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Chiral perturbation theory for nucleon generalized parton distributions

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Abstract

We analyze the moments of the isosinglet generalized parton distributions H, E, $\tilde{H}$, $\tilde{E}$ of the nucleon in one-loop order of heavy-baryon chiral perturbation theory. We discuss in detail the construction of the operators in the effective theory that are required to obtain all corrections to a given order in the chiral power counting. The results will serve to improve the extrapolation of lattice results to the chiral limit.

1 Introduction

In recent years one has learned that many aspects of hadron structure can be described in the unifying framework of generalized parton distributions (GPDs). This framework allows one to combine information which comes from very different sources in an efficient and model-independent manner. The field was pioneered in [1, 2, 3] and has evolved to considerable complexity, reviewed for instance in [4, 5, 6, 7]. As GPDs can be analyzed using standard operator product expansion techniques [1, 8], their moments can be and have been calculated in lattice QCD [9]. Lattice calculations of well-measured quantities can be used to check the accuracy of the method, which may then be employed to evaluate quantities that are much harder to determine experimentally. This complementarity is especially valuable in the context of GPDs, because experimental measurements as e.g. in [10] may not be sufficient to determine these functions of three kinematic variables in a model-independent way. Moreover, several moments of GPDs admit a physically intuitive interpretation in terms of the spatial and spin structure of hadrons, see e.g. [2, 11, 12, 13].

A notorious problem of lattice QCD is the need for various extrapolations from the actual simulations with finite lattice spacing, finite volume and unphysically heavy quarks to the continuum, infinite volume and physical quark masses. Simple phenomenological fits are often still sufficient in view of the general size of uncertainties, but with increasing numerical precision more reliable methods have to be applied. Chiral perturbation theory (ChPT) provides such a method [14]. Describing the exact low-energy limit of QCD it predicts the functional form for the dependence of observables on the finite volume and the pion mass [15] and also the finite lattice spacing [16]. At a given order in the expansion parameter, ChPT defines a number of low-energy constants which determine each of these limits. Some of these constants are typically known from independent sources, and the remaining ones have to be determined from fits to the lattice data. The task of ChPT is thus to provide the corresponding functional expressions for a sufficient number of observables. In this paper we contribute to this endeavor by analyzing the moments of the isoscalar nucleon GPDs H, E, $\tilde{H}$ and $\tilde{E}$ in one-loop order.

The analysis of pion GPDs in ChPT has been performed in several papers [17, 18, 19]. In the case of the nucleon GPDs, the chiral corrections have been calculated for the lowest moments [20, 17, 21] in the framework of heavy-baryon ChPT, which performs an expansion in the inverse nucleon mass $1/M$. Due to the kinematic limit taken in this scheme, the sum and difference of the incoming and outgoing nucleon momenta p^μ and p'^μ are of different order in $1/M$. As a consequence, the nth moment of a nucleon GPD contains terms up to nth order in the $1/M$ expansion. Given the rapidly growing number of low-energy constants in higher orders of ChPT, it has been assumed that the chiral corrections can only be calculated for the terms of lowest order in $1/M$, i.e. for the form factors accompanied by the smallest number of vectors $(p'-p)^\mu$. This would be a serious setback for the program sketched above. The aim of the present paper is to show that the situation is much better. In particular, we find that the corrections of order $O(m_\pi)$ and $O(m_\pi^2)$ to *all* form factors parameterizing the moments of chiral-even isoscalar nucleon GPDs come from one-loop diagrams in ChPT and the corresponding higher-order tree-level insertions.

This paper is organized as follows. In Section 2 we recall the relation between moments of nucleon GPDs and matrix elements of twist-two operators and rewrite it in a form suitable for the $1/M$ expansion. In Section 3 we discuss the construction of twist-two operators in heavy-baryon ChPT and give a general power-counting scheme for their contribution to a given nucleon matrix element. In Sections 4 and 5 we identify the operators that contribute to moments of GPDs at lowest order in the chiral expansion and give the results of the corresponding loop calculations. We summarize our findings in Section 6.

2 Generalized parton distributions in the nucleon

The nucleon GPDs can be introduced as matrix elements of nonlocal operators. In this paper we limit ourselves to the chiral-even isoscalar quark GPDs, which are defined by

$$\int \frac{d\lambda}{4\pi} e^{ix\lambda(aP)} \langle p'|\, \bar{q}(-\tfrac{1}{2}\lambda a)\, \not{a}\, q(\tfrac{1}{2}\lambda a)\,|p\rangle = \frac{1}{2aP}\, \bar{u}(p') \left[\not{a}\, H(x,\xi,t) + \frac{i\sigma^{\mu\nu} a_\mu \Delta_\nu}{2M}\, E(x,\xi,t) \right] u(p)\,,$$

$$\int \frac{d\lambda}{4\pi} e^{ix\lambda(aP)} \langle p'|\, \bar{q}(-\tfrac{1}{2}\lambda a)\, \not{a}\gamma_5\, q(\tfrac{1}{2}\lambda a)\,|p\rangle = \frac{1}{2aP}\, \bar{u}(p') \left[\not{a}\gamma_5\, \widetilde{H}(x,\xi,t) + \frac{a\Delta}{2M}\gamma_5\, \widetilde{E}(x,\xi,t) \right] u(p)\,, \quad (1)$$

where a sum over u and d quark fields on the l.h.s. is understood, so that $H = H^u + H^d$ etc. Here a is a light-like auxiliary vector, M is the nucleon mass, and we use the standard notations for the kinematical variables

$$P = \frac{1}{2}(p+p'), \qquad \Delta = p' - p, \qquad t = \Delta^2, \qquad \xi = -\frac{\Delta a}{2Pa}\,. \quad (2)$$

As usual, Wilson lines between the quark fields are to be inserted in (1) if one is not working in the light-cone gauge $a^\mu A_\mu = 0$. The x-moments of the nucleon GPDs are related to the matrix elements of the local twist-two operators

$$\mathcal{O}_{\mu_1\mu_2\ldots\mu_n} = \mathbf{S}\, \bar{q}\gamma_{\mu_1} i\overleftrightarrow{D}_{\mu_2} \ldots i\overleftrightarrow{D}_{\mu_n}\, q\,, \qquad \widetilde{\mathcal{O}}_{\mu_1\mu_2\ldots\mu_n} = \mathbf{S}\, \bar{q}\gamma_{\mu_1}\gamma_5\, i\overleftrightarrow{D}_{\mu_2} \ldots i\overleftrightarrow{D}_{\mu_n}\, q\,, \quad (3)$$

where $\overleftrightarrow{D}^\mu = \frac{1}{2}(\overrightarrow{D}^\mu - \overleftarrow{D}^\mu)$ and $\mathbf{S}$ denotes the symmetrization of all uncontracted Lorentz indices and the subtraction of traces, e.g. $\mathbf{S}\, t_{\mu\nu} = \frac{1}{2}(t_{\mu\nu} + t_{\nu\mu}) - \frac{1}{4} g_{\mu\nu} t^\lambda{}_\lambda$ for a tensor of rank two. It is convenient to contract all open Lorentz indices with the auxiliary vector a,

$$\mathcal{O}_{\mu_1\ldots\mu_n} \;\to\; \mathcal{O}_n(a) = a^{\mu_1} \ldots a^{\mu_n}\, \mathcal{O}_{\mu_1\ldots\mu_n}\,, \quad (4)$$

and in analogy for $\widetilde{\mathcal{O}}$. The matrix elements of the operators (3) can be parameterized as [4, 6]

$$\langle p'|\mathcal{O}_n(a)|p\rangle = \sum_{\substack{k=0\\ \text{even}}}^{n-1} (aP)^{n-k-1}\, (a\Delta)^k\; \bar{u}(p') \left[\not{a}\, A_{n,k}(t) + \frac{i\sigma^{\mu\nu} a_\mu \Delta_\nu}{2M}\, B_{n,k}(t) \right] u(p)$$
$$+ \operatorname{mod}(n+1,2)\, (a\Delta)^n\, \frac{1}{M}\, \bar{u}(p') u(p)\, C_n(t)\,,$$
$$\langle p'|\widetilde{\mathcal{O}}_n(a)|p\rangle = \sum_{\substack{k=0\\ \text{even}}}^{n-1} (aP)^{n-k-1}\, (a\Delta)^k\; \bar{u}(p') \left[\not{a}\gamma_5\, \widetilde{A}_{n,k}(t) + \frac{a\Delta}{2M}\gamma_5\, \widetilde{B}_{n,k}(t) \right] u(p)\,. \quad (5)$$

The moments of the above GPDs are polynomials in ξ^2,

$$\int_{-1}^{1} dx\, x^{n-1}\, H(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{n-1} (2\xi)^k\, A_{n,k}(t) + \operatorname{mod}(n+1,2)\, (2\xi)^n C_n(t)\,,$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{n-1} (2\xi)^k\, B_{n,k}(t) - \operatorname{mod}(n+1,2)\, (2\xi)^n C_n(t)\,,$$

$$\int_{-1}^{1} dx\, x^{n-1}\, \widetilde{H}(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, \widetilde{A}_{n,k}(t)\,,$$
$$\int_{-1}^{1} dx\, x^{n-1}\, \widetilde{E}(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, \widetilde{B}_{n,k}(t)\,. \tag{6}$$

The restriction to even k in (5) and (6) is a consequence of time reversal invariance.

To calculate the chiral corrections to the nucleons form factors we shall use the formalism of heavy-baryon chiral perturbation theory, which treats the nucleon as an infinitely heavy particle and performs a corresponding non-relativistic expansion [22]. The evaluation of nucleon form factors in heavy-baryon ChPT is simplified if one works in the Breit frame [23]. It is defined by the condition $\vec{P} = 0$, so that the incoming and outgoing nucleons have opposite spatial momenta $\vec{p}\,' = -\vec{p} = \vec{\Delta}/2$ and the same energy $p_0' = p_0 = M\gamma$, where

$$\gamma = \sqrt{1 - \Delta^2/4M^2}\,. \tag{7}$$

In the heavy-baryon formalism the baryon has a additional quantum number, the velocity v, which in the Breit frame is $v = (1,0,0,0)$. The incoming and outgoing nucleon momenta are thus given by $p = M\gamma v - \Delta/2$ and $p' = M\gamma v + \Delta/2$.

The Dirac algebra simplifies considerably in the heavy-baryon formulation. All Dirac bilinears can be expressed in terms of the velocity v_μ and the spin operator

$$S_\mu = \frac{i}{2}\gamma_5 \sigma_{\mu\nu}\, v^\nu. \tag{8}$$

Using that $(v\Delta) = (vS) = 0$, one finds in particular

$$\overline{u}(p')u(p) = \gamma\, \bar{u}_v(p')\, u_v(p)\,,$$
$$\overline{u}(p')\gamma_\mu u(p) = v_\mu\, \bar{u}_v(p')\, u_v(p) + \frac{1}{M}\, \bar{u}_v(p')\, [S_\mu, (S\Delta)]\, u_v(p)\,,$$
$$\frac{i}{2M}\, \overline{u}(p')\, \sigma_{\mu\nu}\Delta^\nu u(p) = v_\mu \frac{\Delta^2}{4M^2}\, \bar{u}_v(p')\, u_v(p) + \frac{1}{M}\, \bar{u}_v(p')\, [S_\mu, (S\Delta)]\, u_v(p)\,,$$
$$\overline{u}(p')\gamma_\mu\gamma_5 u(p) = 2\gamma\, \bar{u}_v(p') S_\mu u_v(p) + \frac{\Delta_\mu}{2M^2(1+\gamma)}\, \bar{u}_v(p')\, (S\Delta)\, u_v(p)\,,$$
$$\overline{u}(p')\gamma_5 u(p) = \frac{1}{M}\, \bar{u}_v(p')\, (S\Delta)\, u_v(p)\,, \tag{9}$$

where the spinors

$$u_v(p) = \mathcal{N}^{-1}\, \frac{1+\not{v}}{2}\, u(p), \qquad\qquad u_v(p') = \mathcal{N}^{-1}\, \frac{1+\not{v}}{2}\, u(p') \tag{10}$$

with

$$\mathcal{N} = \sqrt{\frac{M + vp}{2M}} = \sqrt{\frac{M + vp'}{2M}} = \sqrt{\frac{1+\gamma}{2}} \tag{11}$$

are normalized as $\bar{u}_v(p,s')\,u_v(p,s) = 2M\delta_{s's}$. With (9) one obtains the following representation for the matrix elements (5) in the Breit frame:

$$\begin{aligned}\langle p'|\mathcal{O}_n(a)|p\rangle &= \sum_{k=0}^{n}(M\gamma)^{n-k-1}\,(av)^{n-k}\,(a\Delta)^{k-1} \\ &\quad\times \bar{u}_v(p')\Big[(a\Delta)\,E_{n,k}(t) + \gamma\,[(aS),(S\Delta)]\,M_{n,k-1}(t)\Big]\,u_v(p)\,, \\ \langle p'|\widetilde{\mathcal{O}}_n(a)|p\rangle &= \sum_{k=1}^{n}(M\gamma)^{n-k}\,(av)^{n-k}\,(a\Delta)^{k-1} \\ &\quad\times \bar{u}_v(p')\left[2\gamma\,(aS)\,\widetilde{E}_{n,k-1}(t) + \frac{(a\Delta)(S\Delta)}{2M^2}\,\widetilde{M}_{n,k-1}(t)\right]u_v(p)\,, \end{aligned} \tag{12}$$

with

$$\begin{aligned} E_{n,k}(t) &= A_{n,k}(t) + \frac{\Delta^2}{4M^2}B_{n,k}(t) \qquad \text{for } k<n\,, \qquad\qquad E_{n,n}(t) = \gamma^2 C_n(t)\,, \\ M_{n,k}(t) &= A_{n,k}(t) + B_{n,k}(t)\,, \\ \widetilde{E}_{n,k}(t) &= \widetilde{A}_{n,k}(t)\,, \\ \widetilde{M}_{n,k}(t) &= \frac{1}{1+\gamma}\,\widetilde{A}_{n,k}(t) + \widetilde{B}_{n,k}\,. \end{aligned} \tag{13}$$

The definition of the E_n and $\widetilde{E}_n$ is conventional but might be confusing as E_n is not the nth moment of $E(x,\xi,t)$ etc. We nevertheless use this notation, in order to make it easier to compare our results with those in the literature. Notice that according to (5) the terms with $E_{n,k}$ in (12) are only nonzero if k is even, whereas those with $M_{n,k-1}$, $\widetilde{E}_{n,k-1}$ and $\widetilde{M}_{n,k-1}$ are only nonzero if k is odd. We will evaluate these form factors in heavy-baryon ChPT. It is straightforward to transform back to the original form factors using

$$\begin{aligned} A_{n,k}(t) &= \frac{1}{\gamma^2}\left[E_{n,k}(t) - \frac{\Delta^2}{4M^2}M_{n,k}(t)\right], \qquad\qquad B_{n,k}(t) = \frac{1}{\gamma^2}\Big[M_{n,k}(t) - E_{n,k}(t)\Big]\,, \\ \widetilde{B}_{n,k}(t) &= \widetilde{M}_{n,k}(t) - \frac{1}{1+\gamma}\,\widetilde{E}_{n,k}(t)\,. \end{aligned} \tag{14}$$

3 Twist-two matrix elements in heavy-baryon ChPT

Heavy-baryon ChPT combines the techniques of chiral perturbation theory and of heavy-quark effective field theory [22] (for a detailed review see Ref. [24]). The effective Lagrangian describes the pion-nucleon interactions in the limit when $m_\pi, q \ll M$, where q is a generic momentum. In this situation the velocity v of the nucleon is preserved in the process. One introduces the nucleon field with velocity v as [22]

$$N(x) = e^{-iM_0 vx}\big(N_v(x) + n_v(x)\big)\,, \tag{15}$$

where M_0 is the nucleon mass in the chiral limit. The fields $N_v(x)$, $n_v(x)$ respectively contain the large and small components of the nucleon field and satisfy $\not{v}N_v = N_v$, $\not{v}n_v = -n_v$. Their Fourier transform depends on the residual nucleon momentum, i.e. the original nucleon momentum minus

$M_0 v$. Integrating out the field $n_v(x)$, one obtains an effective Lagrangian for the pion-nucleon system which involves the nucleon field $N_v(x)$ and pion field $\pi(x)$,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N}\,, \tag{16}$$

where

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots\,, \qquad\qquad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \ldots \tag{17}$$

are expanded in powers of q. The explicit expressions for the lowest-order terms read [24]

$$\begin{aligned}
\mathcal{L}_\pi^{(2)} &= \frac{F^2}{4}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger + (\chi^\dagger U + U^\dagger \chi)\right)\,, \\
\mathcal{L}_{\pi N}^{(1)} &= \overline{N}_v \left\{ i(v\nabla) + g_A (Su) \right\} N_v\,, \\
\mathcal{L}_{\pi N}^{(2)} &= \overline{N}_v \left\{ \frac{(v\nabla)^2 - \nabla^2}{2M_0} - \frac{i g_A}{2M_0}\left\{(\nabla S), (vu)\right\} + c_1\,\mathrm{Tr}\left(u^\dagger \chi u^\dagger + u \chi^\dagger u\right) \right. \\
&\qquad \left. + \left(c_2 - \frac{g_A^2}{8M_0}\right)(vu)^2 + c_3\, u_\mu u^\mu + \left(c_4 + \frac{1}{4M_0}\right)[S^\mu, S^\nu]\, u_\mu u_\nu \right\} N_v
\end{aligned} \tag{18}$$

with $U = u^2 = \exp\{i\pi^a\tau^a/F\}$, the covariant derivative $\nabla_\mu = \partial_\mu + \Gamma_\mu$, and

$$\begin{aligned}
\Gamma_\mu &= \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right) = \frac{i}{4F^2}\,\epsilon^{abc}\,\pi^a\,\partial_\mu \pi^b \tau^c + O(\pi^4)\,, \\
u_\mu &= i\left(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger\right) = -\frac{1}{F}\,\partial_\mu \pi^a \tau^a + O(\pi^3)\,.
\end{aligned} \tag{19}$$

The trace Tr and the Pauli matrices τ^a refer to isospin space. As is done in current lattice QCD calculations, we assume isospin symmetry to be exact here, neglecting the difference between u- and d-quark masses. The leading-order parameters appearing in (18) are the pion decay constant F (normalized to $F \approx 92$ MeV) and the nucleon axial-vector coupling g_A, both taken in the chiral limit. The field χ implements the explicit breaking of chiral symmetry by the quark masses, and in the isospin limit can be replaced by $\chi \to m^2\,\mathbb{1}$, where m is the bare pion and $\mathbb{1}$ the unit matrix in isospin space. Estimates of the low-energy constants c_i in the second-order Lagrangian $\mathcal{L}_{\pi N}^{(2)}$, which are of order $1/M$, can be found in [25]. We note that $\mathcal{L}_{\pi N}^{(2)}$ induces corrections to the nucleon propagator, which we treat as insertions on a nucleon line. They read $-i\left((vl)^2 - l^2\right)/(2M_0)$ and $4ic_1 m^2$, where l is the residual nucleon momentum, and are to be multiplied with a nucleon propagator $i/(vl + i0)$ from $\mathcal{L}_{\pi N}^{(1)}$ on either side. The pion-nucleon vertices following from $\mathcal{L}_{\pi N}^{(2)}$ can be found in Appendix A of [24].

In the following subsection we discuss how to construct the operators in the effective theory that match the twist-two quark operators (3). Nucleon matrix elements in the Breit frame are then obtained as [26]

$$\langle p'|\mathcal{O}|p\rangle = \mathcal{N}^2 Z_N\, \overline{u}_v(p')\, G_{\mathcal{O}}(r', r)\, u_v(p)\,, \tag{20}$$

with the spinors u_v and normalization $\mathcal{N}$ given in (10) and (11). Here $G_{\mathcal{O}}(r', r)$ is the truncated Green function for external heavy-baryon fields $\overline{N}_v$, N_v and the operator $\mathcal{O}$ in the effective theory. The residual momenta of the incoming and outgoing nucleon are given by

$$r = p - M_0 v = wv - \Delta/2\,, \qquad\qquad r' = p' - M_0 v = wv + \Delta/2 \tag{21}$$

with

$$w = M(\gamma - 1) + \delta M = -\frac{\Delta^2}{8M} - 4c_1 m^2 + O(q^3)\,, \tag{22}$$

where $\delta M = M - M_0$ is the nucleon mass shift. Finally, Z_N is the heavy-baryon field renormalization constant,

$$Z_N = 1 - \frac{3m^2 g_A^2}{2(4\pi F)^2} - \frac{9m^2 g_A^2}{4(4\pi F)^2}\log\frac{m^2}{\mu^2} - 8m^2\, d_{28}^r(\mu) + O(q^3)\,, \tag{23}$$

where $d_{28}^r(\mu)$ is a low-energy constant in the Lagrangian $\mathcal{L}_{\pi N}^{(3)}$. As explained in [27] the corresponding operator is required for renormalization but does not appear in physical matrix elements. The value of $d_{28}^r(\mu)$ can therefore be chosen freely (with different choices resulting in different values for other low-energy constants), and in [26] it was chosen such that it compensates the $\log(m^2/\mu^2)$ term in (23) at the *physical* value of m. Since we are interested in the pion mass dependence of matrix elements, we must explicitly keep the logarithmic term in Z_N. For further discussion we refer to Section 3.2.

3.1 Construction of effective operators

We now discuss how to construct the isoscalar local twist-two operators in the effective theory that match the quark-gluon operators $\mathcal{O}(a)$ defined in (3) and (4). The relevant operators in the effective theory can be divided into two groups: operators $\mathcal{O}_\pi$ which contain only pion fields (and couple to the nucleon via pion loops) and operators $\mathcal{O}_{\pi N}$ which are bilinear in the nucleon field. The matching of operators thus takes the form

$$\mathcal{O}(a) \;\cong\; \mathcal{O}_\pi(a) + \mathcal{O}_{\pi N}(a)\,, \qquad\qquad \widetilde{\mathcal{O}}(a) \;\cong\; \widetilde{\mathcal{O}}_{\pi N}(a)\,, \tag{24}$$

where we have taken into account that there is no isoscalar pion operator of negative parity (i.e. no $\widetilde{\mathcal{O}}_\pi(a)$). The pion isoscalar operators $\mathcal{O}_\pi(a)$ have been analyzed in several papers [28, 17, 18, 19] and we shall simply use their results.

Let us now list the building blocks for constructing the operators $\mathcal{O}_{\pi N}(a)$ and $\widetilde{\mathcal{O}}_{\pi N}(a)$, which we collectively denote by $Q(a)$, omitting the subscript πN for ease of writing. They should be bilinear in the nucleon field and should be tensors that have n indices contracted with the auxiliary vector a according to (4). To build tensors we have the following objects with Lorentz indices at our disposal: the velocity vector v_μ, the spin vector S_μ, the derivative ∂_μ, and the antisymmetric tensor $\epsilon_{\mu\nu\lambda\rho}$. We recall that any Dirac matrix structure can be reduced to an expression containing the spin operator S_μ, and that the metric tensor $g_{\mu\nu}$ can be omitted in the construction because the twist-two operators are traceless. Using the identities

$$\{S_\mu, S_\nu\} = \frac{1}{2}(v_\mu v_\nu - g_{\mu\nu})\,, \qquad\qquad [S_\mu, S_\nu] = i\epsilon_{\mu\nu\lambda\rho}\, v^\lambda S^\rho \tag{25}$$

we can impose that S_μ should appear at most linearly, or quadratically as the commutator $[S_\mu, S_\nu]$. Concerning the derivative ∂_μ, we find it useful to have it acting either on single nonlinear pion fields u, $u^\dagger$ in the combinations Γ_μ or u_μ given in (19), or as a total derivative on the product of all fields, or in the antisymmetric form $\overleftrightarrow{\partial}_\mu = \frac{1}{2}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)$ on the product of all fields to its right or to its left. This will make it easy to keep track of factors Δ_μ in the corresponding matrix elements, which play a particular role as we shall see. To give operators with the correct chiral transformation behavior, the derivative $\overleftrightarrow{\partial}$ must appear in the covariant combination $\overleftrightarrow{\nabla}_\mu = \overleftrightarrow{\partial}_\mu + \Gamma_\mu$. The fields and derivatives used in our construction are then any number of u_μ, $\overleftrightarrow{\nabla}_\mu$ and $\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$ between the nucleon fields $\overline{N}_v$ and N_v, and any number of total derivatives ∂_μ acting on the operator as a whole. In the sense of (19) we henceforth refer to ∂_μ, $\overleftrightarrow{\nabla}_\mu$ and u_μ as "derivatives". They have chiral dimension 1, whereas $\chi_\pm$ has chiral dimension 2 and will not appear at the order of the chiral expansion we limit ourselves to.

We can decompose the pion-nucleon operators $Q_n(a)$ as

$$Q_n(a) = \sum_{k=0}^{n} M^{n-k-1} \, (av)^{n-k} Q_{n,k}(a) \,, \tag{26}$$

where $Q_{n,k}(a) = a_{\mu_1} \ldots a_{\mu_k} \, Q_n^{\mu_1 \ldots \mu_k}$ does not contain any factors (av). The k external vectors a in $Q_{n,k}(a)$ can be contracted only with derivatives ∂_μ, $\overleftrightarrow{\nabla}_\mu$, u_μ and the spin vector S_μ, or with the antisymmetric tensor. There can be at most one factor (aS) as discussed after (25), so that $Q_{n,k}(a)$ has to contain at least $k-1$ derivatives. We can hence write[1]

$$Q_{n,k} = M Q_{n,k,-1} + Q_{n,k,0} + \frac{1}{M} \, Q_{n,k,1} + \ldots \,, \tag{27}$$

where the operator $Q_{n,k,i}$ has chiral dimension $k+i$. Note that due to parity the number of factors S_μ, u_μ and $\epsilon_{\mu\nu\lambda\rho}$ must be even for $\mathcal{O}$ and odd for $\widetilde{\mathcal{O}}$. We remark that the contraction of a with the ϵ-tensor involves at least two derivatives, given that we chose to replace its simultaneous contraction with v_λ and S_ρ by $[S_\mu, S_\nu]$ using (25). As a consequence, the antisymmetric tensor does not appear in the operators with lowest chiral dimension for a given k.

3.2 Tree-level insertions

At tree level, the matrix elements of the effective operators between two nucleon states are easy to calculate. Since u_μ and Γ_μ involve at least one or two pion fields according to (19), derivatives in the effective operators are to be replaced as $\partial_\mu \to i\Delta_\mu$, $u_\mu \to 0$, and $\overleftrightarrow{\nabla}_\mu \ \to -iwv_\mu$ with w given in (22). Notice that, while generically the derivative $\overleftrightarrow{\nabla}_\mu$ counts as $O(q)$ in the chiral expansion, the kinematics of the external nucleon momenta forces wv_μ to be of order $O(q^2)$. As a result, the leading-order contributions of the operator $Q_{n,k}$ to the form factors in (12) come from the terms with maximum number of factors Δ_μ and no factor wv_μ. With (26) one readily obtains

$$\begin{aligned}
\langle p'|\mathcal{O}_{n,k}(a)|p\rangle &\overset{\text{LO}}{=} (a\Delta)^{k-1} \, \bar{u}_v(p') \Big[(a\Delta) \, E^{(0)}_{n,k} + [(aS),(S\Delta)] \, M^{(0)}_{n,k-1} \Big] \, u_v(p) \,, \\
\langle p'|\widetilde{\mathcal{O}}_{n,k}(a)|p\rangle &\overset{\text{LO}}{=} (a\Delta)^{k-1} \, \bar{u}_v(p') \left[2M(aS) \, \widetilde{E}^{(0)}_{n,k-1} + \frac{(a\Delta)(S\Delta)}{2M} \, \widetilde{M}^{(0)}_{n,k-1} \right] u_v(p) \,,
\end{aligned} \tag{28}$$

where the superscript on each form factor indicates the term of order $O(q^0)$ in its chiral expansion. At this order, the form factors $E_{n,k}$ and $M_{n,k-1}$ of the vector GPD are related to the matrix element of the operator $\mathcal{O}_{n,k,0}$, since the nucleon matrix element of the operator $\mathcal{O}_{n,k,-1}$ is zero at tree level. As explained above, this operator contains a factor (aS), which due to parity must be accompanied by the axial field u_μ and hence does not contribute to tree-level matrix elements without external pions. For the axial vector GPDs one finds that the form factor $\widetilde{E}_{n,k-1}$ ($\widetilde{M}_{n,k-1}$) receives its leading contribution from the operator $\widetilde{\mathcal{O}}_{n,k,-1}$ ($\widetilde{\mathcal{O}}_{n,k,1}$), given the required number of factors Δ_μ in (28).

Beyond leading order, tree-level insertions contribute to the form factors starting at order $O(q^2)$. Contributions proportional to Δ^2 are due to operators with ∂^2 or to a factor w from operators with $\overleftrightarrow{\nabla}$, or to the kinematic factors γ in (12) and $\mathcal{N}$ in (20). Contributions proportional to m^2 are due to operators with χ_+ or with $\overleftrightarrow{\nabla}$ and from the wave function renormalization constant Z_N in (20).

[1]Instead of M one could also use M_0 or F in (26) and (27), since all are of the same order in chiral power counting. We find powers of M most convenient, because they also appear in the form factor decompositions (12).

In the results of the following sections we explicitly include the terms proportional to g_A^2 in the expansion (23) of Z_N, whereas contributions from d_{28}^r are lumped into the coefficients describing the m^2 corrections due to tree-level insertions.

3.3 Loop contributions

Let us now consider a loop diagram with the insertion of the operator $Q_n(a)$. One easily finds that the term $M^{n-k-1}\,(av)^{n-k}\,Q_{n,k}(a)$ in the sum (26) can contribute to the form factors in (12) which are accompanied by at least $n-k$ powers of (av), i.e. to $E_{n,m}$, $M_{n,m-1}$, $\widetilde{E}_{n,m-1}$ and $\widetilde{M}_{n,m-1}$ with $m \le k$. Chiral counting determines which terms can contribute to a given order. Namely, the contribution of the operator $Q_{n,k,i}$ in a loop diagram has chiral dimension

$$D_{k,i} = 4L + (k+i) + \sum_{j=1}^{N_\pi} \dim V_\pi(j) + \sum_{j=1}^{N_{\pi N}} \dim V_{\pi N}(j) - 2I_\pi - I_N\,, \tag{29}$$

where L is the number of loops and $(k+i)$ is the chiral dimension of the operator insertion. $V_\pi(j)$ and $V_{\pi N}(j)$ respectively denote the jth vertex from $\mathcal{L}_\pi$ and $\mathcal{L}_{\pi N}$ in the graph. N_π and $N_{\pi N}$ are the corresponding total numbers of vertices, and I_π and I_N are the numbers of pion and nucleon propagators.[2] Using the relation $L = I_\pi + I_N - N_\pi - N_{\pi N}$ (see e.g. [24]) and the fact that for our specific diagrams $I_N = N_{\pi N}$, we can rewrite this expression as a sum of positive terms, which makes it easy to identify the different contributions at a given order:

$$D_{k,i} = 2L - 1 + k + (i+1) + \sum_{j=1}^{N_\pi} \big(\dim V_\pi(j) - 2\big) + \sum_{j=1}^{N_{\pi N}} \big(\dim V_{\pi N}(j) - 1\big)\,. \tag{30}$$

For each vertex we can insert either the lowest or any higher order, i.e. $\dim V_\pi(j) = 2, 4, \dots$ and $\dim V_{\pi N}(j) = 1, 2, \dots$. Note that a loop diagram with chiral dimension $D_{k,i}$ generates contributions to a nucleon matrix element of order $O(q^d)$ with $d \ge D_{k,i}$. This is on one hand because of the explicit factors $\mathcal{N}$ and Z_N in (20), and on the other hand because the sum $r^\mu + r'^\mu = 2wv^\mu$ is of order $O(q^2)$ and thus one order higher than the generic power associated with residual nucleon momenta.

The form factors enter a matrix element multiplied by factors $(a\Delta)$ or $(S\Delta)$ as given in (12). Taking these into account, one finds that the chiral correction from $Q_{n,k,i}$ to $E_{n,m}$ and $M_{n,m-1}$ has at least order $D_{k,i} - m$, while for the form factors $\widetilde{E}_{n,m-1}$ and $\widetilde{M}_{n,m-1}$ it has at least order $D_{k,i} - m + 1$ and $D_{k,i} - m - 1$, respectively. This is a main result of our paper and allows one to determine which operators need to be considered to calculate the corrections to a form factor to a given order in the chiral expansion. Because $D_{k,i}$ contains a term k and because of the constraint $k - m \ge 0$, the number of loops and the order of the chiral Lagrangian required to calculate the lowest-order corrections for a given form factor do *not* grow with m. Instead, a growing number of factors Δ_μ accompanying a form factor in the nucleon matrix element requires a growing number of derivatives in the operator $Q_{n,k}$.

As an application of our general result we find that the form factors $E_{n,m}$ and $M_{n,m-1}$ can receive corrections starting at order

- $O(q)$ from one-loop diagrams with insertion of the operator $Q_{n,m,-1}$ and leading-order (LO) pion-nucleon vertices,

[2]Note that a nucleon propagator correction from a higher-order Lagrangian counts as one (nucleon-nucleon) vertex with two nucleon propagators on either side, see the discussion after (19).

Table 1: Four-vectors and their products appearing in the numerators of the loop graphs of Fig. 1. NN vertices (arising from nucleon propagator corrections) are not explicitly shown in the graphs.

derivatives in operator insertion	∂_μ	Δ_μ
	$\overleftrightarrow{\nabla}_\mu$	l_μ and wv_μ
	u_μ	l_μ
vertices	πNN at LO	Sl
	πNN at NLO	$(vl)(Sl) \pm (vl)(S\Delta)$
	NN at NLO	$(vl)^2 - l^2 \pm l\Delta - \Delta^2/4$

- $O(q^2)$ from the one-loop diagrams with insertion of the operators $Q_{n,m,0}$ and $Q_{n,m+1,-1}$ and LO pion-nucleon vertices, and from one-loop diagrams with insertion of the operator $Q_{n,m,-1}$ and one next-to-leading order (NLO) pion-nucleon vertex or nucleon propagator correction.

In turn, the form factor $\widetilde{E}_{n,m-1}$ receives corrections starting at order $O(q^2)$ from one-loop diagrams with leading-order vertices and insertion of the operator $Q_{n,m,-1}$. For $\widetilde{M}_{n,m-1}$ the discussion of corrections up to order $O(q^2)$ is more involved and will be given in Section 4.1.

To conclude the discussion of power counting, we consider the contribution to the form factors $E_{n,m}$ and $M_{n,m-1}$ of loop graphs with the insertion of the pion operators $\mathcal{O}_\pi(a)$, see (24). Repeating the above argument and taking into account that now $I_N = N_{\pi N} - 1$, one finds that such diagrams have chiral dimension

$$D_\pi = 2L - 1 + \dim \mathcal{O}_\pi + \sum_{j=1}^{N_\pi} \bigl(\dim V_\pi(j) - 2 \bigr) + \sum_{j=1}^{N_{\pi N}} \bigl(\dim V_{\pi N}(j) - 1 \bigr) . \tag{31}$$

Given that the leading operator $\mathcal{O}_\pi^n(a)$ contributing to $\mathcal{O}_n(a)$ has the chiral dimension n, one finds that it can contribute to the form factors $E_{n,m}$ and $M_{n,m-1}$ starting at order $O(q^{n-m+1})$. Note that because of charge conjugation invariance the isoscalar pion operators $\mathcal{O}_\pi^n(a)$ have even n and that due to time reversal invariance the form factors $E_{n,k}$ and $M_{n,k}$ vanish for odd k. Together with our power-counting formula one thus finds that $E_{n,n}$ gets contributions from $\mathcal{O}_\pi^n(a)$ starting at order $O(q)$ and $M_{n,n-2}$ starting at order $O(q^2)$. All other corrections from operators $\mathcal{O}_\pi(a)$ to form factors $E_{n,k}$ and $M_{n,k}$ start at $O(q^3)$.

Let us now take a closer look at the one-loop graphs with pion-nucleon operator insertions, which are shown in Fig. 1. With our construction of operators explained in Section 3.1 we can readily analyze the origin of factors Δ_μ, whose number determines to which form factor a graph will contribute. Using $(v\Delta) = (vS) = 0$ and the form (18) of the LO and NLO pion-nucleon Lagrangian, we find that the numerators of the loop integrals are composed as specified in Table 1. The denominators of the pion and nucleon propagators respectively are $(l^2 - m^2 + i0)$ and $(lv + w + i0)$, so that the loop integration turns tensors $l_{\mu_1} \dots l_{\mu_j}$ into tensors constructed from v_μ and $g_{\mu\nu}$. A factor Δ_μ that can be contracted with a^μ or S^μ (i.e. is not contracted to Δ^2) can hence only originate from total derivatives ∂_μ in the operator insertion and from an NLO pion-nucleon vertex or nucleon propagator correction. We will see that this reduces considerably the number of operators contributing to the leading chiral corrections of nucleon GPDs.

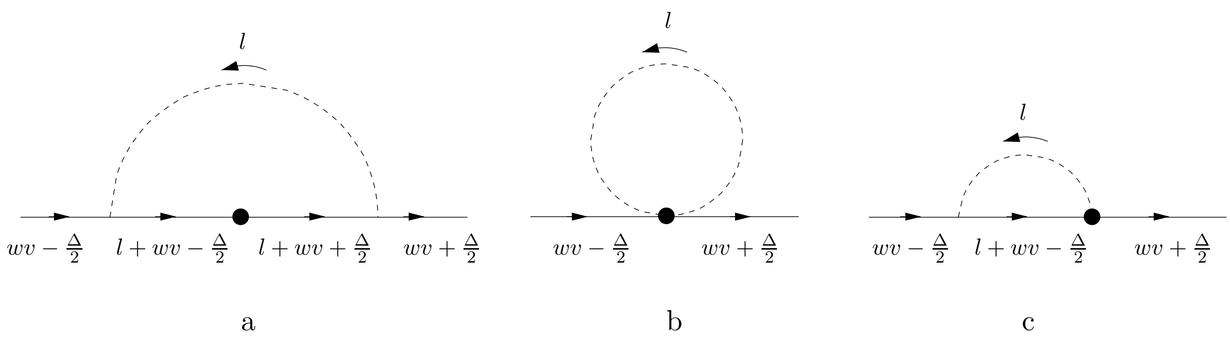

Figure 1: One-loop graphs with the insertion of a pion-nucleon operator $\mathcal{O}_{\pi N}(a)$ or $\widetilde{\mathcal{O}}_{\pi N}(a)$, denoted by a black blob. Not shown is the analog of graph c with residual momentum $l + wv + \Delta/2$ of the intermediate nucleon line.

4 Chiral corrections up to order $O(q^2)$

4.1 Axial-vector operators

Using the formalism developed in the previous section, we now evaluate the form factors up to relative order $O(q^2)$. Let us start by giving the operators in $\widetilde{O}_{n,k,i}$ that have the maximum number of total derivatives ∂_μ contracted with a^μ or S^μ. It will turn out that these are required to produce the factors of $(a\Delta)$ and $(S\Delta)$ in the form factor decomposition (12). With the constraints of parity invariance, we find

$$\begin{aligned}\widetilde{\mathcal{O}}_{n,k,-1}(a) &= \tilde{b}_{n,k}\,(ia\partial)^{k-1}\,\overline{N}_v(aS)N_v + \dots\,,\\ \widetilde{\mathcal{O}}_{n,k,1}(a) &= \tilde{c}_{n,k}\,(ia\partial)^{k}(i\partial_\mu)\,\overline{N}_v\,S^\mu N_v + \dots\,,\end{aligned} \tag{32}$$

where the ... stand for operators with fewer total derivatives. One has $\widetilde{E}^{(0)}_{n,k-1} = \tilde{b}_{n,k}/2$ and $\widetilde{M}^{(0)}_{n,k-1} = 2\tilde{c}_{n,k}$ for the tree-level contributions at order $O(q^0)$. From the time-reversal constraints on the form factors it follows that the low-energy constants $\tilde{b}_{n,k}$ and $\tilde{c}_{n,k}$ are zero for even k.

As derived in Section 3.3, the leading chiral corrections to $\widetilde{E}_{n,k-1}$ come from one-loop graphs with LO pion-nucleon vertices and the operator $\widetilde{O}_{n,k,-1}$. Since this operator does not contain pion fields, one needs to calculate only graph a in Fig. 1. One finds

$$\widetilde{E}_{n,k}(t) = \widetilde{E}^{(0)}_{n,k}\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\left[\log\frac{m^2}{\mu^2} + 1\right]\right\} + \widetilde{E}^{(2,m)}_{n,k} m^2 + \widetilde{E}^{(2,t)}_{n,k} t + O(q^3)\,, \tag{33}$$

where the terms going with m^2 and t originate from tree-level insertions as discussed at the end of Section 3.2. Here and in the following we use the subtraction scheme of [14] for the loop graphs, subtracting $1/\epsilon + \log(4\pi) + \psi(2)$ for each $1/\epsilon$ pole in $4 - 2\epsilon$ dimensions. The renormalization scale is denoted by μ, and the μ dependence of the logarithm in (33) cancels against the μ dependence of $\widetilde{E}^{(2,m)}_{n,k}$, which we have not displayed for simplicity. Note that the bare parameters m, F, g_A can be replaced with their counterparts at the physical point within the precision of our result. Since the nonanalytic corrections in (33) are independent of the moment indices n and k, they apply to the entire nucleon GPD $\widetilde{H}(x,\xi,t)$,

$$\widetilde{H}(x,\xi,t) = \widetilde{H}^{(0)}(x,\xi)\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\left[\log\frac{m^2}{\mu^2} + 1\right]\right\} + m^2\,\widetilde{H}^{(2,m)}(x,\xi) + t\,\widetilde{H}^{(2,t)}(x,\xi) + O(q^3)\,. \tag{34}$$

Let us now consider the chiral corrections for $\widetilde{M}_{n,k-1}$. It follows from (12) that the relevant diagrams have to produce a factor $(a\Delta)^k(S\Delta)$. By power counting, the form factor $\widetilde{M}_{n,k-1}$ could receive corrections of order $O(q^0)$ from diagrams with LO vertices and the operator insertion $\widetilde{\mathcal{O}}_{n,k,-1}$. Similarly, corrections of order $O(q)$ could come from the diagrams with LO vertices and insertion of $\widetilde{\mathcal{O}}_{n,k+1,-1}$ or $\widetilde{\mathcal{O}}_{n,k,0}$, and from diagrams with insertion of $\widetilde{\mathcal{O}}_{n,k,-1}$ and one NLO pion-nucleon vertex or nucleon propagator correction. One finds no operator in $\widetilde{\mathcal{O}}_{n,k,0}$ that has k or more partial derivatives contracted with a^μ or S^μ, and the same holds of course for $\widetilde{\mathcal{O}}_{n,k,-1}$. According to our discussion in Section 3.3 the graphs just discussed can thus produce at most k vectors Δ_μ (not counting those appearing in Δ^2) and hence do not contribute to $\widetilde{M}_{n,k-1}$. At order $O(q^2)$ there is a number of possibilities:

1. graphs with LO vertices and insertion of $\widetilde{\mathcal{O}}_{n,k+2,-1}$, $\widetilde{\mathcal{O}}_{n,k+1,0}$ or $\widetilde{\mathcal{O}}_{n,k,1}$. The insertion of $\widetilde{\mathcal{O}}_{n,k+1,0}$ does not produce a sufficient number of factors Δ_μ, whereas insertion of $\widetilde{\mathcal{O}}_{n,k+2,-1}$ gives a factor $(a\Delta)^{k+1}(aS)$, which contributes to the form factor $\widetilde{E}_{n,k+1}$ but not to $\widetilde{M}_{n,k-1}$. A correction to $\widetilde{M}_{n,k-1}$ is obtained from insertion of the operator $\widetilde{\mathcal{O}}_{n,k,1}$ given in (32), which already provides the tree-level term of this form factor. Only the loop graph in Fig. 1a is nonzero for this insertion, and the result is analogous to the one for the contribution of $\widetilde{\mathcal{O}}_{n,k,-1}$ to $\widetilde{E}_{n,k-1}$.

2. graphs with one NLO vertex or propagator correction and insertion of $\widetilde{\mathcal{O}}_{n,k+1,-1}$ or $\widetilde{\mathcal{O}}_{n,k,0}$. Insertion of $\widetilde{\mathcal{O}}_{n,k,0}$ does again not provide enough factors of Δ_μ, whereas graphs with $\widetilde{\mathcal{O}}_{n,k+1,-1}$ give zero due to time reversal invariance. This can be seen by direct calculation, or by noting that $\widetilde{M}_{n,k-1}$ is only nonzero for odd k, whereas the coefficient $\tilde{b}_{n,k+1}$ is only nonzero for even k, as remarked below (32).

3. graphs with insertion of $\widetilde{\mathcal{O}}_{n,k,-1}$ and (i) two loops with LO vertices, or (ii) one loop with two NLO pion-nucleon vertices or nucleon propagator corrections, or (iii) one loop with one NNLO pion-nucleon vertex or nucleon propagator correction, or (iv) one loop with a pion propagator correction from $\mathcal{L}_\pi^{(4)}$. The operator insertion provides $k-1$ factors of Δ_μ, so that two more factors must be provided by the vertices or propagator corrections (without being contracted to Δ^2). This is not possible in case (i), because the LO pion-nucleon vertices only involve pion momenta and the pion momenta in a two-loop graph can be parameterized such that they are independent of Δ (as in the one-loop graphs of Fig. 1). Likewise, a pion propagator correction in case (iv) does not depend on Δ and can therefore not contribute. In cases (ii) and (iii) one obtains nonzero contributions from the graph in Fig. 1a. The NNLO vertices and propagator corrections follow from the Lagrangian $\mathcal{L}_{\pi N}^{(3)}$ given in [27]. We find that the only term providing the two required factors of Δ_μ is the πNN vertex generated by

$$-\frac{g_A}{4M_0^2}\,\overline{N}_v\left\{(\overleftarrow{\nabla}S)(u\overrightarrow{\nabla})+(\overleftarrow{\nabla}u)(S\overrightarrow{\nabla})\right\}N_v\,. \tag{35}$$

Note that this vertex does not introduce a new low-energy constant, similarly to the term proportional to g_A in $\mathcal{L}_{\pi N}^{(2)}$, which generates the πNN coupling at NLO. These terms arise from the $1/M_0$ expansion of the leading-order relativistic pion-nucleon Lagrangian $\overline{N}\,(i\not{\nabla}-M_0+\frac{1}{2}g_A\not{u}\gamma_5)\,N$, see e.g. [24].

Putting everything together, we obtain

$$\begin{aligned}\widetilde{M}_{n,k}(t)=\widetilde{M}^{(0)}_{n,k}\left\{1-\frac{3m^2g_A^2}{(4\pi F)^2}\left[\log\frac{m^2}{\mu^2}+1\right]\right\}-\widetilde{E}^{(0)}_{n,k}\,\frac{m^2g_A^2}{(4\pi F)^2}\,\log\frac{m^2}{\mu^2}\\+\widetilde{M}^{(2,m)}_{n,k}m^2+\widetilde{M}^{(2,t)}_{n,k}t+O(q^3)\,,\end{aligned} \tag{36}$$

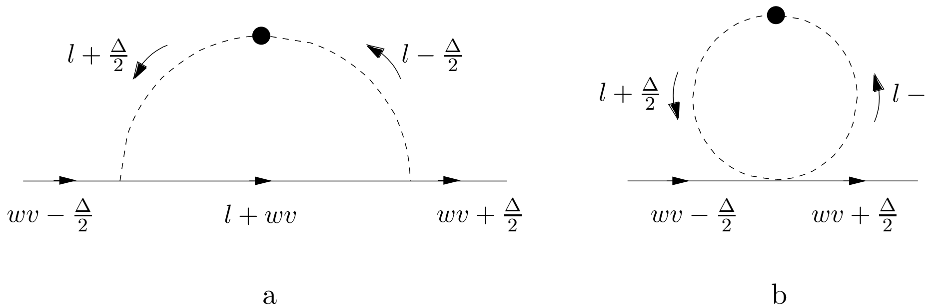

Figure 2: One-loop graphs with the insertion of the pion operator $\mathcal{O}^n_\pi(a)$, denoted by a black blob.

where the terms going with m^2 and t are due to tree-level insertions. With (33), (6) and (14) one can write for the isoscalar quark GPD $\widetilde{E}(x,\xi,t)$

$$\widetilde{E}(x,\xi,t) = \widetilde{E}^{(0)}(x,\xi)\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\left[\log\frac{m^2}{\mu^2} + 1\right]\right\} - \widetilde{H}^{(0)}(x,\xi)\,\frac{m^2 g_A^2}{(4\pi F)^2}\,\log\frac{m^2}{\mu^2} + m^2\,\widetilde{E}^{(2,m)}(x,\xi) + t\,\widetilde{E}^{(2,t)}(x,\xi) + O(q^3)\,. \qquad (37)$$

4.2 Vector operators

The analysis of the vector operators proceeds along similar lines. The operator $\mathcal{O}_{n,k,0}$ reads

$$\mathcal{O}_{n,k,0}(a) = b_{n,k}\,(ia\partial)^k\,\overline{N}_v\,N_v + c_{n,k}\,(ia\partial)^{k-1}\,(i\partial_\mu)\,\overline{N}_v\,[(Sa),S^\mu]N_v + \ldots\,, \qquad (38)$$

where the ... denote operators with fewer total derivatives. One finds $E^{(0)}_{n,k} = b_{n,k}$ and $M^{(0)}_{n,k-1} = -c_{n,k}$ for the leading-order tree-level insertions, which implies $b_{n,k} = c_{n,k+1} = 0$ for odd k.

According to (12) the graphs that give chiral corrections to $E_{n,k}$ or $M_{n,k-1}$ must produce k factors of Δ_μ contracted with a^μ or S^μ. With the constraints from parity invariance one finds that the operator $\mathcal{O}_{n,k,-1}$ does not contain terms which have $k-1$ or more total derivatives contracted with a^μ or S^μ. With the results of Section 3.3 this implies that $E_{n,k}$ and $M_{n,k-1}$ do not receive corrections from pion-nucleon operator insertions at order $O(q)$, and that corresponding corrections at order $O(q^2)$ can only come from the diagram in Fig. 1a with LO vertices. One finds that the one-loop contribution to the form factor $E_{n,k}$ is canceled by the terms proportional to g_A^2 in the wave function renormalization constant (23). For the form factor $M_{n,k}$ one obtains a correction

$$M^{(0)}_{n,k}\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\,\log\frac{m^2}{\mu^2}\right\}\,. \qquad (39)$$

We note that for $n=1$, $k=0$ this implies a chiral logarithm for the isoscalar magnetic form factor $G_{M,s}(t)$,

$$G_{M,s}(t) = \mu_s^{(0)}\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\,\log\frac{m^2}{\mu^2}\right\} + G^{(2,m)}_{M,s}m^2 + G^{(2,t)}_{M,s}t + O(q^3)\,, \qquad (40)$$

where $\mu_s^{(0)}$ is the isoscalar magnetic moment of the nucleon in the chiral limit and where we have added analytic terms due to tree-level insertions. The form (40) is consistent with the result of the relativistic calculation in [29].

One finally has to evaluate corrections due to the diagrams in Fig. 2 with insertion of the pion operator $\mathcal{O}^n_\pi(a)$, where n is even. We use the representation of this operator given in [19],[3]

$$\mathcal{O}^n_\pi(a) = F^2 \sum_{\substack{k=0\\ \text{even}}}^{n-2} \tilde{a}_{n,k}\, (ia\partial)^k \,\mathrm{Tr}\left[(aL)\,(2ia\overleftrightarrow{\partial})^{n-k-2}(aL) + (aR)\,(2ia\overleftrightarrow{\partial})^{n-k-2}(aR)\right] \tag{41}$$

with $L_\mu = U^\dagger \partial_\mu U$ and $R_\mu = U\partial_\mu U^\dagger$. As discussed in Section 3.3, the corrections to $M_{n,k}$ start at order $O(q^2)$ for $k = n-2$ and at higher order otherwise. They are due to diagrams with LO vertices, so that only the graph in Fig. 2a contributes. This is because the leading-order $\pi\pi NN$ vertex corresponds to an isovector transition of the nucleon, as follows from (18) and the expansion of Γ in (19). Combining the result with the correction in (39) and adding analytic terms from tree-level insertions, we obtain

$$M_{n,k}(t) = M^{(0)}_{n,k}\left\{1 - \frac{3m^2 g_A^2}{(4\pi F)^2}\log\frac{m^2}{\mu^2}\right\} + \delta_{k,n-2}\, M^{(2,\pi)}_n(t) + M^{(2,m)}_{n,k} m^2 + M^{(2,t)}_{n,k} t + O(q^3)\,, \tag{42}$$

where k is even and

$$\begin{aligned} M^{(2,\pi)}_n(t) &= \frac{3g_A^2}{(4\pi F)^2}\sum_{\substack{j=0\\ \text{even}}}^{n-2} \tilde{a}_{n,n-j-2}\int_{-1}^{1} d\eta\left[\frac{\partial^2}{\partial\eta^2}\,\eta^j(1-\eta^2)\right] m^2(\eta)\,\log\frac{m^2(\eta)}{\mu^2} \\ &= \frac{3g_A^2}{(4\pi F)^2}\sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j} j(j-1)\, A^{\pi\,(0)}_{n,n-j}\int_{-1}^{1} d\eta\,\eta^{j-2}\, m^2(\eta)\,\log\frac{m^2(\eta)}{\mu^2} \end{aligned} \tag{43}$$

with

$$m^2(\eta) = m^2 - \frac{t}{4}(1-\eta^2)\,. \tag{44}$$

Here $A^{\pi\,(0)}_{n,k}$ is the chiral limit of the form factors $A^\pi_{n,k}(t)$ describing the moments of the pion isoscalar GPD,

$$\int_{-1}^{1} dx\, x^{n-1} H^{I=0}_\pi(x,\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{n} (2\xi)^k A^\pi_{n,k}(t). \tag{45}$$

The relation to the low-energy constants $\tilde{a}_{n,k}$ reads [19]

$$A^{\pi\,(0)}_{n,k} = 2^{n-k}\left[\tilde{a}_{n,k-2} - \tilde{a}_{n,k}\right], \tag{46}$$

which implies

$$\tilde{a}_{n,n-k} = -\sum_{\substack{j=k\\ \text{even}}}^{n} 2^{-j} A^{\pi\,(0)}_{n,n-j} \qquad \text{for } k>0\,, \qquad\qquad \sum_{\substack{j=0\\ \text{even}}}^{n} 2^{-j} A^{\pi\,(0)}_{n,n-j} = 0\,. \tag{47}$$

The corrections to $E_{n,k}$ start at order $O(q)$ for $k = n$ and at order $O(q^3)$ or higher otherwise. At one-loop order we obtain $O(q)$ corrections to $E_{n,n}$ from graphs involving only LO vertices. Corrections of

[3]Note that the normalization of the twist-two operators (3) used here differs from that in [19] by a factor of 2. The coefficients $\tilde{a}_{n,k}$ have the same normalization here and in [19].

order $O(q^2)$ involve either graphs with one NLO vertex or propagator correction, or graphs with LO vertices and the subleading part wv of the residual nucleon momenta, see the discussion after (30). Our final result including analytic terms from tree-level insertions is

$$E_{n,k}(t) = E_{n,k}^{(0)} + \delta_{n,k}\Big[E_n^{(1,\pi)}(t) + E_n^{(2,\pi)}(t)\Big] + E_{n,k}^{(2,m)} m^2 + E_{n,k}^{(2,t)} t + O(q^3)\,, \tag{48}$$

where the order $O(q)$ correction reads

$$\begin{aligned} E_n^{(1,\pi)}(t) &= -\,M(2m^2-t)\,\frac{3\pi g_A^2}{8(4\pi F)^2} \sum_{\substack{j=0\\ \text{even}}}^{n-2} \tilde{a}_{n,n-j-2} \int_{-1}^{1} d\eta\, \frac{\eta^j(1-\eta^2)}{m(\eta)} \\ &= M(2m^2-t)\,\frac{3\pi g_A^2}{8(4\pi F)^2} \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\, A_{n,n-j}^{\pi(0)} \int_{-1}^{1} d\eta\, \frac{1-\eta^j}{m(\eta)}\,, \end{aligned} \tag{49}$$

and the order $O(q^2)$ term is

$$\begin{aligned} E_n^{(2,\pi)}(t) = {} & \frac{3m^2 g_A^2}{(4\pi F)^2}\, \log\frac{m^2}{\mu^2} \sum_{\substack{j=0\\ \text{even}}}^{n-2} \tilde{a}_{n,n-j-2} \\ & + \frac{6}{(4\pi F)^2} \sum_{\substack{j=0\\ \text{even}}}^{n-2} \tilde{a}_{n,n-j-2} \int_{-1}^{1} d\eta\, \eta^j (1-\eta^2) \left\{ \frac{g_A^2}{32}\left[2t\left(\log\frac{m^2(\eta)}{\mu^2}+1\right) - \frac{(t-2m^2)^2}{m^2(\eta)}\right]\right. \\ & \left. + M\left[c_1 m^2\left(\log\frac{m^2(\eta)}{\mu^2}+1\right) - \frac{3}{4}c_2\, m^2(\eta)\,\log\frac{m^2(\eta)}{\mu^2} - c_3\, m^2(\eta)\left(\log\frac{m^2(\eta)}{\mu^2}+\frac{1}{2}\right)\right]\right\} \\ = {} & -\,\frac{3m^2 g_A^2}{2(4\pi F)^2}\, \log\frac{m^2}{\mu^2} \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j} j\, A_{n,n-j}^{\pi(0)} \\ & - \frac{6}{(4\pi F)^2} \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\, A_{n,n-j}^{\pi(0)} \int_{-1}^{1} d\eta\, (1-\eta^j) \left\{ \frac{g_A^2}{32}\left[2t\left(\log\frac{m^2(\eta)}{\mu^2}+1\right) - \frac{(t-2m^2)^2}{m^2(\eta)}\right]\right. \\ & \left. + M\left[c_1 m^2\left(\log\frac{m^2(\eta)}{\mu^2}+1\right) - \frac{3}{4}c_2\, m^2(\eta)\,\log\frac{m^2(\eta)}{\mu^2} - c_3\, m^2(\eta)\left(\log\frac{m^2(\eta)}{\mu^2}+\frac{1}{2}\right)\right]\right\} \end{aligned} \tag{50}$$

The integrals over η in (43), (49) and (50) are elementary, but we have not found a simple closed form of the result for general n. In the next section we give explicit results for the values and derivatives at $t = 0$ of the form factors.

Our result for the form factor $\widetilde{M}_{n,k}(t)$ disagrees with [30], where it was taken for granted that the only operators which contribute at order $O(q^2)$ are those which already appear at tree-level in the same form factor. As our analysis shows, this holds indeed in many cases but not in all. For all other form factors our results agree with [30] where comparable.[4] For $n = 2$ our results for the vector operators also agree with those of Belitsky and Ji [21].[5]

[4] Note that [30] gives the correction to $E_{n,n}$ at order $O(q)$ but not at order $O(q^2)$.

[5] When comparing results, one must take into account that [21] uses $\overline{\text{MS}}$ renormalization, where for each pole in $4-2\epsilon$ dimensions one subtracts $1/\epsilon + \log(4\pi) + \psi(1)$, whereas we use the scheme of [14] and subtract $1/\epsilon + \log(4\pi) + \psi(2)$.

5 Results for moments of GPDs

We now transform the results of the previous section to the basis of the form factors $A_{n,k}$, $B_{n,k}$, C_n and $\widetilde{A}_{n,k}$, $\widetilde{B}_{n,k}$ corresponding to moments of GPDs in the conventional parameterization. We give the values and derivatives of these form factors at $t = 0$, which allows us to obtain closed expressions. Furthermore, these quantities are of most immediate interest in studies of GPDs on the lattice.

With our results (33), (36), (42), (48) and the conversion formulae (13), (14) one obtains for the form factors at $t = 0$

$$
\begin{aligned}
\widetilde{A}_{n,k}(0) &= \widetilde{A}^{(0)}_{n,k} \left\{ 1 - \frac{3m^2 g_A^2}{(4\pi F)^2} \left[\log\frac{m^2}{\mu^2} + 1 \right] \right\} + \widetilde{A}^{(2,m)}_{n,k} m^2 + O(m^3)\,, \\
\widetilde{B}_{n,k}(0) &= \widetilde{B}^{(0)}_{n,k} \left\{ 1 - \frac{3m^2 g_A^2}{(4\pi F)^2} \left[\log\frac{m^2}{\mu^2} + 1 \right] \right\} - \widetilde{A}^{(0)}_{n,k} \frac{m^2 g_A^2}{(4\pi F)^2} \log\frac{m^2}{\mu^2} + \widetilde{B}^{(2,m)}_{n,k} m^2 + O(m^3)\,, \\
A_{n,k}(0) &= A^{(0)}_{n,k} + A^{(2,m)}_{n,k} m^2 + O(m^3)\,, \\
B_{n,k}(0) &= B^{(0)}_{n,k} - \big(A^{(0)}_{n,k} + B^{(0)}_{n,k} \big) \frac{3m^2 g_A^2}{(4\pi F)^2} \log\frac{m^2}{\mu^2} + \delta_{k,n-2}\, M^{(2,\pi)}_n(0) + B^{(2,m)}_{n,k} m^2 + O(m^3)\,, \\
C_n(0) &= C^{(0)}_n + E^{(1,\pi)}_n(0) + E^{(2,\pi)}_n(0) + C^{(2,m)}_n m^2 + O(m^3)
\end{aligned}
\tag{51}
$$

with coefficients related to those in Section 4 by $\widetilde{A}^{(0)}_{n,k} = \widetilde{E}^{(0)}_{n,k}$, $\widetilde{B}^{(0)}_{n,k} = \widetilde{M}^{(0)}_{n,k} - \frac{1}{2}\widetilde{E}^{(0)}_{n,k}$, $A^{(0)}_{n,k} = E^{(0)}_{n,k}$, $B^{(0)}_{n,k} = M^{(0)}_{n,k} - E^{(0)}_{n,k}$, $C^{(0)}_n = E^{(0)}_{n,n}$ and by analogous relations for the coefficients with superscript $(2,m)$. Setting m, g_A, F to their physical values and choosing $\mu = M$, one finds that the corrections from loop graphs with nucleon operator insertions in (51) are moderately large, with $3m^2 g_A^2\,(4\pi F)^{-2}\,[\log(m^2/\mu^2) + 1] \approx -0.20$ and $m^2 g_A^2\,(4\pi F)^{-2} \log(m^2/\mu^2) \approx -0.09$. In the case of $B_{n,k}$ this loop correction can be substantial if $|B_{n,k}| \ll |A_{n,k}|$, which is empirically found for the electromagnetic form factors (i.e. the case $n = 1$) and also in lattice evaluations [9] for the moments with $n = 2$. The contributions to $B_{n,n-2}(0)$ and $C_n(0)$ from loop graphs with pion operator insertions are

$$
\begin{aligned}
M^{(2,\pi)}_n(0) &= \frac{6m^2 g_A^2}{(4\pi F)^2} \log\frac{m^2}{\mu^2} \sum_{\substack{j=2 \\ \text{even}}}^{n} 2^{-j} j\, A^{\pi\,(0)}_{n,n-j}\,, \\
E^{(1,\pi)}_n(0) &= \frac{3\pi m M g_A^2}{2\,(4\pi F)^2} \sum_{\substack{j=2 \\ \text{even}}}^{n} 2^{-j} \frac{j}{j+1} A^{\pi\,(0)}_{n,n-j}\,, \\
E^{(2,\pi)}_n(0) &= -\frac{3m^2 g_A^2}{2\,(4\pi F)^2} \log\frac{m^2}{\mu^2} \sum_{\substack{j=2 \\ \text{even}}}^{n} 2^{-j} j\, A^{\pi\,(0)}_{n,n-j} \\
&\quad + \frac{12m^2}{(4\pi F)^2} \left\{ \frac{g_A^2}{8} - M \left[c_1 \left(\log\frac{m^2}{\mu^2} + 1 \right) - \frac{3}{4} c_2 \log\frac{m^2}{\mu^2} - c_3 \left(\log\frac{m^2}{\mu^2} + \frac{1}{2} \right) \right] \right\} \sum_{\substack{j=2 \\ \text{even}}}^{n} 2^{-j} \frac{j}{j+1} A^{\pi\,(0)}_{n,n-j}\,.
\end{aligned}
\tag{52}
$$

Setting M, m, g_A, F to their physical values, choosing $\mu = M$, and taking the estimates $c_1 \approx -0.9\ \text{GeV}^{-1}$, $c_2 \approx 3.3\ \text{GeV}^{-1}$, $c_3 \approx -4.7\ \text{GeV}^{-1}$ from [25] we find $M^{(2,\pi)}_2(0) \approx -0.27\, A^{\pi\,(0)}_{2,0}$ and $E^{(1,\pi)}_2(0) + E^{(2,\pi)}_2(0) \approx (0.12 + 0.17)\, A^{\pi\,(0)}_{2,0}$. At the physical point, the order $O(m)$ correction is hence not very large. The full size of the order $O(m^2)$ corrections depends of course on the analytic terms in (51), whose values are not known.

The derivatives of the form factors at $t = 0$ obtain nonanalytic contributions only from the pion operator insertions. Writing $\partial_t A(0) = \big[\frac{\partial}{\partial t} A(t)\big]_{t=0}$ etc. we have

$$
\begin{aligned}
\partial_t \widetilde{A}_{n,k}(0) &= \widetilde{E}_{n,k}^{(2,t)} + O(m)\,, \\
\partial_t \widetilde{B}_{n,k}(0) &= \widetilde{M}_{n,k}^{(2,t)} - \widetilde{E}_{n,k}^{(2,t)}/2 - \widetilde{E}_{n,k}^{(0)}/(32M^2) + O(m)\,, \\
\partial_t A_{n,k}(0) &= E_{n,k}^{(2,t)} - \big(M_{n,k}^{(0)} - E_{n,k}^{(0)}\big)/(4M^2) + O(m)\,, \\
\partial_t B_{n,k}(0) &= \delta_{k,n-2}\, \partial_t M_n^{(2,\pi)}(0) + M_{n,k}^{(2,t)} - E_{n,k}^{(2,t)} + \big(M_{n,k}^{(0)} - E_{n,k}^{(0)}\big)/(4M^2) + O(m)\,, \\
\partial_t C_n(0) &= \partial_t E_n^{(1,\pi)}(0) + \partial_t E_n^{(2,\pi)}(0) + E_{n,n}^{(2,t)} + E_{n,n}^{(0)}/(4M^2) + O(m)
\end{aligned}
\tag{53}
$$

with

$$
\begin{aligned}
\partial_t M_n^{(2,\pi)}(0) &= -\frac{3g_A^2}{(4\pi F)^2}\left[\log\frac{m^2}{\mu^2} + 1\right] \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\frac{j}{j+1}\, A_{n,n-j}^{\pi\,(0)}\,, \\
\partial_t E_n^{(1,\pi)}(0) &= -\frac{M}{m}\,\frac{\pi g_A^2}{8(4\pi F)^2} \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\frac{j\,(5j+14)}{(j+1)(j+3)}\, A_{n,n-j}^{\pi\,(0)}\,, \\
\partial_t E_n^{(2,\pi)}(0) &= -\frac{3g_A^2}{4(4\pi F)^2}\left[\log\frac{m^2}{\mu^2} + 3\right] \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\frac{j}{j+1}\, A_{n,n-j}^{\pi\,(0)} \\
&\quad + \frac{2}{(4\pi F)^2}\left\{\frac{g_A^2}{8} + M\left[c_1 - \frac{3}{4}c_2\left(\log\frac{m^2}{\mu^2} + 1\right) - c_3\left(\log\frac{m^2}{\mu^2} + \frac{3}{2}\right)\right]\right\} \sum_{\substack{j=2\\ \text{even}}}^{n} 2^{-j}\frac{j\,(j+4)}{(j+1)(j+3)}\, A_{n,n-j}^{\pi\,(0)}\,.
\end{aligned}
\tag{54}
$$

Note that in the chiral limit the derivative $\partial_t B_{n,n-2}(0) \sim \partial_t M_n^{(2,\pi)}(0)$ diverges as $\log(m^2/\mu^2)$ and $\partial_t C_n(0) \sim \partial_t E_n^{(1,\pi)}(0)$ as $1/m$. With the parameters specified above, one finds $\partial_t M_2^{(2,\pi)}(0) \approx 1.7\ \text{GeV}^{-2} A_{2,0}^{\pi\,(0)}$ and $\partial_t E_2^{(1,\pi)}(0) + \partial_t E_2^{(2,\pi)}(0) \approx -(2.5 + 1.2)\ \text{GeV}^{-2} A_{2,0}^{\pi\,(0)}$. Numerically, the term $\partial_t E_n^{(1,\pi)}(0)$ is thus important but not extremely large at the physical point.

6 Summary

Using heavy-baryon chiral perturbation theory, we have calculated the chiral corrections up to order $O(q^2)$ for the form factors which parameterize moments of nucleon GPDs. We have restricted ourselves to vector and axial-vector quark distributions in the isosinglet combination. Our results generalize trivially to the corresponding gluon GPDs, which have the same quantum numbers and therefore the same corresponding operators in the effective theory (except for the values of the matching constants). Our method is also applicable to operators of different tensor or flavor structure.

The moments of GPDs contain terms of different order in $1/M$, ranging from M^{n-1} to M^{-1}. We have shown that, due to the way in which factors v_μ and Δ_μ arise in the calculation, the number of loops and the order in the expansion of the effective Lagrangian required to calculate a form factor to a given order $O(q^d)$ does not grow with the number of factors Δ_μ that accompany the form factor in the nucleon matrix element. A general power-counting formula is given after (30). In the case of the form factors $\widetilde{M}_{n,k}(t)$, calculation of the order $O(q^2)$ correction requires the pion-nucleon Lagrangian up to third order.

We have found that the form factors $\widetilde{E}_{n,k}(t)$ and $\widetilde{M}_{n,k}(t)$ receive corrections of order $O(q^2)$ which, apart from analytic terms, are independent of the moment indices and independent of t. The same holds for the one-loop corrections to $M_{n,k}(t)$ with $k < n-2$, whereas the corresponding corrections for $E_{n,k}$ with $k < n$ are zero. The form factors $M_{n,n-2}$ receive additional corrections at order $O(q^2)$ from one-loop graphs with the insertion of pion operators, and $E_{n,n}$ receives corresponding contributions starting at order $O(q)$.

For the form factors parameterizing moments of isoscalar GPDs, we find that $B_{n,k}$, $\widetilde{A}_{n,k}$ and $\widetilde{B}_{n,k}$ at $t = 0$ receive nonanalytic corrections of the form $m^2 \log(m^2/\mu^2)$ from loops with nucleon operator insertions. No such corrections are found for $A_{n,k}$ and C_n. The form factors $B_{n,n-2}$ at $t = 0$ receive in addition $m^2 \log(m^2/\mu^2)$ corrections from loop graphs with pion operator insertions, and the corresponding nonanalytic contributions to C_n give a term proportional to m. To leading chiral order, loop graphs with pion operator insertions are the only source of nonanalytic m^2 dependence for the derivatives of the form factors at $t = 0$. The derivative of $M_{n,n-2}$ diverges like $\log(m^2/\mu^2)$ in the chiral limit, and the derivative of C_n like $1/m$.

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