

# Chiral Perturbation Theory

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**Abstract.** An introduction to the basic ideas and methods of Chiral Perturbation Theory is presented. Several phenomenological applications of the effective Lagrangian technique to strong, electromagnetic and weak interactions are discussed.

To be published in Reports on Progress in Physics

February 1995

## 1. Introduction

Quantum Chromodynamics (QCD) is nowadays the established theory of the strong interactions. Owing to its asymptotic-free nature (Gross and Wilczek 1973, Politzer 1973), perturbation theory can be applied at short distances; the resulting predictions have achieved a remarkable success, explaining a wide range of phenomena where large momentum transfers are involved. In the low-energy domain, however, the growing of the running QCD coupling and the associated confinement of quarks and gluons make very difficult to perform a thorough analysis of the QCD dynamics in terms of these fundamental degrees of freedom. A description in terms of the hadronic asymptotic states seems more adequate; unfortunately, given the richness of the hadronic spectrum, this is also a formidable task.

At very low energies, a great simplification of the strong-interaction dynamics occurs. Below the resonance region ( $E < M_\rho$ ), the hadronic spectrum only contains an octet of very light pseudoscalar particles ( $\pi, K, \eta$ ), whose interactions can be easily understood with global symmetry considerations. This has allowed the development of a powerful theoretical framework, Chiral Perturbation Theory (ChPT), to systematically analyze the low-energy implications of the QCD symmetries. This formalism is based on two key ingredients: the chiral symmetry properties of QCD and the concept of effective field theory.

The pseudoscalar octet can be identified with the multiplet of (approximately) massless Goldstone bosons associated with the spontaneous breakdown of chiral symmetry. Goldstone particles obey low-energy theorems, which result in the known predictions of Current Algebra and PCAC (Adler and Dashen 1968, de Alfaro *et al* 1973). Moreover, since there is mass gap separating the light Goldstone states from the rest of the hadronic spectrum, one can build an effective field theory, incorporating the right symmetry requirements, with Goldstone particles as the only dynamical degrees of freedom (Weinberg 1967a, Cronin 1967, Schwinger 1967, Wess and Zumino 1967, Dashen and Weinstein 1969, Gasirowicz and Geffen 1969). This leads to a great simplification of Current Algebra calculations and, what is more important, allows for a systematic investigation of higher-order corrections in the perturbative field-theory sense (Weinberg 1979, Gasser and Leutwyler 1984, 1985).

Effective field theories are the appropriate theoretical tool to describe low-energy physics, where *low* is defined with respect to some energy scale  $\Lambda$ . They only take explicitly into account the relevant degrees of freedom, i.e. those states with  $m \ll \Lambda$ , while the heavier excitations with  $M \gg \Lambda$  are integrated out from the action. One gets in this way a string of non-renormalizable interactions among the light states, which can be organized as an expansion in powers of energy/ $\Lambda$ . The information on the heavier degrees of freedom is then contained in the couplings of the resulting low-energy Lagrangian. Although effective field theories contain an infinite number of terms, renormalizability is not an issue since, at a given order in the energy expansion, the low-energy theory is specified by a finite number of couplings; this allows for an

order-by-order renormalization.

A simple example of effective field theory is provided by QED at very low energies,  $E_\gamma \ll m_e$ . In this limit, one can describe the light-by-light scattering using an effective Lagrangian in terms of the electromagnetic field only. Gauge, Lorentz and Parity invariance constrain the possible structures present in the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{a}{m_e^4}(F^{\mu\nu}F_{\mu\nu})^2 + \frac{b}{m_e^4}F^{\mu\nu}F_{\nu\sigma}F^{\sigma\rho}F_{\rho\mu} + \mathcal{O}(F^6/m_e^8). \quad (1.1)$$

In the low-energy regime, all the information on the original QED dynamics is embodied in the values of the two low-energy couplings  $a$  and  $b$ . The values of these constants can be computed, by explicitly integrating out the electron field from the original QED generating functional (or equivalently, by computing the relevant light-by-light box diagrams). One then gets the well-known result (Euler 1936, Euler and Heisenberg 1936):

$$a = -\frac{\alpha^2}{36}, \quad b = \frac{7\alpha^2}{90}. \quad (1.2)$$

The important point to realize is that, even in the absence of an explicit computation of the couplings  $a$  and  $b$ , the Lagrangian (1.1) contains non-trivial information, which is a consequence of the imposed symmetries. The dominant contributions to the amplitudes for different low-energy photon reactions like  $\gamma\gamma \rightarrow 2\gamma, 4\gamma, \dots$  can be directly obtained from  $\mathcal{L}_{\text{eff}}$ . Moreover, the order of magnitude of the constants  $a, b$  can also be easily estimated through a naïve counting of powers of the electromagnetic coupling and combinatorial and loop  $[1/(16\pi^2)]$  factors.

The previous example is somehow academic, since perturbation theory in powers of  $\alpha$  works extremely well in QED. However, the effective Lagrangian (1.1) would be valid even if the fine structure constant were big; the only difference would then be that we would not be able to perturbatively compute the couplings  $a$  and  $b$ .

In QCD, due to confinement, the quark and gluon fields are not asymptotic states. Moreover, we do not know how to derive the hadronic interactions directly from the fundamental QCD Lagrangian. However, we do know the symmetry properties of the strong interactions; therefore, we can write an effective field theory in terms of the hadronic asymptotic states, and parametrize the unknown dynamical information in a few couplings.

The theoretical basis of effective field theories can be formulated (Weinberg 1979, Leutwyler 1994a) as a **theorem**: *for a given set of asymptotic states, perturbation theory with the most general Lagrangian containing all terms allowed by the assumed symmetries will yield the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetries.*

In the following, I will present an overview of ChPT. The chiral symmetry of the QCD Lagrangian is discussed in section 2. The ChPT formalism is presented in sections 3 and 4, where the lowest-order and next-to-leading-order terms in the chiral expansion are analyzed. Section 5 contains a few selected phenomenological

applications. The role of the lowest-mass resonances on the Goldstone interactions is studied in section 6 and the relation between the effective Lagrangian and the underlying fundamental QCD theory is discussed in section 7, which summarizes recent attempts to calculate the chiral couplings. The effective realization of the non-leptonic  $\Delta S = 1$  interactions and a brief overview of the application of the chiral techniques to non-leptonic  $K$  decays is given in section 8.

Section 9 presents the ChPT formalism in the baryon sector. Some issues concerning the  $U(1)_A$  anomaly and the strong-CP problem are analyzed in section 10. The broad range of application of the ChPT techniques is finally illustrated in sections 11 and 12, which briefly discuss the low-energy interactions of an hypothetical light Higgs boson and the Goldstone dynamics associated with the Standard Model electroweak symmetry breaking. A few summarizing comments are collected in section 13.

This report has grown out of a previous set of lectures (Pich 1994); therefore, rather than giving an exhaustive and updated summary of the field, it attempts to provide a more pedagogical introduction. I have made extensive use of excellent reviews (Bijnens 1993a, Ecker 1993, 1995, Gasser 1990, Leutwyler 1991, 1994c, Meißner 1993, de Rafael 1995) and books (Donoghue, Golowich and Holstein 1992, Georgi 1984, Meißner 1992) already existing in the literature. Further details on particular subjects can be found in those references.

## 2. Chiral symmetry

In the absence of quark masses, the QCD Lagrangian [ $q = \text{column}(u, d, \dots)$ ]

$$\mathcal{L}_{QCD}^0 = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R \quad (2.1)$$

is invariant under independent *global*  $G \equiv SU(N_f)_L \otimes SU(N_f)_R$  transformations<sup>†</sup> of the left- and right-handed quarks in flavour space:

$$q_L \xrightarrow{G} g_L q_L, \quad q_R \xrightarrow{G} g_R q_R, \quad g_{L,R} \in SU(N_f)_{L,R}. \quad (2.2)$$

The Noether currents associated with the chiral group  $G$  are [ $\lambda_a$  are Gell-Mann's matrices with  $\text{Tr}(\lambda_a \lambda_b) = 2\delta_{ab}$ ]:

$$J_X^{a\mu} = \bar{q}_X \gamma^\mu \frac{\lambda_a}{2} q_X, \quad (X = L, R; \quad a = 1, \dots, 8). \quad (2.3)$$

The corresponding Noether charges  $Q_X^a = \int d^3x J_X^{a0}(x)$  satisfy the familiar commutation relations

$$[Q_X^a, Q_Y^b] = i\delta_{XY} f_{abc} Q_X^c, \quad (2.4)$$

<sup>†</sup> Actually, the Lagrangian (2.1) has a larger  $U(N_f)_L \otimes U(N_f)_R$  global symmetry. However, the  $U(1)_A$  part is broken by quantum effects [ $U(1)_A$  anomaly], while the quark-number symmetry  $U(1)_V$  is trivially realized in the meson sector.

which were the starting point of the Current Algebra methods of the sixties.

This chiral symmetry, which should be approximately good in the light quark sector ( $u, d, s$ ), is however not seen in the hadronic spectrum. Although hadrons can be nicely classified in  $SU(3)_V$  representations, degenerate multiplets with opposite parity do not exist. Moreover, the octet of pseudoscalar mesons happens to be much lighter than all the other hadronic states. To be consistent with this experimental fact, the ground state of the theory (the vacuum) should not be symmetric under the chiral group. The  $SU(3)_L \otimes SU(3)_R$  symmetry spontaneously breaks down to  $SU(3)_{L+R}$  and, according to Goldstone's (1961) theorem, an octet of pseudoscalar massless bosons appears in the theory.

More specifically, let us consider a Noether charge  $Q$ , and assume the existence of an operator  $O$  that satisfies

$$\langle 0|[Q, O]|0\rangle \neq 0; \quad (2.5)$$

this is clearly only possible if  $Q|0\rangle \neq 0$ . Goldstone's theorem then tells us that there exists a massless state  $|G\rangle$  such that

$$\langle 0|J^0|G\rangle \langle G|O|0\rangle \neq 0. \quad (2.6)$$

The quantum numbers of the Goldstone boson are dictated by those of  $J^0$  and  $O$ . The quantity in the left-hand side of equation (2.5) is called the order parameter of the spontaneous symmetry breakdown.

Since there are eight broken axial generators of the chiral group,  $Q_A^a = Q_R^a - Q_L^a$ , there should be eight pseudoscalar Goldstone states  $|G^a\rangle$ , which we can identify with the eight lightest hadronic states ( $\pi^+$ ,  $\pi^-$ ,  $\pi^0$ ,  $\eta$ ,  $K^+$ ,  $K^-$ ,  $K^0$  and  $\bar{K}^0$ ); their small masses being generated by the quark-mass matrix, which explicitly breaks the global symmetry of the QCD Lagrangian. The corresponding  $O^a$  must be pseudoscalar operators. The simplest possibility are  $O^a = \bar{q}\gamma_5\lambda_a q$ , which satisfy

$$\langle 0|[Q_A^a, \bar{q}\gamma_5\lambda_b q]|0\rangle = -\frac{1}{2} \langle 0|\bar{q}\{\lambda_a, \lambda_b\}q|0\rangle = -\frac{2}{3} \delta_{ab} \langle 0|\bar{q}q|0\rangle. \quad (2.7)$$

The quark condensate

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{s}s|0\rangle \neq 0 \quad (2.8)$$

is then the natural order parameter of Spontaneous Chiral Symmetry Breaking (SCSB).

### 3. Effective chiral Lagrangian at lowest order

The Goldstone nature of the pseudoscalar mesons implies strong constraints on their interactions, which can be most easily analyzed on the basis of an effective Lagrangian. Since there is a mass gap separating the pseudoscalar octet from the rest

of the hadronic spectrum, we can build an effective field theory containing only the Goldstone modes. Our basic assumption is the pattern of SCSB:

$$G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{\text{SCSB}} H \equiv SU(3)_V. \quad (3.1)$$

Let us denote  $\phi^a$  ( $a = 1, \dots, 8$ ) the coordinates describing the Goldstone fields in the coset space  $G/H$ , and choose a coset representative  $\bar{\xi}(\phi) \equiv (\xi_L(\phi), \xi_R(\phi)) \in G$ . The change of the Goldstone coordinates under a chiral transformation  $g \equiv (g_L, g_R) \in G$  is given by

$$\xi_L(\phi) \xrightarrow{G} g_L \xi_L(\phi) h^\dagger(\phi, g), \quad \xi_R(\phi) \xrightarrow{G} g_R \xi_R(\phi) h^\dagger(\phi, g), \quad (3.2)$$

where  $h(\phi, g) \in H$  is a compensating transformation which is needed to return to the given choice of coset representative  $\bar{\xi}$ ; in general,  $h$  depends both on  $\phi$  and  $g$ . Since the same transformation  $h(\phi, g)$  occurs in the left and right sectors (the two chiral sectors can be related by a parity transformation, which obviously leaves  $H$  invariant), we can get rid of it by combining the two chiral relations in (3.2) into the simpler form

$$U(\phi) \xrightarrow{G} g_R U(\phi) g_L^\dagger, \quad U(\phi) \equiv \xi_R(\phi) \xi_L^\dagger(\phi). \quad (3.3)$$

Moreover, without loss of generality, we can take a canonical choice of coset representative such that  $\xi_R(\phi) = \xi_L^\dagger(\phi) \equiv u(\phi)$ . The  $3 \times 3$  unitary matrix

$$U(\phi) = u(\phi)^2 = \exp \left\{ i\sqrt{2}\Phi/f \right\} \quad (3.4)$$

gives a very convenient parametrization of the Goldstone fields

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}} \vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \quad (3.5)$$

Notice that  $U(\phi)$  transforms linearly under the chiral group, but the induced transformation on the Goldstone fields  $\vec{\phi}$  is highly non-linear.

To get a low-energy effective Lagrangian realization of QCD, for the light-quark sector ( $u, d, s$ ), we should write the most general Lagrangian involving the matrix  $U(\phi)$ , which is consistent with chiral symmetry. The Lagrangian can be organized in terms of increasing powers of momentum or, equivalently, in terms of an increasing number of derivatives (parity conservation requires an even number of derivatives):

$$\mathcal{L}_{\text{eff}}(U) = \sum_n \mathcal{L}_{2n}. \quad (3.6)$$

In the low-energy domain we are interested in, the terms with a minimum number of derivatives will dominate.

Due to the unitarity of the  $U$  matrix,  $UU^\dagger = I$ , at least two derivatives are required to generate a non-trivial interaction. To lowest order, the effective chiral Lagrangian is uniquely given by the term

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu U^\dagger \partial^\mu U \rangle, \quad (3.7)$$

where  $\langle A \rangle$  denotes the trace of the matrix  $A$ .

Expanding  $U(\phi)$  in a power series in  $\Phi$ , one obtains the Goldstone kinetic terms plus a tower of interactions involving an increasing number of pseudoscalars. The requirement that the kinetic terms are properly normalized fixes the global coefficient  $f^2/4$  in equation (3.7). All interactions among the Goldstones can then be predicted in terms of the single coupling  $f$ :

$$\mathcal{L}_2 = \frac{1}{2} \langle \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{1}{12f^2} \langle (\Phi \overleftrightarrow{\partial}_\mu \Phi) (\Phi \overleftrightarrow{\partial}^\mu \Phi) \rangle + \mathcal{O}(\Phi^6/f^4). \quad (3.8)$$

To compute the  $\pi\pi$  scattering amplitude, for instance, is now a trivial perturbative exercise. One gets the well-known (Weinberg 1966) result [ $t \equiv (p'_+ - p_+)^2$ ]

$$T(\pi^+\pi^0 \rightarrow \pi^+\pi^0) = \frac{t}{f^2}. \quad (3.9)$$

Similar results can be obtained for  $\pi\pi \rightarrow 4\pi, 6\pi, 8\pi, \dots$ . The non-linearity of the effective Lagrangian relates amplitudes with different numbers of Goldstone bosons, allowing for absolute predictions in terms of  $f$ .

The effective field theory technique becomes much more powerful if one introduces couplings to external classical fields. Let us consider an extended QCD Lagrangian, with quark couplings to external Hermitian matrix-valued fields  $v_\mu, a_\mu, s, p$ :

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^0 + \bar{q} \gamma^\mu (v_\mu + \gamma_5 a_\mu) q - \bar{q} (s - i\gamma_5 p) q. \quad (3.10)$$

The external fields will allow us to compute the effective realization of general Green functions of quark currents in a very straightforward way. Moreover, they can be used to incorporate the electromagnetic and semileptonic weak interactions, and the explicit breaking of chiral symmetry through the quark masses:

$$\begin{aligned} r_\mu &\equiv v_\mu + a_\mu = eQ A_\mu + \dots \\ \ell_\mu &\equiv v_\mu - a_\mu = eQ A_\mu + \frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^\dagger T_+ + \text{h.c.}) + \dots \\ s &= \mathcal{M} + \dots \end{aligned} \quad (3.11)$$

Here,  $Q$  and  $\mathcal{M}$  denote the quark-charge and quark-mass matrices, respectively,

$$Q = \frac{1}{3} \text{diag}(2, -1, -1), \quad \mathcal{M} = \text{diag}(m_u, m_d, m_s), \quad (3.12)$$

and  $T_+$  is a  $3 \times 3$  matrix containing the relevant Cabibbo–Kobayashi–Maskawa factors

$$T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3.13)$$

The Lagrangian (3.10) is invariant under the following set of *local*  $SU(3)_L \otimes SU(3)_R$  transformations:

$$\begin{aligned} q_L &\longrightarrow g_L q_L, & q_R &\longrightarrow g_R q_R, & s + ip &\longrightarrow g_R (s + ip) g_L^\dagger, \\ \ell_\mu &\longrightarrow g_L \ell_\mu g_L^\dagger + ig_L \partial_\mu g_L^\dagger, & r_\mu &\longrightarrow g_R r_\mu g_R^\dagger + ig_R \partial_\mu g_R^\dagger. \end{aligned} \quad (3.14)$$

We can use this symmetry to build a generalized effective Lagrangian for the Goldstone bosons, in the presence of external sources. Note that to respect the local invariance, the gauge fields  $v_\mu$ ,  $a_\mu$  can only appear through the covariant derivatives

$$D_\mu U = \partial_\mu U - ir_\mu U + iU\ell_\mu, \quad D_\mu U^\dagger = \partial_\mu U^\dagger + iU^\dagger r_\mu - i\ell_\mu U^\dagger, \quad (3.15)$$

and through the field strength tensors

$$F_L^{\mu\nu} = \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu], \quad F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]. \quad (3.16)$$

At lowest order in momenta, the more general effective Lagrangian consistent with Lorentz invariance and (local) chiral symmetry is of the form (Gasser and Leutwyler 1985)

$$\mathcal{L}_2 = \frac{f^2}{4} \langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \rangle, \quad (3.17)$$

where

$$\chi = 2B_0(s + ip), \quad (3.18)$$

and  $B_0$  is a constant, which, like  $f$ , is not fixed by symmetry requirements alone.

Once special directions in flavour space, like the ones in equation (3.11), are selected for the external fields, chiral symmetry is of course explicitly broken. The important point is that (3.17) then breaks the symmetry in exactly the same way as the fundamental short-distance Lagrangian (3.10) does.

The power of the external field technique becomes obvious when computing the chiral Noether currents. The Green functions are obtained as functional derivatives of the generating functional  $Z[v, a, s, p]$ , defined via the path-integral formula

$$\exp\{iZ\} = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}G_\mu \exp\left\{i \int d^4x \mathcal{L}_{QCD}\right\} = \int \mathcal{D}U \exp\left\{i \int d^4x \mathcal{L}_{\text{eff}}\right\}. \quad (3.19)$$

At lowest order in momenta, the generating functional reduces to the classical action  $S_2 = \int d^4x \mathcal{L}_2$ ; therefore, the currents can be trivially computed by taking the appropriate derivatives with respect to the external fields:

$$\begin{aligned} J_L^\mu &\doteq \frac{\delta S_2}{\delta \ell_\mu} = \frac{i}{2} f^2 D_\mu U^\dagger U = \frac{f}{\sqrt{2}} D_\mu \Phi - \frac{i}{2} \left( \Phi \overleftrightarrow{D}^\mu \Phi \right) + \mathcal{O}(\Phi^3/f), \\ J_R^\mu &\doteq \frac{\delta S_2}{\delta r_\mu} = \frac{i}{2} f^2 D_\mu U U^\dagger = -\frac{f}{\sqrt{2}} D_\mu \Phi - \frac{i}{2} \left( \Phi \overleftrightarrow{D}^\mu \Phi \right) + \mathcal{O}(\Phi^3/f). \end{aligned} \quad (3.20)$$

The physical meaning of the chiral coupling  $f$  is now obvious; at  $\mathcal{O}(p^2)$ ,  $f$  equals the pion decay constant,  $f = f_\pi = 92.4$  MeV, defined as

$$\langle 0 | (J_A^\mu)^{12} | \pi^+ \rangle \equiv i\sqrt{2} f_\pi p^\mu. \quad (3.21)$$



Similarly, by taking derivatives with respect to the external scalar and pseudoscalar sources,

$$\begin{aligned}\bar{q}_L^j q_R^i &\doteq -\frac{\delta S_2}{\delta (s - ip)^{ji}} = -\frac{f^2}{2} B_0 U(\phi)^{ij}, \\ \bar{q}_R^j q_L^i &\doteq -\frac{\delta S_2}{\delta (s + ip)^{ji}} = -\frac{f^2}{2} B_0 U(\phi)^{\dagger ij},\end{aligned}\tag{3.22}$$

we learn that the constant  $B_0$  is related to the quark condensate:

$$\langle 0 | \bar{q}^j q^i | 0 \rangle = -f^2 B_0 \delta^{ij}.\tag{3.23}$$

The Goldstone bosons, parametrized by the matrix  $U(\phi)$ , correspond to the zero-energy excitations over this vacuum condensate.

Taking  $s = \mathcal{M}$  and  $p = 0$ , the  $\chi$  term in equation (3.17) gives rise to a quadratic pseudoscalar-mass term plus additional interactions proportional to the quark masses. Expanding in powers of  $\Phi$  (and dropping an irrelevant constant), one has:

$$\frac{f^2}{4} 2B_0 \langle \mathcal{M}(U + U^\dagger) \rangle = B_0 \left\{ -\langle \mathcal{M}\Phi^2 \rangle + \frac{1}{6f^2} \langle \mathcal{M}\Phi^4 \rangle + \mathcal{O}(\Phi^6/f^4) \right\}.\tag{3.24}$$

The explicit evaluation of the trace in the quadratic mass term provides the relation between the physical meson masses and the quark masses:

$$\begin{aligned}M_{\pi^\pm}^2 &= 2\hat{m}B_0, & M_{\pi^0}^2 &= 2\hat{m}B_0 - \varepsilon + \mathcal{O}(\varepsilon^2), \\ M_{K^\pm}^2 &= (m_u + m_s)B_0, & M_{K^0}^2 &= (m_d + m_s)B_0, \\ M_{\eta_8}^2 &= \frac{2}{3}(\hat{m} + 2m_s)B_0 + \varepsilon + \mathcal{O}(\varepsilon^2),\end{aligned}\tag{3.25}$$

where<sup>‡</sup>

$$\hat{m} = \frac{1}{2}(m_u + m_d), \quad \varepsilon = \frac{B_0 (m_u - m_d)^2}{4 (m_s - \hat{m})}.\tag{3.26}$$

Chiral symmetry relates the magnitude of the meson and quark masses to the size of the quark condensate. Using the result (3.23), one gets from the first equation in (3.25) the well-known relation (Gell-Mann, Oakes and Renner 1968)

$$f_\pi^2 M_\pi^2 = -\hat{m} \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle.\tag{3.27}$$

Taking out the common  $B_0$  factor, equations (3.25) imply the old Current Algebra mass ratios (Gell-Mann, Oakes and Renner 1968, Weinberg 1977),

$$\frac{M_{\pi^\pm}^2}{2\hat{m}} = \frac{M_{K^+}^2}{(m_u + m_s)} = \frac{M_{K^0}^2}{(m_d + m_s)} \approx \frac{3M_{\eta_8}^2}{(2\hat{m} + 4m_s)},\tag{3.28}$$

<sup>‡</sup> The  $\mathcal{O}(\varepsilon)$  corrections to  $M_{\pi^0}^2$  and  $M_{\eta_8}^2$  originate from a small mixing term between the  $\pi^0$  and  $\eta_8$  fields:  $-B_0 \langle \mathcal{M}\Phi^2 \rangle \longrightarrow -(B_0/\sqrt{3})(m_u - m_d)\pi^0\eta_8$ . The diagonalization of the quadratic  $\pi^0, \eta_8$  mass matrix, gives the mass eigenstates,  $\pi^0 = \cos\delta\phi^3 + \sin\delta\phi^8$  and  $\eta_8 = -\sin\delta\phi^3 + \cos\delta\phi^8$ , where  $\tan(2\delta) = \sqrt{3}(m_d - m_u)/(2(m_s - \hat{m}))$ .

and (up to  $\mathcal{O}(m_u - m_d)$  corrections) the Gell-Mann (1962)–Okubo (1962) mass relation,

$$3M_{\eta_8}^2 = 4M_K^2 - M_\pi^2. \quad (3.29)$$

Note that the chiral Lagrangian automatically implies the successful quadratic Gell-Mann–Okubo mass relation, and not a linear one. Since  $B_0 m_q \propto M_\phi^2$ , the external field  $\chi$  is counted as  $\mathcal{O}(p^2)$  in the chiral expansion.

Although chiral symmetry alone cannot fix the absolute values of the quark masses, it gives information about quark-mass ratios. Neglecting the tiny  $\mathcal{O}(\varepsilon)$  effects, one gets the relations

$$\frac{m_d - m_u}{m_d + m_u} = \frac{(M_{K^0}^2 - M_{K^+}^2) - (M_{\pi^0}^2 - M_{\pi^+}^2)}{M_{\pi^0}^2} = 0.29, \quad (3.30)$$

$$\frac{m_s - \hat{m}}{2\hat{m}} = \frac{M_{K^0}^2 - M_{\pi^0}^2}{M_{\pi^0}^2} = 12.6. \quad (3.31)$$

In equation (3.30) we have subtracted the pion square-mass difference, to take into account the electromagnetic contribution to the pseudoscalar-meson self-energies; in the chiral limit ( $m_u = m_d = m_s = 0$ ), this contribution is proportional to the square of the meson charge and it is the same for  $K^+$  and  $\pi^+$  (Dashen 1969). The mass formulae (3.30) and (3.31) imply the quark-mass ratios advocated by Weinberg (1977):

$$m_u : m_d : m_s = 0.55 : 1 : 20.3. \quad (3.32)$$

Quark-mass corrections are therefore dominated by  $m_s$ , which is large compared with  $m_u, m_d$ . Notice that the difference  $m_d - m_u$  is not small compared with the individual up- and down-quark masses; in spite of that, isospin turns out to be a very good symmetry, because isospin-breaking effects are governed by the small ratio  $(m_d - m_u)/m_s$ .

The  $\Phi^4$  interactions in equation (3.24) introduce mass corrections to the  $\pi\pi$  scattering amplitude (3.9),

$$T(\pi^+\pi^0 \rightarrow \pi^+\pi^0) = \frac{t - M_\pi^2}{f_\pi^2}, \quad (3.33)$$

in perfect agreement with the Current Algebra result (Weinberg 1966). Since  $f = f_\pi$  is fixed from pion decay, this result is now an absolute prediction of chiral symmetry.

The lowest-order chiral Lagrangian (3.17) encodes in a very compact way all the Current Algebra results obtained in the sixties (Adler and Dashen 1968, de Alfaro *et al* 1973). The nice feature of the chiral approach is its elegant simplicity. Moreover, as we will see in the next section, the effective field theory method allows us to estimate higher-order corrections in a systematic way.

#### 4. ChPT at $\mathcal{O}(p^4)$

At next-to-leading order in momenta,  $\mathcal{O}(p^4)$ , the computation of the generating functional  $Z[v, a, s, p]$  involves three different ingredients:

1. The most general effective chiral Lagrangian of  $O(p^4)$ ,  $\mathcal{L}_4$ , to be considered at tree level.
2. One-loop graphs associated with the lowest-order Lagrangian  $\mathcal{L}_2$ .
3. The Wess–Zumino (1971)–Witten (1983) functional to account for the chiral anomaly.

#### 4.1. $O(p^4)$ Lagrangian

At  $O(p^4)$ , the most general<sup>§</sup> Lagrangian, invariant under parity, charge conjugation and the local chiral transformations (3.14), is given by (Gasser and Leutwyler 1985)

$$\begin{aligned}
\mathcal{L}_4 = & L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi^\dagger U \rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U (U^\dagger \chi + \chi^\dagger U) \rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\
& + L_7 \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\
& - iL_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle \\
& + H_1 \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle.
\end{aligned} \tag{4.1}$$

The terms proportional to  $H_1$  and  $H_2$  do not contain the pseudoscalar fields and are therefore not directly measurable. Thus, at  $O(p^4)$  we need ten additional coupling constants  $L_i$  to determine the low-energy behaviour of the Green functions. These constants parametrize our ignorance about the details of the underlying QCD dynamics. In principle, all the chiral couplings are calculable functions of  $\Lambda_{QCD}$  and the heavy-quark masses. At the present time, however, our main source of information about these couplings is low-energy phenomenology.

#### 4.2. Chiral loops

ChPT is a quantum field theory, perfectly defined through equation (3.19). As such, we must take into account quantum loops with Goldstone-boson propagators in the internal lines. The chiral loops generate non-polynomial contributions, with logarithms and threshold factors, as required by unitarity.

The loop integrals are homogeneous functions of the external momenta and the pseudoscalar masses occurring in the propagators. A simple dimensional counting shows that, for a general connected diagram with  $N_d$  vertices of  $O(p^d)$  ( $d = 2, 4, \dots$ ) and  $L$  loops, the overall chiral dimension is given by (Weinberg 1979)

$$D = 2L + 2 + \sum_d N_d (d - 2). \tag{4.2}$$

<sup>§</sup> Since we will only need  $\mathcal{L}_4$  at tree level, the general expression of this Lagrangian has been simplified, using the  $O(p^2)$  equations of motion obeyed by  $U$ . Moreover, a  $3 \times 3$  matrix relation has been used to reduce the number of independent terms. For the two-flavour case, not all of these terms are independent (Gasser and Leutwyler 1984, 1985).

Each loop adds two powers of momenta; this power suppression of loop diagrams is at the basis of low-energy expansions, such as ChPT. The leading  $D = 2$  contributions are obtained with  $L = 0$  and  $d = 2$ , i.e. only tree-level graphs with  $\mathcal{L}_2$  insertions. At  $\mathcal{O}(p^4)$ , we have tree-level contributions from  $\mathcal{L}_4$  ( $L = 0$ ,  $d = 4$ ,  $N_4 = 1$ ) and one-loop graphs with the lowest-order Lagrangian  $\mathcal{L}_2$  ( $L = 1$ ,  $d = 2$ ).

The Goldstone loops are divergent and need to be renormalized. Although effective field theories are non-renormalizable (i.e. an infinite number of counter-terms is required), order by order in the momentum expansion they define a perfectly renormalizable theory. If we use a regularization which preserves the symmetries of the Lagrangian<sup>‡</sup>, such as dimensional regularization, the counter-terms needed to renormalize the theory will be necessarily symmetric. Since by construction the full effective Lagrangian contains all terms permitted by the symmetry, the divergences can then be absorbed in a renormalization of the coupling constants occurring in the Lagrangian. At one loop (in  $\mathcal{L}_2$ ), the ChPT divergences are  $\mathcal{O}(p^4)$  and are therefore renormalized by the low-energy couplings in equation (4.1):

$$L_i = L_i^r(\mu) + \Gamma_i \lambda, \quad H_i = H_i^r(\mu) + \tilde{\Gamma}_i \lambda, \quad (4.3)$$

where

$$\lambda = \frac{\mu^{d-4}}{16\pi^2} \left\{ \frac{1}{d-4} - \frac{1}{2} [\log(4\pi) + \Gamma'(1) + 1] \right\}. \quad (4.4)$$

The explicit calculation of the one-loop generating functional  $Z_4$  (Gasser and Leutwyler 1985) gives:

$$\begin{aligned} \Gamma_1 &= \frac{3}{32}, & \Gamma_2 &= \frac{3}{16}, & \Gamma_3 &= 0, & \Gamma_4 &= \frac{1}{8}, & \Gamma_5 &= \frac{3}{8}, & \Gamma_6 &= \frac{11}{144}, \\ \Gamma_7 &= 0, & \Gamma_8 &= \frac{5}{48}, & \Gamma_9 &= \frac{1}{4}, & \Gamma_{10} &= -\frac{1}{4}, & \tilde{\Gamma}_1 &= -\frac{1}{8}, & \tilde{\Gamma}_2 &= \frac{5}{24}. \end{aligned} \quad (4.5)$$

The renormalized couplings  $L_i^r(\mu)$  depend on the arbitrary scale of dimensional regularization  $\mu$ . This scale dependence is of course cancelled by that of the loop amplitude, in any measurable quantity.

A typical  $\mathcal{O}(p^4)$  amplitude will then consist of a non-polynomial part, coming from the loop computation, plus a polynomial in momenta and pseudoscalar masses, which depends on the unknown constants  $L_i$ . The non-polynomial part (the so-called chiral logarithms) is completely predicted as a function of the lowest-order coupling  $f$  and the Goldstone masses.

This chiral structure can be easily understood in terms of dispersion relations. Given the lowest-order Lagrangian  $\mathcal{L}_2$ , the non-trivial analytic behaviour associated with some physical intermediate state is calculable without the introduction of new arbitrary chiral coefficients. Analyticity then allows us to reconstruct the

<sup>‡</sup> A rather comprehensive analysis of different regularization schemes in ChPT has been given by Espriu and Matias (1994).

full amplitude, through a dispersive integral, up to a subtraction polynomial. ChPT generates (perturbatively) the correct dispersion integrals and organizes the subtraction polynomials in a derivative expansion.

ChPT is an expansion in powers of momenta over some typical hadronic scale, usually called the scale of chiral symmetry breaking  $\Lambda_\chi$ . The variation of the loop contribution under a rescaling of  $\mu$ , by say  $e$ , provides a natural order-of-magnitude estimate\* of  $\Lambda_\chi$  (Weinberg 1979, Manohar and Georgi 1984):  $\Lambda_\chi \sim 4\pi f_\pi \sim 1.2 \text{ GeV}$ .

### 4.3. The chiral anomaly

Although the QCD Lagrangian (3.10) is invariant under local chiral transformations, this is no longer true for the associated generating functional. The anomalies of the fermionic determinant break chiral symmetry at the quantum level (Adler 1969, Bardeen 1969, Bell and Jackiw 1969). The fermionic determinant can always be defined with the convention that  $Z[v, a, s, p]$  is invariant under vector transformations. Under an infinitesimal chiral transformation

$$g_{L,R} = 1 + i\alpha \mp i\beta + \dots \quad (4.6)$$

the anomalous change of the generating functional is then given by (Bardeen 1969):

$$\delta Z[v, a, s, p] = -\frac{N_C}{16\pi^2} \int d^4x \langle \beta(x) \Omega(x) \rangle, \quad (4.7)$$

$$\Omega(x) = \varepsilon^{\mu\nu\sigma\rho} \left[ v_{\mu\nu} v_{\sigma\rho} + \frac{4}{3} \nabla_\mu a_\nu \nabla_\sigma a_\rho + \frac{2}{3} i \{v_{\mu\nu}, a_\sigma a_\rho\} + \frac{8}{3} i a_\sigma v_{\mu\nu} a_\rho + \frac{4}{3} a_\mu a_\nu a_\sigma a_\rho \right], \quad (4.8)$$

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i [v_\mu, v_\nu], \quad \nabla_\mu a_\nu = \partial_\mu a_\nu - i [v_\mu, a_\nu]. \quad (4.9)$$

( $N_C = 3$  is the number of colours, and  $\varepsilon_{0123} = 1$ ). Note that  $\Omega(x)$  only depends on the external fields  $v_\mu$  and  $a_\mu$ . This anomalous variation of  $Z$  is an  $O(p^4)$  effect, in the chiral counting.

So far, we have been imposing chiral symmetry to construct the effective ChPT Lagrangian. Since chiral symmetry is explicitly violated by the anomaly at the fundamental QCD level, we need to add a functional  $Z_A$  with the property that its change under a chiral gauge transformation reproduces (4.7). Such a functional was first constructed by Wess and Zumino (1971), and reformulated in a nice geometrical way by Witten (1983). It has the explicit form:

$$\begin{aligned} S[U, \ell, r]_{WZW} = & -\frac{iN_C}{240\pi^2} \int d\sigma^{ijklm} \langle \Sigma_i^L \Sigma_j^L \Sigma_k^L \Sigma_l^L \Sigma_m^L \rangle \\ & - \frac{iN_C}{48\pi^2} \int d^4x \varepsilon_{\mu\nu\alpha\beta} (W(U, \ell, r)^{\mu\nu\alpha\beta} - W(\mathbf{1}, \ell, r)^{\mu\nu\alpha\beta}), \end{aligned} \quad (4.10)$$

\* Since the loop amplitude increases with the number of possible Goldstone mesons in the internal lines, this estimate results in a slight dependence of  $\Lambda_\chi$  on the number of light-quark flavours  $N_f$  (Soldate and Sundrum 1990, Chivukula *et al* 1993):  $\Lambda_\chi \sim 4\pi f_\pi / \sqrt{N_f}$ .

$$\begin{aligned}
W(U, \ell, r)_{\mu\nu\alpha\beta} = & \left\langle U\ell_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta + \frac{1}{4}U\ell_\mu U^\dagger r_\nu U\ell_\alpha U^\dagger r_\beta + iU\partial_\mu\ell_\nu\ell_\alpha U^\dagger r_\beta \right. \\
& + i\partial_\mu r_\nu U\ell_\alpha U^\dagger r_\beta - i\Sigma_\mu^L\ell_\nu U^\dagger r_\alpha U\ell_\beta + \Sigma_\mu^L U^\dagger\partial_\nu r_\alpha U\ell_\beta \\
& - \Sigma_\mu^L\Sigma_\nu^L U^\dagger r_\alpha U\ell_\beta + \Sigma_\mu^L\ell_\nu\partial_\alpha\ell_\beta + \Sigma_\mu^L\partial_\nu\ell_\alpha\ell_\beta - i\Sigma_\mu^L\ell_\nu\ell_\alpha\ell_\beta \\
& \left. + \frac{1}{2}\Sigma_\mu^L\ell_\nu\Sigma_\alpha^L\ell_\beta - i\Sigma_\mu^L\Sigma_\nu^L\Sigma_\alpha^L\ell_\beta \right\rangle - (L \leftrightarrow R), \tag{4.11}
\end{aligned}$$

where

$$\Sigma_\mu^L = U^\dagger\partial_\mu U, \quad \Sigma_\mu^R = U\partial_\mu U^\dagger, \tag{4.12}$$

and  $(L \leftrightarrow R)$  stands for the interchanges  $U \leftrightarrow U^\dagger$ ,  $\ell_\mu \leftrightarrow r_\mu$  and  $\Sigma_\mu^L \leftrightarrow \Sigma_\mu^R$ . The integration in the first term of equation (4.10) is over a five-dimensional manifold whose boundary is four-dimensional Minkowski space. The integrand is a surface term; therefore both the first and the second terms of  $S_{WZW}$  are  $\mathcal{O}(p^4)$ , according to the chiral counting rules.

Since anomalies have a short-distance origin, their effect is completely calculable. The translation from the fundamental quark–gluon level to the effective chiral level is unaffected by hadronization problems. In spite of its considerable complexity, the anomalous action (4.10) has no free parameters.

The anomaly functional gives rise to interactions that break the intrinsic parity. It is responsible for the  $\pi^0 \rightarrow 2\gamma$ ,  $\eta \rightarrow 2\gamma$  decays, and the  $\gamma 3\pi$ ,  $\gamma\pi^+\pi^-\eta$  interactions; a detailed analysis of these processes has been given by Bijnens (1993a). The five-dimensional surface term generates interactions among five or more Goldstone bosons.

## 5. Low-energy phenomenology at $\mathcal{O}(p^4)$

At lowest order in momenta, the predictive power of the chiral Lagrangian was really impressive; with only two low-energy couplings, it was possible to describe all Green functions associated with the pseudoscalar-meson interactions. The symmetry constraints become less powerful at higher orders. Ten additional constants appear in the  $\mathcal{L}_4$  Lagrangian, and many more<sup>||</sup> would be needed at  $\mathcal{O}(p^6)$ . Higher-order terms in the chiral expansion are much more sensitive to the non-trivial aspects of the underlying QCD dynamics.

With  $p \lesssim M_K (M_\pi)$ , we expect  $\mathcal{O}(p^4)$  corrections to the lowest-order amplitudes at the level of  $p^2/\Lambda_\chi^2 \lesssim 20\%$  (2%). We need to include those corrections if we aim to increase the accuracy of the ChPT predictions beyond this level. Although the number of free constants in  $\mathcal{L}_4$  looks quite big, only a few of them contribute to a given observable. In the absence of external fields, for instance, the Lagrangian reduces to the first three terms; elastic  $\pi\pi$  and  $\pi K$  scatterings are then sensitive

<sup>||</sup> According to a recent analysis (Fearing and Scherer 1994),  $\mathcal{L}_6$  involves 111 (32) independent terms of even (odd) intrinsic parity.

to  $L_{1,2,3}$ . The two-derivative couplings  $L_{4,5}$  generate mass corrections to the meson decay constants (and mass-dependent wave-function renormalizations). Pseudoscalar masses are affected by the non-derivative terms  $L_{6,7,8}$ ;  $L_9$  is mainly responsible for the charged-meson electromagnetic radius and  $L_{10}$ , finally, only contributes to amplitudes with at least two external vector or axial-vector fields, like the radiative semileptonic decay  $\pi \rightarrow e\nu\gamma$ .

Table 1 (Bijnens, Ecker and Gasser 1994) summarizes the present status of the phenomenological determination of the constants  $L_i$  (Gasser and Leutwyler 1985, Bijnens and Cornet 1988, Bijnens 1990, Riggenbach *et al* 1991, Bijnens, Colangelo and Gasser 1994). The quoted numbers correspond to the renormalized couplings, at a scale  $\mu = M_\rho$ . The values of these couplings at any other renormalization scale can be trivially obtained, through the logarithmic running implied by (4.3):

$$L_i^r(\mu_2) = L_i^r(\mu_1) + \frac{\Gamma_i}{(4\pi)^2} \log\left(\frac{\mu_1}{\mu_2}\right). \quad (5.1)$$

**Table 1.** Phenomenological values of the renormalized couplings  $L_i^r(M_\rho)$ . The last column shows the source used to extract this information.

$i$	$L_i^r(M_\rho) \times 10^3$	Source
1	$0.4 \pm 0.3$	$K_{e4}, \pi\pi \rightarrow \pi\pi$
2	$1.4 \pm 0.3$	$K_{e4}, \pi\pi \rightarrow \pi\pi$
3	$-3.5 \pm 1.1$	$K_{e4}, \pi\pi \rightarrow \pi\pi$
4	$-0.3 \pm 0.5$	Zweig rule
5	$1.4 \pm 0.5$	$F_K : F_\pi$
6	$-0.2 \pm 0.3$	Zweig rule
7	$-0.4 \pm 0.2$	Gell-Mann–Okubo, $L_5, L_8$
8	$0.9 \pm 0.3$	$M_{K^0} - M_{K^+}, L_5, (m_s - \hat{m}) : (m_d - m_u)$
9	$6.9 \pm 0.7$	$\langle r^2 \rangle_V^\pi$
10	$-5.5 \pm 0.7$	$\pi \rightarrow e\nu\gamma$

Comparing the Lagrangians  $\mathcal{L}_2$  and  $\mathcal{L}_4$ , one can make an estimate of the expected size of the couplings  $L_i$  in terms of the scale of SCSB. Taking  $\Lambda_\chi \sim 4\pi f_\pi \sim 1.2 \text{ GeV}$ , one would get

$$L_i \sim \frac{f_\pi^2/4}{\Lambda_\chi^2} \sim \frac{1}{4(4\pi)^2} \sim 2 \times 10^{-3}, \quad (5.2)$$

in reasonable agreement with the phenomenological values quoted in table 1. This indicates a good convergence of the momentum expansion below the resonance region, i.e.  $p < M_\rho$ .

The chiral Lagrangian allows us to make a good book-keeping of phenomenological information with a few couplings. Once these couplings have been fixed, we can predict many other quantities. In addition, the information contained in table 1 is very useful to easily test different QCD-inspired models. Given any particular model aiming

to correctly describe QCD at low energies, we no longer need to make an extensive phenomenological analysis to test its reliability; it suffices to calculate the low-energy couplings predicted by the model, and compare them with the values in table 1.

An exhaustive description of the chiral phenomenology at  $O(p^4)$  is beyond the scope of these review. Instead, I will just present a few examples to illustrate both the power and limitations of the ChPT techniques.

### 5.1. Decay constants

In the isospin limit ( $m_u = m_d = \hat{m}$ ), the  $O(p^4)$  calculation of the meson-decay constants gives (Gasser and Leutwyler 1985):

$$\begin{aligned} f_\pi &= f \left\{ 1 - 2\mu_\pi - \mu_K + \frac{4M_\pi^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu) \right\}, \\ f_K &= f \left\{ 1 - \frac{3}{4}\mu_\pi - \frac{3}{2}\mu_K - \frac{3}{4}\mu_{\eta_8} + \frac{4M_K^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu) \right\}, \\ f_{\eta_8} &= f \left\{ 1 - 3\mu_K + \frac{4M_{\eta_8}^2}{f^2} L_5^r(\mu) + \frac{8M_K^2 + 4M_\pi^2}{f^2} L_4^r(\mu) \right\}, \end{aligned} \quad (5.3)$$

where

$$\mu_P \equiv \frac{M_P^2}{32\pi^2 f^2} \log \left( \frac{M_P^2}{\mu^2} \right). \quad (5.4)$$

The result depends on two  $O(p^4)$  couplings,  $L_4$  and  $L_5$ . The  $L_4$  term generates a universal shift of all meson-decay constants,  $\delta f^2 = 16L_4 B_0 \langle \mathcal{M} \rangle$ , which can be eliminated taking ratios. From the experimental value (Leutwyler and Roos 1984)

$$\frac{f_K}{f_\pi} = 1.22 \pm 0.01, \quad (5.5)$$

one can then fix  $L_5(\mu)$ ; this gives the result quoted in table 1. Moreover, one gets the absolute prediction (Gasser and Leutwyler 1985)

$$\frac{f_{\eta_8}}{f_\pi} = 1.3 \pm 0.05. \quad (5.6)$$

Taking into account isospin violations, one can also predict (Gasser and Leutwyler 1985) a tiny difference between  $f_{K^\pm}$  and  $f_{K^0}$ , proportional to  $m_d - m_u$ .

### 5.2. Electromagnetic form factors

At  $O(p^2)$  the electromagnetic coupling of the Goldstone bosons is just the minimal one, obtained through the covariant derivative. The next-order corrections generate a momentum-dependent form factor:

$$F_V^{\phi^\pm}(q^2) = 1 + \frac{1}{6} \langle r^2 \rangle_V^{\phi^\pm} q^2 + \dots \quad ; \quad F_V^{\phi^0}(q^2) = \frac{1}{6} \langle r^2 \rangle_V^{\phi^0} q^2 + \dots \quad (5.7)$$



The meson electromagnetic radius  $\langle r^2 \rangle_V^\phi$  gets local contributions from the  $L_9$  term, plus logarithmic loop corrections (Gasser and Leutwyler 1985):

$$\begin{aligned}\langle r^2 \rangle_V^{\pi^\pm} &= \frac{12L_9^r(\mu)}{f^2} - \frac{1}{32\pi^2 f^2} \left\{ 2 \log \left( \frac{M_\pi^2}{\mu^2} \right) + \log \left( \frac{M_K^2}{\mu^2} \right) + 3 \right\}, \\ \langle r^2 \rangle_V^{K^0} &= -\frac{1}{16\pi^2 f^2} \log \left( \frac{M_K}{M_\pi} \right), \\ \langle r^2 \rangle_V^{K^\pm} &= \langle r^2 \rangle_V^{\pi^\pm} + \langle r^2 \rangle_V^{K^0}.\end{aligned}\tag{5.8}$$

Since neutral bosons do not couple to the photon at tree level,  $\langle r^2 \rangle_V^{K^0}$  only gets a loop contribution, which is moreover finite (there cannot be any divergence because there exists no counter-term to renormalize it). The predicted value,  $\langle r^2 \rangle_V^{K^0} = -0.04 \pm 0.03 \text{ fm}^2$ , is in perfect agreement with the experimental determination (Molzon *et al* 1978)  $\langle r^2 \rangle_V^{K^0} = -0.054 \pm 0.026 \text{ fm}^2$ .

The measured electromagnetic pion radius,  $\langle r^2 \rangle_V^{\pi^\pm} = 0.439 \pm 0.008 \text{ fm}^2$  (Amendolia *et al* 1986), is used as input to estimate the coupling  $L_9$ . This observable provides a good example of the importance of higher-order local terms in the chiral expansion (Leutwyler 1989). If one tries to ignore the  $L_9$  contribution, using instead some *physical* cut-off  $p_{\text{max}}$  to regularize the loops, one needs  $p_{\text{max}} \sim 60 \text{ GeV}$ , in order to reproduce the experimental value; this is clearly nonsense. The pion charge radius is dominated by the  $L_9^r(\mu)$  contribution, for any reasonable value of  $\mu$ .

The measured  $K^+$  charge radius (Dally *et al* 1982),  $\langle r^2 \rangle_V^{K^\pm} = 0.28 \pm 0.07 \text{ fm}^2$ , has a larger experimental uncertainty. Within present errors, it is in agreement with the parameter-free relation in equation (5.8).

### 5.3. $K_{l3}$ decays

The semileptonic decays  $K^+ \rightarrow \pi^0 l^+ \nu_l$  and  $K^0 \rightarrow \pi^- l^+ \nu_l$  are governed by the corresponding hadronic matrix elements of the vector current [ $t \equiv (P_K - P_\pi)^2$ ],

$$\langle \pi | \bar{s} \gamma^\mu u | K \rangle = C_{K\pi} \left[ (P_K + P_\pi)^\mu f_+^{K\pi}(t) + (P_K - P_\pi)^\mu f_-^{K\pi}(t) \right], \tag{5.9}$$

where  $C_{K^+\pi^0} = 1/\sqrt{2}$ ,  $C_{K^0\pi^-} = 1$ . At lowest order, the two form factors reduce to trivial constants:  $f_+^{K\pi}(t) = 1$  and  $f_-^{K\pi}(t) = 0$ . There is however a sizeable correction to  $f_+^{K^+\pi^0}(t)$ , due to  $\pi^0\eta$  mixing, which is proportional to  $(m_d - m_u)$ ,

$$f_+^{K^+\pi^0}(0) = 1 + \frac{3}{4} \frac{m_d - m_u}{m_s - \hat{m}} = 1.017. \tag{5.10}$$

This number should be compared with the experimental ratio

$$\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.028 \pm 0.010. \tag{5.11}$$

The  $O(p^4)$  corrections to  $f_+^{K\pi}(0)$  can be expressed in a parameter-free manner in terms of the physical meson masses (Gasser and Leutwyler 1985). Including those

contributions, one gets the more precise values

$$f_+^{K^0\pi^-}(0) = 0.977, \quad \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022, \quad (5.12)$$

which are in perfect agreement with the experimental result (5.11). The accurate ChPT calculation of these quantities allows us to extract (Leutwyler and Roos 1984) the most precise determination of the Cabibbo–Kobayashi–Maskawa matrix element  $V_{us}$ :

$$|V_{us}| = 0.2196 \pm 0.0023. \quad (5.13)$$

At  $\mathcal{O}(p^4)$ , the form factors get momentum-dependent contributions. Since  $L_9$  is the only unknown chiral coupling occurring in  $f_+^{K\pi}(t)$  at this order, the slope  $\lambda_+$  of this form factor can be fully predicted:

$$\lambda_+ \equiv \frac{1}{6} \langle r^2 \rangle_V^{K\pi} M_\pi^2 = 0.031 \pm 0.003. \quad (5.14)$$

This number is in excellent agreement with the experimental determinations (Particle Data Group 1994),  $\lambda_+ = 0.0300 \pm 0.0016$  ( $K_{e3}^0$ ) and  $\lambda_+ = 0.0286 \pm 0.0022$  ( $K_{e3}^\pm$ ).

Instead of  $f_-^{K\pi}(t)$ , it is usual to parametrize the experimental results in terms of the so-called scalar form factor

$$f_0^{K\pi}(t) = f_+^{K\pi}(t) + \frac{t}{M_K^2 - M_\pi^2} f_-^{K\pi}(t). \quad (5.15)$$

The slope of this form factor is determined by the constant  $L_5$ , which in turn is fixed by  $f_K/f_\pi$ . One gets the result (Gasser and Leutwyler 1985):

$$\lambda_0 \equiv \frac{1}{6} \langle r^2 \rangle_S^{K\pi} M_\pi^2 = 0.017 \pm 0.004. \quad (5.16)$$

The experimental situation concerning the value of this slope is far from clear; while an older high-statistics measurement (Donaldson *et al* 1974),  $\lambda_0 = 0.019 \pm 0.004$ , confirmed the theoretical expectations, more recent experiments find higher values, which disagree with this result. Cho *et al* (1980), for instance, report  $\lambda_0 = 0.046 \pm 0.006$ , which differs from (5.16) by more than 4 standard deviations. The Particle Data Group (1994) quotes a world average  $\lambda_0 = 0.025 \pm 0.006$ .

#### 5.4. Meson and quark masses

The relations (3.25) get modified at  $\mathcal{O}(p^4)$ . The additional contributions depend on the low-energy constants  $L_4$ ,  $L_5$ ,  $L_6$ ,  $L_7$  and  $L_8$ . It is possible, however, to obtain one relation between the quark and meson masses, which does not contain any of the  $\mathcal{O}(p^4)$  couplings. The dimensionless ratios

$$Q_1 \equiv \frac{M_K^2}{M_\pi^2}, \quad Q_2 \equiv \frac{(M_{K^0}^2 - M_{K^+}^2)_{\text{QCD}}}{M_K^2 - M_\pi^2}, \quad (5.17)$$

get the same  $O(p^4)$  correction (Gasser and Leutwyler 1985):

$$Q_1 = \frac{m_s + \hat{m}}{2\hat{m}} \{1 + \Delta_M\}, \quad Q_2 = \frac{m_d - m_u}{m_s - \hat{m}} \{1 + \Delta_M\}, \quad (5.18)$$

where

$$\Delta_M = -\mu_\pi + \mu_{\eta_8} + \frac{8}{f^2} (M_K^2 - M_\pi^2) [2L_8^r(\mu) - L_5^r(\mu)]. \quad (5.19)$$

Therefore, at this order, the ratio  $Q_1/Q_2$  is just given by the corresponding ratio of quark masses,

$$Q^2 \equiv \frac{Q_1}{Q_2} = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}. \quad (5.20)$$

To a good approximation, equation (5.20) can be written as an ellipse,

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2} \left(\frac{m_s}{m_d}\right)^2 = 1, \quad (5.21)$$

which constrains the quark-mass ratios. The meson masses in (5.17) refer to pure QCD; using the Dashen (1969) theorem  $(\Delta M_K^2 - \Delta M_\pi^2)_{\text{em}} \equiv (M_{K^+}^2 - M_{K^0}^2 - M_{\pi^+}^2 + M_{\pi^0}^2)_{\text{em}} = 0$  to correct for the electromagnetic contributions, the observed values of the meson masses give  $Q = 24$ .

Obviously, the quark-mass ratios (3.32), obtained at  $O(p^2)$ , satisfy this elliptic constraint. At  $O(p^4)$ , however, it is not possible to make a separate determination of  $m_u/m_d$  and  $m_s/m_d$  without having additional information on some of the  $L_i$  couplings.

A useful quantity is the deviation of the Gell-Mann–Okubo relation,

$$\Delta_{\text{GMO}} \equiv \frac{4M_K^2 - 3M_{\eta_8}^2 - M_\pi^2}{M_{\eta_8}^2 - M_\pi^2}. \quad (5.22)$$

Neglecting the mass difference  $m_d - m_u$ , one gets (Gasser and Leutwyler 1985)

$$\begin{aligned} \Delta_{\text{GMO}} &= \frac{-2(4M_K^2\mu_K - 3M_{\eta_8}^2\mu_{\eta_8} - M_\pi^2\mu_\pi)}{M_{\eta_8}^2 - M_\pi^2} \\ &\quad - \frac{6}{f^2} (M_{\eta_8}^2 - M_\pi^2) [12L_7^r(\mu) + 6L_8^r(\mu) - L_5^r(\mu)]. \end{aligned} \quad (5.23)$$

Experimentally, correcting the masses for electromagnetic effects,  $\Delta_{\text{GMO}} = 0.21$ . Since  $L_5$  is already known, this allows the combination  $2L_7 + L_8$  to be fixed.

In order to determine the individual quark-mass ratios from equations (5.18), we would need to fix the constant  $L_8$ . However, there is no way to find an observable that isolates this coupling. The reason is an accidental symmetry of the Lagrangian  $\mathcal{L}_2 + \mathcal{L}_4$ , which remains invariant under the following simultaneous change (Kaplan and Manohar 1986) of the quark-mass matrix and some of the chiral couplings:

$$\begin{aligned} \mathcal{M}' &= \alpha \mathcal{M} + \beta (\mathcal{M}^\dagger)^{-1} \det \mathcal{M}, & B'_0 &= B_0/\alpha, \\ L'_6 &= L_6 - \zeta, & L'_7 &= L_7 - \zeta, & L'_8 &= L_8 + 2\zeta, \end{aligned} \quad (5.24)$$

where  $\alpha$  and  $\beta$  are arbitrary constants, and  $\zeta = \beta f^2 / (32\alpha B_0)$ . The only information on the quark-mass matrix  $\mathcal{M}$  that we used to construct the effective Lagrangian was that it transforms as  $\mathcal{M} \rightarrow g_R \mathcal{M} g_L^\dagger$ . The matrix  $\mathcal{M}'$  transforms in the same manner; therefore, symmetry alone does not allow us to distinguish between  $\mathcal{M}$  and  $\mathcal{M}'$ . Since only the product  $B_0 \mathcal{M}$  appears in the Lagrangian,  $\alpha$  merely changes the value of the constant  $B_0$ . The term proportional to  $\beta$  is a correction of  $\mathcal{O}(\mathcal{M}^2)$ ; when inserted in  $\mathcal{L}_2$ , it generates a contribution to  $\mathcal{L}_4$ , which is reabsorbed by the redefinition of the  $\mathcal{O}(p^4)$  couplings. All chiral predictions will be invariant under the transformation (5.24); therefore it is not possible to separately determine the values of the quark masses and the constants  $B_0$ ,  $L_6$ ,  $L_7$  and  $L_8$ . We can only fix those combinations of chiral couplings and masses that remain invariant under (5.24).

Notice that (5.24) is certainly not a symmetry of the underlying QCD Lagrangian. The accidental symmetry arises in the effective theory because we are not making use of the explicit form of the QCD Lagrangian; only its symmetry properties under chiral rotations have been taken into account. For instance, the matrix elements of the scalar and pseudoscalar currents involve the physical quark masses and are not invariant under (5.24); if we had a low-energy probe of those currents (such as a very light Higgs particle), we could directly fix  $L_8$  in exactly the same way as we have determined  $L_5$  using the weak interactions to test the axial-current matrix elements (Leutwyler 1994b).

We can resolve the ambiguity by obtaining one additional information from outside the pseudoscalar-meson chiral Lagrangian framework. For instance, by analyzing the isospin breaking in the baryon mass spectrum and the  $\rho$ - $\omega$  mixing (Gasser and Leutwyler 1982), it is possible to fix the ratio

$$R \equiv \frac{m_s - \hat{m}}{m_d - m_u} = 43.7 \pm 2.7. \quad (5.25)$$

Inserting this number in (5.20), one gets (Gasser and Leutwyler 1985)

$$\frac{m_s}{\hat{m}} = 25.7 \pm 2.6, \quad \frac{m_d - m_u}{2\hat{m}} = 0.28 \pm 0.03. \quad (5.26)$$

Moreover, one can now determine  $L_8$  from (5.18), and therefore fix  $L_7$  with equation (5.23); one gets then the values quoted in table 1.

The error in (5.26) includes an educated guess of the uncertainties associated with higher-order corrections and electromagnetic effects. It has been pointed out recently that the Dashen theorem receives large  $\mathcal{O}(e^2 \mathcal{M})$  corrections which tend to increase the electromagnetic contribution to the kaon mass difference. The 1-loop logarithmic corrections are known to be sizeable (Langacker and Pagels 1973, Maltman and Kotchan 1990, Urech 1995, Neufeld and Rupertsberger 1995), but the numerical result depends on the scale used to evaluate the logarithms. The magnitude of the non-logarithmic contribution has been recently estimated by two groups; although they use a rather different framework, they get similar results:  $(\Delta M_K^2 - \Delta M_\pi^2)_{\text{em}} = (1.0 \pm 0.1) \times 10^{-3} \text{ GeV}^2$  (Donoghue, Holstein and Wyler 1992)

and  $(1.3 \pm 0.4) \times 10^{-3} \text{ GeV}^2$  (Bijnens 1993b). A lower number is obtained if one assumes that the  $L_7$  coupling is dominated by the  $\eta'$  contribution (see section 6); using the measured  $\eta$ - $\eta'$  mixing angle and equation (5.25), one gets then from  $Q_2$ :  $(\Delta M_K^2 - \Delta M_\pi^2)_{\text{em}} = -(0.1 \pm 1.0) \times 10^{-3} \text{ GeV}^2$  (Leutwyler 1990, 1994b, Urech 1995). In view of the present uncertainties, we can take the conservative range  $(\Delta M_K^2 - \Delta M_\pi^2)_{\text{em}} = (0.75 \pm 0.75) \times 10^{-3} \text{ GeV}^2$ , which implies  $Q = 22.7 \pm 1.4$ . The corresponding quark mass ratios are:

$$\frac{m_s}{\hat{m}} = 22.6 \pm 3.3, \quad \frac{m_d - m_u}{2\hat{m}} = 0.25 \pm 0.04. \quad (5.27)$$

## 6. The role of resonances in ChPT

It seems rather natural to expect that the lowest-mass resonances, such as  $\rho$  mesons, should have an important impact on the physics of the pseudoscalar bosons. In particular, the low-energy singularities due to the exchange of those resonances should generate sizeable contributions to the chiral couplings. This can be easily understood, making a Taylor expansion of the  $\rho$  propagator:

$$\frac{1}{p^2 - M_\rho^2} = \frac{-1}{M_\rho^2} \left\{ 1 + \frac{p^2}{M_\rho^2} + \dots \right\}, \quad (p^2 < M_\rho^2). \quad (6.1)$$

Below the  $\rho$ -mass scale, the singularity associated with the pole of the resonance propagator is replaced by the corresponding momentum expansion. The exchange of virtual  $\rho$  mesons should result in derivative Goldstone couplings proportional to powers of  $1/M_\rho^2$ .

A systematic analysis of the role of resonances in the ChPT Lagrangian has been performed by Ecker *et al* (1989a) [see also Donoghue *et al* 1989]. One writes first a general chiral-invariant Lagrangian  $\mathcal{L}(U, V, A, S, P)$ , describing the couplings of meson resonances of the type  $V(1^{--})$ ,  $A(1^{++})$ ,  $S(0^{++})$  and  $P(0^{-+})$  to the Goldstone bosons, at lowest-order in derivatives. The coupling constants of this Lagrangian are phenomenologically extracted from physics at the resonance-mass scale. One has then an effective chiral theory defined in the intermediate-energy region. The generating functional (3.19) is given in this theory by the path-integral formula

$$\exp \{iZ\} = \int \mathcal{D}U(\phi) \mathcal{D}V \mathcal{D}A \mathcal{D}S \mathcal{D}P \exp \left\{ i \int d^4x \mathcal{L}(U, V, A, S, P) \right\}. \quad (6.2)$$

The integration of the resonance fields results in a low-energy theory with only Goldstone bosons, i.e. the usual ChPT Lagrangian. At lowest-order, this integration can be explicitly performed by expanding around the classical solution for the resonance fields.

The formal procedure to introduce higher-mass states in the chiral Lagrangian was first discussed by Coleman *et al* (1969) and Callan *et al* (1969). The wanted

ingredient for a non-linear representation of the chiral group is the compensating  $SU(3)_V$  transformation  $h(\phi, g)$  which appears under the action of  $G$  on the coset representative  $u(\phi)$  [see equations (3.2) to (3.4)]:

$$u(\phi) \xrightarrow{G} g_R u(\phi) h^\dagger(\phi, g) = h(\phi, g) u(\phi) g_L^\dagger. \quad (6.3)$$

In practice, we shall only be interested in resonances transforming as octets or singlets under  $SU(3)_V$ . Denoting the resonance multiplets generically by  $R = \vec{\lambda}\vec{R}/\sqrt{2}$  (octet) and  $R_1$  (singlet), the non-linear realization of  $G$  is given by

$$R \xrightarrow{G} h(\phi, g) R h(\phi, g)^\dagger, \quad R_1 \xrightarrow{G} R_1. \quad (6.4)$$

Since the action of  $G$  on the octet field  $R$  is local, we are led to define a covariant derivative

$$\nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R], \quad (6.5)$$

with

$$\Gamma_\mu = \frac{1}{2} \{u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - i\ell_\mu) u^\dagger\} \quad (6.6)$$

ensuring the proper transformation

$$\nabla_\mu R \xrightarrow{G} h(\phi, g) \nabla_\mu R h(\phi, g)^\dagger. \quad (6.7)$$

Without external fields,  $\Gamma_\mu$  is the usual natural connection on coset space.

To determine the resonance-exchange contributions to the effective chiral Lagrangian, we need the lowest-order couplings to the pseudoscalar Goldstones which are linear in the resonance fields. It is useful to define objects transforming as  $SU(3)_V$  octets:

$$\begin{aligned} u_\mu &\equiv iu^\dagger D_\mu U u^\dagger = u_\mu^\dagger, \\ \chi_\pm &\equiv u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \\ f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u. \end{aligned} \quad (6.8)$$

Invoking  $P$  and  $C$  invariance, the relevant lowest-order Lagrangian can be written as (Ecker *et al* 1989a)

$$\mathcal{L}_R = \sum_{R=V,A,S,P} \{\mathcal{L}_{\text{Kin}}(R) + \mathcal{L}_2(R)\}, \quad (6.9)$$

with kinetic terms\*

$$\begin{aligned} \mathcal{L}_{\text{Kin}}(R=V, A) &= -\frac{1}{2} \langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\mu\nu} R^{\mu\nu} \rangle - \frac{1}{2} \partial^\lambda R_{1,\lambda\mu} \partial_\nu R_1^{\nu\mu} + \frac{M_{R_1}^2}{4} R_{1,\mu\nu} R_1^{\mu\nu}, \\ \mathcal{L}_{\text{Kin}}(R=S, P) &= \frac{1}{2} \langle \nabla^\mu R \nabla_\mu R - M_R^2 R^2 \rangle + \frac{1}{2} \partial^\mu R_1 \partial_\mu R_1 - \frac{M_{R_1}^2}{2} R_1^2, \end{aligned} \quad (6.10)$$

\* The vector and axial-vector mesons are described in terms of antisymmetric tensor fields  $V_{\mu\nu}$  and  $A_{\mu\nu}$  (Gasser and Leutwyler 1984, Ecker *et al* 1989a) instead of the more familiar vector fields.

where  $M_R$ ,  $M_{R_1}$  are the corresponding masses in the chiral limit. The interactions  $\mathcal{L}_2(R)$  read

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle, \quad (6.11)$$

$$\mathcal{L}_2[A(1^{++})] = \frac{F_A}{2\sqrt{2}} \langle A_{\mu\nu} f_-^{\mu\nu} \rangle, \quad (6.12)$$

$$\mathcal{L}_2[S(0^{++})] = c_d \langle S u_\mu u^\mu \rangle + c_m \langle S \chi_+ \rangle + \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi_+ \rangle, \quad (6.13)$$

$$\mathcal{L}_2[P(0^{-+})] = id_m \langle P \chi_- \rangle + i\tilde{d}_m P_1 \langle \chi_- \rangle. \quad (6.14)$$

All coupling constants are real. The octet fields are written in the usual matrix notation

$$V_{\mu\nu} = \frac{\vec{\lambda}}{\sqrt{2}} \vec{V}_{\mu\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho_{\mu\nu}^0 + \frac{1}{\sqrt{6}}\omega_{8,\mu\nu} & \rho_{\mu\nu}^+ & K_{\mu\nu}^{*+} \\ \rho_{\mu\nu}^- & -\frac{1}{\sqrt{2}}\rho_{\mu\nu}^0 + \frac{1}{\sqrt{6}}\omega_{8,\mu\nu} & K_{\mu\nu}^{*0} \\ K_{\mu\nu}^{*-} & \bar{K}_{\mu\nu}^{*0} & -\frac{2}{\sqrt{6}}\omega_{8,\mu\nu} \end{pmatrix}, \quad (6.15)$$

and similarly for the other octets. We observe that for  $V$  and  $A$  only octets can couple whereas both octets and singlets appear for  $S$  and  $P$  (always to lowest order  $p^2$ ).

From the measured decay rates for  $\rho^0 \rightarrow e^+e^-$  and  $\rho \rightarrow 2\pi$ , one can determine the vector couplings  $|F_V| = 154$  MeV and  $|G_V| = 69$  MeV. Since the pions are far from being soft, chiral corrections to  $G_V$  are expected to be important. We can estimate the size of these corrections from the electromagnetic form factor of the pion, which is known to be well-reproduced by vector-meson dominance (VMD):

$$F_V^{\pi^\pm}(t) \approx \frac{M_\rho^2}{M_\rho^2 - t}, \quad (6.16)$$

i.e.  $\langle r^2 \rangle_V^{\pi^\pm} \approx 6/M_\rho^2 = 0.4 \text{ fm}^2$ , to be compared with the measured value  $\langle r^2 \rangle_V^{\pi^\pm} = 0.439 \pm 0.008 \text{ fm}^2$ . The exchange of a  $\rho$  meson between the  $G_V$  and  $F_V$  vertices, generates a contribution to the electromagnetic pion radius (Ecker *et al* 1989a):  $\langle r^2 \rangle_V^{\pi^\pm} = 6F_V G_V / (f_\pi^2 M_V^2)$ . Taking,  $M_V = M_\rho$ , the success of the naïve VMD formula (6.16) requires  $G_V F_V > 0$  and  $|G_V| \approx |f_\pi^2 / F_V| = 55$  MeV. Including also the contribution from chiral loops (Gasser and Leutwyler 1985), reduces this estimate to  $|G_V| = 53$  MeV, which is the value we shall adopt. The axial parameters can be fixed using the old Weinberg (1967b) sum rules:  $F_A^2 = F_V^2 - f_\pi^2 = (123 \text{ MeV})^2$  and  $M_A^2 = M_V^2 F_V^2 / F_A^2 = (968 \text{ MeV})^2$ .

$V$  exchange generates contributions to  $L_1$ ,  $L_2$ ,  $L_3$ ,  $L_9$  and  $L_{10}$ , while  $A$  exchange only contributes to  $L_{10}$  (Ecker *et al* 1989a):

$$\begin{aligned} L_1^V &= \frac{G_V^2}{8M_V^2}, & L_2^V &= 2L_1^V, & L_3^V &= -6L_1^V, \\ L_9^V &= \frac{F_V G_V}{2M_V^2}, & L_{10}^{V+A} &= -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}. \end{aligned} \quad (6.17)$$

The resulting values of the  $L_i$  couplings (Ecker *et al* 1989a) are summarized in table 2, which compares the different resonance-exchange contributions with the phenomenologically determined values of  $L_i^r(M_\rho)$ . The results shown in the table clearly establish a chiral version of vector (and axial-vector) meson dominance: whenever they can contribute at all,  $V$  and  $A$  exchange seem to completely dominate the relevant coupling constants.

**Table 2.**  $V$ ,  $A$ ,  $S$ ,  $S_1$  and  $\eta_1$  contributions to the coupling constants  $L_i^r$  in units of  $10^{-3}$ . The last column shows the results obtained using the relations (6.19).

i	$L_i^r(M_\rho)$	$V$	$A$	$S$	$S_1$	$\eta_1$	Total	Total <sup>c)</sup>
1	$0.4 \pm 0.3$	0.6	0	-0.2	$0.2^b)$	0	0.6	0.9
2	$1.4 \pm 0.3$	1.2	0	0	0	0	1.2	1.8
3	$-3.5 \pm 1.1$	-3.6	0	0.6	0	0	-3.0	-4.9
4	$-0.3 \pm 0.5$	0	0	-0.5	$0.5^b)$	0	0.0	0.0
5	$1.4 \pm 0.5$	0	0	$1.4^a)$	0	0	1.4	1.4
6	$-0.2 \pm 0.3$	0	0	-0.3	$0.3^b)$	0	0.0	0.0
7	$-0.4 \pm 0.2$	0	0	0	0	-0.3	-0.3	-0.3
8	$0.9 \pm 0.3$	0	0	$0.9^a)$	0	0	0.9	0.9
9	$6.9 \pm 0.7$	$6.9^a)$	0	0	0	0	6.9	7.3
10	$-5.5 \pm 0.7$	-10.0	4.0	0	0	0	-6.0	-5.5

<sup>a)</sup> Input.

<sup>b)</sup> Large- $N_C$  estimate.

<sup>c)</sup> With (6.19)

There are different phenomenologically successful models in the literature for  $V$  and  $A$  resonances [tensor-field description (Gasser and Leutwyler 1984, Ecker *et al* 1989a), massive Yang–Mills (Meißner 1988), hidden gauge formulation (Bando *et al* 1988), etc.]. It can be shown (Ecker *et al* 1989b) that all models are equivalent (i.e. they give the same contributions to the  $L_i$ ), provided they incorporate the appropriate QCD constraints at high energies. Moreover, with additional QCD-inspired assumptions of high-energy behaviour, such as an unsubtracted dispersion relation for the pion electromagnetic form factor, all  $V$  and  $A$  couplings can be expressed in terms of  $f_\pi$  and  $M_V$  only (Ecker *et al* 1989b):

$$F_V = \sqrt{2}f_\pi, \quad G_V = f_\pi/\sqrt{2}, \quad F_A = f_\pi, \quad M_A = \sqrt{2}M_V. \quad (6.18)$$

In that case, one has

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2). \quad (6.19)$$

The last column in table 2 shows the predicted numerical values of the  $L_i$  couplings, using the relations (6.19).

The exchange of scalar resonances generates the contributions (Ecker *et al* 1989a):



$$\begin{aligned}
L_1^{S+S_1} &= -\frac{c_d^2}{6M_S^2} + \frac{\tilde{c}_d^2}{2M_{S_1}^2}, & L_3^S &= \frac{c_d^2}{2M_S^2}, \\
L_4^{S+S_1} &= -\frac{c_d c_m}{3M_S^2} + \frac{\tilde{c}_d \tilde{c}_m}{M_{S_1}^2}, & L_5^S &= \frac{c_d c_m}{M_S^2}, \\
L_6^{S+S_1} &= -\frac{c_m^2}{6M_S^2} + \frac{\tilde{c}_m^2}{2M_{S_1}^2}, & L_8^S &= \frac{c_m^2}{2M_S^2}.
\end{aligned} \tag{6.20}$$

Since the experimental information is quite scarce in the scalar sector, one needs to assume that the couplings  $L_5$  and  $L_8$  are due exclusively to scalar-octet exchange, to determine the scalar-octet couplings  $c_d$  and  $c_m$ . Taking  $M_S = M_{a_0} = 983$  MeV, the scalar-octet contributions to the other  $L_i$  ( $i = 1, 3, 4, 6$ ) are then fixed. Moreover, one can then predict  $\Gamma(a_0 \rightarrow \eta\pi) = 59$  MeV, in good agreement with the experimental value  $\Gamma(a_0 \rightarrow \eta\pi) \approx \Gamma(a_0) = (57 \pm 11)$  MeV. The  $S_1$ -exchange contributions can be expressed in terms of the octet parameters using large- $N_C$  arguments. For  $N_C = \infty$ ,  $M_{S_1} = M_S$ ,  $|\tilde{c}_d| = |c_d|/\sqrt{3}$  and  $|\tilde{c}_m| = |c_m|/\sqrt{3}$  (Ecker *et al* 1989a); therefore, octet- and singlet-scalar exchange cancel in  $L_1$ ,  $L_4$  and  $L_6$ . Although the results in table 2 cannot be considered as a proof for scalar dominance, they provide at least a convincing demonstration of its consistency.

Neglecting the higher-mass  $0^{-+}$  resonances, the only remaining meson-exchange is the one associated with the  $\eta_1$ , which generates a sizeable contribution to  $L_7$  (Gasser and Leutwyler 1985, Ecker *et al* 1989a):

$$L_7^\eta = -\frac{\tilde{d}_m^2}{2M_{\eta_1}^2}. \tag{6.21}$$

The magnitude of this contribution can be calculated from the quark-mass expansion of  $M_\eta^2$  and  $M_{\eta'}^2$ , which fixes the  $\eta_1$  parameters in the large  $N_c$  limit (Ecker *et al* 1989a):  $M_{\eta_1} = 804$  MeV,  $|\tilde{d}_m| = 20$  MeV. The final result for  $L_7$  is in close agreement with its phenomenological value.

The combined resonance contributions appear to saturate the  $L_i^r$  almost entirely (Ecker *et al* 1989a). Within the uncertainties of the approach, there is no need for invoking any additional contributions. Although the comparison has been made for  $\mu = M_\rho$ , a similar conclusion would apply for any value of  $\mu$  in the low-lying resonance region between 0.5 and 1 GeV.

## 7. Short-distance estimates of ChPT parameters

All chiral couplings are in principle calculable from QCD. They are functions of  $\Lambda_{QCD}$  and the heavy-quark masses  $m_c$ ,  $m_b$ ,  $m_t$ . Unfortunately, we are not able at present to make such a first-principle computation. Although the integral over the quark fields in (3.19) can be done explicitly, we do not know how to perform analytically the remaining integration over the gluon fields. A perturbative evaluation of the gluonic contribution would obviously fail in reproducing the correct dynamics of

SCSB. A possible way out is to parametrize phenomenologically the SCSB and make a weak gluon-field expansion around the resulting physical vacuum.

The simplest parametrization (Espriu *et al* 1990) is obtained by adding to the QCD Lagrangian the chiral invariant term

$$\Delta\mathcal{L}_{\text{QCD}} = -M_Q (\bar{q}_R U q_L + \bar{q}_L U^\dagger q_R), \quad (7.1)$$

which serves to introduce the  $U$  field, and a mass parameter  $M_Q$ , which regulates the infra-red behaviour of the low-energy effective action. In the presence of this term the operator  $\bar{q}q$  acquires a vacuum expectation value; therefore, (7.1) is an effective way to generate the order parameter due to SCSB. Making a chiral rotation of the quark fields,  $Q_L \equiv u(\phi)q_L$ ,  $Q_R \equiv u(\phi)^\dagger q_R$ , with  $U = u^2$ , the interaction (7.1) reduces to a mass-term for the *dressed* quarks  $Q$ ; the parameter  $M_Q$  can then be interpreted as a *constituent-quark mass*.

The derivation of the low-energy effective chiral Lagrangian within this framework has been extensively discussed by Espriu *et al* (1990). In the chiral and large- $N_C$  limits, and including the leading gluonic contributions, one gets:

$$\begin{aligned} 8L_1 = 4L_2 = L_9 &= \frac{N_C}{48\pi^2} [1 + \text{O}(1/M_Q^6)], \\ L_3 = L_{10} &= -\frac{N_C}{96\pi^2} \left[ 1 + \frac{\pi^2}{5N_C} \frac{\langle \frac{\alpha_s}{\pi} GG \rangle}{M_Q^4} + \text{O}(1/M_Q^6) \right]. \end{aligned} \quad (7.2)$$

Due to dimensional reasons, the leading contributions to the  $\text{O}(p^4)$  couplings only depend on  $N_C$  and geometrical factors. It is remarkable that  $L_1$ ,  $L_2$  and  $L_9$  do not get any gluonic correction at this order; this result is independent of the way SCSB has been parametrized ( $M_Q$  can be taken to be infinite). Table 3 compares the predictions obtained with only the leading term in (7.2) (i.e. neglecting the gluonic correction) with the phenomenological determination of the  $L_i$  couplings. The numerical agreement is quite impressive; both the order of magnitude and the sign are correctly reproduced (notice that this is just a free-quark result!). Moreover, the gluonic corrections shift the values of  $L_3$  and  $L_{10}$  in the right direction, making them more negative.

**Table 3.** Leading-order ( $\alpha_s = 0$ ) predictions for the  $L_i$ 's, within the QCD-inspired model (7.1). The phenomenological values are shown in the second row for comparison. All numbers are given in units of  $10^{-3}$ .

	$L_1$	$L_2$	$L_3$	$L_9$	$L_{10}$
$L_i^{\text{th}}(\alpha_s = 0)$	0.79	1.58	-3.17	6.33	-3.17
$L_i^r(M_\rho)$	$0.4 \pm 0.3$	$1.4 \pm 0.3$	$-3.5 \pm 1.1$	$6.9 \pm 0.7$	$-5.5 \pm 0.7$

The results (7.2) obey almost all relations in (6.19). Comparing the predictions for  $L_{1,2,9}$  in the VMD approach of equation (6.19) with the QCD-inspired ones in (7.2), one gets a quite good estimate of the  $\rho$  mass:

$$M_V = 2\sqrt{2}\pi f = 821 \text{ MeV}. \quad (7.3)$$

Is it quite easy to prove that the interaction (7.1) is equivalent to the mean-field approximation of the Nambu–Jona-Lasinio (1961) model, where SCSB is triggered by four-quark operators. It has been conjectured recently (Bijnens, Bruno and de Rafael 1993) that integrating out the quark and gluon fields of QCD, down to some intermediate scale  $\Lambda_\chi$ , gives rise to an extended Nambu–Jona-Lasinio Lagrangian. By introducing collective fields (to be identified later with the Goldstone fields and  $S, V, A$  resonances) the model can be transformed into a Lagrangian bilinear in the quark fields, which can therefore be integrated out. One then gets an effective Lagrangian, describing the couplings of the pseudoscalar bosons to vector, axial-vector and scalar resonances. Extending the analysis beyond the mean-field approximation, Bijnens, Bruno and de Rafael (1993) obtain predictions for 20 measurable quantities, including the  $L_i$ 's, in terms of only 4 parameters. The quality of the fits is quite impressive. Since the model contains all resonances that are known to saturate the  $L_i$  couplings, it is not surprising that one gets an improvement of the mean-field-approximation results, specially for the constants  $L_5$  and  $L_8$ , which are sensitive to scalar exchange. What is more important, this analysis clarifies a potential problem of double-counting: in certain limits the model approaches either the pure quark-loop predictions (7.2) or the VMD results (6.19), but in general it interpolates between these two cases.

## 8. $\Delta S = 1$ non-leptonic weak interactions

The Standard Model predicts strangeness-changing transitions with  $\Delta S = 1$  via  $W$ -exchange between two weak charged currents. At low energies ( $E \ll M_W$ ), the heavy fields  $W, Z, t, b, c$  can be integrated out; using standard operator-product-expansion techniques, the non-leptonic  $\Delta S = 1$  weak interactions are described by an effective Hamiltonian (Gilman and Wise 1979)

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i + \text{h.c.}, \quad (8.1)$$

which is a sum of local four-quark operators, constructed with the light ( $u, d, s$ ) quark fields only,

$$\begin{aligned} Q_1 &\equiv 4 (\bar{s}_L \gamma^\mu d_L) (\bar{u}_L \gamma_\mu u_L), & Q_2 &\equiv 4 (\bar{s}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu d_L), \\ Q_3 &\equiv 4 (\bar{s}_L \gamma^\mu d_L) \sum_{q=u,d,s} (\bar{q}_L \gamma_\mu q_L), & Q_4 &\equiv 4 \sum_{q=u,d,s} (\bar{s}_L \gamma^\mu q_L) (\bar{q}_L \gamma_\mu d_L), \\ Q_5 &\equiv 4 (\bar{s}_L \gamma^\mu d_L) \sum_{q=u,d,s} (\bar{q}_R \gamma_\mu q_R), & Q_6 &\equiv -8 \sum_{q=u,d,s} (\bar{s}_L q_R) (\bar{q}_R d_L), \end{aligned} \quad (8.2)$$

modulated by Wilson coefficients  $C_i(\mu)$ , which are functions of the heavy  $W, t, b, c$  masses and an overall renormalization scale  $\mu$ . Only five of these operators are independent, since  $Q_4 = -Q_1 + Q_2 + Q_3$ . From the point of view of chiral  $SU(3)_L \otimes SU(3)_R$  and isospin quantum numbers,  $Q_- \equiv Q_2 - Q_1$  and  $Q_i$  ( $i = 3, 4, 5, 6$ )

transform as  $(8_L, 1_R)$  and induce  $|\Delta I| = \frac{1}{2}$  transitions, while  $Q_1 + 2/3Q_2 - 1/3Q_3$  transforms like  $(27_L, 1_R)$  and induces both  $|\Delta I| = \frac{1}{2}$  and  $|\Delta I| = \frac{3}{2}$  transitions.

The effect of  $\Delta S = 1$  non-leptonic weak interactions can be incorporated in the low-energy chiral theory (Cronin 1967), as a perturbation to the strong effective Lagrangian  $\mathcal{L}_{\text{eff}}(U)$ . At lowest order in the number of derivatives, the most general effective bosonic Lagrangian, with the same  $SU(3)_L \otimes SU(3)_R$  transformation properties as the short-distance Hamiltonian (8.1), contains two terms<sup>¶</sup>:

$$\mathcal{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \langle \lambda L_\mu L^\mu \rangle + g_{27} \left( L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + \text{h.c.} \right\}, \quad (8.3)$$

where

$$\lambda = (\lambda_6 - i\lambda_7)/2, \quad L_\mu = if^2 U^\dagger D_\mu U. \quad (8.4)$$

The chiral couplings  $g_8$  and  $g_{27}$  measure the strength of the two parts in the effective Hamiltonian (8.1) transforming as  $(8_L, 1_R)$  and  $(27_L, 1_R)$ , respectively, under chiral rotations. Their values can be extracted from  $K \rightarrow 2\pi$  decays (Pich *et al* 1986):

$$|g_8| \approx 5.1, \quad g_{27}/g_8 \approx 1/18. \quad (8.5)$$

The huge difference between these two couplings shows the well-known enhancement of the octet  $|\Delta I| = \frac{1}{2}$  transitions.

Using the effective Lagrangian (8.3), the calculation of hadronic weak decays becomes a straightforward perturbative problem. The highly non-trivial QCD dynamics has been parametrized in terms of the two chiral couplings. Of course, the interesting problem that remains to be solved is to compute  $g_8$  and  $g_{27}$  from the underlying QCD theory and, therefore, to gain a dynamical understanding of the so-called  $|\Delta I| = \frac{1}{2}$  rule. Although this is a very difficult task, considerable progress has been achieved recently (Pich and de Rafael 1991a, Jamin and Pich 1994). Applying the QCD-inspired model of equation (7.1) to the weak sector, a quite successful estimate of these two couplings has been obtained; a very detailed description of this calculation, and a comparison with other approaches, has been given by Pich and de Rafael (1991a).

Once the couplings  $g_8$  and  $g_{27}$  have been phenomenologically fixed to the values in (8.5), other decays like  $K \rightarrow 3\pi$  or  $K \rightarrow 2\pi\gamma$  can be easily predicted at  $\mathcal{O}(p^2)$ . As in the strong sector, one reproduces in this way the successful soft-pion relations of Current Algebra. However, the data are already accurate enough for the next-order corrections to be sizeable. Moreover, many transitions do not occur at  $\mathcal{O}(p^2)$ . For instance, due to a mismatch between the minimum number of powers of momenta required by gauge invariance and the powers of momenta that the lowest-order effective Lagrangian can provide, the amplitude for any non-leptonic radiative  $K$ -decay with at most one pion in the final state ( $K \rightarrow \gamma\gamma, K \rightarrow \gamma l^+ l^-, K \rightarrow \pi\gamma\gamma, K \rightarrow \pi l^+ l^-, \dots$ ) vanishes to lowest

<sup>¶</sup> One can build an additional octet term with the external  $\chi$  field,  $\langle \lambda (U^\dagger \chi + \chi^\dagger U) \rangle$ ; however, this term does not contribute to on-shell amplitudes.

order in ChPT (Ecker, Pich and de Rafael 1987a, 1987b, 1988). These decays are then sensitive to the non-trivial quantum field theory aspects of ChPT.

Unfortunately, at  $O(p^4)$  there is a very large number of possible terms, satisfying the appropriate  $(8_L, 1_R)$  and  $(27_L, 1_R)$  transformation properties (Kambor *et al* 1990). Using the  $O(p^2)$  equations of motion obeyed by  $U$  to reduce the number of terms, 35 independent structures (plus 2 contact terms involving external fields only) remain in the octet sector alone (Kambor *et al* 1990, Ecker 1990, Esposito-Farèse 1991). Restricting the attention to those terms that contribute to non-leptonic amplitudes where the only external gauge fields are photons, still leaves 22 relevant octet terms (Ecker, Kambor and Wyler 1993). Clearly, the predictive power of a completely general chiral analysis, using only symmetry constraints, is rather limited. Nevertheless, as we are going to see, it is still possible to make predictions.

Due to the complicated interplay of electroweak and strong interactions, the low-energy constants of the weak non-leptonic chiral Lagrangian encode a much richer information than in the pure strong sector. These chiral couplings contain both long- and short-distance contributions, and some of them (like  $g_8$ ) have in addition a CP-violating imaginary part. Genuine short-distance physics, such as the electroweak penguin operators, have their corresponding effective realization in the chiral Lagrangian. Moreover, there are four  $O(p^4)$  terms containing an  $\varepsilon_{\mu\nu\alpha\beta}$  tensor, which get a direct (probably dominant) contribution from the chiral anomaly (Ecker, Neufeld and Pich 1992, Bijmans, Ecker and Pich 1992).

In recent years, there have been several attempts to estimate these low-energy couplings using different approximations, such as factorization (Fajfer and Gérard 1989, Cheng 1990, Pich and de Rafael 1991a), weak-deformation model (Ecker, Pich and de Rafael 1990), effective-action approach (Pich and de Rafael 1991a, Bruno and Prades 1993), or resonance exchange (Isidori and Pugliese 1992, Ecker, Kambor and Wyler 1993). Although more work in this direction is certainly needed, a qualitative picture of the size of the different couplings is already emerging.

### 8.1. $K \rightarrow 2\pi, 3\pi$ decays

Imposing isospin and Bose symmetries, and keeping terms up to  $O(p^4)$ , a general parametrization (Devlin and Dickey 1979) of the  $K \rightarrow 3\pi$  amplitudes involves ten measurable parameters:  $\alpha_i, \beta_i, \zeta_i, \xi_i, \gamma_3$  and  $\xi'_3$ , where  $i = 1, 3$  refers to the  $\Delta I = \frac{1}{2}, \frac{3}{2}$  pieces. At  $O(p^2)$ , the quadratic slope parameters  $\zeta_i, \xi_i$  and  $\xi'_3$  vanish; therefore the lowest-order Lagrangian (8.3) predicts five  $K \rightarrow 3\pi$  parameters in terms of the two couplings  $g_8$  and  $g_{27}$ , extracted from  $K \rightarrow 2\pi$ . These predictions give the right qualitative pattern, but there are sizeable differences with the measured amplitudes. Moreover, non-zero values for some of the slope parameters have been clearly established experimentally.

The agreement is substantially improved at  $O(p^4)$  (Kambor *et al* 1991). In spite of the large number of unknown couplings in the general effective  $\Delta S = 1$  Lagrangian, only 7 combinations of these weak chiral constants are relevant for describing the

$K \rightarrow 2\pi$  and  $K \rightarrow 3\pi$  amplitudes (Kambor *et al* 1992). Therefore, one has 7 parameters for 12 observables, which results in 5 relations. The extent to which these relations are satisfied provides a non-trivial test of chiral symmetry at the four-derivative level. The results of such a test (Kambor *et al* 1992) are shown in table 4, where the 5 conditions have been formulated as predictions for the 5 slope parameters. The comparison is very successful for the two  $\Delta I = \frac{1}{2}$  parameters, but the data are not good enough to say anything conclusive about the other three  $\Delta I = \frac{3}{2}$  predictions.

**Table 4.** Predicted and measured values of the quadratic slope parameters in the  $K \rightarrow 3\pi$  amplitudes (Kambor *et al* 1992). All values are given in units of  $10^{-8}$ .

Parameter	Experimental value	Prediction
$\zeta_1$	$-0.47 \pm 0.15$	$-0.47 \pm 0.18$
$\xi_1$	$-1.51 \pm 0.30$	$-1.58 \pm 0.19$
$\zeta_3$	$-0.21 \pm 0.08$	$-0.011 \pm 0.006$
$\xi_3$	$-0.12 \pm 0.17$	$0.092 \pm 0.030$
$\xi'_3$	$-0.21 \pm 0.51$	$-0.033 \pm 0.077$

The  $O(p^4)$  analysis of these decays has also clarified the role of long-distance effects ( $\pi\pi$  rescattering) in the dynamical enhancement of  $\Delta I = \frac{1}{2}$  amplitudes. The  $O(p^4)$  corrections give indeed a sizeable constructive contribution, which results (Kambor *et al* 1991) in a fitted value for  $|g_8|$  that is about 30% smaller than the lowest-order determination (8.5). While this certainly goes in the right direction, it also shows that the bulk of the enhancement mechanism comes from a different source.

## 8.2. Radiative $K$ Decays

Owing to the constraints of electromagnetic gauge invariance, radiative  $K$  decays with at most one pion in the final state do not occur at  $O(p^2)$ . Moreover, only a few terms of the octet  $O(p^4)$  Lagrangian are relevant for this kind of processes (Ecker, Pich and de Rafael 1987a, 1987b, 1988):

$$\begin{aligned} \mathcal{L}_4^{\Delta S=1,em} \doteq & -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \left\{ -\frac{ie}{f^2} F^{\mu\nu} [w_1 \langle Q\lambda L_\mu L_\nu \rangle + w_2 \langle QL_\mu \lambda L_\nu \rangle] \right. \\ & \left. + e^2 f^2 w_4 F^{\mu\nu} F_{\mu\nu} \langle \lambda Q U^\dagger Q U \rangle + \text{h.c.} \right\}. \end{aligned} \quad (8.6)$$

The small number of unknown chiral couplings allows us to derive useful relations among different processes and to obtain definite predictions. The absence of a tree-level  $O(p^2)$  contribution makes the final results very sensitive to the loop structure of the amplitudes.

### 8.2.1. $K_S \rightarrow \gamma\gamma$ .

The symmetry constraints do not allow any direct tree-level  $K_1^0 \gamma\gamma$  coupling at  $O(p^4)$  ( $K_{1,2}^0$  refer to the CP-even and CP-odd eigenstates, respectively). This decay proceeds then through a loop of charged pions as shown in figure 1 (there are

similar diagrams with charged kaons in the loop, but their sum is proportional to  $M_{K^0}^2 - M_{K^+}^2$  and therefore can be neglected). Since there are no possible counter-terms to renormalize divergences, the one-loop amplitude is necessarily finite. Although each of the four diagrams in figure 1 is quadratically divergent, these divergences cancel in the sum. The resulting prediction (D'Ambrosio and Espriu 1986, Goity 1987) is in very good agreement with the experimental measurement (Burkhardt *et al* 1987):

$$\text{Br}(K_S \rightarrow \gamma\gamma) = \begin{cases} 2.0 \times 10^{-6} & \text{(theory)} \\ (2.4 \pm 1.2) \times 10^{-6} & \text{(experiment)} \end{cases}. \quad (8.7)$$

### 8.2.2. $K_{L,S} \rightarrow \mu^+\mu^-$ .

There are well-known short-distance contributions (electroweak penguins and box diagrams) to the decay  $K_L \rightarrow \mu^+\mu^-$ . However, this transition is dominated by long-distance physics. The main contribution proceeds through a two-photon intermediate state:  $K_2^0 \rightarrow \gamma^*\gamma^* \rightarrow \mu^+\mu^-$ . Contrary to  $K_1^0 \rightarrow \gamma\gamma$ , the prediction for the  $K_2^0 \rightarrow \gamma\gamma$  decay is very uncertain, because the first non-zero contribution occurs<sup>‡</sup> at  $O(p^6)$ . That makes very difficult any attempt to predict the  $K_L \rightarrow \mu^+\mu^-$  amplitude.

The situation is completely different for the  $K_S$  decay. A straightforward chiral analysis (Ecker and Pich 1991) shows that, at lowest order in momenta, the only allowed tree-level  $K^0\mu^+\mu^-$  coupling corresponds to the CP-odd state  $K_2^0$ . Therefore, the  $K_1^0 \rightarrow \mu^+\mu^-$  transition can only be generated by a finite non-local two-loop contribution. The calculation has been performed recently (Ecker and Pich 1991), with the result:

$$\frac{\Gamma(K_S \rightarrow \mu^+\mu^-)}{\Gamma(K_S \rightarrow \gamma\gamma)} = 2 \times 10^{-6}, \quad \frac{\Gamma(K_S \rightarrow e^+e^-)}{\Gamma(K_S \rightarrow \gamma\gamma)} = 8 \times 10^{-9}, \quad (8.8)$$

well below the present experimental upper limits. Although, in view of the smallness of the predicted ratios, this calculation seems quite academic, it has important implications for CP-violation studies.

The longitudinal muon polarization  $\mathcal{P}_L$  in the decay  $K_L \rightarrow \mu^+\mu^-$  is an interesting measure of CP violation. As for every CP-violating observable in the neutral kaon system, there are in general two different kinds of contributions to  $\mathcal{P}_L$ : indirect CP violation through the small  $K_1^0$  admixture of the  $K_L$  ( $\varepsilon$  effect), and direct CP violation in the  $K_2^0 \rightarrow \mu^+\mu^-$  decay amplitude.

In the Standard Model, the direct-CP-violating amplitude is induced by Higgs exchange with an effective one-loop flavour-changing  $\bar{s}dH$  coupling (Botella and Lim 1986). The present lower bound on the Higgs mass,  $M_H > 58.4$  GeV (95% C.L.), implies a conservative upper limit  $|\mathcal{P}_{L,\text{Direct}}| < 10^{-4}$ . Much larger values,  $\mathcal{P}_L \sim O(10^{-2})$ , appear quite naturally in various extensions of the Standard Model (Geng

<sup>‡</sup> At  $O(p^4)$ , this decay proceeds through a tree-level  $K_2^0 \rightarrow \pi^0, \eta$  transition, followed by  $\pi^0, \eta \rightarrow \gamma\gamma$  vertices. Because of the Gell-Mann-Okubo relation, the sum of the  $\pi^0$  and  $\eta$  contributions cancels exactly to lowest order. The decay amplitude is then very sensitive to  $SU(3)$  breaking.

and Ng 1990, Mohapatra 1993). It is worth emphasizing that  $\mathcal{P}_L$  is especially sensitive to the presence of light scalars with CP-violating Yukawa couplings. Thus,  $\mathcal{P}_L$  seems to be a good signature to look for new physics beyond the Standard Model; for this to be the case, however, it is very important to have a good quantitative understanding of the Standard Model prediction to allow us to infer, from a measurement of  $\mathcal{P}_L$ , the existence of a new CP-violation mechanism.

The chiral calculation of the  $K_1^0 \rightarrow \mu^+ \mu^-$  amplitude allows us to make a reliable estimate\*\* of the contribution to  $\mathcal{P}_L$  due to  $K^0 - \bar{K}^0$  mixing (Ecker and Pich 1991):

$$1.9 < |\mathcal{P}_{L,\varepsilon}| \times 10^3 \left( \frac{2 \times 10^{-6}}{\text{Br}(K_S \rightarrow \gamma\gamma)} \right)^{1/2} < 2.5. \quad (8.9)$$

Taking into account the present experimental errors in  $\text{Br}(K_S \rightarrow \gamma\gamma)$  and the inherent theoretical uncertainties due to uncalculated higher-order corrections, one can conclude that experimental indications for  $|\mathcal{P}_L| > 5 \times 10^{-3}$  would constitute clear evidence for additional mechanisms of CP violation beyond the Standard Model.

### 8.2.3. $K_L \rightarrow \pi^0 \gamma \gamma$ .

Assuming CP conservation, the most general form of the amplitude for  $K_2^0 \rightarrow \pi^0 \gamma \gamma$  depends on two independent invariant amplitudes  $A$  and  $B$  (Ecker, Pich and de Rafael 1988),

$$\begin{aligned} \mathcal{A}[K_L(p_K) \rightarrow \pi^0(p_0) \gamma(q_1) \gamma(q_2)] = & \epsilon_\mu(q_1) \epsilon_\nu(q_2) \left\{ \frac{A(y, z)}{M_K^2} \left( q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu} \right) \right. \\ & \left. + \frac{2B(y, z)}{M_K^4} \left( p_K \cdot q_1 q_2^\mu p_K^\nu + p_K \cdot q_2 q_1^\nu p_K^\mu - q_1 \cdot q_2 p_K^\mu p_K^\nu - p_K \cdot q_1 p_K \cdot q_2 g^{\mu\nu} \right) \right\}, \end{aligned} \quad (8.10)$$

where  $y \equiv |p_K \cdot (q_1 - q_2)|/M_K^2$  and  $z = (q_1 + q_2)^2/M_K^2$ .

Only the amplitude  $A(y, z)$  is non-vanishing to lowest non-trivial order,  $\mathcal{O}(p^4)$ , in ChPT. Again, the symmetry constraints do not allow any tree-level contribution from  $\mathcal{O}(p^4)$  terms in the Lagrangian. The  $A(y, z)$  amplitude is therefore determined by a finite loop calculation (Ecker, Pich and de Rafael 1987b). The relevant Feynman diagrams are analogous to the ones in figure 1, but with an additional  $\pi^0$  line emerging from the weak vertex; charged kaon loops also give a small contribution in this case. Due to the large absorptive  $\pi^+ \pi^-$  contribution, the spectrum in the invariant mass of the two photons is predicted (Ecker, Pich and de Rafael 1987b, Capiello and D'Ambrosio 1988) to have a very characteristic behaviour (dotted line in figure 3), peaked at high values of  $m_{\gamma\gamma}$ . The agreement with the measured two-photon distribution (Barr *et al* 1992), shown in figure 4, is remarkably good. However, the  $\mathcal{O}(p^4)$  prediction for the rate (Ecker, Pich and de Rafael 1987b, Capiello and

\*\* Taking only the absorptive parts of the  $K_{1,2} \rightarrow \mu^+ \mu^-$  amplitudes into account, a value  $|\mathcal{P}_{L,\varepsilon}| \approx 7 \times 10^{-4}$  was estimated previously (Herczeg 1983). However, this is only one out of four contributions to  $\mathcal{P}_L$  (Ecker and Pich 1991), which could all interfere constructively with unknown magnitudes.



D'Ambrosio 1988),  $\text{Br}(K_L \rightarrow \pi^0 \gamma \gamma) = 0.67 \times 10^{-6}$ , is smaller than the experimental value:

$$\text{Br}(K_L \rightarrow \pi^0 \gamma \gamma) = \begin{cases} (1.7 \pm 0.3) \times 10^{-6} & (\text{Barr } et \text{ al } 1992), \\ (2.2 \pm 1.0) \times 10^{-6} & (\text{Papadimitriou } et \text{ al } 1991). \end{cases} \quad (8.11)$$

Since the effect of the amplitude  $B(y, z)$  first appears at  $\mathcal{O}(p^6)$ , one should worry about the size of the next-order corrections. A naïve VMD estimate through the decay chain  $K_L \rightarrow \pi^0, \eta, \eta' \rightarrow V \gamma \rightarrow \pi^0 \gamma \gamma$  (Sehgal 1988, Morozumi and Iwasaki 1989, Flynn and Randall 1989, Heiliger and Sehgal 1993) results in a sizeable contribution to  $B(y, z)$  (Ecker, Pich and de Rafael 1990),

$$A(y, z)|_{\text{VMD}} = \tilde{a}_V \left( 3 - z + \frac{M_\pi^2}{M_K^2} \right), \quad B(y, z)|_{\text{VMD}} = -2\tilde{a}_V, \quad (8.12)$$

$$\tilde{a}_V \equiv -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \frac{M_K^2 \alpha}{\pi} a_V, \quad (8.13)$$

with  $a_V \approx 0.32$ . However, this type of calculation predicts a photon spectrum peaked at low values of  $m_{\gamma\gamma}$ , in strong disagreement with experiment. As first emphasized by Ecker, Pich and de Rafael (1990), there are also so-called direct weak contributions associated with  $V$  exchange, which cannot be written as a strong VMD amplitude with an external weak transition. Model-dependent estimates of this direct contribution (Ecker, Pich and de Rafael 1990) suggest a strong cancellation with the naïve vector-meson-exchange effect, i.e.  $|a_V| < 0.32$ ; but the final result is unfortunately quite uncertain.

A detailed calculation of the most important  $\mathcal{O}(p^6)$  corrections has been performed recently (Cohen, Ecker and Pich 1993). In addition to the VMD contribution, the unitarity corrections associated with the two-pion intermediate state (i.e.  $K_L \rightarrow \pi^0 \pi^+ \pi^- \rightarrow \pi^0 \gamma \gamma$ ) have been included<sup>††</sup>. Figure 3 shows the resulting photon spectrum for  $a_V = 0$  (dashed curve) and  $a_V = -0.9$  (full curve). The corresponding branching ratio is:

$$\text{Br}(K_L \rightarrow \pi^0 \gamma \gamma) = \begin{cases} 0.67 \times 10^{-6}, & \mathcal{O}(p^4), \\ 0.83 \times 10^{-6}, & \mathcal{O}(p^6), a_V = 0, \\ 1.60 \times 10^{-6}, & \mathcal{O}(p^6), a_V = -0.9. \end{cases} \quad (8.14)$$

The unitarity corrections by themselves raise the rate only moderately. Moreover, they produce an even more pronounced peaking of the spectrum at large  $m_{\gamma\gamma}$ , which tends to ruin the success of the  $\mathcal{O}(p^4)$  prediction. The addition of the  $V$  exchange contribution restores again the agreement. Both the experimental rate and the spectrum can be simultaneously reproduced with  $a_V = -0.9$ . A more complete unitarization of the  $\pi$ - $\pi$  intermediate states (Kambor and Holstein 1994), including the experimental  $\gamma\gamma \rightarrow \pi^0 \pi^0$  amplitude, increases the  $K_L \rightarrow \pi^0 \gamma \gamma$  decay width some 10%, leading to a slightly smaller value of  $|a_V|$ .

<sup>††</sup> The charged-pion loop has also been computed by Capiello *et al* (1993).

8.2.4.  $K \rightarrow \pi l^+ l^-$ .

In contrast to the previous processes, the  $\mathcal{O}(p^4)$  calculation of  $K^+ \rightarrow \pi^+ l^+ l^-$  and  $K_S \rightarrow \pi^0 l^+ l^-$  involves a divergent loop, which is renormalized by the  $\mathcal{O}(p^4)$  Lagrangian. The decay amplitudes can then be written (Ecker, Pich and de Rafael 1987a) as the sum of a calculable loop contribution plus an unknown combination of chiral couplings,

$$\begin{aligned} w_+ &= -\frac{1}{3}(4\pi)^2[w_1^r + 2w_2^r - 12L_9^r] - \frac{1}{3}\log(M_K M_\pi/\mu^2), \\ w_S &= -\frac{1}{3}(4\pi)^2[w_1^r - w_2^r] - \frac{1}{3}\log(M_K^2/\mu^2), \end{aligned} \quad (8.15)$$

where  $w_+$ ,  $w_S$  refer to the decay of the  $K^+$  and  $K_S$  respectively. These constants are expected to be of order 1 by naïve power-counting arguments. The logarithms have been included to compensate the renormalization-scale dependence of the chiral couplings, so that  $w_+$ ,  $w_S$  are observable quantities. If the final amplitudes are required to transform as octets, then  $w_2 = 4L_9$ , implying  $w_S = w_+ + \log(M_\pi/M_K)/3$ . It should be emphasized that this relation goes beyond the usual requirement of chiral invariance.

The measured  $K^+ \rightarrow \pi^+ e^+ e^-$  decay rate determines two possible solutions for  $w_+$  (Ecker, Pich and de Rafael 1987a). The two-fold ambiguity can be solved, looking to the shape of the invariant-mass distribution of the final lepton pair, which is regulated by the same parameter  $w_+$ . A fit to the BNL-E777 data (Alliegro *et al* 1992) gives

$$w_+ = 0.89_{-0.14}^{+0.24}, \quad (8.16)$$

in agreement with model-dependent theoretical estimates (Ecker, Pich and de Rafael 1990, Bruno and Prades 1993). Once  $w_+$  has been fixed, one can make predictions (Ecker, Pich and de Rafael 1987a) for the rates and Dalitz-plot distributions of the related modes  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ ,  $K_S \rightarrow \pi^0 e^+ e^-$  and  $K_S \rightarrow \pi^0 \mu^+ \mu^-$ .

8.2.5.  $K_L \rightarrow \pi^0 e^+ e^-$ .

The rare decay  $K_L \rightarrow \pi^0 e^+ e^-$  is an interesting process in looking for new CP-violating signatures. If CP were an exact symmetry, only the CP-even state  $K_1^0$  could decay via one-photon emission, while the decay of the CP-odd state  $K_2^0$  would proceed through a two-photon intermediate state and, therefore, its decay amplitude would be suppressed by an additional power of  $\alpha$ . When CP-violation is taken into account, however, an  $\mathcal{O}(\alpha)$   $K_L \rightarrow \pi^0 e^+ e^-$  decay amplitude is induced, both through the small  $K_1^0$  component of the  $K_L$  ( $\varepsilon$  effect) and through direct CP-violation in the  $K_2^0 \rightarrow \pi^0 e^+ e^-$  transition. The electromagnetic suppression of the CP-conserving amplitude then makes it plausible that this decay is dominated by the CP-violating contributions.

The short-distance analysis of the product of weak and electromagnetic currents allows a reliable calculation of the direct CP-violating  $K_2^0 \rightarrow \pi^0 e^+ e^-$  amplitude. The corresponding branching ratio has been estimated (Buras *et al* 1994) to be around

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) \Big|_{\text{Direct}} \simeq 6 \times 10^{-12}, \quad (8.17)$$

the exact number depending on the values of  $m_t$  and the quark-mixing angles.

The indirect CP-violating amplitude induced by the  $K_1^0$  component of the  $K_L$  is given by the  $K_S \rightarrow \pi^0 e^+ e^-$  amplitude times the CP-mixing parameter  $\varepsilon$ . Using the octet relation between  $w_+$  and  $w_S$ , the determination of the parameter  $\omega_+$  in (8.16) implies

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) \Big|_{\text{Indirect}} \leq 1.6 \times 10^{-12}. \quad (8.18)$$

Comparing this value with (8.17), we see that the direct CP-violating contribution is expected to be bigger than the indirect one. This is very different from the situation in  $K \rightarrow \pi\pi$ , where the contribution due to mixing completely dominates.

The present experimental upper bound (Harris *et al* 1993),

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) \Big|_{\text{Exp}} < 4.3 \times 10^{-9} \quad (90\% \text{C.L.}), \quad (8.19)$$

is still far away from the expected Standard Model signal, but the prospects for getting the needed sensitivity of around  $10^{-12}$  in the next few years are rather encouraging. To be able to interpret a future experimental measurement of this decay as a CP-violating signature, it is first necessary, however, to pin down the actual size of the two-photon exchange CP-conserving amplitude.

Using the computed  $K_L \rightarrow \pi^0 \gamma \gamma$  amplitude, one can estimate the two-photon exchange contribution to  $K_L \rightarrow \pi^0 e^+ e^-$ , by taking the absorptive part due to the two-photon discontinuity as an educated guess of the actual size of the complete amplitude. At  $\mathcal{O}(p^4)$ , the  $K_L \rightarrow \pi^0 e^+ e^-$  decay amplitude is strongly suppressed (it is proportional to  $m_e$ ), owing to the helicity structure of the  $A(y, z)$  term (Donoghue, Holstein and Valencia 1987, Ecker, Pich and de Rafael 1988):

$$\text{Br}(K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+ e^-) \Big|_{\mathcal{O}(p^4)} \sim 5 \times 10^{-15}. \quad (8.20)$$

This helicity suppression is, however, no longer true at the next order in the chiral expansion. The  $\mathcal{O}(p^6)$  estimate of the amplitude  $B(y, z)$  (Cohen, Ecker and Pich 1993) gives rise to

$$\text{Br}(K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 e^+ e^-) \Big|_{\mathcal{O}(p^6)} \sim \begin{cases} 0.3 \times 10^{-12}, & a_V = 0, \\ 1.8 \times 10^{-12}, & a_V = -0.9. \end{cases} \quad (8.21)$$

Although the rate increases with  $|a_V|$ , there is some destructive interference between the unitarity corrections of  $\mathcal{O}(p^6)$  and the  $V$ -exchange contribution (for  $a_V = -0.9$ ). To get a more accurate estimate, it would be necessary to make a careful fit to the  $K_L \rightarrow \pi^0 \gamma \gamma$  data, taking the experimental acceptance into account, to extract the actual value of  $a_V$ .

Thus, the decay width seems to be dominated by the CP-violating amplitude, but the CP-conserving contribution could also be important. Notice that if both amplitudes were comparable there would be a sizeable CP-violating energy asymmetry between the  $e^-$  and the  $e^+$  distributions (Sehgal 1988, Heiliger and Sehgal 1993, Donoghue and Gabbiani 1994).

### 8.3. The chiral anomaly in non-leptonic $K$ decays

The chiral anomaly also appears in the non-leptonic weak interactions. A systematic study of all non-leptonic  $K$  decays where the anomaly contributes at leading order,  $\mathcal{O}(p^4)$ , has been performed recently (Ecker, Neufeld and Pich 1992). Only radiative  $K$  decays are sensitive to the anomaly in the non-leptonic sector.

The manifestations of the anomaly can be grouped in two different classes of anomalous amplitudes: reducible and direct contributions. The reducible amplitudes arise from the contraction of meson lines between a weak non-leptonic  $\Delta S = 1$  vertex and the Wess–Zumino–Witten functional (4.10). In the octet limit, all reducible anomalous amplitudes of  $\mathcal{O}(p^4)$  can be predicted in terms of the coupling  $g_8$ . The direct anomalous contributions are generated through the contraction of the  $W$  boson field between a strong Green function on one side and the Wess–Zumino–Witten functional on the other. Their computation is not straightforward, because of the presence of strongly interacting fields on both sides of the  $W$ . Nevertheless, due to the non-renormalization theorem of the chiral anomaly (Adler and Bardeen 1969), the bosonized form of the direct anomalous amplitudes can be fully predicted (Bijnens, Ecker and Pich 1992). In spite of its anomalous origin, this contribution is chiral-invariant. The anomaly turns out to contribute to all possible octet terms of  $\mathcal{L}_4^{\Delta S=1}$  proportional to the  $\varepsilon_{\mu\nu\alpha\beta}$  tensor. Unfortunately, the coefficients of these terms get also non-factorizable contributions of non-anomalous origin, which cannot be computed in a model-independent way. Therefore, the final predictions can only be parametrized in terms of four dimensionless chiral couplings, which are expected to be positive and of order one.

The most frequent *anomalous* decays  $K^+ \rightarrow \pi^+\pi^0\gamma$  and  $K_L \rightarrow \pi^+\pi^-\gamma$  share the remarkable feature that the normally dominant bremsstrahlung amplitude is strongly suppressed, making the experimental verification of the anomalous amplitude substantially easier. This suppression has different origins:  $K^+ \rightarrow \pi^+\pi^0$  proceeds through the small 27-plet part of the non-leptonic weak interactions, whereas  $K_L \rightarrow \pi^+\pi^-$  is CP-violating. The remaining non-leptonic  $K$  decays with direct anomalous contributions are either suppressed by phase space [ $K^+ \rightarrow \pi^+\pi^0\pi^0\gamma(\gamma)$ ,  $K^+ \rightarrow \pi^+\pi^+\pi^-\gamma(\gamma)$ ,  $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ ,  $K_S \rightarrow \pi^+\pi^-\pi^0\gamma(\gamma)$ ] or by the presence of an extra photon in the final state [ $K^+ \rightarrow \pi^+\pi^0\gamma\gamma$ ,  $K_L \rightarrow \pi^+\pi^-\gamma\gamma$ ].

## 9. Baryons

The inclusion of baryons in the low-energy effective field theory follows the same procedure used in section 6 to incorporate the mesonic resonances. The octet of baryon fields is collected in a  $3 \times 3$  matrix

$$B(x) \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}, \quad (9.1)$$

which under  $SU(3)_L \otimes SU(3)_R$  transforms non-linearly

$$B \xrightarrow{G} h(\phi, g) B h^\dagger(\phi, g). \quad (9.2)$$

We look for the most general chiral-invariant effective Lagrangian one can write in terms of the matrices  $B(x)$ ,  $\bar{B}(x) \equiv B(x)^\dagger \gamma_0$ , and  $u(\phi)$ . We can easily write down the lowest-order baryon-meson Lagrangian for Green functions with at most two baryons:

$$\mathcal{L}_1^{(B)} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle, \quad (9.3)$$

where

$$\nabla_\mu B \equiv \partial_\mu B + [\Gamma_\mu, B], \quad (9.4)$$

with  $\Gamma_\mu$  defined in equation (6.6). The covariant derivative incorporates the correct minimal coupling to the electromagnetic field,  $\langle \bar{B} \gamma^\mu [v_\mu, B] \rangle \doteq e A_\mu \langle \bar{B} \gamma^\mu [Q, B] \rangle$ , and interactions with the pseudoscalar mesons, such as the so-called Weinberg term  $i \langle \bar{B} \gamma^\mu [[\Phi, \partial_\mu \Phi], B] \rangle / (4f^2)$ .

$M_B$  is a common mass of the baryon octet, due to the SCSB; it is the mass that the baryons would have if the  $u$ ,  $d$  and  $s$  quarks were exactly massless. The fact that the baryon masses do not vanish in the chiral limit and, moreover, are not small parameters compared with  $\Lambda_\chi$  complicates the chiral analysis of the baryon sector. Notice that from the point of view of chiral power counting  $\nabla_\mu B$  and  $M_B B$  are  $\mathcal{O}(1)$ , but  $i \gamma^\mu \nabla_\mu B - M_B B$  is  $\mathcal{O}(p)$ .

The last two terms in (9.3) contain the baryonic coupling to the external axial source  $a_\mu$ , and  $\bar{B} \Phi B$  interactions. If one restricts the discussion to the two flavour sector ( $u, d$ ), only the sum of the  $D$  and  $F$  terms is relevant [ $\bar{N} \equiv (\bar{p}, \bar{n})$ ,  $\Pi \equiv \vec{\tau} \vec{\pi} / \sqrt{2}$ ]:

$$\begin{aligned} \mathcal{L}_1^{(N)} = & \langle \bar{N} i \gamma^\mu \nabla_\mu N \rangle - M_N \langle \bar{N} N \rangle \\ & + (D + F) \left\langle \bar{N} \gamma^\mu \gamma_5 \left\{ a_\mu - \frac{1}{\sqrt{2}f} \partial_\mu \Pi + \frac{i}{\sqrt{2}f} [v_\mu, \Pi] + \mathcal{O}(\Pi^2) \right\} N \right\rangle. \end{aligned} \quad (9.5)$$

Thus,  $D+F$  is the usual nucleon axial-vector coupling constant measured in  $n \rightarrow pe^- \bar{\nu}_e$  decay,

$$D + F = g_A = 1.257 \pm 0.003, \quad (9.6)$$

and the strength of the  $\pi NN$  interaction is fixed in terms of  $g_A$ :

$$T(N \rightarrow N \pi^i) = -i g_{\pi NN} \bar{u}(p') \gamma_5 \tau^i u(p), \quad g_{\pi NN} = \frac{g_A M_N}{f} = 12.8. \quad (9.7)$$

Equation (9.7) is the well-known Goldberger–Treiman (1958) relation, which is well satisfied by the measured value  $g_{\pi NN} \approx 13.3 \pm 0.3$ . The same coupling  $g_A$  determines many other interactions with the pion fields, like the Kroll–Ruderman (1954)  $\bar{N} \pi \gamma N$  term.

With 3 light flavours, the baryon axial-vector currents get different contributions from the  $D$  and  $F$  terms. Therefore, using semileptonic hyperon decays, one can make a separate determination of the two couplings. A fit to the experimental data, neglecting higher-order corrections, gives (Luty and White 1993):

$$D = 0.85 \pm 0.06, \quad F = 0.52 \pm 0.04. \quad (9.8)$$

The baryon vector currents have of course their canonical form at zero momentum transfer. Whereas the quark axial-vector currents are renormalized by the strong interaction, giving rise to the  $D \pm F$  factors, the unbroken  $SU(3)_V$  symmetry protects the vector currents.

Baryon mass splittings appear at higher orders in the chiral expansion. The possible lowest  $O(\mathcal{M})$  interactions induced in the effective meson-baryon Lagrangian are

$$\mathcal{L}_{\mathcal{M}}^{(B)} = -b_0 \langle \chi_+ \rangle \langle \bar{B}B \rangle - b_1 \langle \bar{B}\chi_+ B \rangle - b_2 \langle \bar{B}B\chi_+ \rangle, \quad (9.9)$$

where  $b_0$ ,  $b_1$  and  $b_2$  are coupling constants with dimensions of an inverse mass. The  $b_0$  term gives an overall contribution to the common baryon mass  $M_B$ , and therefore cannot be extracted from baryon mass splittings. The other two couplings can be easily determined from the measured masses, with the result (for  $m_u = m_d = \hat{m}$ )

$$b_1 = \frac{M_{\Xi} - M_{\Sigma}}{4(M_K^2 - M_{\pi}^2)} = 0.14 \text{ GeV}^{-1}, \quad b_2 = \frac{M_N - M_{\Sigma}}{4(M_K^2 - M_{\pi}^2)} = -0.28 \text{ GeV}^{-1}. \quad (9.10)$$

The Lagrangian  $\mathcal{L}_{\mathcal{M}}^{(B)}$  implies the Gell-Mann (1962)–Okubo (1962) baryon mass relation

$$M_{\Sigma} + 3M_{\Lambda} = 2(M_{\Xi} + M_N), \quad (9.11)$$

which is experimentally satisfied to better than 1%.

### 9.1. Loops

Goldstone loops generate non-analytic corrections to the lowest-order results (Li and Pagels 1971, Langacker and Pagels 1973, Pagels 1975). Due to the different Lorentz structure of meson and baryon fields, the baryon chiral expansion contains terms of  $O(p^n)$  for each positive integer  $n$ , unlike in the mesonic sector where the expansion proceeds in steps of two powers of  $p$ . This implies additional types of non-analyticity in the baryonic amplitudes. The baryon masses, for instance, get calculable corrections of order  $M_{\pi}^3$ ,

$$\delta M_N \sim -\frac{3g_A^2 M_{\pi}^3}{32\pi f^2}, \quad (9.12)$$

i.e. a non-analytic (in the quark masses) contribution proportional to  $\mathcal{M}^{3/2}$  (Langacker and Pagels 1973).

The structure of the 1-loop generating functional has been analyzed in the  $SU(2)$  case by Gasser, Sainio and Švark (1988), and later extended to  $SU(3)$  by Krause (1990). The presence of the large mass scale  $M_B$  gives rise to a very complicated power counting. In the meson sector, contributions from  $n$ -loop graphs are suppressed by  $(p^2)^n$  and, therefore, there is a one-to-one correspondence between the loop and small momentum expansions. If baryons are present, however, the loops can produce powers of the heavy baryon mass instead of powers of the low external momenta; the chiral power of the loop contribution is then reduced. An amplitude with given chiral dimension may receive contributions from diagrams with an arbitrary number of loops. In particular, the coupling constants of the baryon Lagrangian get renormalized in every order of the loop expansion. Thus, the evaluation of one-loop graphs associated with  $\mathcal{L}_1^{(B)}$  produces divergences of  $O(1)$  and  $O(p)$  which renormalize the lowest-order parameters  $M_B$ ,  $D$  and  $F$ ; they give in addition contributions of  $O(p^2)$  and  $O(p^3)$ , which renormalize higher-order terms in the chiral Lagrangian. After appropriate mass and coupling constant renormalization, higher loops start again to contribute at  $O(p^2)$ .

### 9.2. Heavy baryon ChPT

Since  $M_B/\Lambda_\chi \sim O(1)$ , higher-loop contributions are not necessarily suppressed. Thus, in the presence of baryons, the standard chiral expansion is not only complicated but in addition its convergence is suspect. The problem can be circumvented by taking the limit  $p/M_B \ll 1$  and making an additional expansion in inverse powers of  $M_B$ . Using heavy quark effective theory techniques (Isgur and Wise 1989, Grinstein 1990, Eichten and Hill 1990, Georgi 1990), developed for the study of bottom physics, Jenkins and Manohar (1991a, 1991b, 1992a) have reformulated baryon ChPT in such a way as to transfer  $M_B$  from the propagators to the vertices (as an inverse scale).

The heavy baryon Lagrangian describes the interactions of a heavy static baryon with low-momentum pions. The velocity of the baryon is nearly unchanged when it exchanges some small momentum with the pion. The baryon momentum can be written as

$$p^\mu = M_B v^\mu + k^\mu, \quad (9.13)$$

with  $v^\mu$  the four-velocity satisfying  $v^2 = 1$ , and  $k^\mu$  a small off-shell momentum ( $k \cdot v \ll \Lambda_\chi$ ). The effective theory can be formulated in a Lorentz covariant way by defining velocity-dependent fields (Georgi 1990)

$$B_v(x) \equiv e^{iM_B v_\mu x^\mu} P_v^+ B(x), \quad P_v^+ \equiv \frac{1 + \not{v}}{2}. \quad (9.14)$$

The projection operator  $P_v^+$  projects onto the particle portion of the spinor, i.e.  $B_v$  is a two-component spinor. In the baryon rest frame  $v = (1, 0, 0, 0)$  and  $B_v$  corresponds to the usual non-relativistic projection of the Dirac spinor into the upper-component Pauli spinor. The new baryon fields obey a modified Dirac equation,

$$i\not{\partial} B_v = 0, \quad (9.15)$$

which no longer contains a baryon mass term. Derivatives acting on the field  $B_v$  produce factors of  $k$ , rather than  $p$ , so that higher derivative terms in the effective theory are suppressed by powers of  $k/\Lambda_\chi$ , which is small. Moreover, factors of  $M_B$  cannot occur in any loop. Thus, the heavy baryon Lagrangian has a consistent power-counting expansion (Weinberg 1990, Ecker 1994a): the chiral dimension increases with the number of loops and the lowest-order coupling constants are not renormalized by higher-order loops.

All Lorentz tensors made from spinors can be written in terms of  $v^\mu$  and a spin operator  $S_v^\mu$ , that acts on the baryon fields, defined through the properties (Jenkins and Manohar 1991a)

$$v \cdot S_v = 0, \quad S_v^2 B_v = -\frac{3}{4} B_v, \quad (9.16)$$

$$\{S_v^\lambda, S_v^\sigma\} = \frac{1}{2} (v^\lambda v^\sigma - g^{\lambda\sigma}), \quad [S_v^\lambda, S_v^\sigma] = i\epsilon^{\lambda\sigma\alpha\beta} v_\alpha (S_v)_\beta. \quad (9.17)$$

In the baryon rest frame,  $S_v$  reduces to the usual spin operator  $\vec{\sigma}/2$ .

In the heavy baryon formulation, the equivalent of the lowest-order Lagrangian  $\mathcal{L}_1^{(B)} + \mathcal{L}_M^{(B)}$  is given by (Jenkins and Manohar 1991a)

$$\begin{aligned} \mathcal{L}_v = & \langle \bar{B}_v i v^\mu \nabla_\mu B_v \rangle + D \langle \bar{B}_v S_v^\mu \{ \xi_\mu, B_v \} \rangle + F \langle \bar{B}_v S_v^\mu [ \xi_\mu, B ] \rangle \\ & - b_0 \langle \chi_+ \rangle \langle \bar{B}_v B_v \rangle - b_1 \langle \bar{B}_v \chi_+ B_v \rangle - b_2 \langle \bar{B}_v B_v \chi_+ \rangle. \end{aligned} \quad (9.18)$$

The  $1/M_B$  effects (and the antibaryon spinor components) in the original Dirac theory can be reproduced in the effective theory by including higher-dimension operators suppressed by powers of  $1/M_B$ . Since  $M_B \sim \Lambda_\chi$ , the  $1/M_B$  and  $1/\Lambda_\chi$  expansions can be combined into a single derivative expansion.

A complete analysis of the heavy baryon generating functional of  $\mathcal{O}(p^3)$  has been recently performed (Ecker 1994b) in the  $SU(2)$  case. The non-analytic pieces have been fully calculated. Moreover, the  $\mathcal{O}(p^2)$  and some  $\mathcal{O}(p^3)$  couplings have been either determined from phenomenology or estimated from resonance exchange (Bernard *et al* 1992, 1994, Jenkins 1992a). For some particular observables, like the nucleon electromagnetic polarizabilities,  $\mathcal{O}(p^4)$  calculations already exist (Bernard *et al* 1993a, 1994). A recent summary of chiral predictions compared to experimental data has been given by Meißner (1994).

The status of the three-flavour theory is less satisfactory. While a complete one-loop analysis is still lacking, rather large non-analytic corrections associated with kaon loops have been found in several observables (Bijnens *et al* 1985, Jenkins and Manohar 1991a, 1992b, Jenkins 1992a). For instance, taking into account the non-analytic one-loop contributions evaluated at a scale  $\mu = M_\rho$ , the fit to the semileptonic hyperon data gives (Luty and White 1993):

$$D = 0.60 \pm 0.03, \quad F = 0.36 \pm 0.02. \quad (9.19)$$

The difference with the lowest-order determination in equation (9.8) is rather large. Notice, however, that the contributions from local terms in the chiral Lagrangian have



been neglected; thus, it is not clear how meaningful those results are. In fact, a fit to the  $\pi N$   $\sigma$ -term (Gasser *et al* 1991) and to the baryon masses reveals (Bernard *et al* 1993b) that there are large cancellations between the strange loops and the counterterms.

Jenkins and Manohar (1991b) have advocated the inclusion of the spin- $\frac{3}{2}$  decuplet baryons as explicit degrees of freedom in the low-energy Lagrangian. Since the octet–decuplet mass splitting is not very large ( $\Delta \approx 300$  MeV), one can expect significant contributions from these close-by baryon states. They find that, in the limit  $\Delta \rightarrow 0$ , decuplet loops tend to compensate the large octet-loop contributions, improving the convergence of the chiral expansion. This approach has been, however, criticized by recent analyses (Bernard *et al* 1993b, Luty and White 1993), which show that setting  $\Delta = 0$  gives a very poor approximation to the decuplet contribution. In the usual Lagrangian without decuplet fields, the decuplet effects are already contained in the chiral couplings. The Jenkins–Manohar approach is nothing else than a way to make an estimate of the decuplet contribution to those couplings (and sum some higher-order corrections). However, their results are still incomplete because they have not taken into account all possible terms in the Lagrangian, at the considered order. Clearly, a complete analysis of the three-flavour theory is needed in order to clarify these issues.

### 9.3. Non-leptonic hyperon decays

Neglecting the small ( $27_L, 1_R$ ) contribution, the lowest-order  $\Delta S = 1$  non-leptonic effective Lagrangian involving baryons contains two terms (Manohar and Georgi 1984, Georgi 1984):

$$\mathcal{L}_{\Delta S=1}^{(B)} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \{ h_D \langle \bar{B} \{ u \lambda u^\dagger, B \} \rangle + h_F \langle \bar{B} [ u \lambda u^\dagger, B ] \rangle \}. \quad (9.20)$$

The invariant matrix elements for non-leptonic hyperon decays are conventionally parametrized as

$$T(B_i \rightarrow B_j \pi) = \bar{u}_{B_j} \left[ A_{ij}^{(S)} + \gamma_5 A_{ij}^{(P)} \right] u_{B_i}, \quad (9.21)$$

where  $A_{ij}^{(S)}$  and  $A_{ij}^{(P)}$  are the  $S$ - and  $P$ -wave amplitudes, respectively. The Lagrangian (9.20) implies that the tree-level  $S$ -wave amplitudes obey the Lee (1964)–Sugawara (1964) relation:

$$\sqrt{\frac{3}{2}} A_{\Sigma^- n}^{(S)} + A_{\Lambda p}^{(S)} + 2A_{\Xi^- \Lambda}^{(S)} = 0, \quad (9.22)$$

which is well satisfied experimentally. A similar relation for the  $P$ -wave amplitudes does not exist, since these involve pole diagrams in which the baryon changes strangeness before or after pion emission.

Fitting the parameters  $h_D$  and  $h_F$  to the measured  $S$ -wave amplitudes, one obtains a very bad description of the  $A_{ij}^{(P)}$  amplitudes. Therefore, higher-order corrections are

crucial in order to understand these decays. In fact, keeping only the non-analytic contributions evaluated at  $\mu = 1$  GeV, Bijmens *et al* (1985) found very large 1-loop corrections, which spoil the successful Lee-Sugawara relation for the  $S$ -waves, and are even larger than the tree-level result for some  $P$ -wave amplitudes.

The inclusion of the baryon decuplet (Jenkins 1992b, Jenkins and Manohar 1992a) seems to improve the convergence of the chiral expansion. There is again a large cancellation between the octet and decuplet loop contributions, reducing considerably the total loop correction. For the  $S$ -wave amplitudes one gets a very good fit, with a quite small correction to the relation (9.22). Although a satisfactory description of the  $P$ -wave amplitudes is still not obtained, one finds that in this case the tree-level result consists of two terms which tend to cancel to a large extent, for the parameter values determined by the  $S$ -wave fit; normal-size chiral logarithmic corrections are then of order one compared to the tree-level amplitudes. Thus, the missing contributions from the relevant local terms in the effective Lagrangian could easily explain the measured amplitudes.

Strangeness-changing radiative hyperon decays have been also analyzed within the chiral framework at the 1-loop level (Jenkins *et al* 1993, Neufeld 1993).

## 10. Large- $N_C$ limit, $U(1)_A$ anomaly and strong CP violation

In the large- $N_C$  limit the  $U(1)_A$  anomaly (Adler 1969, Adler and Bardeen 1969, Bell and Jackiw 1969) is absent. The massless QCD Lagrangian (2.1) has then a larger  $U(3)_L \otimes U(3)_R$  chiral symmetry, and there are nine Goldstone bosons associated with the SCSB to the diagonal subgroup  $U(3)_V$ . These Goldstone excitations can be conveniently collected in the  $3 \times 3$  unitary matrix

$$\tilde{U}(\phi) \equiv \langle 0|\tilde{U}|0\rangle U(\phi) \equiv \langle 0|\tilde{U}|0\rangle \exp\left\{i\sqrt{2}\tilde{\Phi}/f\right\}, \quad \tilde{\Phi} \equiv \frac{\eta_1}{\sqrt{3}}I_3 + \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi}, \quad (10.1)$$

where we have explicitly factor out from the  $\tilde{U}(\phi)$  matrix its vacuum expectation value. Under the chiral group,  $\tilde{U}(\phi)$  transforms as  $\tilde{U} \rightarrow g_R \tilde{U} g_L^\dagger$  ( $g_{R,L} \in U(3)_{R,L}$ ). To lowest order in the chiral expansion, the interactions of the nine Goldstone bosons are described by the Lagrangian (3.17) with  $\tilde{U}(\phi)$  instead of  $U(\phi)$ . Notice that the  $\eta_1$  kinetic term in  $\langle D_\mu \tilde{U} D_\mu \tilde{U}^\dagger \rangle$  decouples from the  $\phi$ 's and the  $\eta_1$  particle becomes stable in the chiral limit (Di Vecchia *et al* 1981).

To lowest non-trivial order in  $1/N_C$ , the chiral symmetry breaking effect induced by the  $U(1)_A$  anomaly can be taken into account in the effective low-energy theory, through the term (Di Vecchia and Veneziano 1980, Witten 1980, Rosenzweig *et al* 1980)

$$\mathcal{L}_{U(1)_A} = -\frac{f^2}{4} \frac{a}{N_C} \left\{ \frac{i}{2} \left[ \log(\det \tilde{U}) - \log(\det \tilde{U}^\dagger) \right] \right\}^2, \quad (10.2)$$

which breaks  $U(3)_L \otimes U(3)_R$  but preserves  $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$ . The parameter  $a$  has dimensions of mass squared and, with the factor  $1/N_C$  pulled out, is booked to be

of  $O(1)$  in the large- $N_C$  counting rules. Its value is not fixed by symmetry requirements alone; it depends crucially on the dynamics of instantons. In the presence of the term (10.2), the  $\eta_1$ -field becomes massive even in the chiral limit:  $M_{\eta_1^2} = 3a/N_C + O(\mathcal{M})$ .

Deeply related to the  $U(1)_A$  anomaly is the possible presence of an additional term in the QCD Lagrangian,

$$\mathcal{L}_\theta = \theta_0 \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G_a^{\mu\nu}(x) G_a^{\rho\sigma}(x), \quad (10.3)$$

with  $\theta_0$ , the so-called vacuum angle, a hitherto unknown parameter. This term violates  $P$ ,  $T$  and  $CP$  and may lead to observable effects in flavour conserving transitions. Owing to the  $U(1)_A$ -anomaly, the  $\theta_0$ -vacuum is not invariant under  $U(1)_A$  transformations,  $g_R = g_L^\dagger = e^{i\beta} I_3$  ( $N_f = 3$  is the number of light-quark flavours):

$$\theta_0 \longrightarrow \theta_0 - 2N_f\beta. \quad (10.4)$$

To simplify the discussion, let us fix the external scalar and pseudoscalar fields in (3.10) to the values  $s = \widetilde{\mathcal{M}}$  and  $p = 0$ , where  $\widetilde{\mathcal{M}}$  denotes the full mass matrix emerging from the Yukawa couplings of the light quarks in the electroweak sector. In full generality,  $\widetilde{\mathcal{M}}$  is non-diagonal and non-Hermitian. However, with the help of an appropriate  $SU(3)_L \otimes SU(3)_R$  transformation one can always restrict  $\widetilde{\mathcal{M}}$  to the form

$$\widetilde{\mathcal{M}} = \exp\left\{\frac{i}{3} \arg(\det \widetilde{\mathcal{M}})\right\} \mathcal{M}, \quad (10.5)$$

with  $\mathcal{M}$  the diagonal (and positive-definite) quark-mass matrix (3.12). The phase  $\arg(\det \widetilde{\mathcal{M}})$  could be reabsorbed by a simple  $U(1)_A$  rotation of the quark fields; however, because of the  $U(1)_A$ -anomaly, the  $U(1)_A$  rotation which eliminates  $\arg(\det \widetilde{\mathcal{M}})$  from the mass term generates a new  $\theta_0$ -vacuum

$$\theta \equiv \theta_0 + \arg(\det \widetilde{\mathcal{M}}). \quad (10.6)$$

The combination  $\theta$  remains invariant under arbitrary  $U(1)_A$  transformations.

In order to analyze the implications of the  $\theta$  term on the effective chiral theory, it is convenient to put the full  $\theta$ -dependence on the quark-mass matrix. Performing an appropriate chiral transformation, we can eliminate the term (10.3) from the QCD Lagrangian, and write the mass matrix in the form  $\widetilde{\mathcal{M}} = \exp\{i\theta/3\} \mathcal{M}$ . In the absence of the anomaly term (10.2), the  $\theta$  phase could be reabsorbed by the  $U(1)_A$  transformation

$$\widetilde{U} \longrightarrow e^{i\theta/6} \widetilde{U} e^{i\theta/6}. \quad (10.7)$$

In the presence of the  $U(1)_A$  anomaly, and hence the term (10.2), this transformation generates new physical interactions:

$$\mathcal{L}_2 + \mathcal{L}_{U(1)_A} = \frac{f^2}{4} \langle D_\mu \widetilde{U} D^\mu \widetilde{U}^\dagger \rangle - V(\widetilde{U}), \quad (10.8)$$

$$V(\widetilde{U}) = -\frac{f^2}{4} \left\{ \langle \widetilde{\chi}^\dagger \widetilde{U} + \widetilde{U}^\dagger \widetilde{\chi} \rangle - \frac{a}{N_C} \left\{ \frac{i}{2} \left[ \log \left( \frac{\det \widetilde{U}}{\det \widetilde{U}^\dagger} \right) \right] - \theta \right\}^2 \right\}, \quad (10.9)$$

where now the matrix  $\tilde{\chi} = 2\tilde{B}_0\mathcal{M} \equiv \text{diag}(\tilde{\chi}_u^2, \tilde{\chi}_d^2, \tilde{\chi}_s^2)$  is real, positive and diagonal.

If the term proportional to  $a/N_C$  were absent, we could take without loss of generality  $\langle 0|\tilde{U}|0\rangle = 1$  and the diagonal entries  $\tilde{\chi}_i^2$  would correspond to the Goldstone boson masses. In the presence of the anomaly, however, we should minimize the potential energy  $V(\tilde{U})$  in order to fix  $\langle 0|\tilde{U}|0\rangle$ . With  $\tilde{\chi}$  diagonal,  $\langle 0|\tilde{U}|0\rangle$  can be restricted to be diagonal as well and of the form

$$\langle 0|\tilde{U}|0\rangle = \text{diag}(e^{-i\varphi_u}, e^{-i\varphi_d}, e^{-i\varphi_s}). \quad (10.10)$$

The minimization conditions  $\partial V/\partial\varphi_i = 0$  restrict the  $\varphi_i$ 's to satisfy the Dashen (1971)–Nuyts (1971) equations:

$$\tilde{\chi}_i^2 \sin\varphi_i = \frac{a}{N_C} \left( \theta - \sum_j \varphi_j \right) \equiv \frac{a}{N_C} \bar{\theta}, \quad (i = u, d, s). \quad (10.11)$$

The  $\varphi_i$ 's appearing in the effective Lagrangian can be reabsorbed in Hermitian matrices  $\chi$  and  $H$  defined by

$$\langle 0|\tilde{U}^\dagger|0\rangle \tilde{\chi} \equiv \chi + iH, \quad \tilde{\chi}^\dagger \langle 0|\tilde{U}|0\rangle \equiv \chi - iH. \quad (10.12)$$

Equations (10.11) fix  $H$  to be proportional to the unit matrix:  $H = \frac{a}{N_C} \bar{\theta} I_3$ .

The effective bosonic Lagrangian as a functional of  $U(\phi)$ , with  $\langle 0|U|0\rangle = 1$ , is then (Pich and de Rafael 1991b):

$$\begin{aligned} \mathcal{L}_2 + \mathcal{L}_{U(1)_A} = & \frac{f^2}{4} \left\{ \langle D_\mu U D^\mu U^\dagger + \chi(U + U^\dagger) \rangle - \frac{a}{N_C} \left\{ \bar{\theta}^2 - \frac{1}{4} \left[ \log \left( \frac{\det U}{\det U^\dagger} \right) \right]^2 \right\} \right. \\ & \left. - i \frac{a}{N_C} \bar{\theta} \left\{ \langle U - U^\dagger \rangle - \log \left( \frac{\det U}{\det U^\dagger} \right) \right\} \right\}. \end{aligned} \quad (10.13)$$

The diagonalization of the quadratic piece of the Lagrangian gives rise to the physical fields. In the isospin limit ( $m_u = m_d = \hat{m}$ ), only the  $\eta_1$  and  $\eta_8$  mix:

$$\eta = \eta_8 \cos\theta_P - \eta_1 \sin\theta_P, \quad \eta' = \eta_8 \sin\theta_P + \eta_1 \cos\theta_P. \quad (10.14)$$

From the measured pseudoscalar-mass spectrum, one can get the values of the mixing angle and the parameter  $a$ :  $\theta_P \approx -20^\circ$ ,  $a = M_\eta^2 + M_{\eta'}^2 - 2M_K^2 = 0.726 \text{ GeV}^2$ . Since  $\tilde{\chi}_u^2, \tilde{\chi}_d^2 \ll \tilde{\chi}_s^2, a/N_C$  and  $\hat{m} \ll m_s$ , equations (10.11) imply the approximate relation

$$\frac{a}{N_C} \bar{\theta} \approx \frac{\theta}{\sum_i \tilde{\chi}_i^{-2} + N_C/a} \approx \frac{1}{2} M_\pi^2 \theta. \quad (10.15)$$

The last term in (10.13) generates strong CP-violating transitions between pseudoscalar particles. In particular it induces the phase-space allowed decays  $\eta_{1,8} \rightarrow \pi\pi$ . Comparing the prediction  $\text{Br}(\eta \rightarrow \pi^+\pi^-) = 1.8 \times 10^2 \theta^2$  with the present experimental upper bound,  $\text{Br}(\eta \rightarrow \pi^+\pi^-) < 1.5 \times 10^{-3}$ , one gets the limit  $|\theta| < 3 \times 10^{-3}$  (Pich and de Rafael 1991b).

### 10.1. Baryon electric dipole moments

It is completely straightforward to extend the previous analysis to the baryon sector (Pich and de Rafael 1991b). One simply writes the matrix  $\tilde{U}(\phi)$  in terms of the canonical coset representative,

$$\tilde{U}(\phi) = \tilde{\xi}_R(\phi) \tilde{\xi}_L^\dagger(\phi), \quad \tilde{\xi}_L(\phi) = \langle 0 | \tilde{\xi}_L | 0 \rangle u(\phi)^\dagger, \quad \tilde{\xi}_R(\phi) = \langle 0 | \tilde{\xi}_R | 0 \rangle u(\phi), \quad (10.16)$$

with  $\langle 0 | u | 0 \rangle = 1$  and  $U(\phi) = u(\phi)^2$ .

From equation (10.12) it follows that

$$\langle 0 | \tilde{\xi}_R^\dagger | 0 \rangle \tilde{\chi} \langle 0 | \tilde{\xi}_L | 0 \rangle = \langle 0 | \tilde{\xi}_L | 0 \rangle \langle 0 | \tilde{\xi}_R^\dagger | 0 \rangle \tilde{\chi} = \chi + iH, \quad (10.17)$$

where we have used the fact that  $\tilde{\chi}$  is diagonal and, without loss of generality,  $\langle 0 | \tilde{\xi}_L | 0 \rangle$  and  $\langle 0 | \tilde{\xi}_R | 0 \rangle$  can be restricted to be diagonal as well.

The lowest-order baryon Lagrangian is directly obtained from equation (9.3), making the obvious replacements

$$u_\mu \longrightarrow \tilde{\xi}_\mu \equiv i \left\{ \tilde{\xi}_R^\dagger (\partial_\mu - ir_\mu) \tilde{\xi}_R - \tilde{\xi}_L^\dagger (\partial_\mu - il_\mu) \tilde{\xi}_L \right\}, \quad (10.18)$$

$$\Gamma_\mu \longrightarrow \tilde{\Gamma}_\mu \equiv \frac{1}{2} \left\{ \tilde{\xi}_R^\dagger (\partial_\mu - ir_\mu) \tilde{\xi}_R + \tilde{\xi}_L^\dagger (\partial_\mu - il_\mu) \tilde{\xi}_L \right\}, \quad (10.19)$$

and adding the additional singlet piece

$$\Delta \mathcal{L}_1^{(B)} = g_S \langle \tilde{\xi}_\mu \rangle \langle \bar{B} \gamma^\mu \gamma_5 B \rangle. \quad (10.20)$$

The  $\mathcal{O}(\mathcal{M})$  interactions are given by the Lagrangian (9.9), with the change

$$\chi_+ \longrightarrow \tilde{\chi}_+ \equiv \tilde{\xi}_R^\dagger \tilde{\chi} \tilde{\xi}_L + \tilde{\xi}_L^\dagger \tilde{\chi}^\dagger \tilde{\xi}_R. \quad (10.21)$$

Equation (10.17) implies that

$$\tilde{\chi}_+ = \chi_+ + i \frac{a}{N_C} \bar{\theta} (U^\dagger - U). \quad (10.22)$$

Inserting this relation into  $\mathcal{L}_{\mathcal{M}}^{(B)}$ , leads to a CP non-conserving meson–baryon interaction term modulated by the coupling  $a\bar{\theta}/N_C$ :

$$\mathcal{L}_{\bar{\theta}}^{(B)} = -i \frac{a}{N_C} \bar{\theta} \left\{ b_0 \langle U^\dagger - U \rangle \langle \bar{B} B \rangle + b_1 \langle \bar{B} (U^\dagger - U) B \rangle + b_2 \langle \bar{B} B (U^\dagger - U) \rangle \right\}. \quad (10.23)$$

At the one-loop level, the Lagrangian  $\mathcal{L}_{\bar{\theta}}^{(B)}$  generates baryon electric dipole moments. The logarithmic chiral contribution is fully calculable. For the neutron electric dipole moment one obtains (Pich and de Rafael 1991b):

$$d_n^\gamma = \frac{a\bar{\theta}}{N_c} \frac{e}{16\pi^2 f_\pi^2} \left\{ \frac{M_\Xi - M_\Sigma}{m_K^2 - m_\pi^2} (D + F) \log \left( \frac{M_N^2}{m_\pi^2} \right) + \frac{M_\Sigma - M_N}{m_K^2 - m_\pi^2} (D - F) \log \left( \frac{M_\Sigma^2}{m_K^2} \right) \right\} \quad (10.24)$$

in agreement with the old Current Algebra result (Crewther *et al* 1979, Di Vecchia 1980). Taking into account the uncertainty associated with the unknown contribution from higher-order local terms in the baryon Lagrangian, one finally gets (Pich and de Rafael 1991b):

$$d_n^\gamma = (3.3 \pm 1.8) \times 10^{-16} \theta \text{ e cm}. \quad (10.25)$$

From a comparison between this result and the experimental (95% CL) upper limit  $d_n^\gamma < 11 \times 10^{-26} \text{ e cm}$  (Altarev *et al* 1992), one concludes that

$$|\theta| < 7 \times 10^{-10}. \quad (10.26)$$

## 11. Interactions of a light Higgs

An hypothetical light Higgs particle provides a good example of the broad range of application of the chiral techniques. Its hadronic couplings are fixed by low-energy theorems which relate the  $\phi \rightarrow \phi' h^0$  transition with a zero-momentum Higgs to the corresponding  $\phi \rightarrow \phi'$  coupling (Gunion *et al* 1990). Although, within the Standard Model, the possibility of a light Higgs boson is already excluded, an extended scalar sector with additional degrees of freedom could easily avoid the present experimental limits.

The quark–Higgs interaction can be written down in the general form

$$\mathcal{L}_{h^0 \bar{q}q} = -\frac{h^0}{v} \{k_d \bar{d} M_d d + k_u \bar{u} M_u u\}, \quad (11.1)$$

where  $v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$ ,  $M_u$  and  $M_d$  are the diagonal mass matrices for up- and down-type quarks respectively, and the couplings  $k_u$  and  $k_d$  depend on the model considered. In the Standard Model,  $k_u = k_d = 1$ , while in the usual two-Higgs-doublet models (without tree-level flavour-changing neutral currents)  $k_d = k_u = \cos \alpha / \sin \beta$  (model I) or  $k_d = -\sin \alpha / \cos \beta$ ,  $k_u = \cos \alpha / \sin \beta$  (model II), where  $\alpha$  and  $\beta$  are functions of the parameters of the scalar potential.

The Yukawa interactions of the light-quark flavours can be trivially incorporated into the low-energy chiral Lagrangian through the external scalar field  $s$ , together with the light-quark-mass matrix  $\mathcal{M}$ :

$$s = \mathcal{M} \left\{ 1 + \frac{h^0}{v} (k_d A + k_u B) \right\}, \quad (11.2)$$

where  $A \equiv \text{diag}(0, 1, 1)$  and  $B \equiv \text{diag}(1, 0, 0)$ . It remains to compute the contribution from the heavy flavours  $c$ ,  $b$ ,  $t$ . Their Yukawa interactions induce a Higgs–gluon coupling through heavy-quark loops (Shifman *et al* 1978),

$$\mathcal{L}_{h^0 GG} = \frac{\alpha_s}{12\pi} (n_d k_d + n_u k_u) \frac{h^0}{v} G_{\mu\nu}^a G_a^{\mu\nu}. \quad (11.3)$$

Here,  $n_d = 1$  and  $n_u = 2$  are the number of heavy quarks of type down and up respectively. The operator  $G_{\mu\nu}^a G_a^{\mu\nu}$  can be related to the trace of the energy-momentum tensor; in the three light-flavour theory, one has

$$\Theta_\mu^\mu = \frac{\beta_1 \alpha_s}{4\pi} G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q} \mathcal{M} q, \quad (11.4)$$

where  $\beta_1 = -9/2$  is the first coefficient of the QCD  $\beta$ -function. To obtain the low-energy representation of  $\mathcal{L}_{h^0 GG}$  it therefore suffices to replace  $\Theta_\mu^\mu$  and  $\bar{q} \mathcal{M} q$  by their corresponding expressions in the effective chiral Lagrangian theory. One gets (Chivukula *et al* 1989, Leutwyler and Shifman 1990, Prades and Pich 1990),

$$\mathcal{L}_{h^0 GG}^{\text{eff}} = \xi \frac{h^0 f^2}{v} \left\{ \langle D_\mu U^\dagger D^\mu U \rangle + 3B_0 \langle U^\dagger \mathcal{M} + \mathcal{M} U \rangle \right\}. \quad (11.5)$$

The information on the heavy quarks, which survives in the low-energy limit, is contained in the coefficient  $\xi \equiv -(n_d k_d + n_u k_u)/(3\beta_1) = 2(k_d + 2k_u)/27$ .

Using the chiral formalism, the present experimental constraints on a very light neutral scalar have been investigated, in the context of two-Higgs-doublet models. A Higgs in the mass range  $2m_\mu < M_{h^0} < 2M_\pi$  can be excluded (within model II), analyzing the decay  $\eta \rightarrow \pi^0 h^0$  (Prades and Pich 1990). A more general analysis (Pich *et al* 1992), using the light-Higgs production channels  $Z \rightarrow Z^* h^0$ ,  $\eta' \rightarrow \eta h^0$ ,  $\eta \rightarrow \pi^0 h^0$  and  $\pi \rightarrow \nu h^0$ , allows us to exclude a large area in the parameter space  $(\alpha, \beta, M_{h^0})$  of both models (I and II) for  $M_{h^0} < 2m_\mu$ .

## 12. Effective theory at the electroweak scale

In spite of the spectacular success of the Standard Model, we still do not really understand the dynamics underlying the electroweak symmetry breaking  $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{QED}}$ . The Higgs mechanism provides a renormalizable way to generate the  $W$  and  $Z$  masses and, therefore, their longitudinal degrees of freedom. However, an experimental verification of this mechanism is still lacking.

The scalar sector of the Standard Model Lagrangian can be written in the form

$$\mathcal{L}(\Phi) = \frac{1}{2} \langle D^\mu \Sigma^\dagger D_\mu \Sigma \rangle - \frac{\lambda}{16} (\langle \Sigma^\dagger \Sigma \rangle - v^2)^2, \quad (12.1)$$

where

$$\Sigma \equiv \begin{pmatrix} \Phi^0 & -\Phi^+ \\ \Phi^- & \Phi^{0*} \end{pmatrix} \quad (12.2)$$

and  $D_\mu \Sigma$  is the usual gauge-covariant derivative

$$D_\mu \Sigma \equiv \partial_\mu \Sigma + ig \frac{\vec{\tau}}{2} \vec{W}_\mu \Sigma - ig' \Sigma \frac{\tau_3}{2} B_\mu. \quad (12.3)$$

In the limit where the coupling  $g'$  is neglected,  $\mathcal{L}(\Phi)$  is invariant under global  $G \equiv SU(2)_L \otimes SU(2)_C$  transformations ( $SU(2)_C$  is the so-called custodial-symmetry group),

$$\Sigma \xrightarrow{G} g_L \Sigma g_C^\dagger, \quad g_{L,C} \in SU(2)_{L,C}. \quad (12.4)$$

Performing a polar decomposition,

$$\Sigma(x) = \frac{1}{\sqrt{2}} (v + H(x)) U(\phi(x)), \quad U(\phi(x)) = \exp\left(i\vec{\tau}\vec{\phi}(x)/v\right), \quad (12.5)$$

in terms of the Higgs field  $H$  and the Goldstones  $\vec{\phi}$ , and taking the limit  $\lambda \gg 1$  (heavy Higgs), we can rewrite  $\mathcal{L}(\Phi)$  in the standard chiral form (Appelquist and Bernard 1980):

$$\mathcal{L}(\Phi) = \frac{v^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle + \mathcal{O}\left(\frac{H}{v}\right). \quad (12.6)$$

In the unitary gauge  $U = 1$ , this  $\mathcal{O}(p^2)$  Lagrangian reduces to the usual bilinear gauge-mass term.

Equation (12.6) is the universal model-independent interaction of the Goldstone bosons induced by the assumed pattern of SCSB,  $SU(2)_L \otimes SU(2)_C \longrightarrow SU(2)_{L+C}$ . The scattering of electroweak Goldstone bosons (or equivalently longitudinal gauge bosons) is then described by the same formulae as the scattering of pions, changing  $f$  by  $v$  (Cornwall *et al* 1974, Lee *et al* 1977, Chanowitz and Gaillard 1985). To the extent that the present data are still not sensitive to the virtual Higgs effects, we have only tested up to now the symmetry properties of the scalar sector encoded in equation (12.6).

In order to really prove the particular scalar dynamics of the Standard Model, we need to test the model-dependent part involving the Higgs field  $H$ . If the Higgs turns out to be too heavy to be directly produced (or if it does not exist at all!), one could still investigate the higher-order effects by applying the standard chiral-expansion techniques in a completely straightforward way (Appelquist 1980, Appelquist and Bernard 1980, Longhitano 1980). The Standard Model gives definite predictions for the corresponding chiral couplings of the  $\mathcal{O}(p^4)$  Lagrangian, which could be tested in future experiments<sup>‡‡</sup>. It remains to be seen if the experimental determination of the higher-order electroweak chiral couplings will confirm the renormalizable Standard Model Lagrangian, or will constitute an evidence of new physics

<sup>‡‡</sup> There is a large number of publications devoted to this subject: Dobado and Herrero 1989, Donoghue and Ramirez 1990, Holdom and Terning 1990, Dawson and Valencia 1991, Dobado *et al* 1991, Georgi 1991, Golden and Randall 1991, Holdom 1991, Espriu and Herrero 1992, De Rujula *et al* 1992, Dobado and Peláez 1994, Herrero and Ruiz-Morales 1994, Espriu and Matias 1995. Many other relevant references can be traced back from these papers.



### 13. Summary

ChPT is a powerful tool to study the low-energy interactions of the pseudoscalar-meson octet. This effective Lagrangian framework incorporates all the constraints implied by the chiral symmetry of the underlying Lagrangian at the quark–gluon level, allowing for a clear distinction between genuine aspects of the Standard Model and additional assumptions of variable credibility, usually related to the problem of long-distance dynamics. The low-energy amplitudes of the Standard Model are calculable in ChPT, except for some coupling constants which are not restricted by chiral symmetry. These constants reflect our lack of understanding of the QCD confinement mechanism and must be determined experimentally for the time being. Further progress in QCD can only improve our knowledge of these chiral constants, but it cannot modify the low-energy structure of the amplitudes.

ChPT provides a convenient language to improve our understanding of the long-distance dynamics. Once the chiral couplings are experimentally known, one can test different dynamical models, by comparing the predictions that they give for those couplings with their phenomenologically determined values. The final goal would be, of course, to derive the low-energy chiral constants from the Standard Model Lagrangian itself. Although this is a very difficult problem, the recent attempts done in this direction look quite promising.

It is important to emphasize that:

1. ChPT is not a model. The effective Lagrangian generates the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetries. Therefore, ChPT is the effective theory of the Standard Model at low energies.
2. The experimental verification of the ChPT predictions does not provide a test of the detailed dynamics of the Standard Model; only the implications of the underlying symmetries are being proved. Any other model with identical chiral-symmetry properties would give rise to the same effective Lagrangian, but with different values for the low-energy couplings.
3. The dynamical information on the underlying fundamental Lagrangian is encoded in the chiral couplings. In order to actually test the non-trivial low-energy dynamics of the Standard Model, one needs first to know the Standard Model predictions for the chiral couplings.

In this report I have presented the basic formalism of ChPT and some selected phenomenological applications. The ChPT techniques can be applied in many more situations: any system involving Goldstone bosons can be studied in a similar way.

### Acknowledgments

I am grateful to G. Ecker, E. Oset and E. de Rafael for useful discussions and comments. This work has been supported in part by CICYT (Spain) under grant AEN-93-0234.

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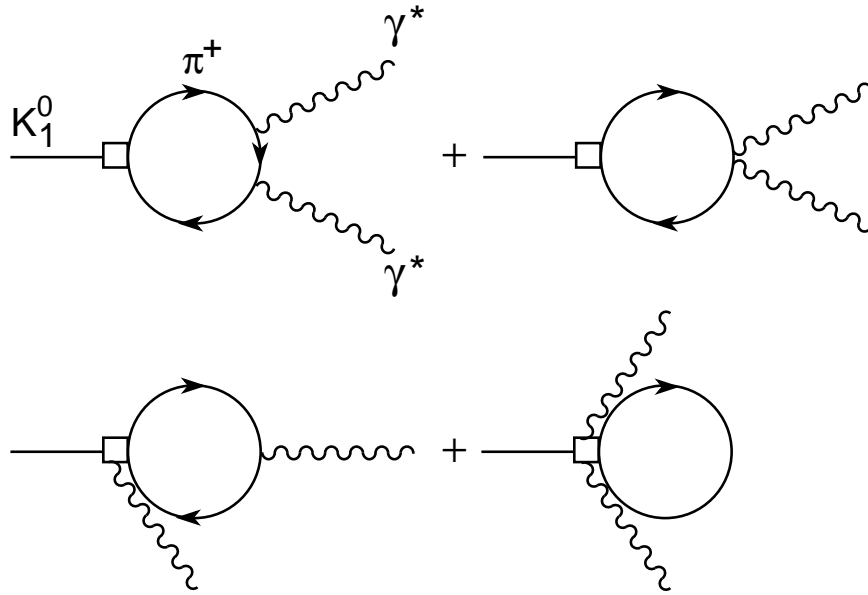
## Figure captions

**Figure 1.** Feynman diagrams for  $K_1^0 \rightarrow \gamma^* \gamma^*$ .

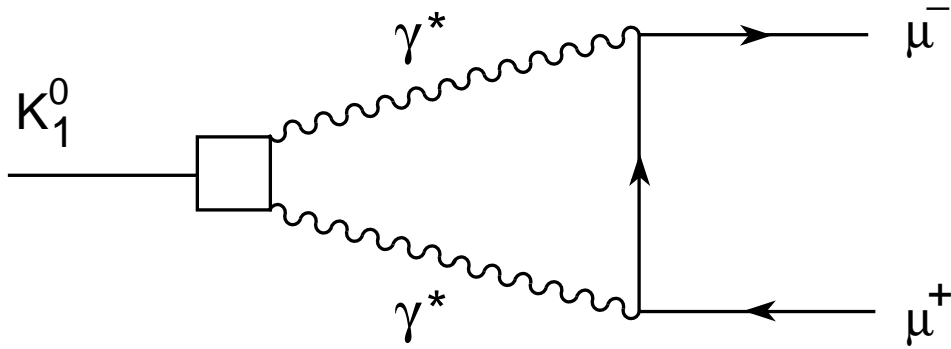
**Figure 2.** Feynman diagram for the  $K_1^0 \rightarrow \mu^+ \mu^-$  decay. The  $K_1^0 \gamma^* \gamma^*$  vertex is generated through the one-loop diagrams shown in figure 1.

**Figure 3.**  $2\gamma$ -invariant-mass distribution for  $K_L \rightarrow \pi^0 \gamma \gamma$ :  $O(p^4)$  ( $\cdots\cdots$ ),  $O(p^6)$  with  $a_V = 0$  (----),  $O(p^6)$  with  $a_V = -0.9$  (—). The spectrum is normalized to the 50 unambiguous events of NA31 (Barr *et al* 1992), without acceptance corrections.

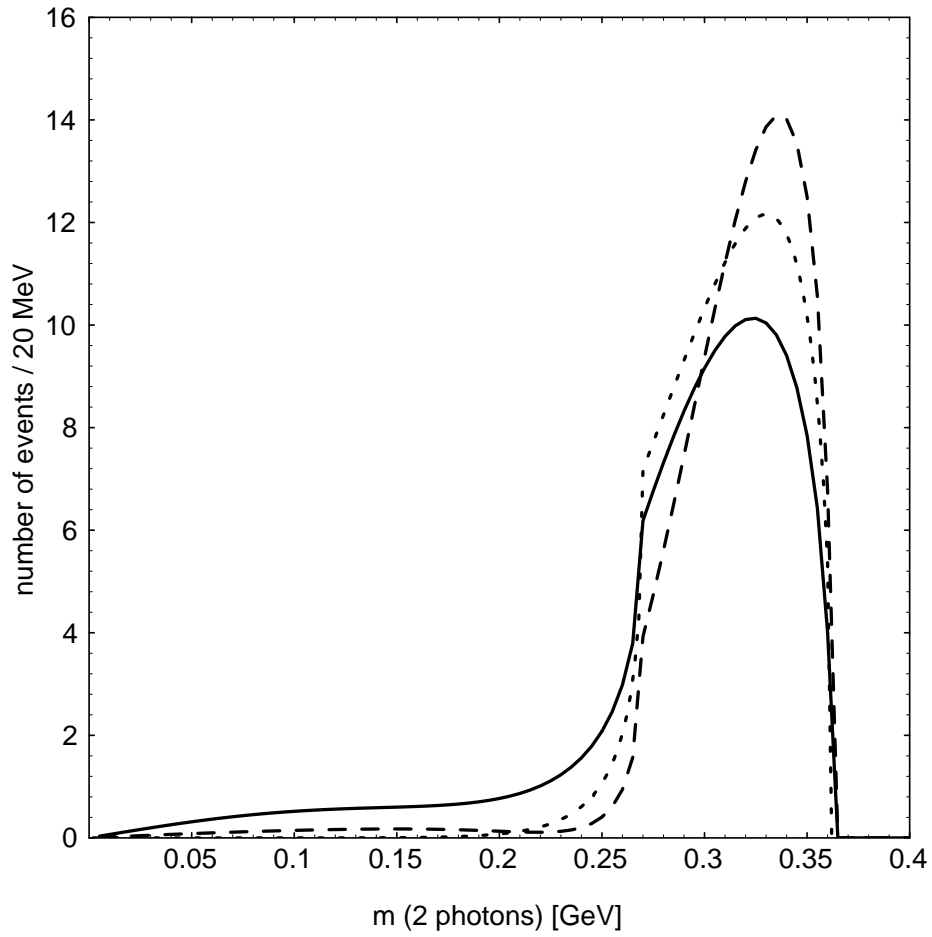
**Figure 4.** Measured (Barr *et al* 1992)  $2\gamma$ -invariant-mass distribution for  $K_L \rightarrow \pi^0 \gamma \gamma$  (—), and estimated background (----). The experimental acceptance is given by the crosses. The dotted line simulates the  $O(p^4)$  ChPT prediction.



**Figure 1.** Feynman diagrams for  $K_1^0 \rightarrow \gamma^* \gamma^*$ .

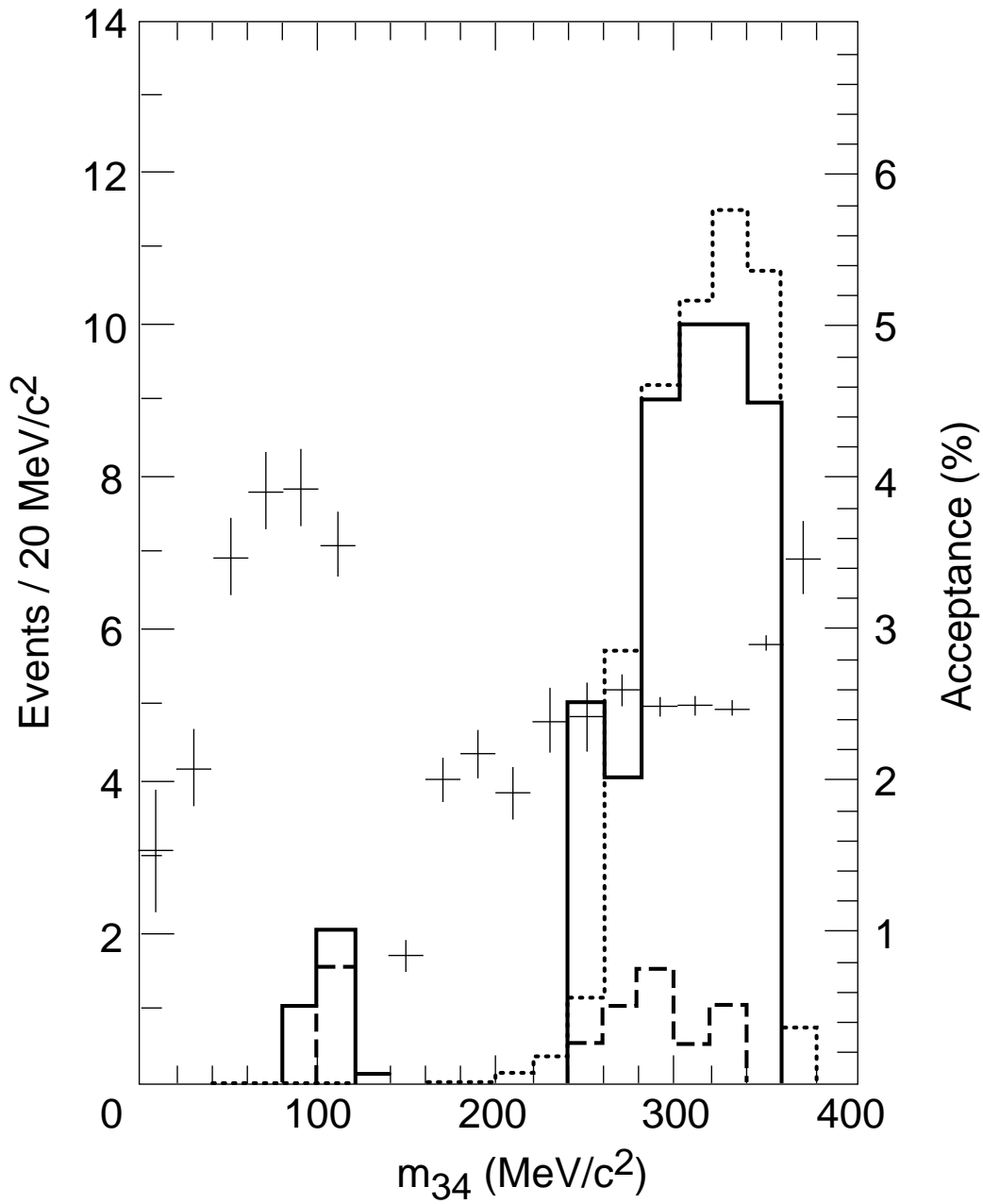


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