Paper

Chattering Free Sliding Mode Control in Magnetic Levitation System

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It is well known that sliding mode control (SMC) is capable of tackling systems with uncertainties. However, the discontinuous control signal causes a significant problem of chattering. In this paper, a new and simple approach to chattering free SMC methodology is proposed. The main purpose is to eliminate the chattering phenomenon. As a result, the chattering is eliminated and error performance of sliding mode control is improved. The reduction of the chattering of sliding mode control is achieved by using a distance function which measure the distance between the trajectory of state errors and the sliding surface as the corrective control term instead of discontinuous sign function. Experimental study carried out on a magnetic levitation system is presented. Experiments verified that the proposed control has the advantage of less chattering in SMC.

Keywords: sliding mode control, distance measurement, magnetic levitation, linear system

1. Introduction

Variable structure control with sliding mode, which is commonly known as sliding mode control (SMC), is a nonlinear control strategy that is well known for its robustness characteristics [1] and has been developed and applied to closed-loop control systems for the last three decades [2][3]. The main feature of SMC is that it uses a high-speed switching control law to drive the system states from any initial state onto a user-specified surface in the state space (the so-called sliding surface), and to maintain the states on the surface for all subsequent time. This method is well known for its robustness to disturbance and parameter variations [4]–[6]. Conventionally, the SMC is based on the state-space approach. That is, one first constructs a Lyapunov function and then tries to find a control law to make the derivative of the Lyapunov function negative definite.

In the design of the SMC law, it is assumed that the control can be switched from one value to another infinitely fast. However, this is impossible to achieve in practical systems because finite time delays are present for control computation, and limitations exist in the physical actuators. This nonideal switching results in a major problem, i.e., the chattering phenomenon [7][8]. This phenomenon is not only highly undesirable by itself but it may also excite the high-frequency unmodeled dynamics which could result in unforeseen instability, and can also cause damage to actuators or the plant. Hence, it has received considerable attention from the research community [8]. To reduce the chattering, some researchers have proposed to use the saturation function or sigmoid function for replacing the sign nonlinearity [8][9].

In this paper, a new and simple control strategy based SMC is proposed to deal with the problem of eliminating the chattering effect. The proposed control strategy is based on the concept of point to hyperplane distance, define a distance function which only need to measure the distance between the trajectory of state errors and the sliding surface to generate the corrective control instead of using other function.

On the other hand, magnetic levitation systems have practical importance in many engineering systems such as frictionless bearings, levitation of high speed trains, and vibration isolation tables in semiconductor manufacturing [10]. Therefore, the performance of the proposed control strategy is then demonstrated through experimental studies on a magnetic levitation system. The experimental results show that this control approach effectively suppresses the vibration action of the magnet.

This paper is organized as follows: the problem formulation is presented in Section 2. Subsequently, the explanation of new proposed strategy will be given in Section 3. Section 4 explains the experimental apparatus of magnetic levitation system. Then in Section 5, experimental studies are carried out to demonstrate the validity of the proposed control schemes. Finally, conclusions of the design scheme is given in Section 6.

2. Problem Formulation

The sliding mode control based on the state-space formulation is presented in this section. First let us consider a linear system that defined as

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t) \cdots\cdots (1)$$

Here, $\boldsymbol{x}(t) = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is the state vector and $u(t)$ is the control input. $A \in R^{n\times n}$, $\boldsymbol{b} \in R^{n\times 1}$ are appropriate matrix and vector. We further assume that

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the above system (1) is controllable and observable.

The major steps in the design of a sliding mode controller are (i) to construct a sliding surface that represents a desired system dynamics, and (ii) to develop a switching control law such that a sliding mode exists on every point of the sliding surface, and any states outside the surface is driven to reach the surface in finite time.

The control objective is to determine a control law $u(t)$ such that the state vector $\boldsymbol{x}(t)$ asymptotically tracks a given bounded desired state vector $\boldsymbol{x}_d(t) = [x_d, \dot{x}_d, \cdots, x_d^{(n-1)}]^T$.

To begin with, let the tracking error be defined as $e(t) = x_d(t) - x(t)$, and the tracking error vector be defined as

$$\boldsymbol{e}(t) = \boldsymbol{x}_d(t) - \boldsymbol{x}(t) = \left[e, \dot{e}, \cdots, e^{(n-1)}\right]^T \cdots\cdots (2)$$

Then a sliding surface in the space of the error state can be defined as

$$\begin{aligned} S(t) &= c_1 e + c_2 \dot{e} + \cdots + c_{n-1} e^{(n-2)} + c_n e^{(n-1)} \\ &= \boldsymbol{c}^T \boldsymbol{e}(t) \cdots\cdots (3) \end{aligned}$$

where $\boldsymbol{c} = [c_1, c_2, \cdots, c_{n-1}, c_n]^T$ is chosen such that $c_n = 1$ and the coefficients $c_1, \cdots, c_{n-1}$ are describing the dynamics of the sliding surface $S(t) = 0$. Any states that reach this surface will then remain on it for all subsequent time, and a sliding mode or sliding motion is said to occur.

When a system is in the sliding mode, its dynamics is solely governed by the dynamics of the sliding surface. Thus, the coefficients $c_1, \cdots, c_{n-1}$ have to be chosen such that the system in a sliding motion produces the desired behavior. This can be done by ensuring the roots of the characteristic polynomial (Hurwitz polynomial) describing the sliding surface

$$p(\lambda) = \lambda^{n-1} + c_{n-1}\lambda^{n-2} + \cdots + c_1 \cdots\cdots (4)$$

where λ denotes the complex variable, have negative real parts with desirable pole placement.

On the other hand, the process of SMC can be divided into two phases, i.e., the approaching phase with $S(t) \neq 0$ and the sliding phase with $S(t) = 0$. A sufficient condition to guarantee that the trajectory of the error vector $\boldsymbol{e}(t)$ will translate from the approaching phase to the sliding phase is to select the control strategy such that

$$S(t)\dot{S}(t) \leq -\eta\,|S(t)| \cdots\cdots (5)$$

where η is a small positive constant, and (5) is called *reaching condition* [7]. Corresponding to two phases, two types of control law can be derived separately. In the sliding phase, we have $S(t) = 0$ and $\dot{S}(t) = 0$, then the equivalent control $u_{eq}(t)$ which will force the system dynamics to stay on the sliding surface is chosen such that

$$\begin{aligned} \dot{S}(t) &= \boldsymbol{c}^T \dot{\boldsymbol{e}}(t) \\ &= \boldsymbol{c}^T \left[\dot{\boldsymbol{x}}_d(t) - \dot{\boldsymbol{x}}(t)\right] \\ &= \boldsymbol{c}^T \dot{\boldsymbol{x}}_d(t) - \boldsymbol{c}^T A\boldsymbol{x}(t) - \boldsymbol{c}^T \boldsymbol{b} u_{eq}(t) \;=\; 0 \\ &\qquad \cdots\cdots (6) \end{aligned}$$

then

$$u_{eq}(t) = -\left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1}\left[\boldsymbol{c}^T A\boldsymbol{x}(t) - \boldsymbol{c}^T\dot{\boldsymbol{x}}_d(t)\right] \cdots\cdots (7)$$

In the approaching phase, where $S(t) \neq 0$, in order to satisfy the *reaching condition* (5), a corrective control term (the so-called switching function) $u_c(t)$ must be added.

First, let the Lyapunov function be selected as below

$$V(t) = \frac{S(t)^2}{2} \cdots\cdots (8)$$

It can be noted that this function is positive definite. It is aimed that the derivative of the Lyapunov function is negative definite. This can be assured if one can assure that

$$\dot{S}(t) = -k\,sign(S(t)) \cdots\cdots (9)$$

where k is positive gain constant, and $sign(S(t))$ is defined as

$$sign(S(t)) = \begin{cases} +1, & \text{if } S(t) > 0 \\ 0, & \text{if } S(t) = 0 \\ -1, & \text{if } S(t) < 0 \end{cases} \cdots\cdots (10)$$

Taking the derivative of (8) and substitude (9) into it, the following equation is obtained

$$\begin{aligned} \dot{V}(t) &= S(t)\dot{S}(t) \\ &= S(t)\left[-k\,sign(S(t))\right] \\ &= -kS(t)sign(S(t)) \cdots\cdots (11) \end{aligned}$$

Furthermore, when $k \geq \eta$ is chosen, the *reaching condition* (5) is satisfied.

Again, the time derivative of (3) can be represented as

$$\dot{S}(t) = \boldsymbol{c}^T\dot{\boldsymbol{x}}_d(t) - \boldsymbol{c}^T A\boldsymbol{x}(t) - \boldsymbol{c}^T\boldsymbol{b}u(t) \cdots\cdots (12)$$

Then substitude (12) into the right hand side of (9) and the control input signal can be written as

$$\begin{aligned} u(t) &= -\left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1}\left[\boldsymbol{c}^T A\boldsymbol{x}(t) - \boldsymbol{c}^T\dot{\boldsymbol{x}}_d(t)\right] \\ &\quad + \left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1} k\,sign(S(t)) \\ &= u_{eq}(t) + u_c(t) \cdots\cdots (13) \end{aligned}$$

where

$$u_c(t) = \left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1} k\,sign(S(t)) \cdots\cdots (14)$$

is the corrective control. Putting

$$K = \left(\boldsymbol{c}^T\boldsymbol{b}\right)^{-1} k$$

then the final form of corrective control can be rewritten as

$$u_c(t) = K\,sign(S(t)) \cdots\cdots (15)$$

Here, K is called as switching gain.

3. Chattering Elimination

The controller of (13) exhibits high frequency oscillations in its output, causing a problem known as the chattering phenomenon. Chattering is highly undesirable because it can excite the high frequency dynamics of the system and can also cause damage to actuators or the plant. For its elimination, it is suggested to use a saturation or a shifted sigmoid function instead of the sign function.

In this section, a new and simple control method using the concept of point to hyperplane distance is proposed to suppress the chattering phenomenon. This alternative is to define a distance function $h(t)$ for calculate the distance between the trajectory of state errors and the sliding surface to generate the corrective control instead of the other functions.

For preliminary, we first discuss the concept of point to hyperplane distance.

From Fig. 1, given a plane

$$n_1 z_1 + n_2 z_2 + n_3 z_3 = k_3 \quad \cdots (16)$$

and a point $\boldsymbol{p}(p_1, p_2, p_3)$, the normal to the plane is given by

$$\boldsymbol{n} = [n_1,\ n_2,\ n_3]^T \quad \cdots (17)$$

and a vector from the plane to the point is given by

$$\boldsymbol{v} = -[z_1 - p_1,\ z_2 - p_2,\ z_3 - p_3]^T \quad \cdots (18)$$

Projecting $\boldsymbol{v}$ onto $\boldsymbol{n}$ gives the distance H from the point to the plane as

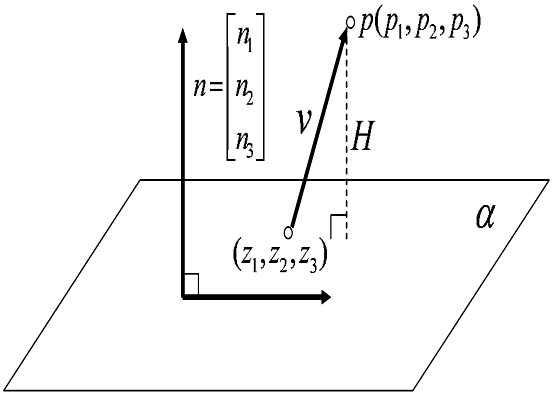

Fig. 1. Distance between a point and a plane.

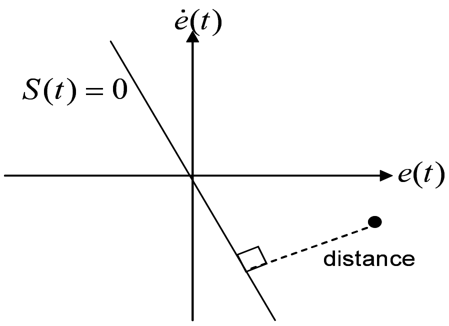

Fig. 2. Distance of state error trajectory to sliding surface $S(t) = 0$.

$$H = \frac{|\boldsymbol{n} \cdot \boldsymbol{v}|}{|\boldsymbol{n}|} = \frac{|n_1 p_1 + n_2 p_2 + n_3 p_3 - k_3|}{\sqrt{n_1^2 + n_2^2 + n_3^2}} \quad \cdots (19)$$

Consequently, when considering m-dimensional hyperplane, the distance function can be rewritten as below

$$h(\boldsymbol{p}, \alpha) = \frac{|\boldsymbol{n} \cdot \boldsymbol{p} - k_m|}{|\boldsymbol{n}|} = \frac{|n_1 p_1 + n_2 p_2 + \cdots + n_m p_m - k_m|}{\sqrt{n_1^2 + n_2^2 + \cdots + n_m^2}} \quad \cdots (20)$$

where α express the hyperplane, and this time $\boldsymbol{n} = [n_1, n_2, \cdots, n_m]^T$, $\boldsymbol{p} = [p_1, p_2, \cdots, p_m]^T$ are represented.

Hence, according to (2) and (3), the distance function that we use in this paper can be expressed as

$$h(t) = \frac{\left|c_1 e(t) + c_2 \dot{e}(t) + \cdots + c_n e^{(n-1)}(t)\right|}{\sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}} \quad \cdots (21)$$

and this (21) is to measure the distance of state error trajectories to sliding surface $S(t) = 0$ (Fig. 2).

Dropping the absolute value signs gives the signed distance

$$\bar{h}(t) = \frac{c_1 e(t) + c_2 \dot{e}(t) + \cdots + c_n e^{(n-1)}(t)}{\sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}} \cdots (22)$$

which is negative if trajectory of $\boldsymbol{e}(t)$ is on the side $S(t) < 0$ and positive if it is on the opposite side $S(t) > 0$.

Consequently, (22) also can be expressed as follow

$$\bar{h}(t) = h(t) sign(S(t)) \quad \cdots (23)$$

Let the proposed corrective control be defined as

$$u_c(t) = K_h \bar{h}(t) = K_h h(t) sign(S(t)) \quad \cdots (24)$$

where

$$K_h = (\boldsymbol{c}^T \boldsymbol{b})^{-1} k_h \cdots (25)$$

is defined and k_h is a positive constant which defined as weight of distance function for improving the control effect. Moreover, $K_h h(t)$ can be considered as the switching gain of the proposed corrective control in here. Meanwhile, consider the Lyapunov candidate function as

$$V(t) = \frac{S(t)^2}{2} \quad \cdots (26)$$

Thus, the time derivative of $V(t)$ by means of (7), (12), (24) and (25) becomes

$$\begin{aligned} \dot{V}(t) &= S(t)\dot{S}(t) \\ &= S(t)\left[\boldsymbol{c}^T \dot{\boldsymbol{x}}_d(t) - \boldsymbol{c}^T A \boldsymbol{x}(t) - \boldsymbol{c}^T \boldsymbol{b} u(t)\right] \\ &= S(t)\left[\boldsymbol{c}^T \dot{\boldsymbol{x}}_d(t) - \boldsymbol{c}^T A \boldsymbol{x}(t) - \boldsymbol{c}^T \boldsymbol{b}[u_{eq}(t) + u_c(t)]\right] \\ &= S(t)\left[-k_h h(t) sign(S(t))\right] \\ &= -k_h h(t) S(t) sign(S(t)) \cdots (27) \end{aligned}$$

If $k_h h(t) \geq \eta$ is chosen, then

$$-k_h h(t)S(t)sign(S(t)) \leq -\eta\,|S(t)| \cdots\cdots\cdots (28)$$

and the *reaching condition* (5) is satisfied.

In this paper, for the chattering elimination, (24) is utilized. Intuitively, using this presented method, the switching gain becomes small when the state error trajectories approach to the sliding surface, and when the state error trajectories reach the surface, the distance becomes zero i.e. the switching gain becomes zero. Thus the chattering phenomenon can be avoided.

4. Experimental Apparatus and Control Model

The experimental apparatus, shown in Fig. 3, consists of upper and lower drive coils that produce a magnetic field in response to a DC current. One or two magnets travel along a precision glass guide rod. By energizing the lower coil, a single magnet is levitated through a repulsive magnetic force. As current in the coil increases, the field strength increases and the levitated magnet height is increased. For the upper coil, the levitating force is attractive. Two magnets may be controlled simultaneously by stacking them on the glass rod. Two laser-based sensors measure the magnet positions. The lower sensor is typically used to measure a given magnet's position in proximity to the lower coil, and the upper one for proximity to the upper coil.

In this paper, we only consider using the lower drive coil to control one magnet as we consider to use single-input single-output plant for our experimental studies. Consequently from Fig. 3, the following equation of motion for the levitation system can be simply yield according to force balance analysis in the vertical plane

$$m\ddot{y}_1 = F_u - mg \cdots\cdots\cdots (29)$$

where m is the mass of the levitation magnet in kilograms, y_1 is the distance of the levitation magnet in meters, g is gravity, and F_u is the magnetic control force in newtons. The magnetic force term is modeled as having the following form.

$$F_u = \frac{i_1}{a_1(y_1 + a_2)^N} \cdots\cdots\cdots (30)$$

where a_1, a_2, and N are constants. Typically $3 < N < 4.5$. i_1 is current of coil.

Here, we replace the coil current i_1 to the more general term, denoted as u_1. The general term may be a digital word, voltage, or current and is presumed to be linearly proportional to the coil current. The coefficient a_1 must of course be consistently scaled with the units of u_1. In this paper, we consider u_1 to be a voltage, therefore (30) are redefined as

$$F_u = \frac{u_1}{a_1(y_1 + a_2)^N} \cdots\cdots\cdots (31)$$

and $a_1 = 27926$, $a_2 = 0.062$, $N = 4$ can be determined by numerical modeling of the magnetic configuration.

On the other hand, for small motions, the system may be modeled as being linear. Hence, above system can be simply linearized at the equilibrium operating point $y = 2$ cm magnet height as following state-space representation (see Appendix)

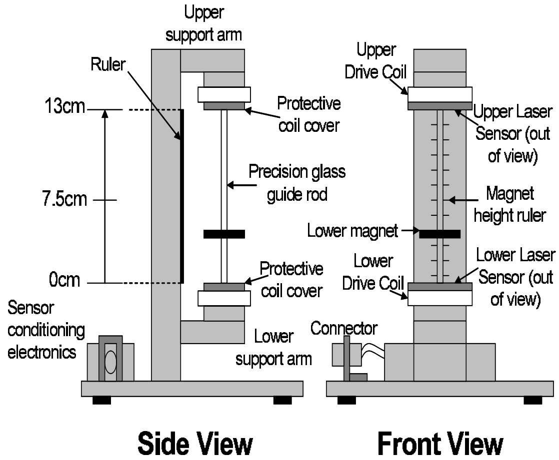

Fig. 3. Schematic diagram of magnetic levitation system (11).

Fig. 4. The practical hardware structure of the experimental system.

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t) \cdots\cdots\cdots (32)$$

where

$$\boldsymbol{x}(t) = \begin{bmatrix} y_1^* \\ \dot{y}_1^* \end{bmatrix}, A = \begin{bmatrix} 0 & 1 \\ -478.54 & 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 0 \\ 6.5456 \end{bmatrix}$$

The essential components for the real-time control system are a M56000 processor family's DSP board, a host PC, a magnetic levitation apparatus, servo/actuator interfaces, servo amplifiers, and auxiliary power supplies. The DSP is capable of executing control laws at high sampling rate of 1.1kHz allowing the implementation to be modeled as being in continuous or discrete time. The 16-bit dual-channel A/D and D/A acquistition systems are mounted on the system board. The Fig. 4 is the practical hardware structure of the experimental system.

5. Experimental studies

The magnetic levitation system which introduced in previous section is used to verify the effectiveness of the proposed SMC control strategy. The system is tested using the proposed control strategy and compared with the

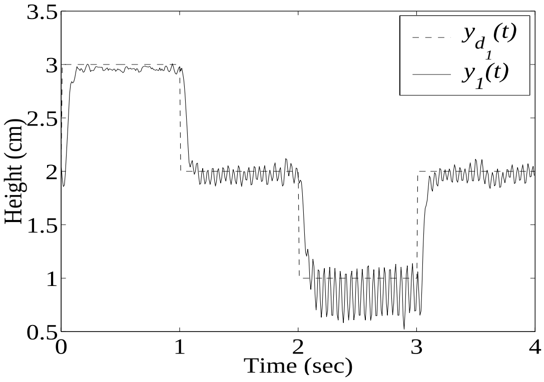

Fig. 5. System output for SMC using sign function: $K = 2$.

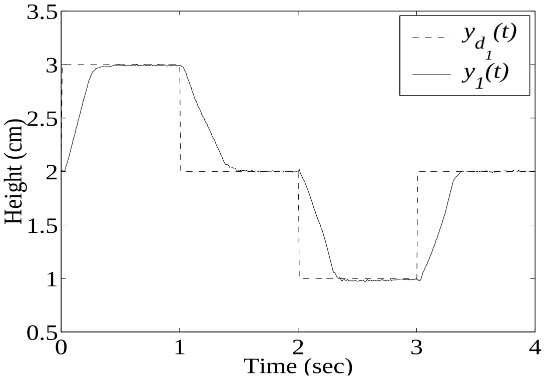

Fig. 6. System output for SMC using sign function: $K = 0.5$.

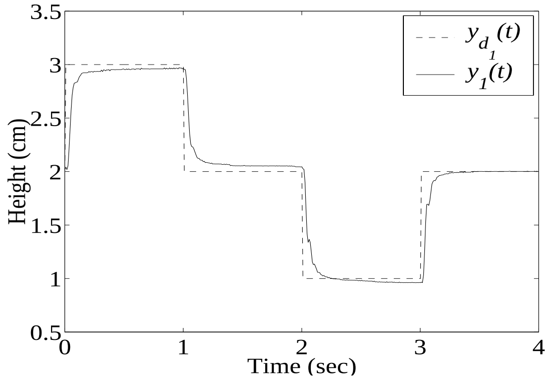

Fig. 7. System output for SMC using distance function: $K_h = 350$.

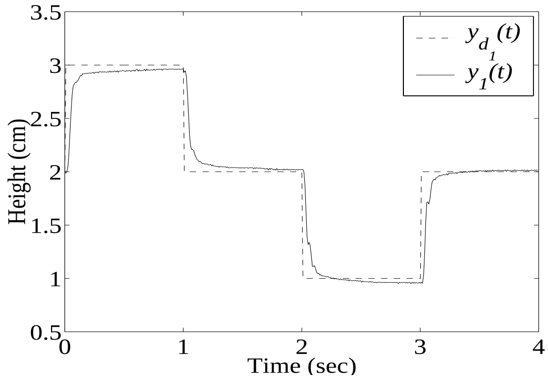

Fig. 8. System output for SMC using distance function: $K_h = 400$.

other two control strategies, i.e., conventional method which using sign function and shifted sigmoid function.

The three controllers that implemented are defined as *Proposed corrective control*:

$$u_c(t) = K_h h(t) sign(S(t))$$

Corrective control with sign function:

$$u_c(t) = K sign(S(t))$$

Corrective control with shifted sigmoid function:

$$u_c(t) = \frac{2K}{1 + e^{-\mu S(t)}} - K$$

respectively, where μ is the gradient of shifted sigmoid function.

During the experiment, the sliding surface parameters, $c_1 = 50$, $c_2 = 1$ were used, where the sliding function is expressed as

$$S(t) = c_1 e_1(t) + c_2 e_2(t)$$
$$e_1(t) = y_{d_1}(t) - y_1(t), \quad e_2(t) = \dot{y}_{d_1}(t) - \dot{y}_1(t)$$

All these controllers are implemented at a sampling rate of 0.565kHz.

The results of using the conventional method sign function are shown in Fig. 5 and Fig. 6 with different value of switching gain K, where the dotted line shows the desired output. Figure 5 shows the result when switching gain $K = 2$ is set. Highly chattering action occurs especially at the period from 2s to 3s, which is highly undesirable. The result showed a highly oscillatory response when a high switching gain of 2 is used. The chattering problem can be improved by using a lower switching gain $K = 0.5$ where shown in Fig. 6. However, from Fig. 6 slow convergent speed of system output to desired output can be observed at every step changes.

The results obtained using the proposed control strategy are shown in Fig. 7 and Fig. 8 where the respective values of K_h are set as $K_h = 350$ and $K_h = 400$. The system response showed perfect tracking with no any oscillations. Comparing the results to that of the proposed control strategy, it can be said that the proposed control strategy gave the better performance than using the conventional method sign function. Notice that, the value of K_h is extremely large compared to K. It is because the distance function $h(t)$ generates a very small value due to the unit of state error $e(t)$ is in meters.

Other result is shown in Fig. 9 where a larger $K_h = 1000$ is set. Oscillations can be observed at the period of 2s till 3s. From these results, a appropriate value of K_h must be selected such as 400 for this experimental study when using the proposed control method.

Figure 10 shows the result of using shifted sigmoid function when switching gain $K = 2$ and $\mu = 10$ is set. According to Fig. 10, the chattering phenomenon is

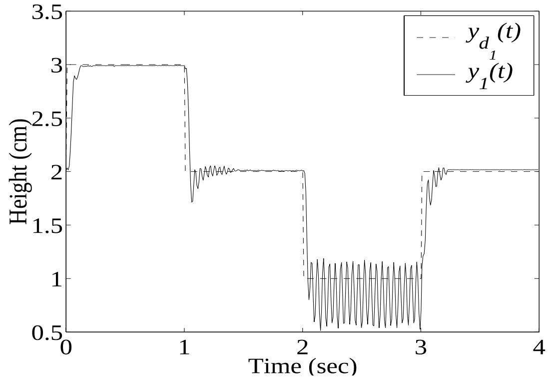

Fig. 9. System output for SMC using distance function: $K_h = 1000$.

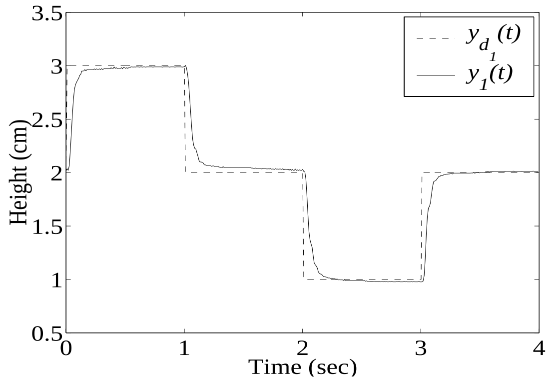

Fig. 10. System output for SMC using shifted sigmoid function: $K = 2$, $\mu = 10$.

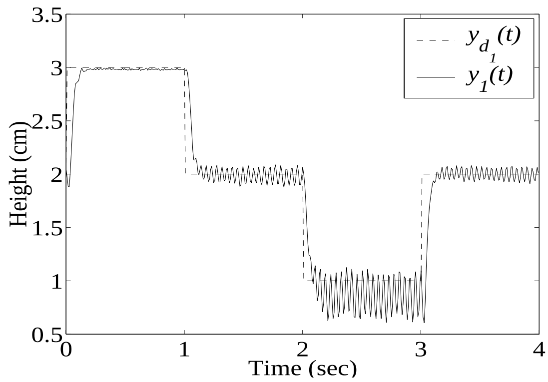

Fig. 11. System output for SMC using shifted sigmoid function: $K = 2$, $\mu = 50$.

eliminated by using the shifted sigmoid function. Furthermore, comparing Fig. 8 to Fig. 10, the performance of proposed control strategy is by not means inferior to performance of using shifted sigmoid function.

Meanwhile, Fig. 11 is the result of using shifted sigmoid function where its gradient value is $\mu = 50$. Chattering is unable to be eliminated completely, when $\mu = 50$ is used. In terms of these points, the value of μ including the switching gain K are the factors which may influence the control performance when shifted sigmoid function is utilized. For the proposed control strategy (distance function), only K_h is necessary to consider.

6. Conclusions

In this paper, a new contribution to the solution to the chattering elimination problem in SMC is presented. The proposed algorithm is simple, only need to use a defined distance function which to measure the distance between the trajectory of state errors and the sliding surface as the corrective control term instead of the conventional method sign function.

Experimental comparison between a classical controller: sign switching function/shifted sigmoid function and the proposed controller: distance switching function for controlling a magnetic levitation system was investigated. The experimental results show that this control approach effectively suppresses the chattering phenomenon.

In this research, we have considered the design of SMC on continuous time. Since PC is utilized to operate the designed controller, we will deal with discrete time design in the future work.

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References

(1) V.I. Utkin: "Variable structure systems with sliding mode", *IEEE Trans. Auto. Control*, Vol.22, pp.212–222 (1977)
(2) J.Y. Hung, W. Gao, and J.C. Hung: "Variable structure control: A survey", *IEEE Trans. Industr. Electron.*, Vol.40, pp.2–22 (1993)
(3) W. Gao, Y. Wang, and A. Homaifa: "Discrete-time variable structure control systems", *IEEE Trans. Industr. Electron.*, Vol.42, pp.117–122 (1995)
(4) B. Drazenovic: "The invariance conditions in variable structure systems", *Automatica*, Vol.15, pp.287–295 (1969)
(5) D.S. Yoo and M.J. Chung: "A variable structure control with simple adaptation for upper bound on the norm of the uncertainties", *IEEE Trans. Auto. Control*, Vol.37, pp.860–864 (1992)
(6) S.H. Zak and S. Hui: "On variable structure output feedback controllers for uncertain dynamic systems", *IEEE Trans. Auto. Control*, Vol.38, pp.1509–1512 (1993)
(7) L.C. Fu and T.L. Liao: "Globally stable robust tracking of nonlinear system using variable structure control and application to a robotic manipulator", *IEEE Trans. Auto. Control*, Vol.34, pp.1345–1350 (1990)
(8) J.E. Slotine and S.S. Sastry: "Tracking control of nonlinear systems using sliding surface with application to robotic manipulators", *Int. J. Control*, Vol.38, pp.465–492 (1983)
(9) W. Tang, A.N. Michael, and H.W. Hale: "On structure and stability of interconnected dynamical systems", *IEEE Trans. Circuits Syst.*, Vol.27, pp.391–404 (1980)
(10) D. Cho, Y. Kato, and D. Spilman: "Sliding mode and classical controllers in magnetic levitation systems", *IEEE Control Systems Magazine*, Vol.13, pp.42–48 (1993)
(11) T.R. Parks: Manual for Model 730 Magnetic Levitation System, Educational Control Products (1991)

Appendix

The equation of motion for the magnetic levitation system is given as

$$m\ddot{y}_1 = F_u - mg \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(A1)}$$

$$F_u = \frac{u_1}{a_1(y_1 + a_2)^4} \cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(A2)}$$

When the nonlinear term (right hand side) of (A1) is presented as $\alpha(y_1, u_1, t)$, we have

$$m\ddot{y}_1 = \alpha(y_1, u_1, t) = F_u - mg \cdots\cdots\cdots\cdots \text{(A3)}$$

Then the linearized equation of motion is found by calculating

$$\begin{aligned}\alpha(y_1, u_1, t) &- \alpha(y_{10}, u_{10}, t) \\ &= \left.\frac{\partial \alpha}{\partial y_1}\right|_{y_{10},u_{10}} (y_1 - y_{10}) \\ &\quad + \left.\frac{\partial \alpha}{\partial u_1}\right|_{y_{10},u_{10}} (u_1 - u_{10}) \cdots\cdots\cdots \text{(A4)}\end{aligned}$$

where y_{10}, u_{10} are the magnet position and control effort that define the operating point. For the purposes of control design, we shall choose the operating point to be at an equilibrium so that

$$\alpha(y_{10}, u_{10}, t) = F_u - mg|_{y_{10},u_{10}} = 0 \cdots\cdots \text{(A5)}$$

Evaluating (A4) and using (A5) we have

$$\begin{aligned}m\ddot{y}_1 &= -\frac{4u_{10}}{a_1(y_{10} + a_2)^5}(y_1 - y_{10}) \\ &\quad + \frac{1}{a_1(y_{10} + a_2)^4}(u_1 - u_{10}) \cdots\cdots\cdots \text{(A6)}\end{aligned}$$

which may be rewritten as

$$m\ddot{y}_1^* + k_1' y_1^* = k_2' u_1^* \cdots\cdots\cdots\cdots\cdots\cdots\cdots \text{(A7)}$$

where

$$y_1^* = y_1 - y_{10}, \quad u_1^* = u_1 - u_{10}$$

$$k_1' = \frac{4u_{10}}{a_1(y_{10} + a_2)^5}, \quad k_2' = \frac{1}{a_1(y_{10} + a_2)^4}$$

From (A5) we may solve for the equilibrium control input values as

$$u_{10} = a_1 mg(y_{10} + a_2)^4 \cdots\cdots\cdots\cdots\cdots\cdots \text{(A8)}$$

Hence, according to the values of $a_1 = 27926$, $a_2 = 0.062$, $y_{10} = 0.02m$, $m = 0.121kg$, $g = 9.81ms^{-2}$

$$\begin{aligned}\begin{bmatrix} \dot{y}_1^* \\ \ddot{y}_1^* \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -478.54 & 0 \end{bmatrix} \begin{bmatrix} y_1^* \\ \dot{y}_1^* \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 6.5456 \end{bmatrix} u_1^* \cdots\cdots\cdots\cdots\cdots \text{(A9)}\end{aligned}$$

can be obtained.

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