Chiral Effective Theory of Dark Matter Direct Detection

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Abstract

We present the effective field theory for dark matter interactions with the visible sector that is valid at scales of $\mathcal{O}(1\,\text{GeV})$. Starting with an effective theory describing the interactions of fermionic and scalar dark matter with quarks, gluons and photons via higher dimension operators that would arise from dimension-five and dimension-six operators above electroweak scale, we perform a nonperturbative matching onto a heavy baryon chiral perturbation theory that describes dark matter interactions with light mesons and nucleons. This is then used to obtain the coefficients of the nuclear response functions using a chiral effective theory description of nuclear forces. Our results consistently keep the leading contributions in chiral counting for each of the initial Wilson coefficients.

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I. INTRODUCTION

Dark Matter (DM) scattering in direct detection lends itself well to an Effective Field Theory (EFT) description [1–17]. DM scattering on nuclei can be taken to be nonrelativistic,

since, in order to be gravitationally bound in the DM halo, the DM velocity needs to be below about 600 km/s. The typical DM velocity in the halo is thus $|\vec{v}_\chi| \sim 10^{-3}$. The maximal recoil momentum transfer depends on the reduced mass of the DM-nucleus system and on the range of recoil energies, E_R, that the experiments are measuring. The recoil energy is typically kept in the range of a few keV to few tens of keV, while the heaviest nuclei have masses of $m_A \sim 100$ GeV. This gives a maximal momentum transfer of

$$q_{\max} \lesssim 200 \text{ MeV.} \tag{1}$$

This is also a typical size of the momenta exchanged between the nucleons bound inside the nucleus. The maximal recoil momentum is much smaller than the proton and neutron masses, $q \ll m_N$, so that the nucleons remain nonrelativistic also after scattering and the nucleus does not break apart. One can then use the chiral EFT (ChEFT) approach to nuclear forces to organize different terms using an expansion in $q/\Lambda_{\text{ChEFT}} \sim m_\pi/\Lambda_{\text{ChEFT}} \sim 0.3$.

In this paper we perform such a systematic treatment of DM direct detection. We start from an EFT that describes couplings of DM to quarks, gluons and photons through higher dimension operators, keeping only the terms that would arise from dimension-five and dimension-six operators above the electroweak scale. We then match nonperturbatively onto a theory that describes DM interactions with light mesons, i.e., Chiral Perturbation Theory (ChPT) with DM, and to a theory that also includes DM interactions with protons and neutrons, i.e., Heavy Baryon Chiral Perturbation Theory (HBChPT). A single insertion of DM interaction with either a light meson, or with a nucleon, then induces the scattering of DM on the nucleus. We are able to compare the parametric sizes of different contributions by using chiral counting within ChEFT of nuclear forces. We keep the leading contributions in chiral counting and calculate the resulting coefficients that multiply the nuclear response functions of Ref. [2, 6], treating q^2 as an external parameter.

The EFT description of DM – nucleus scattering is valid if the mediators between the DM and the visible sector are heavier than $\mathcal{O}(1\text{GeV})$, and therefore covers a wide range of UV-complete theories of DM. Our expressions extend previous results on direct detection scattering rates. We cover both fermionic and scalar DM, systematically keeping the leading terms in chiral counting. Special care is needed, for instance, in the evaluation of the product of axial-vector DM and vector quark currents, as well as the product of vector DM and axial-vector quark currents. These products vanish in the long wavelength limit where both

the relative velocity between DM and nucleus, Δv, and the momentum exchange, q, are becoming arbitrarily small ($\Delta v, q \to 0$). The leading contributions thus follow from higher orders in a derivative expansion of the interactions.

The chiral counting also allows for a systematic assignment of uncertainties on the predictions. Since we restrict the analysis to the leading order in chiral counting, the errors on the predictions are expected to be of $\mathcal{O}(30\%)$. Furthermore, we use the chiral counting to discuss higher-order corrections in the direct detection rates. The short-distance scattering on two nucleons is, for instance, suppressed by $\mathcal{O}(q^3)$ compared to the scattering on a single nucleon. However, the long-distance corrections due to DM scattering on a pion exchanged between two nucleons can already start at $\mathcal{O}(q)$ [3, 9].

This paper is organized as follows. In Sections II-IV we focus on fermionic DM, while we give the results for scalar DM in Section V. In Section II we first introduce the EFT for DM coupling to quarks, gluons and photons through higher dimension operators. We treat the DM mass as heavy, $m_\chi \gg q$, leading to a Heavy Dark Matter Effective Theory (HDMET). The DM interactions with mesons and nucleons are constructed in Section III, while Section IV contains the calculation of the form factors for the nuclear response functions. The analysis is repeated for scalar DM in Section V. We draw our conclusions in Section VI. In Appendix A we give the translation of our results to the basis of Ref. [2, 6], while in Appendix C we provide the values of the required low-energy constants. Appendix B contains further details on DM interactions with mesons and nucleons.

II. NONRELATIVISTIC DARK MATTER INTERACTIONS

We first focus on fermionic DM and its interactions with quarks, gluons and photons at the scale $\mu \sim 1$ GeV. These interactions are generated by mediators that couple to both the DM and the visible sector. The DM interactions can be described by an EFT as long as the mediators are much heavier than $\mathcal{O}(1\text{GeV})$,

$$\mathcal{L}_\chi = \sum_{a,d} \hat{\mathcal{C}}_a^{(d)} \mathcal{Q}_a^{(d)}, \qquad \text{where} \quad \hat{\mathcal{C}}_a^{(d)} = \frac{\mathcal{C}_a^{(d)}}{\Lambda^{d-4}}. \tag{2}$$

Here, the $\mathcal{C}_a^{(d)}$ are dimensionless Wilson coefficients, while Λ can be identified with the mediator mass. For later convenience of notation we also introduced dimensionful Wilson coefficients, $\hat{\mathcal{C}}_a^{(d)}$. In our analysis we only keep those operators that would arise from

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dimension-five and dimension-six operators above the scale of electroweak symmetry breaking [18].

We first consider the case where DM is relativistic. There are two dimension-five operators,

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi)F_{\mu\nu}\,, \tag{3}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. The magnetic dipole operator $\mathcal{Q}_1^{(5)}$ is CP even, while the electric dipole operator $\mathcal{Q}_2^{(5)}$ is CP odd. The dimension-six operators are

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q), \qquad\qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q), \tag{4}$$

$$\mathcal{Q}_{3,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,, \qquad\qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,, \tag{5}$$

and we also include a subset of the dimension-seven operators, namely

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \tag{6}$$

$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \tag{7}$$

$$\mathcal{Q}_{5,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}q)\,, \qquad\qquad \mathcal{Q}_{6,q}^{(7)} = m_q(\bar{\chi}i\gamma_5\chi)(\bar{q}q)\,, \tag{8}$$

$$\mathcal{Q}_{7,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}i\gamma_5 q)\,, \qquad\qquad \mathcal{Q}_{8,q}^{(7)} = m_q(\bar{\chi}\gamma_5\chi)(\bar{q}\gamma_5 q)\,. \tag{9}$$

Here, $q = u, d, s$ denote the light quarks (we limit ourselves to flavor conserving operators), $G_{\mu\nu}^a$ is the QCD field strength tensor, while $\widetilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ is its dual, and $a = 1,\ldots,8$ are the adjoint color indices. The strong coupling constant α_s is taken at $\mu \sim 1$ GeV. We also assumed that DM is a Dirac fermion in the expressions above. However, our results will also apply for a Majorana fermion DM with the exception that the operators $\mathcal{Q}_{1,2}^{(5)}$ and $Q_{1,q;3,q}^{(6)}$ vanish in this case, and with straightforward modifications in the matching onto the nonrelativistic theory, see Appendix D. Matching the UV theory to the EFT may then require the inclusion of higher dimension operators which is beyond the scope of the present paper. In (8), (9) we included a factor of quark mass, m_q, in the definitions of the operators because it arises from the flavor structure of many of the models of DM. In our analysis we keep the operators involving scalar currents of the form $m_q(\bar{\chi}\chi)(\bar{q}q)$, but not those involving tensor currents, such as $m_q(\bar{\chi}\sigma_{\mu\nu}\chi)(\bar{q}\sigma^{\mu\nu}q)$, etc. The former can arise from the dimension-five UV operator $(\bar{\chi}\chi)H^\dagger H$ by integrating out the Higgs at the electroweak scale, see [18]. The latter requires a dimension-seven operator in the UV, such as $(\bar{\chi}\sigma_{\mu\nu}\chi)(\bar{Q}_L\sigma^{\mu\nu}u_R)H$.

The DM in the galactic halo is nonrelativistic with a typical velocity $|\vec{v}_\chi| \sim 10^{-3}$ so that the momenta exchanges are much smaller than the DM mass, $q \ll m_\chi$. DM scattering in direct detection experiments is thus described by a Heavy Dark Matter Effective Theory (HDMET) in which the DM mass is integrated out [12, 18, 19], giving an expansion in $1/m_\chi$. The leading term in the Lagrangian then describes the motion of DM in the limit of infinite DM mass. To derive it we factor out of the DM field χ the large momenta due to the propagation of the heavy DM mass, defining (here χ is a Dirac fermion, for Majorana fermions see Appendix D)

$$\chi(x) = e^{-im_\chi v \cdot x}\big(\chi_v(x) + X_v(x)\big), \tag{10}$$

where

$$\chi_v(x) = e^{im_\chi v \cdot x}\frac{1 + \slashed{v}}{2}\chi(x), \qquad X_v(x) = e^{im_\chi v \cdot x}\frac{1 - \slashed{v}}{2}\chi(x). \tag{11}$$

This defines the heavy-particle field $\chi_v(x)$ in analogy to the heavy quark field in Heavy Quark Effective Theory [20–23]. The remaining x dependence is due to the soft momenta. For instance, direct detection scattering changes the soft momentum of the DM by q but does not change the DM velocity label v. The velocity label v^μ can be identified with either the incoming or outgoing DM velocity four-vector, or any other velocity four-vector that is nonrelativistically close to these two. In the following section we will identify v^μ with the lab frame velocity so that $v^\mu = (1, \vec{0})$; but, for now, we leave it in its four-vector form.

The "small-component" field X_v describes the antiparticle modes. To excite an antiparticle mode requires the absorption of a hard momentum of order $\mathcal{O}(2m_\chi)$. In building HDMET the antiparticle modes are integrated out, giving the tree-level relation [20]

$$\chi = e^{-im_\chi v \cdot x}\left(1 + \frac{i\slashed{\partial}_\perp}{iv \cdot \partial + 2m_\chi - i\epsilon}\right)\chi_v, \tag{12}$$

where $\gamma_\perp^\mu = \gamma^\mu - v^\mu\slashed{v}$. The HDMET Lagrangian is thus given by

$$\mathcal{L}_{\text{HDMET}} = \bar{\chi}_v(iv \cdot \partial)\chi_v + \frac{1}{2m_\chi}\bar{\chi}_v(i\partial_\perp)^2\chi_v + \cdots + \mathcal{L}_{\chi_v}. \tag{13}$$

The first term is the leading-order (LO) HDMET Lagrangian and contains no explicit dependence on m_χ. The coefficient of the $\mathcal{O}(1/m_\chi)$ term is fixed by reparametrization invariance [24], and the ellipsis denotes higher-order terms. The effective Lagrangian $\mathcal{L}_{\chi_v}$ gives the interactions of DM with the SM. The expansion in powers of $1/m_\chi$ and $1/\Lambda$ can be made

explicit by defining

$$\mathcal{L}_{\chi_v} = \sum_{d,m} \hat{\mathcal{C}}_a^{(d,m)} \mathcal{Q}_a^{(d,m)}, \qquad \text{where} \qquad \hat{\mathcal{C}}_a^{(d,m)} = \frac{\mathcal{C}_a^{(d,m)}}{\Lambda^{d-m-4} m_\chi^m} \,. \tag{14}$$

Here, the operators $\mathcal{Q}_a^{(d,m)}$ arise as the terms of order $1/m_\chi^m$ in the HDMET expansion of the UV operators $\mathcal{Q}_a^{(d)}$. For instance, we have (neglecting radiative corrections to the matching conditions)

$$\bar{\chi}\chi \to \bar{\chi}_v\chi_v + \cdots , \tag{15}$$

$$\bar{\chi}i\gamma_5\chi \to \frac{1}{m_\chi}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \cdots , \tag{16}$$

$$\bar{\chi}\gamma^\mu\chi \to v^\mu\bar{\chi}_v\chi_v + \frac{1}{2m_\chi}\bar{\chi}_v i\overset{\leftrightarrow}{\partial}{}_\perp^\mu\chi_v + \frac{1}{2m_\chi}\partial_\nu\big(\bar{\chi}_v\sigma_\perp^{\mu\nu}\chi_v\big) + \cdots , \tag{17}$$

$$\bar{\chi}\gamma^\mu\gamma_5\chi \to 2\bar{\chi}_v S_\chi^\mu\chi_v - \frac{i}{m_\chi}v^\mu\bar{\chi}_v S_\chi\cdot\overset{\leftrightarrow}{\partial}\chi_v + \cdots , \tag{18}$$

$$\bar{\chi}\sigma^{\mu\nu}\chi \to \bar{\chi}_v\sigma_\perp^{\mu\nu}\chi_v + \frac{1}{2m_\chi}\Big(\bar{\chi}_v iv^{[\mu}\sigma_\perp^{\nu]\rho}\overset{\leftrightarrow}{\partial}_\rho\chi_v - v^{[\mu}\partial^{\nu]}\bar{\chi}_v\chi_v\Big) + \cdots , \tag{19}$$

$$\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi \to 2\bar{\chi}_v S_\chi^{[\mu}v^{\nu]}\chi_v + \cdots , \tag{20}$$

where $\sigma_\perp^{\mu\nu} = i[\gamma_\perp^\mu, \gamma_\perp^\nu]/2$, $\bar{\chi}_v\overset{\leftrightarrow}{\partial}{}^\mu\chi_v = \bar{\chi}_v(\partial^\mu\chi_v) - (\partial^\mu\bar{\chi}_v)\chi_v$, and $S^\mu = \gamma_\perp^\mu\gamma_5/2$ is the spin operator. The square brackets in the last line denote antisymmetrization in the enclosed indices, while the ellipses denote higher orders in $1/m_\chi$.

We group the operators in HDMET in terms of their $d - m$ values and only display those $1/m_\chi$-suppressed operators that will be needed to obtain all LO terms in chiral EFT description of DM scattering on nuclei. The two dimension-five operators in (3) get replaced by the HDMET operators

$$\mathcal{Q}_1^{(5,0)} = \frac{e}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}(\bar{\chi}_v S_\chi^\alpha v^\beta\chi_v)F^{\mu\nu}, \qquad\qquad \mathcal{Q}_2^{(5,0)} = \frac{e}{2\pi^2}(\bar{\chi}_v S_\chi^\mu v^\nu\chi_v)F_{\mu\nu}, \tag{21}$$

$$\mathcal{Q}_1^{(6,1)} = \frac{ie}{8\pi^2}\big(\bar{\chi}_v v^\mu\sigma_\perp^{\nu\rho}\overset{\leftrightarrow}{\partial}_\rho\chi_v\big)F_{\mu\nu}, \qquad\qquad \mathcal{Q}_2^{(6,1)} = -\frac{e}{8\pi^2}\big(v^\mu\partial^\nu\bar{\chi}_v\chi_v\big)F_{\mu\nu}. \tag{22}$$

We used the relation

$$\bar{\chi}_v\sigma_\perp^{\mu\nu}\chi_v = -2\epsilon^{\mu\nu\alpha\beta}v_\alpha\big(\bar{\chi}_v S_{\chi,\beta}\chi_v\big), \tag{23}$$

where $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor, with $\epsilon^{0123} = 1$. If the matching from the UV theory of DM interactions is done at tree level at $\mu \sim m_\chi$, we have the following relations [18]

$$\mathcal{C}_1^{(5)} \overset{\text{tree}}{=} \mathcal{C}_1^{(5,0)} \overset{\text{tree}}{=} \mathcal{C}_1^{(6,1)} \overset{\text{tree}}{=} \mathcal{C}_1^{(6,1)}, \qquad \mathcal{C}_2^{(5)} \overset{\text{tree}}{=} \mathcal{C}_2^{(5,0)}, \tag{24}$$

so that below $\mu \sim m_\chi$ the operators always appear in the combination

$$\mathcal{Q}_1^{(5,0)} + \frac{1}{m_\chi}\left(\mathcal{Q}_1^{(6,1)} + \mathcal{Q}_2^{(6,1)}\right) + \cdots . \tag{25}$$

The relations (24) would receive corrections if the matching is performed at loop level. Note that in our analysis we will not need the $1/m_\chi$ corrections to the CP odd operator $\mathcal{Q}_2^{(5,0)}$.

The dimension-six operators to LO in $1/m_\chi$ are

$$\mathcal{Q}_{1,q}^{(6,0)} = (\bar{\chi}_v \chi_v)(\bar{q}\slashed{v}q), \qquad\qquad \mathcal{Q}_{2,q}^{(6,0)} = 2(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{q}\gamma^\mu q), \tag{26}$$

$$\mathcal{Q}_{3,q}^{(6,0)} = (\bar{\chi}_v \chi_v)(\bar{q}\slashed{v}\gamma_5 q), \qquad\qquad \mathcal{Q}_{4,q}^{(6,0)} = 2(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{q}\gamma^\mu \gamma_5 q). \tag{27}$$

The $1/m_\chi$-suppressed operators that we need to consider[1] are

$$\mathcal{Q}_{1,q}^{(7,1)} = \frac{1}{2}(\bar{\chi}_v i\overset{\leftrightarrow}{\partial}{}_\perp^\mu \chi_v)(\bar{q}\gamma_\mu q), \qquad\qquad \mathcal{Q}_{2,q}^{(7,1)} = -i(\bar{\chi}_v S_\chi \cdot \overset{\leftrightarrow}{\partial} \chi_v)(\bar{q}\slashed{v}q), \tag{28}$$

$$\mathcal{Q}_{3,q}^{(7,1)} = \frac{1}{2}(\bar{\chi}_v i\overset{\leftrightarrow}{\partial}{}_\perp^\mu \chi_v)(\bar{q}\gamma_\mu \gamma_5 q), \qquad\qquad \mathcal{Q}_{4,q}^{(7,1)} = -i(\bar{\chi}_v S_\chi \cdot \overset{\leftrightarrow}{\partial} \chi_v)(\bar{q}\slashed{v}\gamma_5 q), \tag{29}$$

$$\mathcal{Q}_{5,q}^{(7,1)} = \frac{1}{2}\partial_\nu(\bar{\chi}_v \sigma_\perp^{\mu\nu} \chi_v)(\bar{q}\gamma_\mu q), \qquad\qquad \mathcal{Q}_{6,q}^{(7,1)} = \frac{1}{2}\partial_\nu(\bar{\chi}_v \sigma_\perp^{\mu\nu} \chi_v)(\bar{q}\gamma_\mu \gamma_5 q), \tag{30}$$

where our convention is that the derivatives act only within the brackets or on the nearest bracket. For matching from the UV theory at scale $\mu \sim m_\chi$, we would have the following relations

$$\mathcal{C}_{i,q}^{(6)} \overset{\text{tree}}{=} \mathcal{C}_{i,q}^{(6,0)} = \mathcal{C}_{i,q}^{(7,1)} \overset{\text{tree}}{=} \mathcal{C}_{(i+9)/2,q}^{(7,1)}, \quad i = 1,3; \qquad \mathcal{C}_{i,q}^{(6)} \overset{\text{tree}}{=} \mathcal{C}_{i,q}^{(6,0)} = \mathcal{C}_{i,q}^{(7,1)}, \quad i = 2,4. \tag{31}$$

Note that the equality denoted by "tree" is only valid for tree-level matching, while the remaining relations are valid to all orders due to reparametrization invariance, cf. Eqs. (B10) and (B11). Hence, in the EFT below $\mu \sim m_\chi$, the following linear combinations of operators would appear with the same coefficient,

$$\begin{gathered} \mathcal{Q}_{1,q}^{(6,0)} + \frac{1}{m_\chi}\left(\mathcal{Q}_{1,q}^{(7,1)} + \mathcal{Q}_{5,q}^{(7,1)}\right) + \cdots, \qquad \mathcal{Q}_{2,q}^{(6,0)} + \frac{1}{m_\chi}\mathcal{Q}_{2,q}^{(7,1)} + \cdots, \\ \mathcal{Q}_{3,q}^{(6,0)} + \frac{1}{m_\chi}\left(\mathcal{Q}_{3,q}^{(7,1)} + \mathcal{Q}_{6,q}^{(7,1)}\right) + \cdots, \qquad \mathcal{Q}_{4,q}^{(6,0)} + \frac{1}{m_\chi}\mathcal{Q}_{4,q}^{(7,1)} + \cdots, \end{gathered} \tag{32}$$

with the ellipses denoting higher-order terms. Note that the coefficient in front of $\mathcal{Q}_{5,q}^{(7,1)}$ and $\mathcal{Q}_{6,q}^{(7,1)}$ in the two sums can differ from unity at loop level in the matching.

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[1] In fact only the operators in (29) and (30) will enter the phenomenological analysis but we keep the other operators for completeness and transparency of notation.

The relevant dimension-seven operators (6)-(9) involve scalar and pseudoscalar DM currents. The HDMET scalar current operator starts at $\mathcal{O}(1/m_\chi^0)$, while the pseudoscalar current starts at $\mathcal{O}(1/m_\chi)$. We thus define the HDMET operators

$$\mathcal{Q}_1^{(7,0)} = \frac{\alpha_s}{12\pi}(\bar{\chi}_v\chi_v)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad \mathcal{Q}_2^{(8,1)} = \frac{\alpha_s}{12\pi}\partial_\rho\big(\bar{\chi}_v S_\chi^\rho \chi_v\big)G^{a\mu\nu}G_{\mu\nu}^a\,, \tag{33}$$

$$\mathcal{Q}_3^{(7,0)} = \frac{\alpha_s}{8\pi}(\bar{\chi}_v\chi_v)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad \mathcal{Q}_4^{(8,1)} = \frac{\alpha_s}{8\pi}\partial_\rho\big(\bar{\chi}_v S_\chi^\rho \chi_v\big)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \tag{34}$$

$$\mathcal{Q}_{5,q}^{(7,0)} = m_q(\bar{\chi}_v\chi_v)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(8,1)} = m_q\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big)(\bar{q}q)\,, \tag{35}$$

$$\mathcal{Q}_{7,q}^{(7,0)} = m_q(\bar{\chi}_v\chi_v)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(8,1)} = -m_q\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big)(\bar{q}i\gamma_5 q)\,, \tag{36}$$

so that we have the following tree-level matching conditions

$$\mathcal{C}_i^{(7)} \overset{\text{tree}}{=} \mathcal{C}_i^{(7,0)}\,, \quad i=1,3,5,7\,; \qquad \mathcal{C}_i^{(7)} \overset{\text{tree}}{=} \mathcal{C}_i^{(8,1)}\,, \quad i=2,4,6,8\,. \tag{37}$$

III. DARK MATTER INTERACTIONS WITH MESONS AND NUCLEONS

A. QCD with external currents

As far as QCD interactions are concerned the DM currents can be viewed as classical external fields. The quark level DM-SM interaction Lagrangian can thus be written in a form familiar from the ChPT literature [25] as

$$\begin{aligned}
\mathcal{L} = {}& \mathcal{L}_{\text{QCD}}^0 + s_G(x)\frac{\alpha_s}{12\pi}G_{\mu\nu}^a G^{a\mu\nu} + \theta(x)\frac{\alpha_s}{8\pi}G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \\
&+ \bar{q}(x)\gamma^\mu\big[\nu_\mu(x) + \gamma_5 a_\mu(x)\big]q(x) - \bar{q}(x)\big[s(x) - i\gamma_5 p(x)\big]q(x),
\end{aligned} \tag{38}$$

where $q = (u,d,s)$ is a vector of light quark fields. Here $\mathcal{L}_{\text{QCD}}^0$ is the QCD+QED Lagrangian in the limit of zero quark masses and no interactions with DM. We treat the quark masses and insertions of DM currents as perturbations. They are collected in six spurions which, for relativistic DM (4)-(9), are given by

$$\nu_\mu(x) = -e\bar{Q}_q A_\mu^e + \nu_{\chi,\mu} = -e\bar{Q}_q A_\mu^e + \bar{\mathcal{C}}_1^{(6)}\big(\bar{\chi}\gamma_\mu\chi\big) + \bar{\mathcal{C}}_2^{(6)}\big(\bar{\chi}\gamma_\mu\gamma_5\chi\big), \tag{39}$$

$$a_\mu(x) = \bar{\mathcal{C}}_3^{(6)}\big(\bar{\chi}\gamma_\mu\chi\big) + \bar{\mathcal{C}}_4^{(6)}\big(\bar{\chi}\gamma_\mu\gamma_5\chi\big), \tag{40}$$

$$s(x) = \mathcal{M}_q + s_\chi = \mathcal{M}_q - \mathcal{M}_q\bar{\mathcal{C}}_5^{(7)}\big(\bar{\chi}\chi\big) - \mathcal{M}_q\bar{\mathcal{C}}_6^{(7)}\big(\bar{\chi}i\gamma_5\chi\big), \tag{41}$$

$$p(x) = \mathcal{M}_q\bar{\mathcal{C}}_7^{(7)}\big(\bar{\chi}\chi\big) - \mathcal{M}_q\bar{\mathcal{C}}_8^{(7)}\big(\bar{\chi}i\gamma_5\chi\big), \tag{42}$$

$$s_G(x) = \hat{\mathcal{C}}_1^{(7)}\big(\bar{\chi}\chi\big) + \hat{\mathcal{C}}_2^{(7)}\big(\bar{\chi}i\gamma_5\chi\big), \tag{43}$$

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$$\theta(x) = \hat{\mathcal{C}}_3^{(7)}\big(\bar{\chi}\chi\big) + \hat{\mathcal{C}}_4^{(7)}\big(\bar{\chi}i\gamma_5\chi\big). \tag{44}$$

Here, we introduced 3×3 diagonal matrices of Wilson coefficients and electromagnetic charges

$$\bar{\mathcal{C}}_i^{(d)} = \text{diag}\big(\hat{\mathcal{C}}_{i,u}^{(d)}, \hat{\mathcal{C}}_{i,d}^{(d)}, \hat{\mathcal{C}}_{i,s}^{(d)}\big),$$
$$\bar{Q}_q = \text{diag}\big(Q_q\big) = \text{diag}(2/3, -1/3, -1/3). \tag{45}$$

The general HDMET expressions for spurions are somewhat lengthier,

$$\nu^\mu(x) = -e\bar{Q}_q A^{e,\mu} + \nu_\chi^\mu = -e\bar{Q}_q A^{e,\mu} + \bar{\mathcal{C}}_1^{(6,0)} v^\mu \bar{\chi}_v \chi_v + \frac{1}{2}\bar{\mathcal{C}}_1^{(7,1)}\bar{\chi}_v i\overleftrightarrow{\partial}_\perp^\mu \chi_v$$
$$+ \frac{1}{2}\bar{\mathcal{C}}_5^{(7,1)}\partial_\nu\big(\bar{\chi}\sigma_\perp^{\mu\nu}\chi_v\big) + 2\bar{\mathcal{C}}_2^{(6,0)}\bar{\chi}_v S_\chi^\mu \chi_v - i\bar{\mathcal{C}}_2^{(7,1)} v^\mu \bar{\chi}_v S_\chi \cdot \overleftrightarrow{\partial}\chi_v + \cdots, \tag{46}$$

$$a^\mu(x) = \bar{\mathcal{C}}_3^{(6,0)} v^\mu \bar{\chi}_v \chi_v + \frac{1}{2}\bar{\mathcal{C}}_3^{(7,1)}\bar{\chi}_v i\overleftrightarrow{\partial}_\perp^\mu \chi_v + \frac{1}{2}\bar{\mathcal{C}}_6^{(7,1)}\partial_\nu\big(\bar{\chi}\sigma_\perp^{\mu\nu}\chi_v\big) + 2\bar{\mathcal{C}}_4^{(6,0)}\bar{\chi}_v S_\chi^\mu \chi_v$$
$$- i\bar{\mathcal{C}}_4^{(7,1)} v^\mu \bar{\chi}_v S_\chi \cdot \overleftrightarrow{\partial}\chi_v + \cdots, \tag{47}$$

$$s(x) = \mathcal{M}_q + s_\chi = \mathcal{M}_q - \mathcal{M}_q \bar{\mathcal{C}}_5^{(7,0)}\big(\bar{\chi}_v\chi_v\big) - \mathcal{M}_q \bar{\mathcal{C}}_6^{(8,1)}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \cdots, \tag{48}$$

$$p(x) = \mathcal{M}_q \bar{\mathcal{C}}_7^{(7,0)}\big(\bar{\chi}_v\chi_v\big) - \mathcal{M}_q \bar{\mathcal{C}}_8^{(8,1)}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \cdots, \tag{49}$$

$$s_G(x) = \hat{\mathcal{C}}_1^{(7,0)}\big(\bar{\chi}_v\chi_v\big) + \hat{\mathcal{C}}_2^{(8,1)}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \cdots, \tag{50}$$

$$\theta(x) = \hat{\mathcal{C}}_3^{(7,0)}\big(\bar{\chi}_v\chi_v\big) + \hat{\mathcal{C}}_4^{(8,1)}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \cdots, \tag{51}$$

where the $\bar{\mathcal{C}}_i^{(d,m)}$ are defined in analogy to Eq. (45) and the ellipses denote higher orders in the $1/m_\chi$ expansion. The scalar spurion $s(x)$ contains the diagonal quark matrix, $\mathcal{M}_q = \text{diag}(m_q)$, as well as the DM scalar current s_χ. Similarly, the vector current contains a contribution due to quarks interacting with the QED gauge field, $e\bar{Q}_q A_\mu^e$, as well as the DM vector current $\nu_{\chi,\mu}$. All the remaining spurions vanish in the limit of vanishing DM interactions. The chiral counting of spurions is $\nu_\mu, a_\mu, s_G, \theta \sim \mathcal{O}(p^0)$, and $s, p \sim \mathcal{O}(p^2)$. However, in HDMET the contributions from the pseudoscalar DM current only start at $\mathcal{O}(p)$ in s_G, θ and at $\mathcal{O}(p^3)$ in s, p.

The QCD Lagrangian $\mathcal{L}_{\text{QCD}}^0$ exhibits a global chiral $U(3)_L \times U(3)_R$ symmetry that is spontaneously broken to the vectorial $U(3)_V$ at low energies (the anomalous $U(1)_A$ can be included because of the shift symmetry in θ, see below). The combined Lagrangian (38), composed of the spurion terms and the QCD Lagrangian, is still formally invariant under the local chiral transformations

$$q(x) \to V_R(x)\frac{1}{2}(1 + \gamma_5)q(x) + V_L(x)\frac{1}{2}(1 - \gamma_5)q(x), \tag{52}$$

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if the spurions transform simultaneously as

$$\nu_\mu + a_\mu \to V_R(\nu_\mu + a_\mu)V_R^\dagger + iV_R\partial_\mu V_R^\dagger, \tag{53}$$

$$\nu_\mu - a_\mu \to V_L(\nu_\mu - a_\mu)V_L^\dagger + iV_L\partial_\mu V_L^\dagger, \tag{54}$$

$$s + ip \to V_R(s + ip)V_L^\dagger, \tag{55}$$

$$s_G \to s_G. \tag{56}$$

The $\theta(x)$ undergoes a shift transformation such that it cancels the contribution due to the anomalous $U(1)_A$ axial part of the transformations (52). For chiral transformations $V_{L,R}(x) = \exp\big(i\alpha(x) \mp i\beta(x)\big)$ this gives [25]

$$\theta \to \theta - 2\,\mathrm{Tr}(\beta). \tag{57}$$

Since the DM currents can be viewed as classical external fields as far as the QCD interactions are concerned, we can use the $U(1)_A$ transformation with

$$\beta(x) = \frac{\theta(x)}{2}\frac{\mathcal{M}_q^{-1}}{\mathrm{Tr}(\mathcal{M}_q^{-1})}, \tag{58}$$

to eliminate the θ term in Eq. (38) and move it to the axial and pseudo-scalar currents [26]. After the transformation, the Lagrangian is given by

$$\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\mathrm{QCD}}^0 + s_G(x)\frac{\alpha_s}{12\pi}G_{\mu\nu}^a G^{a\mu\nu} + \bar{q}(x)\gamma^\mu\big[\nu_\mu(x) + \gamma_5 a_\mu'(x)\big]q(x) \\
& - \bar{q}(x)\big[s(x) - i\gamma_5 p'(x)\big]q(x),
\end{aligned} \tag{59}$$

where

$$a_\mu' = a_\mu + \frac{\partial_\mu\theta}{2}\frac{\mathcal{M}_q^{-1}}{\mathrm{Tr}(\mathcal{M}_q^{-1})}, \qquad p' = p + \frac{\theta}{\mathrm{Tr}(\mathcal{M}_q^{-1})} \tag{60}$$

where we kept only terms linear in the spurions, so that in this approximation $s' = s$. We have omitted the primes on the transformed quark fields in (59). The primed spurions a_μ' and p' obey the same transformation laws as the unprimed equivalents in (53)-(55).

B. Chiral perturbation theory for dark matter interactions

The formal invariance of $\mathcal{L}$ in (38) under the local transformations (52) constrains the allowed DM interactions with pions and nucleons. We start with the ChPT Lagrangian for DM–pion interactions which needs to be formally invariant under the transformations (52),

thus limiting the possible spurion insertions. As usual, the ChPT is organized in terms of a derivative expansion. The pseudo-Nambu-Goldstone bosons (PNGBs) are collected in the Hermitian matrix $\Pi \equiv \sum_a \lambda_a \pi_a$, given by

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \tag{61}$$

where λ_a are the Gell-Mann matrices normalized as $\mathrm{Tr}(\lambda_a \lambda_b) = \delta_{ab}$. We do not include η' in the ChPT Lagrangian due to its large mass, which therefore contributes to the DM-nucleon contact terms. We thus also ignore $\eta - \eta'$ mixing, so that $\eta_8 \simeq \eta$.

The PNGB degrees of freedom parametrize the coset space $(SU(3)_L \times SU(3)_R)/SU(3)_V$. We use the exponential parametrization of the coset space, given by the matrix $U(x)$. Under chiral transformations

$$U \to V_R U V_L^\dagger. \tag{62}$$

The U matrix is unitary, $UU^\dagger = U^\dagger U = 1$. Since the DM $\theta(x)$ current has been moved to the axial and scalar currents, it is consistent to impose the condition[2] $\det U = 1$. Thus, in our convention the U matrix is

$$U(x) = \exp\left(i\sqrt{2}\Pi/f\right), \tag{63}$$

where $f \simeq 92$ MeV equals the pion decay constant at leading order in ChPT (experimentally, we have $f_\pi = 92.21(14)$ MeV [27]). Note that under a parity transformation $\pi_a \to -\pi_a$, and thus $U \to U^\dagger$.

The ChPT Lagrangian at LO, i.e., at $\mathcal{O}(p^2)$, is given by [25]

$$\mathcal{L}_{\mathrm{ChPT}}^{(2)} = \frac{f^2}{4}\,\mathrm{Tr}\left(\nabla_\mu U^\dagger \nabla^\mu U\right) + \frac{B_0 f^2}{2}\,\mathrm{Tr}\left[(s - ip')U + (s + ip')U^\dagger\right], \tag{64}$$

where B_0 is a low-energy constant. To $\mathcal{O}(m_q)$ it is given by the quark condensate, and equals $\langle \bar{q}q \rangle \simeq -f^2 B_0$. Using quark condensate from [28] and the LO relation $f = f_\pi$ one has $B_0 = 2.666(57)$ GeV, evaluated at the scale $\mu = 2$ GeV. The covariant derivative in (64) is defined as

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a'_\mu)U + iU(v_\mu - a'_\mu), \tag{65}$$

---

[2] Alternatively, we could have worked with untransformed interaction Lagrangian (38) in which case the condition $\det U = e^{-i\theta(x)}$ would need to be imposed [25].

so that under chiral transformations

$$\nabla_\mu U \to V_R \nabla_\mu U V_L^\dagger. \tag{66}$$

Each of the terms in (64) can be multiplied by an arbitrary function of s_G.

To obtain the leading DM interactions with the pseudoscalar mesons we expand (64) up to linear order in the DM currents. The zeroth order term gives the usual LO ChPT Lagrangian

$$\mathcal{L}_{\text{ChPT}}^{(2),\text{QCD}} = \frac{f^2}{4} \operatorname{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{B_0 f^2}{2} \operatorname{Tr}\left[\mathcal{M}_q(U + U^\dagger)\right], \tag{67}$$

while the QED interactions are

$$\mathcal{L}_{\text{ChPT}}^{(2),\text{QED}} = i \frac{e f^2}{2} A^{e,\mu} \operatorname{Tr}\left[(U \partial_\mu U^\dagger + U^\dagger \partial_\mu U) \bar{Q}_q\right]. \tag{68}$$

The linear terms give the interactions of PNGBs with DM as

$$
\begin{aligned}
\mathcal{L}_{\chi,\text{ChPT}} = & -\frac{i f^2}{2} \operatorname{Tr}\left[(U \partial_\mu U^\dagger + U^\dagger \partial_\mu U)\nu_\chi^\mu + (U \partial_\mu U^\dagger - U^\dagger \partial_\mu U)a^\mu\right] \\
& + \frac{B_0 f^2}{2} \operatorname{Tr}\left[s_\chi(U + U^\dagger) - ip(U - U^\dagger) - \frac{i\theta}{\operatorname{Tr}(\mathcal{M}_q^{-1})}(U - U^\dagger)\right] + S_G(x) s_G \\
& - \frac{i f^2}{4} \frac{\partial^\mu \theta}{\operatorname{Tr}(\mathcal{M}_q^{-1})} \operatorname{Tr}\left[(U \partial_\mu U^\dagger - U^\dagger \partial_\mu U)\mathcal{M}_q^{-1}\right].
\end{aligned} \tag{69}
$$

The scalar function $S_G(x)$ multiplying s_G is chirally invariant. To fix it to quadratic order in the derivative expansion we require that the trace of the QCD energy-momentum tensor be reproduced in the chiral effective theory. The general quadratic expansion has the form

$$S_G(x) = a_1 \frac{f^2}{4} \operatorname{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + a_2 \frac{B_0 f^2}{2} \operatorname{Tr}\left[\mathcal{M}_q(U + U^\dagger)\right]. \tag{70}$$

From the trace of the QCD energy momentum tensor, given at quark level by $\theta_\mu^\mu = -\frac{9}{8\pi}\alpha_s G_{\mu\nu}^a G^{a\mu\nu} + \sum_q m_q \bar{q}q$, and at leading order in the ChPT expansion by $\theta_\mu^{\mu\,\text{eff}} = -\operatorname{Tr}(\partial_\mu \Pi \partial^\mu \Pi) + 4 B_0 \operatorname{Tr}(\mathcal{M}_q \Pi^2)$, one obtains the LO expressions for the low-energy coefficients $a_{1,2}$ [29],

$$a_1 = \frac{2}{3} a_2 = \frac{4}{27}. \tag{71}$$

Expanding (69) to first nonzero order in PNGB fields for each of the spurions gives

$$
\begin{aligned}
\mathcal{L}_{\text{ChPT}}^{\text{DM}} \supset & \, i \operatorname{Tr}\left([\partial_\mu \Pi, \Pi]\nu_\chi^\mu\right) - \sqrt{2} f \operatorname{Tr}\left(\partial^\mu \Pi\, a_\mu\right) - B_0 \operatorname{Tr}\left(s_\chi \Pi^2\right) + \sqrt{2} B_0 f \operatorname{Tr}\left(\Pi p\right) \\
& + \left[\frac{2}{27} \operatorname{Tr}\left(\partial_\mu \Pi \partial^\mu \Pi\right) - \frac{6}{27} B_0 \operatorname{Tr}\left(\mathcal{M}_q \Pi^2\right)\right] s_G + f \frac{\theta}{\sqrt{2}} \frac{\operatorname{Tr}(\partial^2 \Pi \mathcal{M}_q^{-1})}{\operatorname{Tr}(\mathcal{M}_q^{-1})} + \cdots,
\end{aligned} \tag{72}
$$

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where the ellipses denote terms with more PNGBs. The p, a_μ, and θ spurions are flavor diagonal. The corresponding traces, $\mathrm{Tr}\left(\partial^\mu \Pi\, a_\mu\right)$, $\mathrm{Tr}\left(\Pi\, p\right)$, and $\mathrm{Tr}(\partial^2 \Pi \mathcal{M}_q^{-1})$ therefore lead to couplings of DM axial and scalar currents to a single π^0 or η. In contrast, the ν_χ^μ, s_χ, s_G, and θ DM currents couple to at least two PNGBs. They thus enter the ChPT description of the DM-nucleon scattering for the first time at one-loop level.

Note that in (72) we do not display the terms that contribute to the DM mass. Due to chiral symmetry breaking the DM mass term is

$$\mathcal{L}_\chi \supset -m_\chi(\bar\chi\chi) - (\bar\chi\chi)\sum_q B_0 f^2 m_q \hat{\mathcal{C}}_{5,q}^{(7)} - (\bar\chi i\gamma_5 \chi)\sum_q B_0 f^2 m_q \hat{\mathcal{C}}_{6,q}^{(7)} + \cdots , \qquad (73)$$

where we have displayed only the corrections to DM mass, δm_χ, due to scalar-spurion contribution in Eq. (69), with ellipsis denoting similar terms due to the s_G and θ spurions. Keeping only the leading terms in $\delta m_\chi/m_\chi$, the last term in (73) can be eliminated by a small axial rotation of the DM field. The second term, however, modifies the DM mass by a term of order $\Lambda_{\mathrm{QCD}}^4/\Lambda^3$. This is a small correction for all intents and purposes. For $\Lambda \gtrsim v_{\mathrm{EW}}$ one has $\delta m_\chi \lesssim 1$ eV. Similar comments apply to corrections due to s_G and θ spurions.

The various external DM currents in $\mathcal{L}_{\chi,\mathrm{ChPT}}$ (69) have different chiral dimensions. We thus organize the DM–meson interactions in terms of their overall chiral suppression, including the derivative suppression of the DM currents when expanded in $1/m_\chi$,

$$\mathcal{L}_{\chi,\mathrm{ChPT}} = \mathcal{L}_{\chi,\mathrm{ChPT}}^{(1)} + \mathcal{L}_{\chi,\mathrm{ChPT}}^{(2)} + \mathcal{L}_{\chi,\mathrm{ChPT}}^{(3)} + \cdots . \qquad (74)$$

Keeping only the leading terms in chiral counting for each of the Wilson coefficients in (14) gives

$$\mathcal{L}_{\chi,\mathrm{ChPT}}^{(1)} = -\frac{if^2}{2}(\bar\chi_v\chi_v)v^\mu \,\mathrm{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\bar{\mathcal{C}}_1^{(6,0)} + (U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\bar{\mathcal{C}}_3^{(6,0)}\right] \\ - if^2(\bar\chi_v S_\chi^\mu \chi_v)\,\mathrm{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\bar{\mathcal{C}}_2^{(6,0)} + (U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\bar{\mathcal{C}}_4^{(6,0)}\right] , \qquad (75)$$

$$\mathcal{L}_{\chi,\mathrm{ChPT}}^{(2)} \supset -\frac{B_0 f^2}{2}(\bar\chi_v\chi_v)\,\mathrm{Tr}\left[(U+U^\dagger)\mathcal{M}_q \bar{\mathcal{C}}_5^{(7,0)} + i(U-U^\dagger)\mathcal{M}_q \bar{\mathcal{C}}_7^{(7,0)}\right] \\ + \frac{f^2}{27}(\bar\chi_v\chi_v)\left[\mathrm{Tr}\left(\partial_\mu U^\dagger\partial^\mu U\right) + 3B_0\,\mathrm{Tr}\left[\mathcal{M}_q(U+U^\dagger)\right]\right]\hat{\mathcal{C}}_1^{(7,0)} \\ - \frac{if^2}{4m_\chi}(\bar\chi_v i\overset{\leftrightarrow}{\partial}{}^\mu_\perp \chi_v)\,\mathrm{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\bar{\mathcal{C}}_1^{(6,0)} + (U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\bar{\mathcal{C}}_3^{(6,0)}\right] \\ + \frac{if^2}{4}\frac{(\bar\chi_v\chi_v)}{\mathrm{Tr}(\mathcal{M}_q^{-1})}\,\mathrm{Tr}\left[\partial^\mu(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\mathcal{M}_q^{-1}\right]\hat{\mathcal{C}}_3^{(7,0)} , \qquad (76)$$

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$$\mathcal{L}^{(3)}_{\chi,\text{ChPT}} \supset -\frac{B_0 f^2}{2}\partial_\mu(\bar{\chi}_v S^\mu_\chi \chi_v)\,\text{Tr}\left[(U+U^\dagger)\mathcal{M}_q\bar{\mathcal{C}}^{(8,1)}_6 - i(U-U^\dagger)\mathcal{M}_q\bar{\mathcal{C}}^{(8,1)}_8\right]$$
$$+\frac{f^2}{27}\partial_\nu(\bar{\chi}_v S^\nu_\chi \chi_v)\left[\text{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + 3B_0\,\text{Tr}\left[\mathcal{M}_q(U+U^\dagger)\right]\right]\hat{\mathcal{C}}^{(8,1)}_2 \qquad (77)$$
$$+\frac{if^2}{4}\frac{\partial_\nu(\bar{\chi}_v S^\nu_\chi \chi_v)}{\text{Tr}(\mathcal{M}_q^{-1})}\,\text{Tr}\left[\partial^\mu(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\mathcal{M}_q^{-1}\right]\hat{\mathcal{C}}^{(8,1)}_4\,,$$

In (76) we also kept part of the formally subleading terms proportional to $\bar{\mathcal{C}}^{(6,0)}_1$ and $\mathcal{C}^{(6,0)}_3$ because there is a cancellation with $\mathcal{L}^{(1)}_{\chi,\text{ChPT}}$ that occurs after the expansion in meson fields. The Lagrangians (75)-(77) expanded to first nonzero order in the meson fields are collected in (B14)-(B16).

C. Heavy baryon chiral perturbation theory

In order to describe the DM interactions including nucleons we use Heavy Baryon Chiral Perturbation Theory (HBChPT) [30]. This is the appropriate effective field theory as long as $q \sim m_\pi \ll m_N$, where m_N is the nucleon mass and q the typical momentum exchange. The baryon momentum can be split into

$$p^\mu = m_N v^\mu + k^\mu, \qquad (78)$$

where v^μ is the four-velocity of the nucleon, while the soft momentum $k^\mu \sim \mathcal{O}(q)$ gives the off-shellness of the nucleon. The large momentum component due to the inertia of the heavy baryon can be factored out from the dynamics. Generalizing to the baryon octet, we introduce the HBChPT baryon field

$$B_v(x) = \exp(im_N \slashed{v} v_\mu x^\mu)B(x), \qquad (79)$$

where m_N and v are the baryon mass and velocity, respectively. Some useful properties of the field B_v are $\frac{1}{2}(1+\slashed{v})B_v = B_v$, $\bar{B}_v\gamma_5 B_v = 0$, $\bar{B}_v\gamma_\mu B_v = v_\mu\bar{B}_v B_v$, and $\bar{B}_v\gamma^\mu\gamma_5 B_v = 2\bar{B}_v S^\mu_N B_v$, where S^μ_N is the spin operator satisfying

$$v\cdot S_N = 0\,, \quad S^2_N B_v = -\tfrac{3}{4}B_v\,, \quad \{S^\mu_N, S^\nu_N\} = \tfrac{1}{2}\left(v^\mu v^\nu - g^{\mu\nu}\right), \quad [S^\mu_N, S^\nu_N] = -i\epsilon^{\mu\nu\lambda\sigma}v_\lambda S_{N,\sigma}\,. \qquad (80)$$

As in HDMET, v^μ is just a label and is not changed by the QCD interactions or by DM scattering that only lead to exchanges of soft momenta of $\mathcal{O}(q)$. In the lab frame we have $v^\mu = (1, \vec{0})$.

The octet of baryons forms a 3×3 matrix

$$B_v = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & \Sigma_v^+ & p_v \\ \Sigma_v^- & -\frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & n_v \\ \Xi_v^- & \Xi_v^0 & -\frac{2}{\sqrt{6}}\Lambda_v. \end{pmatrix}. \tag{81}$$

For tree-level contributions to DM-nucleon scattering, i.e. working at LO, we need to keep only the p_v and n_v entries of the B_v matrix, while the remaining entries can be set to zero. In order to write down HBChPT it is useful to define the square root of the matrix U,

$$U(x) = \xi(x)^2. \tag{82}$$

The $\xi(x)$ transforms under chiral rotations as[3]

$$\xi(x) \to V_R(x)\xi(x)V^\dagger(x) = V(x)\xi(x)V_L^\dagger(x). \tag{83}$$

This equation defines the vector transformation $V(x)$, an element of the group $SU(3)_V$ that remains unbroken after the spontaneous breaking of the chiral $SU(3)_L \times SU(3)_R$ symmetry. From the scalar and pseudoscalar spurions appearing in (59) we can construct a quantity that transforms as an adjoint of $SU(3)_V$,

$$\xi^\dagger(s + ip')\xi^\dagger \to V(x)\xi^\dagger(s + ip')\xi^\dagger V(x)^\dagger. \tag{84}$$

A related parity-even spurion,

$$s_+ \equiv \xi^\dagger(s + ip')\xi^\dagger + \xi(s - ip')\xi, \tag{85}$$

is thus also in the adjoint of $SU(3)_V$,

$$s_+ \to V s_+ V^\dagger. \tag{86}$$

Note that s_+ contains, in addition to the DM scalar, pseudoscalar, and θ currents, a contribution from the quark masses. For later convenience we define a parity-even spurion that vanishes in the limit of zero DM currents,

$$s_+^\chi \equiv \xi^\dagger(s_\chi + ip)\xi^\dagger + \xi(s_\chi - ip)\xi + \frac{i\theta}{\text{Tr}(\mathcal{M}_q^{-1})}\left(U^\dagger - U\right) + \cdots. \tag{87}$$

---

[3] This differs from [30] and follows [25]. The convention of [30] is obtained by the replacement $\xi \to \xi^\dagger$.

The ellipses denote terms that involve more than one insertion of the DM currents. One therefore has

$$s_+ = s_+^\chi + \xi^\dagger \mathcal{M}_q \xi^\dagger + \xi \mathcal{M}_q \xi , \tag{88}$$

where the last two terms arise purely from the QCD Lagrangian.

From the a_μ' and ν_μ spurions in (59) we can form axial, A_μ, and vector, V_μ, currents that transform under chiral rotations as

$$V_\mu \to V V_\mu V^\dagger + i V \partial_\mu V^\dagger, \qquad A_\mu \to V A_\mu V^\dagger. \tag{89}$$

They are sums of DM and SM currents,

$$V_\mu = V_\mu^\chi + i V_\mu^\xi, \qquad A_\mu = A_\mu^\chi + A_\mu^\xi, \tag{90}$$

where[4]

$$V_\mu^\chi = \frac{1}{2}\big[\xi^\dagger(\nu_{\chi,\mu} + a_\mu')\xi + \xi(\nu_{\chi,\mu} - a_\mu')\xi^\dagger\big], \tag{91}$$

$$V_\mu^\xi = \frac{1}{2}\big(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\big) + i\frac{eA_\mu^e}{2}\big(\xi^\dagger\bar{Q}_q\xi + \xi\bar{Q}_q\xi^\dagger\big), \tag{92}$$

and

$$A_\mu^\chi = \frac{1}{2}\big[\xi^\dagger(\nu_{\chi,\mu} + a_\mu')\xi - \xi(\nu_{\chi,\mu} - a_\mu')\xi^\dagger\big], \tag{93}$$

$$A_\mu^\xi = \frac{i}{2}\big(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\big) - \frac{eA_\mu^e}{2}\big(\xi^\dagger\bar{Q}_q\xi - \xi\bar{Q}_q\xi^\dagger\big). \tag{94}$$

The V_μ^ξ and A_μ^ξ are pure QCD and QED currents, while the dependence on the DM currents is included in V_μ^χ and A_μ^χ.

The pNGBs can be factored out of the baryon fields B_v so that they transform as

$$B_v \to V B_v V^\dagger. \tag{95}$$

We define the covariant derivative by

$$\nabla_\mu B_v = \partial_\mu B_v - i[V_\mu, B_v] = \partial_\mu B_v - i[V_\mu^\chi, B_v] + [V_\mu^\xi, B_v]. \tag{96}$$

Under chiral rotations it transforms as $\nabla_\mu B_v \to V \nabla_\mu B_v V^\dagger$.

---

[4] Our convention for the QED covariant derivative is $D_\mu = \partial_\mu + ieQA_\mu^e$, where Q is the charge of the particle in terms of the positron charge, and A_μ^e is the photon field.

With the above notation in hand we can write down the HBChPT Lagrangian. The $\mathcal{O}(p)$ terms are[5]

$$
\begin{aligned}
\mathcal{L}_{\text{HBChPT}}^{(1)} =& i \operatorname{Tr}\left(\bar{B}_v v \cdot \nabla B_v\right) - \frac{2m_G}{27} \operatorname{Tr}\left(\bar{B}_v B_v\right) s_G + 2D \operatorname{Tr}\left(\bar{B}_v S_N^\mu \{A_\mu, B_v\}\right) \\
& + 2F \operatorname{Tr}\left(\bar{B}_v S_N^\mu [A_\mu, B_v]\right) + 2G \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v\right) \operatorname{Tr}\left(A_\mu\right) + \operatorname{Tr}(V_\mu) \operatorname{Tr}\left(\bar{B}_v v^\mu B_v\right).
\end{aligned}
\tag{97}
$$

We included the dimension-seven s_G contribution, formally of $\mathcal{O}(p^0)$, in the $\mathcal{O}(p)$ Lagrangian. Note that the last two terms do not appear in [30] since the QCD and QED parts of the currents vanish, $\operatorname{Tr} V_\mu^\xi = \operatorname{Tr} A_\mu^\xi = 0$, while in our case $\operatorname{Tr} V_\mu = \operatorname{Tr}(\nu_\mu)$ and $\operatorname{Tr} A_\mu = \operatorname{Tr} a_\mu + \partial_\mu \theta/2$ can be nonzero, depending on the DM interactions. The coefficient of the last term is fixed by requiring that the vector current $\bar{q}\gamma_\mu q$ counts the number of valence quarks in the baryons.

The scalar and pseudoscalar spurions first appear in the $\mathcal{O}(p^2)$ HBChPT Lagrangian. The terms relevant for our analysis are

$$
\begin{aligned}
\mathcal{L}_{\text{HBChPT}}^{(2)} \supset & \, b_D \operatorname{Tr} \bar{B}_v \{s_+, B_v\} + b_F \operatorname{Tr} \bar{B}_v [s_+, B_v] + b_0 \operatorname{Tr}\left(\bar{B}_v B_v\right) \operatorname{Tr}\left(s_+^\chi\right) - \frac{\operatorname{Tr} \bar{B}_v \nabla^2 B_v}{2m_N} \\
& + \frac{1}{2m_N} \operatorname{Tr}\left(V_\mu\right) \operatorname{Tr}\left(\bar{B}_v i \overset{\leftrightarrow}{\nabla}^\mu B_v\right) - \frac{G}{m_N} \operatorname{Tr}\left(\bar{B}_v S_N \cdot i \overset{\leftrightarrow}{\nabla} B_v\right) \operatorname{Tr}\left(v \cdot A\right) \\
& - \frac{(D+F)}{2m_N} \operatorname{Tr}\left(\bar{B}_v \{S_N \cdot i \overset{\leftrightarrow}{\nabla}, v \cdot A\} B_v\right) - \frac{(D-F)}{m_N} \operatorname{Tr}\left[(\bar{B}_v S_N \cdot i \overset{\leftrightarrow}{\nabla} B_v)(v \cdot A)\right] \\
& - i\epsilon^{\alpha\beta\lambda\sigma} v_\alpha \Big[g_4 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} \nabla_\lambda \nabla_\sigma B_v\right) - ig_5 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v \nabla_\lambda V_\sigma\right) \\
& \hspace{4cm} + ig_4' \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v\right) \partial_\lambda \operatorname{Tr}(V_\sigma) \Big] + \cdots,
\end{aligned}
\tag{98}
$$

where we used reparametrization invariance to fix some of the low-energy constants, see Eq. (B12). The remaining constants are given in Table I. They are related to the proton and neutron magnetic moments, the nucleon sigma terms, $\sigma_{u,d}^p$, and the axial-vector matrix elements, Δq_p, as detailed in Appendix C. The complete expression for $\mathcal{L}_{\text{HBChPT}}^{(2)}$ is given in Appendix B 1.

The DM spurions in the above Lagrangian can be expanded in terms of the PNGB fields. Keeping only the first nonzero terms, one has

$$
V_\mu^\chi = \nu_{\chi,\mu} - \frac{i}{\sqrt{2}f}[\Pi, a_\mu] - \frac{i\partial_\mu\theta\,[\Pi, \mathcal{M}_q^{-1}]}{2\sqrt{2}f\,\operatorname{Tr}(\mathcal{M}_q^{-1})} + \cdots,
\tag{99}
$$

---

[5] Here we use the notation p for the typical momenta exchange, $p \sim q$, since this is the usual notation in ChPT. It reduces the confusion with the quark indices.

LE constant	value	LE constant	value
D	$0.812(30)$	b_D	1.4 ± 0.8
F	$0.462(14)$	b_F	-1.8 ± 0.8
G	$-0.376(28)$	g_4	$4.70/m_N$
m_G	$848(14)\,\mathrm{MeV}$	g_4'	$1.03/m_N$
b_0	-3.7 ± 1.4	g_5	$5.95(6)/m_N$

Table I: Numerical values for the low-energy constants relevant for leading order DM scattering in ChEFT. Scale-dependent quantities are defined in the $\overline{\text{MS}}$ scheme at $2\,\mathrm{GeV}$. For more details and references, see the main text.

$$A_\mu^\chi = a_\mu - \frac{i}{\sqrt{2}f}[\Pi, \nu_{\chi,\mu}] + \frac{\partial_\mu \theta}{2} \frac{\mathcal{M}_q^{-1}}{\mathrm{Tr}(\mathcal{M}_q^{-1})} + \cdots, \tag{100}$$

$$s_+^\chi = 2s_\chi + \frac{\sqrt{2}}{f}\{\Pi, p\} + \frac{2\sqrt{2}}{f} \frac{\theta\,\Pi}{\mathrm{Tr}(\mathcal{M}_q^{-1})} + \cdots. \tag{101}$$

In our analysis we need the pure QCD interactions as well as the interactions of nucleons with DM. Setting the DM currents to zero in $\mathcal{L}_{\text{HBChPT}}$ gives the pure QCD part of HBChPT. This has the following chiral expansion, $\mathcal{L}_{\text{HBChPT}}^{\text{QCD}} = \mathcal{L}_{\text{HBChPT}}^{(1),\text{QCD}} + \mathcal{L}_{\text{HBChPT}}^{(2),\text{QCD}} + \cdots$, where

$$\mathcal{L}_{\text{HBChPT}}^{(1),\text{QCD}} = i\,\mathrm{Tr}\left(\bar{B}_v v\cdot\nabla^\xi B_v\right) + 2D\,\mathrm{Tr}\left(\bar{B}_v S_N^\mu \{A_\mu^\xi, B_v\}\right) + 2F\,\mathrm{Tr}\left(\bar{B}_v S_N^\mu [A_\mu^\xi, B_v]\right), \tag{102}$$

$$\mathcal{L}_{\text{HBChPT}}^{(2),\text{QCD}} \supset b_D\,\mathrm{Tr}\,\bar{B}_v\{\xi^\dagger\mathcal{M}_q\xi^\dagger + \xi\mathcal{M}_q\xi, B_v\} + b_F\,\mathrm{Tr}\,\bar{B}_v[\xi^\dagger\mathcal{M}_q\xi^\dagger + \xi\mathcal{M}_q\xi, B_v] + \cdots. \tag{103}$$

The QCD part of the HBChPT covariant derivative is

$$\nabla_\mu^\xi B_v = \partial_\mu B_v + [V_\mu^\xi, B_v]. \tag{104}$$

The interactions between DM and nucleons have a chiral expansion that starts at $\mathcal{O}(p^0)$, $\mathcal{L}_{\chi,\text{HBChPT}} = \mathcal{L}_{\chi,\text{HBChPT}}^{(0)} + \mathcal{L}_{\chi,\text{HBChPT}}^{(1)} + \mathcal{L}_{\chi,\text{HBChPT}}^{(2)} + \cdots$. In our analysis we need terms up to $\mathcal{O}(p^3)$. Keeping only the leading terms for each of the Wilson coefficients in (14), the HBChPT interaction Lagrangians are

$$\begin{aligned}
\mathcal{L}_{\chi,\text{HBChPT}}^{(0)} &= (\bar{\chi}_v\chi_v)\left(\frac{1}{2}\,\mathrm{Tr}\,\bar{B}_v\left[(\xi^\dagger\bar{\mathcal{C}}_1^{(6,0)}\xi + \xi\bar{\mathcal{C}}_1^{(6,0)}\xi^\dagger), B_v\right] + \mathrm{Tr}\,\bar{B}_v B_v\,\mathrm{Tr}\,\bar{\mathcal{C}}_1^{(6,0)}\right) \\
&\quad + 2(\bar{\chi}_v S_\chi^\mu \chi_v)\Big(D\,\mathrm{Tr}\,\bar{B}_v S_N^\mu\{\xi^\dagger\bar{\mathcal{C}}_4^{(6,0)}\xi + \xi\bar{\mathcal{C}}_4^{(6,0)}\xi^\dagger, B_v\} \\
&\quad + F\,\mathrm{Tr}\,\bar{B}_v S_N^\mu[\xi^\dagger\bar{\mathcal{C}}_4^{(6,0)}\xi + \xi\bar{\mathcal{C}}_4^{(6,0)}\xi^\dagger, B_v] + 2G\,\mathrm{Tr}\,\bar{B}_v S_N^\mu B_v\,\mathrm{Tr}\,\bar{\mathcal{C}}_4^{(6,0)}\Big) \\
&\quad - \frac{2}{27}m_G\,(\bar{\chi}_v\chi_v)\,\mathrm{Tr}(\bar{B}_v B_v)\,\hat{\mathcal{C}}_1^{(7,0)},
\end{aligned} \tag{105}$$

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$$\mathcal{L}^{(1)}_{\chi,\text{HBChPT}} \supset 2(\bar\chi_v S_{\chi,\mu}\chi_v)\sum_q \tilde{J}^{V\mu,\text{NLO}}_q \hat{\mathcal{C}}^{(6,0)}_{2,q} + (\bar\chi_v\chi_v)\sum_q v\cdot\tilde{J}^{A,\text{NLO}}_{q,\mu}\hat{\mathcal{C}}^{(6,0)}_{3,q}$$

$$- i(\bar\chi_v S_\chi\cdot\overset{\leftrightarrow}{\partial}\chi_v)\Big(\frac{1}{2}\operatorname{Tr}\bar B_v\big[(\xi^\dagger\bar{\mathcal{C}}^{(7,1)}_2\xi + \xi\bar{\mathcal{C}}^{(7,1)}_2\xi^\dagger), B_v\big] + \operatorname{Tr}\bar B_v B_v\operatorname{Tr}\bar{\mathcal{C}}^{(7,1)}_2\Big)$$

$$+ \frac{i}{2}\Big[\bar\chi_v\overset{\leftrightarrow}{\partial}{}^\mu_\perp\chi_v - i\partial_\nu\big(\bar\chi_v\sigma^{\mu\nu}_\perp\chi_v\big)\Big]\Big(\frac{1}{2}\operatorname{Tr}\bar B_v\big[v^\mu(\xi^\dagger\bar{\mathcal{C}}^{(7,1)}_3\xi - \xi\bar{\mathcal{C}}^{(7,1)}_3\xi^\dagger), B_v\big]$$

$$+ D\operatorname{Tr}\bar B_v S^\mu_N\{\xi^\dagger\bar{\mathcal{C}}^{(7,1)}_3\xi + \xi\bar{\mathcal{C}}^{(7,1)}_3\xi^\dagger, B_v\}$$

$$+ F\operatorname{Tr}\bar B_v S^\mu_N\big[\xi^\dagger\bar{\mathcal{C}}^{(7,1)}_3\xi + \xi\bar{\mathcal{C}}^{(7,1)}_3\xi^\dagger, B_v\big] + 2G\operatorname{Tr}\bar B_v S^\mu_N B_v\operatorname{Tr}\bar{\mathcal{C}}^{(7,1)}_3\Big)$$

$$- \frac{2}{27}m_G\,\partial_\mu(\bar\chi_v S^\mu_\chi\chi_v)\operatorname{Tr}(\bar B_v B_v)\hat{\mathcal{C}}^{(8,1)}_2$$

$$- (\bar\chi_v\chi_v)\frac{\hat{\mathcal{C}}^{(7,0)}_3}{2\operatorname{Tr}(\mathcal{M}^{-1}_q)}\Big\{\frac{1}{2}v\cdot\partial\operatorname{Tr}\bar B_v\big[(\xi^\dagger\mathcal{M}^{-1}_q\xi - \xi\mathcal{M}^{-1}_q\xi^\dagger), B_v\big]$$

$$+ \partial_\mu\Big(D\operatorname{Tr}\bar B_v S^\mu_N\{\xi^\dagger\mathcal{M}^{-1}_q\xi + \xi\mathcal{M}^{-1}_q\xi^\dagger, B_v\}$$

$$+ F\operatorname{Tr}\bar B_v S^\mu_N\big[\xi^\dagger\mathcal{M}^{-1}_q\xi + \xi\mathcal{M}^{-1}_q\xi^\dagger, B_v\big] + 2G\operatorname{Tr}(\mathcal{M}^{-1}_q)\operatorname{Tr}\bar B_v S^\mu_N B_v\Big)\Big\}, \tag{106}$$

$$\mathcal{L}^{(2)}_{\chi,\text{HBChPT}} \supset -(\bar\chi_v\chi_v)\Big[b_0\operatorname{Tr}(\bar B_v B_v)\operatorname{Tr}\mathcal{M}_q\big(\bar{\mathcal{C}}^{(7,0)}_5(U^\dagger + U) - i\bar{\mathcal{C}}^{(7,0)}_7(U^\dagger - U)\big)$$

$$+ b_D\operatorname{Tr}\bar B_v\{\xi^\dagger\mathcal{M}_q\big(\bar{\mathcal{C}}^{(7,0)}_5 - i\bar{\mathcal{C}}^{(7,0)}_7\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}^{(7,0)}_5 + i\bar{\mathcal{C}}^{(7,0)}_7\big)\xi, B_v\}$$

$$+ b_F\operatorname{Tr}\bar B_v\big[\xi^\dagger\mathcal{M}_q\big(\bar{\mathcal{C}}^{(7,0)}_5 - i\bar{\mathcal{C}}^{(7,0)}_7\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}^{(7,0)}_5 + i\bar{\mathcal{C}}^{(7,0)}_7\big)\xi, B_v\big]\Big]$$

$$- \partial_\nu(\bar\chi_v S^\nu_\chi\chi_v)\frac{\hat{\mathcal{C}}^{(8,1)}_4}{2\operatorname{Tr}(\mathcal{M}^{-1}_q)}\Big\{\frac{1}{2}v\cdot\partial\operatorname{Tr}\bar B_v\big[(\xi^\dagger\mathcal{M}^{-1}_q\xi - \xi\mathcal{M}^{-1}_q\xi^\dagger), B_v\big]$$

$$+ \partial_\mu\Big(D\operatorname{Tr}\bar B_v S^\mu_N\{\xi^\dagger\mathcal{M}^{-1}_q\xi + \xi\mathcal{M}^{-1}_q\xi^\dagger, B_v\}$$

$$+ F\operatorname{Tr}\bar B_v S^\mu_N\big[\xi^\dagger\mathcal{M}^{-1}_q\xi + \xi\mathcal{M}^{-1}_q\xi^\dagger, B_v\big] + 2G\operatorname{Tr}(\mathcal{M}^{-1}_q)\operatorname{Tr}\bar B_v S^\mu_N B_v\Big)\Big\}, \tag{107}$$

$$\mathcal{L}^{(3)}_{\chi,\text{HBChPT}} \supset -\partial_\mu(\bar\chi_v S^\mu_\chi\chi_v)\Big[b_0\operatorname{Tr}(\bar B_v B_v)\operatorname{Tr}\mathcal{M}_q\big(\bar{\mathcal{C}}^{(8,1)}_6(U^\dagger + U) + i\bar{\mathcal{C}}^{(8,1)}_8(U^\dagger - U)\big)$$

$$+ b_D\operatorname{Tr}\bar B_v\{\xi^\dagger\mathcal{M}_q\big(\bar{\mathcal{C}}^{(8,1)}_6 + i\bar{\mathcal{C}}^{(8,1)}_8\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}^{(8,1)}_6 - i\bar{\mathcal{C}}^{(8,1)}_8\big)\xi, B_v\}$$

$$+ b_F\operatorname{Tr}\bar B_v\big[\xi^\dagger\mathcal{M}_q\big(\bar{\mathcal{C}}^{(8,1)}_6 + i\bar{\mathcal{C}}^{(8,1)}_8\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}^{(8,1)}_6 - i\bar{\mathcal{C}}^{(8,1)}_8\big)\xi, B_v\big]\Big]. \tag{108}$$

The expressions for the NLO currents $\tilde{J}^{V,\text{NLO}}_{q,\mu}$ and $\tilde{J}^{A,\text{NLO}}_{q,\mu}$ appearing in (106) can be found in (B39) and (B42). The diagonal matrix of Wilson coefficients $\bar{\mathcal{C}}_i$ was defined in (45). The DM HBChPT Lagrangians (106)-(108), expanded in the meson fields, are collected in Section B 2.

IV. DISCUSSION AND MATCHING ONTO NUCLEAR CHIRAL EFT

We are now in a position to calculate the scattering of DM on nuclei using a chiral EFT description of nuclear forces. We first briefly review the results of the previous two sections, keeping only the essential ingredients, and introduce a simplified notation. We rewrite the HDMET interaction Lagrangian (13) as

$$\mathcal{L}_\chi = \mathcal{L}_\chi^{(5)} + \mathcal{L}_\chi^{(6)} + \mathcal{L}_\chi^{(7)} + \cdots , \tag{109}$$

where we collect in each Lagrangian $\mathcal{L}_\chi^{(d)}$ the terms that would come from relativistic DM operators with dimensionality d in (2)-(9), $\mathcal{L}_\chi^{(d)} = \sum_{a,m} \hat{\mathcal{C}}_a^{(d+m,m)} \mathcal{Q}_a^{(d+m,m)}$. We work to tree-level order in the matching at the scale $\mu \sim m_\chi$. The Wilson coefficients then satisfy the relations (24), (31), (37), so that we have

$$\mathcal{L}_\chi^{(5)} = \hat{\mathcal{C}}_1^{(5,0)} \frac{e}{8\pi^2} F_{\mu\nu} J_\chi^{T,\mu\nu} + \hat{\mathcal{C}}_2^{(5,0)} \frac{e}{8\pi^2} F_{\mu\nu} J_\chi^{AT,\mu\nu}. \tag{110}$$

$$\begin{aligned}
\mathcal{L}_\chi^{(6)} = {} & J_\chi^{V,\mu} \sum_{q=u,d,s} \left[\hat{\mathcal{C}}_{1,q}^{(6,0)} \left(\bar{q}\gamma_\mu q \right) + \hat{\mathcal{C}}_{3,q}^{(6,0)} \left(\bar{q}\gamma_\mu\gamma_5 q \right) \right] \\
& + J_\chi^{A,\mu} \sum_{q=u,d,s} \left[\hat{\mathcal{C}}_{2,q}^{(6,0)} \left(\bar{q}\gamma_\mu q \right) + \hat{\mathcal{C}}_{4,q}^{(6,0)} \left(\bar{q}\gamma_\mu\gamma_5 q \right) \right].
\end{aligned} \tag{111}$$

$$\begin{aligned}
\mathcal{L}_\chi^{(7)} = {} & J_\chi^S \sum_{q=u,d,s} \left[\hat{\mathcal{C}}_{5,q}^{(7,0)} m_q \left(\bar{q}q \right) + \hat{\mathcal{C}}_{7,q}^{(7,0)} m_q \left(\bar{q}i\gamma_5 q \right) \right. \\
& \left. + \hat{\mathcal{C}}_1^{(7,0)} \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{a\mu\nu} + \hat{\mathcal{C}}_3^{(7,0)} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \right] \\
& + m_\chi J_\chi^P \sum_{q=u,d,s} \left[\hat{\mathcal{C}}_{6,q}^{(8,1)} m_q \left(\bar{q}q \right) - \hat{\mathcal{C}}_{8,q}^{(8,1)} m_q \left(\bar{q}i\gamma_5 q \right) \right. \\
& \left. + \hat{\mathcal{C}}_2^{(8,1)} \frac{\alpha_s}{12\pi} G_{\mu\nu}^a G^{a\mu\nu} + \hat{\mathcal{C}}_4^{(8,1)} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} \right].
\end{aligned} \tag{112}$$

Note that, for tree-level matching, the HDMET interactions are simply a product of the DM and SM currents, with the DM currents taken outside the sums over quark flavors. The DM currents are given by

$$J_\chi^{T,\mu\nu} = \bar{\chi}_v \sigma_\perp^{\mu\nu} \chi_v + \frac{1}{2m_\chi} \left(\bar{\chi}_v i v^{[\mu} \sigma_\perp^{\nu]\rho} \overleftrightarrow{\partial}_\rho \chi_v - v^{[\mu}\partial^{\nu]} \bar{\chi}_v \chi_v \right) + \ldots \overset{\text{tree}}{=} \bar{\chi}\sigma^{\mu\nu}\chi , \tag{113}$$

$$J_\chi^{AT,\mu\nu} = 2\bar{\chi}_v S_\chi^{[\mu} v^{\nu]} \chi_v + \cdots \overset{\text{tree}}{=} \bar{\chi}\sigma^{\mu\nu} i\gamma_5\chi, \tag{114}$$

$$J_\chi^{V,\mu} = v^\mu \bar{\chi}_v \chi_v + \frac{1}{2m_\chi}\bar{\chi}_v i\overleftrightarrow{\partial}_\perp^\mu \chi_v + \frac{1}{2m_\chi}\partial_\nu \left(\bar{\chi}\sigma_\perp^{\mu\nu}\chi_v \right) + \cdots \overset{\text{tree}}{=} \bar{\chi}\gamma^\mu\chi , \tag{115}$$

$$J_\chi^{A,\mu} = 2\bar{\chi}_v S_\chi^\mu \chi_v - \frac{i}{m_\chi} v^\mu \bar{\chi}_v S_\chi \cdot \overleftrightarrow{\partial} \chi_v + \cdots \overset{\text{tree}}{=} \bar{\chi}\gamma^\mu\gamma_5\chi , \tag{116}$$

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$$J_\chi^S = \bar{\chi}_v \chi_v + \cdots \overset{\text{tree}}{=} \bar{\chi}\chi\,, \tag{117}$$

$$J_\chi^P = \frac{1}{m_\chi}\partial_\mu \bar{\chi}_v S_\chi^\mu \chi_v + \cdots \overset{\text{tree}}{=} \bar{\chi} i\gamma_5 \chi\,. \tag{118}$$

The notation in (110)-(112) will prove useful we discuss the leading contributions in chiral counting for each of the Wilson coefficients, as it makes it easy to see where the q/m_χ-suppressed terms come from. For matching at higher loop orders at scale $\mu \sim m_\chi$ one could generalize the above notation by making the DM currents quark-flavor dependent and move them inside the quark flavor sums, although the notation would not be simpler than in (14).[6]

The Lagrangian $\mathcal{L}_\chi^{(5)}$, Eq. (110), contains only QED interactions of DM with the SM. On the other hand, we have seen in the previous two sections that the $\mathcal{L}_\chi^{(6,7)}$ interactions involving quarks and gluons, Eq. (111) and (112), match onto an effective Lagrangian with mesons and nucleons, $\mathcal{L}_{\chi,\text{ChPT}} + \mathcal{L}_{\chi,\text{HBChPT}}$. Here $\mathcal{L}_{\chi,\text{ChPT}}$ contains only the light pseudoscalar mesons π, K, and η as QCD asymptotic states, while $\mathcal{L}_{\chi,\text{HBChPT}}$ contains, in addition, the protons and neutrons. One can organize the different terms using chiral counting since the momentum transfer is small, $q \leq q_{\max} \ll 4\pi f_\pi$ (cf. Eq. (1)), where f_π is the pion decay constant. The chiral expansion corresponds to an expansion in momenta exchanges, $p \sim q$, where the meson masses are counted as $m_\pi \sim \mathcal{O}(p)$. As a consequence the quark masses scale as $m_q \sim \mathcal{O}(p^2)$ since $m_q \propto m_\pi^2$. The interactions of DM with mesons start at $\mathcal{O}(p)$,

$$\mathcal{L}_{\chi,\text{ChPT}} = \mathcal{L}_{\chi,\text{ChPT}}^{(1)} + \mathcal{L}_{\chi,\text{ChPT}}^{(2)} + \cdots\,, \tag{121}$$

while the interactions of DM with nucleons start at $\mathcal{O}(p^0)$

$$\mathcal{L}_{\chi,\text{HBChPT}} = \mathcal{L}_{\chi,\text{HBChPT}}^{(0)} + \mathcal{L}_{\chi,\text{HBChPT}}^{(1)} + \mathcal{L}_{\chi,\text{HBChPT}}^{(2)} + \cdots\,. \tag{122}$$

The QCD interactions among pions have an expansion in p^2, while the interactions between pions and nucleons have an expansion in p,

$$\mathcal{L}_{\text{ChPT}}^{\text{QCD}} = \mathcal{L}_{\text{ChPT}}^{(2)} + \mathcal{L}_{\text{ChPT}}^{(4)} + \cdots\,, \qquad \mathcal{L}_{\text{HBChPT}}^{\text{QCD}} = \mathcal{L}_{\text{HBChPT}}^{(1)} + \mathcal{L}_{\text{HBChPT}}^{(2)} + \cdots\,. \tag{123}$$

---

[6] For instance one could define

$$J_{\chi,j,q}^{V,\mu} = v^\mu \big(\bar{\chi}_v \chi_v\big) + \Big[r_{j,q}(\bar{\chi}_v i\overset{\leftrightarrow}{\partial}_\perp^\mu \chi_v) + r_{(j+9)/2,q}\partial_\nu\big(\bar{\chi}\sigma_\perp^{\mu\nu}\chi_v\big) \Big]/2, \qquad j = 1,3, \tag{119}$$

$$J_{\chi,j,q}^{A,\mu} = 2\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) - r_{j,q} v^\mu(\bar{\chi}_v iS\cdot\overset{\leftrightarrow}{\partial}\chi_v), \qquad j = 2,4, \tag{120}$$

with $r_{j,q} = \hat{\mathcal{C}}_{j,q}^{(7,1)}/\hat{\mathcal{C}}_{1,q}^{(6,0)}$.

The explicit forms of the above Lagrangians are given in (67), (68), (75)-(77), (102), (103), and (105)-(108). The LO QCD interactions are schematically

$$
\begin{aligned}
\mathcal{L}_{\mathrm{ChPT}}^{(2)} &\sim (\partial_\mu \pi)^2 + (\pi \partial_\mu \pi)^2 + \cdots , \\
\mathcal{L}_{\mathrm{HBChPT}}^{(1)} &\sim \bar{N} v \cdot \partial N + \bar{N} N v \cdot \partial \pi + \bar{N} S_N^\mu N \pi \partial_\mu \pi + \cdots ,
\end{aligned}
\tag{124}
$$

where we expanded in the meson fields, π, with N denoting a nucleon field.

The leading few terms in chiral counting for the DM–meson interactions are

$$
\begin{aligned}
\mathcal{L}_{\chi,\mathrm{ChPT}}^{(1)} = J_\chi^{V,\mu} \sum_q \left(J_{q,\mu}^V \hat{\mathcal{C}}_{1,q}^{(6,0)} + J_{q,\mu}^A \hat{\mathcal{C}}_{3,q}^{(6,0)} \right) \\
+ J_\chi^{A,\mu} \sum_q \left(J_{q,\mu}^V \hat{\mathcal{C}}_{2,q}^{(6,0)} + J_{q,\mu}^A \hat{\mathcal{C}}_{4,q}^{(6,0)} \right) ,
\end{aligned}
\tag{125}
$$

$$
\mathcal{L}_{\chi,\mathrm{ChPT}}^{(2)} = J_\chi^S \left(\sum_q J_q^S \hat{\mathcal{C}}_{5,q}^{(7,0)} + \sum_q J_q^P \hat{\mathcal{C}}_{7,q}^{(7,0)} + J^G \hat{\mathcal{C}}_1^{(7,0)} + J^\theta \hat{\mathcal{C}}_3^{(7,0)} \right) ,
\tag{126}
$$

$$
\mathcal{L}_{\chi,\mathrm{ChPT}}^{(3)} = m_\chi J_\chi^P \left(\sum_q J_q^S \hat{\mathcal{C}}_{6,q}^{(8,1)} - \sum_q J_q^P \hat{\mathcal{C}}_{8,q}^{(8,1)} + J^G \hat{\mathcal{C}}_2^{(8,1)} + J^\theta \hat{\mathcal{C}}_4^{(8,1)} \right) .
\tag{127}
$$

Here, we used tree-level matching expressions, and have thus factored out the DM currents (115)-(118). We took into account the scaling $J_\chi^P \sim \mathcal{O}(p)$, c.f. Eq. (118), while all the other DM currents are $\mathcal{O}(p^0)$. The quark level currents were hadronized into the corresponding mesonic currents,

$$
\begin{aligned}
\bar{q}\gamma_\mu q &\to J_{q,\mu}^V \sim \pi \partial_\mu \pi + \cdots , & \bar{q}\gamma_\mu \gamma_5 q &\to J_{q,\mu}^A \sim \partial_\mu \pi + \cdots , \\
m_q \bar{q}q &\to J_q^S \sim m_q \pi^2 + \cdots , & \bar{q}i\gamma_5 q &\to J_q^P \sim m_q \pi + \cdots , \\
\frac{\alpha_s}{12\pi} G_{\mu\nu} G^{\mu\nu} &\to J^G \sim \partial^2 \pi^2 + \cdots , & \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} &\to J^\theta \sim \partial^2 \pi + \cdots .
\end{aligned}
\tag{128}
$$

Again, we showed their schematic structure when expanded in the meson fields, keeping only the first nonzero terms. The full form of the currents are given in Appendix B.

The DM–nucleon interactions, keeping only the leading terms in chiral counting for each effective operator, are given by

$$
\mathcal{L}_{\chi,\mathrm{HBChPT}}^{(0)} = \sum_q \left(J_\chi^V \cdot \tilde{J}_q^V \, \hat{\mathcal{C}}_{1,q}^{(6,0)} + J_\chi^A \cdot \tilde{J}_q^A \, \hat{\mathcal{C}}_{4,q}^{(6,0)} \right) + J_\chi^S \tilde{J}^G \hat{\mathcal{C}}_1^{(7,0)},
\tag{129}
$$

$$
\begin{aligned}
\mathcal{L}_{\chi,\mathrm{HBChPT}}^{(1)} \supset \sum_q \left(J_\chi^A \cdot \tilde{J}_q^V \, \hat{\mathcal{C}}_{2,q}^{(6,0)} + J_\chi^V \cdot \tilde{J}_q^A \, \hat{\mathcal{C}}_{3,q}^{(6,0)} \right) + m_\chi J_\chi^P \tilde{J}^G \, \hat{\mathcal{C}}_2^{(8,1)} \\
+ J_\chi^S \tilde{J}^\theta \, \hat{\mathcal{C}}_3^{(7,0)},
\end{aligned}
\tag{130}
$$

$$
\mathcal{L}_{\chi,\mathrm{HBChPT}}^{(2)} \supset J_\chi^S \sum_q \left(\tilde{J}_q^S \, \hat{\mathcal{C}}_{5,q}^{(7,0)} + \tilde{J}_q^P \hat{\mathcal{C}}_{7,q}^{(7,0)} \right) + m_\chi J_\chi^P \, \tilde{J}^\theta \, \hat{\mathcal{C}}_4^{(8,1)},
\tag{131}
$$

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$$\mathcal{L}^{(3)}_{\chi,\text{HBChPT}} \supset m_\chi J^P_\chi \sum_q \left(\tilde{J}^S_q \, \hat{\mathcal{C}}^{(8,1)}_{6,q} - \tilde{J}^P_q \, \hat{\mathcal{C}}^{(8,1)}_{8,q} \right), \tag{132}$$

where in each term one should keep only the leading nonzero terms in the HDMET expansion of the DM currents. The explicit form of the Lagrangians are given in Eqs. (105)-(108), while the expressions expanded in meson fields are given in Eqs. (B17)-(B22). The quark-level currents get hadronized to nucleon currents. They are schematically

$$
\begin{aligned}
\bar{q}\gamma^\mu q \to \tilde{J}^{V,\mu}_q &\sim v^\mu \bar{N}N + \cdots, & \bar{q}\gamma^\mu\gamma_5 q \to \tilde{J}^{A,\mu}_q &\sim \bar{N}S^\mu_N N + \cdots, \\
m_q \bar{q}q \to \tilde{J}^S_q &\sim m_q \bar{N}N + \cdots, & m_q \bar{q}i\gamma_5 q \to \tilde{J}^P_q &\sim m_q \bar{N}N\pi + \cdots, \\
\frac{\alpha_s}{12\pi}GG \to \tilde{J}^G &\sim \bar{N}N + \cdots, & \frac{\alpha_s}{8\pi}G\tilde{G} \to \tilde{J}^\theta &\sim q^\mu \bar{N}S_{N,\mu}N + \cdots,
\end{aligned}
\tag{133}
$$

with their explicit forms given in Eqs. (B33)-(B38), Eqs. (B39)-(B42), and Eqs. (B46)-(B57). Using the expressions (115)-(118) for the DM currents expanded in $1/m_\chi$, and the fact that $v \cdot S_N = 0$, $v \cdot S_\chi = 0$, we see that the $\mathcal{O}(p^0)$ terms cancel in the products $J^V_\chi \cdot \tilde{J}^A_q$ and $J^A_\chi \cdot \tilde{J}^V_q$. These are then part of $\mathcal{L}^{(1)}_{\chi,\text{HBChPT}}$, see Eq. (130). Schematically, we have for the products of currents in (129)-(132)

$$
\begin{aligned}
J^V_\chi \cdot \tilde{J}^V_q &\sim J^S_\chi \tilde{J}^G \sim (\bar{\chi}_v\chi_v)(\bar{N}N), & J^A_\chi \cdot \tilde{J}^A_q &\sim (\bar{\chi}_v S_\chi \chi_v) \cdot (\bar{N}S_N N), \\
J^A_\chi \cdot \tilde{J}^V_q &\sim J^P_\chi \tilde{J}^G \sim (\bar{\chi}_v \partial \cdot S_\chi \chi_v)(\bar{N}N), & J^V_\chi \cdot \tilde{J}^A_q &\sim J^S_\chi \tilde{J}^\theta \sim (\bar{\chi}_v\chi_v)(\bar{N}\partial \cdot S_N N), \\
J^S_\chi \tilde{J}^S_q &\sim m_q (\bar{\chi}_v\chi_v)(\bar{N}N), \quad J^S_\chi \tilde{J}^P_q \sim m_q (\bar{\chi}_v\chi_v)(\bar{N}N)\pi, & J^P_\chi \tilde{J}^\theta &\sim (\bar{\chi}_v\partial \cdot S_\chi\chi_v)(\bar{N}\partial \cdot S_N N), \\
J^P_\chi \tilde{J}^S_q &\sim (\bar{\chi}_v\partial \cdot S_\chi\chi_v)m_q(\bar{N}N), & J^P_\chi \tilde{J}^P_q &\sim (\bar{\chi}_v\partial \cdot S_\chi\chi_v)m_q(\bar{N}N)\pi,
\end{aligned}
\tag{134}
$$

where in addition the $J^A_\chi \cdot \tilde{J}^V_q$ and $J^V_\chi \cdot \tilde{J}^A_q$ contain the operator $\epsilon^{\alpha\beta\mu\nu}v_\alpha q_\beta(\bar{\chi}_v S_\mu\chi_v)(\bar{N}S_\nu N)$. In accordance with the chiral counting, the products of currents (134) entering $\mathcal{L}^{(d)}_{\chi,\text{HBChPT}}$ have chiral dimension d, that is, they either have d derivatives, or have $d-2$ derivatives and one factor of $m_q \sim \mathcal{O}(p^2)$. Note that the hadronization of the pseudoscalar current, $\bar{q}i\gamma_5 q$, requires the emission of at least one meson.

A. DM–nucleus scattering in chiral EFT

The above DM–nucleon interactions are the building blocks for predicting the DM–nucleus scattering rates using the ChEFT-based description of nuclear forces. DM scattering is described by a single insertion of the interaction Lagrangian $\mathcal{L}_\chi$, Eq. (2), in the scattering

amplitude. Our goal is to obtain the leading contribution to the DM-nucleus scattering rate for all the interactions in Eq. (3)-(9). Each of the operators in (3)-(9) induces both a coupling of DM to the light mesons only and a coupling of DM to nucleons and mesons. In order to gauge the importance of each of these two types of contributions we use the chiral counting for nuclear forces within ChEFT.

The ChEFT description of nuclear forces is based on Weinberg's insight that the N-body nucleon potentials can be obtained from N-nucleon irreducible amplitudes [31, 32]. The N-nucleon irreducible amplitudes consist of those diagrams that cannot be disconnected by cutting N nucleon lines, i.e., there must be at least one pion exchange. The internal pion and nucleon propagators are off-shell by $E \sim \mathcal{O}(p) \sim \mathcal{O}(m_\pi)$. As such they allow for a consistent chiral counting. The properties of the nucleus can then be obtained by solving the Schrödinger equation involving the 2, 3, ..., N-nucleon potentials. This is equivalent to resumming the reducible diagrams where some of the internal nucleon lines are close to being on-shell, with $E \sim \mathcal{O}(p^2/m_N)$.

We are interested in DM scattering on a nucleus with atomic number A. The scattering operator follows from a sum of A-nucleon irreducible amplitudes, $M_{A,\chi}$, with one insertion of the DM interaction. A given A-nucleon irreducible amplitude scales as $M_{A,\chi} \sim (p/\Lambda_{\mathrm{ChEFT}})^\nu$, with [3, 32, 33]

$$\nu = 4 - A - 2C + 2L + \sum_i V_i \epsilon_i + \epsilon_\chi, \tag{135}$$

for a diagram with C connected parts, L loops, V_i strong-interaction vertices of type i, and one DM interaction vertex. The effective chiral dimension ϵ_i of the vertex of type i is given by $\epsilon_i = d_i + n_i/2 - 2$, where d_i is the chiral dimension of the vertex and n_i the number of nucleon legs attached to the vertex. We explicitly isolated the contribution ϵ_χ due to the external DM current since each amplitude will only have one such insertion [3]. For instance, the effective chiral dimension of a vertex from $\mathcal{L}^{(d)}_{\chi,\mathrm{ChPT}}$ is $\epsilon_\chi = d - 2$, while the DM-nucleon interactions in $\mathcal{L}^{(d)}_{\chi,\mathrm{HBChPT}}$ have effective chiral dimension $\epsilon_\chi = d - 1$. The leading QCD interactions from $\mathcal{L}^{(1)}_{\mathrm{HBChPT}}$ and $\mathcal{L}^{(2)}_{\mathrm{ChPT}}$ have $\epsilon_i = 0$. This means that one can insert an arbitrary number of these strong vertices without affecting the p^ν power scaling.

The chiral loop counting in irreducible amplitudes suggests that the cut-off of the ChEFT is the same as in ChPT, $\Lambda_{\mathrm{ChEFT}} \sim \Lambda_\chi \sim 4\pi f_\pi \sim 1$ GeV. The resummation of the bubble diagrams in the reducible amplitudes, on the other hand, leads to the appearance of the
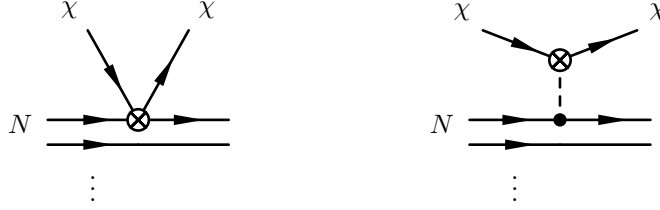
Figure 1: The leading order diagrams for the DM-nucleus scattering. The effective DM–nucleon and DM–meson interactions is denoted by a circle, the dashed lines denote mesons, and the dots represent the remaining $A - 2$ nucleon lines.

observed shallow bound states for $p\,m_N/\Lambda_{\mathrm{ChEFT}}^2 \sim 1$, and thus for $\Lambda_{\mathrm{ChEFT}} \sim 0.5$ GeV.[7] Conservatively, we will use in the numerical estimates $p/\Lambda_{\mathrm{ChEFT}} \sim m_\pi/\Lambda_{\mathrm{ChEFT}} \sim 0.3$.

The LO diagrams for DM-nucleon scattering are shown in Fig. 1. The left diagram gives the leading contribution for the hadronization of the $\bar{q}\gamma_\mu q$, $\bar{q}q$, and GG currents. The right diagram is the leading contribution for the hadronization of the $\bar{q}i\gamma_5 q$ current (the insertion is the mesonic J^P current), in which case the left diagram is absent. Finally, for $\bar{q}\gamma_\mu\gamma_5 q$ and $G\tilde{G}$ both the left and the right diagrams are leading and contribute at the same order (for $J_\chi^V \cdot J_q^A$ the left diagram dominates). In terms of the p^ν scaling we have for the leading contributions proportional to the $\hat{\mathcal{C}}_a^{(n,m)}$ Wilson coefficients

$$
\begin{aligned}
\nu_{\min} &= \nu_{\mathrm{LO}}[J_\chi^V \cdot \tilde{J}_q^V] = \nu_{\mathrm{LO}}[J_\chi^A \cdot \tilde{J}_q^A] = \nu_{\mathrm{LO}}[J_\chi^S \tilde{J}^G], \\
\nu_{\min} + 1 &= \nu_{\mathrm{LO}}[J_\chi^V \cdot \tilde{J}_q^A] = \nu_{\mathrm{LO}}[J_\chi^A \cdot \tilde{J}_q^V] = \nu_{\mathrm{LO}}[J_\chi^P \tilde{J}^G] = \nu_{\mathrm{LO}}[J_\chi^S \tilde{J}^\theta] = \nu_{\mathrm{LO}}[J_\chi^S J_q^P], \\
\nu_{\min} + 2 &= \nu_{\mathrm{LO}}[J_\chi^S \tilde{J}_q^S] \;\;= \nu_{\mathrm{LO}}[J_\chi^P \tilde{J}^\theta] = \nu_{\mathrm{LO}}[J_\chi^P J_q^P], \\
\nu_{\min} + 3 &= \nu_{\mathrm{LO}}[J_\chi^P \tilde{J}_q^S].
\end{aligned}
\tag{136}
$$

Here $\nu_{\min} = 3 - 3A$ simply reflects our normalization of the A-nucleon state, where A is the atomic number of the nucleus. In the brackets we displayed the leading products of currents

---

[7] This scaling would imply that the nucleon mass is parametrically larger than Λ_{ChEFT}, so that $p/m_N \sim \mathcal{O}(p^2)$, where $p \sim m_\pi$ [32, 34]. In the derivation of the nuclear potentials using ChEFT one counts $p/m_N \sim \mathcal{O}(p)$, the same as in HBChPT [35]. The Weinberg's counting is fully consistent when deriving the nuclear potentials. Renormalization of the potentials when solving the Schrödinger equation, however, may require counterterms of formally higher chiral order [33–36]. For instance, divergences due to iterations of leading-order interactions may not be absorbed by the leading-order operators themselves [33]. While conceptually discomforting, this problem is numerically small when using momentum cut-off regularization for modes above ~ 1GeV. The alternative KSW counting [37–39], treating the NLO corrections perturbatively, is fully consistent, but leads to poorly convergent results [40].

that multiply the $\hat{\mathcal{C}}_a^{(n,m)}$ in Eqs. (129)-(132). As already mentioned above, for most products of currents the left diagram in Fig. 1 gives the dominant contribution. The resulting $p^{\nu_{\mathrm{LO}}}$ suppression then follows directly from the chiral suppression of the corresponding interaction Lagrangian, $\mathcal{L}_{\chi,\mathrm{HBChPT}}^{(0,1,2,3)}$. The exceptions are $J_\chi^S J_q^P$ and $J_\chi^P J_q^P$, for which the right diagram in Fig. 1 dominates. These products have a chiral suppression that is smaller than naively expected from the dimensionality of the corresponding term in $\mathcal{L}_{\chi,\mathrm{HBChPT}}^{(0,1,2,3)}$ since the single pion exchange reduces ν_{LO} by one. From the general counting rule (135) a similar conclusion would be reached also for the product $J_\chi^V \cdot \tilde{J}_q^A$. However, in this case the single pion coupling to the DM current vanishes due to vector current conservation, so that the formally leading contribution from the right diagram in Fig. 1 is zero. Special cases are $J_\chi^A \cdot \tilde{J}_q^A$, $J_\chi^P \cdot \tilde{J}_q^\theta$, and $J_\chi^S \cdot \tilde{J}_q^\theta$, for which both diagrams in Fig. 1 contribute at the same order.

Note that at LO in chiral counting DM interacts with a single nucleon, either directly through the short distance operator, or through a single pion exchange. An interesting question is at which order in p the two-body interactions do become important. Examples of the relevant subleading contributions are shown in Fig. 2. The first two diagrams are due to DM coupling to a short distance two-nucleon current. These contributions always scale as $p^{\nu_{\mathrm{LO}}+3}$. There are also contributions, shown in the third diagram of Fig. 2, where the DM attaches to the meson exchanged between two nucleons, leading to long-distance two-nucleon currents. For DM interactions originating from $\mathcal{O}_{2,q}^{(6)} \sim J_\chi^A \cdot J_q^V$, $\mathcal{O}_{5,q}^{(7)} \sim J_\chi^S \cdot J_q^S$, and $\mathcal{O}_{6,q}^{(7)} \sim J_\chi^P \cdot J_q^S$, this contribution scales as $p^{\nu_{\mathrm{LO}}+1}$, while for DM interactions originating from $\mathcal{O}_{1,q}^{(6)} \sim J_\chi^V \cdot J_q^V$ it scales as $p^{\nu_{\mathrm{LO}}+2}$. In these cases the long distance two-nucleon contributions are parametrically larger than the short distance ones. For the remaining operators the long-distance contributions are of the same order or power suppressed compared to the short-distance ones.

In addition, there are higher-order corrections that involve single-nucleon interactions with DM. The last diagram in Fig. 2 shows an example of such an one-loop contribution. In addition there are also power suppressed single-nucleon current insertions. These include the counterterms that cancel the 1-loop divergences. For the DM interactions $J_\chi^A \cdot J_q^V$, $J_\chi^S \cdot J_q^S$, and $J_\chi^P \cdot J_q^S$, the one-loop contributions scale as $p^{\nu_{\mathrm{LO}}+1}$, and are, together with the long-distance pion exchange from the third diagram in Fig. 2, the leading chiral corrections.

In this work we are satisfied with LO matching and neglect relative $\mathcal{O}(p)$-suppressed terms. Our results thus have a relative $\mathcal{O}(q/\Lambda_{\mathrm{ChEFT}}) \sim 30\%$ accuracy. At this order the
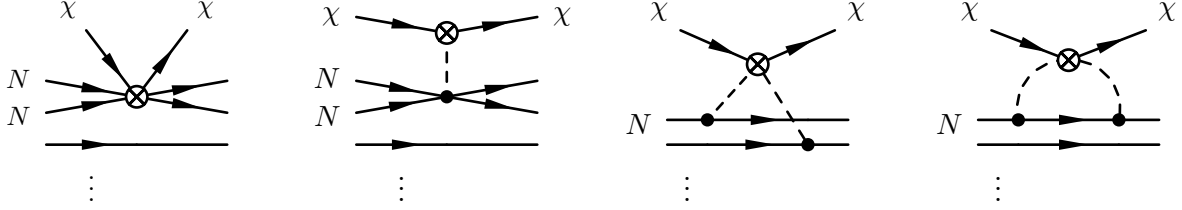
Figure 2: Sample NLO diagrams for the DM-nucleon scattering inside nuclei. The effective DM–nucleon or DM–meson interaction is denoted by a box, the dashed lines denote mesons.

effective DM interactions involve only single nucleon currents. At NLO, i.e., at relative $\mathcal{O}((q/\Lambda_{\mathrm{ChEFT}})^2) \sim 10\%$ accuracy, the DM is still interacting with a single nucleon current for almost all DM–nucleon effective operators. The exceptions are the DM–nucleon interactions $J_\chi^A \cdot J_q^V$, $J_\chi^S \cdot J_q^S$, and $J_\chi^P \cdot J_q^S$. For these the two-nucleon contributions are a long-distance effect so that the corrections are still calculable in ChPT. The results for scalar quark currents in the case of Xe are available in [3, 9], and are of the expected size. The genuine short-distance two-nucleon currents, for which one would require lattice QCD calculations, appear only at NNNLO in chiral counting, i.e., below few-percent accuracy.

B. Form factors for dark matter–nucleon interactions

We can use the formalism in the previous section to calculate the form factors for the DM–nucleon interactions. We perform the leading order matching, shown in Fig. 1. The hadronized $\bar{q}\gamma_\mu q$, $\bar{q}q$, $G\tilde{G}$, and GG currents receive contributions from the left diagram, the $\bar{q}i\gamma_5 q$ current from the right diagram, while $\bar{q}\gamma_\mu\gamma_5 q$ receives contributions from both diagrams. The expanded hadronic currents are collected in Appendix B 4. We include the contributions from single π^0 and η exchanges in the q^2-dependent coefficients of the nonrelativistic operators defined below. The momenta exchanges are small enough that the DM-nucleon interactions cannot lead to the dissociation of nuclei and the production of on-shell pions.

The resulting effective Lagrangian is

$$\mathcal{L}_{\mathrm{eff}} = \sum_{i,d} \left(c_{i,p}^{(d)}(q^2)\, Q_{i,p}^{(d)} + c_{i,n}^{(d)}(q^2)\, Q_{i,n}^{(d)} \right), \tag{137}$$

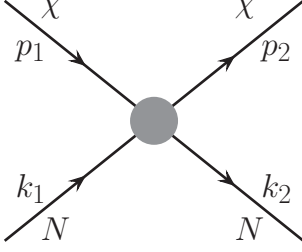with d counting the number of derivatives in the operators. It is understood that $\mathcal{L}_{\mathrm{eff}}$ is to

28

Figure 3: The kinematics of DM scattering on nucleons, $\chi(p_1)N(k_1) \to \chi(p_2)N(k_2)$.

be used only at tree level. The operator basis is, for $d = 0$,

$$Q_{1,p}^{(0)} = (\bar{\chi}_v\chi_v)(\bar{p}_v p_v), \qquad\qquad Q_{2,p}^{(0)} = (\bar{\chi}_v S_\chi^\mu \chi_v)(\bar{p}_v S_{N,\mu} p_v), \qquad (138)$$

with a similar set of operators for neutrons, with $p \to n$. The $Q_{1,p}^{(0)}$ operator will induce spin-independent DM scattering on the nucleus, while $Q_{2,p}^{(0)}$ will induce the spin-dependent scattering. In our analysis we also include all the $d = 1$ operators,

$$Q_{1,p}^{(1)} = (\bar{\chi}_v\chi_v)(\bar{p}_v iq\cdot S_N p_v), \qquad\qquad Q_{2,p}^{(1)} = (\bar{\chi}_v iq\cdot S_\chi \chi_v)(\bar{p}_v p_v), \qquad (139)$$

$$Q_{3,p}^{(1)} = m_N(\bar{\chi}_v\chi_v)(\bar{p}_v\, v_\perp\cdot S_N\, p_v), \qquad Q_{4,p}^{(1)} = m_N(\bar{\chi}_v\, v_\perp\cdot S_\chi\, \chi_v)(\bar{p}_v p_v), \qquad (140)$$

$$Q_{5,p}^{(1)} = i\epsilon^{\alpha\beta\mu\nu}v_\alpha q_\beta(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{p}_v S_{N,\nu} p_v), \quad Q_{6,p}^{(1)} = m_N\epsilon^{\alpha\beta\mu\nu}v_\alpha v_{\perp,\beta}(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{p}_v S_{N,\nu} p_v). \qquad (141)$$

The related operators for neutrons are obtained with a $p \to n$ replacement. From the $d = 2$ set of operators we need only

$$Q_{1,p}^{(2)} = (\bar{\chi}_v iq\cdot S_\chi \chi_v)(\bar{p}_v\, iq\cdot S_N\, p_v), \quad Q_{2,p}^{(2)} = im_N\,\epsilon^{\alpha\beta\mu\nu}v_\alpha q_\beta v_{\perp,\mu}(\bar{\chi}_v S_{\chi,\nu}\chi_v)(\bar{p}_v p_v). \qquad (142)$$

Above, we have defined several kinematic quantities for DM-nucleon scattering, $\chi(p_1)N(k_1) \to \chi(p_2)N(k_2)$, see Fig. 3. The momentum exchange is

$$q^\mu \equiv k_2^\mu - k_1^\mu = p_1^\mu - p_2^\mu, \qquad q^\mu = (q^0, \vec{q}). \qquad (143)$$

The definition of momentum exchange three vector is thus[8]

$$\vec{q} = \vec{k}_2 - \vec{k}_1 = \vec{p}_1 - \vec{p}_2. \qquad (144)$$

---

[8] This differs by a sign from [6], a difference that we will keep track of in our definitions of the NR operators.

It is also useful to define the four-component perpendicular relative velocity (see also [6])

$$v_\perp^\mu = \frac{1}{2}\left(\frac{p_1^\mu}{m_\chi} + \frac{p_2^\mu}{m_\chi} - \frac{k_1^\mu}{m_N} - \frac{k_2^\mu}{m_N}\right) = \Delta v^\mu - \frac{q^\mu}{2\mu_N}, \tag{145}$$

where Δv^μ is the initial relative velocity between DM and nucleon,

$$\Delta v^\mu = \frac{p_1^\mu}{m_\chi} - \frac{k_1^\mu}{m_N}, \tag{146}$$

and $\mu_N = m_\chi m_N/(m_\chi + m_N)$ the reduced mass of the DM–nucleon system (we work in the isospin limit, $m_N = m_p = m_n$). Note the difference in our notation between v^μ, the HBChPT velocity label, and Δv^μ, the initial relative velocity. In the lab frame we have $v^\mu = (1, \vec{0})$, while $\Delta v^\mu \sim \mathcal{O}(\Lambda_{\rm QCD}/m_N)$ arises primarily due to the movement of nucleons inside the nucleus. The perpendicular relative velocity obeys $v_\perp \cdot q = 0$. Furthermore, in the lab frame one has $v_\perp^\mu = (0, \vec{v}_\perp)$ so that also $\vec{v}_\perp \cdot \vec{q} = 0$.

The $d = 0$ Wilson coefficients for interactions of DM with protons are given by

$$c_{1,p}^{(0)} = 2\hat{\mathcal{C}}_{1,u}^{(6,0)} + \hat{\mathcal{C}}_{1,d}^{(6,0)} - \frac{2m_G}{27}\hat{\mathcal{C}}_1^{(7,0)} + \sigma_u^p \hat{\mathcal{C}}_{5,u}^{(7,0)} + \sigma_d^p \hat{\mathcal{C}}_{5,d}^{(7,0)} + \sigma_s \hat{\mathcal{C}}_{5,s}^{(7,0)} - \frac{\alpha Q_p}{2\pi m_\chi}\hat{\mathcal{C}}_1^{(5,0)}, \tag{147}$$

$$c_{2,p}^{(0)} = 4\left(\Delta u_p \hat{\mathcal{C}}_{4,u}^{(6,0)} + \Delta d_p \hat{\mathcal{C}}_{4,d}^{(6,0)} + \Delta s \hat{\mathcal{C}}_{4,s}^{(6,0)}\right) + \frac{2\alpha}{\pi}\frac{\mu_p}{m_N}\hat{\mathcal{C}}_1^{(5,0)}, \tag{148}$$

while the contributions for the neutrons are obtained through the replacement $p \to n$, $u \leftrightarrow d$. For convenience of notation we assumed that HDMET matching was done at tree level (see the end of this Section for the general case). The above results can then be used directly also for the relativistic form of the DM EFT (3)-(9) by simply replacing $\hat{C}_i^{(d,0)} \to \hat{C}_i^{(d)}$. The terms proportional to $\hat{\mathcal{C}}_1^{(5,0)}$ come from a single photon exchange. For the photon propagator we used that $v \cdot q = \mathcal{O}(q^2)$, so that $(q^2 - (v \cdot q)^2)/q^2 = 1 + \mathcal{O}(q^2)$. The low-energy constant m_G is the gluon contribution to the nucleon mass. The remaining HBChPT constants have been converted to nucleon sigma terms, $\sigma_{u,d}^p$, axial vector matrix elements, Δq_p, and nuclear magnetic moments, μ_N, using the leading-order expressions in Appendix C. Their values are given in Appendix C and are collected in Tab. II. $Q_{p(n)} = 1(0)$ is the proton (neutron) electric charge.

LE constant	value	LE constant	value	LE constant	value
$\Delta u_p = \Delta d_n$	$0.897(27)$	$\Delta d_p = \Delta u_n$	$-0.376(27)$	Δs	$-0.026(4)$
μ_p	2.79	μ_n	-1.91	μ_s	$-0.073(19)$
$B_0 m_u$	$(6200 \pm 400)\,\mathrm{MeV}^2$	$B_0 m_d$	$(13300 \pm 400)\,\mathrm{MeV}^2$	$B_0 m_s$	$(0.27 \pm 0.01)\,\mathrm{GeV}^2$
σ_u^p	$(17 \pm 5)\,\mathrm{MeV}$	σ_d^p	$(32 \pm 10)\,\mathrm{MeV}$	σ_s	$(41.3 \pm 7.7)\,\mathrm{MeV}$
σ_u^n	$(15 \pm 5)\,\mathrm{MeV}$	σ_d^n	$(36 \pm 10)\,\mathrm{MeV}$		
g_A	$1.2723(23)$	m_G	$848(14)\,\mathrm{MeV}$		

Table II: Numerical input values for the non-perturbative constants. For more details and references, see Appendix C.

The $d = 1$ Wilson coefficients are

$$
\begin{aligned}
c_{1,p}^{(1)} = {} & \frac{B_0\, g_A}{m_\pi^2 - q^2}\big(m_u \hat{\mathcal{C}}_{7,u}^{(7,0)} - m_d \hat{\mathcal{C}}_{7,d}^{(7,0)}\big) \\
& + \frac{B_0}{3}\frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 - q^2}\big(m_u \hat{\mathcal{C}}_{7,u}^{(7,0)} + m_d \hat{\mathcal{C}}_{7,d}^{(7,0)} - 2m_s \hat{\mathcal{C}}_{7,s}^{(7,0)}\big) \\
& - \tilde{m}\bigg[\frac{\Delta u_p}{m_u} + \frac{\Delta d_p}{m_d} + \frac{\Delta s}{m_s} + \frac{g_A}{2}\left(\frac{1}{m_u} - \frac{1}{m_d}\right)\frac{q^2}{m_\pi^2 - q^2} \\
& + \frac{1}{6}\big(\Delta u_p + \Delta d_p - 2\Delta s\big)\left(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s}\right)\frac{q^2}{m_\eta^2 - q^2}\bigg]\hat{\mathcal{C}}_3^{(7,0)}\,,
\end{aligned}
\tag{149}
$$

$$
c_{2,p}^{(1)} = -\sigma_u^p \hat{\mathcal{C}}_{6,u}^{(8,1)} - \sigma_d^p \hat{\mathcal{C}}_{6,d}^{(8,1)} - \sigma_s \hat{\mathcal{C}}_{6,s}^{(8,1)} + \frac{2m_G}{27}\hat{\mathcal{C}}_2^{(8,1)} - \frac{2\alpha Q_p}{\pi q^2}\hat{\mathcal{C}}_2^{(5,0)}\,,
\tag{150}
$$

$$
c_{3,p}^{(1)} = \frac{2}{m_N}\Big[\Delta u_p \hat{\mathcal{C}}_{3,u}^{(6,0)} + \Delta d_p \hat{\mathcal{C}}_{3,d}^{(6,0)} + \Delta s\, \hat{\mathcal{C}}_{3,s}^{(6,0)}\Big]\,,
\tag{151}
$$

$$
c_{4,p}^{(1)} = -\frac{2}{m_N}\Big[2\,\hat{\mathcal{C}}_{2,u}^{(6,0)} + \hat{\mathcal{C}}_{2,d}^{(6,0)}\Big]\,,
\tag{152}
$$

$$
\begin{aligned}
c_{5,p}^{(1)} = {} & \frac{4\hat{\mu}_u^p}{m_N}\hat{\mathcal{C}}_{2,u}^{(6,0)} + \frac{2\hat{\mu}_d^p}{m_N}\hat{\mathcal{C}}_{2,d}^{(6,0)} - \frac{6}{m_N}\mu_s\,\hat{\mathcal{C}}_{2,s}^{(6,0)} \\
& + \frac{2}{m_\chi}\Big(\Delta u_p \hat{\mathcal{C}}_{3,u}^{(6,0)} + \Delta d_p \hat{\mathcal{C}}_{3,d}^{(6,0)} + \Delta s\, \hat{\mathcal{C}}_{3,s}^{(6,0)}\Big)\,,
\end{aligned}
\tag{153}
$$

$$
c_{6,p}^{(1)} = 0\,,
\tag{154}
$$

while the Wilson coefficients for the interactions of DM with neutrons, c_i^n, are obtained by the replacements $p \to n, u \leftrightarrow d$. We have defined

$$
\tilde{m} = \left(\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right)^{-1}\,.
\tag{155}
$$

The above results apply to the relativistic form of the DM EFT (3)-(9) by replacing $\hat{C}_i^{(d,0)} \to \hat{C}_i^{(d)}$ and $\hat{C}_i^{(8,1)} \to \hat{C}_i^{(7)}$, as long as the matching to HDMET was performed at tree level

(see the end of this Section for the general case). In Eq. (149) and in Eq. (156) below we use $\Delta u_p - \Delta d_p = \Delta d_n - \Delta u_n = g_A$ as this combination is determined more precisely, see Eq. (C16). The B_0 coefficient is related to the quark condensate so that $B_0 m_q \sim m_\pi^2$, see Eq. (C27). We have also defined the contributions to the proton and neutron magnetic moments from the u- and d-quark currents, $\hat{\mu}_u^p = \hat{\mu}_d^n = 1.84, \hat{\mu}_d^p = \hat{\mu}_u^n = -1.03$ (see Eq. (C26)), while μ_s is the s quark contribution to the proton and neutron magnetic moments, see Eq. (C23).

The $d = 2$ Wilson coefficients are

$$
\begin{aligned}
c_{1,p}^{(2)} = {} & \frac{2\alpha}{\pi q^2} \frac{\mu_p}{m_N} \hat{\mathcal{C}}_1^{(5,0)} - \frac{2}{3} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 - q^2} \left(\hat{\mathcal{C}}_{4,u}^{(6,0)} + \hat{\mathcal{C}}_{4,d}^{(6,0)} - 2\hat{\mathcal{C}}_{4,s}^{(6,0)} \right) \\
& - \frac{2g_A}{m_\pi^2 - q^2} \left(\hat{\mathcal{C}}_{4,u}^{(6,0)} - \hat{\mathcal{C}}_{4,d}^{(6,0)} \right) + \frac{B_0}{m_\chi} \frac{g_A}{m_\pi^2 - q^2} \left(m_u \hat{\mathcal{C}}_{8,u}^{(7,0)} - m_d \hat{\mathcal{C}}_{8,d}^{(7,0)} \right) \\
& + \frac{B_0}{3m_\chi} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 - q^2} \left(m_u \hat{\mathcal{C}}_{8,u}^{(7,0)} + m_d \hat{\mathcal{C}}_{8,d}^{(7,0)} - 2m_s \hat{\mathcal{C}}_{8,s}^{(7,0)} \right) \\
& + \tilde{m} \left[\frac{\Delta u_p}{m_u} + \frac{\Delta d_p}{m_d} + \frac{\Delta s}{m_s} + \frac{g_A}{2} \left(\frac{1}{m_u} - \frac{1}{m_d} \right) \frac{q^2}{m_\pi^2 - q^2} \right. \\
& \left. + \frac{1}{6} (\Delta u_p + \Delta d_p - 2\Delta s) \left(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s} \right) \frac{q^2}{m_\eta^2 - q^2} \right] \hat{\mathcal{C}}_4^{(8,1)},
\end{aligned}
\tag{156}
$$

$$
c_{2,p}^{(2)} = - \frac{2\alpha Q_p}{\pi m_N q^2} \hat{\mathcal{C}}_1^{(5,0)},
\tag{157}
$$

while the expressions for the neutron follow from the replacements $p \to n, u \leftrightarrow d$. As before the above results also apply to the relativistic form of the DM EFT (3)-(9) by replacing $\hat{C}_i^{(d,0)} \to \hat{C}_i^{(d)}, \hat{\mathcal{C}}_4^{(8,1)} \to \hat{\mathcal{C}}_4^{(7)}/m_\chi$ (for tree level matching to HDMET). Note that, due to the photon $1/q^2$ pole, the $\hat{\mathcal{C}}_1^{(5,0)}$ contributions are of the same order as in (147), (149), even though they multiply operators that are $\mathcal{O}(q^2)$ suppressed. Similarly, due to the meson poles, the contributions proportional to the Wilson coefficients $\hat{\mathcal{C}}_{4,q}^{(6,0)}$ in $c_{1,p}^{(2)}$, coming from the right diagram in Fig. 1 are of the same chiral order as the $\hat{\mathcal{C}}_{4,q}^{(6,0)}$ terms in $c_{2,p}^{(0)}$, coming from the left diagram in Fig. 1.

The coefficients $c_{1,N}^{(1)}, c_{2,p}^{(1)}, c_{1,N}^{(2)}$ have a q^2-dependence from pion, η, and photon exchanges, i.e., they are non-local at the scale $q \sim m_\pi$. This signals that the above effective description of DM–nucleon interactions is not an effective field theory in the usual sense, and $\mathcal{L}_{\text{eff}}$ from (137) may only be used at tree level. The effective description does make sense, though, since the pion and the η cannot be kinematically produced and never appear as asymptotic states. In the scattering process q^2 is spacelike, so that one never reaches the

pion or η pole in the above expressions. The single photon exchange similarly leads to a classical potential for the DM-proton interactions.

The above results apply with trivial changes also if the matching to HDMET is performed beyond tree level. In that case one needs to replace $\hat{\mathcal{C}}_1^{(5,0)} \to m_\chi \hat{\mathcal{C}}_2^{(6,1)}$ in (147), $\hat{\mathcal{C}}_1^{(5,0)} \to m_\chi \hat{\mathcal{C}}_1^{(6,1)}$ in (157), $\hat{\mathcal{C}}_{3,q}^{(6,0)} \to m_\chi \hat{\mathcal{C}}_{3,q}^{(7,1)}$ in (151), $\hat{\mathcal{C}}_{3,q}^{(6,0)} \to m_\chi \hat{\mathcal{C}}_{6,q}^{(7,1)}$ in (153), $\hat{\mathcal{C}}_{8,q}^{(7,0)} \to m_\chi \hat{\mathcal{C}}_{8,q}^{(8,1)}$ in (156).

Using the results of Ref. [6] for the nuclear response in DM direct detection, the cross section for DM scattering on the nucleus is given by[9]

$$\frac{d\sigma}{dE_R} = \frac{m_A}{2\pi|\vec{v}_\chi|^2} \frac{1}{(2J_\chi + 1)} \frac{1}{(2J_A + 1)} \sum_{\text{spins}} |\mathcal{M}|_{\text{NR}}^2, \tag{158}$$

where E_R is the recoil energy of the nucleus, m_A its mass, and $\vec{v}_\chi$ the initial DM velocity in the lab frame. The non-vanishing contributions to the matrix element squared are [6]

$$\frac{1}{2J_\chi + 1} \frac{1}{2J_A + 1} \sum_{\text{spins}} |\mathcal{M}|_{\text{NR}}^2 = \frac{4\pi}{2J_A + 1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \left\{ R_M^{\tau\tau'} W_M^{\tau\tau'}(q) + R_{\Sigma''}^{\tau\tau'} W_{\Sigma''}^{\tau\tau'}(q) \right.$$
$$\left. + R_{\Sigma'}^{\tau\tau'} W_{\Sigma'}^{\tau\tau'}(q) + \frac{\vec{q}^{\,2}}{m_N^2} \left[R_\Delta^{\tau\tau'} W_\Delta^{\tau\tau'}(q) + R_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q) \right] \right\}, \tag{159}$$

where $J_\chi = 1/2$ is the spin of DM, and J_A is the spin of the target nucleus. The nonrelativistic matrix element $\mathcal{M}_{\text{NR}}$ has the same normalization as the one in [6]. The coefficients $R_i^{\tau\tau'}$ depend on $\vec{v}_T^{\perp 2}$, $\vec{q}^{\,2}/m_N^2$, as well as on the coefficients $c_{i,N}^{(d)}$ in (147)-(157), and are given by [6]

$$R_M^{\tau\tau'} = c_{1,\tau}^{(0)} c_{1,\tau'}^{(0)} + \frac{m_N^2}{4} \left[\frac{\vec{q}^{\,2}}{m_N^2} c_{2,\tau}^{(1)} c_{2,\tau'}^{(1)} + \vec{v}_T^{\perp 2} \left(c_{4,\tau}^{(1)} c_{4,\tau'}^{(1)} + \vec{q}^{\,2} c_{2,\tau}^{(2)} c_{2,\tau'}^{(2)} \right) \right], \tag{160}$$

$$R_{\Sigma''}^{\tau\tau'} = \frac{1}{16} \left[c_{2,\tau}^{(0)} c_{2,\tau'}^{(0)} + \vec{q}^{\,2} \left(c_{2,\tau}^{(0)} c_{1,\tau'}^{(2)} + c_{1,\tau}^{(2)} c_{2,\tau'}^{(0)} + 4 c_{1,\tau}^{(1)} c_{1,\tau'}^{(1)} \right) + \vec{q}^{\,4} c_{1,\tau}^{(2)} c_{1,\tau'}^{(2)} \right], \tag{161}$$

$$R_{\Sigma'}^{\tau\tau'} = \frac{m_N^2}{8} \vec{v}_T^{\perp 2} c_{3,\tau}^{(1)} c_{3,\tau'}^{(1)} + \frac{1}{16} \left(c_{2,\tau}^{(0)} c_{2,\tau'}^{(0)} + \vec{q}^{\,2} c_{5,\tau}^{(1)} c_{5,\tau'}^{(1)} \right), \tag{162}$$

$$R_\Delta^{\tau\tau'} = \frac{m_N^2}{4} \left(c_{4,\tau}^{(1)} c_{4,\tau'}^{(1)} + \vec{q}^{\,2} c_{2,\tau}^{(2)} c_{2,\tau'}^{(2)} \right), \tag{163}$$

$$R_{\Delta\Sigma'}^{\tau\tau'} = \frac{m_N^2}{4} \left(c_{4,\tau}^{(1)} c_{5,\tau'}^{(1)} - c_{2,\tau}^{(2)} c_{2,\tau'}^{(0)} \right). \tag{164}$$

Note that (160)-(164) are already specific to the case of fermionic DM, $J_\chi = 1/2$ (see [6] for the general expression). Above,

$$\vec{v}_T^\perp = \vec{v}_\chi - \vec{q}/(2\mu_{\chi A}), \tag{165}$$

---

[9] For the reader's convenience we translate our notation to the basis of [6] in Appendix A.

is the component of initial DM velocity in the lab frame, $\vec{v}_\chi$, that is perpendicular to $\vec{q}$, in complete analogy with the single nucleon case (145). The typical value is $|\vec{v}_T^\perp| \sim 10^{-3}$. Here $\mu_{\chi A} = 1/(1/m_\chi + 1/m_A)$ is the reduced mass of the DM and the nucleus. The sum in (159) is over isospin values $\tau = 0, 1$. The Wilson coefficients $c_{i,\tau}^{(d)}$ are related to the proton and neutron Wilson coefficients through

$$c_{i,0}^{(d)} = \frac{1}{2}\big(c_{i,p}^{(d)} + c_{i,n}^{(d)}\big), \qquad c_{i,1}^{(d)} = \frac{1}{2}\big(c_{i,p}^{(d)} - c_{i,n}^{(d)}\big). \tag{166}$$

V. SCALAR DARK MATTER

The above results are easily extended to the case of scalar DM.[10] For relativistic scalar DM, denoted by φ, the effective interactions with the SM start at dimension six,

$$\mathcal{L}_\varphi = \hat{\mathcal{C}}_a^{(6)} \mathcal{Q}_a^{(6)} + \cdots , \qquad \text{where} \quad \hat{\mathcal{C}}_a^{(6)} = \frac{\mathcal{C}_a^{(6)}}{\Lambda^2} , \tag{167}$$

where ellipses denote higher dimension operators. The dimension-six operators are

$$\mathcal{Q}_{1,q}^{(6)} = \big(\varphi^* i \overset{\leftrightarrow}{\partial}_\mu \varphi\big)(\bar{q}\gamma^\mu q), \qquad\qquad \mathcal{Q}_{2,q}^{(6)} = \big(\varphi^* i \overset{\leftrightarrow}{\partial}_\mu \varphi\big)(\bar{q}\gamma^\mu \gamma_5 q), \tag{168}$$

$$\mathcal{Q}_{3,q}^{(6)} = m_q(\varphi^*\varphi)(\bar{q}q) , \qquad\qquad\qquad \mathcal{Q}_{4,q}^{(6)} = m_q(\varphi^*\varphi)(\bar{q}i\gamma_5 q) , \tag{169}$$

$$\mathcal{Q}_5^{(6)} = \frac{\alpha_s}{12\pi}(\varphi^*\varphi)G^{a\mu\nu}G_{\mu\nu}^a , \qquad\qquad \mathcal{Q}_6^{(6)} = \frac{\alpha_s}{8\pi}(\varphi^*\varphi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a . \tag{170}$$

$$\mathcal{Q}_7^{(6)} = i\frac{e}{8\pi^2}\big(\partial_\mu\varphi^*\partial_\nu\varphi\big)F^{\mu\nu} , \tag{171}$$

Here $\overset{\leftrightarrow}{\partial}_\mu$ is defined through $\phi_1\overset{\leftrightarrow}{\partial}_\mu\phi_2 = \phi_1\partial_\mu\phi_2 - (\partial_\mu\phi_1)\phi_2$, and $q = u, d, s$ again denote the light quarks. The strong coupling constant α_s is taken at $\mu \sim 1$ GeV. The operator $\mathcal{Q}_6^{(6)}$ is CP-odd, while the other operators are CP-even. Note that because there are also leptonic equivalents to the operators $\mathcal{Q}_{1,q}^{(6)}$ which we do not include in the analysis, the inclusion of $\mathcal{Q}_7^{(6)}$ is not redundant (the equations of motion relate $\partial^\mu F_{\mu\nu} = \sum_f eQ_f \bar{f}\gamma_\nu f$ where f are both quarks and leptons).

In (168)-(171) we kept the leading operators that one would get from a UV theory of complex scalar DM for each of the chiral and flavor structures. At dimension six there are also the Rayleigh operators $(\varphi^*\varphi)F^{\mu\nu}F_{\mu\nu}$ and $(\varphi^*\varphi)F^{\mu\nu}\widetilde{F}_{\mu\nu}$ which, however, lead to

---

[10] For operators, spurions, and Wilson coefficients we adopt the same notation for scalar DM as for fermionic DM. No confusion should arise as this abuse of notation is restricted to this section.

scattering rates suppressed by a factor of α compared to $\mathcal{Q}_7^{(6)}$ [41]. For real scalar DM the operators $\mathcal{Q}_{1,q}^{(6)}$, $\mathcal{Q}_{2,q}^{(6)}$, and $\mathcal{Q}_7^{(6)}$ vanish, and one would need to consider subleading operators. In this paper we limit ourselves to the case of complex scalar DM.[11]

The next step is to consider scalar DM interactions with the visible sector in HDMET. To derive it we factor out the large momenta,

$$\varphi(x) = \frac{1}{\sqrt{2m_\varphi}} e^{-im_\varphi v \cdot x} \varphi_v . \tag{172}$$

The HDMET for scalar DM is thus

$$\mathcal{L}_{\text{HDMET}} = \varphi_v^* iv \cdot \partial \varphi_v + \frac{1}{2m_\varphi} \varphi_v^* (i\partial_\perp)^2 \varphi_v + \cdots + \mathcal{L}_{\varphi_v} . \tag{173}$$

The first term is the LO HDMET for scalar fields. The $1/m_\varphi$ term is fixed by reparametrization invariance [24], while the ellipses denote the higher-order terms. The interaction Lagrangian $\mathcal{L}_{\varphi_v}$ is also expanded in $1/m_\varphi$,

$$\mathcal{L}_{\varphi_v} = \sum_{d,m} \hat{\mathcal{C}}_a^{(d,m)} \mathcal{Q}_a^{(d,m)}, \qquad \text{where} \qquad \hat{\mathcal{C}}_a^{(d,m)} = \frac{\mathcal{C}_a^{(d,m)}|_{n_f=5}}{\Lambda^{d-m-4} m_\varphi^m} . \tag{174}$$

As for fermionic DM, the operators $\mathcal{Q}_a^{(d,m)}$ arise as the terms of order $1/m_\chi^m$ in the HDMET expansion of the UV operators $\mathcal{Q}_a^{(d)}$ in (168)-(171). Because of the derivatives acting on scalar fields the index m can also be negative, since

$$i\big(\varphi^* \overset{\leftrightarrow}{\partial}_\mu \varphi\big) \to 2m_\varphi v_\mu \big(\varphi_v^* \varphi_v\big) + i\big(\varphi_v^* \overset{\leftrightarrow}{\partial}_\mu \varphi_v\big) + \cdots , \tag{175}$$

$$\big(\partial_{[\mu} \varphi^* \partial_{\nu]} \varphi\big) \to im_\varphi v_{[\mu} \partial_{\nu]} \big(\varphi_v^* \varphi_v\big) + \big(\partial_{[\mu} \varphi_v^* \partial_{\nu]} \varphi_v\big) + \cdots . \tag{176}$$

We thus have three HDMET operators that start at dimension five

$$\mathcal{Q}_{1,q}^{(5,-1)} = 2(\varphi_v^* \varphi_v)(\bar{q}\slashed{v}q), \qquad\qquad \mathcal{Q}_{2,q}^{(5,-1)} = 2(\varphi_v^* \varphi_v)(\bar{q}\slashed{v}\gamma_5 q), \tag{177}$$

$$\mathcal{Q}_3^{(5,-1)} = -\frac{e}{8\pi^2} v_\mu \partial_\nu \big(\varphi_v^* \varphi_v\big) F^{\mu\nu}. \tag{178}$$

The relevant dimension-six operators are

$$\mathcal{Q}_{1,q}^{(6,0)} = \big(\varphi_v^* i\overset{\leftrightarrow}{\partial}_\mu \varphi_v\big)(\bar{q}\gamma^\mu q), \qquad\qquad \mathcal{Q}_{2,q}^{(6,0)} = \big(\varphi_v^* i\overset{\leftrightarrow}{\partial}_\mu \varphi_v\big)(\bar{q}\gamma^\mu \gamma_5 q), \tag{179}$$

---

[11] We have also neglected the contributions of operators of dimension seven and higher that are promoted to dimension five or six in going to HDMET, e.g., $\partial^\mu \varphi^* \partial_\mu \varphi\, \bar{q}q \to m_\varphi^2 \varphi_v^* \varphi_v\, \bar{q}q$. These operators are suppressed by additional powers of m_φ/Λ compared to the operators that we consider.

$$\mathcal{Q}_{3,q}^{(6,0)} = m_q(\varphi_v^*\varphi_v)(\bar{q}q)\,, \qquad\qquad \mathcal{Q}_{4,q}^{(6,0)} = m_q(\varphi_v^*\varphi_v)(\bar{q}i\gamma_5 q)\,, \qquad (180)$$

$$\mathcal{Q}_5^{(6,0)} = \frac{\alpha_s}{12\pi}(\varphi_v^*\varphi_v)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_6^{(6,0)} = \frac{\alpha_s}{8\pi}(\varphi_v^*\varphi_v)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad (181)$$

$$\mathcal{Q}_7^{(6,0)} = i\frac{e}{8\pi^2}\left(\partial_\mu\varphi_v^*\partial_\nu\varphi_v\right)F^{\mu\nu}. \qquad (182)$$

These are simple extensions of the relativistic operators in (168)-(171), but the derivatives are now all $\mathcal{O}(q)$ since they act on HDMET fields. Reparametrization invariance fixes

$$\mathcal{C}_{1,q}^{(5,-1)} = \mathcal{C}_{1,q}^{(6,0)}\,, \qquad \mathcal{C}_{2,q}^{(5,-1)} = \mathcal{C}_{2,q}^{(6,0)}\,, \qquad \mathcal{C}_3^{(5,-1)} = \mathcal{C}_7^{(6,0)} \qquad (183)$$

to all loop orders in the matching at the scale m_χ. In the Lagrangian (174) the operators thus always appear in the linear combinations

$$m_\varphi\mathcal{Q}_{1,q}^{(5,-1)} + \mathcal{Q}_{1,q}^{(6,0)}\,, \qquad m_\varphi\mathcal{Q}_{2,q}^{(5,-1)} + \mathcal{Q}_{2,q}^{(6,0)}\,, \qquad m_\varphi\mathcal{Q}_3^{(5,-1)} + \mathcal{Q}_7^{(6,0)}. \qquad (184)$$

It is now easy to obtain the ChPT and HBChPT Lagrangians. The external spurions in the QCD Lagrangian (38) are, for relativistic DM,

$$v_\mu(x) = -e\bar{Q}_q A_\mu^e + v_{\chi,\mu} = -e\bar{Q}_q A_\mu^e + \bar{\mathcal{C}}_1^{(6)}\big(\varphi^* i\overleftrightarrow{\partial}_\mu\varphi\big), \qquad (185)$$

$$a_\mu(x) = \bar{\mathcal{C}}_2^{(6)}\big(\varphi^* i\overleftrightarrow{\partial}_\mu\varphi\big), \qquad (186)$$

$$s(x) = \mathcal{M}_q + s_\chi = \mathcal{M}_q - \mathcal{M}_q\bar{\mathcal{C}}_3^{(6)}\big(\varphi^*\varphi\big), \qquad (187)$$

$$p(x) = \mathcal{M}_q\bar{\mathcal{C}}_4^{(6)}\big(\varphi^*\varphi\big), \qquad (188)$$

$$s_G(x) = \hat{\mathcal{C}}_5^{(6)}\big(\varphi^*\varphi\big), \qquad (189)$$

$$\theta(x) = \hat{\mathcal{C}}_6^{(6)}\big(\varphi^*\varphi\big). \qquad (190)$$

For HDMET the external spurions are thus

$$v_\mu(x) = -e\bar{Q}_q A_\mu^e + v_{\chi,\mu} = -e\bar{Q}_q A_\mu^e + 2v^\mu\bar{\mathcal{C}}_1^{(5,-1)}(\varphi_v^*\varphi_v) + \bar{\mathcal{C}}_1^{(6,0)}\big(\varphi_v^* i\overleftrightarrow{\partial}_\mu\varphi_v\big) + \cdots\,, \qquad (191)$$

$$a_\mu(x) = 2v^\mu\bar{\mathcal{C}}_2^{(5,-1)}(\varphi_v^*\varphi_v) + \bar{\mathcal{C}}_2^{(6,0)}\big(\varphi_v^* i\overleftrightarrow{\partial}_\mu\varphi_v\big) + \cdots\,, \qquad (192)$$

$$s(x) = \mathcal{M}_q + s_\chi = \mathcal{M}_q - \mathcal{M}_q\bar{\mathcal{C}}_3^{(6,0)}\big(\varphi_v^*\varphi_v\big) + \cdots\,, \qquad (193)$$

$$p(x) = \mathcal{M}_q\bar{\mathcal{C}}_4^{(6,0)}\big(\varphi_v^*\varphi_v\big) + \cdots\,, \qquad (194)$$

$$s_G(x) = \hat{\mathcal{C}}_5^{(6,0)}\big(\varphi_v^*\varphi_v\big) + \cdots\,, \qquad (195)$$

$$\theta(x) = \hat{\mathcal{C}}_6^{(6,0)}\big(\varphi_v^*\varphi_v\big) + \cdots\,, \qquad (196)$$

with ellipses denoting higher order terms. We use the same notation for Wilson coefficients as in (45). Note that the derivatives act on HDMET fields and are thus $\mathcal{O}(q)$.

From this we can immediately obtain the ChPT interactions for scalar DM,

$$\mathcal{L}^{(1)}_{\varphi,\mathrm{ChPT}} = -if^2 m_\varphi (\varphi_v^* \varphi_v) v^\mu \,\mathrm{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\,\bar{\mathcal{C}}_1^{(6,0)} + (U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\,\bar{\mathcal{C}}_2^{(6,0)}\right], \quad (197)$$

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\varphi,\mathrm{ChPT}} \supset &-\frac{B_0 f^2}{2}(\varphi_v^* \varphi_v)\,\mathrm{Tr}\left[(U + U^\dagger)\mathcal{M}_q \bar{\mathcal{C}}_3^{(6,0)} + i(U - U^\dagger)\mathcal{M}_q \bar{\mathcal{C}}_4^{(6,0)}\right] \\
&+ (\varphi_v^* \varphi_v)\left[\frac{2}{27}\,\mathrm{Tr}\left(\partial_\mu \Pi \partial^\mu \Pi\right) - \frac{6}{27} B_0\,\mathrm{Tr}\left(\mathcal{M}_q \Pi^2\right)\right]\hat{\mathcal{C}}_5^{(6,0)} \\
&- \frac{if^2}{2}\big(\varphi_v^* i\overset{\leftrightarrow}{\partial}_\mu \varphi_v\big)\,\mathrm{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\,\bar{\mathcal{C}}_1^{(6,0)} + (U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\,\bar{\mathcal{C}}_2^{(6,0)}\right].
\end{aligned}
\quad (198)
$$

In (197) we used the relations $\bar{\mathcal{C}}_1^{(5,-1)} = m_\varphi \bar{\mathcal{C}}_1^{(6,0)}$ and $\bar{\mathcal{C}}_2^{(5,-1)} = m_\varphi \bar{\mathcal{C}}_2^{(6,0)}$, valid to all orders in perturbation theory (cf. Eq. (183)), in order to explicitly show the m_φ dependence. Compared to the fermionic DM ChPT Lagrangians in (75)-(77), there are fewer terms in (197)-(198), as there is no equivalent of the pseudoscalar and axial-vector DM currents for scalar DM. Working at LO we thus do not need to consider $\mathcal{L}^{(3)}_{\varphi,\mathrm{ChPT}}$ at all. Note that in $\mathcal{L}^{(2)}_{\chi,\mathrm{ChPT}}$ we need to keep the $\mathcal{O}(p^2)$ terms proportional to $\bar{\mathcal{C}}_{1,2}^{(6,0)}$, even though these Wilson coefficients appear already in $\mathcal{L}^{(1)}_{\varphi,\mathrm{ChPT}}$. Both of these terms give contributions of the same order in ChEFT since (197) gives $v\cdot q \sim \mathcal{O}(p^2)$ suppressed contributions for typical external momenta.

The HBChPT interaction Lagrangians for scalar DM are

$$
\begin{aligned}
\mathcal{L}^{(0)}_{\varphi,\mathrm{HBChPT}} = &\, m_\varphi (\varphi_v^* \varphi_v)\Big(\mathrm{Tr}\,\bar{B}_v\big[(\xi^\dagger \bar{\mathcal{C}}_1^{(6,0)}\xi + \xi\bar{\mathcal{C}}_1^{(6,0)}\xi^\dagger), B_v\big] + 2\,\mathrm{Tr}\,\bar{B}_v B_v\,\mathrm{Tr}\,\bar{\mathcal{C}}_1^{(6,0)}\Big) \\
&- \frac{2}{27} m_G\,(\varphi_v^* \varphi_v)\,\mathrm{Tr}(\bar{B}_v B_v)\,\hat{\mathcal{C}}_5^{(6,0)},
\end{aligned}
\quad (199)
$$

$$
\begin{aligned}
\mathcal{L}^{(1)}_{\varphi,\mathrm{HBChPT}} \supset &\, 2m_\varphi(\varphi_v^* \varphi_v)\sum_q v\cdot \tilde{J}^{A,\mathrm{NLO}}_{q,\mu}\hat{\mathcal{C}}_{2,q}^{(6,0)} \\
&+ i(\varphi_v^* \overset{\leftrightarrow}{\partial}_\mu \varphi_v)\Big(\frac{1}{2}\,\mathrm{Tr}\,\bar{B}_v\big[v^\mu(\xi^\dagger \bar{\mathcal{C}}_2^{(6,0)}\xi - \xi\bar{\mathcal{C}}_2^{(6,0)}\xi^\dagger), B_v\big] \\
&\qquad + D\,\mathrm{Tr}\,\bar{B}_v S_N^\mu\{\xi^\dagger \bar{\mathcal{C}}_2^{(6,0)}\xi + \xi\bar{\mathcal{C}}_2^{(6,0)}\xi^\dagger, B_v\} \\
&\qquad + F\,\mathrm{Tr}\,\bar{B}_v S_N^\mu\big[\xi^\dagger \bar{\mathcal{C}}_2^{(6,0)}\xi + \xi\bar{\mathcal{C}}_2^{(6,0)}\xi^\dagger, B_v\big] + 2G\,\mathrm{Tr}\,\bar{B}_v S_N^\mu B_v\,\mathrm{Tr}\,\bar{\mathcal{C}}_2^{(6,0)}\Big) \\
&- (\varphi_v^* \varphi_v)\frac{\hat{\mathcal{C}}_6^{(6,0)}}{2\,\mathrm{Tr}(\mathcal{M}_q^{-1})}\Big\{\frac{1}{2}v\cdot\partial\,\mathrm{Tr}\,\bar{B}_v\big[(\xi^\dagger \mathcal{M}_q^{-1}\xi - \xi\mathcal{M}_q^{-1}\xi^\dagger), B_v\big] \\
&\qquad + \partial_\mu\Big(D\,\mathrm{Tr}\,\bar{B}_v S_N^\mu\{\xi^\dagger \mathcal{M}_q^{-1}\xi + \xi\mathcal{M}_q^{-1}\xi^\dagger, B_v\} \\
&\qquad + F\,\mathrm{Tr}\,\bar{B}_v S_N^\mu\big[\xi^\dagger \mathcal{M}_q^{-1}\xi + \xi\mathcal{M}_q^{-1}\xi^\dagger, B_v\big] + 2G\,\mathrm{Tr}(\mathcal{M}_q^{-1})\,\mathrm{Tr}\,\bar{B}_v S_N^\mu B_v\Big)\Big\},
\end{aligned}
\quad (200)
$$

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\varphi,\mathrm{HBChPT}} \supset &-(\varphi_v^* \varphi_v)\Big[b_0\,\mathrm{Tr}(\bar{B}_v B_v)\,\mathrm{Tr}\,\mathcal{M}_q\big(\bar{\mathcal{C}}_3^{(6,0)}(U^\dagger + U) - i\bar{\mathcal{C}}_4^{(6,0)}(U^\dagger - U)\big) \\
&+ b_D\,\mathrm{Tr}\,\bar{B}_v\{\xi^\dagger \mathcal{M}_q\big(\bar{\mathcal{C}}_3^{(6,0)} - i\bar{\mathcal{C}}_4^{(6,0)}\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}_3^{(6,0)} + i\bar{\mathcal{C}}_4^{(6,0)}\big)\xi, B_v\} \\
&+ b_F\,\mathrm{Tr}\,\bar{B}_v\big[\xi^\dagger \mathcal{M}_q\big(\bar{\mathcal{C}}_3^{(6,0)} - i\bar{\mathcal{C}}_4^{(6,0)}\big)\xi^\dagger + \xi\mathcal{M}_q\big(\bar{\mathcal{C}}_3^{(6,0)} + i\bar{\mathcal{C}}_4^{(6,0)}\big)\xi, B_v\big]\Big].
\end{aligned}
\quad (201)
$$

The contribution of $\mathcal{O}(p^0)$ to the Wilson coefficient $\bar{\mathcal{C}}_2^{(5,-1)}$ cancels because $v \cdot \tilde{J}_{q,\mu}^{A,\mathrm{LO}} = 0$, so that the leading contributions are given by $\mathcal{L}_{\varphi,\mathrm{HBChPT}}^{(1)}$ in (200), where we used the all-order relation $\bar{\mathcal{C}}_2^{(5,-1)} = m_\varphi \bar{\mathcal{C}}_2^{(6,0)}$, Eq. (183), to make the dependence on m_φ explicit. The expression for the NLO axial-vector current $\tilde{J}_{q,\mu}^{A,\mathrm{NLO}}$ is given in (B42). The diagonal matrix of Wilson coefficients $\bar{\mathcal{C}}_i$ was defined in (45). The A-nucleon irreducible amplitudes follow the same scaling within ChEFT as for fermionic DM, Eq. (135), with the trivial replacement $\epsilon_\chi \to \epsilon_\varphi$. Here, the effective chiral dimension ϵ_φ is the same as for the fermionic DM, as we did not include the dimension of the external DM fields in its definition, and we have $\epsilon_\varphi = d - 2$ for $\mathcal{L}_{\varphi,\mathrm{ChPT}}^{(d)}$ and $\epsilon_\varphi = d - 1$ for $\mathcal{L}_{\varphi,\mathrm{HBChPT}}^{(d)}$.

Accounting for the effect of π^0 and η exchange through q-dependent Wilson coefficients results in an effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \sum_{i,d} \left(c_{i,p}^{(d)}(q^2) \, Q_{i,p}^{(d)} + c_{i,n}^{(d)}(q^2) \, Q_{i,n}^{(d)} \right), \tag{202}$$

where d denotes the number of derivatives. For scalar DM we have, for $d = 0, 1$,

$$Q_{1,p}^{(0)} = \left(\varphi_v^* \varphi_v \right) \left(\bar{p}_v p_v \right), \tag{203}$$

$$Q_{1,p}^{(1)} = \left(\varphi_v^* \varphi_v \right) \left(\bar{p}_v i q \cdot S_N p_v \right), \qquad Q_{2,p}^{(1)} = m_N \left(\varphi_v^* \varphi_v \right) \left(\bar{p}_v \, v_\perp \cdot S_N \, p_v \right), \tag{204}$$

with a similar set of operators for neutrons, with $p \to n$. Unlike fermionic DM we do not need the $d = 2$ operators when working to leading order. For fermionic DM, photon exchange and couplings of the DM spin to axial-vector and pseudoscalar quark currents lead to momentum-suppressed operators in the nonrelativistic limit. Because of the enhancement by the photon and pion poles, respectively, these contributions were of leading order. No such terms are possible for scalar DM as it does not carry spin. The leading contributions from the operators (168)-(171) are thus already captured by the nonrelativistic operators (203) and (204) with up to one derivative. The matching calculation gives, for scalar DM,

$$
\begin{aligned}
c_{1,p}^{(0)} = {}& 2m_\varphi \left(2\hat{\mathcal{C}}_{1,u}^{(6,0)} + \hat{\mathcal{C}}_{1,d}^{(6,0)} \right) - \frac{2m_G}{27} \hat{\mathcal{C}}_5^{(6,0)} \\
& + \sigma_u^p \hat{\mathcal{C}}_{3,u}^{(6,0)} + \sigma_d^p \hat{\mathcal{C}}_{3,d}^{(6,0)} + \sigma_s \hat{\mathcal{C}}_{3,s}^{(6,0)} - \frac{\alpha Q_p}{2\pi} m_\varphi \hat{\mathcal{C}}_7^{(6,0)} ,
\end{aligned}
\tag{205}
$$

$$c_{1,p}^{(1)} = \frac{B_0 \, g_A}{m_\pi^2 + \vec{q}^{\,2}} \left(m_u \hat{\mathcal{C}}_{4,u}^{(6,0)} - m_d \hat{\mathcal{C}}_{4,d}^{(6,0)} \right)$$
$$+ \frac{B_0}{3} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 + \vec{q}^{\,2}} \left(m_u \hat{\mathcal{C}}_{4,u}^{(6,0)} + m_d \hat{\mathcal{C}}_{4,d}^{(6,0)} - 2m_s \hat{\mathcal{C}}_{4,s}^{(6,0)} \right)$$
$$- \tilde{m} \left[\frac{\Delta u_p}{m_u} + \frac{\Delta d_p}{m_d} + \frac{\Delta s}{m_s} - \frac{g_A}{2} \left(\frac{1}{m_u} - \frac{1}{m_d} \right) \frac{\vec{q}^{\,2}}{m_\pi^2 + \vec{q}^{\,2}} \right.$$
$$\left. - \frac{1}{6} (\Delta u_p + \Delta d_p - 2\Delta s) \left(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s} \right) \frac{\vec{q}^{\,2}}{m_\eta^2 + \vec{q}^{\,2}} \right] \hat{\mathcal{C}}_6^{(6,0)} , \quad (206)$$

$$c_{2,p}^{(1)} = \frac{4m_\varphi}{m_N} \left[\Delta u_p \, \hat{\mathcal{C}}_{2,u}^{(6,0)} + \Delta d_p \, \hat{\mathcal{C}}_{2,d}^{(6,0)} + \Delta s \, \hat{\mathcal{C}}_{2,s}^{(6,0)} \right]. \quad (207)$$

Starting from the EFT for relativistic DM (168)-(171), the above results can be used by simply replacing $\hat{\mathcal{C}}_i^{(6,0)} \to \hat{\mathcal{C}}_i^{(6)} = \mathcal{C}_i^{(6)}/\Lambda^2$. Because of the reparametrization invariance relation (183), they are also valid if the masses of DM and mediators are comparable, in which case the matching to HDMET is done at the same time as the mediators are being integrated out, i.e., at $\mu \sim \Lambda \sim m_\varphi$. Note that $B_0 m_q \sim \mathcal{O}(m_\pi^2)$, with the explicit relations given in (C27). In terms of the p^ν scaling we have for the leading contributions proportional to the $\hat{\mathcal{C}}_a^{(n,m)}$ Wilson coefficients

$$\nu_{\min} = \nu_{\mathrm{LO}}[J_\varphi^V \cdot \tilde{J}_q^V] = \nu_{\mathrm{LO}}[J_\varphi^S \tilde{J}^G] ,$$
$$\nu_{\min} + 1 = \nu_{\mathrm{LO}}[J_\varphi^V \cdot \tilde{J}_q^A] = \nu_{\mathrm{LO}}[J_\varphi^S \tilde{J}^\theta] = \nu_{\mathrm{LO}}[J_\varphi^S J_q^P] , \quad (208)$$
$$\nu_{\min} + 2 = \nu_{\mathrm{LO}}[J_\varphi^S \tilde{J}_q^S] ,$$

where we follow the same notation as for the case of fermionic DM for ease of comparison. The vector and scalar DM currents are $J_\varphi^V = i\varphi^* \overleftrightarrow{\partial}_\mu \varphi$ and $J_\varphi^S = \varphi^* \varphi$, respectively, with their HDMET decomposition given in (177), (178). The leading contributions for the $J_\varphi^S J_q^P$ interaction comes from the right diagram in Fig. 1. The pion exchange reduces the chiral scaling of the resulting amplitude by one, compared to the contact interaction. For all the other operators the leading contribution comes from the left diagram in Fig. 1, so that the chiral scaling is given by the chiral dimension of the corresponding HBChPT Lagrangian, $\mathcal{L}_{\varphi,\mathrm{HBChPT}}^{(d)}$. As before, $\nu_{\min} = 3 - 3A$ simply reflects our normalization of the A-nucleon state. Note that the results (205)-(206) are valid for matching at $\mu \sim m_\varphi$ to all loop orders, but only to leading order in the chiral expansion.

The cross section for scalar DM scattering on the nucleus is [6]

$$\frac{d\sigma}{dE_R} = \frac{m_A}{2\pi |\vec{v}_\chi|^2} \frac{4\pi}{2J_A + 1} \left[\sum_{\tau,\tau'=0,1} R_M^{\tau\tau'} W_M^{\tau\tau'}(q) + R_{\Sigma''}^{\tau\tau'} W_{\Sigma''}^{\tau\tau'}(q) + R_{\Sigma'}^{\tau\tau'} W_{\Sigma'}^{\tau\tau'}(q) \right], \quad (209)$$

where E_R is the recoil energy of the nucleus, m_A its mass, $\vec{v}_\chi$ the initial DM velocity in the lab frame, and $W_i^{\tau\tau'}(q)$ the nuclear response functions. The coefficients multiplying them are given by

$$R_M^{\tau\tau'} = c_{1,\tau}^{(0)} c_{1,\tau'}^{(0)}, \qquad R_{\Sigma''}^{\tau\tau'} = \frac{1}{4}\vec{q}^{\,4} c_{1,\tau}^{(1)} c_{1,\tau'}^{(1)}, \qquad R_{\Sigma'}^{\tau\tau'} = \frac{m_N^2}{8}\vec{v}_T^{\perp 2} c_{2,\tau}^{(1)} c_{2,\tau'}^{(1)}. \qquad (210)$$

The perpendicular velocity $\vec{v}_T^\perp$ is defined in (165), while the relations between the coefficients in (205)-(207) and the coefficients in the isospin basis $c_{i,\tau}^{(d)}$ are given in (166).

VI. CONCLUSIONS

Dark Matter scattering in direct detection is naturally described by an EFT if the mediators are heavier than about $\sim 1\,\text{GeV}$. We performed the leading order matching between the EFT with quark, gluons and photons as the external states and the EFT that describes DM interactions with light mesons and nucleons. We covered both fermionic and scalar DM and analyzed the operators that correspond to interactions between the visible and DM sector up to and including dimension-six operators above the electroweak scale. The resulting EFT was then used to obtain the coefficients that multiply the nuclear response functions, see, e.g., Ref. [6]. Our main results for fermionic DM are given in (147)-(157). With these one can go directly from the EFT with quarks, gluons and photons, Eqs. (3)-(9) to the nuclear response functions and DM scattering rates. The results for scalar DM are given in (205)-(206). The translation to the notation of Ref. [6] for fermionic DM is given in Appendix A, in Eqs. (A13)-(A21). Note that only a subset of 9 out of 14 possible nonrelativistic operators with up to two derivatives is generated in our set-up.

For each of the initial operators coupling DM to quarks and gluons we derived the leading contributions when they hadronize. In order to compare the size of different potential contributions we used chiral power counting in the ChEFT of nuclear forces, where we counted the momentum exchange between DM and the nucleus as $|\vec{q}| \sim m_\pi \sim \mathcal{O}(200\,\text{MeV})$.

Using this counting one can see, for instance, that for fermionic DM the axial-axial operator induces two different leading contributions to the spin-dependent scattering rate. The first contribution is due to the scattering of DM on a single nucleon, while the second contribution arises from a pion exchange between DM and the nucleon. The pion exchange contribution involves a nonrelativistic operator with two derivatives, $Q_{1,N}^{(2)}$ in Eq. (142) that

would naively give a $\mathcal{O}(q^2)$ suppressed contribution to the scattering amplitude. Its contribution is, however, enhanced by the pion pole $1/(m_\pi^2 + |\vec{q}|^2)$, giving a contribution of $\mathcal{O}(|\vec{q}|^2/m_\pi^2) \sim \mathcal{O}(1)$ for $|\vec{q}| \sim m_\pi$. For this reason we needed to keep the nonrelativistic operators with up to two derivatives.

Similar arguments apply to all the operators in Eqs. (3)-(9). Pion exchange is the leading contribution to the scattering amplitude for operators with pseudoscalar quark currents, while it is of the same order as the contact interactions with nucleons for the axial-axial operator as well as those operators coupling the DM current to $G\tilde{G}$. For the remaining operators, the DM-nucleon contact interactions give the leading contributions. Moreover, obtaining contributions of leading order in chiral counting requires some care for the case of vector and axial-vector quark currents, since these need to be expanded to NLO in chiral counting when they are multiplied by axial-vector and vector DM currents, respectively.

The EFT we constructed in this paper is valid at $\mu \sim 1\,\text{GeV}$. A different EFT analysis, valid all the way up to the scale of the mediator much above the electroweak scale, can be useful when relating direct detection to processes at much higher energies, the DM searches at the LHC [42–49] or signals from DM annihilation [50–55]. When relating these with direct detection it is important to use simplified models [56–60] and even to include loop corrections [61–69]. In the present work we completed the final step of this program, explicitly connecting the EFT describing DM interactions with quarks and gluons with nuclear physics.

Our results assume that that there are no large cancellations between different Wilson coefficients in the UV. In the presence of cancellations one would need to include terms of higher order in the chiral expansion. For instance, the pseudoscalar-pseudoscalar UV operator $\mathcal{Q}_{8,q}^{(7)}$ in Eq. (9) contributes to the $(q \cdot S_\chi)(q \cdot S_N)$ nonrelativistic operator in Eq. (142). This contribution vanishes, however, if $m_u \hat{\mathcal{C}}_{8,u}^{(7,0)} = m_d \hat{\mathcal{C}}_{8,d}^{(7,0)} = m_s \hat{\mathcal{C}}_{8,s}^{(7,0)}$, cf., Eq. (156). The leading contribution to the DM-nucleon scattering would then come from a contact term of higher order in chiral counting which could be viewed as due to η' exchange. For contributions of this type one could easily extend our analysis and include the η' exchange contributions by multiplying each term in $\mathcal{L}_{\text{ChPT}} + \mathcal{L}_{\text{HBChPT}}$ with an arbitrary function of the η' field. However, since the mass of the η' is comparable with the cut-off of the theory, it is consistent to integrate it out, as we did. The same is true for the scalar-pseudoscalar UV operator $\mathcal{Q}_{7,q}^{(7)}$ in Eq. (9) whose contributions to the $(q \cdot S_N)$ nonrelativistic operator in Eq. (139) also vanish

in the limit $m_u \hat{\mathcal{C}}_{7,u}^{(7,0)} = m_d \hat{\mathcal{C}}_{7,d}^{(7,0)} = m_s \hat{\mathcal{C}}_{7,s}^{(7,0)}$. A somewhat different situation is encountered for the axialvector-axialvector operator $\mathcal{Q}_{4,q}^{(6)}$ in Eq. (5). Its contributions to $Q_{1,N}^{(2)}$ vanish if $\hat{\mathcal{C}}_{4,u}^{(6,0)} = \hat{\mathcal{C}}_{4,d}^{(6,0)} = \hat{\mathcal{C}}_{4,s}^{(6,0)}$. However, in this case the contact contributions of $\mathcal{Q}_{4,q}^{(6)}$ to $Q_{2,N}^{(0)}$ would still be nonzero, see Eq. (148), and would be leading over the η' exchange contributions.

Similar situations can arise for all the other operators in (3)-(9), where, through fine-tuning in the UV theory, one can cancel the leading contributions in chiral counting. In such situations it would be important to extend our analysis to higher orders in chiral counting, as well as to analyze whether or not such fine-tunings are stable under quantum corrections in the UV. We postpone such an analysis to future work [18].

Note that our analysis, while valid within the assumed power counting, does not capture the leading contributions for all theories of DM, even without considering fine tuning. For instance, dimension-seven Rayleigh operators can be leading for Majorana DM [70, 71]. For this particular case the EFT analysis is already available [41], while a more complete analysis of dimension-seven and higher dimension operators is still called for.

Appendix A: Relation to the basis of Anand et al.

In this appendix we relate our nonrelativistic basis to the operator basis from Ref. [6]. The operators in (138)-(142) are products of nonrelativistic DM and nucleon currents, although still given in a Lorentz covariant notation. We now pass to a manifestly nonrelativistic notation[12], for which we use the operator basis from Ref. [6]. The operators with up to two

---

[12] Our metric convention for the Lorentz vectors is $p^\mu = (p^0, \vec{p})$, $p_\mu = (p^0, -\vec{p})$.

derivatives are

$$\mathcal{O}_1^N = \mathbb{1}_\chi \mathbb{1}_N \,, \qquad\qquad\qquad \mathcal{O}_2^N = \left(v_\perp\right)^2 \mathbb{1}_\chi \mathbb{1}_N \,, \tag{A1}$$

$$\mathcal{O}_3^N = \mathbb{1}_\chi \vec{S}_N \cdot \left(\vec{v}_\perp \times \frac{i\vec{q}}{m_N}\right) \,, \qquad \mathcal{O}_4^N = \vec{S}_\chi \cdot \vec{S}_N \,, \tag{A2}$$

$$\mathcal{O}_5^N = \vec{S}_\chi \cdot \left(\vec{v}_\perp \times \frac{i\vec{q}}{m_N}\right) \mathbb{1}_N \,, \qquad \mathcal{O}_6^N = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right), \tag{A3}$$

$$\mathcal{O}_7^N = \mathbb{1}_\chi \left(\vec{S}_N \cdot \vec{v}_\perp\right) \,, \qquad\qquad \mathcal{O}_8^N = \left(\vec{S}_\chi \cdot \vec{v}_\perp\right) \mathbb{1}_N \,, \tag{A4}$$

$$\mathcal{O}_9^N = \vec{S}_\chi \cdot \left(\frac{i\vec{q}}{m_N} \times \vec{S}_N\right) \,, \qquad \mathcal{O}_{10}^N = -\mathbb{1}_\chi \left(\vec{S}_N \cdot \frac{i\vec{q}}{m_N}\right) \,, \tag{A5}$$

$$\mathcal{O}_{11}^N = -\left(\vec{S}_\chi \cdot \frac{i\vec{q}}{m_N}\right) \mathbb{1}_N \,, \qquad \mathcal{O}_{12}^N = \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}_\perp\right) \,, \tag{A6}$$

$$\mathcal{O}_{13}^N = -\left(\vec{S}_\chi \cdot \vec{v}_\perp\right)\left(\vec{S}_N \cdot \frac{i\vec{q}}{m_N}\right) \,, \qquad \mathcal{O}_{14}^N = -\left(\vec{S}_\chi \cdot \frac{i\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \vec{v}_\perp\right) \,, \tag{A7}$$

with $N = p, n$. Note that each insertion of $\vec{q}$ is accompanied with a factor of $1/m_N$, so that all of the above operators have the same dimensionality. The minus signs and order changes in the cross products for the definitions of some of the operators compensate the relative sign difference between our convention for the momentum exchange (143) and the one in [6].

The Wilson coefficients are in this basis given by

$$c_{\text{NR},1}^N = c_{1,N}^{(0)} \,, \qquad\qquad c_{\text{NR},2}^N = 0 \,, \qquad\qquad c_{\text{NR},3}^N = 0 \,, \tag{A8}$$

$$c_{\text{NR},4}^N = -c_{2,N}^{(0)} \,, \qquad c_{\text{NR},5}^N = m_N^2 c_{2,N}^{(2)} \,, \qquad c_{\text{NR},6}^N = -m_N^2 c_{1,N}^{(2)} \,, \tag{A9}$$

$$c_{\text{NR},7}^N = -m_N c_{3,N}^{(1)} \,, \qquad c_{\text{NR},8}^N = -m_N c_{4,N}^{(1)} \,, \qquad c_{\text{NR},9}^N = m_N c_{5,N}^{(1)} \,, \tag{A10}$$

$$c_{\text{NR},10}^N = m_N c_{1,N}^{(1)} \,, \qquad c_{\text{NR},11}^N = m_N c_{2,N}^{(1)} \,, \qquad c_{\text{NR},12}^N = -m_N c_{6,N}^{(1)} \,. \tag{A11}$$

$$c_{\text{NR},13}^N = 0 \,, \qquad\qquad c_{\text{NR},14}^N = 0 \,. \tag{A12}$$

With this dictionary one can go directly from the EFT with quark, gluons and photons as external states, (3)-(9), to the nuclear response functions, using the coefficients in (147)-(157). In the expressions (149)-(157) one also needs to replace $q^2 \to -\vec{q}^{\,2}$, so that, e.g., the propagators due to pion exchange are proportional to $1/(m_\pi^2 + \vec{q}^{\,2})$. For tree-level matching onto HDMET we thus have in terms of the operators (3)-(9)

$$c_{\text{NR},1}^p = 2\hat{\mathcal{C}}_{1,u}^{(6)} + \hat{\mathcal{C}}_{1,d}^{(6)} - \frac{2m_G}{27}\hat{\mathcal{C}}_1^{(7)} + \sigma_u^p \hat{\mathcal{C}}_{5,u}^{(7)} + \sigma_d^p \hat{\mathcal{C}}_{5,d}^{(7)} + \sigma_s \hat{\mathcal{C}}_{5,s}^{(7)} - \frac{\alpha Q_p}{2\pi m_\chi}\hat{\mathcal{C}}_1^{(5)} \,, \tag{A13}$$

$$c_{\text{NR},4}^p = -4\left(\Delta u_p \hat{\mathcal{C}}_{4,u}^{(6)} + \Delta d_p \hat{\mathcal{C}}_{4,d}^{(6)} + \Delta s \hat{\mathcal{C}}_{4,s}^{(6)}\right) - \frac{2\alpha}{\pi}\frac{\mu_p}{m_N}\hat{\mathcal{C}}_1^{(5)} \,, \tag{A14}$$

$$c_{\text{NR},5}^p = \frac{2\alpha Q_p m_N}{\pi \vec{q}^{\,2}}\hat{\mathcal{C}}_1^{(5)} \,, \tag{A15}$$

$$c_{\text{NR},6}^p = m_N^2 \left\{ \frac{2\alpha}{\pi \vec{q}^2} \frac{\mu_p}{m_N} \hat{\mathcal{C}}_1^{(5)} + \frac{2}{3} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 + \vec{q}^2} \left(\hat{\mathcal{C}}_{4,u}^{(6)} + \hat{\mathcal{C}}_{4,d}^{(6)} - 2\hat{\mathcal{C}}_{4,s}^{(6)} \right) \right.$$

$$+ \frac{2g_A}{m_\pi^2 + \vec{q}^2} \left(\hat{\mathcal{C}}_{4,u}^{(6)} - \hat{\mathcal{C}}_{4,d}^{(6)} \right) - \frac{B_0}{m_\chi} \frac{g_A}{m_\pi^2 + \vec{q}^2} \left(m_u \hat{\mathcal{C}}_{8,u}^{(7)} - m_d \hat{\mathcal{C}}_{8,d}^{(7)} \right)$$

$$- \frac{B_0}{3 m_\chi} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 + \vec{q}^2} \left(m_u \hat{\mathcal{C}}_{8,u}^{(7)} + m_d \hat{\mathcal{C}}_{8,d}^{(7)} - 2 m_s \hat{\mathcal{C}}_{8,s}^{(7)} \right) \qquad \text{(A16)}$$

$$- \frac{\tilde{m}}{m_\chi} \left[\frac{\Delta u_p}{m_u} + \frac{\Delta d_p}{m_d} + \frac{\Delta s}{m_s} - \frac{g_A}{2} \left(\frac{1}{m_u} - \frac{1}{m_d} \right) \frac{\vec{q}^2}{m_\pi^2 + \vec{q}^2} \right.$$

$$\left. \left. - \frac{1}{6} \left(\Delta u_p + \Delta d_p - 2\Delta s \right) \left(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s} \right) \frac{\vec{q}^2}{m_\eta^2 + \vec{q}^2} \right] \hat{\mathcal{C}}_4^{(7)} \right\},$$

$$c_{\text{NR},7}^p = -2 \left(\Delta u_p \hat{\mathcal{C}}_{3,u}^{(6)} + \Delta d_p \hat{\mathcal{C}}_{3,d}^{(6)} + \Delta s \hat{\mathcal{C}}_{3,s}^{(6)} \right), \qquad \text{(A17)}$$

$$c_{\text{NR},8}^p = 4 \hat{\mathcal{C}}_{2,u}^{(6)} + 2 \hat{\mathcal{C}}_{2,d}^{(6)}, \qquad \text{(A18)}$$

$$c_{\text{NR},9}^p = 4 \hat{\mu}_u^p \hat{\mathcal{C}}_{2,u}^{(6)} + 2 \hat{\mu}_d^p \hat{\mathcal{C}}_{2,d}^{(6)} - 6 \mu_s \hat{\mathcal{C}}_{2,s}^{(6)} + \frac{2 m_N}{m_\chi} \left(\Delta u_p \hat{\mathcal{C}}_{3,u}^{(6)} + \Delta d_p \hat{\mathcal{C}}_{3,d}^{(6)} + \Delta s \hat{\mathcal{C}}_{3,s}^{(6)} \right), \qquad \text{(A19)}$$

$$c_{\text{NR},10}^p = m_N \left\{ \frac{B_0 \, g_A}{m_\pi^2 + \vec{q}^2} \left(m_u \hat{\mathcal{C}}_{7,u}^{(7)} - m_d \hat{\mathcal{C}}_{7,d}^{(7)} \right) \right.$$

$$+ \frac{B_0}{3} \frac{(\Delta u_p + \Delta d_p - 2\Delta s)}{m_\eta^2 + \vec{q}^2} \left(m_u \hat{\mathcal{C}}_{7,u}^{(7)} + m_d \hat{\mathcal{C}}_{7,d}^{(7)} - 2 m_s \hat{\mathcal{C}}_{7,s}^{(7)} \right)$$

$$- \tilde{m} \left[\frac{\Delta u_p}{m_u} + \frac{\Delta d_p}{m_d} + \frac{\Delta s}{m_s} - \frac{g_A}{2} \left(\frac{1}{m_u} - \frac{1}{m_d} \right) \frac{\vec{q}^2}{m_\pi^2 + \vec{q}^2} \right. \qquad \text{(A20)}$$

$$\left. \left. - \frac{1}{6} \left(\Delta u_p + \Delta d_p - 2\Delta s \right) \left(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s} \right) \frac{\vec{q}^2}{m_\eta^2 + \vec{q}^2} \right] \hat{\mathcal{C}}_3^{(7)} \right\},$$

$$c_{\text{NR},11}^p = \frac{m_N}{m_\chi} \left(- \sigma_u^p \hat{\mathcal{C}}_{6,u}^{(7)} - \sigma_d^p \hat{\mathcal{C}}_{6,d}^{(7)} - \sigma_s \hat{\mathcal{C}}_{6,s}^{(7)} + \frac{2 m_G}{27} \hat{\mathcal{C}}_2^{(7)} + \frac{2\alpha Q_p m_\chi}{\pi \vec{q}^2} \hat{\mathcal{C}}_2^{(5)} \right), \qquad \text{(A21)}$$

while the remaining coefficients are zero. The coefficients for neutrons are obtained by replacing $p \to n$, $u \leftrightarrow d$. The Wilson coefficients of $\mathcal{O}_2^N, \mathcal{O}_3^N, \mathcal{O}_{13}^N, \mathcal{O}_{14}^N$ are zero in our framework as a result of the fact that we limited our discussion to the operators (21)-(30) that can be generated from UV physics described by dimension-five and dimension-six operators above the electroweak scale [18]. These Wilson coefficients are expected to be generated if either one works to higher orders in q or if higher dimension operators are included in the UV.

The above expressions extend the results in Ref. [5], where estimates for $c_{\text{NR},6}^N$ and $c_{\text{NR},10}^N$ were obtained without using the chiral expansion and thus do not contain the pion pole contributions. A chiral expansion was performed in Ref. [9]. Our expressions involving the axial quark current agree with Ref. [9], as do the expressions for the pseudoscalar quark currents in the limits where the results of Ref. [9] are applicable, i.e., for either isospin

triplet or flavor octet flavor structures.

Appendix B: Further details on chiral dark matter interactions

In this appendix we give further details on the ChPT and HBChPT Lagrangians that describe DM interactions.

1. HBChPT Lagrangian at second order

We first give the full form of the HCBhPT Lagrangian at $\mathcal{O}(p^2)$, including DM interactions. The terms relevant for our analysis were shown already in Eq. (98). The complete form of the Lagrangian is (see also [30, 72])

$$\mathcal{L}^{(2)}_{\text{HBChPT}} = \mathcal{L}^{(2)}_{ps} + \mathcal{L}^{(2)}_{V} + \mathcal{L}^{(2)}_{A} + \mathcal{L}^{(2)}_{S}, \tag{B1}$$

where we split the contributions proportional to different spurions as denoted by subscripts (except for $\mathcal{L}^{(2)}_{S}$ that collects terms that involve the nuclear spin operator). The $\mathcal{L}^{(2)}_{ps}$ contains the scalar and pseudoscalar spurions,

$$\mathcal{L}^{(2)}_{ps} = b_D \operatorname{Tr} \bar{B}_v \{s_+, B_v\} + b_F \operatorname{Tr} \bar{B}_v [s_+, B_v] + b_0 \operatorname{Tr} \left(\bar{B}_v B_v \right) \operatorname{Tr} \left(s^\chi_+ \right). \tag{B2}$$

The terms with the vector current, V^μ, are

$$\begin{aligned}
\mathcal{L}^{(2)}_{V} =& c_1 \operatorname{Tr} \left(\bar{B}_v \nabla^2 B_v \right) - c'_1 \operatorname{Tr} \left(V_\mu \right) \operatorname{Tr} \left(\bar{B}_v \, i \overset{\leftrightarrow}{\nabla}{}^\mu B_v \right) - c'_2 \operatorname{Tr} \left(\partial \cdot V \right) \operatorname{Tr} \left(\bar{B}_v B_v \right) \\
& - c'_3 \operatorname{Tr} \left(\bar{B}_v B_v \right) v \cdot \partial \operatorname{Tr} \left(v \cdot V \right) + c'_4 \left(\operatorname{Tr} V_\mu \right)^2 \operatorname{Tr} (\bar{B}_v B_v) \\
& + c'_5 \left(\operatorname{Tr} v \cdot V \right)^2 \operatorname{Tr} (\bar{B}_v B_v),
\end{aligned} \tag{B3}$$

where[13] $(\bar{B}_v \overset{\leftrightarrow}{\nabla}{}^\mu B_v) \equiv \bar{B}_v \nabla^\mu B_v - \bar{B}_v \overset{\leftarrow}{\nabla}{}^\mu B_v$. The c'_i terms vanish if there are no DM currents in V_μ, as then $\operatorname{Tr} V_\mu = \operatorname{Tr} V^\xi_\mu = 0$; they were thus omitted in [30, 72]. Note also that, in general, the DM current $\nu_{\chi,\mu}$, Eq. (39), is not conserved, $\partial_\mu \nu^\mu_\chi \neq 0$, so that $\partial \cdot V \neq 0$. Because the vector quark currents are conserved, one does have $c'_2 = c'_3 = 0$, see Section B 4. The

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[13] We use the convention that $\overset{\leftrightarrow}{\nabla}{}^\mu$ acts only inside the brackets. In c'_1 there is thus not derivative acting on $\operatorname{Tr}(V_\mu)$.

terms involving the axial-vector current A^μ but not the spin operator are

$$
\begin{aligned}
\mathcal{L}_A^{(2)} =& d_1 \operatorname{Tr}\left(\bar{B}_v A^2 B_v\right) + d_2 \operatorname{Tr}\left(\bar{B}_v (v\cdot A)^2 B_v\right) + d_3 \operatorname{Tr}\left(\bar{B}_v B_v A^2\right) \\
&+ d_4 \operatorname{Tr}\left(\bar{B}_v B_v (v\cdot A)^2\right) + d_5 \operatorname{Tr}\left(\bar{B}_v B_v\right) \operatorname{Tr}\left(A^2\right) + d_6 \operatorname{Tr}\left(\bar{B}_v B_v\right) \operatorname{Tr}\left((v\cdot A)^2\right) \\
&+ d_7 \operatorname{Tr}\left(\bar{B}_v A_\mu\right) \operatorname{Tr}\left(A^\mu B_v\right) + d_8 \operatorname{Tr}\left(\bar{B}_v v\cdot A\right) \operatorname{Tr}\left(v\cdot A B_v\right) \\
&+ d_9 \operatorname{Tr}\left(\bar{B}_v A_\mu B_v A^\mu\right) + d_{10} \operatorname{Tr}\left(\bar{B}_v v\cdot A B_v v\cdot A\right) \\
&+ d_1' \operatorname{Tr}\left(\bar{B}_v B_v\right) \left(\operatorname{Tr} A_\mu\right)^2 + d_2' \operatorname{Tr}\left(\bar{B}_v B_v\right) \left(\operatorname{Tr} v\cdot A\right)^2 \\
&+ d_3' \operatorname{Tr}\left(\bar{B}_v A_\mu B_v\right) \operatorname{Tr}\left(A^\mu\right) + d_4' \operatorname{Tr}\left(\bar{B}_v v\cdot A B_v\right) \operatorname{Tr}\left(v\cdot A\right) \\
&+ d_5' \operatorname{Tr}\left(\bar{B}_v B_v A_\mu\right) \operatorname{Tr}\left(A^\mu\right) + d_6' \operatorname{Tr}\left(\bar{B}_v B_v v\cdot A\right) \operatorname{Tr}\left(v\cdot A\right).
\end{aligned}
\tag{B4}
$$

For the d_7 and d_8 terms, the contraction of Dirac indices is understood across the two traces. The d_i' terms vanish in the limit of vanishing DM currents, and were omitted in [72].

Finally, the terms involving the spin operator S_v^μ are

$$
\mathcal{L}_S^{(2)} = \mathcal{L}_{S_0}^{(2)} + \mathcal{L}_{S'}^{(2)} + \mathcal{L}_{S,\epsilon}^{(2)},
\tag{B5}
$$

where

$$
\begin{aligned}
\mathcal{L}_{S_0}^{(2)} =& f_1 \operatorname{Tr}\left(\bar{B}_v \{v\cdot i\overleftrightarrow{\nabla}, S_N\cdot A\} B_v\right) + f_2 \operatorname{Tr}\left(\bar{B}_v \{S_N\cdot i\overleftrightarrow{\nabla}, v\cdot A\} B_v\right) \\
&+ i f_3 \operatorname{Tr}\left(\bar{B}_v [S_N\cdot i\overleftrightarrow{\nabla}, v\cdot A] B_v\right) + f_4 \operatorname{Tr}\left[(\bar{B}_v v\cdot i\overleftrightarrow{\nabla} S_N^\nu B_v) A_\nu\right] \\
&+ f_5 \operatorname{Tr}\left[(\bar{B}_v S_N\cdot i\overleftrightarrow{\nabla} B_v)(v\cdot A)\right] + f_6 \operatorname{Tr}\left(\bar{B}_v (S_N\cdot A)^2 B_v\right) \\
&+ f_7 \operatorname{Tr}\left(\bar{B}_v S_N^\nu B_v v\cdot\nabla A_\nu\right) + f_8 \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v \nabla_\mu v\cdot A\right),
\end{aligned}
\tag{B6}
$$

while

$$
\begin{aligned}
\mathcal{L}_{S'}^{(2)} =& f_1' \operatorname{Tr}\left(\bar{B}_v v\cdot i\overleftrightarrow{\nabla} S_N^\mu B_v\right) \operatorname{Tr}\left(A_\mu\right) + f_2' \operatorname{Tr}\left(\bar{B}_v S_N\cdot i\overleftrightarrow{\nabla} B_v\right) \operatorname{Tr}\left(v\cdot A\right) \\
&+ f_3' \operatorname{Tr}\left(\bar{B}_v (S_N\cdot A) B_v\right) \operatorname{Tr}\left(v\cdot V\right) + f_4' \operatorname{Tr}\left(\bar{B}_v S_N^\mu (v\cdot A) B_v\right) \operatorname{Tr}\left(V_\mu\right) \\
&+ f_5' \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v\right) \operatorname{Tr}\left(A_\mu\right) \operatorname{Tr}\left(v\cdot V\right) + f_6' \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v\right) \operatorname{Tr}\left(V_\mu\right) \operatorname{Tr}\left(v\cdot A\right) \\
&+ f_7' \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v\right) v\cdot\partial \operatorname{Tr}\left(A_\mu\right) + f_8' \operatorname{Tr}\left(\bar{B}_v S_N^\mu B_v\right) \partial_\mu \operatorname{Tr}\left(v\cdot A\right),
\end{aligned}
\tag{B7}
$$

and

$$
\begin{aligned}
\mathcal{L}_{S,\epsilon}^{(2)} =& -i\epsilon^{\alpha\beta\lambda\sigma} v_\alpha \Big[g_1 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} A_\lambda A_\sigma B_v\right) + i g_2 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} A_\lambda B_v A_\sigma\right) \\
&+ g_3 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v A_\lambda A_\sigma\right) + g_4 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} \nabla_\lambda \nabla_\sigma B_v\right) \\
&- i g_5 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v \nabla_\lambda V_\sigma\right) + g_6 \operatorname{Tr}\left(\bar{B}_v S_{N\beta} A_\lambda\right) \operatorname{Tr}\left(A_\sigma B_v\right) \\
&+ i g_1' \operatorname{Tr}\left(\bar{B}_v S_{N\beta} A_\lambda B_v\right) \operatorname{Tr}(A_\sigma) + i g_2' \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v A_\lambda\right) \operatorname{Tr}(A_\sigma) \\
&+ g_3' \operatorname{Tr}\left(\bar{B}_v S_{N\beta} \overleftrightarrow{\nabla}_\lambda B_v\right) \operatorname{Tr}(V_\sigma) + i g_4' \operatorname{Tr}\left(\bar{B}_v S_{N\beta} B_v\right) \partial_\lambda \operatorname{Tr}(V_\sigma) \Big].
\end{aligned}
\tag{B8}
$$

In writing the above Lagrangian we imposed invariance of QCD under parity. Equations of motion for the baryon fields were used to trade $\text{Tr}\left(\bar{B}_v(v\cdot\nabla)^2 B_v\right)$, $\text{Tr}\left(v\cdot V\right)\text{Tr}\left(\bar{B}_v v\cdot\nabla B_v\right)$, $\text{Tr}\left(\bar{B}_v(S_N\cdot A)(v\cdot\nabla)B_v\right)$ in favor of the other terms in (B3)-(B8). This differs from the convention used in [30, 72]. We also used the relations in Eq. (80) to simplify the terms involving the spin operators. Note that the $f_{1,2}$ terms in (B6) are multiplied by i, correcting a typographical error in [30] (see also [73]).

Note that the last term in the first line of (B1) contains only the DM part of the scalar current, while the QCD part has already been absorbed in the definition of the B_v masses. This term and the d_i', f_i', g_1' terms do not appear in [30] since the traces of QCD vector and axial currents vanish

The terms that contain at most one insertion of DM current are

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\text{HBChPT}} \supset\ & b_D\,\text{Tr}\,\bar{B}_v\{s_+,B_v\} + b_F\,\text{Tr}\,\bar{B}_v[s_+,B_v] + b_0\,\text{Tr}\left(\bar{B}_v B_v\right)\text{Tr}\left(s_+^\chi\right), \\
& + c_1\,\text{Tr}\,\bar{B}_v\nabla^2 B_v - c_1'\,\text{Tr}\left(V_\mu\right)\text{Tr}\left(\bar{B}_v i\overset{\leftrightarrow}{\nabla}{}^\mu B_v\right) - c_2'\,\text{Tr}\left(\partial\cdot V\right)\text{Tr}\left(\bar{B}_v B_v\right) \\
& - c_3'\,\text{Tr}\left(\bar{B}_v B_v\right)v\cdot\partial\,\text{Tr}\left(v\cdot V\right) + f_1\,\text{Tr}\left(\bar{B}_v\{v\cdot i\overset{\leftrightarrow}{\nabla},S_N\cdot A\}B_v\right) \\
& + f_2\,\text{Tr}\left(\bar{B}_v\{S_N\cdot i\overset{\leftrightarrow}{\nabla},v\cdot A\}B_v\right) + if_3\,\text{Tr}\left(\bar{B}_v[S_N\cdot i\overset{\leftrightarrow}{\nabla},v\cdot A]B_v\right) \\
& + f_4\,\text{Tr}\left[\left(\bar{B}_v v\cdot i\overset{\leftrightarrow}{\nabla}S_N^\nu B_v\right)A_\nu\right] + f_5\,\text{Tr}\left[\left(\bar{B}_v S_N\cdot i\overset{\leftrightarrow}{\nabla}B_v\right)(v\cdot A)\right] \\
& + f_7\,\text{Tr}\left(\bar{B}_v S_N^\nu B_v\, v\cdot\nabla A_\nu\right) + f_8\,\text{Tr}\left(\bar{B}_v S_N^\mu B_v\,\nabla_\mu v\cdot A\right) \\
& + f_1'\,\text{Tr}\left(\bar{B}_v v\cdot i\overset{\leftrightarrow}{\nabla}S_N^\mu B_v\right)\text{Tr}\left(A_\mu\right) + f_2'\,\text{Tr}\left(\bar{B}_v S_N\cdot i\overset{\leftrightarrow}{\nabla}B_v\right)\text{Tr}\left(v\cdot A\right) \\
& + f_7'\,\text{Tr}\left(\bar{B}_v S_N^\mu B_v\right)v\cdot\partial\,\text{Tr}\left(A_\mu\right) + f_8'\,\text{Tr}\left(\bar{B}_v S_N^\mu B_v\right)\partial_\mu\text{Tr}\left(v\cdot A\right) \\
& - i\epsilon^{\alpha\beta\lambda\sigma}v_\alpha\Big[g_4\,\text{Tr}\left(\bar{B}_v S_{N\beta}\nabla_\lambda\nabla_\sigma B_v\right) - ig_5\,\text{Tr}\left(\bar{B}_v S_{N\beta}B_v\nabla_\lambda V_\sigma\right) \\
& \quad + g_3'\,\text{Tr}\left(\bar{B}_v S_{N\beta}\overset{\leftrightarrow}{\nabla}_\lambda B_v\right)\text{Tr}(V_\sigma) + ig_4'\,\text{Tr}\left(\bar{B}_v S_{N\beta}B_v\right)\partial_\lambda\text{Tr}(V_\sigma)\Big] + \cdots,
\end{aligned}
\tag{B9}
$$

where we kept only the terms that are nonzero once expanded up to linear order in the meson fields. Not all of these terms are needed for our ChEFT analysis, though. The reduced set of relevant terms is given in (98).

The coefficients $m_G, D, F, G, b_i, b_i', c_i, c_i', d_i, d_i', f_i, f_i', g_i, g_i'$ are real low-energy constants. Some of these coefficients are fixed by the fact that the theory needs to be invariant under infinitesimal Lorentz transformations [24, 74],

$$
B_v \to e^{i\varepsilon\cdot x}B_v, \qquad v\cdot\nabla \to v\cdot\nabla + \frac{1}{m_N}\varepsilon\cdot\nabla_\perp, \qquad \nabla_\perp^\mu \to \nabla_\perp^\mu - \frac{1}{m_N}\varepsilon^\mu(v\cdot\nabla). \tag{B10}
$$

To lowest order the above transformation effectively corresponds to reparametrization invariance under the shift of the label momentum $v^\mu \to v^\mu + \varepsilon^\mu/m_N$, but they also shift the

external currents,[14]

$$A^\mu \to A^\mu + v^\mu \frac{\varepsilon \cdot A}{m_N} - \varepsilon^\mu \frac{v \cdot A}{m_N}, \qquad V^\mu \to V^\mu + v^\mu \frac{\varepsilon \cdot V}{m_N} - \varepsilon^\mu \frac{v \cdot V}{m_N}. \tag{B11}$$

Reparametrization invariance then leads to the relations [75] (see also [8, 74])

$$c_1 = c_1' = -\frac{1}{2m_N}, \qquad f_2 = -\frac{1}{2m_N}(D+F), \qquad f_5 = -\frac{1}{m_N}(D-F),$$

$$f_2' = -\frac{1}{m_N}G, \qquad g_3' = 0. \tag{B12}$$

In addition, the conservation of the quark vector current and the Lorentz structure of the matrix element for quark axial vector current give

$$c_2' = c_3' = 0, \qquad f_3 = f_8 = f_8' = 0, \tag{B13}$$

respectively, see Section B 4.

We discuss the numerical values of the remaining parameters that are relevant for DM phenomenology in Sec. C.

2. ChPT and HBChPT Lagrangians expanded in meson fields

Here we give the DM interaction Lagrangian in ChPT, Eqs. (125)-(127) and HBChPT, Eqs. (129)-(131), expanded up to linear order in the meson fields. Unlike in the main text, the expressions in this subsection are valid beyond tree level matching onto HDMET. The $\mathcal{O}(p)$ ChPT Lagrangian for DM interactions with mesons is

$$\mathcal{L}_{\chi,\text{ChPT}}^{(1)} = 2f(\bar{\chi}_v iq \cdot S_\chi \chi_v)\Big[\big(-\hat{\mathcal{C}}_{4,u}^{(6,0)} + \hat{\mathcal{C}}_{4,d}^{(6,0)}\big)\pi^0 - \big(\hat{\mathcal{C}}_{4,u}^{(6,0)} + \hat{\mathcal{C}}_{4,d}^{(6,0)} - 2\hat{\mathcal{C}}_{4,s}^{(6,0)}\big)\frac{\eta}{\sqrt{3}}\Big] + \cdots, \tag{B14}$$

with $q^\mu = p_1^\mu - p_2^\mu$ the difference of incoming and outgoing DM momenta, while the ellipses denote terms with two or more mesons. It comes from the product $J_\chi^A \cdot J_q^A$ in (125), while the contributions from $J_\chi^V \cdot J_q^V$ and $J_\chi^A \cdot J_q^V$ start only at $\mathcal{O}(\pi^2)$. Note that the formally leading term in (B14) from $J_\chi^V \cdot J_q^A$ in (125), i.e., from the first line in (75), cancels exactly due to vector current conservation against the corresponding $1/m_\chi$ suppressed contribution to (B15) from the third line in (76). We thus do not display these two contributions.

---

[14] This can also be used to show the equality in (31) imposed by reparametrization invariance, but now $\bar{f}\gamma_\mu f$ and $\bar{f}\gamma_\mu\gamma_5 f$ are to be treated as external currents.

The $\mathcal{O}(p^2)$ and $\mathcal{O}(p^3)$ DM ChPT Lagrangians are

$$\mathcal{L}^{(2)}_{\chi,\mathrm{ChPT}} = (\bar{\chi}_v \chi_v)\Big\{ B_0 f\Big[\big(m_u \hat{\mathcal{C}}^{(7,0)}_{7,u} - m_d \hat{\mathcal{C}}^{(7,0)}_{7,d}\big)\pi^0$$
$$+ \big(m_u \hat{\mathcal{C}}^{(7,0)}_{7,u} + m_d \hat{\mathcal{C}}^{(7,0)}_{7,d} - 2m_s \hat{\mathcal{C}}^{(7,0)}_{7,s}\big)\frac{\eta}{\sqrt{3}}\Big] \tag{B15}$$
$$- \frac{f\tilde{m}}{2}q^2\Big[\Big(\frac{1}{m_u} - \frac{1}{m_d}\Big)\pi^0 + \Big(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s}\Big)\frac{\eta}{\sqrt{3}}\Big]\hat{\mathcal{C}}^{(7,0)}_3 \Big\} + \cdots,$$

$$\mathcal{L}^{(3)}_{\chi,\mathrm{ChPT}} = (\bar{\chi}_v iq\cdot S_\chi \chi_v)\Big\{ B_0 f\Big[\big(m_u \hat{\mathcal{C}}^{(8,1)}_{8,u} - m_d \hat{\mathcal{C}}^{(8,1)}_{8,d}\big)\pi^0$$
$$+ \big(m_u \hat{\mathcal{C}}^{(8,1)}_{8,u} + m_d \hat{\mathcal{C}}^{(8,1)}_{8,d} - 2m_s \hat{\mathcal{C}}^{(8,1)}_{8,s}\big)\frac{\eta}{\sqrt{3}}\Big] + \tag{B16}$$
$$+ \frac{f\tilde{m}}{2}q^2\Big[\Big(\frac{1}{m_u} - \frac{1}{m_d}\Big)\pi^0 + \Big(\frac{1}{m_u} + \frac{1}{m_d} - \frac{2}{m_s}\Big)\frac{\eta}{\sqrt{3}}\Big]\hat{\mathcal{C}}^{(8,1)}_4 \Big\} + \cdots.$$

The terms shown above come from $J^S_\chi \cdot J^P_q$ and $J^S_\chi \cdot J^\theta$ in (126), and from $J^P_\chi \cdot J^P_q$ and $J^P_\chi \cdot J^\theta$ in (127), respectively. The contributions from $J^{S,P}_\chi \cdot J^S_q$ and $J^{S,P}_\chi \cdot J^G$, by contrast, start only at $\mathcal{O}(\pi^2)$. The unexpanded versions of (B14)-(B16) are given in (75)-(77).

Expanding the $\mathcal{O}(p^0)$ HBChPT interaction Lagrangian with DM, Eq. (129), to lowest order in meson fields gives

$$\mathcal{L}^{(0)}_{\chi,\mathrm{HBChPT}} \supset (\bar{\chi}_v \chi_v)(\bar{p}_v p_v)\Big(2\hat{\mathcal{C}}^{(6,0)}_{1,u} + \hat{\mathcal{C}}^{(6,0)}_{1,d} - \frac{2m_G}{27}\hat{\mathcal{C}}^{(7,0)}_1\Big)$$
$$+ 4(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{p}_v S^\mu_N p_v)\Big((D+F+G)\hat{\mathcal{C}}^{(6,0)}_{4,u} + G\hat{\mathcal{C}}^{(6,0)}_{4,d} + (D-F+G)\hat{\mathcal{C}}^{(6,0)}_{4,s}\Big)$$
$$+ (p_v \leftrightarrow n_v, u \leftrightarrow d). \tag{B17}$$

Here the p_v and n_v are the HBChPT fields for protons and neutrons. The $\mathcal{O}(p)$ Lagrangian, Eq. (130), is given by

$$\mathcal{L}^{(1)}_{\chi,\mathrm{HBChPT}} \supset -(\bar{\chi}_v v_\perp \cdot S_\chi \chi_v)(\bar{p}_v p_v) \times 2\Big[2\hat{\mathcal{C}}^{(6,0)}_{2,u} + \hat{\mathcal{C}}^{(6,0)}_{2,d}\Big]$$
$$+ (\bar{\chi}_v \chi_v)(\bar{p}_v v_\perp \cdot S_N p_v) \times 2m_\chi\Big[(D+F+G)\hat{\mathcal{C}}^{(7,1)}_{3,u} + G\hat{\mathcal{C}}^{(7,1)}_{3,d} + (D-F+G)\mathcal{C}^{(7,1)}_{3,s}\Big]$$
$$+ (\bar{\chi}_v iq\cdot S_\chi \chi_v)(\bar{p}_v p_v) \times \frac{2}{27}m_G\hat{\mathcal{C}}^{(8,1)}_2$$
$$- (\bar{\chi}_v \chi_v)(\bar{p}_v iq\cdot S_N p_v) \times \Big[D\Big(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_s}\Big) + F\Big(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_s}\Big) + G\Big]\hat{\mathcal{C}}^{(7,0)}_3$$
$$+ 2i\epsilon^{\alpha\beta\mu\nu}v_\alpha q_\beta(\bar{\chi}_v S_{\chi,\mu}\chi_v)(\bar{p}_v S_{N,\nu}p_v) \times \Big[(g_4 - g'_4)\hat{\mathcal{C}}^{(6,0)}_{2,u} - g'_4\hat{\mathcal{C}}^{(6,0)}_{2,d}$$
$$- (g_4 - g_5 + g'_4)\hat{\mathcal{C}}^{(6,0)}_{2,s} + \Big((D+F+G)\hat{\mathcal{C}}^{(7,1)}_{6,u} + G\hat{\mathcal{C}}^{(7,1)}_{6,d} + (D-F+G)\hat{\mathcal{C}}^{(7,1)}_{6,s}\Big)\Big]$$
$$+ (p_v \leftrightarrow n_v, u \leftrightarrow d), \tag{B18}$$

where we used that some terms vanish due to Eq. (B13). The four-component perpendicular relative velocity $v_\perp^\mu$ is defined in (145). In the derivation of the above HBChPT Lagrangian we also used the relations

$$p_{1,2}^\mu = \frac{1}{2}\big[\pm q^\mu + (p_1^\mu + p_2^\mu)\big] = \frac{1}{2}\big[\pm q^\mu + 2m_\chi v_\perp^\mu + \frac{m_\chi}{m_N}(k_1^\mu + k_2^\mu)\big], \tag{B19}$$

$$k_{1,2}^\mu = \frac{1}{2}\big[\mp q^\mu + (k_1^\mu + k_2^\mu)\big] = \frac{1}{2}\big[\mp q^\mu - 2m_N v_\perp^\mu + \frac{m_N}{m_\chi}(p_1^\mu + p_2^\mu)\big], \tag{B20}$$

the relation (23), as well as the relation (31) imposed by reparametrization invariance.

The $\mathcal{O}(p^2)$ DM–nucleon interaction Lagrangian, Eq. (131), expanded for each of the Wilson coefficients to the first nontrivial order in meson fields, is given by

$$
\begin{aligned}
\mathcal{L}_{\chi,\text{HBChPT}}^{(2)} \supset\ & (\bar\chi_v\chi_v)(\bar p_v p_v)\times 2\Big[-(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{5,u}^{(7,0)} - b_0 m_d\hat{\mathcal{C}}_{5,d}^{(7,0)} \\
& \qquad\qquad\qquad\qquad\qquad - (b_0 + b_D - b_F)m_s\hat{\mathcal{C}}_{5,s}^{(7,0)}\Big] \\
& - (-1)^{Q_p}(\bar\chi_v\chi_v)(\bar p_v p_v)\frac{\pi^0}{f}\times 2\Big[(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{7,u}^{(7,0)} - b_0 m_d\hat{\mathcal{C}}_{7,d}^{(7,0)}\Big] \\
& + (\bar\chi_v\chi_v)(\bar p_v p_v)\frac{\eta}{\sqrt{3}f}\times 2\Big[(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{7,u}^{(7,0)} + b_0 m_d\hat{\mathcal{C}}_{7,d}^{(7,0)} \\
& \qquad\qquad\qquad\qquad\qquad - 2(b_0 + b_D - b_F)m_s\hat{\mathcal{C}}_{7,s}^{(7,0)}\Big] \\
& + (\bar\chi_v iq\cdot S_\chi\chi_v)(\bar p_v iq\cdot S_N p_v)\times\Big[D\Big(\frac{\tilde m}{m_u} + \frac{\tilde m}{m_s}\Big) + F\Big(\frac{\tilde m}{m_u} - \frac{\tilde m}{m_s}\Big) + G\Big]\hat{\mathcal{C}}_4^{(8,1)} \\
& + (p_v \leftrightarrow n_v, u \leftrightarrow d),
\end{aligned}
\tag{B21}
$$

while the $\mathcal{O}(p^3)$ DM–nucleon interaction Lagrangian, Eq. (132), is given by

$$
\begin{aligned}
\mathcal{L}_{\chi,\text{HBChPT}}^{(3)} \supset\ & (\bar\chi_v iq\cdot S_\chi\chi_v)(\bar p_v p_v)\times 2\Big[(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{6,u}^{(8,1)} + b_0 m_d\hat{\mathcal{C}}_{6,d}^{(8,1)} \\
& \qquad\qquad\qquad\qquad\qquad + (b_0 + b_D - b_F)m_s\hat{\mathcal{C}}_{6,s}^{(8,1)}\Big] \\
& - (-1)^{Q_p}(\bar\chi_v iq\cdot S_\chi\chi_v)(\bar p_v p_v)\frac{\pi^0}{f}\times 2\Big[(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{8,u}^{(8,1)} - b_0 m_d\hat{\mathcal{C}}_{8,d}^{(8,1)}\Big] \\
& + (\bar\chi_v iq\cdot S_\chi\chi_v)(\bar p_v p_v)\frac{\eta}{\sqrt{3}f}\times 2\Big[(b_0 + b_D + b_F)m_u\hat{\mathcal{C}}_{8,u}^{(8,1)} + b_0 m_d\hat{\mathcal{C}}_{8,d}^{(8,1)} \\
& \qquad\qquad\qquad\qquad\qquad - 2(b_0 + b_D - b_F)m_s\hat{\mathcal{C}}_{8,s}^{(8,1)}\Big] \\
& + (p_v \leftrightarrow n_v, u \leftrightarrow d).
\end{aligned}
\tag{B22}
$$

The final ingredient that we need is the leading HBChPT chiral Lagrangian without DM fields (124)

$$\mathcal{L}_{\text{HBChPT}}^{(1),\text{QCD}} \supset \frac{(D+F)}{f}\big(\bar p_v\, iq\cdot S_N p_v - \bar n_v\, iq\cdot S_N n_v\big)\pi^0 - \frac{D-3F}{\sqrt{3}f}\big(\bar n\, iq\cdot S_N n + \bar p\, iq\cdot S_N p\big)\eta + \cdots,$$
$$\tag{B23}$$

where we expanded to linear order in meson fields, and only display the couplings to the neutral mesons. As in the rest of the paper, $q^\mu = k_2^\mu - k_1^\mu$ is the difference of final and initial nucleon momenta. The single photon interactions with neutrons and protons are given by

$$\mathcal{L}_{\text{HBChPT}}^{\text{QED}} \supset -e\,\bar{p}_v\left(v^\mu + \frac{\tilde{k}_{12}^\mu}{2m_N}\right)A_\mu^e\,p_v + \frac{e}{2m_N}\left(\mu_p\bar{p}_v(\sigma_\perp^{\mu\nu}iq_\mu)p_v + \mu_n\bar{n}_v(\sigma_\perp^{\mu\nu}iq_\mu)n_v\right)A_\nu^e\,, \quad \text{(B24)}$$

where $\mu_p = 2.79$ and $\mu_n = -1.91$ are the proton and neutron magnetic moments in units of nuclear magnetons, respectively, and $\tilde{k}_{12}^\mu$ is defined below in Eq. (B51).

3. Explicit form of hadronic currents

It is straightforward to give the explicit expressions for the different currents appearing in (125)-(127),

$$J_q^S = -\frac{B_0 f^2}{2}\,\text{Tr}\left[(U + U^\dagger)m_q\mathbb{1}_q\right], \qquad J_q^P = -\frac{B_0 f^2}{2}\,\text{Tr}\left[i(U - U^\dagger)m_q\mathbb{1}_q\right], \qquad \text{(B25)}$$

$$J_{q,\mu}^V = -\frac{if^2}{2}\,\text{Tr}\left[(U\partial_\mu U^\dagger + U^\dagger\partial_\mu U)\mathbb{1}_q\right], \qquad J_{q,\mu}^A = -\frac{if^2}{2}\,\text{Tr}\left[(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U)\mathbb{1}_q\right], \quad \text{(B26)}$$

$$J^G = \frac{f^2}{27}\left[\text{Tr}\left(\partial_\mu U^\dagger\partial^\mu U\right) + 3B_0\,\text{Tr}\left[\mathcal{M}_q(U + U^\dagger)\right]\right], \qquad \text{(B27)}$$

$$J^\theta = \frac{if^2}{4\,\text{Tr}(\mathcal{M}_q^{-1})}\,\text{Tr}\left[\partial^\mu\left(U\partial_\mu U^\dagger - U^\dagger\partial_\mu U\right)\mathcal{M}_q^{-1}\right]. \qquad \text{(B28)}$$

Expanding the currents to first nonzero order in meson fields and dropping the constant terms in J_q^S gives

$$J_{q,\mu}^V = i\,\text{Tr}\left([\partial_\mu\Pi, \Pi]\mathbb{1}_q\right) + \cdots, \qquad J_{q,\mu}^A = -\sqrt{2}f\,\text{Tr}\left(\partial_\mu\Pi\,\mathbb{1}_q\right) + \cdots, \quad \text{(B29)}$$

$$J_q^S = B_0\,\text{Tr}\left(\Pi^2 m_q\mathbb{1}_q\right) + \cdots, \qquad J_q^P = \sqrt{2}B_0 f m_q\,\text{Tr}(\Pi\,\mathbb{1}_q) + \cdots, \quad \text{(B30)}$$

$$J^G = \tfrac{2}{27}\,\text{Tr}\left(\partial_\mu\Pi\partial^\mu\Pi\right) - \tfrac{6}{27}B_0\,\text{Tr}\left(\mathcal{M}_q\Pi^2\right) + \cdots, \qquad \text{(B31)}$$

$$J^\theta = \frac{f}{\sqrt{2}\,\text{Tr}(\mathcal{M}_q^{-1})}\,\text{Tr}\left[\partial^2\Pi\mathcal{M}_q^{-1}\right] + \cdots. \qquad \text{(B32)}$$

Here we defined $\mathbb{1}_u = \text{diag}(1,0,0)$, $\mathbb{1}_d = \text{diag}(0,1,0)$, $\mathbb{1}_s = \text{diag}(0,0,1)$. The explicit forms of the pseudoscalar and axial-vector currents in terms of the π^0 and η fields are given in (B43) and (B44).

In (122) we have expanded the DM-nucleon interactions in terms of their chiral scaling. The LO expressions for the currents in (129), (130), (131), are

$$\tilde{J}_q^{V\mu,\text{LO}} = \frac{1}{2}\,\text{Tr}\,\bar{B}_v\left[v^\mu(\xi^\dagger\mathbb{1}_q\xi + \xi\mathbb{1}_q\xi^\dagger), B_v\right] + D\,\text{Tr}\,\bar{B}_v S_N^\mu\{\xi^\dagger\mathbb{1}_q\xi - \xi\mathbb{1}_q\xi^\dagger, B_v\}$$
$$+ F\,\text{Tr}\,\bar{B}_v S_N^\mu\left[\xi^\dagger\mathbb{1}_q\xi - \xi\mathbb{1}_q\xi^\dagger, B_v\right] + \text{Tr}\,\bar{B}_v v^\mu B_v\,, \qquad \text{(B33)}$$

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$$\tilde{J}_q^{A\mu,\text{LO}} = \frac{1}{2}\operatorname{Tr}\bar{B}_v\left[v^\mu(\xi^\dagger\mathbb{1}_q\xi - \xi\mathbb{1}_q\xi^\dagger), B_v\right] + D\operatorname{Tr}\bar{B}_v S_N^\mu\{\xi^\dagger\mathbb{1}_q\xi + \xi\mathbb{1}_q\xi^\dagger, B_v\}$$
$$+ F\operatorname{Tr}\bar{B}_v S_N^\mu\left[\xi^\dagger\mathbb{1}_q\xi + \xi\mathbb{1}_q\xi^\dagger, B_v\right] + 2G\operatorname{Tr}\bar{B}_v S_N^\mu B_v\,, \tag{B34}$$

$$\tilde{J}^{G,\text{LO}} = -\frac{2m_G}{27}\operatorname{Tr}\bar{B}_v B_v\,, \tag{B35}$$

$$\tilde{J}^{\theta,\text{LO}} = -\frac{1}{2\operatorname{Tr}(\mathcal{M}_q^{-1})}\left\{\frac{1}{2}v\cdot\partial\operatorname{Tr}\bar{B}_v\left[(\xi^\dagger\mathcal{M}_q^{-1}\xi - \xi\mathcal{M}_q^{-1}\xi^\dagger), B_v\right]\right.$$
$$+ \partial_\mu\left(D\operatorname{Tr}\bar{B}_v S_N^\mu\{\xi^\dagger\mathcal{M}_q^{-1}\xi + \xi\mathcal{M}_q^{-1}\xi^\dagger, B_v\}\right.$$
$$\left.\left. + F\operatorname{Tr}\bar{B}_v S_N^\mu\left[\xi^\dagger\mathcal{M}_q^{-1}\xi + \xi\mathcal{M}_q^{-1}\xi^\dagger, B_v\right] + 2G\operatorname{Tr}(\mathcal{M}_q^{-1})\operatorname{Tr}\bar{B}_v S_N^\mu B_v\right)\right\}\,, \tag{B36}$$

$$\tilde{J}_q^{S,\text{LO}} = -b_0\operatorname{Tr}(\bar{B}_v B_v)\operatorname{Tr}\left((U^\dagger + U)m_q\mathbb{1}_q\right) - b_D\operatorname{Tr}\bar{B}_v\{\xi^\dagger m_q\mathbb{1}_q\xi^\dagger + \xi m_q\mathbb{1}_q\xi, B_v\}$$
$$- b_F\operatorname{Tr}\bar{B}_v\left[\xi^\dagger m_q\mathbb{1}_q\xi^\dagger + \xi m_q\mathbb{1}_q\xi, B_v\right]\,, \tag{B37}$$

$$\tilde{J}_q^{P,\text{LO}} = b_0\operatorname{Tr}(\bar{B}_v B_v)\operatorname{Tr}\left((U^\dagger - U)im_q\mathbb{1}_q\right) + b_D\operatorname{Tr}\bar{B}_v\{\xi^\dagger im_q\mathbb{1}_q\xi^\dagger - \xi im_q\mathbb{1}_q\xi, B_v\}$$
$$+ b_F\operatorname{Tr}\bar{B}_v\left[\xi^\dagger im_q\mathbb{1}_q\xi^\dagger - \xi im_q\mathbb{1}_q\xi, B_v\right]\,. \tag{B38}$$

When contracting with the nonrelativistic DM currents we also need the expression for the QCD vector current $\tilde{J}_q^{V\mu}$ and axial-vector current $\tilde{J}_q^{A\mu}$ to NLO in the chiral expansion, i.e., to $\mathcal{O}(p)$. The NLO contributions to $\tilde{J}_q^{V\mu}$ are

$$\tilde{J}_q^{V\mu,\text{NLO}} \supset -ic_1\left(\operatorname{Tr}\bar{B}_v\left[\mathbb{1}_q^\xi, \nabla^{\xi,\mu}B_v\right] - \operatorname{Tr}\bar{B}_v\overleftarrow{\nabla}^{\xi,\mu}\left[\mathbb{1}_q^\xi, B_v\right]\right) - c_1'\operatorname{Tr}\left(\bar{B}_v i\overleftrightarrow{\nabla}^{\xi,\mu}B_v\right)$$
$$+ c_2'\partial^\mu\operatorname{Tr}\left(\bar{B}_v B_v\right) + c_3'v^\mu v\cdot\partial\operatorname{Tr}\left(\bar{B}_v B_v\right)$$
$$+ \epsilon^{\alpha\beta\lambda\mu}v_\alpha\left(g_4\operatorname{Tr}\bar{B}_v S_{N\beta}\overleftarrow{\nabla}_\lambda^\xi\left[\mathbb{1}_q^\xi, B_v\right] + g_4\operatorname{Tr}\bar{B}_v S_{N\beta}\left[\mathbb{1}_q^\xi, \nabla_\lambda^\xi B_v\right]\right.$$
$$+ g_5\,\partial_\lambda\operatorname{Tr}\left(\bar{B}_v S_{N\beta}B_v\mathbb{1}_q^\xi\right) - ig_3'\operatorname{Tr}\bar{B}_v S_{N\beta}\overleftrightarrow{\nabla}_\lambda^\xi B_v$$
$$\left. - g_4'\partial_\lambda\operatorname{Tr}\bar{B}_v S_{N\beta}B_v\right) + \cdots\,, \tag{B39}$$

where $\bar{B}_v\overleftarrow{\nabla}_\mu^\xi = \partial_\mu\bar{B}_v - [\bar{B}_v, V_\mu^\xi] = \partial_\mu\bar{B}_v + [V_\mu^\xi, \bar{B}_v]$, and we used the abbreviation $\mathbb{1}_q^\xi \equiv \frac{1}{2}(\xi^\dagger\mathbb{1}_q\xi + \xi\mathbb{1}_q\xi^\dagger)$. The ellipses denote terms that, when expanded in terms of meson fields, start at linear order or higher. Keeping only the terms that do not involve the meson fields gives

$$\tilde{J}_q^{V\mu,\text{NLO}} \supset -ic_1\operatorname{Tr}\bar{B}_v\overleftrightarrow{\partial}^\mu\left[\mathbb{1}_q, B_v\right] - ic_1'\operatorname{Tr}\bar{B}_v\overleftrightarrow{\partial}^\mu B_v + c_2'\partial^\mu\operatorname{Tr}\bar{B}_v B_v + c_3'v^\mu v\cdot\partial\operatorname{Tr}\bar{B}_v B_v$$
$$+ \epsilon^{\alpha\beta\lambda\mu}v_\alpha\left[g_4\partial_\lambda\operatorname{Tr}\bar{B}_v S_{N\beta}\left[\mathbb{1}_q, B_v\right] + g_5\,\partial_\lambda\operatorname{Tr}\left(\bar{B}_v S_{N\beta}B_v\mathbb{1}_q\right)\right.$$
$$\left. - ig_3'\operatorname{Tr}\bar{B}_v S_{N\beta}\overleftrightarrow{\partial}_\lambda B_v - g_4'\partial_\lambda\operatorname{Tr}\bar{B}_v S_{N\beta}B_v\right]\,, \tag{B40}$$

with $\overset{\leftrightarrow}{\partial}_\mu$ defined through $\phi_1 \overset{\leftrightarrow}{\partial}_\mu \phi_2 = \phi_1 \partial_\mu \phi_2 - (\partial_\mu \phi_1)\phi_2$, as before. The axial current at NLO is

$$
\begin{aligned}
\tilde{J}_q^{A\mu,\mathrm{NLO}} \supset\; & 2if_1 \Big[\mathrm{Tr}\left(\bar{B}_v \mathbb{1}_q^\xi v\cdot \nabla^\xi S_N^\mu B_v\right) - \mathrm{Tr}\left(\bar{B}_v S_N^\mu v\cdot \overset{\leftarrow}{\nabla}^\xi \mathbb{1}_q^\xi B_v\right)\Big] \\
& + 2if_2 v^\mu \Big[\mathrm{Tr}\left(\bar{B}_v \mathbb{1}_q^\xi S_N \cdot \nabla^\xi B_v\right) - \mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftarrow}{\nabla}^\xi \mathbb{1}_q^\xi B_v\right)\Big] \\
& + 2f_3 v^\mu \Big[\mathrm{Tr}\left(\bar{B}_v \mathbb{1}_q^\xi S_N \cdot \nabla^\xi B_v\right) + \mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftarrow}{\nabla}^\xi \mathbb{1}_q^\xi B_v\right)\Big] \\
& + if_4 \,\mathrm{Tr}\left[\left(\bar{B}_v v\cdot \overset{\leftrightarrow}{\nabla}^\xi S_N^\mu B_v\right)\mathbb{1}_q^\xi\right] + if_5 v^\mu \,\mathrm{Tr}\left[\left(\bar{B}_v S_N \cdot \overset{\leftrightarrow}{\nabla}^\xi B_v\right)\mathbb{1}_q^\xi\right] \\
& - f_7 \,\mathrm{Tr}\left(\mathbb{1}_q^\xi v\cdot \nabla^\xi \bar{B}_v S_N^\mu B_v\right) - f_8 v^\mu \,\mathrm{Tr}\left(\mathbb{1}_q^\xi \nabla_\nu^\xi \bar{B}_v S_N^\nu B_v\right) \\
& + if_1' \,\mathrm{Tr}\left(\bar{B}_v v\cdot \overset{\leftrightarrow}{\nabla}^\xi S_N^\mu B_v\right) + if_2' v^\mu \,\mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftrightarrow}{\nabla}^\xi B_v\right) \\
& - f_7' v\cdot\partial \,\mathrm{Tr}\left(\bar{B}_v \, S_N^\mu B_v\right) - f_8' v^\mu \,\partial_\nu \,\mathrm{Tr}\left(\bar{B}_v S_N^\nu \, B_v\right) + \cdots .
\end{aligned}
\tag{B41}
$$

Expanding in meson fields gives

$$
\begin{aligned}
\tilde{J}_q^{A\mu,\mathrm{NLO}} \supset\; & 2if_1 \,\mathrm{Tr}\left(\bar{B}_v v\cdot \overset{\leftrightarrow}{\partial} S_N^\mu \mathbb{1}_q B_v\right) + if_4 \,\mathrm{Tr}\left(\bar{B}_v v\cdot \overset{\leftrightarrow}{\partial} S_N^\mu B_v \mathbb{1}_q\right) - f_7 \,\mathrm{Tr}\left(v\cdot \partial \bar{B}_v S_N^\mu B_v \mathbb{1}_q\right) \\
& + if_1' \,\mathrm{Tr}\left(\bar{B}_v v \cdot \overset{\leftrightarrow}{\partial} S_N^\mu B_v\right) - f_7' v\cdot\partial \,\mathrm{Tr}\left(\bar{B}_v \, S_N^\mu B_v\right) \\
& + v^\mu \Big[2if_2 \,\mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftrightarrow}{\partial} \mathbb{1}_q B_v\right) + 2f_3 \partial_\nu \,\mathrm{Tr}\left(\bar{B}_v \mathbb{1}_q S_N^\nu B_v\right) + if_5 \,\mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftrightarrow}{\partial} B_v \mathbb{1}_q\right) \\
& - f_8 \partial_\nu \,\mathrm{Tr}\left(\bar{B}_v S_N^\nu B_v \mathbb{1}_q\right) + if_2' \,\mathrm{Tr}\left(\bar{B}_v S_N \cdot \overset{\leftrightarrow}{\partial} B_v\right) - f_8' \partial_\nu \,\mathrm{Tr}\left(\bar{B}_v S_N^\nu \, B_v\right) \Big] + \cdots .
\end{aligned}
\tag{B42}
$$

4. Quark currents expanded in meson fields

In this subsection we collect the expanded results for the nucleon and meson currents in term of meson fields, keeping only the lowest orders. The expanded expressions have been used in Section IV B to match onto the chiral effective theory of nuclear forces. For this calculation we need single meson exchanges for the hadronized versions of the $\bar{q}\gamma_\mu\gamma_5 q$, $\bar{q}i\gamma_5 q$, and $G_{\mu\nu}^a G^{a\mu\nu}$ currents. The corresponding DM-meson interactions in $\mathcal{L}_{\chi,\mathrm{ChPT}}^{(1,2,3)}$, Eqs. (125)-(127), contain the mesonic currents given in (B25)-(B28). Expanding in meson fields to the first nonzero order one has for the axial currents

$$
(J_{u,d}^A)_\mu = f\left(\mp \partial_\mu \pi^0 - \frac{\partial_\mu \eta}{\sqrt{3}}\right) + \cdots , \qquad (J_s^A)_\mu = \frac{2f}{\sqrt{3}}\partial_\mu \eta + \cdots .
\tag{B43}
$$

while the pseudoscalar currents are

$$
J_{u,d}^P = B_0 f m_{u,d}\left(\pm \pi^0 + \frac{1}{\sqrt{3}}\eta\right) + \cdots , \qquad J_s^P = -\frac{2}{\sqrt{3}} B_0 f m_s \eta + \cdots .
\tag{B44}
$$

The contribution of the J^θ current is

$$J^\theta = \frac{f}{2}\left[\left(\frac{\tilde{m}}{m_u} - \frac{\tilde{m}}{m_d}\right)\partial^2\pi^0 + \left(\frac{\tilde{m}}{m_u} + \frac{\tilde{m}}{m_d} - \frac{2\tilde{m}}{m_s}\right)\frac{\partial^2\eta}{\sqrt{3}}\right]. \tag{B45}$$

Expanding the nucleon currents (B33)-(B38) to the first nonzero order in meson fields gives for the $q = u, d$ quark currents

$$\tilde{J}_q^{V,\mu} = \left(v^\mu + \frac{\tilde{k}_{12}^\mu}{2m_N}\right)\left(\bar{N}_q N_q + \bar{N}N\right) + ic_2' q^\mu \bar{N}N \tag{B46}$$
$$+ i\epsilon^{\alpha\beta\lambda\mu}v_\alpha q_\lambda\left(g_4\bar{N}_q S_{N\beta}N_q - g_4'\bar{N}S_{N\beta}N\right) + \cdots,$$

$$\tilde{J}_q^{A,\mu} = 2(D+F)\bar{N}_q\left(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{k}_{12}{\cdot}S_N\right)N_q + 2G\bar{N}\left(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{k}_{12}{\cdot}S_N\right)N \tag{B47}$$
$$+ iv^\mu\left(2f_3\bar{N}_q q{\cdot}S_N N_q - f_8'\bar{N}q{\cdot}S_N N\right) + \cdots,$$

$$\tilde{J}_q^S = -2b_0 m_q\bar{N}N - 2(b_D + b_F)m_q\bar{N}_q N_q + \cdots, \tag{B48}$$

$$\tilde{J}_q^P = \frac{2m_q}{f}\left[b_0\bar{N}N\left(\pm\pi^0 + \frac{\eta}{\sqrt{3}}\right) + (b_D + b_F)\bar{N}_q N_q\left(\pm\pi^0 + \frac{\eta}{\sqrt{3}}\right)\right] + \cdots. \tag{B49}$$

In $\tilde{J}_{q,\mu}^V$ and $\tilde{J}_{q,\mu}^A$ we keep the $\mathcal{O}(p)$ terms from (B39), (B40), and do not display the $v\cdot q$-suppressed terms, while for $\tilde{J}_q^P$ we display only the couplings to neutral mesons. The plus(minus) sign in (B49) is for $q = u(d)$, and there is no summation over repeated q indices. Here $N = (p_v, n_v)$ is the nucleon isospin doublet, so that the up and down components are

$$N_u = p_v, \quad N_d = n_v, \quad\text{while}\quad \bar{N}N = \bar{p}_v p_v + \bar{n}_v n_v. \tag{B50}$$

To shorten the notation we introduced

$$\tilde{k}_{12}^\mu = \tilde{k}_1^\mu + \tilde{k}_2^\mu, \qquad q^\mu = -\tilde{k}_1^\mu + \tilde{k}_2^\mu, \tag{B51}$$

with $\tilde{k}_{1,2}^\mu = k_{1,2}^\mu - m_N v_N^\mu$ the soft nucleon momenta. The expression for the momentum transfer q^μ coincides with the definition in (143).

The corresponding strange-quark currents are given by

$$\tilde{J}_s^{V,\mu} = ic_2' q^\mu \bar{N}N - i(g_4 - g_5 + g_4')\epsilon^{\alpha\beta\lambda\mu}v_\alpha q_\lambda\bar{N}S_{N\beta}N + \cdots, \tag{B52}$$

$$\tilde{J}_s^{A,\mu} = 2(D - F + G)\bar{N}\left(S_N^\mu - \frac{v^\mu}{2m_N}\tilde{k}_{12}{\cdot}S_N\right)N - i(f_8 + f_8')v^\mu\bar{N}q{\cdot}S_N N + \cdots, \tag{B53}$$

$$\tilde{J}_s^S = -2\left(b_0 + b_D - b_F\right)m_s\bar{N}N + \cdots, \tag{B54}$$

$$\tilde{J}_s^P = -\frac{4}{f}\left(b_0 + b_D - b_F\right)m_s\bar{N}N\frac{\eta}{\sqrt{3}} + \cdots, \tag{B55}$$

where for $\tilde{J}_s^P$ we again display only the couplings to the neutral mesons, and do not show the $v \cdot q$-suppressed terms in $\tilde{J}_s^{V,\mu}$ and $\tilde{J}_s^{A,\mu}$. Note that in order to obtain the above expressions we have used the reparametrisation-invariance relations (B12).

The conservation of the vector current, $q \cdot \tilde{J}_{q,s}^V = 0$, requires $c_2' = 0$. Comparing the most general parametrization of the matrix element for the axial-vector current, $\langle N(k_2)|\bar{q}\gamma^\mu\gamma_5 q|N(k_1)\rangle = \bar{u}_N\big(F_A(q^2)\gamma^\mu\gamma_5 + F_{P'}(q^2)q^\mu/(2m_N)\gamma_5\big)u_N$, with its nonrelativistic decomposition (16), (18), requires $f_3 = f_8 = f_8' = 0$.

The gluonic GG and $G\tilde{G}$ currents hadronize to

$$\tilde{J}^G = -\frac{2m_G}{27}\bar{N}N, \tag{B56}$$

$$\tilde{J}^\theta = -\left[D\left(\frac{\tilde{m}}{m_u}+\frac{\tilde{m}}{m_s}\right)+F\left(\frac{\tilde{m}}{m_u}-\frac{\tilde{m}}{m_s}\right)+G\right]\bar{p}_v i q \cdot S_N p_v \tag{B57}$$

$$+\, p \to n, u \to d + \cdots .$$

The values of the low-energy constants $D, F, G, m_G, b_0, b_D, b_F, g_4, g_4', g_5$ are discussed in the following section and are collected in Table I.

Appendix C: Values of low energy constants

In this appendix we derive numerical values for the low-energy coefficients $m_G, D, F, G, b_D, b_F, b_0, g_4, g_4', g_5$. The nonperturbative coefficient m_G is the gluonic contribution to the nucleon mass,

$$m_G\bar{u}_B u_B = -\frac{9\alpha_s}{8\pi}\langle B_v|G_{\mu\nu}G^{\mu\nu}|B_v\rangle. \tag{C1}$$

This can be estimated from the trace of the stress-energy tensor $\theta_\mu^\mu = -9\alpha_s/(8\pi)G_{\mu\nu}G^{\mu\nu} + \sum_{u,d,s}m_q\bar{q}q$, giving

$$m_G = m_B - \sum_q \sigma_q^B, \tag{C2}$$

where $\sigma_q^B\bar{u}_B u_B = \langle B_v|m_q\bar{q}q|B_v\rangle$, with u_B the heavy baryon spinor.[15] A common notation is also $\sigma_q^N = m_N f_{Tq}^{(N)}$, where $N = p, n$. Taking the naive average of the most recent lattice QCD

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[15] We use the conventional HQET normalization for the fields and states, $\langle B(\vec{k}')_{v'}|B(\vec{k})_v\rangle = 2v^0\delta_{vv'}(2\pi)^3\delta^3(\vec{k} - \vec{k}')$, so that $\langle B_v|\bar{B}_v\gamma^\mu B_v|B_v\rangle = \bar{u}_B(v)\gamma^\mu u_B(v) = 2v^\mu$, where the heavy fermion spinors u_B are related to relativistic spinors through $u(p) = \sqrt{m_N}u_B(v)$, see also [76]. Similarly, one has $\langle B_v|\bar{B}_v\gamma^\mu\gamma_5 B_v|B_v\rangle = \bar{u}_B(v)\gamma^\mu\gamma_5 u_B(v) = 2s^\mu$, where s^μ is the heavy baryon spin.

determinations [77–79], we find $\sigma_s = (41.3\pm7.7)$ MeV The matrix elements of u and d quarks are related to the $\sigma_{\pi N}$ term, defined as $\sigma_{\pi N} = \langle N|\bar{m}(\bar{u}u + \bar{d}d)|N\rangle$, where $\bar{m} = (m_u + m_d)/2$. A HBChPT analysis of the πN scattering data gives $\sigma_{\pi N} = 59(7)$ MeV [80], in agreement with $\sigma_{\pi N} = 52(3)(8)$ MeV obtained from a fit to world lattice $N_f = 2 + 1$ QCD data [81]. Including, however, both $\Delta(1232)$ and finite spacing in the fit shifts the central value to $\sigma_{\pi N} = 44$ MeV. We thus use a conservative estimate $\sigma_{\pi N} = (50 \pm 15)$ MeV. Using the expressions in [82] gives $\sigma_u^p = (17 \pm 5)$ MeV, $\sigma_d^p = (32 \pm 10)$ MeV, $\sigma_u^n = (15 \pm 5)$ MeV, $\sigma_d^n = (36 \pm 10)$ MeV. From there we get

$$m_G = (848 \pm 14) \text{ MeV}, \tag{C3}$$

in the isospin limit. While the isospin violation in the σ_q^N values, factoring out the masses, is of $\mathcal{O}(10\%)$, this translates to a very small isospin violation in m_G, of less than 1 MeV. The obtained value of m_G thus applies to both p and n, while for other members of B_v octet it is correct up to flavor $SU(3)$ breaking terms.

The σ_q^N are related to the low-energy constants b_0, b_D, and b_F through

$$\sigma_u^p = -2m_u(b_0 + b_D + b_F), \qquad \sigma_d^n = -2m_d(b_0 + b_D + b_F), \tag{C4}$$

$$\sigma_u^n = -2m_u b_0, \qquad \sigma_d^p = -2m_d b_0, \tag{C5}$$

while

$$\sigma_s = -2m_s(b_0 + b_D - b_F). \tag{C6}$$

The combinations that are well determined are

$$\bar{m}(b_D + b_F) = (-1.41 \pm 0.24)\text{MeV}, \tag{C7}$$

$$2\bar{m}(2b_0 + b_D + b_F) = -\sigma_{\pi N} = (-50 \pm 15)\text{MeV}, \tag{C8}$$

$$2m_s(b_0 + b_D - b_F) = -\sigma_s = (-41.3 \pm 7.7)\text{MeV}. \tag{C9}$$

where we used $\bar{m} = 3.5^{+0.7}_{-0.2}$MeV [27]. For the first line we used the results from [82]

$$\bar{m}(b_D + b_F) = Bc_5(m_d - m_u)\frac{m_d + m_u}{m_d - m_u}, \tag{C10}$$

with $Bc_5(m_d - m_u) = (-0.51 \pm 0.08)$ MeV [82, 83], and $m_u/m_d = 0.47 \pm 0.04$ [27]. The above results can be transcribed to

$$\bar{m}(b_D + b_F) = (-1.41 \pm 0.24)\text{MeV},$$

$$\bar{m}b_0 = (-11.8 \pm 3.8)\text{MeV}, \tag{C11}$$

$$\bar{m}(b_0 + b_D - b_F) = (-0.75 \pm 0.14)\text{MeV},$$

where we used $m_s/\bar{m} = 27.5 \pm 1.0$ [27]. From here we get

$$\bar{m}b_0 = (-12.5\pm3.8)\mathrm{MeV}\,, \qquad \bar{m}b_D = (4.8\pm1.9)\mathrm{MeV}\,, \qquad \bar{m}b_F = (-6.2\pm1.9)\mathrm{MeV}\,. \quad \text{(C12)}$$

Using symmetrized errors on $\bar{m} = 3.5^{+0.7}_{-0.2}\,\mathrm{MeV}$ [27] this gives at the renormalization scale $\mu = 2\,\mathrm{GeV}$

$$b_0 = -3.7 \pm 1.4\,, \qquad b_D = 1.4 \pm 0.8\,, \qquad b_F = -1.8 \pm 0.8\,. \quad \text{(C13)}$$

Note that the errors in the last set of relations are large because of the relatively poorly known $\bar{m}$.

The low-energy constants D, F, G multiplying the axial vector currents can be expressed in terms of the matrix elements

$$2s^{\mu}\Delta q_p = \langle p_v|\bar{q}\gamma^{\mu}\gamma_5 q|p_v\rangle_Q\,, \quad \text{(C14)}$$

where p is a proton state at rest, s^{μ} is the proton spin (or polarization) vector such that $s^2 = -1, s \cdot p = 0$, see, e.g. [84], and the matrix element is evaluated at scale Q. We work in the isospin limit so that (C14) gives also the matrix elements for neutrons with $d \leftrightarrow u$ exchanged,

$$\Delta u \equiv \Delta u_p = \Delta d_n\,, \qquad \Delta d \equiv \Delta d_p = \Delta u_n\,. \quad \text{(C15)}$$

The matrix elements Δq are scale dependent. The non-isosinglet combinations $\Delta u - \Delta d$ and $\Delta u + \Delta d - 2\Delta s$ are scale independent, since they are protected by non-anomalous Ward identities. The isovector combination

$$\Delta u - \Delta d = g_A = 1.2723(23), \quad \text{(C16)}$$

is determined precisely from nuclear β decays [27]. For the remaining two combinations we use lattice QCD determinations [85–90]. Following [91], the averages of lattice QCD results give $\Delta u + \Delta d = 0.521(53)$ and $\Delta s = -0.026(4)$ in $\overline{\mathrm{MS}}$ at $Q = 2\,\mathrm{GeV}$. Combining with Eq. (C16) this gives [91]

$$\Delta u = 0.897(27), \qquad \Delta d = -0.376(27), \qquad \Delta s = -0.026(4), \quad \text{(C17)}$$

all at the scale $Q = 2\,\mathrm{GeV}$. At LO in the chiral expansion we have then

$$2D = \Delta u - 2\Delta d + \Delta s\,, \qquad 2F = \Delta u - \Delta s\,, \qquad G = \Delta d\,, \quad \text{(C18)}$$

so that at $Q = 2$ GeV

$$D = 0.812(30) \,, \qquad F = 0.462(14) \,, \qquad G = -0.376(28) \,. \tag{C19}$$

Note that the scale invariant combination

$$D + F = g_A = 1.2723(23), \tag{C20}$$

is determined more precisely than D and F separately.

Proton and neutron magnetic moments fix the values of the coefficients g_4, g_5, g_4' in Eq. (98). Using the NLO quark vector currents (B39) (cf. also (B46), (B52), (B24)) one obtains

$$\frac{2}{3}(g_4 - g_4')_u + \frac{1}{3}(g_4')_d - \frac{1}{3}(g_5 - g_4 - g_4')_s = \frac{\mu_p}{m_N} \,, \tag{C21}$$

$$-\frac{1}{3}(g_4 - g_4')_d - \frac{2}{3}(g_4')_u - \frac{1}{3}(g_5 - g_4 - g_4')_s = \frac{\mu_n}{m_N} \,, \tag{C22}$$

where $\mu_p = 2.79$ and $\mu_n = -1.91$ are the values for proton and neutron magnetic moments in units of nuclear magnetons $\hat{\mu}_N = e/(2m_N)$ [27]. Above we denoted with subscripts which quark current $\tilde{J}_q^{V,\mu}$ the contributions originate from. The s quark contributions to the proton and neutron magnetic moments are the same in the isospin limit, giving [92] (see also [93])

$$-\frac{1}{3}(g_5 - g_4 - g_4') = \frac{\mu_s}{m_N} = \frac{-0.073(19)}{m_N} \,. \tag{C23}$$

We then have

$$g_4 = \frac{\mu_p - \mu_n}{m_N} = \frac{4.70}{m_N} \,, \qquad g_4' = -\frac{\mu_p + 2\mu_n}{m_N} = \frac{1.03}{m_N} \,, \tag{C24}$$

neglecting the small corrections due to μ_s. For notational convenience we also define

$$\mu_p = \frac{4}{3}\hat{\mu}_u^p - \frac{1}{3}\hat{\mu}_d^p \,, \qquad \mu_n = -\frac{2}{3}\hat{\mu}_d^n + \frac{2}{3}\hat{\mu}_u^n \,, \tag{C25}$$

where $\hat{\mu}_{u,d}^p$ and $\hat{\mu}_{u,d}^n$ are the contributions to the proton and neutron magnetic moments from the u- and d-quark currents (the hats indicate that the quark charges have been factored out from the definitions). Isospin relates contributions to neutron and proton, giving

$$\hat{\mu}_u^p = \hat{\mu}_d^n = 1.84 \,, \qquad \hat{\mu}_d^p = \hat{\mu}_u^n = -1.03 \,. \tag{C26}$$

Note that in the numerics it is advantageous not to use B_0 directly, but rather the numerical values for the products $B_0 m_q$. We can use the relation $2 B_0 \bar{m} = m_\pi^2$ to write

$$
\begin{aligned}
B_0 m_u &= \frac{m_\pi^2}{1 + m_d/m_u} = (6.2 \pm 0.4) \times 10^{-3}\,\mathrm{GeV}^2\,, \\
B_0 m_d &= \frac{m_\pi^2}{1 + m_u/m_d} = (13.3 \pm 0.4) \times 10^{-3}\,\mathrm{GeV}^2\,, \\
B_0 m_s &= \frac{m_\pi^2}{2}\frac{m_s}{\bar{m}} = (0.27 \pm 0.01)\,\mathrm{GeV}^2\,,
\end{aligned}
\tag{C27}
$$

using the ratios $m_u/m_d = 0.47 \pm 0.04$, $m_s/\bar{m} = 27.5 \pm 1.0$ [27], and the charged-pion mass for m_π.

Appendix D: HDMET for Majorana fermions

In this appendix we list the changes that need to be made in our results if DM is a Majorana fermion. The changes to the final results, given for Dirac fermions in (A13)-(A21), are straightforward. The Wilson coefficient $\hat{\mathcal{C}}_{1,2}^{(5)}$ and $\hat{\mathcal{C}}_{1,q;3,q}^{(6)}$ are zero, because the corresponding operators vanish for Majorana DM. All the other Wilson coefficients in (A13)-(A21) need to be multiplied by an extra factor of 2 that arises from matching onto HDMET, as we explain below.[16]

We construct the HDMET for Majorana fermions, following [94], by splitting the small and large components according to[17]

$$
\chi(x) = e^{-im_\chi v \cdot x}\big[\chi_v(x) + X_v(x)\big] + e^{im_\chi v \cdot x}\big[\chi_v^c(x) + X_v^c(x)\big]\,,
\tag{D1}
$$

where $\chi_v = \frac{1}{2}(1 + \not{v})\chi_v$, $X_v = \frac{1}{2}(1 - \not{v})X_v$, while the charge conjugated fields are given by $\chi_v^c = C\bar{\chi}_v^T$, $X_v^c = C\bar{X}_v^T$. Here C is the charge conjugation matrix satisfying $C^\dagger = C^T = C^{-1} = -C$ and $C\gamma_\mu^T C^{-1} = -\gamma_\mu$, for instance, one can choose $C = -i\gamma_2\gamma_0$. The "small-component" field X_v carries momenta of order $\mathcal{O}(2m_\chi)$ and is integrated out. At tree-level one has the relation

$$
\chi = e^{-im_\chi v \cdot x}\left(1 + \frac{i\not{\partial}_\perp}{iv \cdot \partial + 2m_\chi - i\epsilon}\right)\chi_v + e^{im_\chi v \cdot x}\left(1 - \frac{i\not{\partial}_\perp}{iv \cdot \partial - 2m_\chi + i\epsilon}\right)\chi_v^c\,.
\tag{D2}
$$

---

[16] We adopt the same notation, χ, for Majorana DM as we did for Dirac DM. No confusion should arise as this abuse of notation is restricted to this appendix.

[17] Alternatively, one could instead impose $\chi = e^{-im_\chi v \cdot x}\sqrt{2}\big[\chi_v(x) + X_v(x)\big] = e^{im_\chi v \cdot x}\sqrt{2}\big[\chi_v^c(x) + X_v^c(x)\big]$, with $\chi_v = \frac{1}{2}(1 + \not{v})\chi_v$, $X_v = \frac{1}{2}(1 - \not{v})X_v$ [13, 95]. The tree-level matching leads to the same results.

Note that χ is self-conjugate, $\chi^c = \chi$, while the HDMET field χ_v is not self-conjugate. The HDMET field describing Dirac fermion is also not self-conjugate. Still, there is a difference between HDMET describing Majorana and Dirac fermions, since for Majorana fermion the HDMET Lagrangian is symmetric under $v^\mu \to -v^\mu$, $\chi_v \to \chi_v^c$ [13, 94].

The relativistic Lagrangian for Majorana DM, $\mathcal{L} \supset \frac{1}{2}\bar{\chi}i\slashed{\partial}\chi - \frac{1}{2}m_\chi\bar{\chi}\chi$, then already leads to the canonically normalized HDMET Lagrangian

$$\mathcal{L}_{\text{HDMET}} = \bar{\chi}_v(iv\cdot\partial)\chi_v + \frac{1}{2m_\chi}\bar{\chi}_v(i\partial_\perp)^2\chi_v + \cdots + \mathcal{L}_{\chi_v}. \tag{D3}$$

The higher dimension interaction Lagrangian, $\mathcal{L}_{\chi_v}$, is given still by (14). Due to the Majorana nature of χ, however, not all operators enter: the operators that are odd under $v^\mu \to -v^\mu$, $\chi_v \to \chi_v^c$ vanish. Neglecting radiative corrections to the matching conditions we have

$$\bar{\chi}\chi \to 2\bar{\chi}_v\chi_v + \cdots , \tag{D4}$$

$$\bar{\chi}i\gamma_5\chi \to \frac{2}{m_\chi}\partial_\mu\big(\bar{\chi}_v S_\chi^\mu \chi_v\big) + \dots , \tag{D5}$$

$$\bar{\chi}\gamma^\mu\gamma_5\chi \to 4\bar{\chi}_v S_\chi^\mu \chi_v - 2\frac{i}{m_\chi}v^\mu\bar{\chi}_v S_\chi\cdot\overleftrightarrow{\partial}\chi_v + \cdots , \tag{D6}$$

which differs by an extra factor of 2 compared to the Dirac fermion case, while the remaining currents vanish, $\bar{\chi}\gamma^\mu\chi \to 0$, $\bar{\chi}\sigma^{\mu\nu}\chi \to 0$, $\chi\sigma^{\mu\nu}i\gamma_5\chi \to 0$.

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