Chiral $3d$ SU(3) SQCD and $\mathcal{N}=2$ mirror duality

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ABSTRACT: Recently a very interesting three-dimensional $\mathcal{N}=2$ supersymmetric theory with SU(3) global symmetry was discussed by several authors. We denote this model by T_x. This was conjectured to have two dual descriptions, one with explicit supersymmetry and emergent flavor symmetry and the other with explicit flavor symmetry and emergent supersymmetry. We discuss a third description of the model which has both flavor symmetry and supersymmetry manifest. We then investigate models which can be constructed by using T_x as a building block gauging the global symmetry and paying special attention to the global structure of the gauge group. We conjecture several cases of $\mathcal{N}=2$ mirror dualities involving such constructions with the dual being either a simple $\mathcal{N}=2$ Wess-Zumino model or a discrete gauging thereof.

Contents

1 Introduction and discussion

Three dimensions is quantum field theorist's paradise. On the one hand it is easy to build simple asymptotically free field theories which flow to interacting fixed points, and on the other we have a lot of control over many such models. This is to be contrasted with the situation in higher dimensions, where the number of interesting models in the IR with simple UV Lagrangians decreases, and in lower dimensions, where many subtle effects pertaining to the vacuum structure appear. Moreover, in principle, models in three dimensions can be engineered as effective descriptions of real-world condensed matter systems in a lab.

The situation is particularly beneficial with supersymmetric theories, where we have a plethora of exact computations we can perform for $\mathcal{N} = 2$ supersymmetric cases (see e.g. [1–3]). Recently even models with $\mathcal{N} = 1$ supersymmetry have led to exact results [4–8]. Moreover, the progress with understanding the supersymmetric models and also the large-N models, as summarized in [9], has led to a remarkable progress in understanding non supersymmetric Chern-Simons theories, see e.g. [10].

In this note we stay in the $\mathcal{N} = 2$ supersymmetric domain and discuss some effects motivated by recent progress with less supersymmetric theories. In particular, a very interesting model, which modulo contact terms we will call T_x, has been conjectured to have two descriptions [6, 7]. One with manifest $SU(3)$ global symmetry and $\mathcal{N} = 1$ supersymmetry, which enhances to $\mathcal{N} = 2$ in the IR. The other as an $\mathcal{N} = 2$ theory with $U(1) \times SU(2)$ global symmetry enhancing to $SU(3)$ in the IR. The fact that the symmetry enhances to $SU(3)$ was also obtained from geometric considerations in the context of the

$3d/3d$ correspondence [11] in [12, 13]. We will suggest here a third description (from which yet another can be derived using a by now well-known IR duality [14, 15]) which has both $\mathcal{N} = 2$ supersymmetry and SU(3) global symmetry in the UV. The description is an SU(3) Chern-Simons model with level $5/2$ and a single chiral field in the bi-fundamental representation of the gauge and flavor SU(3), supplemented by a baryon superpotential. We will give evidence for the duality by comparing the superconformal index and three-sphere partition function of this model and of the description with non manifest flavor symmetry but manifest supersymmetry.

In the second part of the note we will construct theories which do not have any continuous symmetries by gauging with Chern-Simons terms the diagonal global symmetry of several T_x models. As the model T_x has only matter charged in representations with N-ality zero under SU(3), both gauging SU(3) and SU(3)/$\mathbb{Z}_3$ is possible. In three dimensions the latter possibility leads to models with $\mathbb{Z}_3$ zero-form global symmetry. This is to be contrasted with four dimensions where such a choice of global structure affects the spectrum of line operators as it affects one-form symmetries. See [16] and [17] for recent discussions. The operators which are charged under such discrete symmetries in three dimensions are gauge-invariant monopoles. We will mainly focus on the case with SU(3)/$\mathbb{Z}_3$ gauging and construct theories which have discrete global symmetry. For several examples of lowest possible values of Chern-Simons levels and low number of copies we conjecture that such theories are dual to Wess-Zumino models with $\mathcal{N} = 2$ supersymmetry and cubic superpotential interactions. Note that these interactions preserve a $\mathbb{Z}_3$ symmetry. Performing the gauging with SU(3) will result in such duals with the $\mathbb{Z}_3$ symmetry gauged. The main evidence we give for the conjectured dualities is again by comparing the supersymmetric indices.

We observe several other interesting features from our results. For instance, taking four copies of T_x and gauging SU(3) with level two we obtain evidence that the model is dual to T_x with the SU(3) symmetry emerging in IR. Another observation is that the basic monopole operators in many examples we study here are counted by Catalan numbers, and it would be interesting to understand whether mirror models with such property, that is Catalan numbers counting operators built from fundamental fields, can be considered. Finally, SU(3) plays a special role in our construction. This is mainly because it is easy to construct SU(3)-invariant relevant superpotential with matter in fundamental representations. The group SU(3) plays a special role in four-dimensional $\mathcal{N} = 1$ field theories as well, where the fact that baryons are marginal can lead to large conformal manifolds [18, 19]. Recently such theories were related to compactifications on Riemann surfaces of a certain minimal SCFT in six dimensions [20]. It would be interesting to understand whether the constructions we consider here are useful in that context too.

2 The model T_x

We start by reviewing the known definitions of model T_x and conjecturing a definition with both global symmetry and supersymmetry manifest in the UV theory.
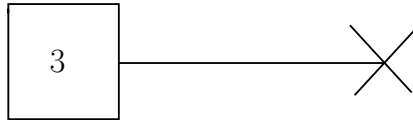
Figure 1. We will denote the theory T_x by such a graph and use it as a building block gauging either SU(3) or SU(3)/$\mathbb{Z}_3$ symmetry with some Chern-Simons level.

2.1 Description A: manifest global symmetry

The first description has $\mathcal{N} = 1$ supersymmetry and manifest SU(3) global symmetry. This is simply a Wess-Zumino model of eight real superfields with superpotential,

$$d_{acb}\chi^a\chi^b\chi^c\,. \tag{2.1}$$

Here $d_{abc} = \text{Tr}\,T_a\{T_b\,,T_c\}$ with T_a the generators of SU(3). It was conjectured in [6, 7] that the supersymmetry of this model enhances to $\mathcal{N} = 2$ and a continuous R-symmetry emerges in the IR CFT.

2.2 Description B: manifest supersymmetry

A second description is the one which has emergent global symmetry but explicit supersymmetry. We will denote this description by T_x. Different descriptions might differ by contact terms and to be precise when referring to model T_x we will refer to the model discussed in this section. The fact that we have manifest $\mathcal{N} = 2$ supersymmetry allows us to utilize various localization techniques to study it [21].

The model is an $\mathcal{N} = 2$ supersymmetric U(1) gauge theory with two chiral fields with the same charge under the U(1). We choose the charge to be one. The global symmetry that we can identify in the Lagrangian is an SU(2) rotating the two chiral fields, on top of which we have the topological U(1) symmetry the monopole operators are charged under. It was claimed in [6, 7] that the symmetry here enhances to SU(3) with the fundamental given by the following decomposition into SU(2) × U(1),

$$\mathbf{3} = \mathbf{2}_{-1} + \mathbf{1}_2\,. \tag{2.2}$$

This can be easily seen by computing the supersymmetric index of the model. The index is the following measure of the spectrum of local operators:

$$\mathcal{I} = \text{Tr}_{\mathbb{S}^2}\left[(-1)^{2J_3}q^{\frac{1}{2}(\Delta+J_3)}\prod_a v_a^{e_a}\prod_b f_b^{q_b}\right]\,. \tag{2.3}$$

Here J_3 is the generator of the SU(2) rotation isometry of $\mathbb{S}^2$, Δ is the conformal dimension, e_a are charges under the a-th Cartan generator of the global symmetry group, and finally the trace is taken in radial quantization. The fugacities f_b are for abelian discrete symmetries $\mathbb{Z}_{n_b}$ and thus are n_b-th roots of unity, while q_b label the elements of the discrete groups. We will use in this paper the notations of the index of [22] (which are explained in [23]). For a review on the derivation of the index expressions the reader can consult [2]. The

index is a function of fugacities for different symmetries and fluxes for global symmetries through $\mathbb{S}^2$.

Without turning on fluxes through the sphere for the global symmetry, the index is given by

$$\mathcal{I}(b, w) = \sum_{m=-\infty}^{\infty} w^m \oint \frac{dh}{2\pi i h} \underline{\mathcal{I}}(hb; m; r) \underline{\mathcal{I}}(hb^{-1}; m; r) . \quad (2.4)$$

Here h is the U(1) gauge symmetry fugacity, b that of the Cartan of SU(2), w of the topological U(1), $\underline{\mathcal{I}}$ the index of a chiral field which is given by

$$\underline{\mathcal{I}}(z; n; r) = \left(q^{\frac{1-r}{2}} z^{-1}\right)^{\frac{|n|}{2}} \prod_{l=0}^{\infty} \frac{1 - (-1)^n z^{-1} q^{\frac{|n|}{2} + \frac{1}{2} r + l}}{1 - (-1)^n z q^{\frac{|n|}{2} + 1 - \frac{1}{2} r + l}} . \quad (2.5)$$

We take r to be the R-charge. The fugacity z is for the U(1) symmetry under which the chiral field is charged and n is the flux through $\mathbb{S}^2$ for this symmetry. Note that the flux has to be properly quantized. The signs in the expressions appear as J_3. For an object of electric charge e in the presence of a magnetic monopole with charge m, J_3 is shifted by $e \cdot m$ (see the discussion in [11]). The signs are important in general and we follow the notations of [22]. The index of T_x was analyzed in [13] and here we will discuss some points which will be important for us. Evaluating the index one obtains,

$$\begin{aligned}\mathcal{I}(b, w) = {} & 1 - (2 + b^2 + b^{-2} + (b + b^{-1})(w + w^{-1}))q \\ & - (3 + b^2 + b^{-2} + (b + b^{-1})(w + w^{-1}))q^2 + \cdots .\end{aligned} \quad (2.6)$$

The term at order q should count marginal operators minus conserved currents [24, 25], and we see this is consistent with having no marginal operators and a current in the adjoint of SU(3). The character of the adjoint is obtained upon taking $b = z_1^{-1/2} z_2^{-1}$ and $w = z_1^{-3/2}$ with z_i parametrizing the Cartan of SU(3) such that $\prod_{l=1}^3 z_l = 1$.

For this theory the choice of R-symmetry for the chiral fields is a gauge symmetry and thus all choices should be equivalent. There is a small subtlety with this statement as we also have a Fayet-Iliopoulos parameter which is the mass for the topological symmetry. In presence of such a term, starting from some choice of R-symmetry and performing a gauge transformation that changes this assignment we produce a contact term between the R-symmetry and the topological one. Only for a particular choice of the contact term, for a given value of the R-symmetry, the theory will enjoy an SU(3) symmetry. Let us exemplify this with the index computation. The index in the presence of fluxes for the global symmetry is,

$$\mathcal{I}(z_l; n, \hat{n}) = \sum_{m \in \mathbb{Z}} w^m \oint \frac{dh}{2\pi i h} h^n \underline{\mathcal{I}}(hb; m + \hat{n}; r) \underline{\mathcal{I}}(hb^{-1}; m - \hat{n}; r) \Big|_{w = z_1^{-3/2}, b = z_1^{-1/2} z_2^{-1}} . \quad (2.7)$$

Here n is the flux of the U(1) topological symmetry and $\hat{n}$ that of the Cartan of the SU(2) symmetry. Note that changing the R-symmetry to $R \to R + 2\alpha Q$ with Q being charge under U(1), amounts to redefining for the chiral fields $h \to q^\alpha h$. Without the FI parameter

this has no effect on the index but with it we produce a term of the form $q^{-\alpha n}$. This looks as a contact term between R-symmetry and the topological symmetry.

The flux can be written, following the map of fugacities we derived here, in terms of the fluxes for the Cartan of SU(3),

$$(n\,,\ \hat{n}) = \left(-\frac{3}{2}m_1\,,\ -\frac{1}{2}m_1 - m_2\right)\,. \tag{2.8}$$

The index computed with arbitrary values of fluxes should be invariant under the action of the Weyl symmetry of SU(3), that is

$$\begin{aligned}\mathcal{I}_x(z_i; m_i) &:= \ \mathcal{I}(z_l; n, \hat{n})|_{n=-\frac{3}{2}m_1,\ \hat{n}=-\frac{1}{2}m_1-m_2}\,,\\ \mathcal{I}_x(z_i; m_i) &= \ \mathcal{I}_x(z_{\sigma(i)}; m_{\sigma(i)}),\qquad \sigma \in S_3\,. \end{aligned} \tag{2.9}$$

Here $m_3 = -m_1 - m_2$ and $z_3 = z_1^{-1}z_2^{-1}$. By computing the index we find that there is invariance if the R-charge is $1/3$ and there is no contact term. We can change the R-charge but then we will need to add a contact term between R-symmetry and the topological symmetry.

Another interesting issue is the following subtlety. Note that, according to the way we define the theory, the magnetic monopole charges for SU(2) and for the topological U(1) are integers. However, if the symmetry enhances to SU(3) we should be able to turn on integer fluxes for the latter as well. According to (2.8), an odd m_1 flux would imply half-integer n and $\hat{n}$ fluxes. To deal with this, whenever we turn on an odd m_1 we need to shift the lattice of fluxes for the gauge symmetry by a half.[1] That is,

$$\begin{aligned}\mathcal{I}_x(z_i; m_i) = \sum_{m\in\mathbb{Z}+\frac{1}{2}(m_1\bmod 2)} w^m \oint \frac{dh}{2\pi i h} h^{-\frac{3}{2}m_1}\underline{\mathcal{I}}\left(hb; m-\frac{1}{2}m_1-m_2; \frac{1}{3}\right) \\ \cdot \underline{\mathcal{I}}\left(hb^{-1}; m+\frac{1}{2}m_1+m_2; \frac{1}{3}\right)\Bigg|_{w=z_1^{-3/2},\ b=z_1^{-1/2}z_2^{-1}}\,. \end{aligned} \tag{2.10}$$

Computing the index, without refining with fugacities for the global symmetry, we obtain:

$$\mathcal{I}_x(1; 0) = 1 - 8q - 9q^2 + 18q^3 + 46q^4 + \cdots\,. \tag{2.11}$$

Further, as we know from description A and as can be inferred from the index computation, the N-ality of all SU(3) representations of states in the theory is 0. This means that we can also turn on fluxes for SU(3) which are shifted by multiples of $1/3$. The content of the model thus allows gauging of both SU(3) and SU(3)/$\mathbb{Z}_3$.

2.3 Description C: manifest global symmetry and supersymmetry

We consider a Wess-Zumino model with nine $\mathcal{N} = 2$ chiral superfields organized into a bi-fundamental chiral Q_{ij} of two SU(3) symmetries, and a superpotential given by the baryon[2]

$$W = \epsilon^{ilm}\epsilon^{jkc}Q_{ij}Q_{lc}Q_{mk}\,. \tag{2.12}$$

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[1]See [26] for a similar discussion in the context of $\mathcal{N} = 4$ theories. Another way to phrase this is that the group rotating the chirals is U(2) = (SU(2) × U(1))/$\mathbb{Z}_2$ and thus if we gauge the U(1) we can have half integer flux for SU(2) as long as U(1) has half integer flux.

[2]Such theories in three dimensions flow to interacting SCFT's, e.g. they have intricate conformal manifolds (see [27]).
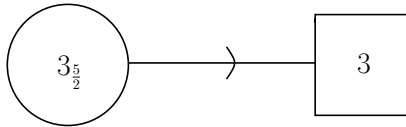
Figure 2. The theory $T_x^{(2)}$ which is dual to T_x with a contact term for the SU(3) global symmetry. We turn on a baryonic superpotential for the chiral field which preserves the non-abelian symmetry. Here we parametrize the gauge node by N of SU(N) and by the level k of the Chern-Simons term as N_k.

We then gauge one of the SU(3) symmetries with a level-5/2 Chern-Simons term. For smaller values of the level the theory will be "bad", that is the partition function will not be well-defined. This might signal either spontaneous breakdown of supersymmetry, or wrong R-symmetry assignments as in [28]. Moreover we need half-integer Chern-Simons level, for otherwise the theory would have a parity anomaly.

The model has manifest SU(3) symmetry and $\mathcal{N} = 2$ supersymmetry and we conjecture it is dual to T_x. As a check one can compute the supersymmetric index and find that at least in expansion in fugacities it matches precisely with the index of T_x. Importantly, the two models have a relative contact Chern-Simons term for the global SU(3) symmetry at level one. To see this we can compute the index in presence of a background monopole flux for the SU(3) symmetry. For example, taking the flux to be $(m_1, m_2, m_3) = (1, -1, 0)$ we obtain, for the model T_x:

$$q^{1/2} \left(z_1^{-1/2} z_2^{-5/2} + z_1^{1/2} z_2^{5/2} \right) - q \left(z_1^{1/2} z_2^{-1/2} + z_2^{1/2} z_1^{-1/2} \right) + \cdots . \qquad (2.13)$$

For the SU(3) gauge theory we obtain instead:

$$q^{1/2} \left(z_1^{1/2} z_2^{-7/2} + z_1^{3/2} z_2^{3/2} \right) - q \left(z_1^{1/2} z_2^{-1/2} + z_2^{-3/2} z_1^{3/2} \right) + \cdots . \qquad (2.14)$$

Since the Chern-Simons term contributes $z_1^{k(2m_1+m_2)} z_2^{k(2m_2+m_1)}$ to the index, where $(m_1, m_2, -m_1 - m_2)$ is the flux and k the level, we can see that the two above expressions differ by a factor of $z_1 z_2^{-1}$, which comes from a background Chern-Simons term at level one. We stress that this model has manifest symmetry and supersymmetry to be contrasted with the other descriptions.[3]

The SU(N) gauge theories with matter in fundamental representations in three dimensions have known duals which descend from dualities in four dimensions and real mass deformations thereof [23, 29]. One can use such dualities to obtain other descriptions with manifest symmetry. For example, following [14] (see also [30]), we know that $\mathcal{N} = 2$ U(3) at level 5/2 with three fundamental chirals is dual to a U(1) gauge theory at level $-5/2$ with three fundamental chirals. Importantly there are also contact terms, one of which is a Chern-Simons term at level one for the topological symmetry. We can gauge the topological symmetry of the pair by putting the Chern-Simons term for the topological symmetry

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[3]However, because of the Chern-Simons term this description does not have manifest time reversal symmetry with the two other descriptions manifestly invariant. We thank Kazuya Yonekura for pointing this fact out to us.

on the U(1) side. The U(3) model then becomes the SU(3) theory we consider [31], and on the dual side (because of the contact term) we are left with a U(1) gauge theory. The baryonic symmetry of the SU(3) theory maps to the topological symmetry on the dual side; turning on a baryonic superpotential amounts to a monopole superpotential on the U(1) side.[4] The reader can consult [15] for this duality and we work out the details in the appendix. We moreover use it to verify the equality of $\mathbb{S}^3$ partition functions of the new description and description B. This is an independent check of the duality.

Let us note that we can consider a generalization of the model by increasing the Chern-Simons level, though we do not have any claim for duality for higher levels. We consider gauging with arbitrary CS term at level $k = l + \frac{1}{2}$, with integer l bigger than one. All such models have SU(3) global symmetries and no $\mathcal{N} = 2$ marginal or relevant deformations. We will denote such models as $T_x^{(l)}$, so that $T_x^{(2)}$ is dual to T_x (adding contact terms). Increasing l the monopoles will obtain higher charges meaning the gauge sector will have weaker coupling. The index for several values of l is:

$$
\begin{aligned}
l = 2: \quad & 1 - 8q - 9q^2 + 18q^3 + 46q^4 + \cdots ; \\
l = 3: \quad & 1 - 8q + 9q^2 + 53q^3 + 28q^4 + \cdots ; \\
l = 4: \quad & 1 - 8q + 9q^2 + 43q^3 - 9q^4 + \cdots .
\end{aligned}
\tag{2.15}
$$

The -8 at order q is the contribution of the conserved currents of the SU(3) global symmetry. For higher values of the Chern-Simons level the first terms in the expansion of the index are as for $l = 4$ and the difference appears at higher powers of q, as it comes from gauge-invariant dressed monopole operators (whose charges scale with the level).

3 Gluing the T_x together and $\mathcal{N} = 2$ mirror duality

We can consider gluing together several copies of T_x by gauging the diagonal SU(3) or SU(3)/$\mathbb{Z}_3$ symmetry with a Chern-Simons term. As the model T_x does not have any marginal or relevant deformations for large enough values of the level of the Chern-Simons term or large enough number of copies, the resulting models will also have no relevant or marginal deformations as the charges of the monopoles of the gauge group increase with the level and the amount of matter. For too low a number of copies and too low a level, the theories are bad in the sense that the partition functions do not converge. However, for high enough levels and number of copies the theories are sensible and might have interesting low-dimension operators. We have studied the models with minimal levels and number of copies possible and in what follows we will report on few examples where we could recognize a dual description. We find three examples with the theories dual to simple $\mathcal{N} = 2$ Wess-Zumino models with cubic interactions.[5] We will moreover discuss a dual of T_x itself which can be obtained via such a construction. The main check we will

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[4]Monopole superpotentials were first discussed in the context of dualities in [32]. See [33–36] for many recent examples of models with monopole superpotentials.

[5]In [13], motivated by geometric considerations, the authors considered gauging subgroups of SU(3) for a single T_x.
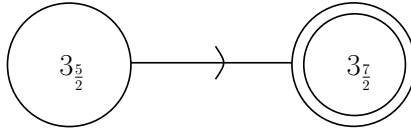
Figure 3. The dual of theory T_x with $SU(3)/\mathbb{Z}_3$ gauged at level $k = 7/2$. The theory is dual to a chiral field with cubic superpotential. The double circle denotes $SU(3)/\mathbb{Z}_3$ gauging with the single circle denoting an $SU(3)$ gauge model.

refer to is the equality of indices. We have verified such equalities in a series expansion in q to several non-trivial orders, but do not have a proof of the identities.

3.1 Single T_x with $SU(3)/\mathbb{Z}_3$ gauged dual to WZ with one chiral field

Let us consider gauging the $SU(3)/\mathbb{Z}_3$ symmetry of a single copy of T_x. The model then will have no continuous global symmetry and will have $\mathbb{Z}_3$ symmetry. With low level of Chern-Simons term the theory is bad, and the lowest level for which we find that the partition function converges is $9/2$. We also find that the index of this model agrees with the index of a single chiral field with cubic superpotential. We conjecture then that T_x with $SU(3)$ gauged at level $9/2$ is a Wess-Zumino model of one chiral field with cubic superpotential. The index is given by,[6]

$$\frac{1}{6} \sum_{l=0}^{2} g^l \sum_{m_1, m_2 \in \mathbb{Z} + \frac{l}{3}} \oint \frac{dz_1}{2\pi i z_1} \frac{dz_2}{2\pi i z_2} \prod_{i \neq j} q^{-\frac{|m_i - m_j|}{4}} \left(1 - (-1)^{m_i - m_j} q^{\frac{|m_i - m_j|}{2}} \frac{z_i}{z_j} \right)$$
$$\cdot (z_1^{2m_1 + m_2} z_2^{2m_2 + m_1})^{9/2} \mathcal{I}_x(z_i; m_i) = \underline{\mathcal{I}} \left(g^{-1}; 0; \frac{2}{3} \right) . \tag{3.1}$$

Here g is a third root of unity, i.e. a fugacity for the $\mathbb{Z}_3$ symmetry. The operators charged under the discrete symmetry are gauge-invariant monopole operators with fractional charge.[7] The theory with gauged $SU(3)$ symmetry is then dual to the $\mathbb{Z}_3$ gauging of the Wess-Zumino model, i.e.

$$\frac{1}{6} \sum_{m_1, m_2 \in \mathbb{Z}} \oint \frac{dz_1}{2\pi i z_1} \frac{dz_2}{2\pi i z_2} \prod_{i \neq j} q^{-\frac{|m_i - m_j|}{4}} \left(1 - (-1)^{m_i - m_j} q^{\frac{|m_i - m_j|}{2}} \frac{z_i}{z_j} \right)$$
$$\cdot (z_1^{2m_1 + m_2} z_2^{2m_2 + m_1})^{9/2} \mathcal{I}_x(z_i; m_i) = \frac{1}{3} \sum_{j=0}^{2} \underline{\mathcal{I}} \left(e^{\frac{2\pi i j}{3}}; 0; \frac{2}{3} \right) . \tag{3.2}$$

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[6]For a discussion of gauging of $SU(N)/\mathbb{Z}_N$ symmetries in the index see e.g. [26]. The index in three dimensions can be obtained as the limit of the lens index in four dimensions [37] which depends in a non-trivial way on the global structure of the gauge group [38]. The differences in monopole operators appearing for different global structures are relatives of differences in line operators, see [17] for a recent discussion in four dimensions.

[7]Weighting different sectors by the discrete symmetry is a three-dimensional avatar of weighting different sectors in the lens index in four dimensions [38].
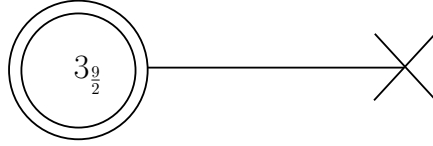
Figure 4. The theory T_x with $SU(3)/\mathbb{Z}_3$ gauged at level $k = 9/2$. The theory is dual to a chiral field with cubic superpotential. The double circle denotes the $SU(3)/\mathbb{Z}_3$ gauging. The different level here and in the previous figure is because the duality involves a relative contact term.

We can use description C to write a quiver (see figure 2), and as descriptions B and C have a relative Chern-Simons contact term for the $SU(3)$ symmetry, the level of the $SU(3)/\mathbb{Z}_3$ Chern-Simons term we need to add is $7/2$.

Note that the mirror dual of a free chiral field is well-known, and is given by a $U(1)$ gauge theory with level half Chern-Simons term plus a single chiral field. The topological symmetry is dual to the $U(1)$ baryonic symmetry rotating the chiral field. We need to turn on a cubic superpotential to break this symmetry, which on the gauge theory side translates to a monopole superpotential. Thus this provides yet another dual of the model we build by gauging the $SU(3)/\mathbb{Z}_3$ symmetry of T_x.

3.2 Five glued T_x dual to WZ with five chiral fields

We consider gauging a diagonal $SU(3)/\mathbb{Z}_3$ symmetry of five copies of T_x at level $3/2$. The index is equal to the one of the Wess-Zumino model with five chiral fields and a general cubic superpotential,

$$W = \sum_{i,j,l=1}^{5} \lambda_{ijl} \Phi_i \Phi_j \Phi_l \,. \tag{3.3}$$

The index is

$$\frac{1}{6} \sum_{l=0}^{2} g^l \sum_{m_1,m_2 \in \mathbb{Z}+\frac{l}{3}} \oint \frac{dz_1}{2\pi i z_1} \frac{dz_2}{2\pi i z_2} \prod_{i \neq j} q^{-\frac{|m_i-m_j|}{4}} \left(1 - (-1)^{m_i-m_j} q^{\frac{|m_i-m_j|}{2}} \frac{z_i}{z_j} \right)$$

$$\cdot (z_1^{2m_1+m_2} z_2^{2m_2+m_1})^{\frac{3}{2}} \mathcal{I}_x(z_i; m_i)^5 = \underline{\mathcal{I}} \left(g^{-1}; 0; \frac{2}{3} \right)^5 \,. \tag{3.4}$$

This suggests that gauging five copies of T_x with $SU(3)/\mathbb{Z}_3$ at level $3/2$ is dual to a Wess-Zumino model of five chiral fields. The model has a $\mathbb{Z}_3$ global symmetry which we can gauge. The precise statement of the conjecture is that the two models flow to the same conformal manifold on a generic point of which we have a $\mathbb{Z}_3$ symmetry explicitly visible in both theories. For special values of the couplings, on one of the sides some additional discrete symmetries might appear; this, as usual in dualities, might or might not be visible on the dual side.

3.3 Eight glued T_x dual to WZ with fourteen chiral fields

We consider gauging a diagonal $SU(3)/\mathbb{Z}_3$ symmetry of eight copies of T_x at level zero. The index is equal to the one of a Wess-Zumino model with fourteen chiral fields and a general
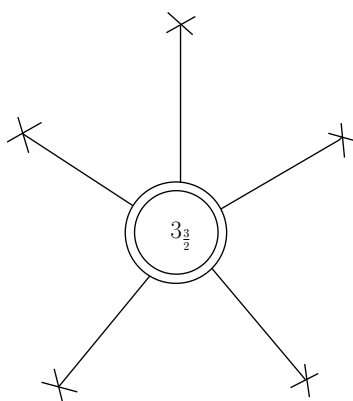
Figure 5. The theory of five copies of T_x with $SU(3)/\mathbb{Z}_3$ gauged at level $k = 3/2$, dual to a WZ model of five chiral fields.
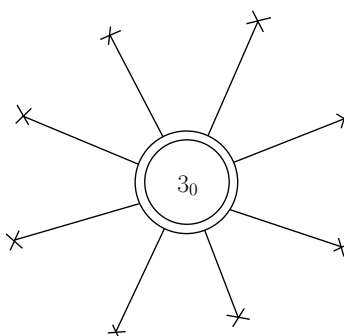

Figure 6. The theory of eight copies of T_x with $SU(3)/\mathbb{Z}_3$ gauged at level zero, dual to a WZ model with fourteen chiral fields.

cubic superpotential,

$$W = \sum_{i,j,l=1}^{14} \lambda_{ijl} \Phi_i \Phi_j \Phi_l \,. \tag{3.5}$$

The index is

$$\frac{1}{6} \sum_{l=0}^{2} g^l \sum_{m_1,m_2 \in \mathbb{Z}+\frac{l}{3}} \oint \frac{dz_1}{2\pi i z_1} \frac{dz_2}{2\pi i z_2} \prod_{i \neq j} q^{-\frac{|m_i - m_j|}{4}}$$

$$\cdot \left(1 - (-1)^{m_i - m_j} q^{\frac{|m_i - m_j|}{2}} \frac{z_i}{z_j} \right) \mathcal{I}_x(z_i; m_i)^8 = \underline{\mathcal{I}} \left(g^{-1}; 0; \frac{2}{3} \right)^{14} \,. \tag{3.6}$$

This suggests that gauging eight copies of T_x with $SU(3)/\mathbb{Z}_3$ at level zero is dual to a Wess-Zumino model of fourteen chiral fields. As in the above example here again the claim of the conjecture is that the two models flow to the same conformal manifold.

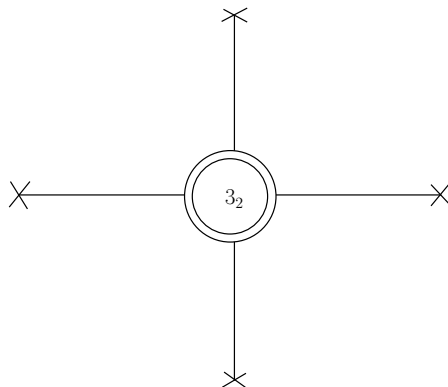In this case the $\mathbb{S}^3$ partition function is converging fast enough, so we can evaluate it

Figure 7. Model of four copies of T_x with $\mathrm{SU}(3)/\mathbb{Z}_3$ gauged at level two, dual to T_x itself.

in both dual frames. The $\mathbb{S}^3$ partition function of T_x is

$$
Z_x(m_1, m_2) = \int_{-\infty}^{\infty} d\sigma e^{2\pi i \sigma(-\frac{3}{2}m_1)} e^{l(\frac{2}{3} + i\sigma - i\frac{1}{2}m_1 - im_2) + l(\frac{2}{3} + i\sigma + i\frac{1}{2}m_1 + im_2)} . \tag{3.7}
$$

Here $l(m)$ is the l-function of Jafferis [39] and m_i are real masses for the $\mathrm{SU}(3)$ symmetry. The duality implies that

$$
\frac{1}{2} \int_{-\infty}^{\infty} d\sigma_1 \int_{-\infty}^{\infty} d\sigma_2 \prod_{i \neq j} |2\sinh\pi(\sigma_i - \sigma_j)| Z_x(\sigma_1, \sigma_2)^8 = \left[e^{l(\frac{1}{3})} \right]^{14} . \tag{3.8}
$$

Note that the half in front of the integral is $1/3!$, the dimension of the Weyl group, times 3, coming from the fact that the gauging is of $\mathrm{SU}(3)/\mathbb{Z}_3$. (See [40] for similar factors in class $\mathcal{S}$ computations.) We find that the equality indeed holds, and the numerical evaluation yields 0.01706 for the first five digits. This is an independent check of the duality.

For eight and more copies of T_x glued there is no need for Chern-Simons terms for the partition functions to converge; thus we can assume that these describe SCFT's and we identified the R-symmetry correctly. The index of these models with s copies of T_x and $\mathrm{SU}(3)/\mathbb{Z}_3$ gauged is given by, for $s = 8, 10, 12, 14, 16$,

$$
1 + C_{\frac{s}{2}} q^{\frac{s/2-3}{3}} + \cdots . \tag{3.9}
$$

Here C_n is the n-th Catalan number, $(2n)!/((n+1)!n!)$. The operators contributing to the leading order are gauge-invariant dressed monopole operators. As we have just seen, in the $s = 8$ case we have a dual description where the basic operators come from chiral fields and it is interesting to understand whether there are duals for higher values of s such that the basic operators do not come from monopoles, a question we leave for future investigation. For higher values of s the index starts with q^2. We can therefore deduce that there are states associated with monopole operators which are counted by the Catalan number, and at q^2 other states appear. The number of new states at q^2, which for high enough s is all the states, is $s(s-3)/2$.

3.4 Four glued T_x dual to T_x

We consider gauging a diagonal $SU(3)/\mathbb{Z}_3$ symmetry of four copies of T_x at level two. The index is equal to the (unrefined) index of a single T_x:

$$
\frac{1}{6}\sum_{l=0}^{2} g^l \sum_{m_1,m_2 \in \mathbb{Z}+\frac{l}{3}} \oint \frac{dz_1}{2\pi i z_1}\frac{dz_2}{2\pi i z_2} \prod_{i\neq j} q^{-\frac{|m_i-m_j|}{4}}\left(1-(-1)^{m_i-m_j}q^{\frac{|m_i-m_j|}{2}}\frac{z_i}{z_j}\right)
$$

$$
\cdot (z_1^{2m_1+m_2} z_2^{2m_2+m_1})^2\, \mathcal{I}_x(z_i;m_i)^4 = \mathcal{I}_x(1;0)\,. \tag{3.10}
$$

Note that this turns out to be independent of g, and the same as the index for a gauged $SU(3)$. This suggests that gauging four copies of T_x with $SU(3)/\mathbb{Z}_3$ at level 2 is dual to T_x. The model has a $\mathbb{Z}_3$ symmetry which is identified with the center of the $SU(3)$ symmetry of T_x. Given that only representations with zero N-ality appear, we do not observe it in the computation. From the above equality of the indices we deduce that the contribution to the index of the gauge theory side at order q is -8. This implies [24, 25] that the IR fixed point has a flavor symmetry of dimension eight which is consistent with the flavor symmetry enhancing to $SU(3)$ in the IR. In fact, assuming that we have identified the R-symmetry correctly (to which we do not observe any counter-evidence), this is a proof that symmetry enhances.

Acknowledgments

A U(1) SQED with monopole superpotential dual of T_x with manifest symmetry and supersymmetry

We start with a $U(1)$ theory at Chern-Simons level $-5/2$ and three chiral fields with charge one. We give R-charge zero to the chiral fields for concreteness. The theory has $SU(3)$ symmetry rotating the fields and a topological $U(1)$. According to [14] this model is dual to a $U(3)$ gauge theory with three fundamental flavors and level $5/2$ Chern-Simons term. The R-charge of the fields is one. We also have a level-one relative contact term for the topological symmetry, a level-one relative Chern-Simons term for $SU(3)$, and a level-3/2 relative mixed Chern-Simons term for the topological $U(1)$ and the R-symmetry. We put all the contact terms on the $U(1)$ side of the duality; the duality implies e.g. that the index of the $U(3)_{5/2}$ theory with three fundamental fields is equal to

$$
w^{-n} q^{-\frac{3}{4}n} \sum_{m\in\mathbb{Z}} w^m \oint \frac{dz}{2\pi i z} z^n z^{-\frac{5}{2}m} \prod_{j=1}^{3} \underline{\mathcal{I}}(zz_j;m;0)\,. \tag{A.1}
$$

We have turned on a magnetic flux n for the topological symmetry. Now we can gauge the topological U(1) symmetry. Since on the U(3) side we do not have any contact terms, we will obtain an SU(3) gauge theory at level $5/2$ and three fundamental chiral fields. On the dual side we perform the analysis using the index. The gauging of the topological symmetry gives

$$\sum_{n \in \mathbb{Z}} c^n \oint \frac{dw}{2\pi i w} \left[w^{-n} q^{-\frac{3}{4} n} \sum_{m \in \mathbb{Z}} w^m \oint \frac{dz}{2\pi i z} z^n z^{-\frac{5}{2} m} \prod_{j=1}^{3} \underline{\mathcal{I}}(z z_j; m; 0) \right]$$
$$= \sum_{n \in \mathbb{Z}} c^n q^{-\frac{3}{4} n} \oint \frac{dz}{2\pi i z} z^{-\frac{3}{2} n} \prod_{j=1}^{3} \underline{\mathcal{I}}(z z_j; n; 0) . \tag{A.2}$$

Here we performed the integral over w which identified the flux of the original gauge symmetry m with the flux of the topological symmetry n. The fugacity c is for the topological symmetry of the new U(1) gauge symmetry. This is dual to $-1/3$ the baryonic symmetry on the SU(3) gauge theory side. We can evaluate this index to be

$$1 - 9q + (c^{-1} - 10c)q^{\frac{3}{2}} - 18c q^{\frac{5}{2}} + (44 + c^{-2})q^3 + \cdots . \tag{A.3}$$

Here the 9 at order q is the conserved current for the baryonic U(1) and for the SU(3). The term with weight c^{-1} at order $q^{\frac{3}{2}}$ is the baryon on the SU(3) side. Remember that the R-charge of the quarks on the SU(3) side is one and thus this is precisely how the baryon contributes. On the U(1) side as this state is charged under the topological symmetry it comes from the monopoles. Now we need to turn on the baryonic superpotential. This amounts on the U(1) side to a monopole superpotential. In the index we need to set $c^{-1} q^{\frac{3}{2}}$ to q. The index then becomes,

$$\sum_{n \in \mathbb{Z}} q^{-\frac{1}{4} n} \oint \frac{dz}{2\pi i z} z^{-\frac{3}{2} n} \prod_{j=1}^{3} \underline{\mathcal{I}}(z z_j; n; 0) = \sum_{n \in \mathbb{Z}} \oint \frac{dz}{2\pi i z} z^{-\frac{3}{2} n} \prod_{j=1}^{3} \underline{\mathcal{I}} \left(z z_j; n; -\frac{1}{3} \right) . \tag{A.4}$$

This is just the index of a U(1) gauge theory at level $-3/2$ plus three charge-one fields with R-charge $-1/3$, and a monopole superpotential. The claim is that the latter is dual to an SU(3) model at level $5/2$ with three fundamental fields with R-charge one. This is nothing but T_x up to contact terms. The index above is

$$1 - 8q - 9q^2 + 18q^3 + 46q^4 + \cdots . \tag{A.5}$$

This agrees with the computations done in the T_x model in an expansion in fugacities. Computing the index, refined with the fugacities and magnetic fluxes for the global symmetry, we can deduce that the model discussed here has a level minus one contact term for the SU(3) symmetry relative to T_x. We can refine the index with fluxes for SU(3) and discover that there is a relative Chern-Simons contact term for that symmetry at level one.

The U(1) description with the monopole superpotential is simple enough to allow for a numerical evaluation of the $\mathbb{S}^3$ partition function. We have checked extensively that the

latter agrees with that of T_x as a function of the real mass parameters for the SU(3) flavor symmetry. The precise equality is

$$Z_x(m_1, m_2) = e^{2\pi i(m_1^2 + m_2^2 + m_1 m_2)} e^{\frac{\pi i 19}{12}} \int_{-\infty}^{\infty} d\sigma e^{-\frac{3}{2}\pi i(\sigma + i\epsilon)^2} e^{\pi(\sigma + i\epsilon)} e^{\sum_{j=1}^{3} l(1 + i(\sigma + i\epsilon + m_j))}.$$
(A.6)

Here Z_x is the partition function of T_x which was given in (3.7). The parameter ϵ is an arbitrarily small positive real number which lifts the contour of integration slightly above the real axis in the complex plane. This is necessary to avoid poles whenever $\sigma = -m_i$. The parameters m_i are real masses for SU(3) and satisfy $\sum_{l=1}^{3} m_l = 0$. The two exponentials in front of the integral are contact terms, with the first being at level one for the SU(3) symmetry and second for the R-symmetry. We also stress that the equality of the partition function of the description we discuss here and description C is a mathematical identity following from [30].[8] Therefore checking its equality with T_x is equivalent to checking the equality of description C with T_x.

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[8]In fact the equality above itself follows from eq. (5.6.18) in [30], which was given a physical interpretation in [41] (see also [42]).

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