

Chemical Dynamics in Condensed Phases

**Relaxation, Transfer and Reactions in
Condensed Molecular Systems**

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