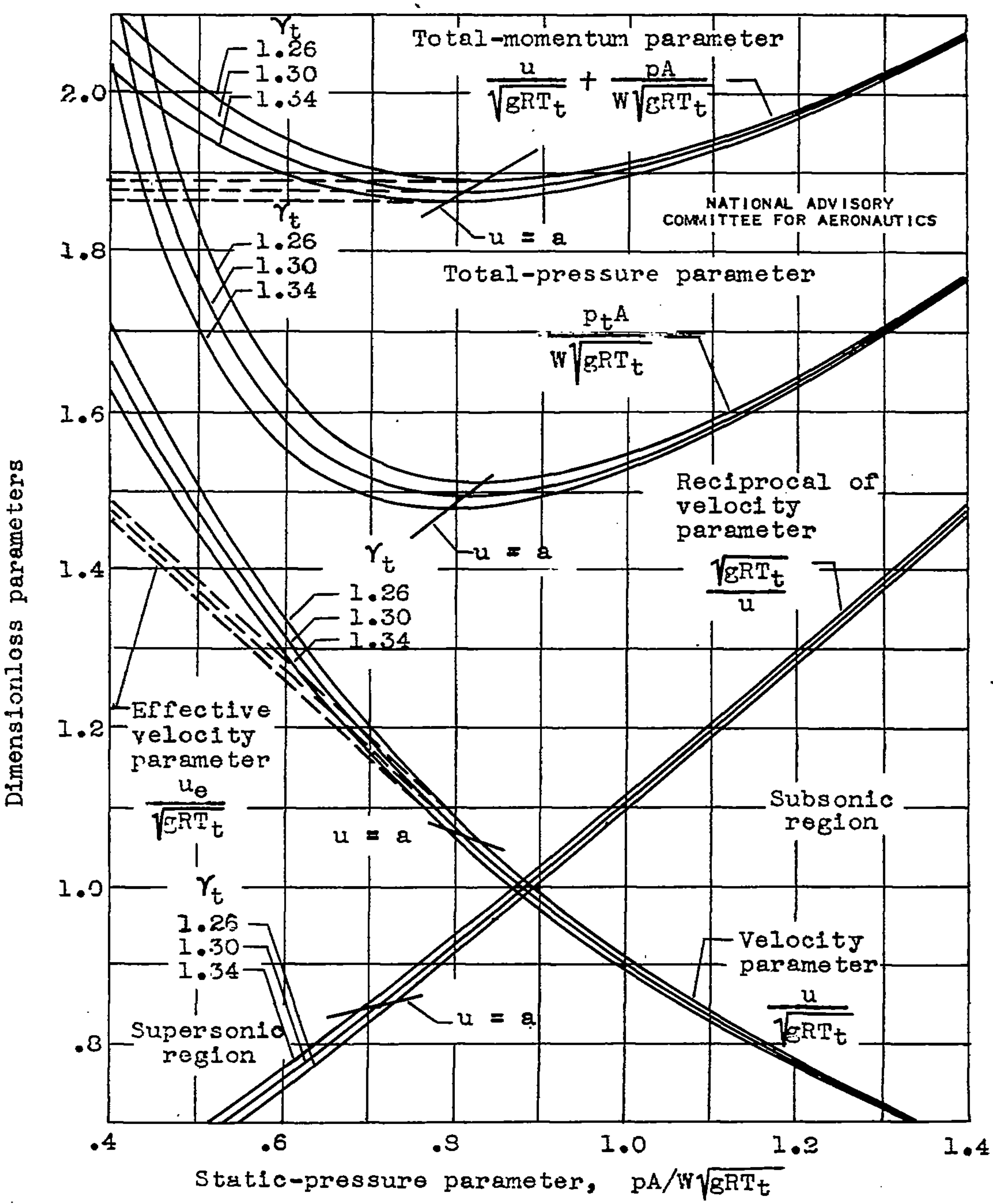
(a) Concluded. Constant ratio of specific heats of 1.40 for range of static-pressure parameter from 0 to 2.2. Enlargement of area shown in figure 2(a).

Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid. (A 17 in. by 22 in. print of this figure is enclosed.)



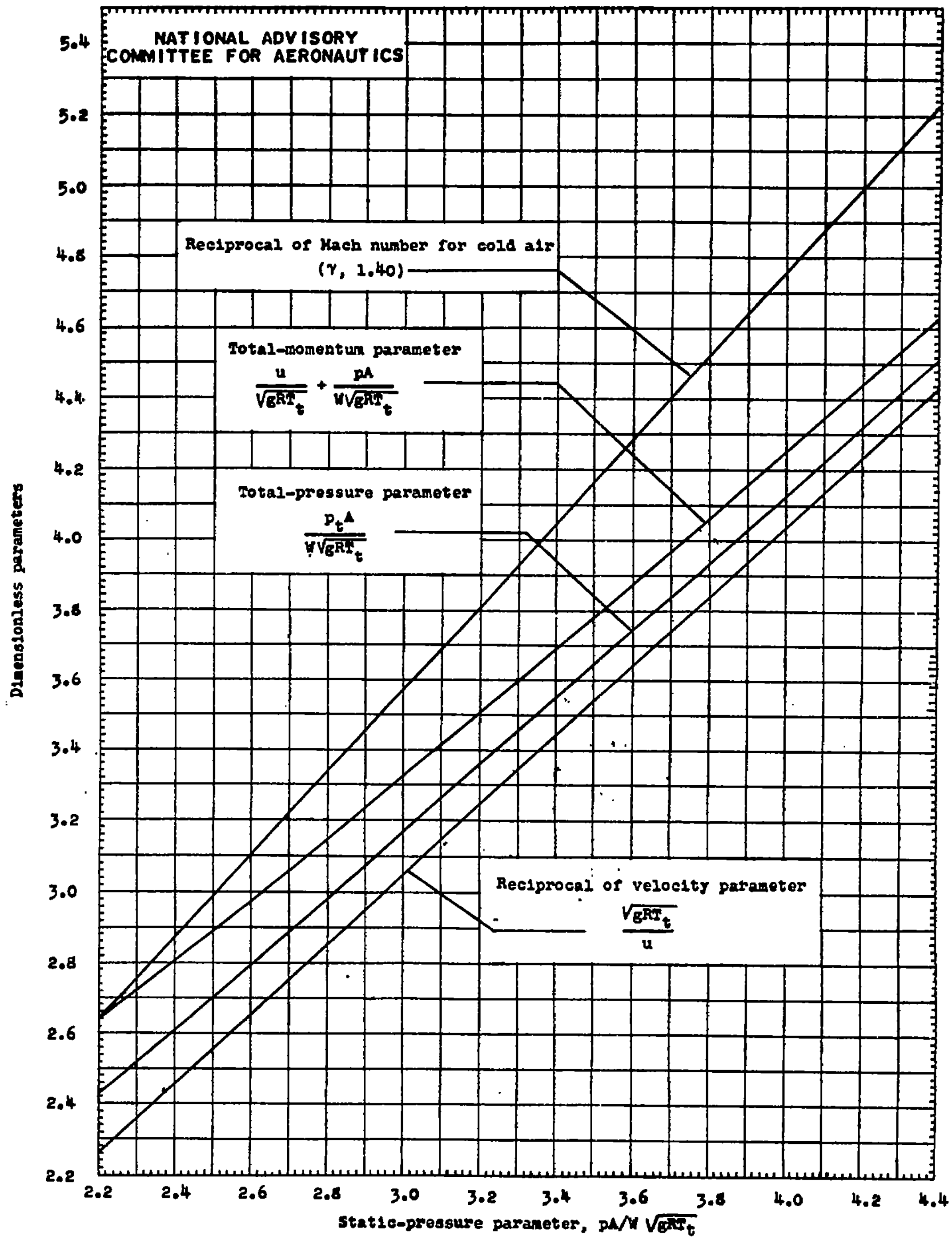
(b) Variable specific heats for range of static-pressure parameter from 0 to 2.2. Over-all plot; enclosed area enlarged in figure 2(b) concluded.

Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid. (A 17 in. by 22 in. print of this figure is enclosed.)



(b) Concluded. Variable specific heats for range of static-pressure parameter from 0 to 2.2. Enlargement of area shown in figure 2(b).

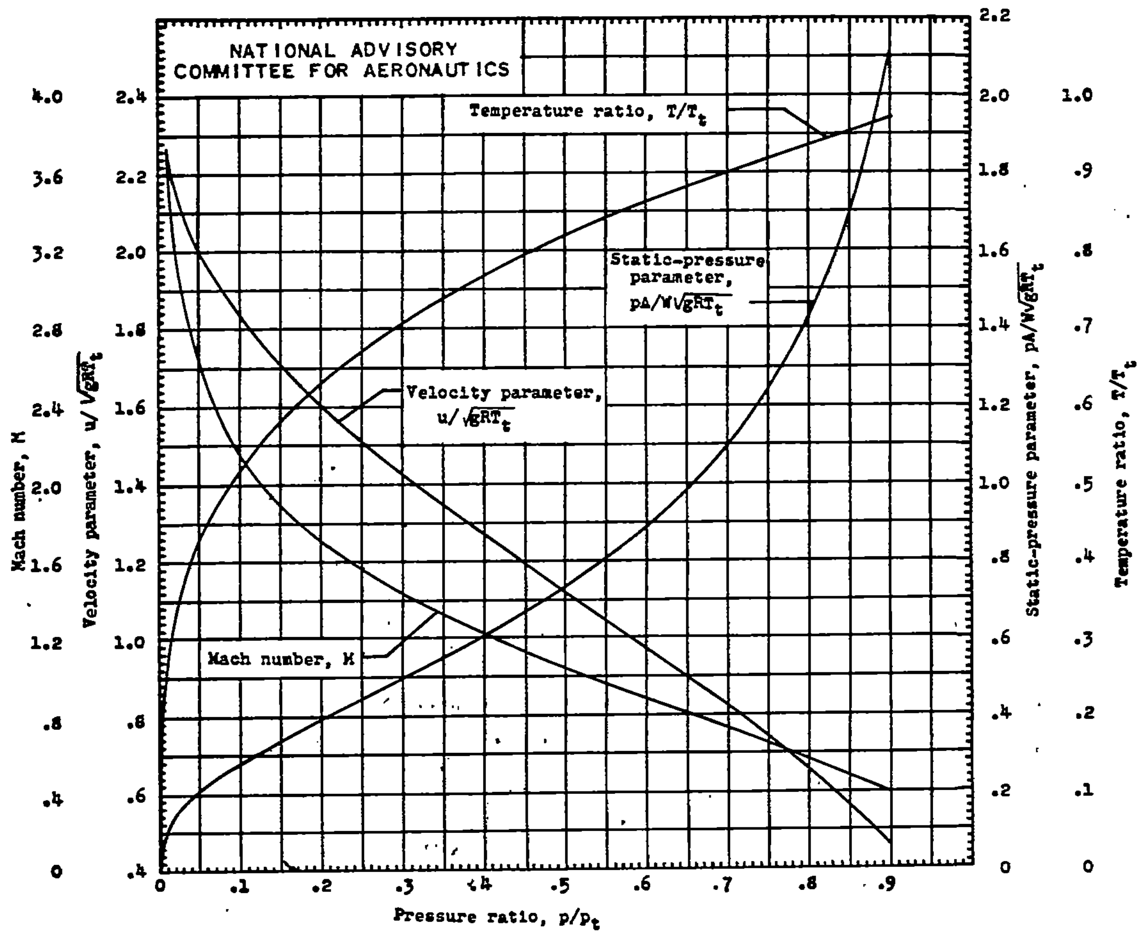
Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid. (A 17 in. by 22 in. print of this figure is enclosed.)



(c) Variable specific heats for range of static-pressure parameter from 2.2 to 4.4. On this figure the curves of total-momentum parameter, total-pressure parameter, and reciprocal of velocity parameter are applicable for all values of γ_t between 1.26 and 1.40.

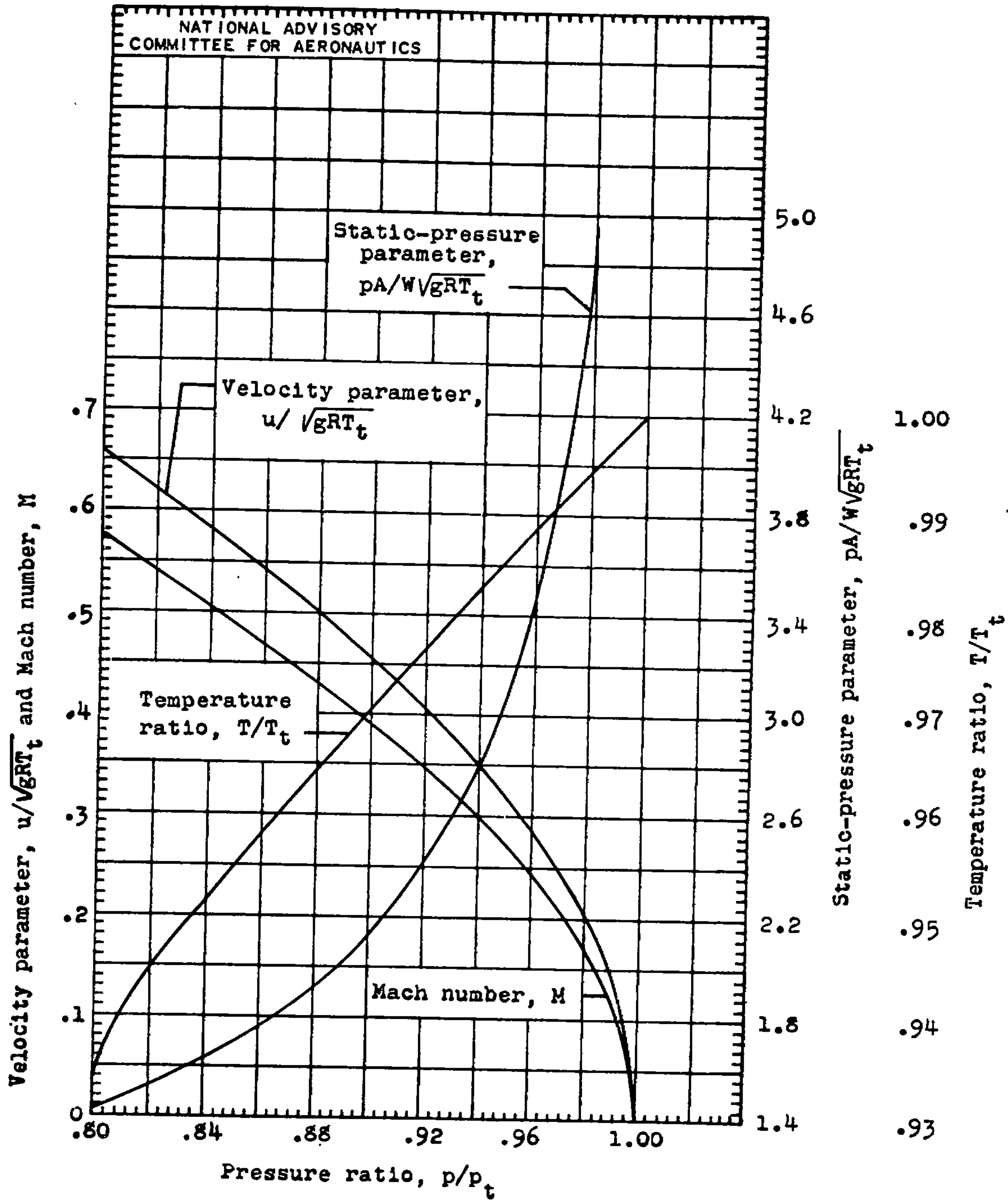
Figure 2. - Concluded. Relations between dimensionless parameters for compressible fluid. (A 17 in. by 22 in. print of this figure is enclosed.)

757



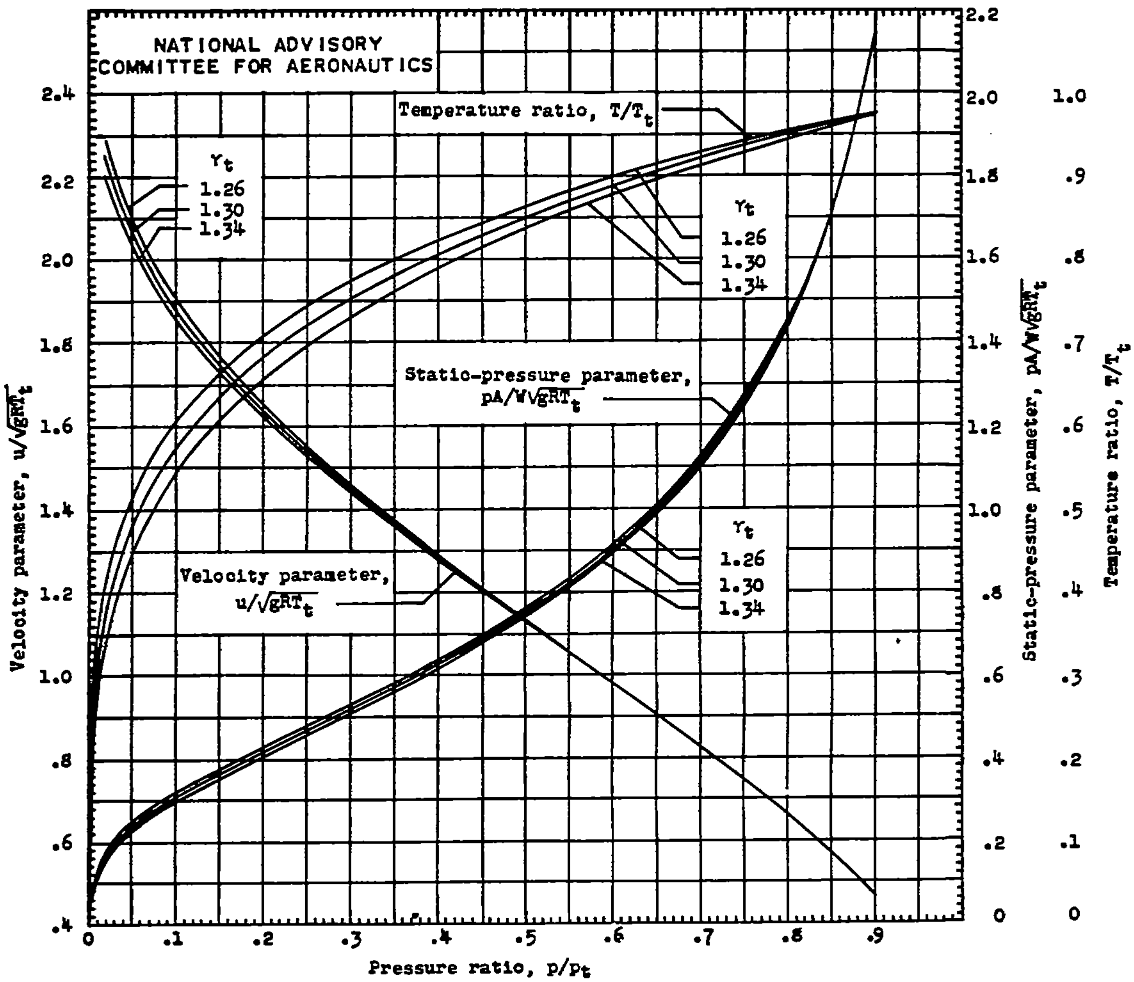
(a) Constant ratio of specific heats; γ, 1.40.

Figure 3. - Relations between flow parameters and pressure ratio. (A 12 in. by 21 in. print of this figure is enclosed.)



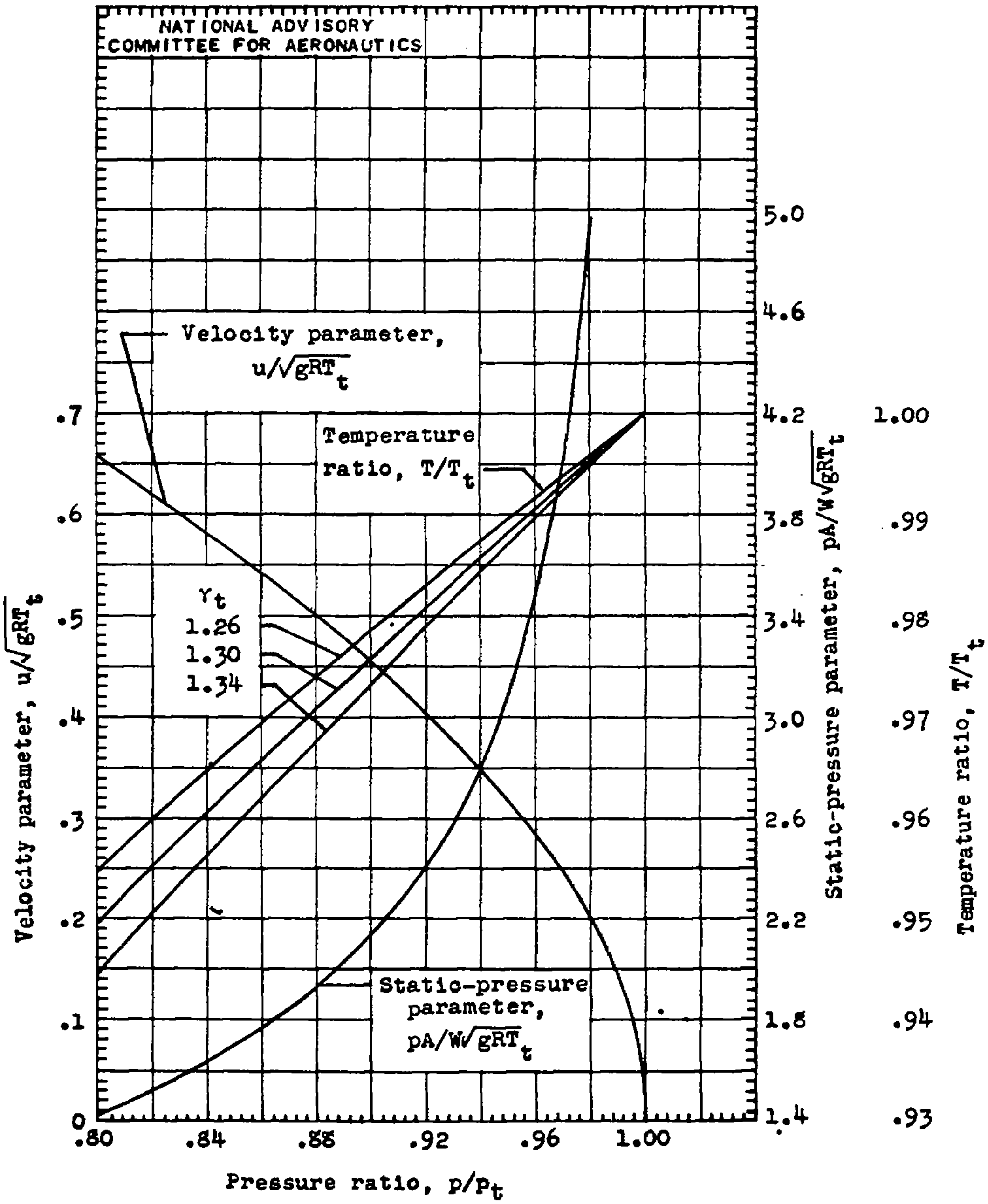
(a) Concluded. Constant ratio of specific heats; γ, 1.40.

Figure 3. - Continued. Relations between flow parameters and pressure ratio. (A 12 in. by 21 in. print of this figure is enclosed.)



(b) Variable specific heats.

Figure 3. - Continued. Relations between flow parameters and pressure ratio. (A 12 in. by 21 in. print of this figure is enclosed.)



(b) Concluded. Variable specific heats.

Figure 3. - Concluded. Relations between flow parameters and pressure ratio. (A 12 in. by 21 in. print of this figure is enclosed.)

757

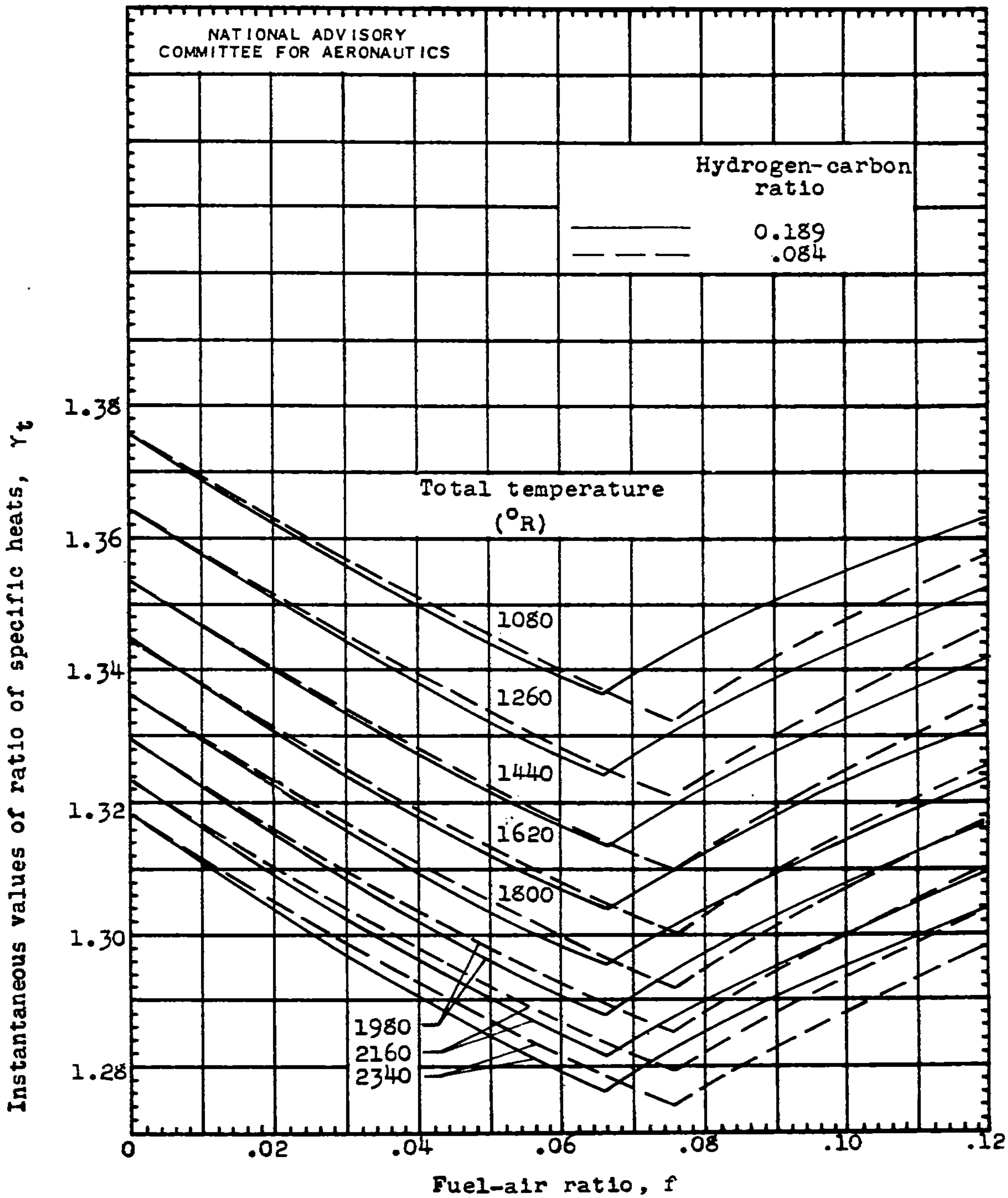


Figure 4. - Instantaneous values of ratio of specific heats γ_t for combustion gas. (Replotted from reference 1.)

757

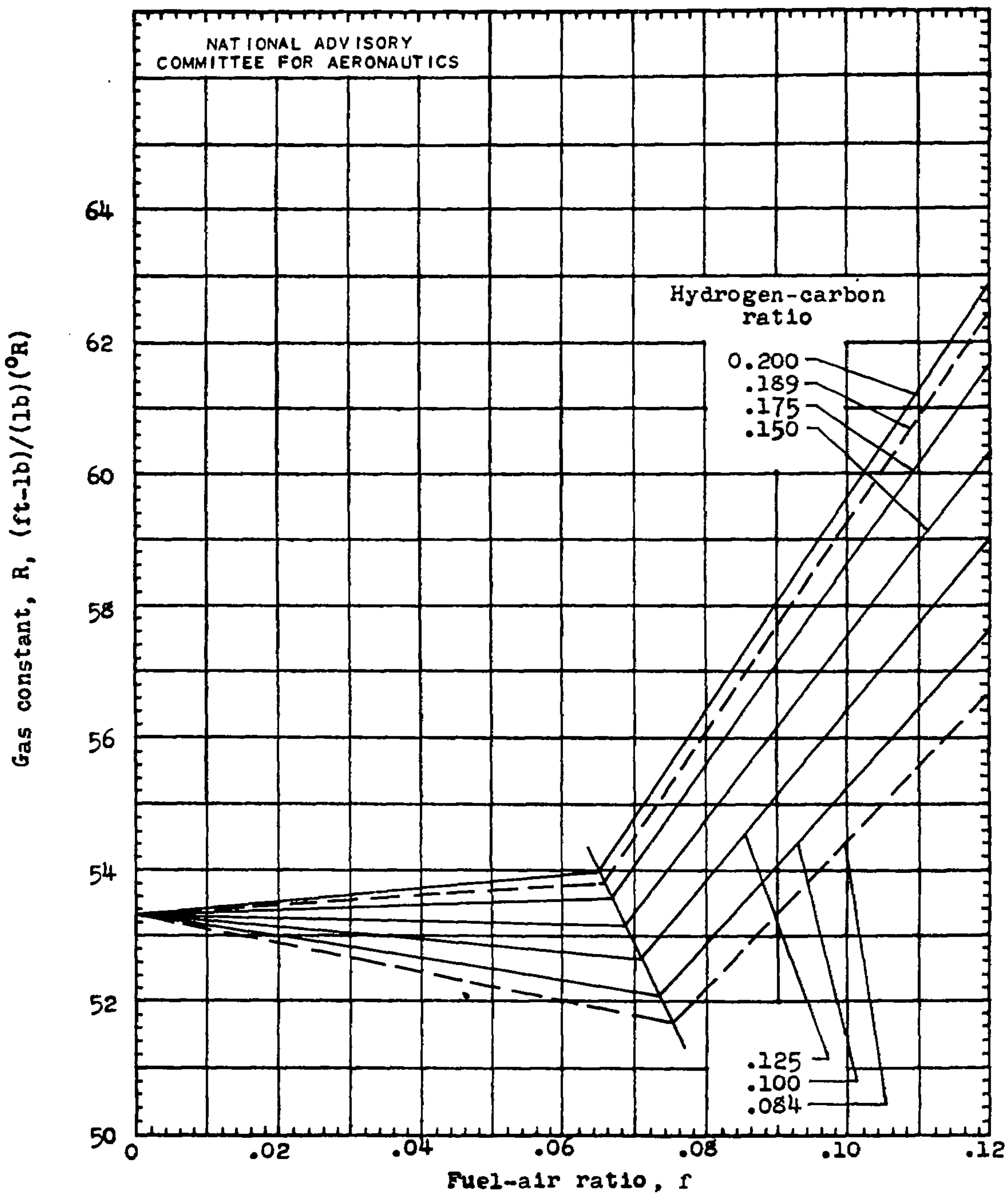


Figure 5. - Gas constant of combustion gas. (Replotted from reference 1.)

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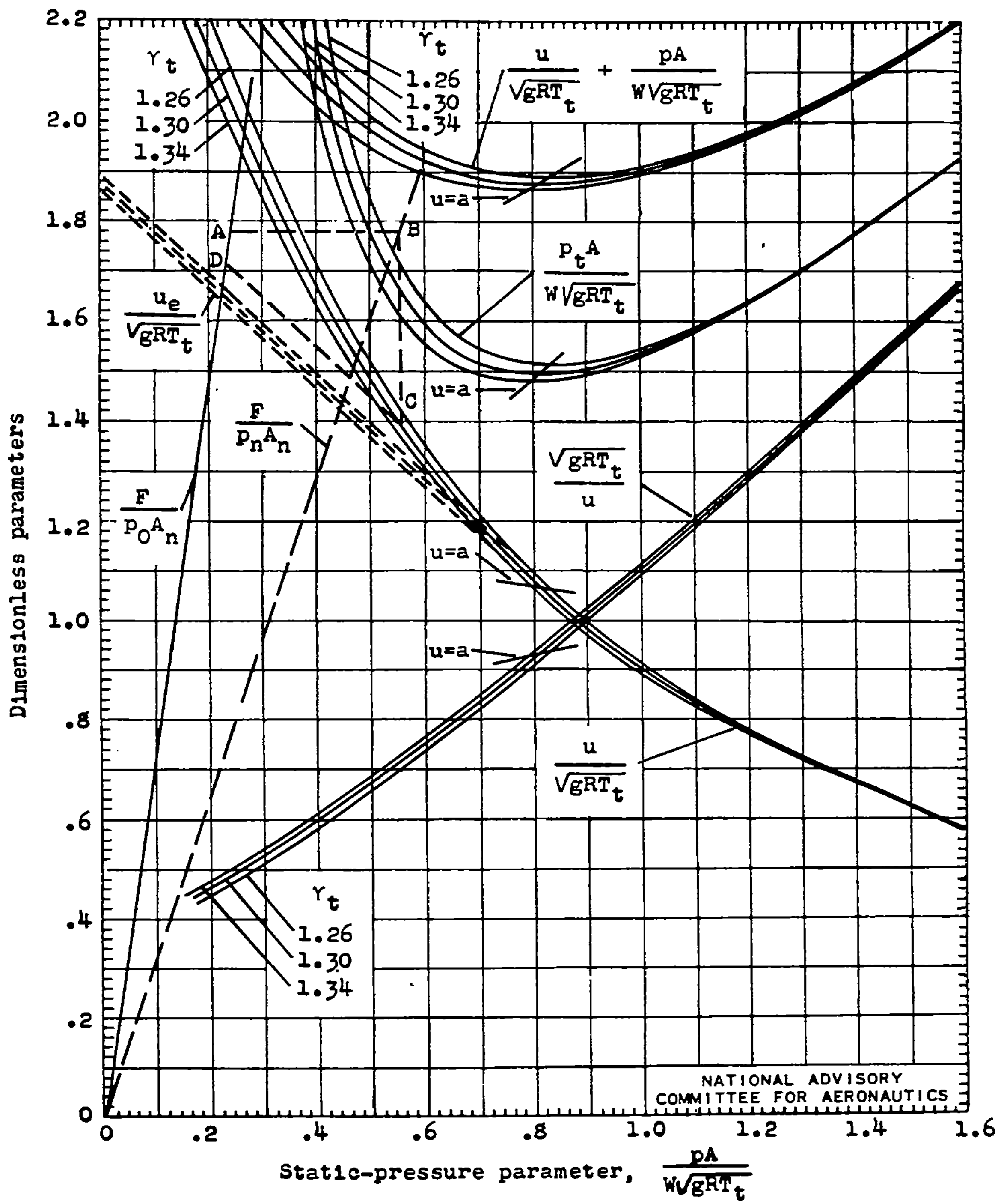


Figure 6. - Use of one-dimensional flow chart to determine total temperature of fluid from jet thrust.

757

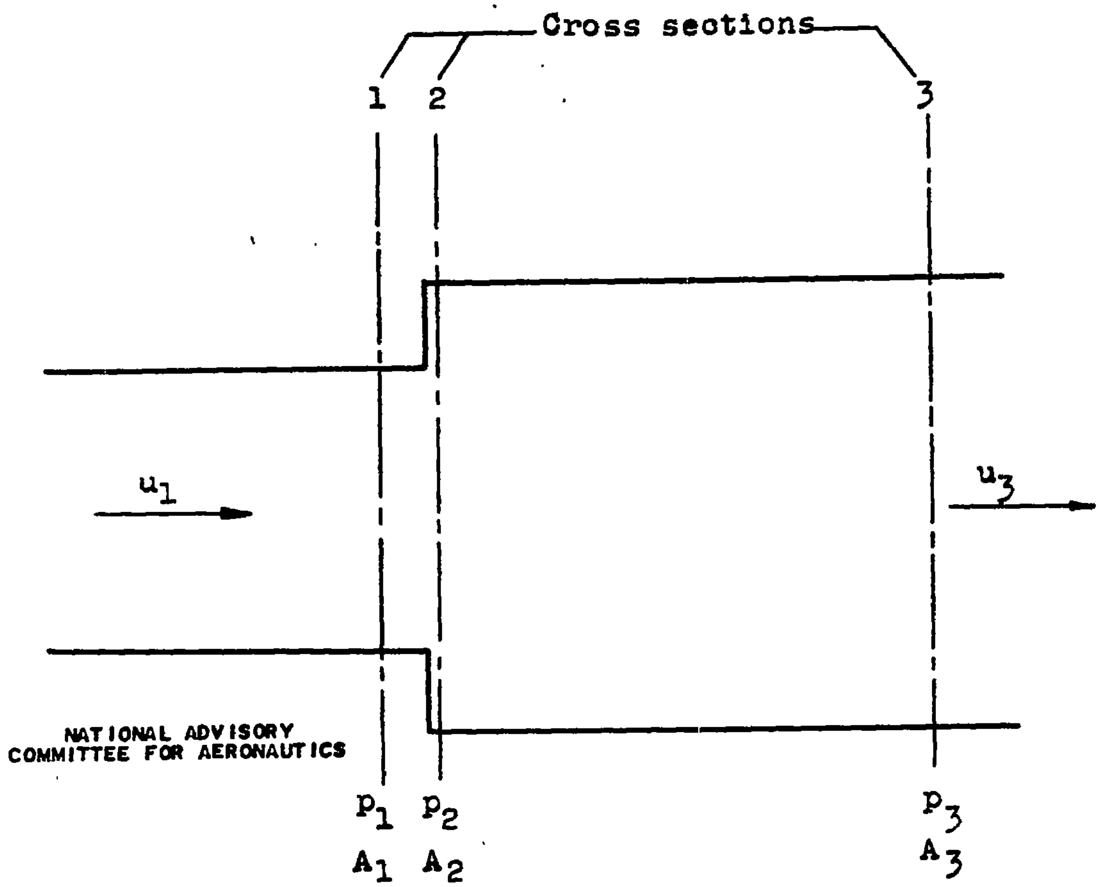


Figure 7. - Flow of compressible fluid through duct with abrupt change of area.

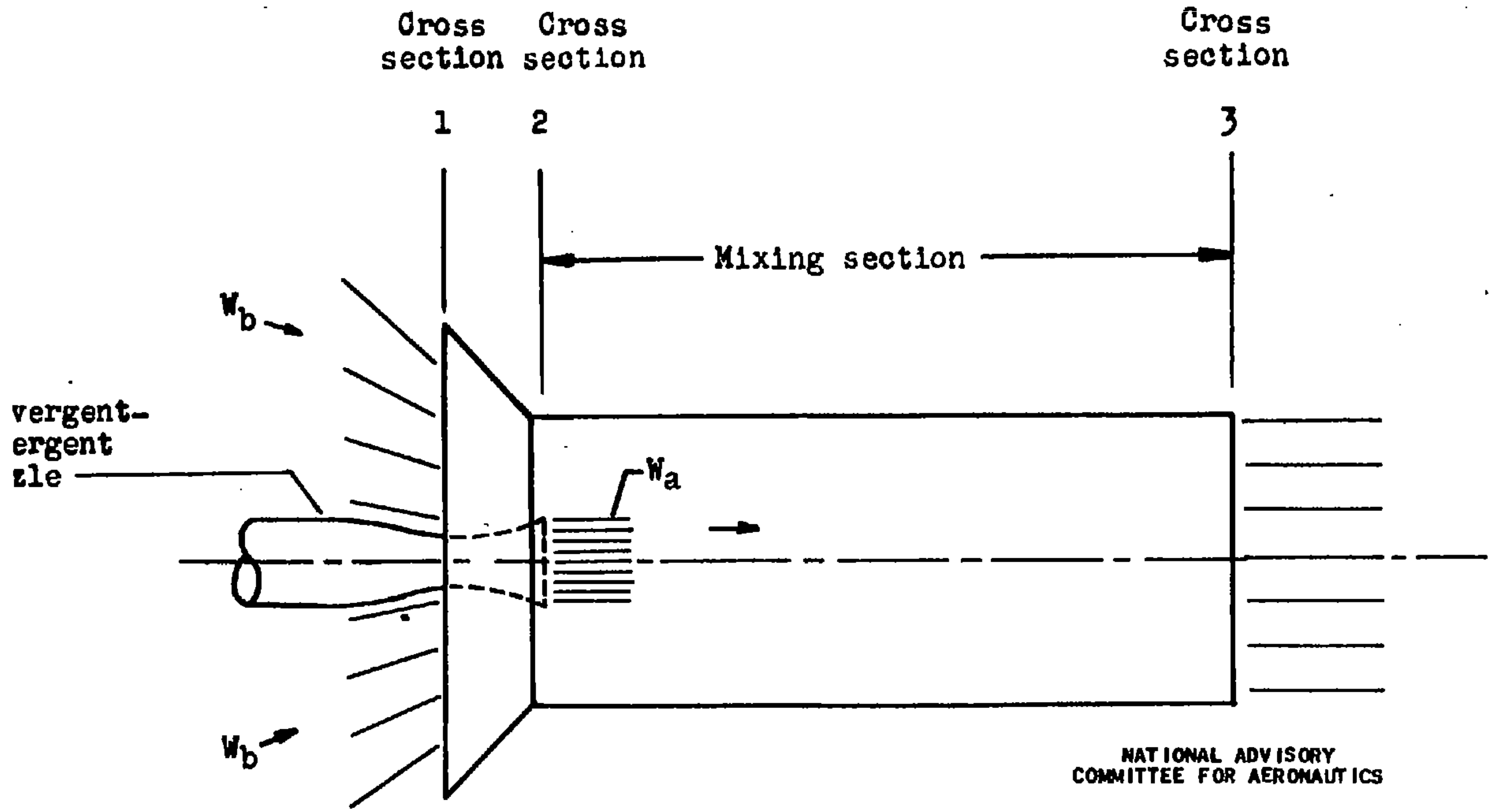


Figure 8. - Diagrammatic sketch of jet pump.

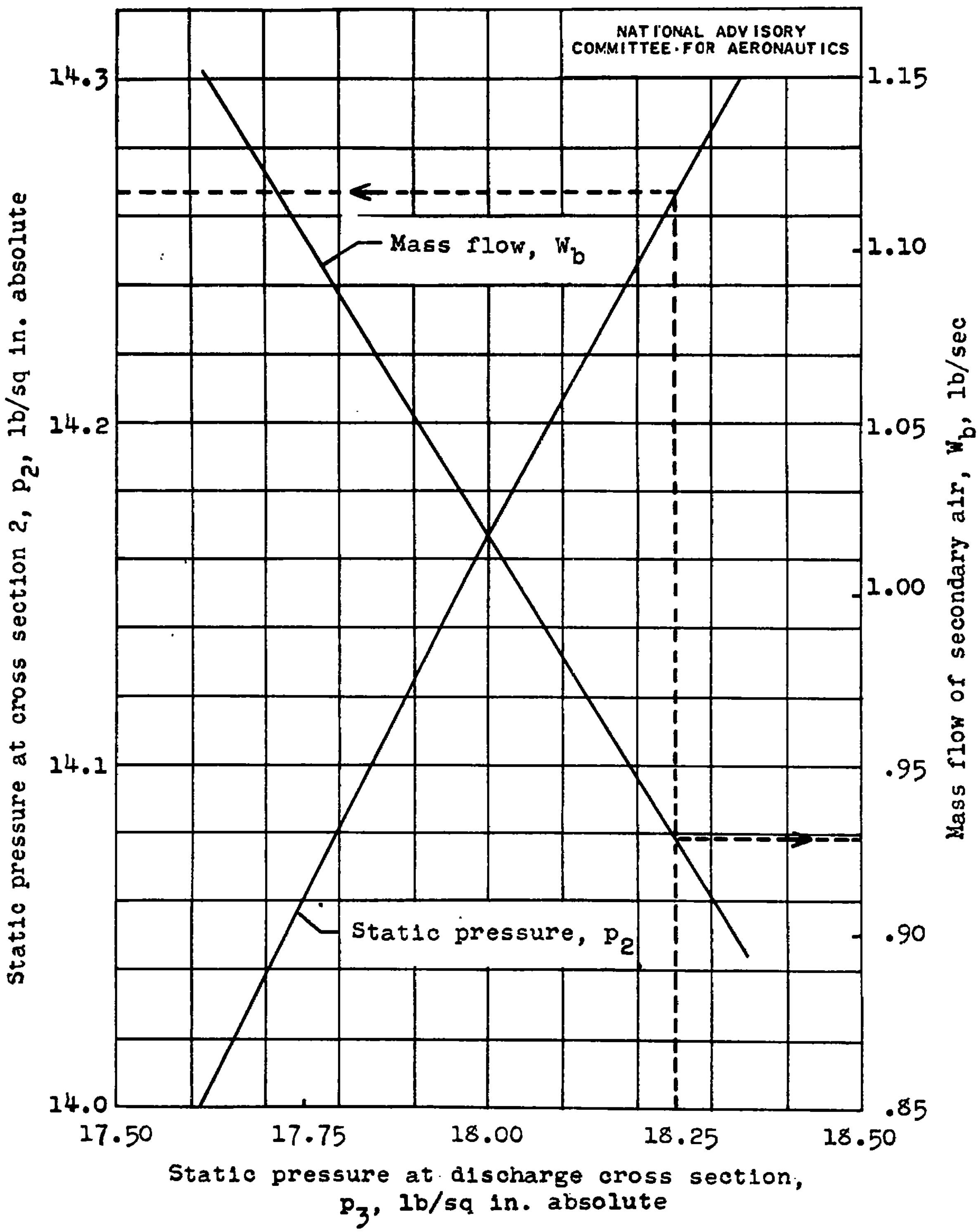
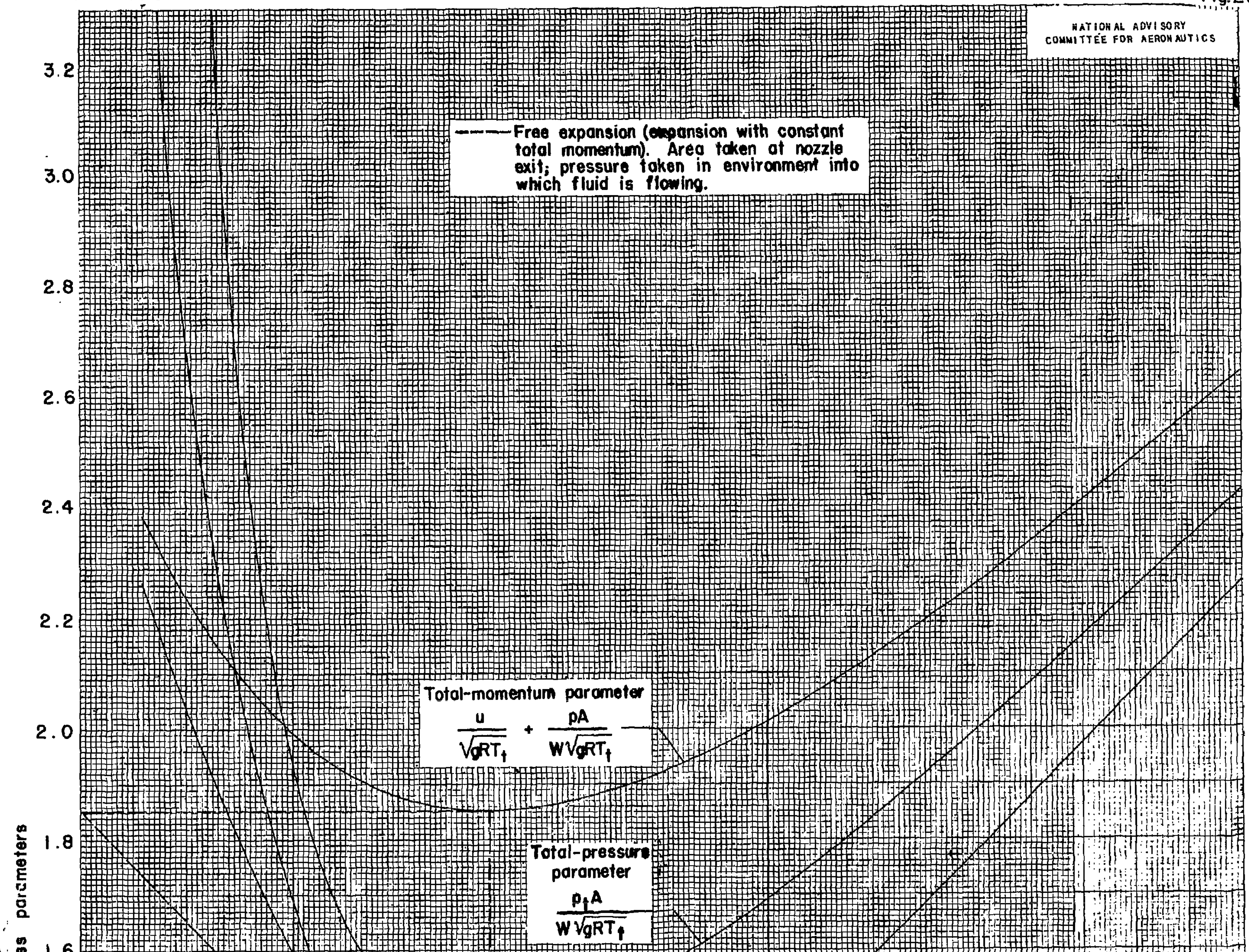


Figure 9. - Effect of static pressure at discharge cross section p_3 on static pressure p_2 at cross section 2 and mass flow of secondary air W_b for jet pump.

Fig. 2a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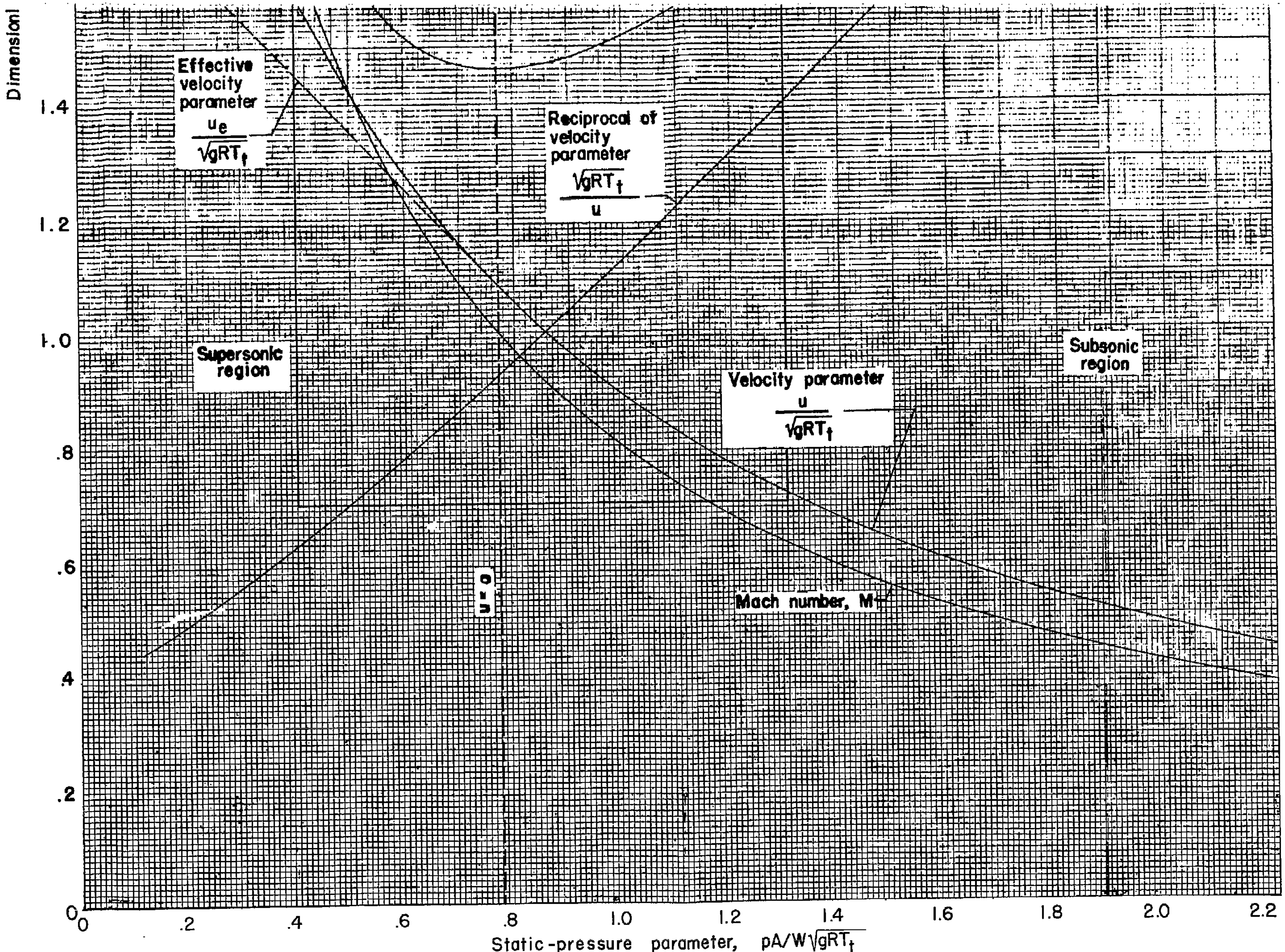
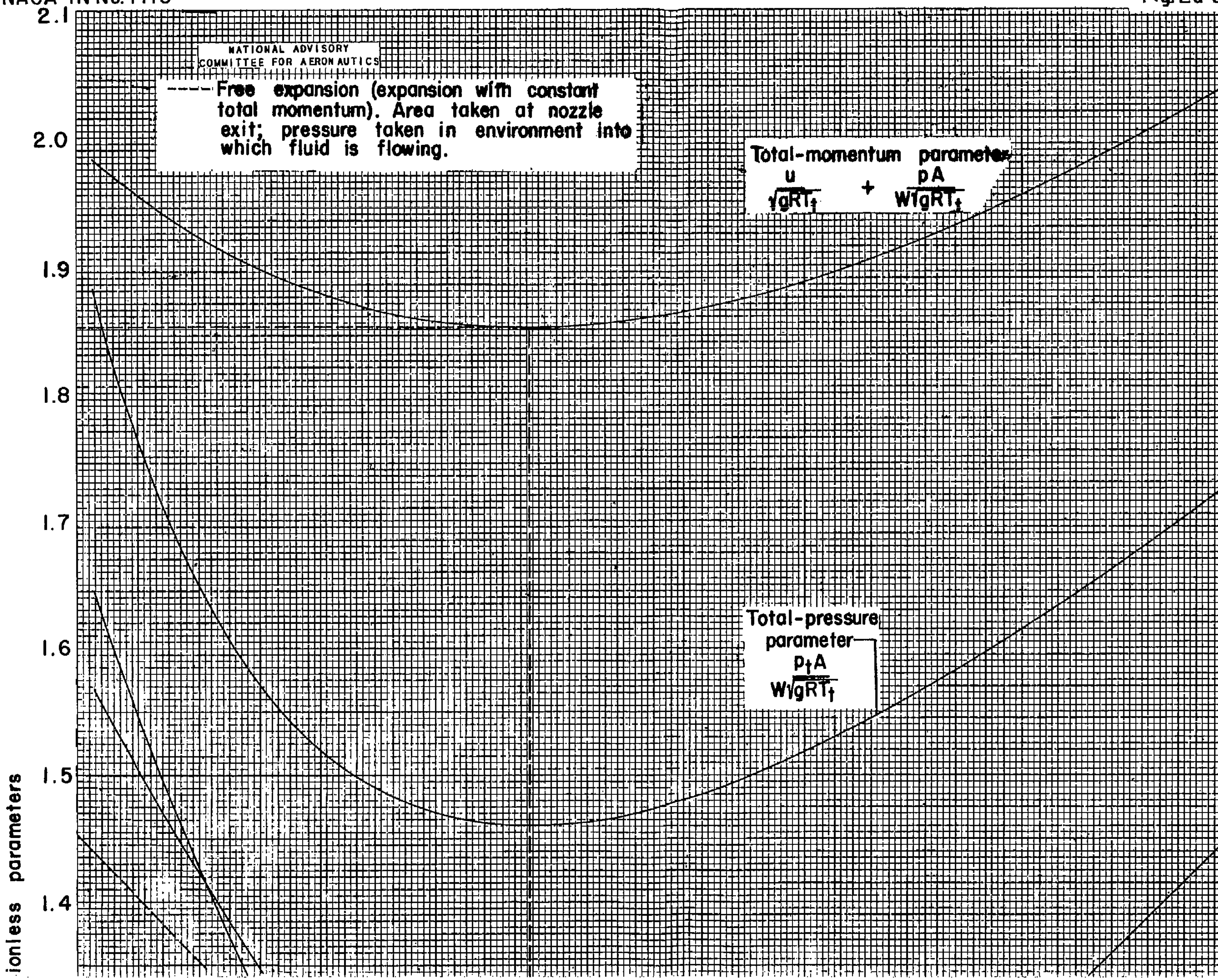
Free expansion (expansion with constant total momentum). Area taken at nozzle exit; pressure taken in environment into which fluid is flowing.
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
Total-momentum parameter
$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$
Total-pressure parameter
$\frac{p_t A}{W\sqrt{gRT_t}}$
ss parameters

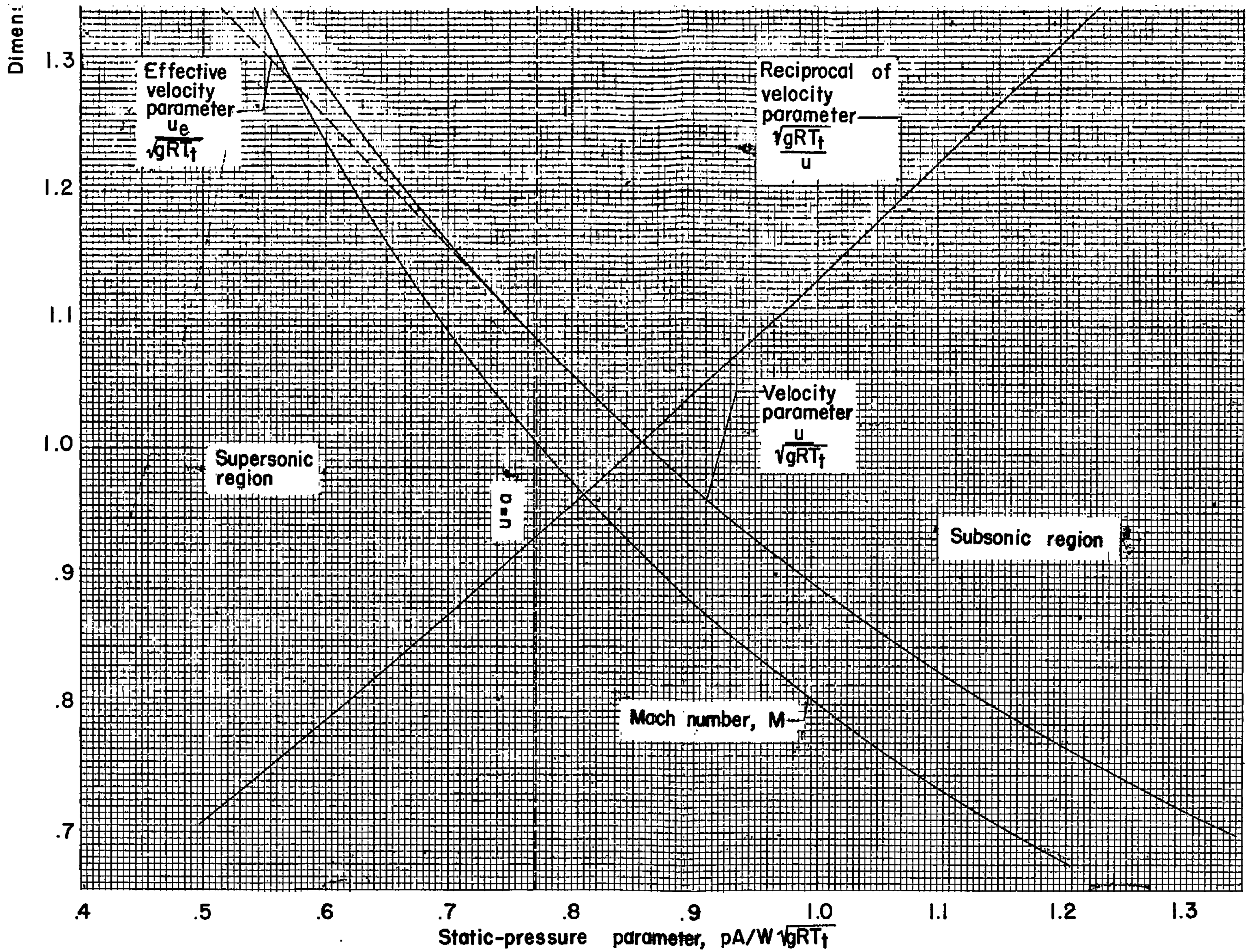


(a) Constant ratio of specific heats of 1.40 for range of static-pressure parameter from 0 to 2.2. Over-all plot; enclosed area enlarged in figure 2(a) Concluded.

Figure 2. - Relations between dimensionless parameters for compressible fluid.

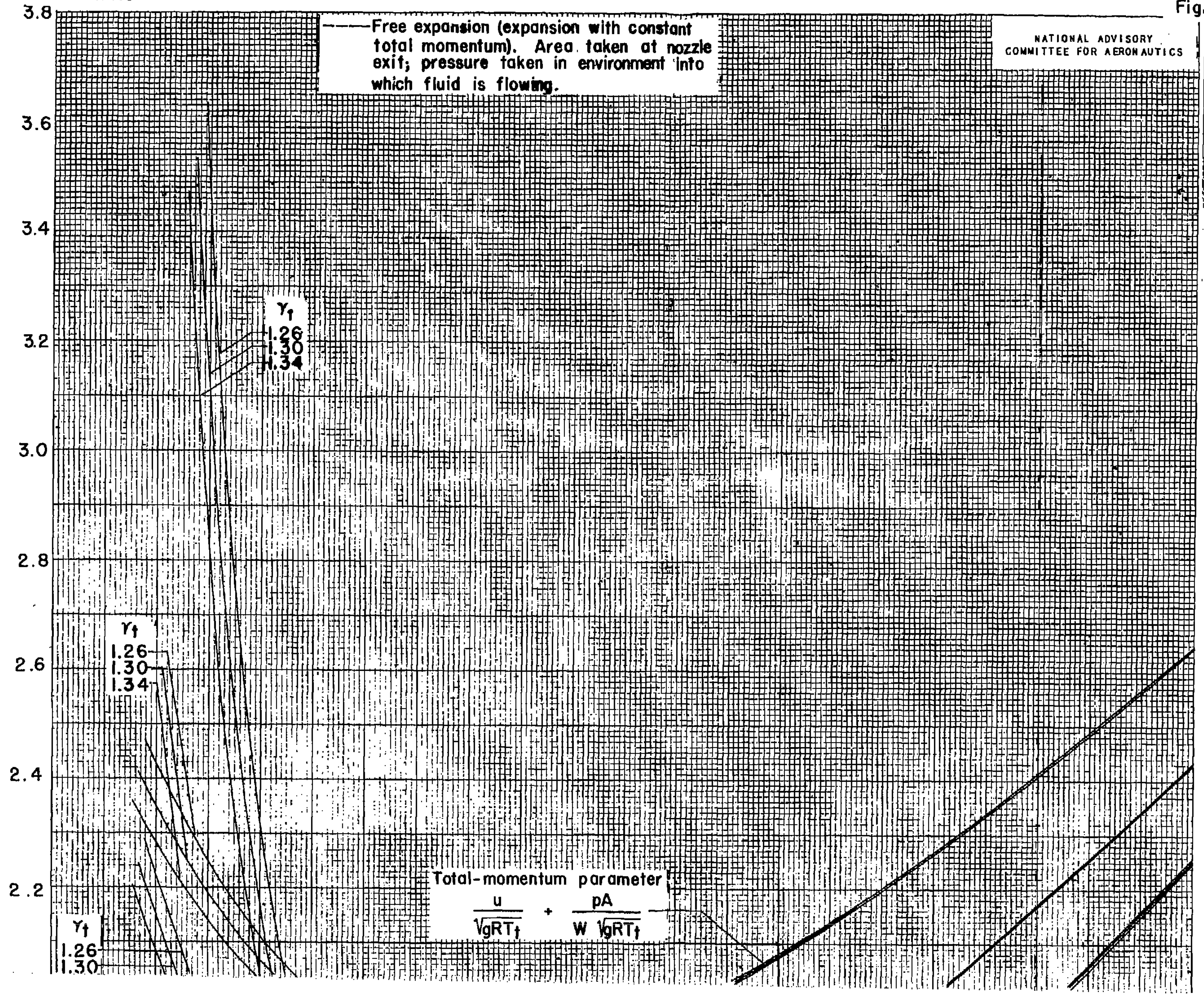
Fig. 2a Concl.

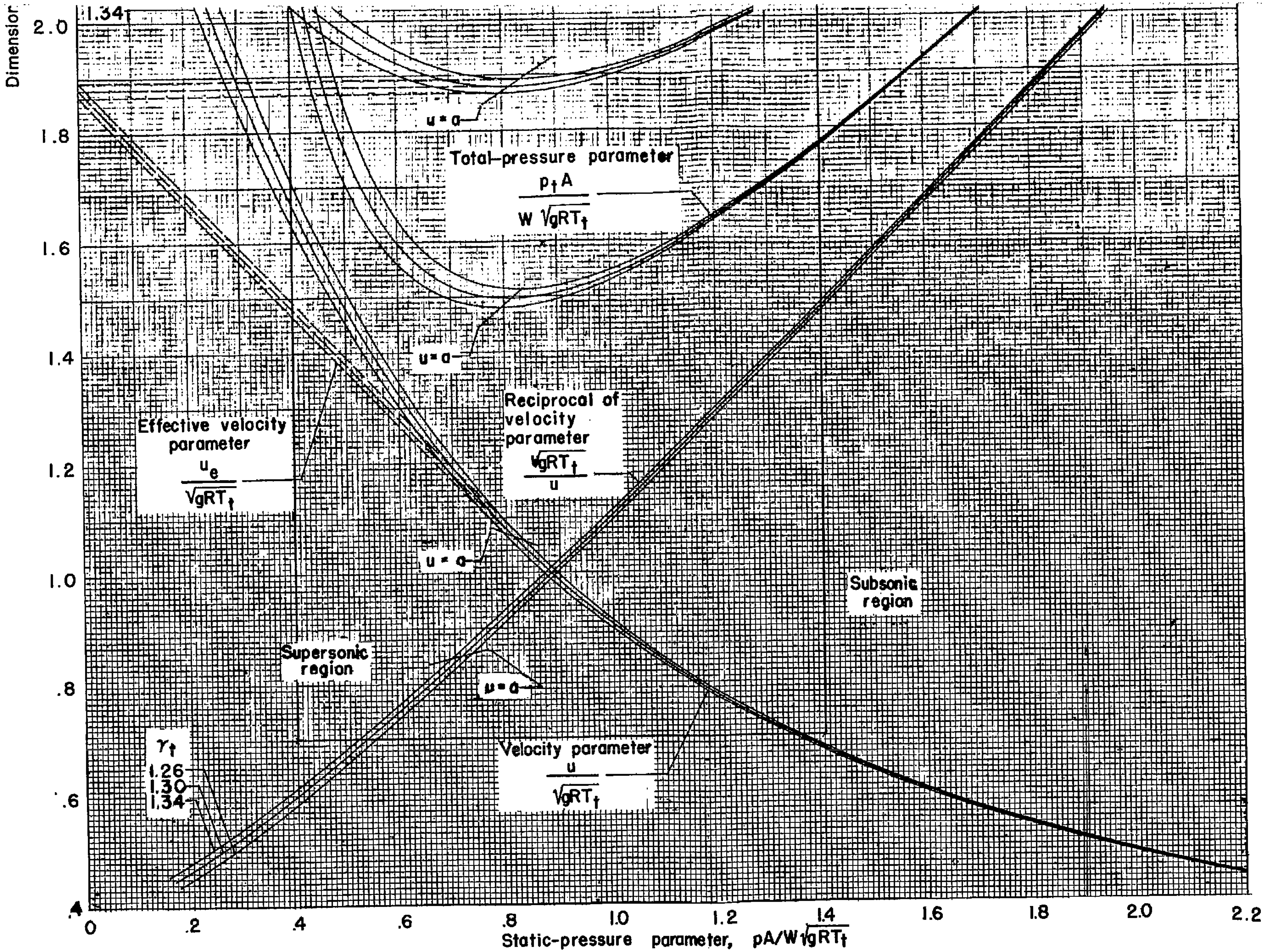

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
---- Free expansion (expansion with constant total momentum). Area taken at nozzle exit; pressure taken in environment into which fluid is flowing.
Total-momentum parameter $\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$
Total-pressure parameter $\frac{p_t A}{W\sqrt{gRT_t}}$
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
ionless parameters



(a) Concluded. Constant ratio of specific heats of 1.40 for range of static-pressure parameter from 0 to 2.2. Enlargement of area shown in figure 2(a).

Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid.

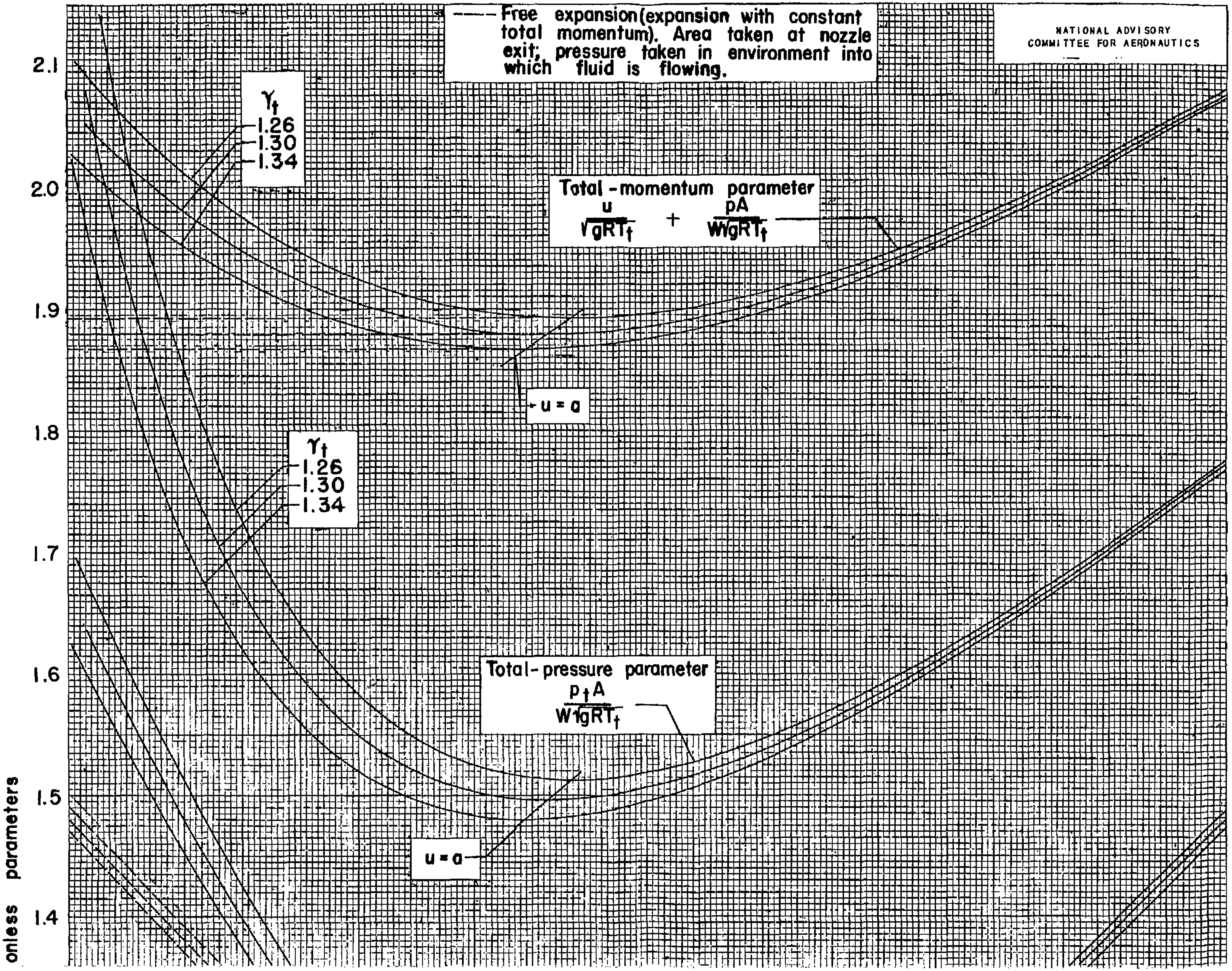

3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
ess parameters
Free expansion (expansion with constant total momentum). Area taken at nozzle exit; pressure taken in environment into which fluid is flowing.
γt
1.26
1.30
1.34
γt
1.26
1.30
1.34
γt
1.26
1.30
Total-momentum parameter
$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$



(b) Variable specific heats for range of static-pressure parameter from 0 to 2.2. Over-all plot; enclosed area enlarged in figure 2 (b) Concluded.

Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid.

Fig. 2b Concl.



—— Free expansion (expansion with constant total momentum). Area taken at nozzle exit; pressure taken in environment into which fluid is flowing.
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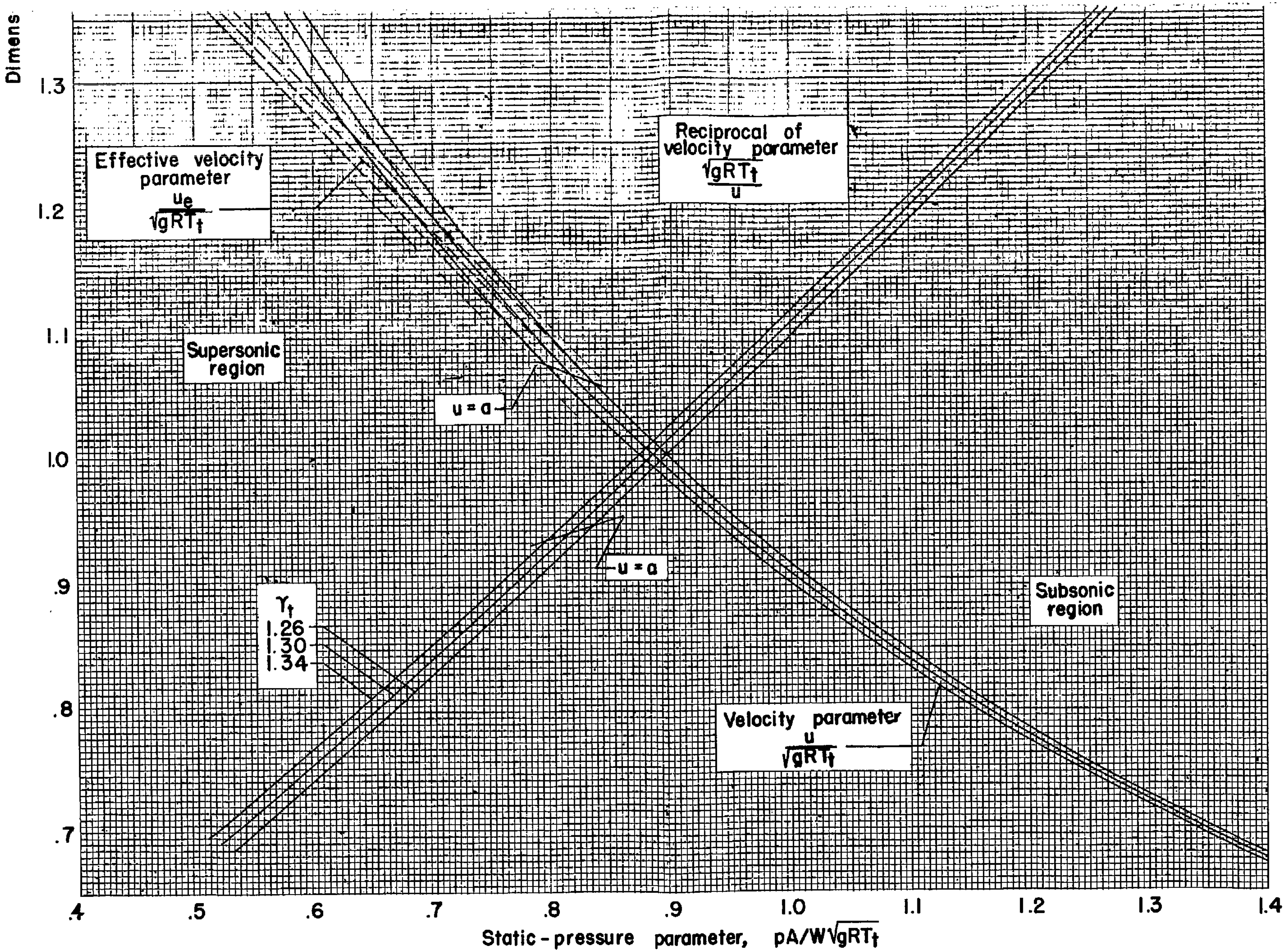
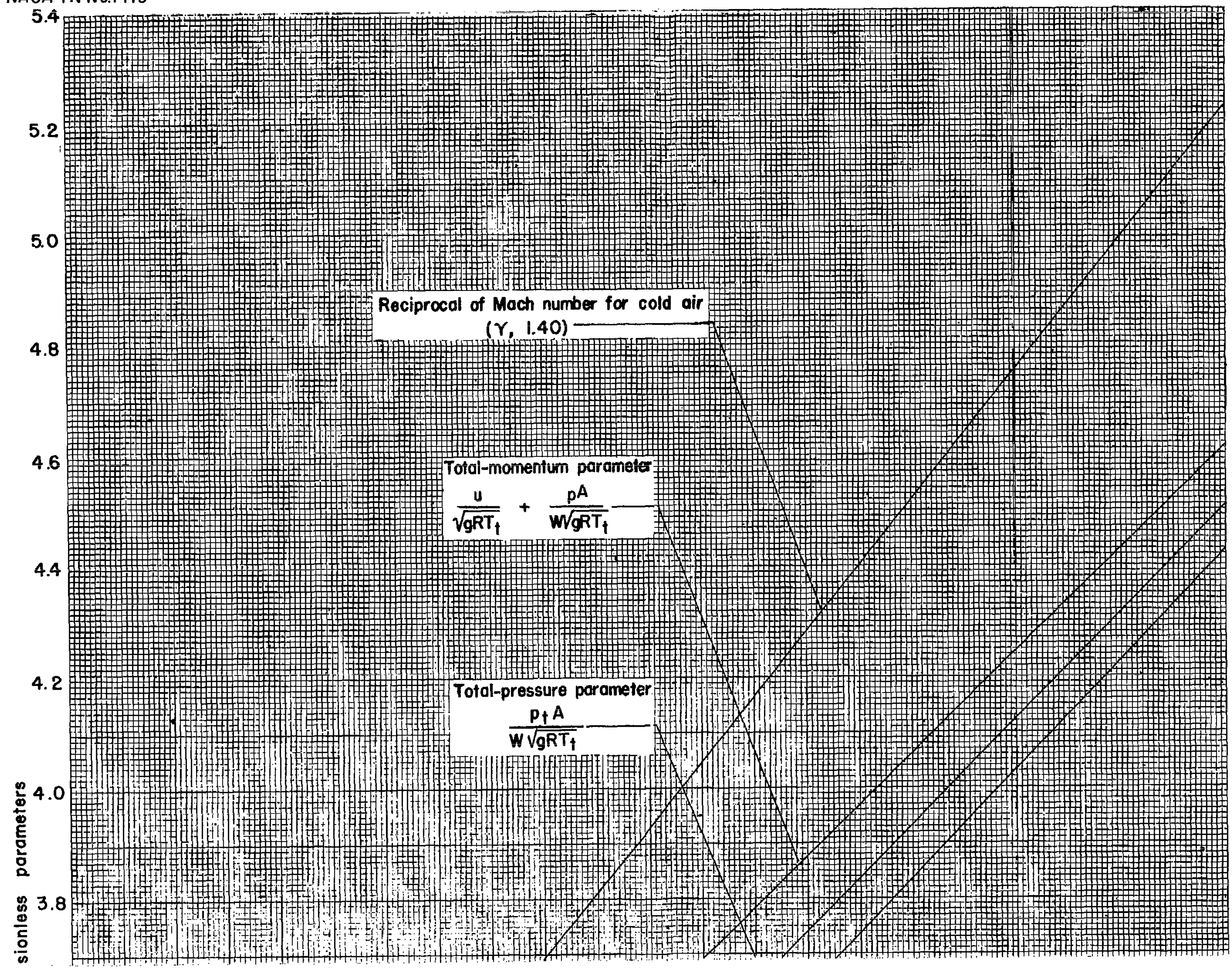
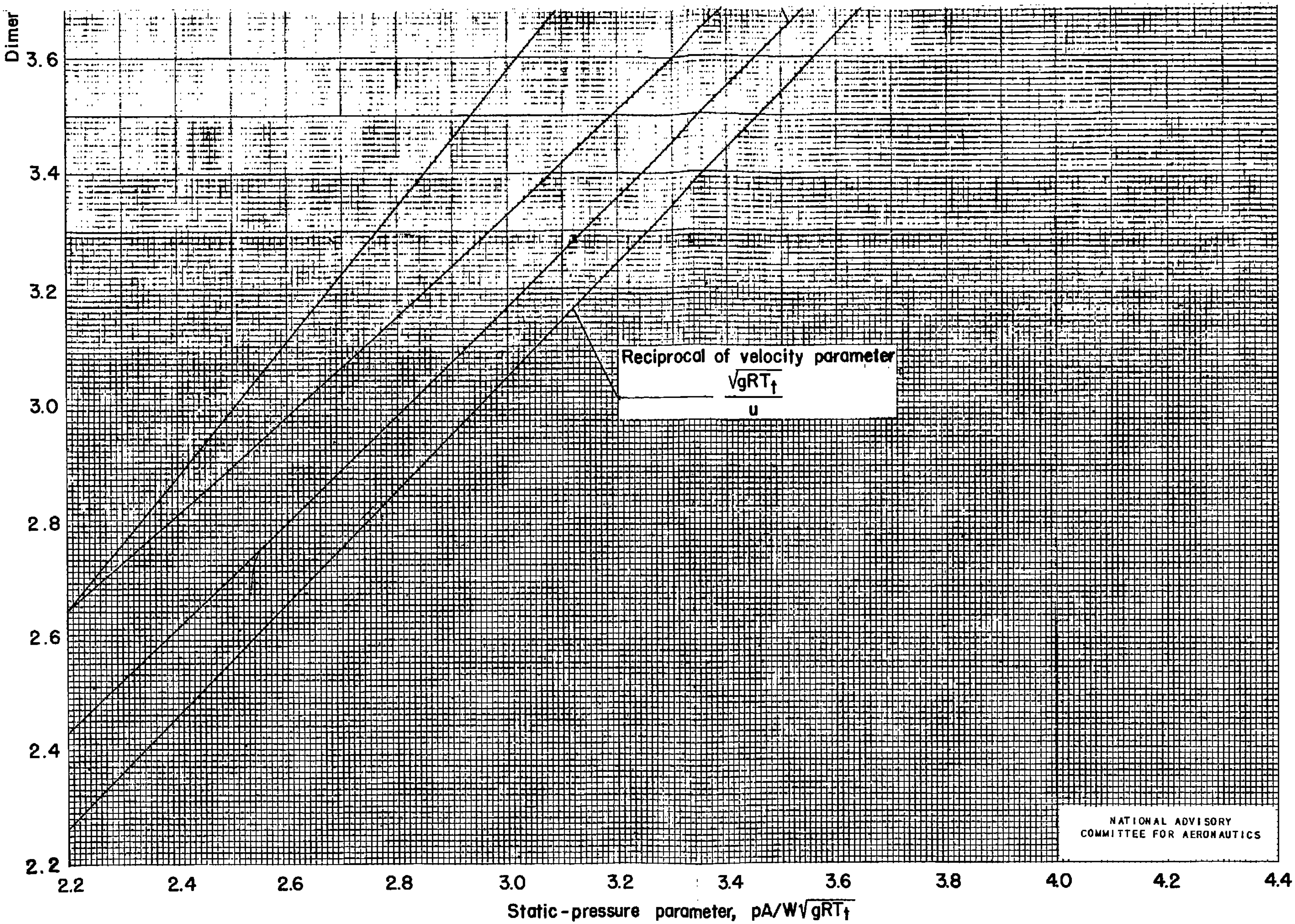
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
γ_t
1.26
1.30
1.34
γ_t
1.26
1.30
1.34
Total-momentum parameter
$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$
u = a
Total-pressure parameter
$\frac{p_t A}{W\sqrt{gRT_t}}$
u = a
onless parameters



(b) Concluded. Variable specific heats for range of static-pressure parameter from 0 to 2.2. Enlargement of area shown in figure 2(b).

Figure 2. - Continued. Relations between dimensionless parameters for compressible fluid.

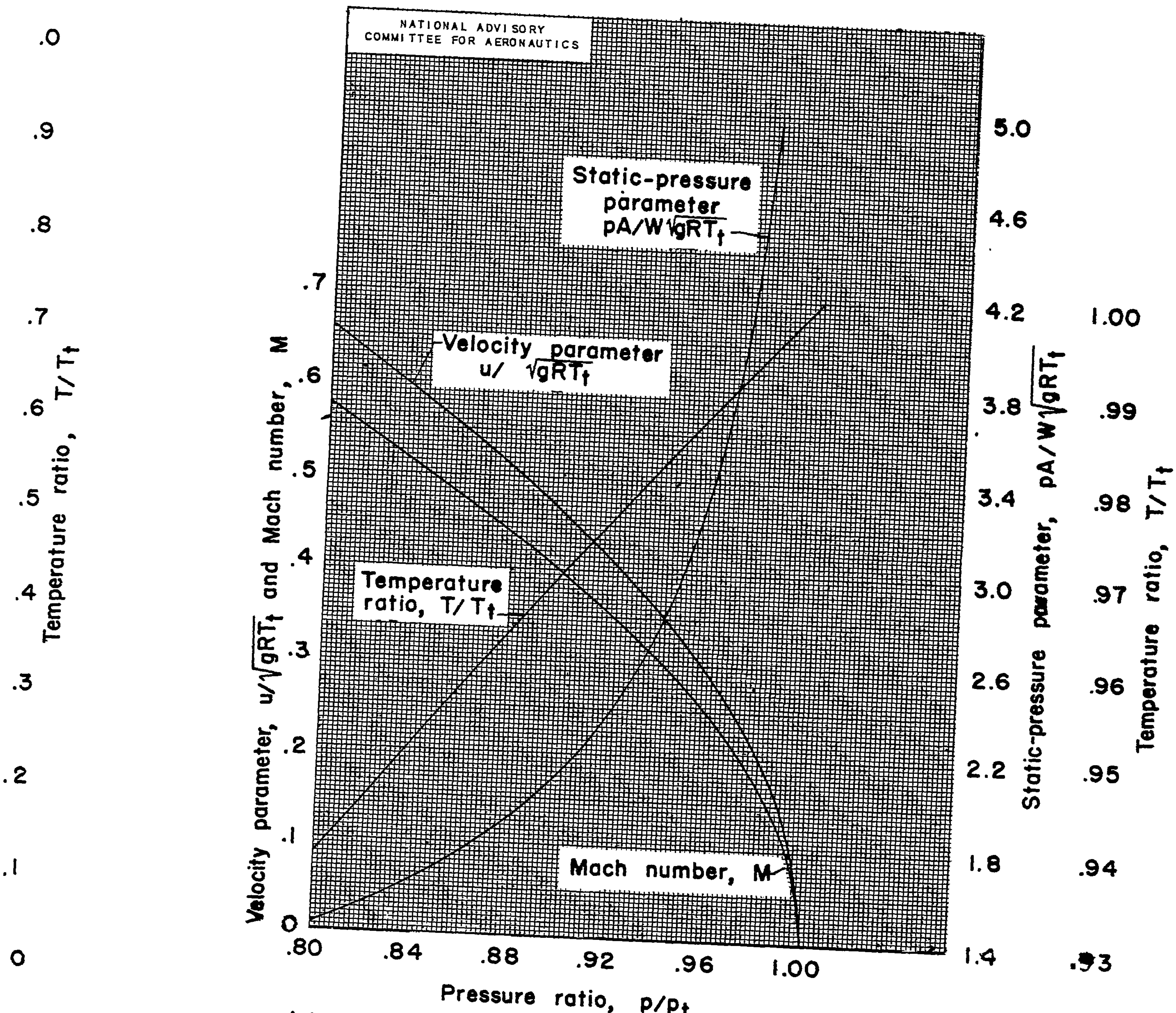

5.4
5.2
5.0
4.8
4.6
4.4
4.2
4.0
3.8
sionless parameters
Reciprocal of Mach number for cold air
(γ, 1.40)
Total-momentum parameter
$\frac{u}{\sqrt{gRT_t}} + \frac{pA}{W\sqrt{gRT_t}}$
Total-pressure parameter
$\frac{p_t A}{W\sqrt{gRT_t}}$



(c) Variable specific heats for range of static-pressure parameter from 2.2 to 4.4.

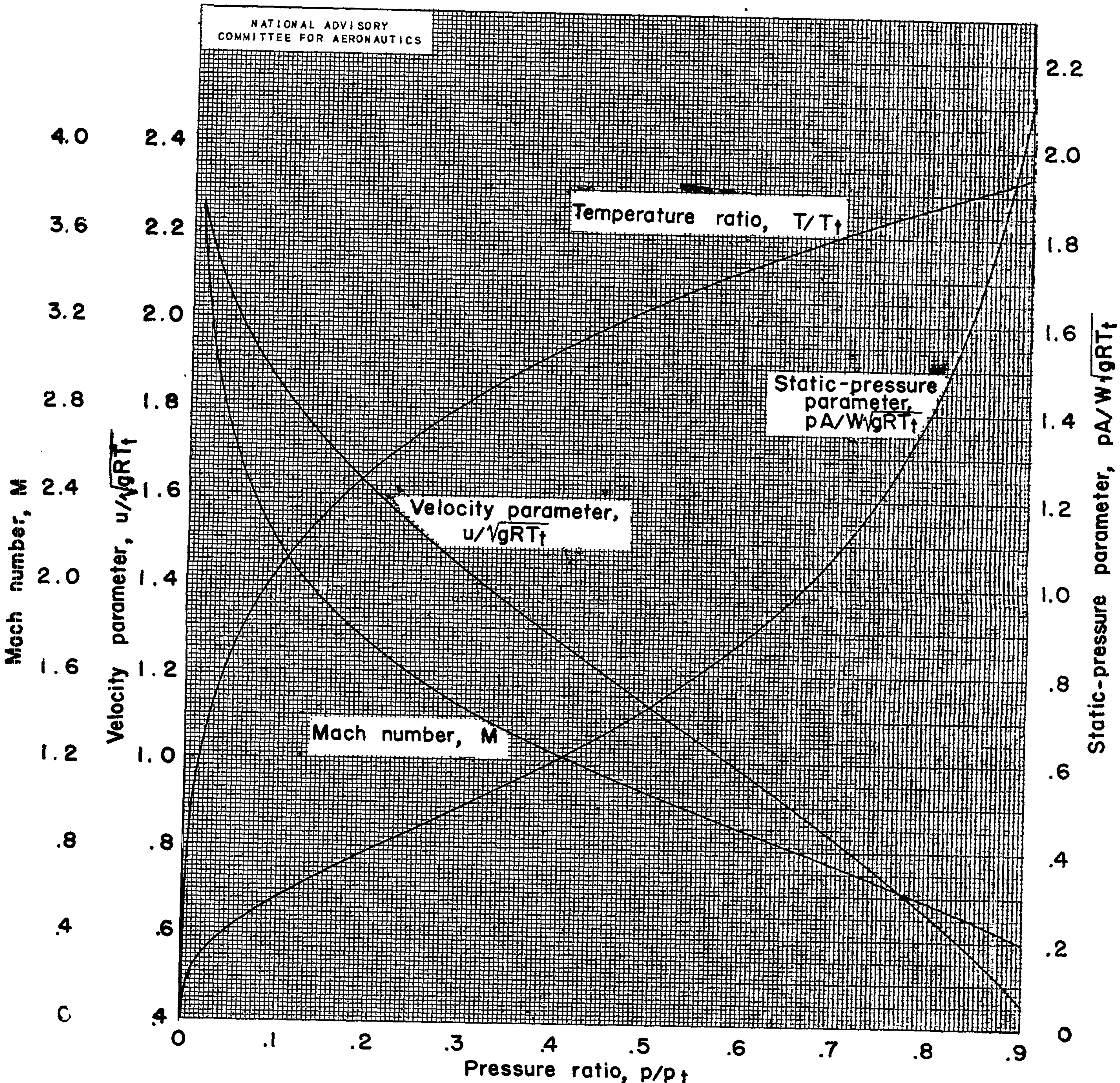
On this figure the curves of total-momentum parameter, total-pressure parameter, and reciprocal of velocity parameter are applicable for all values of γ_t between 1.26 and 1.40.

Figure 2. - Concluded. Relations between dimensionless parameters for compressible fluid.



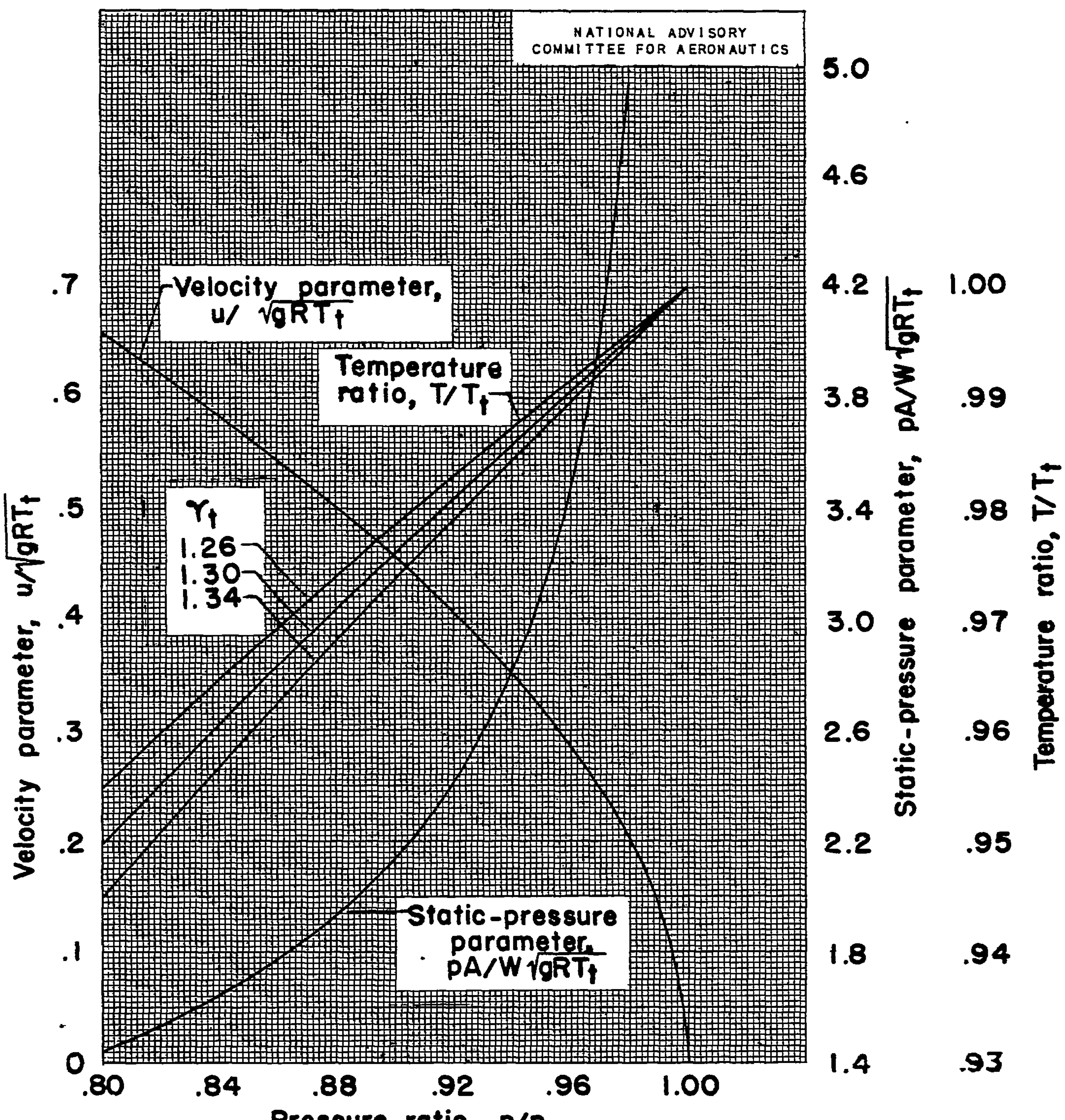
(a) Concluded. Constant ratio of specific heats; γ, 1.40.

Figure 3. - Continued. Relations between flow parameters and pressure ratio.



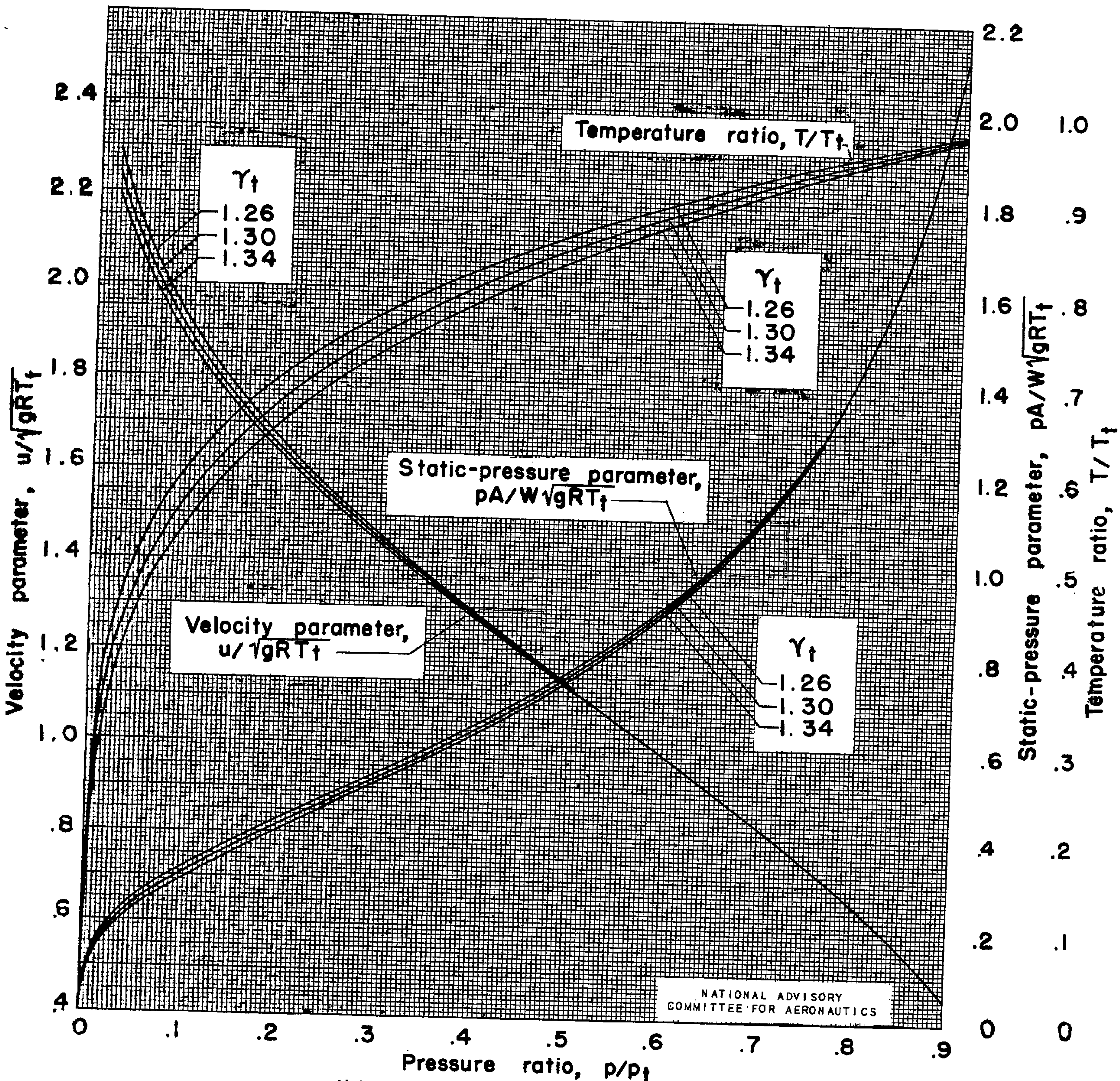
(a) Constant ratio of specific heats; γ, 1.40.

Figure 3.- Relations between flow parameters and pressure ratio.



(b) Concluded. Variable specific heats.

Figure 3. - Concluded. Relations between flow parameters and pressure ratio.



(b) Variable specific heats.

Figure 3.- Continued. Relations between flow parameters and pressure ratio.