Chiral Magnetic Wave at finite baryon density and the electric quadrupole moment of quark-gluon plasma in heavy ion collisions

Yannis Burnier,[1] Dmitri E. Kharzeev,[1, 2] Jinfeng Liao,[2] and Ho-Ung Yee[1]

[1]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA*
[2]*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
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Chiral Magnetic Wave (CMW) is a gapless collective excitation of quark-gluon plasma in the presence of external magnetic field that stems from the interplay of Chiral Magnetic (CME) and Chiral Separation Effects (CSE); it is composed by the waves of the electric and chiral charge densities coupled by the axial anomaly. We consider CMW at finite baryon density and find that it induces the electric quadrupole moment of the quark-gluon plasma produced in heavy ion collisions: the "poles" of the produced fireball (pointing outside of the reaction plane) acquire additional positive electric charge, and the "equator" acquires additional negative charge. We point out that this electric quadrupole deformation lifts the degeneracy between the elliptic flows of positive and negative pions leading to $v_2(\pi^+) < v_2(\pi^-)$, and estimate the magnitude of the effect.

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Introduction.—The axial anomaly has been found to induce the following two phenomena in the quark-gluon plasma subjected to an external magnetic field: the Chiral Magnetic Effect (CME) and the Chiral Separation Effect (CSE). The CME is the phenomenon of electric charge separation along the axis of the applied magnetic field in the presence of fluctuating topological charge [1–5]. The CME in QCD coupled to electromagnetism assumes a chirality asymmetry between left- and right-handed quarks, parametrized by an axial chemical potential μ_A. At finite μ_A, an external magnetic field induces the vector current $j_i = \bar{\psi}\gamma_i\psi$:

$$\boldsymbol{j}_V = \frac{N_c\, e}{2\pi^2}\mu_A \boldsymbol{B}; \qquad (1)$$

in our present convention the electric current is ej_V.

Recently, STAR [6, 7] and PHENIX [8, 9] Collaborations at Relativistic Heavy Ion Collider at BNL reported experimental observation of charge asymmetry fluctuations possibly providing an evidence for CME; this interpretation is still under intense discussion, see e.g. [10, 11] and references therein.

The Chiral Separation Effect (CSE) refers to the separation of chiral charge along the axis of external magnetic field at finite density of vector charge (e.g. at finite baryon number density) [12, 13]. The resulting axial current is given by

$$\boldsymbol{j}_A = \frac{N_c\, e}{2\pi^2}\mu_V \boldsymbol{B}, \qquad (2)$$

where μ_V is the vector chemical potential. Both CME and CSE effects have been proved robust in holographic QCD models in strong coupling regime [14–20] as well as in lattice QCD computations [21, 22]. The effects also persist in relativistic hydrodynamics, as shown in Ref.[23].

Recently, two of us studied the properties of the "Chiral Magnetic Wave" (CMW) [24] stemming from the coupling of the density waves of electric and chiral charge induced by the axial anomaly in the presence of external magnetic field; a related idea has been also discussed in [25]. The CMW is a gapless collective excitation; its existence is a straightforward consequence of the relations eq.(1) and eq.(2). Indeed, consider a local fluctuation of electric charge density; according to eq.(2) it induces a local fluctuation of axial current. This fluctuation of axial current in turn induces a local fluctuation of the axial chemical potential, and thus according to eq.(1) a fluctuation of electric current. The resulting fluctuation of electric charge density completes the cycle leading to the CMW that combines the density waves of electric and chiral charges.

The plasma created in heavy ion collisions possesses a finite baryon density. The CSE [12, 13, 23] then implies the separation of chiral charge: the "poles" of the fireball acquire the chiral charges of opposite sign. The CME current at the opposite poles then according to eq.(1) flows in opposite directions, as argued recently in [26]. In this letter we will show that CMW induces a static quadrupole moment of the electric charge density.

Chiral Magnetic Wave.—The Chiral Magnetic Wave (CMW) is a long wavelength hydrodynamic mode of chiral charge densities; their propagation in space-time is described by the following equation [24]

$$\left(\partial_0 \mp \partial_1 v_\chi - D_L\partial_1^2\right)j^0_{L,R} = 0, \qquad (3)$$

where v_χ is the wave velocity and D_L is the longitudinal diffusion constant.

In the case of N_f quark flavors with electric charges q_f there will be N_f independent CMWs with the velocities and longitudinal diffusion constants determined by q_f, eB and T. In this paper we consider the propagation of u and d flavored CMWs, since there is no net density of

strange quarks in the plasma. The full flavor symmetry $U(2)_f$ contains $U(1)_u \times U(1)_d$ which defines independent $U(1)$ flavor symmetries of u and d quarks. Considering the same triangle anomalies leading to CME and CSE that now involve each of these $U(1)$ symmetries, one obtains

$$\boldsymbol{j}_{V,A}^f = q_f \frac{N_c e}{2\pi^2} \mu_{A,V}^f \boldsymbol{B}, \qquad (4)$$

where μ^f are chemical potentials of $U(1)_f$. From the results of [24] and (4) we then derive N_f independent CMWs of flavored chiral charge densities $j_{L,R}^{0,f}$ with velocities given by

$$v_\chi^f = q_f \frac{N_c eB}{4\pi^2} \left(\frac{\partial \mu_L^f}{\partial j_L^{0,f}} \right) \equiv q_f \frac{N_c eB \alpha^f}{4\pi^2}. \qquad (5)$$

We obtain v_χ^f and D_L^f from the computation in Ref.[24] performed in the framework of Sakai-Sugimoto model in the large N_c quenched approximation. Each quark of flavor f interacts with the magnetic field of effective magnitude $q_f eB$, we replace eB with $q_f eB$ in the arguments of v_χ and D_L as functions of eB:

$$v_\chi^f = v_\chi \left(eB \to q_f eB \right), \quad D_L^f = D_L \left(eB \to q_f eB \right). \qquad (6)$$

We neglect the isospin asymmetry between the u and d flavors at the time of plasma creation, and take the equal initial chemical potentials $\mu_V^f = \mu_B/3$. The shape of the initial "almond" of QCD matter produced in a heavy ion collision is taken by using the phenomenologically successful KLN model [27] based on parton saturation and k_T factorization. $Au - Au$ collisions have been simulated, with realistic Woods-Saxon nuclear densities. The axial chemical potentials at the initial time are set to zero.

We then solve the CMW equation numerically and find that it generates the separation of chiral charge, as shown in Fig.1 – the quark-gluon plasma acquires a "chiral dipole moment".

We evaluate the total electric charge distribution by super-imposing the waves of different flavors weighted by their charges,

$$j_e^0 = \sum_f q_f \left(j_L^{0,f} + j_R^{0,f} \right) \quad . \qquad (7)$$

The resulting distribution is shown in Fig.2; for clarity, we have subtracted the charge density distribution without the CMW. As argued above qualitatively, the quark-gluon plasma indeed acquires an electric quadrupole moment. The "poles" of the produced fireball (pointing outside of the reaction plane) acquire additional positive electric charge, and the "equator" acquires additional negative charge. It is very important to note that this pattern of charge separation does not depend on the orientation of magnetic field. This means that the effect should survive even after the event averaging.
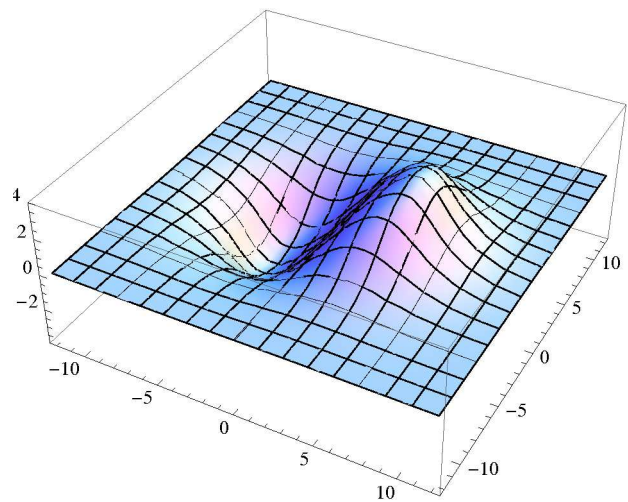

FIG. 1: Chiral charge density in the plane transverse to the beam axis; magnetic field strength $eB = m_\pi^2$, lifetime of magnetic field $\tau = 10$ fm, temperature $T = 165$ MeV, impact parameter $b = 3$ fm.
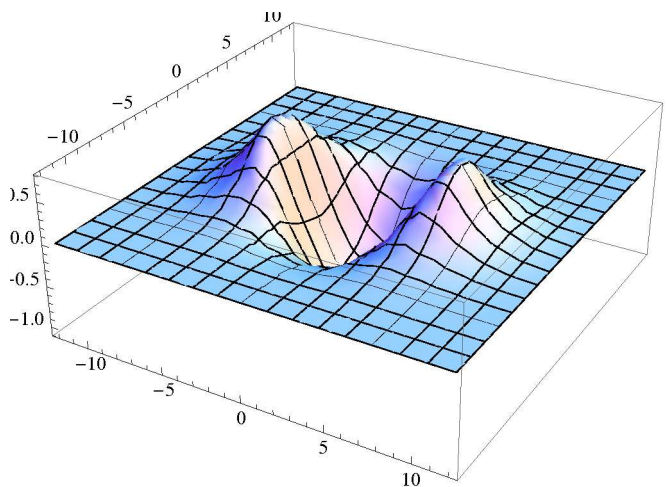

FIG. 2: Electric charge density in the transverse plane (background subtracted, see text); same parameters as in Fig. 1.

From the Electric Quadrupole Moment to Charge-Dependent Elliptic Flow.—The expansion of the quark-gluon plasma produced in heavy ion collisions is characterized by strong anisotropic collective flow driven by the gradients of pressure that transforms the spatial anisotropy of produced matter into the momentum anisotropy of the produced hadrons. Since the fireball of quark-gluon plasma produced in an off-central heavy ion collision has an elliptical almond-like shape, the gradients of pressure make it expand predominantly along the minor axis, i.e. in the reaction plane – this is the "elliptic flow" (for a review, see [28]). As a result, the electric quadrupole deformation of the plasma described above will increase the elliptic flow of negative hadrons, and decrease the elliptic flow of positive hadrons leading to $v_2^+ < v_2^-$. However, the large differences in the ab-

sorption cross sections of antiprotons and protons, and of negative and positive kaons in hadronic matter at finite baryon density are likely to mask or reverse this difference in the hadron resonance "afterburner" phase of a heavy ion collision. On the other hand, the smaller difference in the absorption cross sections of negative and positive pions potentially may make it possible to detect the electric quadrupole moment of the plasma through difference of elliptic flows of pions, $v_2(\pi^+) < v_2(\pi^-)$.

Let us now quantify this statement and estimate the magnitude of the effect. The net electric charge density is a sum of the initial charge density and the modulation caused by the propagating CMW. Keeping the leading term in the multipole expansion, we thus write

$$\rho_e \equiv \bar{\rho}_e - 2q_e \cos(2\phi) = \bar{\rho}_e \left[1 - \left(\frac{q_e}{\bar{\rho}_e}\right) 2\cos(2\phi)\right] \quad (8)$$

Note that the quadrupole term $\sim q_e$ only re-shuffles the initial charges such that the net charge density ρ_e is larger out-of-plane than in-plane.

The bulk evolution of the system should not be significantly affected by the quadrupole deformation as q_e is small compared to the overall density. We therefore assume that the overall elliptic flow is not modified and the charged hadron distribution in the azimuthal angle ϕ with respect to the reaction plane is still given by

$$\frac{dN_{ch}}{d\phi} = N_0 \left[1 + 2v_2 \cos(2\phi)\right]. \quad (9)$$

Nonzero ρ_e translates into the difference of the yields of positive and negative charges. Let us split the total charged hadron number at hadronization N_0 into the numbers of positive and negative hadrons: $N_0 = N_+ + N_-$. With zero ρ_e, one has $N_+ = N_-$. An isotropic $\bar{\rho}_e$ stemming from the baryon stopping in a heavy ion collision leads to non-zero difference $(N_+ - N_-) > 0$ independent of the azimuthal angle ϕ. When there is, in addition, a quadrupole deformation $-2q_e \cos(2\phi)$, the asymmetry $(N_+ - N_-)$ develops an azimuthal angle dependence. In the presence of a strong radial flow, the direction of the momentum of a fluid element coincides with the direction of the radius-vector pointing to the position of this fluid element. Therefore, a quadrupole deformation in coordinate space should translate into a quadrupole deformation in momentum space, and we thus expect a non-zero ϕ-dependent difference between N_+ and N_-:

$$N_+ = \bar{N}_+ - N_q \cos(2\phi); \quad (10)$$
$$N_- = \bar{N}_- + N_q \cos(2\phi). \quad (11)$$

The difference now becomes

$$N_+ - N_- = (\bar{N}_+ - \bar{N}_-)\left[1 - \frac{N_q}{\bar{N}_+ - \bar{N}_-}2\cos(2\phi)\right]. (12)$$

While the $(\bar{N}_+ - \bar{N}_-)$ is due to the charge density $\bar{\rho}_e$, the ϕ-dependent part is due to the quadrupole deformation q_e. Since the two components of net charge density corresponding to the first and second terms in Eq.(8) are transformed into the final plus/minus difference via the same bulk evolution, it is plausible to assume that the ratio between the two components – the quadrupole and the background parts in Eq.(12) – remains the same as in Eq.(8):

$$\frac{N_q}{\bar{N}_+ - \bar{N}_-} = \frac{q_e}{\bar{\rho}_e} \quad (13)$$

This translates the electric quadrupole deformation of the plasma into the charge dependence of the elliptic flow of hadrons. Indeed the ϕ-dependence in Eq.(12) leads to a charge-dependent elliptic flow term:

$$\frac{dN_\pm}{d\phi} = N_\pm \left[1 + 2v_2 \cos(2\phi)\right] \approx \bar{N}_\pm \Big[1 + 2v_2 \cos(2\phi)$$
$$\mp \left(\frac{q_e}{\bar{\mu}_e}\right)\left(\frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-}\right) 2\cos(2\phi)\Big]; \quad (14)$$

in deriving this relation we assumed $v_2 \ll 1$ and $(\bar{N}_+ - \bar{N}_-) \ll \bar{N}_\pm$. The elliptic flow thus becomes charge dependent:

$$v_2^\pm = v_2 \mp \left(\frac{q_e}{\bar{\rho}_e}\right) A_\pm; \quad (15)$$

where $A_\pm \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$ is the charge asymmetry in the plasma.

The magnitude of the effect: numerical simulation.— As described above, we have computed the evolution of the right and left chiral components of the u and d quarks according to equation (3) (at zero rapidity) in a static plasma. For simplicity, we assume the temperature to be uniform within the almond. At the boundary of the plasma, the chiral symmetry is broken and therefore we set $v_\chi = 0$. In the transverse (w.r.t. the magnetic field) direction, we assume a diffusion with a diffusion constant D_T estimated [24] as $D_T = (2\pi T)^{-1}$ within the Sakai-Sugimoto model. The difference in the elliptic flows of positive and negative pions is given, within our approximation, by eq.(15). In Fig.3 we present the ratio $r = 2q_e/\bar{\rho}_e$ of the electric quadrupole moment

$$q_e = \int RdR \, d\phi \, \cos(2\phi) \left(j_e^0(R,\phi) - j_{e,B=0}^0(R,\phi)\right). \quad (16)$$

to the total charge density

$$\bar{\rho}_e = \int RdR \, d\phi \, j_{e,B=0}^0(R,\phi). \quad (17)$$

In this computation we took the impact parameter dependence of magnetic field from [3], with the maximal value $eB|_{max} = m_\pi^2$. To convert this ratio into the difference of the elliptic flows of positive and negative pions

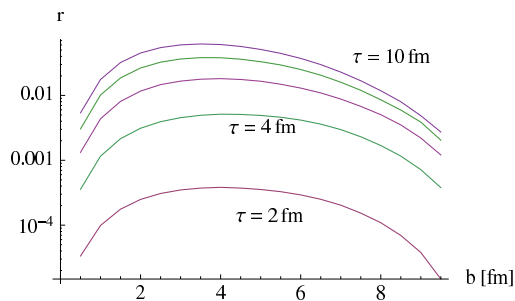FIG. 3: The normalized electric quadrupole moment r, $eB|_{max} = m_\pi^2$, $T = 165$ MeV.

according to Eq.(15) we also have to estimate the electric charge asymmetry $A_\pm$ in the quark-gluon plasma that varies between 0 and 1. We do this using the baryon chemical potential and temperature at freeze-out extracted [29] from the data, and for the energy of $\sqrt{s} = 11$ GeV estimate $A_\pm \simeq 0.3$; note that at finite baryon density the asymmetry in the plasma and at freeze-out may differ. The lifetime of magnetic field in the plasma is still uncertain: the initial pulse of magnetic field rapidly falls of with time [3],[30], but the induction in electrically conducting quark-gluon plasma was estimated to drastically extend the lifetime of the magnetic field according to the Lenz's rule, perhaps making it last for the entire lifetime of the plasma [31]. Choosing for the sake of estimate $\tau = 8$ fm and the corresponding $r \simeq 0.04$ for mid-central collisions in the plot of Fig. 3 and using $A_\pm \simeq 0.3$ we estimate the difference of π^- and π^+ elliptic flows in $AuAu$ collisions at $\sqrt{s} = 11$ GeV as

$$\Delta v_2^{CMW} \equiv v_2(\pi^-) - v_2(\pi^+) \simeq r \, A_\pm \simeq 0.01 \qquad (18)$$

The pion elliptic flow at $\sqrt{s} = 8.7$ GeV in mid-central $Pb - Pb$ collisions is about $v_2 \simeq 0.03$ [32]. Therefore, the difference between the elliptic flows of positive and negative pions may be as big as $\sim 30\%$; this would clearly make it observable.

Uncertainties and outlook.— The main source of uncertainty in our computation is the lifetime of magnetic field that will need to be evaluated numerically within relativistic magnetohydrodynamics (MHD). Our treatment of expansion and evolution of the plasma has been quite crude, and will also need to be refined by an MHD computation. Possible backgrounds to the effect considered here have to be carefully analyzed, and include the Coulomb interaction of produced pions and the difference in absorption of negative and positive pions in dense resonance gas. Since the CMW can propagate only in the chirally symmetric phase, the electric quadrupole deformation of the plasma can provide a signature of chiral symmetry restoration in heavy ion collisions.

[1] D. Kharzeev, Phys. Lett. B **633**, 260 (2006).
[2] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007) [arXiv:0706.1026 [hep-ph]].
[3] D. E. Kharzeev, L. D. McLerran and H .J. Warringa, Nucl. Phys. A **803**, 227 (2008);
[4] K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008) [arXiv:0808.3382 [hep-ph]].
[5] D. E. Kharzeev, Annals Phys. **325**, 205 (2010).
[6] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. Lett. **103**, 251601 (2009) [arXiv:0909.1739 [nucl-ex]].
[7] B. I. Abelev *et al.* [STAR Collaboration], Phys. Rev. C **81**, 054908 (2010) [arXiv:0909.1717 [nucl-ex]].
[8] N. N. Ajitanand, S. Esumi, R. A. Lacey [PHENIX Collaboration], in: Proc. of the RBRC Workshops, vol. 96, 2010: "P- and CP-odd effects in hot and dense matter".
[9] N. N. Ajitanand, R. A. Lacey, A. Taranenko and J. M. Alexander, arXiv:1009.5624 [nucl-ex].
[10] A. Bzdak, V. Koch and J. Liao, Phys. Rev. C **81**, 031901 (2010); Phys. Rev. C **82**, 054902.
[11] D. E. Kharzeev, D. T. Son, Phys. Rev. Lett. **106**, 062301 (2011); [arXiv:1010.0038 [hep-ph]].
[12] D. T. Son and A. R. Zhitnitsky, Phys. Rev. D **70**, 074018 (2004) [arXiv:hep-ph/0405216].
[13] M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2005) [arXiv:hep-ph/0505072].
[14] H. U. Yee, JHEP **0911**, 085 (2009).
[15] V. A. Rubakov, arXiv:1005.1888 [hep-ph].
[16] A. Rebhan, A. Schmitt and S. A. Stricker, JHEP **1001**, 026 (2010) [arXiv:0909.4782 [hep-th]].
[17] A. Gynther, K. Landsteiner, F. Pena-Benitez and A. Rebhan, arXiv:1005.2587 [hep-th].
[18] A. Gorsky, P. N. Kopnin and A. V. Zayakin, arXiv:1003.2293 [hep-ph].
[19] L. Brits, J. Charbonneau, [arXiv:1009.4230 [hep-th]].
[20] I. Amado, K. Landsteiner and F. Pena-Benitez, arXiv:1102.4577 [hep-th].
[21] P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, Phys. Rev. D **80**, 054503 (2009).
[22] M. Abramczyk, T. Blum, G. Petropoulos and R. Zhou, arXiv:0911.1348 [hep-lat].
[23] D. T. Son and P. Surowka, Phys. Rev. Lett. **103**, 191601 (2009) [arXiv:0906.5044 [hep-th]].
[24] D. E. Kharzeev, H. -U. Yee, [arXiv:1012.6026 [hep-th]].
[25] G. M. Newman, JHEP **0601**, 158 (2006).
[26] E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, [arXiv:1101.4954 [hep-ph]].
[27] D. Kharzeev, M. Nardi, Phys. Lett. **B507**, 121-128 (2001); [nucl-th/0012025]. D. Kharzeev and E. Levin, Phys. Lett. B **523** (2001) 79 [arXiv:nucl-th/0108006]; D. Kharzeev, E. Levin, M. Nardi, Phys. Rev. **C71**, 054903 (2005). [hep-ph/0111315].
[28] S. A. Voloshin, A. M. Poskanzer, R. Snellings, [arXiv:0809.2949 [nucl-ex]].
[29] A. Andronic, P. Braun-Munzinger, J. Stachel, Phys. Lett. **B673**, 142-145 (2009). [arXiv:0812.1186 [nucl-th]].
[30] V. Skokov, A. Y. Illarionov, V. Toneev, Int. J. Mod. Phys. **A24**, 5925-5932 (2009).
[31] K. Tuchin, Phys. Rev. **C82**, 034904 (2010).
[32] C. Alt *et al.* [NA49 Collaboration], Phys. Rev. **C68**, 034903 (2003). [nucl-ex/0303001].