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# Chiral Symmetry and the Parity-Violating $NN\pi$ Yukawa Coupling

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## Abstract

We construct the complete  $SU(2)$  parity-violating (PV)  $\pi, N, \Delta$  interaction Lagrangian with one derivative, and calculate the chiral corrections to the PV Yukawa  $NN\pi$  coupling constant  $h_\pi$  through  $\mathcal{O}(1/\Lambda_\chi^3)$  in the leading order of heavy baryon expansion. We discuss the relationship between the renormalized  $h_\pi$ , the measured value of  $h_\pi$ , and the corresponding quantity calculated microscopically from the Standard Model four-quark PV interaction.

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## I. INTRODUCTION

The parity-violating (PV) nucleon-nucleon interaction has been a subject of interest in nuclear and particle physics for some time. To date, PV observables generated by this interaction remain the only experimental windows on the  $\Delta S = 0$ , nonleptonic weak interaction. Since the 1970's, the PV NN interaction has been studied in a variety of processes, including  $\vec{p}-p$  and  $\vec{p}$ -nucleus scattering,  $\gamma$ -decays of light nuclei, the scattering of epithermal neutrons from heavy nuclei, and atomic PV (for a review, see Refs. [1,2]. The on-going interest in the subject has spawned new PV experiments in few-body systems, including high-energy  $\vec{p}-p$  scattering at COSY,  $\vec{n}+p \rightarrow d+\gamma$  at LANSCE [3],  $\gamma+d \rightarrow n+p$  at JLab [4], and the rotation of polarized neutrons in helium at NIST.

The theoretical analysis of these PV observables is complicated by the short range of the low-energy weak interaction. The Compton wavelength of the weak gauge bosons ( $\sim 0.002$  fm) implies that direct  $W^\pm$  and  $Z$  exchange between nucleons is highly suppressed by the short-range repulsive core of the strong NN interaction. In the conventional framework, longer range PV effects arise from the exchange of light mesons between nucleons. One requires the exchange of the  $\pi$ ,  $\rho$  and  $\omega$  in order to saturate the seven spin-isospin channels associated with the quantum numbers of the underlying four-quark strangeness-conserving PV interaction,  $\mathcal{H}_w^{PV}(\Delta S = 0)$  (henceforth, the  $\Delta S = 0$  will be understood). These exchanges are parameterized by PV meson-nucleon couplings,  $h_M$ , whose values may be extracted from experiment. At present, there appear to be discrepancies between the values extracted from different experiments. In particular, the values of the isovector  $\pi NN$  coupling,  $h_\pi$ , and the isoscalar  $\rho NN$  coupling,  $h_\rho^0$  – as extracted from  $\vec{p}-p$  scattering and the  $\gamma$ -decay of  $^{18}\text{F}$ , do not appear to agree with the corresponding values implied by the anapole moment of  $^{133}\text{Cs}$  as measured in atomic PV [5].

The origin of this discrepancy is not understood. One possibility is that the use of  $\rho$  and  $\omega$  exchange to describe the short-range part of the PV NN interaction is inadequate. An alternate approach, using effective field theory (EFT), involves an expansion of the short-range PV NN interaction in a series of four-nucleon contact interactions whose coefficients are *a priori* unknown but in principle could be determined from experiment. The use of  $\rho$  and  $\omega$  exchange amounts to adoption of a model – rather than the use of experiment – to determine the coefficients of the higher-derivative operators in this expansion. Whether or not the application of EFT to nuclear PV can yield a more self-consistent set of PV low-energy constants than the meson-exchange approach remains to be seen. A comprehensive analysis of nuclear PV observables using EFT has yet to be performed.

The least ambiguous element – shared by both approaches – involves the long-range  $\pi$ -exchange interaction. At leading order in the derivative expansion, the PV  $\pi NN$  interaction is a purely isovector, Yukawa interaction. The strength of this interaction is characterized by the same constant –  $h_\pi$  – in both the EFT and meson-exchange approaches. At the level of the Standard Model (SM),  $h_\pi$  is particularly sensitive to the neutral current component of  $\mathcal{H}_w^{PV}$ . In this respect, the result of  $^{18}\text{F}$  PV  $\gamma$ -decay measurement is puzzling:

$$h_\pi = (0.73 \pm 2.3)g_\pi \quad , \quad (1)$$

where  $g_\pi = 3.8 \times 10^{-8}$  gives the scale of the  $h_M$  in the absence of neutral currents [6]. This result is especially significant, since the relevant two-body nuclear parity-mixing matrix element can be obtained by isospin symmetry from the  $\beta$ -decay of  $^{18}\text{Ne}$  [2]. The result in Eq. (1) is, thus, relatively insensitive to the nuclear model. Theoretical calculations of  $h_\pi$  starting from  $\mathcal{H}_W^{PV}$  have been performed using  $\text{SU}(6)_w$  symmetry and the quark model [7,8], the Skyrme model [9], and QCD sum rules [10]. As a benchmark for comparison with experiment, we refer the  $\text{SU}(6)_w$ /quark model analysis of Refs. [7,8]—hereafter referred to as DDH,FCDH\*. These authors quote a “best value” and “reasonable range” for the  $h_M$ :

$$h_\pi(\text{best}) = 7g_\pi \quad (2)$$

$$h_\pi(\text{range}) : (0 \rightarrow 30)g_\pi \quad . \quad (3)$$

where here the “best value” is more aptly described as an educated guess, while the “reasonable range” indicates a set of numbers such that theory would be very hard-pressed to explain were the experimental value not found to be within this band. Nevertheless, the difference between the “best value” of Eq. (2) and the  $^{18}\text{F}$  result would appear to call for an explanation, and in the following note we comment on a possible source of the discrepancy.

In general, the problem of relating the fundamental weak quark-quark interaction to the low-energy constants which parameterize hadronic matrix elements of that interaction is non-trivial. In the framework of EFT, one may define these constants at tree-level in the hadronic effective theory. The quantities extracted from experiment in the conventional analysis, however, are not the tree-level parameters, but rather renormalized couplings. Denoting the latter as  $h_\pi^{EFF}$ , one has

$$h_\pi^{EFF} = Z_N \sqrt{Z_\pi} h_\pi^{BARE} + \Delta h_\pi \quad , \quad (4)$$

where  $h_\pi^{BARE}$  is the coefficient of the leading-order, PV Yukawa interaction in the effective theory,  $\sqrt{Z_N}$  and  $\sqrt{Z_\pi}$  denotes chiral loop renormalizations of the nucleon and pion wavefunctions, respectively, and  $\Delta h_\pi$  denotes contributions from chiral loops and higher-dimension operators to the Yukawa interactions (only the finite parts of these couplings are implied; loop divergences are cancelled by the corresponding pole terms in  $h_\pi^{BARE}$  and the  $Z_{N,\pi}$ ). At leading order in  $1/\Lambda_\chi$ , one has  $Z_{N,\pi} = 1$ ,  $\Delta h_\pi = 0$ , and  $h_\pi^{EFF} = h_\pi^{BARE}$ . The renormalized coupling appears as the coefficient in the one-pion-exchange (OPE) PV NN potential

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\*Note that although the DDH analysis used the symmetry group  $\text{SU}(6)_w$  in order to connect weak vector meson and pion couplings the predictions relating pion couplings alone to hyperon decay data rely only  $\text{SU}(3)$ .

$$\hat{H}_{PV}^{OPE} = i \frac{g_{NN\pi} h_{\pi}^{EFF}}{\sqrt{2}} \left( \frac{\tau_1 \times \tau_2}{2} \right)_z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[ \frac{\vec{p}_1 - \vec{p}_2}{2m_N}, f_{\pi}(r) \right] , \quad (5)$$

where  $g_{NN\pi}$  is the strong  $\pi NN$  coupling and  $f_{\pi}(r) = \exp(-m_{\pi}r)/4\pi r$ . Neglecting the effects of three-body PV forces and  $2\pi$ -exchange interactions, it is  $h_{\pi}^{EFF}$  to which the result in Eq. (1) corresponds.

The relationship between  $h_{\pi}^{EFF}$  and the coupling obtained by computing  $\langle N\pi | \mathcal{H}_W^{PV} | N \rangle$  in a microscopic model is not immediately transparent. In what follows, we make several observations about this relationship. We first show that  $Z_N \sqrt{Z_{\pi}}$  and  $\Delta h_{\pi}$  are substantial, so that  $h_{\pi}^{EFF}$  differs significantly from  $h_{\pi}^{BARE}$ . To that end, we compute all of the chiral corrections to the PV Yukawa interaction through  $\mathcal{O}(1/\Lambda_{\chi}^3)$ , where  $\Lambda_{\chi} = 4\pi F_{\pi}$ . We work to leading order in  $1/m_N$  in heavy baryon chiral perturbation theory (HBChPT). Of particular significance is the dependence of  $\Delta h_{\pi}$  on other low-energy constants parameterizing PV  $2\pi$  production and the PV  $N \rightarrow \pi\pi\Delta$  transition. We subsequently reexamine the  $SU(6)_w$ /quark model calculation of Refs. [7,8] and argue that most – if not all – of the chiral loop effects which renormalize  $h_{\pi}$  are not included in the microscopic calculation. Thus, the relationship between  $h_{\pi}^{EFF}$  and microscopic calculations remains ambiguous at best. This ambiguity is unlikely to be resolved until an unquenched lattice QCD calculation of  $h_{\pi}$  using light quarks becomes tenable. In the meantime, one should not necessarily view a discrepancy between the experimental value of  $h_{\pi}^{EFF}$  and microscopic model calculations as disturbing.

Our discussion of these observations is organized as follows. In Section 2 we summarize our conventions and notation, including the PV chiral Lagrangians relevant to  $h_{\pi}$  renormalization. Section 3 gives a discussion of the loop calculations. In Section 4 we comment on the scale of the loop corrections and provide simple estimates of some of the new PV low-energy constants appearing in the analysis. Section 5 gives our discussion of the relationship between  $h_{\pi}^{EFF}$  and the calculation of Refs. [7,8]. Section 6 summarizes our conclusions. Some technical details are relegated to the Appendices.

## II. NOTATIONS AND CONVENTIONS

We follow standard HBChPT conventions [11,12] and introduce

$$\Sigma = \xi^2 , \quad \xi = \exp\left(\frac{i\pi}{F_{\pi}}\right) , \quad \pi = \frac{1}{2}\pi^a \tau^a \quad (6)$$

with  $F_{\pi} = 92.4$  MeV being the pion decay constant. The chiral vector and axial vector currents are given by

$$\mathcal{D}_{\mu} = D_{\mu} + V_{\mu}$$

$$A_{\mu} = -\frac{i}{2}(\xi D_{\mu}\xi^{\dagger} - \xi^{\dagger} D_{\mu}\xi) = -\frac{D_{\mu}\pi}{F_{\pi}} + O(\pi^3) \quad (7)$$

$$V_{\mu} = \frac{1}{2}(\xi D_{\mu}\xi^{\dagger} + \xi^{\dagger} D_{\mu}\xi) . \quad (8)$$

For the  $\Delta$ , we use the isospurion formalism [13], treating the  $\Delta$  field  $T_\mu^i(x)$  as a vector spinor in both spin and isospin space with the constraint  $\tau^i T_\mu^i(x) = 0$ . The components of this field are

$$T_\mu^3 = -\sqrt{\frac{2}{3}} \begin{pmatrix} \Delta^+ \\ \Delta^0 \end{pmatrix}_\mu, \quad T_\mu^+ = \begin{pmatrix} \Delta^{++} \\ \Delta^+/\sqrt{3} \end{pmatrix}_\mu, \quad T_\mu^- = -\begin{pmatrix} \Delta^0/\sqrt{3} \\ \Delta^- \end{pmatrix}_\mu. \quad (9)$$

The field  $T_\mu^i$  also satisfies the constraints for the ordinary Schwinger-Rarita spin- $\frac{3}{2}$  field,

$$\gamma^\mu T_\mu^i = 0 \quad \text{and} \quad p^\mu T_\mu^i = 0. \quad (10)$$

We eventually convert to the heavy baryon expansion, in which case the latter constraint becomes  $v^\mu T_\mu^i = 0$  with  $v_\mu$  the heavy baryon velocity.

The relativistic parity-conserving (PC) Lagrangian for  $\pi$ ,  $N$ ,  $\Delta$  interactions needed here is

$$\begin{aligned} \mathcal{L}^{PC} = & \frac{F_\pi^2}{4} T \bar{r} D^\mu \Sigma D_\mu \Sigma^\dagger + \bar{N} (i \mathcal{D}_\mu \gamma^\mu - m_N) N + g_A \bar{N} A_\mu \gamma^\mu \gamma_5 N \\ & - T_i^\mu [(i \mathcal{D}_\alpha^{ij} \gamma^\alpha - m_\Delta \delta^{ij}) g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma^\lambda (i \mathcal{D}_\alpha^{ij} \gamma^\alpha - m_\Delta \delta^{ij}) \gamma_\lambda \gamma^\nu \\ & + \frac{g_1}{2} g_{\mu\nu} A_\alpha^{ij} \gamma^\alpha \gamma_5 + \frac{g_2}{2} (\gamma_\mu A_\nu^{ij} + A_\mu^{ij} \gamma_\nu) \gamma_5 + \frac{g_3}{2} \gamma_\mu A_\alpha^{ij} \gamma^\alpha \gamma_5 \gamma_\nu] T_j^\nu \\ & + g_{\pi N \Delta} [\bar{T}_i^\mu (g_{\mu\nu} + z_0 \gamma_\mu \gamma_\nu) \omega_i^\nu N + h.c.] \quad , \end{aligned} \quad (11)$$

where  $\omega_\mu^i = \text{tr}[\tau^i A_\mu]/2$  while  $D_\mu$  and  $\mathcal{D}_\mu$  are the gauge and chiral covariant derivatives, respectively. Explicit expressions for the fields and the transformation properties can be found in [14]. Here,  $z_0$  is an off-shell parameter, which is not relevant in the present work [13].

In order to obtain proper chiral counting for the nucleon, we employ the conventional heavy baryon expansion of  $\mathcal{L}^{PC}$ , and in order to consistently include the  $\Delta$  we follow the small scale expansion proposed in [13]. In this approach energy-momenta and the delta and nucleon mass difference  $\delta$  are both treated as small expansion parameters in chiral power counting. The leading order vertices in this framework can be obtained via  $P_+ \Gamma P_+$  where  $\Gamma$  is the original vertex in the relativistic Lagrangian and

$$P_\pm = \frac{1 \pm \not{v}}{2}. \quad (12)$$

are projection operators for the large, small components of the Dirac wavefunction respectively. We collect some of the relevant terms below:

$$\begin{aligned} \mathcal{L}_v^{PC} = & \bar{N} [i v \cdot D + 2g_A S \cdot A] N - i \bar{T}_i^\mu [i v \cdot D^{ij} - \delta^{ij} \delta + g_1 S \cdot A^{ij}] T_\mu^j \\ & + g_{\pi N \Delta} [\bar{T}_i^\mu \omega_\mu^i N + \bar{N} \omega_\mu^{i\dagger} T_i^\mu] \end{aligned} \quad (13)$$

where  $S_\mu$  is the Pauli-Lubanski spin operator and  $\delta \equiv m_\Delta - m_N$ .

The PV analog of Eq. (11) can be constructed using the chiral fields  $X_{L,R}^a$  defined as [15]:

$$X_L^a = \xi^\dagger \tau^a \xi, \quad X_R^a = \xi \tau^a \xi^\dagger, \quad X_\pm^a = X_L^a \pm X_R^a. \quad (14)$$

We find it convenient to follow the convention in Ref. [15] and separate the PV Lagrangian into its various isospin components.

The hadronic weak interaction has the form

$$\mathcal{H}_w = \frac{G_\mu}{\sqrt{2}} J_\lambda J^\lambda \dagger + \text{h.c.}, \quad (15)$$

where  $J_\lambda$  denotes either a charged or neutral weak current built out of quarks. In the Standard Model, the strangeness conserving charged currents are pure isovector, whereas the neutral currents contain both isovector and isoscalar components. Consequently,  $\mathcal{H}_w$  contains  $\Delta I = 0, 1, 2$  pieces and these channels must all be accounted for in any realistic hadronic effective theory.

We quote here the relativistic Lagrangians, but employ the heavy baryon projections, as described above, in computing loops. It is straightforward to obtain the corresponding heavy baryon Lagrangians from those listed below, so we do not list the PV heavy baryon terms below. For the  $\pi N$  sector we have

$$\mathcal{L}_{\Delta I=0}^{\pi N} = h_V^0 \bar{N} A_\mu \gamma^\mu N \quad (16)$$

$$\begin{aligned} \mathcal{L}_{\Delta I=1}^{\pi N} = & \frac{h_V^1}{2} \bar{N} \gamma^\mu N \text{Tr}(A_\mu X_+^3) - \frac{h_A^1}{2} \bar{N} \gamma^\mu \gamma_5 N \text{Tr}(A_\mu X_-^3) \\ & - \frac{h_\pi}{2\sqrt{2}} F_\pi \bar{N} X_-^3 N \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{L}_{\Delta I=2}^{\pi N} = & h_V^2 \mathcal{I}^{ab} \bar{N} [X_R^a A_\mu X_R^b + X_L^a A_\mu X_L^b] \gamma^\mu N \\ & - \frac{h_A^2}{2} \mathcal{I}^{ab} \bar{N} [X_R^a A_\mu X_R^b - X_L^a A_\mu X_L^b] \gamma^\mu \gamma_5 N, \end{aligned} \quad (18)$$

where  $\mathcal{I}^{ab}$  is a matrix coupling the  $X^{a,b}$  to  $I = 2, I_3 = 0$ . The above Lagrangian was first given by Kaplan and Savage (KS) [15]. However, the coefficients used in our work are slightly different from those of Ref. [15] since our definition of  $A_\mu$  differs by an overall phase.

The term proportional to  $h_\pi$  contains no derivatives. At leading-order in  $1/F_\pi$ , it yields the PV  $NN\pi$  Yukawa coupling traditionally used in meson-exchange models for the PV NN interaction [7,2]. Unlike the PV Yukawa interaction, the vector and axial vector terms in Eqs. (16-18) contain derivative interactions. The terms containing  $h_A^1$  and  $h_A^2$  start off with  $NN\pi\pi$  interactions, while all the other terms start off as  $NN\pi$ . Such derivative interactions have not been included in conventional analyses of nuclear and hadronic PV experiments. Consequently, the experimental constraints on the low-energy constants  $h_V^i, h_A^i$  are unknown.

It is useful to list the first few terms obtained by expanding the Lagrangians in Eqs. (16-18) in  $1/F_\pi$ . For the present purposes, the following terms are needed:

$$\mathcal{L}_{\text{Yukawa}}^{\pi NN} = -ih_\pi(\bar{p}n\pi^+ - \bar{n}p\pi^-)[1 - \frac{1}{3F_\pi^2}(\pi^+\pi^- + \frac{1}{2}\pi^0\pi^0)] \quad (19)$$

$$\mathcal{L}_V^{\pi NN} = -\frac{h_V^0 + 4/3h_V^2}{\sqrt{2}F_\pi}[\bar{p}\gamma^\mu n D_\mu\pi^+ + \bar{n}\gamma^\mu p D_\mu\pi^-] \quad (20)$$

$$\begin{aligned} \mathcal{L}_A^{\pi NN} = & i\frac{h_A^1 + h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 p(\pi^+ D_\mu\pi^- - \pi^- D_\mu\pi^+) \\ & + i\frac{h_A^1 - h_A^2}{F_\pi^2}\bar{n}\gamma^\mu\gamma_5 n(\pi^+ D_\mu\pi^- - \pi^- D_\mu\pi^+) \\ & + i\frac{\sqrt{2}h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 n\pi^+ D_\mu\pi^0 - i\frac{\sqrt{2}h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 n\pi^+ D_\mu\pi^0. \end{aligned} \quad (21)$$

For the PV  $\pi NN$  Yukawa coupling we have also kept terms with three pions.

The corresponding PV Lagrangians involving a  $N \rightarrow \Delta$  transition are somewhat more complicated. We relegate the complete expressions to Appendix A, and give here only the leading terms required for our calculation. As noted in Ref. [14], the one-pion  $\pi N\Delta$  PV Lagrangian vanishes at leading order in the heavy baryon expansion. The two-pion terms are

$$\begin{aligned} \mathcal{L}_A^{\pi N\Delta} = & -\frac{ih_A^{p\Delta^{++}\pi^-\pi^0}}{F_\pi^2}\bar{p}\Delta_\mu^{++}D^\mu\pi^-\pi^0 - \frac{ih_A^{p\Delta^{++}\pi^0\pi^-}}{F_\pi^2}\bar{p}\Delta_\mu^{++}D^\mu\pi^0\pi^- \\ & -\frac{ih_A^{p\Delta^{++}\pi^0\pi^0}}{F_\pi^2}\bar{p}\Delta_\mu^{++}D^\mu\pi^0\pi^0 - \frac{ih_A^{p\Delta^{++}\pi^+\pi^-}}{F_\pi^2}\bar{p}\Delta_\mu^{++}D^\mu\pi^+\pi^- \\ & -\frac{ih_A^{p\Delta^{++}\pi^-\pi^+}}{F_\pi^2}\bar{p}\Delta_\mu^{++}D^\mu\pi^-\pi^+ - \frac{ih_A^{p\Delta^0\pi^+\pi^0}}{F_\pi^2}\bar{p}\Delta_\mu^0D^\mu\pi^+\pi^0 \\ & -\frac{ih_A^{p\Delta^0\pi^0\pi^+}}{F_\pi^2}\bar{p}\Delta_\mu^0D^\mu\pi^0\pi^+ - \frac{ih_A^{p\Delta^-\pi^+\pi^+}}{F_\pi^2}\bar{p}\Delta_\mu^-D^\mu\pi^+\pi^+ \\ & -\frac{ih_A^{n\Delta^{++}\pi^-\pi^-}}{F_\pi^2}\bar{n}\Delta_\mu^{++}D^\mu\pi^-\pi^- - \frac{ih_A^{n\Delta^{++}\pi^-\pi^0}}{F_\pi^2}\bar{n}\Delta_\mu^{++}D^\mu\pi^-\pi^0 \\ & -\frac{ih_A^{n\Delta^{++}\pi^0\pi^-}}{F_\pi^2}\bar{n}\Delta_\mu^{++}D^\mu\pi^0\pi^- - \frac{ih_A^{n\Delta^0\pi^0\pi^0}}{F_\pi^2}\bar{n}\Delta_\mu^0D^\mu\pi^0\pi^0 \\ & -\frac{ih_A^{n\Delta^0\pi^+\pi^-}}{F_\pi^2}\bar{n}\Delta_\mu^0D^\mu\pi^+\pi^- - \frac{ih_A^{n\Delta^0\pi^-\pi^+}}{F_\pi^2}\bar{n}\Delta_\mu^0D^\mu\pi^-\pi^+ \\ & -\frac{ih_A^{n\Delta^-\pi^+\pi^0}}{F_\pi^2}\bar{n}\Delta_\mu^-D^\mu\pi^+\pi^0 - \frac{ih_A^{n\Delta^-\pi^0\pi^+}}{F_\pi^2}\bar{n}\Delta_\mu^-D^\mu\pi^0\pi^+ + \text{h.c.}, \end{aligned} \quad (22)$$

where the couplings  $h_A^{p\Delta^{++}\pi^-\pi^0}$  are defined in terms of the various SU(2) PV low-energy constants in Appendix A.



The PV  $\pi\Delta\Delta$  Lagrangians, also listed in Appendix A, contain terms analogous to the Yukawa,  $V$ , and  $A$  terms in Eqs. (16-18). Since we compute corrections up to one-loop order only, and since the initial and final states are nucleons, the PV  $\pi\pi\Delta\Delta$  terms ( $A$ -type) are not relevant here. The leading, single- $\pi$  Yukawa and  $V$ -type interactions are

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}}^{\pi\Delta\Delta} = & -i\frac{h_\Delta}{\sqrt{3}}(\bar{\Delta}^{++}\Delta^+\pi^+ - \bar{\Delta}^+\Delta^{++}\pi^-) \\ & -i\frac{h_\Delta}{\sqrt{3}}(\bar{\Delta}^0\Delta^-\pi^+ - \bar{\Delta}^-\Delta^0\pi^-) \\ & -i\frac{2h_\Delta}{3}(\bar{\Delta}^+\Delta^0\pi^+ - \bar{\Delta}^0\Delta^+\pi^-)\end{aligned}\quad (23)$$

$$\begin{aligned}\mathcal{L}_V^{\pi\Delta\Delta} = & -\frac{h_V^{\Delta^{++}\Delta^+}}{F_\pi}(\bar{\Delta}^{++}\gamma_\mu\Delta^+D^\mu\pi^+ + \bar{\Delta}^+\gamma_\mu\Delta^{++}D^\mu\pi^-) \\ & -\frac{h_V^{\Delta^+\Delta^0}}{F_\pi}(\bar{\Delta}^+\gamma_\mu\Delta^0D^\mu\pi^+ + \bar{\Delta}^0\gamma_\mu\Delta^+D^\mu\pi^-) \\ & -\frac{h_V^{\Delta^0\Delta^-}}{F_\pi}(\bar{\Delta}^0\gamma_\mu\Delta^-D^\mu\pi^+ + \bar{\Delta}^-\gamma_\mu\Delta^0D^\mu\pi^-)\end{aligned}\quad (24)$$

where the coefficients are given in Appendix A.

One may ask whether there exist additional PV effective interactions that could contribute at the order to which we work. In the pionic sector there exists one CP-conserving, PV Lagrangian:

$$\mathcal{L}_\pi^{\text{PV}} = \epsilon_{ijk}\omega_\mu^i\omega_\nu^j(D^\mu\omega_k^\nu - D^\nu\omega_k^\mu) \quad . \quad (25)$$

At leading order in  $1/F_\pi$ ,  $\mathcal{L}_\pi$  contains five pions. Its lowest order contribution appears at two-loop order at best, so we do not consider it here.

Similarly, one may consider possible contributions from two-derivative operators. There exists one CP-conserving, PV operator:

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[D_\mu A_\nu - D_\nu A_\mu]N \quad . \quad (26)$$

There exist three independent PC, two-derivative operators [16]. For example, one may choose the following three:

$$\frac{1}{\Lambda_\chi}\bar{N}i\gamma_5 D_\mu A^\mu N \quad , \quad (27)$$

$$\frac{1}{\Lambda_\chi}\bar{N}A^\mu A_\mu N \quad , \quad (28)$$

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[A_\mu, A_\nu]N \quad . \quad (29)$$

As we discuss in Appendix B, none of the two-derivative operators in Eqs. (26) - (29) contribute to the renormalization of  $h_\pi$  at the order to which we work in the present analysis.

### III. THE LOOP CORRECTIONS

The leading order loop corrections to the Yukawa interaction of Eq. (19) are generated by the diagrams of Figs. 1-2. As we discuss in Appendix B, the contributions from many of the diagrams which nominally renormalize  $h_\pi$  vanish at the order at which we truncate. In particular, none of the vector ( $V$ -type)  $\pi NN$  and  $\pi\Delta\Delta$  terms contribute to this order. In what follows, we discuss only the non-vanishing Yukawa and  $A$ -type contributions. Details regarding the vanishing of the other contributions appear in Appendix B. Following the conventional practice, we regulate the loop integrals using dimensional regularization. The pole terms proportional to  $1/D - 4$  are cancelled by appropriate counterterms. We identify only the terms nonanalytic in quark masses with the loops. All other analytic terms are indistinguishable from finite parts of the corresponding counterterms.

The nonvanishing contribution from Fig. 1(a) arises from the insertion of the  $3\pi$  part of the Yukawa interaction of Eq. (17). The nonanalytic term is

$$iM_{(a)} = \frac{5}{6} \frac{m_\pi^2}{\Lambda_\chi^2} \ln\left(\frac{\mu}{m_\pi}\right)^2 h_\pi \tau^+ , \quad (30)$$

where  $\Lambda_\chi = 4\pi F_\pi$  and  $\mu$  is the subtraction scale introduced in dimensional regularization. For simplicity, we show here only the contributions for  $n \rightarrow p\pi^-$ . The terms for  $p \rightarrow n\pi^+$  are equal in magnitude and opposite in sign since it is the hermitian conjugate of the  $n \rightarrow p\pi^-$  piece. This property holds to all orders of chiral expansion.

The nonvanishing contribution from Fig. 1 (b) arises from strong vertex correction to the leading order  $\pi NN$  Yukawa interaction:

$$iM_{(b)} = \frac{3}{4} g_A^2 \frac{m_\pi^2}{\Lambda_\chi^2} \ln\left(\frac{\mu}{m_\pi}\right)^2 h_\pi \tau^+ . \quad (31)$$

The terms in Figs. 1(c1)-(c2) are generated by the PV axial  $\pi\pi NN$  couplings proportional to the  $h_A^i$ . We have

$$iM_{(c1)+(c2)} = 2\sqrt{2}\pi g_A \frac{m_\pi^3}{F_\pi \Lambda_\chi^2} h_A^1 \tau^+ . \quad (32)$$

The contribution from  $h_A^2$  to these two diagrams cancels out, leaving only the dependence on  $h_A^1$ . We note that although this term is proportional to  $m_q^{3/2}$  and, thus, nominally suppressed, the coefficient of  $h_A^1$  is fortuitously large ( $\sim 1/4$ ). The two pion vertex in FIG. 1 (d1)-(d2) comes from the chiral connection  $V_\mu$ :

$$iM_{(d1)+(d2)} = -\frac{1}{2} \frac{m_\pi^2}{\Lambda_\chi^2} \ln\left(\frac{\mu}{m_\pi}\right)^2 h_\pi \tau^+ . \quad (33)$$

The leading contribution involving  $\Delta$  intermediate states arises from Fig. 2(a). The corresponding amplitude receives contributions from three different isospin combinations for the  $\Delta$  intermediate states. Their sum reads

$$iM_{2(a)} = -\frac{20}{9} \frac{g_{\pi N \Delta}^2 h_{\Delta}}{\Lambda_{\chi}^2} [(2\delta^2 - m_{\pi}^2) \ln(\frac{\mu}{m_{\pi}})^2 - 4\delta\sqrt{\delta^2 - m_{\pi}^2} \ln \frac{\delta + \sqrt{\delta^2 - m_{\pi}^2}}{m_{\pi}}] \tau^+ . \quad (34)$$

The corrections generated by the PV  $\pi\pi N\Delta$  vertices are

$$iM_{2(b1)+2(b2)} = \frac{2}{3} \frac{g_{\pi N \Delta}}{F_{\pi} \Lambda_{\chi}^2} [(\delta^2 - \frac{3}{2}m_{\pi}^2)\delta \ln(\frac{\mu}{m_{\pi}})^2 - 2(\delta^2 - m_{\pi}^2)^{3/2} \ln \frac{\delta + \sqrt{\delta^2 - m_{\pi}^2}}{m_{\pi}}] h_A^{\Delta} \tau^+ , \quad (35)$$

where  $h_A^{\Delta}$  is defined as

$$h_A^{\Delta} = \frac{1}{\sqrt{3}} (h_A^{n\Delta^0\pi^+\pi^-} + h_A^{p\Delta^+\pi^-\pi^+}) + \sqrt{\frac{2}{3}} (h_A^{n\Delta^+\pi^0\pi^-} - h_A^{p\Delta^0\pi^0\pi^+}) - h_A^{n\Delta^{++}\pi^-\pi^-} - h_A^{p\Delta^-\pi^+\pi^+} . \quad (36)$$

Summing all the non-vanishing loop contributions yields the following expression for  $\Delta h_{\pi}$ :

$$\begin{aligned} \Delta h_{\pi} &= \frac{1}{3} \frac{m_{\pi}^2}{\Lambda_{\chi}^2} \ln(\frac{\mu}{m_{\pi}})^2 h_{\pi} + \frac{3}{4} g_A^2 \frac{m_{\pi}^2}{\Lambda_{\chi}^2} \ln(\frac{\mu}{m_{\pi}})^2 h_{\pi} + 2\sqrt{2}\pi g_A \frac{m_{\pi}^3}{F_{\pi} \Lambda_{\chi}^2} h_A^1 \\ &\quad - \frac{20}{9} \frac{g_{\pi N \Delta}^2 h_{\Delta}}{\Lambda_{\chi}^2} [(2\delta^2 - m_{\pi}^2) \ln(\frac{\mu}{m_{\pi}})^2 - 4\delta\sqrt{\delta^2 - m_{\pi}^2} \ln \frac{\delta + \sqrt{\delta^2 - m_{\pi}^2}}{m_{\pi}}] \\ &\quad + \frac{2}{3} \frac{g_{\pi N \Delta}}{F_{\pi} \Lambda_{\chi}^2} [(\delta^2 - \frac{3}{2}m_{\pi}^2)\delta \ln(\frac{\mu}{m_{\pi}})^2 - 2(\delta^2 - m_{\pi}^2)^{3/2} \ln \frac{\delta + \sqrt{\delta^2 - m_{\pi}^2}}{m_{\pi}}] h_A^{\Delta} \end{aligned} \quad (37)$$

The final nonvanishing corrections arise from  $N$  and  $\pi$  wavefunction renormalization. These corrections, which have been computed previously [17], generate deviations from unity of  $Z_N$  and  $\sqrt{Z_{\pi}}$  appearing in the expression for  $h_{\pi}^{EFF}$  in Eq. (4). In the case of  $Z_N$ , the nonvanishing contributions arise from Figs. 1(e1)-(e2) and 2(c1)-(c2):

$$\begin{aligned} Z_N - 1 &= \frac{9}{4} g_A^2 \frac{m_{\pi}^2}{\Lambda_{\chi}^2} \ln(\frac{\mu}{m_{\pi}})^2 - 4g_{\pi N \Delta}^2 [\frac{2\delta^2 - m_{\pi}^2}{\Lambda_{\chi}^2} \ln(\frac{\mu}{m_{\pi}})^2 \\ &\quad - 4\frac{\delta\sqrt{\delta^2 - m_{\pi}^2}}{\Lambda_{\chi}^2} \ln \frac{\delta + \sqrt{\delta^2 - m_{\pi}^2}}{m_{\pi}}] . \end{aligned} \quad (38)$$

The pion's wavefunction renormalization arises from Fig. 2 (k) [18]:

$$\sqrt{Z_{\pi}} - 1 = -\frac{1}{3} \left( \frac{m_{\pi}}{\Lambda_{\chi}} \right)^2 \ln \left( \frac{\mu}{m_{\pi}} \right)^2 . \quad (39)$$

Numerically, the loop contributions to  $\sqrt{Z_{\pi}}$  are small compared to those entering  $Z_N$ .

Note the one loop renormalization of  $h_\pi$  from the PV Yukawa  $\pi NN$  and  $\pi\Delta\Delta$  vertices is already at the order  $1/\Lambda_\chi^2$ . An additional loop will introduce a factor of  $1/\Lambda_\chi^2$ . Loops containing the axial vector  $NN\pi\pi$  and  $N\Delta\pi\pi$  vertices and one strong  $NN\pi$  or  $N\Delta\pi$  vertex are of  $\mathcal{O}(1/\Lambda_\chi^2 F_\pi)$ . To obtain contributions of  $\mathcal{O}(1/\Lambda_\chi^3)$ , one would require the insertion of operators carrying explicit factors of  $1/\Lambda_\chi$  into one loop graphs. We find no such contributions.

#### IV. THE SCALE OF LOOP CORRECTIONS

We may estimate the numerical importance of the loop corrections to  $h_\pi^{BARE}$  by taking  $\delta = 0.3$  GeV,  $g_A = 1.267$  [19] and  $g_{\pi N\Delta} = 1.05$  [13] and by choosing  $\mu = \Lambda_\chi = 1.16$  GeV<sup>†</sup>. With these inputs, the value of  $Z_N\sqrt{Z_\pi}$  is completely determined. The vertex corrections, which appear as  $\Delta h_\pi$  in Eq. (4), depend on the PV couplings  $h_\pi$ ,  $h_A^1$ ,  $h_\Delta$ , and  $h_A^\Delta$ . We obtain

$$h_\pi^{EFF} = 0.5h_\pi + 0.25h_A^1 - 0.24h_\Delta + 0.079h_A^\Delta. \quad (40)$$

Note that the effect of the wavefunction renormalization corrections is to reduce the dependence on  $h_\pi^{BARE}$  by roughly 50%. In addition, the dependence of  $h_\pi^{EFF}$  on  $h_A^1$  and  $h_\Delta$  is non-negligible. Their coefficients are only a factor of two smaller than that of  $h_\pi^{BARE}$ . Although these contributions arise at  $\mathcal{O}(p^2, p^3)$ , they are fortuitously enhanced numerically. Thus, in a complete analysis of the OPE PV interaction one should not ignore these constants.

At present, one has no direct experimental constraints on the parameters  $h_A^1$ ,  $h_\Delta$ , and  $h_A^\Delta$ , as a comprehensive analysis of hadronic PV data including the full chiral structure of the PV hadronic interaction has yet to be performed. Consequently, one must rely on theoretical input for guidance regarding the scale of the unknown constants. Estimates of  $h_A^1$  are given by the authors of Ref. [15]. These authors observe that the usual pole dominance approximation for P-wave non-leptonic hyperon decays typically underpredicts the experimental amplitudes by a factor of two. The difference may be resolved by the inclusion of local, parity-conserving operators having structures analogous to the  $A$ -type terms in Eq. (17). The requisite size of the  $\Delta S = 1$  contact terms may imply a scale for the analogous  $\Delta I = 1$  PV terms. If so, one might conclude that  $h_A^1$  should be on the order of  $10g_\pi$ . On the other hand, a simple factorization estimate leads to  $h_A^1 \sim 0.2g_\pi$ . While the sign of  $h_A^1$  is fixed in the factorization approximation, the sign of the larger value is undetermined. Thus, it is reasonable to conclude that  $h_A^1$  may be large enough to significantly impact  $h_\pi^{EFF}$ , though considerably more analysis is needed to yield a firm conclusion.

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<sup>†</sup>Since the dependence on  $\mu$  is logarithmic, one may choose other values, such as  $\mu = m_\rho$ , without affecting the numerical results significantly

The  $\pi\Delta\Delta$  Yukawa coupling  $h_\Delta$  has been estimated in Ref. [8] using methods similar to those of Ref. [7]. The authors quote a “best value” of  $h_\Delta = -20g_\pi$ , with a “reasonable range” of  $(-51 \rightarrow 0) \times g_\pi$ .<sup>‡</sup> Naïvely, substitution of the best value into Eq. (40) would increase the value of  $h_\pi^{EFF}$ , whereas the  $^{18}\text{F}$  result would seem to require a reduction. As we argue below, however, the relationship between the couplings computed in Refs. [7,8] and the parameters appearing in Eq. (40) is somewhat ambiguous. Direct substitution of the theoretical value into  $h_\pi^{EFF}$  may not be entirely appropriate.

To date, no theoretical estimate of the  $A$ -type  $\pi\pi N\Delta$  coupling has been performed. A simple estimate of the scale is readily obtained using the factorization approximation. To that end, we work with tree-level form of  $\mathcal{H}_W^{PV}$ . Neglecting short-distance QCD corrections and terms containing strange quarks, one has

$$\mathcal{H}_W^{PV}(\Delta S = 0) = \frac{G_F}{\sqrt{2}} \{ \cos^2 \theta_c \bar{u} \gamma_\lambda (1 - \gamma_5) d d \bar{\gamma}^\lambda (1 - \gamma_5) u \quad (41)$$

$$-2(1 - 2\sin^2 \theta_w) V_\lambda^{(3)} A^{(3)\lambda} + \frac{4}{3} \sin^2 \theta_w V_\lambda^{(0)} A^{(3)\lambda} \} \quad , \quad (42)$$

where  $V_\lambda^{(3)}$  and  $A_\lambda^{(3)}$  denote the third components of the octet of vector and axial vector currents, respectively, and

$$V_\lambda^{(0)} = \frac{1}{2} (\bar{u} \gamma_\lambda u + \bar{d} \gamma_\lambda d) \quad . \quad (43)$$

Consider now the first term in the expression for  $h_A^\Delta$  given in Eq. (36). In the factorization approximation,  $\mathcal{H}_W^{PV}$  contributes only to the antisymmetric combination

$$\frac{1}{2} (h_A^{n\Delta^0\pi^+\pi^-} - h_A^{n\Delta^0\pi^-\pi^+}) \quad . \quad (44)$$

The neutral current contribution to this combination, which arises only from the term containing  $V_\lambda^{(3)}$ , is

$$\sqrt{2} G_F F_\pi^2 (1 - 2\sin^2 \theta_w) C_5^A(n\Delta^0) \approx 2g_\pi C_5^A(n\Delta^0) \quad , \quad (45)$$

where  $C_5^A(n\Delta^0) \sim \mathcal{O}(1)$  is the axial vector  $n \rightarrow \Delta^0$  form factor at the photon point. After Fierz re-ordering, the charged current component of  $\mathcal{H}_W^{PV}$  contributes roughly

$$- (4g_\pi/3) C_5^A(n\Delta^0), \quad (46)$$

yielding a total factorization contribution of about  $(2g_\pi/3) C_5^A(n\Delta^0)$ . Thus, one would expect the scale of the axial vector  $\pi\pi N\Delta$  couplings to be on the order of a few  $\times g_\pi$ .

In the particular case of the combination appearing in  $h_A^\Delta$ , however, the sum of factorization contributions cancels identically. As one sees from the expressions for the  $h_A^{N\Delta\pi\pi}$  given in Appendix A, isospin requires

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<sup>‡</sup>This coupling is denoted  $f_{\Delta\Delta\pi}$  in Ref. [8].

$$h_A^{n\Delta^0\pi^+\pi^-} + h_A^{p\Delta^+\pi^-\pi^+} = 0 \quad . \quad (47)$$

The factorization contributions independently satisfy this sum rule. The second combination of constants appearing in Eq. (36),

$$h_A^{n\Delta^+\pi^0\pi^-} - h_A^{p\Delta^0\pi^0\pi^+} \quad , \quad (48)$$

also vanishes in the factorization approximation, even though the individual couplings do not. The third pair of couplings received no factorization contributions. Thus, one has  $h_A^\Delta = 0$  in this approximation. In principle, non-factorization contributions yield a non-zero value for  $h_A^\Delta$ . Although we have not evaluated these contributions, we do not expect the scale to be significantly larger than the factorization value for the individual  $h_A^{N\Delta\pi\pi}$  couplings. Consequently, we estimate a reasonable range for  $h_A^\Delta$  of  $(0 \rightarrow \text{few}) \times g_\pi$ .

These theoretical estimates suggest considerable ambiguity in the prediction for  $h_\pi^{EFF}$ . In principle, some of this ambiguity might be removed by performing the comprehensive analysis of hadronic PV suggested above, in which the various constants would be determined entirely by experiment. The viability of such a program remains to be seen.

## V. COMPARING WITH MICROSCOPIC CALCULATIONS

The results in Eqs. (37-39) embody the full SU(2) chiral structure at  $\mathcal{O}(p^3)$  of  $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$  at leading order in the pion momentum. Any microscopic calculation of this matrix element which respects the symmetries of QCD should display the dependence on light quark masses appearing in  $h_\pi^{EFF}$ . In principle, an unquenched lattice QCD calculation with light quarks would manifest this chiral structure. In practice, however, unquenched calculations remain difficult, and even quenched calculations require the use of heavy quarks. For a lattice determination of  $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$ , the expressions in Eqs. (37-39) could be used to extrapolate to the light quark limit, much as the chiral structure of baryon mass and magnetic moment can be used for similar extrapolations [20].

In the absence of a first principles QCD calculation, one must rely on symmetries and/or models to obtain the PV  $NN\pi$  coupling. A variety of such approaches have been undertaken, including the SU(6)<sub>w</sub>/quark model calculation of Refs. [7,8], the Skyrme model [9], and QCD sum rules [10]. To date, the DDH/FCDH analysis remains the most comprehensive and has become the benchmark for comparison between experiment and theory. Consequently, we focus on this work as a “case study” in the problem of matching microscopic calculations onto hadronic effective theory.

The DDH/FCDH approach relies heavily on symmetry methods in order to relate the PV  $\Delta S = 0$  matrix elements to experimental  $\Delta S = 1$  nonleptonic hyperon decay amplitudes. All the charged current (CC) contributions to the  $\Delta S = 0, 1$   $B \rightarrow B'M$  amplitudes, where  $M$  is a pseudoscalar meson, can be related using SU(3) arguments. Likewise, the neutral current (NC) component of the effective weak Hamiltonian belonging to the same multiplets as the CC components (*i.e.* those arising from a product of purely left-handed currents) can also be related via SU(3). The remaining NC contributions to

the  $\Delta S = 0$  PV amplitudes are computed using factorization and the MIT bag model. The DDH approach also employs  $SU(6)_w$  symmetry arguments in order to calculate parity-violating vector meson couplings. Although one requires only  $SU(3)$  to determine the pseudoscalar couplings, we refer below to the general  $SU(6)_w$  formalism used in Refs. [7,8].

The general  $SU(6)_w$  analysis employed by DDH/FCDH introduces five reduced matrix elements:  $a_{t,v}$ ,  $b_{t,v}$ , and  $c_v$ . These constants correspond to  $SU(6)_w$  components of the weak Hamiltonian:

$$[(\bar{B}B)_{35} \otimes M_{35}]_{35} \sim c_v \quad (49)$$

$$[(\bar{B}B)_{405} \otimes M_{35}]_{280, \overline{280}} \sim b_t, b_v \quad (50)$$

$$[(\bar{B}B)_{405} \otimes M_{35}]_{280, \overline{280}} \sim a_t, a_v \quad (51)$$

One may represent these different components of  $\mathcal{H}_W^{PV}$  diagrammatically as in Fig. 3. The components shown in Fig. 3a,b correspond to  $b_{t,v}$  and  $c_v$ , respectively. In practice, these contributions are determined entirely from empirical hyperon decay data. The term in Fig. 3a corresponds to  $a_{t,v}$  and is computed in Refs. [7,8] using factorization.

The PV  $NN\pi$  Yukawa coupling can be expressed in terms of these  $SU(6)_w$  reduced matrix elements plus an additional factorization/quark model term. Temporarily neglecting short-distance QCD corrections to  $\mathcal{H}_W^{PV}$ , one has

$$\begin{aligned} \langle p\pi^- | \mathcal{H}_W^{PV} | n \rangle &= \frac{1}{3\sqrt{2}} \tan \theta_c c_v \quad (52) \\ &- \frac{2}{9\sqrt{2}} \csc 2\theta_c \sin^2 \theta_w (2c_v - b_t) + \frac{1}{3} \sin^2 \theta_w y \quad , \end{aligned}$$

where  $\theta_c$  and  $\theta_w$  are the Cabibbo and Weinberg angles, respectively, and  $y$  denotes a Fierz/factorization contribution. The first term on the RHS of Eq. (52) gives the CC contribution, while the remaining terms arise from weak NC's. Including short-distance QCD renormalization of  $\mathcal{H}_W^{PV}$  leads to a modification of Eq. (52):

$$\begin{aligned} \langle p\pi^- | \mathcal{H}_W^{PV} | n \rangle &= \left\{ [1 - 2\sin^2 \theta_w] \gamma(K) + \sin^2 \theta_c \right\} \frac{\rho}{\sin^2 \theta_c} g_\pi \\ &+ \sin^2 \theta_c (B_1 + B_2) \quad , \quad (53) \end{aligned}$$

where

$$g_\pi = \frac{1}{3\sqrt{2}} \tan \theta_c c_v \quad (54)$$

$$B_1 = \frac{4}{9\sqrt{2}} \eta E(K) \left( \frac{1}{\sin \theta_c \cos \theta_c} \right) (b_v/6 - b_t/12 - c_v/2) \quad (55)$$

$$B_2 = \frac{1}{3} F(K) y \quad , \quad (56)$$

and  $\gamma(K)$ ,  $E(K)$  and  $F(K)$  are summed leading log (renormalization group) factors dependent on

$$K = 1 - \frac{\alpha_s(\mu)}{\pi} \left[ 11 - \frac{2}{3} N_f \right] \ln \frac{M_W^2}{\mu^2} \quad . \quad (57)$$

The overall scale factor  $\rho$  appearing in Eq. (53) was introduced in Ref. [7] in order to account for various theoretical uncertainties entering the analysis.

The appearance of  $c_v$ ,  $b_t$ , and  $b_v$  in  $g_\pi$  and  $B_1$  relies on *tree-level*  $SU(6)_w$  symmetry—long-distance chiral corrections of the types shown in Fig. 4 have not been explicitly included. Inclusion of such corrections would necessitate a reanalysis of the  $\Delta S = 1$  amplitudes in much the same way that one treats the octet of baryon axial vector currents [11] or magnetic moments [21]. For example, letting  $A(\Lambda_-^0)$  denote the amplitude for  $\Lambda \rightarrow p\pi^-$  one has at tree-level

$$A(\Lambda_-^0) = \frac{1}{\sqrt{3}} (b_v/6 - b_t/12 - c_v/2) \quad . \quad (58)$$

Including the leading chiral corrections would yield the modification

$$A(\Lambda_-^0) = \frac{1}{\sqrt{3}} \sqrt{Z_\Lambda Z_p Z_\pi} (b_v/6 - b_t/12 - c_v/2) + \Delta A(\Lambda_-^0) \quad , \quad (59)$$

where  $\Delta A(\Lambda_-^0)$  denotes vertex corrections and possible contributions from higher-dimension operators. Similar corrections would appear in the  $SU(6)_w$  symmetry terms in Eqs. (52, 53). Given the absence of these corrections from the DDH/FCDH analysis, the symmetry components  $\langle p\pi^- | \mathcal{H}_W^{PV} | n \rangle$  do not formally embody the subleading chiral structure of  $h_\pi^{EFF}$ . The *numerical* impact of applying chiral corrections to the DDH/FCDH tree-level  $SU(6)_w$  analysis is much less clear, since some of the chiral modifications can be absorbed into renormalized values of the chiral couplings, which are determined empirically. Nevertheless, the potentially sizeable effect of the  $SU(2)$  chiral corrections on  $h_\pi^{EFF}$  should give one pause.

A related issue is the degree to which ambiguities introduced by kaon and  $\eta$  loops in  $SU(3)$  HBChPT could plague an analysis of the  $\Delta S = 1$  amplitudes. Here recent work by Donoghue and Holstein argues that finite nucleon size call for long-distance regularization of such heavy meson loops, which substantially reduces their effects [22]. Results are then similar to what arises from use of a cloudy bag approach to such matrix elements [23]. A comprehensive study of such issues – and their impact on the DDH/FCDH calculation of  $h_\pi$  – goes beyond the scope of the present work. Nevertheless, the sizeable impact of the chiral corrections in  $h_\pi^{EFF}$  and the use of tree-level symmetry arguments in Refs. [7,8] points to a possibly significant mismatch between  $h_\pi^{EFF}$  and  $h_\pi^{DDH}$ .

The remaining terms in the DDH/FCDH analysis – involving the parameters  $\eta$  and  $y$  – are determined by explicit MIT bag model calculations. One may ask whether the latter effectively includes any part of the subleading chiral structure of  $h_\pi^{EFF}$ . In order to address this question, we make three observations:



**1. Sea quarks and gluons generate  $c_v$ .** The parameter  $c_v$  vanishes identically in any quark model in which baryons consist solely of three constituent quarks. The  $\Delta S = 1$  hyperon decay data, however, clearly implies that  $c_v \neq 0$ . In order to obtain a nonzero value in a quark model, one requires the presence of sea quarks and gluons. It is shown in [24] for example, that  $c_v \neq 0$  when gluons are added to the MIT bag model. Similarly, one would expect contributions from the  $q\bar{q}$  pairs in the sea. Since relativistic quark models already contain  $q\bar{q}$  pairs in the form of “Z-graphs” [25], it is likely that disconnected  $q\bar{q}$  insertions (see Fig. 5b) give the dominant sea quark contribution to  $c_v$ . In a chirally corrected analysis of nonleptonic decays, the long-distance part of the disconnected  $q\bar{q}$  insertions appear explicitly in the guise of pseudoscalar loops, while the short-distance contributions are subsumed into the value of  $c_v$  and possible higher dimension operators. “Quenched” quark models without explicit pionic degrees of freedom generally do not include the long-distance physics of disconnected insertions.

**2. The  $m_q$ -dependence is different.** In conventional HBChPT analyses of hadronic observables, one only retains the loop contributions non-analytic in the light quark mass. The constituent quark model (without explicit pions) has a difficult time producing these non-analytic contributions. The simplest, illustrative example is the nucleon isovector charge radius,  $\langle r^2 \rangle_{T=1}$ , which is singular in the chiral limit [26]. This chiral singularity, of the form  $\ln m_\pi^2 \sim \ln m_q$ , is produced by  $\pi$  loops. Relativistic quark models, such as the MIT bag model, yield a finite value for  $\langle r^2 \rangle_{T=1}$  as  $m_q \rightarrow 0$ . One cannot produce the chiral singularity in a quark model without including disconnected  $q\bar{q}$  insertions dressed as mesons.

The corresponding argument in the case of  $h_\pi^{EFF}$  is less direct, but still straightforward. In the limit of a degenerate  $N$  and  $\Delta$ , the non-analytic terms in  $h_\pi^{EFF}$  have quark mass-dependences of the form  $m_q \ln m_q$  or  $m_q^{3/2}$ . As we show in Appendix C, bag model matrix elements of the four quark operators appearing in  $\mathcal{H}_W^{PV}$  have a Taylor series expansion about  $m_q = 0$ . Thus, the parameters  $\eta$  and  $y$  cannot contain the non-analytic structures generated by the diagrams in Figs. 1-2.

**3. Graphs are missing.** This observation is simply a diagrammatic summary of the previous two observations. For simplicity, consider a subset of the quark-level diagrams associated with the appearance of  $h_A^i$  in  $h_\pi^{EFF}$ . Typical contributions to the axial  $NN\pi\pi$  PV vertex are shown in Fig. 5a. The corresponding loop contributions to  $h_\pi^{EFF}$  appear in Fig. 5b,c. Those in Fig. 5b involve disconnected  $q\bar{q}$  insertions, which do not occur in the constituent quark model. The contribution of Fig. 5c involves Z-graphs, which are produced in a relativistic quark model<sup>§</sup>. In principle, the  $3q + q\bar{q}$  intermediate state could contain an  $N\pi$  pair. As argued previously, however, the Z-graphs implicit in the MIT bag model calculation of  $h_\pi$  do not produce the nonanalytic structure of the corresponding  $\pi$  loop. Apparently, only an unquenched quark model, which generates the disconnected insertions of Fig. 5b, could produce the requisite nonanalytic terms.

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<sup>§</sup>e.g., as a correction to the  $b_{t,v}$  terms of Fig. 3b

From this “case study” of the DDH/FCDH calculation of  $h_\pi$ , we conclude that the  $SU(6)_w$ /quark model approach used in Refs. [7,8] does not incorporate the chiral structure of  $h_\pi^{EFF}$ . Were the numerical impact of the chiral corrections negligible, this observation would not be bothersome. The actual impact of the chiral corrections, however, is significant. Thus, it is perhaps not surprising that the  $^{18}\text{F}$  result and the DDH/FCDH “best value” do not agree.

## VI. CONCLUSIONS

With the confirmation of the electroweak sector of the Standard Model at the 1% level or better in a variety of leptonic and semi-leptonic processes, one has little reason to doubt its validity in the purely hadronic domain. Similarly, the predictions of QCD in the perturbative regime have been confirmed with a high degree of confidence. Thus, one may justifiably consider  $\mathcal{H}_W^{PV}$ , the effective Hamiltonian including its perturbative strong interaction correction, to be well understood. Moreover, the precision available with present and future hadronic PV experiments is unlikely to match the levels achieved in leptonic and semileptonic processes. Consequently, one has little hope of detecting small deviations in  $\mathcal{H}_W^{PV}$  from its SM structure due to “new physics”. On the other hand, much about QCD in the non-perturbative regime remains mysterious: the mechanism of confinement, the dynamics of chiral symmetry breaking, the role of sea quarks in the low-energy structure of the nucleon, and so forth. Each of these issues bears on one’s understanding of *matrix elements* of  $\mathcal{H}_W^{PV}$ . In this sense, the low-energy, PV hadronic weak interaction constitutes a probe of the dynamics of low-energy QCD, in a manner analogous to the probe provided by the electromagnetic interaction.

From a phenomenological standpoint, the matrix element one may hope to extract from hadronic PV observables with the least ambiguity is  $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$ . In this study, we have argued that any theoretical interpretation of this matrix element must take into account the consequences of chiral symmetry. Indeed the chiral corrections to the tree-level, PV  $\pi NN$  Yukawa coupling are not small. At  $\mathcal{O}(1/\Lambda_\chi^3)$ , the effective coupling measured in experiments,  $h_\pi^{EFF}$ , depends not only on the bare coupling,  $h_\pi^{BARE}$ , but also on new (and experimentally undetermined) PV low-energy constants,  $h_A^1$ ,  $h_A^\Delta$ , and  $h_\Delta$ , as well. Furthermore, the coefficients of  $h_\pi^{BARE}$ ,  $h_A^1$ , and  $h_\Delta$  are comparable in magnitude. At present, one has only simple theoretical estimates of the magnitudes of the  $h_A^1$  and  $h_A^\Delta$  in addition to the FCDH calculation of  $h_\Delta$ . These estimates suggest that the new PV couplings appearing in  $h_\pi^{EFF}$  could be as large as  $h_\pi^{BARE}$ . Since no experimental constraints have been obtained for the new couplings, there exists considerable latitude in the theoretical expectation for  $h_\pi^{EFF}$ .

For two decades now, the benchmark theoretical calculation of  $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$  has been the  $SU(6)_w$ /quark model approach of Ref. [7], updated in Ref. [8]. We have argued, however, that the DDH/FCDH calculation does not manifest the general strictures of chiral invariance obtained in the present analysis. At the quark level, this chiral structure reflects the importance of the “disconnected”  $q\bar{q}$  components of the sea. While relativistic quark models contain  $q\bar{q}$  sea quark effects in the guise of Z-graphs or lower-component

wavefunctions, the most common “quenched” versions do not include explicit disconnected pairs\*\*. Given the size of the chiral corrections associated in part with the disconnected insertions, it may then be not so surprising to find a possible discrepancy between the experimental value for  $h_\pi^{EFF}$  and the DDH/FCDH “best value”.

Applying chiral corrections to the SU(3) analysis of  $\Delta S = 1$  hyperon decays may help to close the gap between  $h_\pi^{EFF}$  and  $h_\pi^{DDH}$ . Presumably, similar corrections should be applied in other approaches not containing explicit pionic degrees of freedom. In the longer run, one may be able to use the chiral structure of  $h_\pi^{EFF}$  to extrapolate an unquenched lattice calculation with heavy quarks into the physical regime.

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\*\*Some effects of disconnected  $q\bar{q}$  pairs may, however, hide in the effective parameters of the quark model, such as the string tension [27].

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## APPENDIX A: PV LAGRANGIANS

Here we present the full expressions for some of the PV Lagrangians not included in the main body of the paper. The analogues of Eqs. (16-18) are

$$\begin{aligned} \mathcal{L}_{\Delta I=0}^{\pi\Delta N} &= f_1 \epsilon^{abc} \bar{N} i \gamma_5 [X_L^a A_\mu X_L^b + X_R^a A_\mu X_R^b] T_c^\mu \\ &\quad + g_1 \bar{N} [A_\mu, X_-^a]_+ T_a^\mu + g_2 \bar{N} [A_\mu, X_-^a]_- T_a^\mu + \text{h.c.} \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \mathcal{L}_{\Delta I=1}^{\pi\Delta N} &= f_2 \epsilon^{ab3} \bar{N} i \gamma_5 [A_\mu, X_+^a]_+ T_b^\mu + f_3 \epsilon^{ab3} \bar{N} i \gamma_5 [A_\mu, X_+^a]_- T_b^\mu \\ &\quad + \frac{g_3}{2} \bar{N} [(X_L^a A_\mu X_L^3 - X_L^3 A_\mu X_L^a) - (X_R^a A_\mu X_R^3 - X_R^3 A_\mu X_R^a)] T_a^\mu \\ &\quad + \frac{g_4}{2} \{ \bar{N} [3X_L^3 A^\mu (X_L^1 T_\mu^1 + X_L^2 T_\mu^2) + 3(X_L^1 A^\mu X_L^3 T_\mu^1 + X_L^2 A^\mu X_L^3 T_\mu^2) \\ &\quad - 2(X_L^1 A^\mu X_L^1 + X_L^2 A^\mu X_L^2 - 2X_L^3 A^\mu X_L^3) T_\mu^3] - (L \leftrightarrow R) \} + \text{h.c.} \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \mathcal{L}_{\Delta I=2}^{\pi\Delta N} &= f_4 \epsilon^{abd} \mathcal{I}^{cd} \bar{N} i \gamma_5 [X_L^a A_\mu X_L^b + X_R^a A_\mu X_R^b] T_c^\mu \\ &\quad + f_5 \epsilon^{ab3} \bar{N} i \gamma_5 [X_L^a A_\mu X_L^3 + X_L^3 A_\mu X_L^a + (L \leftrightarrow R)] T_b^\mu \\ &\quad + g_5 \mathcal{I}^{ab} \bar{N} [A_\mu, X_-^a]_+ T_b^\mu + g_6 \mathcal{I}^{ab} \bar{N} [A_\mu, X_-^a]_- T_b^\mu + \text{h.c.} \quad , \end{aligned} \quad (\text{A3})$$

where the terms containing  $f_i$  and  $g_i$  start off with one- and two-pion vertices, respectively. In the heavy baryon expansion, the terms containing the  $f_i$  start to contribute at  $\mathcal{O}(1/m_N)$ . The leading order term vanishes since  $P_+ \cdot i\gamma_5 \cdot P_+ = 0$ . Since we work only to lowest order in the  $1/m_N$  expansion, we obtain no contribution from the terms containing the  $f_i$ .

For the pv  $\pi\Delta\Delta$  effective Lagrangians we have

$$\mathcal{L}_{\Delta I=0}^{\pi\Delta} = j_0 \bar{T}^i A_\mu \gamma^\mu T_i \quad , \quad (\text{A4})$$

$$\begin{aligned} \mathcal{L}_{\Delta I=1}^{\pi\Delta} &= \frac{j_1}{2} \bar{T}^i \gamma^\mu T_i T_r(A_\mu X_+^3) - \frac{k_1}{2} \bar{T}^i \gamma^\mu \gamma_5 T_i T_r(A_\mu X_-^3) \\ &\quad - \frac{h_1^1}{2\sqrt{2}} f_\pi \bar{T}^i X_-^3 T_i - \frac{h_2^2}{2\sqrt{2}} f_\pi \{ 3T^3 (X_-^1 T^1 + X_-^2 T^2) + 3(\bar{T}^1 X_-^1 + \bar{T}^2 X_-^2) T^3 \\ &\quad - 2(\bar{T}^1 X_-^3 T^1 + \bar{T}^2 X_-^3 T^2 - 2\bar{T}^3 X_-^3 T^3) \} + j_2 \{ 3[(\bar{T}^3 \gamma^\mu T^1 + \bar{T}^1 \gamma^\mu T^3) T_r(A_\mu X_+^1) \\ &\quad + (\bar{T}^3 \gamma^\mu T^2 + \bar{T}^2 \gamma^\mu T^3) T_r(A_\mu X_+^2)] - 2(\bar{T}^1 \gamma^\mu T^1 + \bar{T}^2 \gamma^\mu T^2 - 2\bar{T}^3 \gamma^\mu T^3) T_r(A_\mu X_+^3) \} \\ &\quad + k_2 \{ 3[(\bar{T}^3 \gamma^\mu \gamma_5 T^1 + \bar{T}^1 \gamma^\mu \gamma_5 T^3) T_r(A_\mu X_-^1) + (\bar{T}^3 \gamma^\mu \gamma_5 T^2 + \bar{T}^2 \gamma^\mu \gamma_5 T^3) T_r(A_\mu X_-^2)] \\ &\quad - 2(\bar{T}^1 \gamma^\mu \gamma_5 T^1 + \bar{T}^2 \gamma^\mu \gamma_5 T^2 - 2\bar{T}^3 \gamma^\mu \gamma_5 T^3) T_r(A_\mu X_-^3) \} \\ &\quad + j_3 \{ \bar{T}^a \gamma^\mu [A_\mu, X_+^a]_+ T^3 + \bar{T}^3 \gamma^\mu [A_\mu, X_+^a]_+ T^a \} \\ &\quad + j_4 \{ \bar{T}^a \gamma^\mu [A_\mu, X_+^a]_- T^3 - \bar{T}^3 \gamma^\mu [A_\mu, X_+^a]_- T^a \} \\ &\quad + k_3 \{ \bar{T}^a \gamma^\mu \gamma_5 [A_\mu, X_-^a]_+ T^3 + \bar{T}^3 \gamma^\mu \gamma_5 [A_\mu, X_-^a]_+ T^a \} \\ &\quad + k_4 \{ \bar{T}^a \gamma^\mu \gamma_5 [A_\mu, X_-^a]_- T^3 - \bar{T}^3 \gamma^\mu \gamma_5 [A_\mu, X_-^a]_- T^a \} \quad , \end{aligned} \quad (\text{A5})$$

$$\begin{aligned}
\mathcal{L}_{\Delta I=2}^{\pi\Delta} = & j_5 \mathcal{I}^{ab} \bar{T}^a \gamma^\mu A_\mu T^b + j_6 \mathcal{I}^{ab} \bar{T}^i [X_R^a A_\mu X_R^b + X_L^a A_\mu X_L^b] \gamma^\mu T_i \\
& + k_5 \mathcal{I}^{ab} \bar{T}^i [X_R^a A_\mu X_R^b - X_L^a A_\mu X_L^b] \gamma^\mu \gamma_5 T_i \\
& + k_6 \epsilon^{ab3} [\bar{T}^3 i \gamma_5 X_+^b T^a + \bar{T}^a i \gamma_5 X_+^b T^3], \tag{A6}
\end{aligned}$$

where we have suppressed the Lorentz indices of the  $\Delta$  field, i.e.,  $\bar{T}^\nu \cdots T_\nu$ . The vertices with  $k_i, h_\Delta$  contain two pions. All other vertices contain one pion when expanded to the leading order. At first sight the leading order term with  $k_6$  in (A6) has no pions. However such a term cancels its hermitian conjugate exactly. The constants  $h_{\pi\Delta}^i$  are the PV  $\pi\Delta\Delta$  Yukawa coupling constants.

In Section 2, the leading terms in the above Lagrangians were expressed in terms of effective  $\pi\pi N\Delta$  and  $\pi\Delta\Delta$  coupling constants. These constants may be expressed in terms of the  $f_i, g_i, k_i, j_i$  and  $h_{\pi\Delta}^i$  as follows:

$$\begin{aligned}
h_A^{p\Delta^{++}\pi^-\pi^0} &= -2g_1 + 2g_2 - g_3 - 3g_4 - \frac{2}{3}g_5 + \frac{2}{3}g_6 \\
h_A^{p\Delta^{++}\pi^0\pi^-} &= 2g_1 + g_3 + 6g_4 + \frac{2}{3}g_5 \\
h_A^{p\Delta^+\pi^0\pi^0} &= -\frac{\sqrt{6}}{9}(6g_2 + 9g_4 + 2g_6) \\
h_A^{p\Delta^+\pi^+\pi^-} &= -\frac{\sqrt{6}}{9}(-6g_1 - 9g_4 + 4g_5 + 6g_6) \\
h_A^{p\Delta^+\pi^-\pi^+} &= -\frac{\sqrt{6}}{9}(6g_1 - 6g_2 - 4g_5 + 4g_6) \\
h_A^{p\Delta^0\pi^+\pi^0} &= -\frac{\sqrt{3}}{9}(6g_1 + 6g_2 - 3g_3 + 9g_4 + 2g_5 + 2g_6) \\
h_A^{p\Delta^0\pi^0\pi^+} &= -\frac{\sqrt{3}}{9}(-6g_1 + 12g_2 + 3g_3 + 18g_4 - 2g_5 - 8g_6) \\
h_A^{p\Delta^-\pi^+\pi^+} &= \frac{\sqrt{2}}{3}(6g_2 - 9g_4 + 2g_6) \\
h_A^{n\Delta^{++}\pi^-\pi^-} &= \frac{\sqrt{2}}{3}(6g_2 - 9g_4 + 2g_6) \\
h_A^{n\Delta^+\pi^-\pi^0} &= -\frac{\sqrt{3}}{9}(6g_1 + 6g_2 + 3g_3 - 9g_4 + 2g_5 + 2g_6) \\
h_A^{n\Delta^+\pi^0\pi^-} &= -\frac{\sqrt{3}}{9}(-6g_1 + 12g_2 - 3g_3 - 18g_4 - 2g_5 - 8g_6) \\
h_A^{n\Delta^0\pi^0\pi^0} &= -\frac{\sqrt{6}}{9}(-6g_2 + 9g_4 - 2g_6) \\
h_A^{n\Delta^0\pi^+\pi^-} &= -\frac{\sqrt{6}}{9}(-6g_1 + 6g_2 + 4g_5 - 4g_6) \\
h_A^{n\Delta^0\pi^-\pi^+} &= -\frac{\sqrt{6}}{9}(6g_1 - 9g_4 - 4g_5 - 6g_6) \\
h_A^{n\Delta^-\pi^+\pi^0} &= 2g_1 - 2g_2 + g_3 + 3g_4 + \frac{2}{3}g_5 - \frac{2}{3}g_6
\end{aligned}$$

$$h_A^{n\Delta^-\pi^0\pi^+} = -2g_1 - g_3 - 6g_4 - \frac{2}{3}g_5 \quad (\text{A7})$$

$$h_\Delta = h_{\pi\Delta}^1 + h_{\pi\Delta}^2$$

$$\begin{aligned} h_V^{\Delta^{++}\Delta^+} &= \frac{1}{\sqrt{6}}(j_0 + \frac{4}{3}j_6) - 2\sqrt{6}j_2 - \frac{2\sqrt{6}}{3}(j_3 + j_4) + \frac{j_5}{3\sqrt{6}} \\ h_V^{\Delta^+\Delta^0} &= \frac{\sqrt{2}}{3}(j_0 + \frac{4}{3}j_6) - \frac{2\sqrt{2}}{9}j_5 \\ h_V^{\Delta^0\Delta^-} &= \frac{1}{\sqrt{6}}(j_0 + \frac{4}{3}j_6) + 2\sqrt{6}j_2 + \frac{2\sqrt{6}}{3}(j_3 + j_4) + \frac{j_5}{3\sqrt{6}} \end{aligned} \quad (\text{A8})$$

It is interesting to note there is only one independent PV Yukawa coupling constant  $h_\Delta$  for  $\pi\Delta\Delta$  interactions.

## APPENDIX B: VANISHING LOOP CONTRIBUTIONS

As noted in Section 3, a large number of graphs which nominally contribute to  $h_\pi^{EFF}$  actually vanish up to  $\mathcal{O}(1/\Lambda_\chi^3)$ . Here, we summarize the the reasons why.

Consider first the corrections due to the PV vector  $\pi NN$  vertices. For FIG. 1 (b) we have

$$iM_{(b)} = i\frac{g_A^2}{\sqrt{2}F_\pi^3}\tau^+(h_v^0 + \frac{4}{3}h_V^2)(v \cdot q) \int \frac{d^D k}{(2\pi)^D} \frac{(S \cdot k)^2}{v \cdot kv \cdot (k+q)(k^2 - m_\pi^2)} \sim \mathcal{O}(1/m_N \Lambda_\chi^3), \quad (\text{B1})$$

where we have used  $v \cdot q \sim \mathcal{O}(1/m_N)$ . Since we are working to leading order in the  $1/m_N$  expansion, this amplitude does not contribute. The PV vector interactions also appear in Figs. 1(j1,j2). The corresponding amplitude is

$$iM_{(j1)+(j2)} = -i\frac{g_A^2}{\sqrt{2}F_\pi^3}\tau^+(h_v^0 + 2h_V^1 - \frac{8}{3}h_V^2) \int \frac{d^D k}{(2\pi)^D} \frac{[(S \cdot k), (S \cdot q)]_+}{v \cdot kv \cdot (k+q)(k^2 - m_\pi^2)} = 0 \quad (\text{B2})$$

This integral vanishes because it is proportional to  $[(S \cdot v), (S \cdot q)]_+$ , which vanishes because  $S \cdot v = 0$ . All other possible insertions of PV vector  $\pi NN$  vertices vanish for similar reasons as either (B1) or (B2). In what follows, we refer only to insertions involving the PV  $\pi NN$  Yukawa and  $\pi\pi NN$  axial couplings.

The propagator corrections in FIG. 1 (g1)-(h2) vanish after integration since their amplitude of (g1)-g2) goes as

$$\sim h_\pi \int \frac{d^D k}{(2\pi)^D} \frac{v \cdot k}{k^2 - m_\pi^2} = 0 \quad . \quad (\text{B3})$$



while the amplitude of (h1)-(h2) goes as

$$\sim h_A^i \int \frac{d^D k}{(2\pi)^D} \frac{S \cdot k}{k^2 - m_\pi^2} = 0 \quad . \quad (\text{B4})$$

The amplitude of FIG. 1 (i1)-(i4) contains a vanishing integral

$$\sim h_\pi \int \frac{d^D k}{(2\pi)^D} \frac{S \cdot k}{v \cdot k (k^2 - m_\pi^2)} = 0 \quad . \quad (\text{B5})$$

Figs. 1 (j1)-(j2) do not contribute for the PV Yukawa coupling  $h_\pi$  due to charge conservation. The remaining non-zero diagrams are Figs. 1 (a)-(f2) where the insertions in loops are of the Yukawa or axial interactions. Figs. (f1), (f2) arises from the insertion of the counter terms of mass and wave function renormalization. Fig. 1 (e1)-(e2) and Fig. 2 (c1)-(c2) contribute to the wave function renormalization in Eq. (38).

Due to the heavy baryon projection  $P_+ \cdot i\gamma_5 \cdot P_+ = 0$  the one pion PV  $\pi N\Delta$  vertex does not contribute in the leading order of heavy baryon expansion. Hence, the chiral loop corrections from FIG. 2 (d1)-(g4) are of higher order. Fig. 2 (h1)-(j2) vanishes after integration for reasons similar to (B2). The remaining, non-vanishing diagrams are discussed explicitly in Section 3.

As pointed out in Section 2, both PC and PV two-derivative operators which conserve CP do not contribute to  $h_\pi$  renormalization. For example, there exists one CP-conserving, PV such operator:

$$\frac{1}{\Lambda_\chi} \bar{N} \sigma^{\mu\nu} [D_\mu A_\nu - D_\nu A_\mu] N \quad . \quad (\text{B6})$$

After expansion, the leading term starts with three pions. It contributes via Figure 1 (a), at the order of  $1/\Lambda_\chi F_\pi^3$ . Moreover the loop integration yields a factor  $g_{\mu\nu}$  and leads to zero after contraction with  $\sigma^{\mu\nu}$ .

Another possibility comes from insertions of PC two-derivative nucleon pion operators. There are three PC operators which conserve CP:

$$\frac{1}{\Lambda_\chi} \bar{N} i\gamma_5 D_\mu A^\mu N \quad , \quad (\text{B7})$$

$$\frac{1}{\Lambda_\chi} \bar{N} A^\mu A_\mu N \quad , \quad (\text{B8})$$

$$\frac{1}{\Lambda_\chi} \bar{N} \sigma^{\mu\nu} [A_\mu, A_\nu] N \quad . \quad (\text{B9})$$

Note the first two operators are symmetric in the Lorentz indices. Only the last one arises from the antisymmetric operators listed in Eq. (29). The first one starts off with one pion. The relevant Feynman diagrams are Figure 1 (c1)-(c2), where the PV vertex is

associated with  $h_A^i$ . Note these diagrams do not contribute at leading order of HBChPT due to the presence of the  $i\gamma_5$ . The remaining two operators start off with two pions. The relevant diagrams are Figure 1 (d1)-(d2). After integration the contribution of the third operator reads

$$\sim h_\pi \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta v^\mu q^\nu m_\pi^2 \ln m_\pi / \Lambda_\chi F_\pi^2 . \quad (\text{B10})$$

So its contribution is zero. In contrast the second operator yields

$$h_\pi (v \cdot q) m_\pi^2 \ln m_\pi / (\Lambda_\chi F_\pi^2) . \quad (\text{B11})$$

Note  $v \cdot q \sim 1/m_N$ . So its contribution is of order  $1/(\Lambda_\chi^3 m_N)$ . In short, none of the two-derivative operators contribute to the renormalization of  $h_\pi$  at the order to which we work.

### APPENDIX C: BAG MODEL INTEGRALS

Here, we show that the four-quark bag model integrals relevant to the calculation of the DDH/FCDH parameters  $\eta$  and  $y$  have a Taylor expansion in light quark mass around  $m_q = 0$ . We write a bag model quark wavefunction as [28,18]

$$\psi(x) = \begin{pmatrix} iu(r)\chi \\ \ell(r)\vec{\sigma} \cdot \vec{r}\chi \end{pmatrix} \exp(-iEt) \quad , \quad (\text{C1})$$

where  $\chi$  denotes a two-component Pauli spinor and where wave function normalization yields

$$\int d^3r (u(r)^2 + \ell(r)^2) = 1 \quad , \quad (\text{C2})$$

where the radial integration runs from 0 to the bag radius,  $R$ . The four quark matrix elements of interest here can depend three different integrals:

$$\int d^3r u(r)^4 \quad , \quad \int d^3r \ell(r)^4 \quad , \quad \int d^3r u(r)^2 \ell(r)^2 \quad . \quad (\text{C3})$$

The quark radial wave functions are

$$u(r) = N j_0\left(\frac{p_n r}{R}\right) \quad (\text{C4})$$

$$\ell(r) = -N \left(\frac{\omega_n - m_q R}{\omega_n + m_q R}\right)^{1/2} j_1\left(\frac{p_n r}{R}\right) \quad , \quad (\text{C5})$$

where

$$\tan p_n = -\frac{p_n}{\omega_n + m_q R - 1} \quad (n = 1, 2, \dots) \quad (\text{C6})$$

$$p_n = \sqrt{\omega_n^2 - m_q^2 R^2} \quad (\text{C7})$$

$$N = \sqrt{\frac{p_n^4}{R^3(2\omega_n^2 - 2\omega_n + m_q R) \sin^2 p_n}} \quad (\text{C8})$$

$$R^4 = \frac{N\omega_n - Z_0}{4\pi B} \quad (\text{C9})$$

$B$  is the bag constant and  $Z_0$  is a phenomenological parameter involved with the center of mass motion of the bag.

For light quarks and lowest eigenmode

$$\omega_0 \approx (2.043 + 0.493m_q R) \quad (\text{C10})$$

$$N \approx 2.27/\sqrt{4\pi R^3} \quad (\text{C11})$$

It is straightforward to show that the bag model integrals in Eq. (C3) have a Taylor expansion about  $m_q = 0$ . The argument proceeds by noting that the quantities  $N$ ,  $R$ ,  $p_n$ ,  $\omega_n$  and the argument of the spherical Bessel functions all have Taylor series in  $m_q$  about  $m_q = 0$ . The existence of this expansion can be seen by an explicit, iterative construction. First, expand  $\omega_n$  and  $R$ :

$$\omega_n = \sum_{k=0}^{\infty} \omega_{n,k} m_q^k \quad (\text{C12})$$

$$R = \sum_{k=0}^{\infty} R_k m_q^k \quad (\text{C13})$$

Now let  $m_q = 0$  in Eqs. (C6,C7). Doing so eliminates all dependence on  $R$  and determines  $\omega_{n,0}$ . Next, set  $m_q = 0$  in Eqs. (C8, C9) with  $\omega_n \rightarrow \omega_{n,0}$ . Doing so determines  $R_0$ . Now expand Eqs. (C6,C7) to first order in  $m_q$ . This step yields  $\omega_{n,1}$  in terms of  $\omega_{n,0}$  and  $R_0$ . Expanding Eqs. (C8, C9) to first order in  $m_q$  then determines  $R_1$  in terms of  $\omega_{n,0}$ ,  $\omega_{n,1}$ , and  $R_0$  and so forth. Note that at any step of the recursion, the argument of any transcendental function is  $\omega_{n,0}$ . Hence, at any order, a solution for the  $\omega_{n,k}$  and  $R_k$  exists.

The expansion of the bag model integrals continues by computing their derivatives with respect to  $m_q$  and using the expansions of  $N$ ,  $R$ , etc. in terms of  $m_q$  as constructed above. Taking  $n$  derivatives of one of the integrals in Eq. (C3) yields new integrals involving powers of  $r/R$  times products of the Bessel functions and their derivatives. Using the standard Bessel function recursion relations, the derivatives of the  $j_k$  can always be expressed in terms of other spherical Bessel functions. Since the  $j_k$  and their derivatives are finite at the origin, and since the radial bag integration is bounded above by  $R$ , the  $n$ th derivative of any of the integrals in Eq. (C3) is finite. Thus, each of the integrals in Eq. (C3) can be expanded in a Taylor series about  $m_q = 0$ .

## Figure Captions

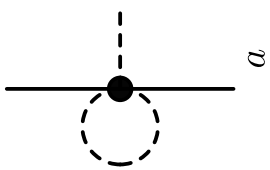
Figure 1. Meson-nucleon intermediate state contributions to the PV  $\pi NN$  vertex  $h_\pi$ . The shaded circle denotes the PV vertex. The solid and dashed lines correspond to the nucleon and pion respectively.

Figure 2. The chiral corrections from  $\Delta$  intermediate state, which is denoted by the double line.

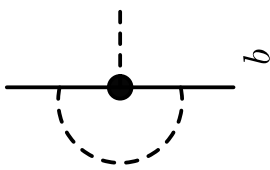
Figure 3. Diagrammatic representation of the  $SU(6)_w$  components of  $\langle B'M | \mathcal{H}_w^{PV}(\Delta S = 0, 1) | B \rangle$ . Figs. 3a-c correspond, respectively, to  $b_{t,v}$ ,  $c_v$ , and  $a_{t,v}$ . The wavy line denotes the action of  $\mathcal{H}_w^{PV}$ .

Figure 4. Chiral corrections to the  $B \rightarrow B'M$  nonleptonic weak decay.

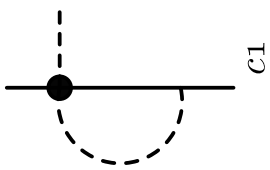
Figure 5. Quark line diagrams for the renormalization of  $h_\pi$  due to the axial PV  $\pi\pi NN$  interaction. As in Fig. 3, the wavy line denotes the action of  $\mathcal{H}_w^{PV}$ . Fig. 5a shows a typical contribution to  $h_A^i$ . Figs 5b,c denote the corresponding loop corrections to  $h_\pi$ . Fig. 5(b) contains the disconnected  $q\bar{q}$  insertions, while Fig. 5(c) gives a Z-graph contribution.



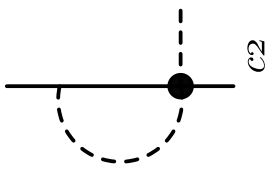
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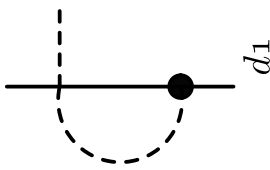
*b*



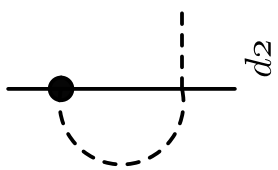
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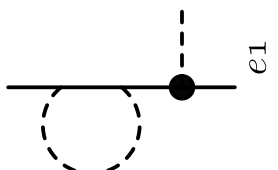
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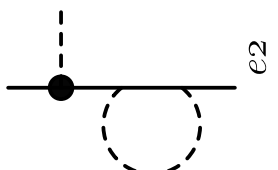
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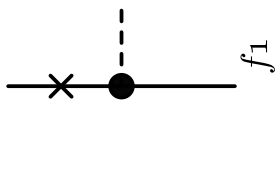
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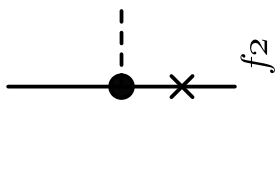
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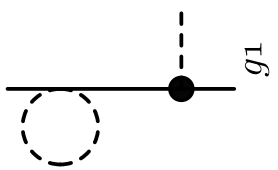
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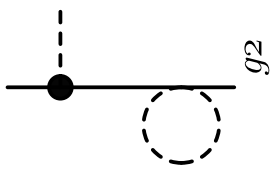
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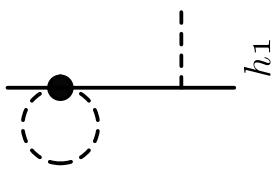
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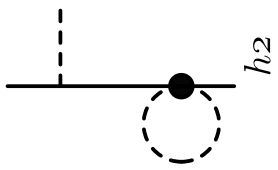
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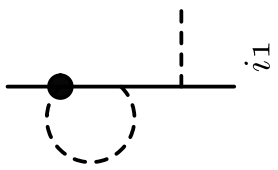
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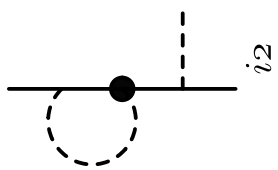
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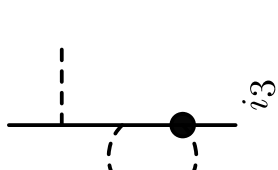
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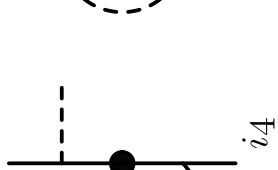
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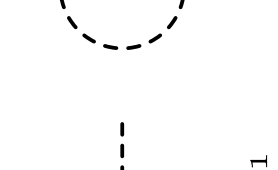
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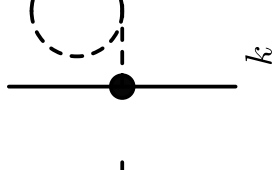
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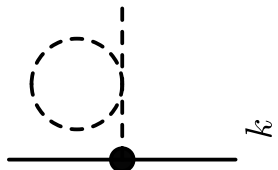
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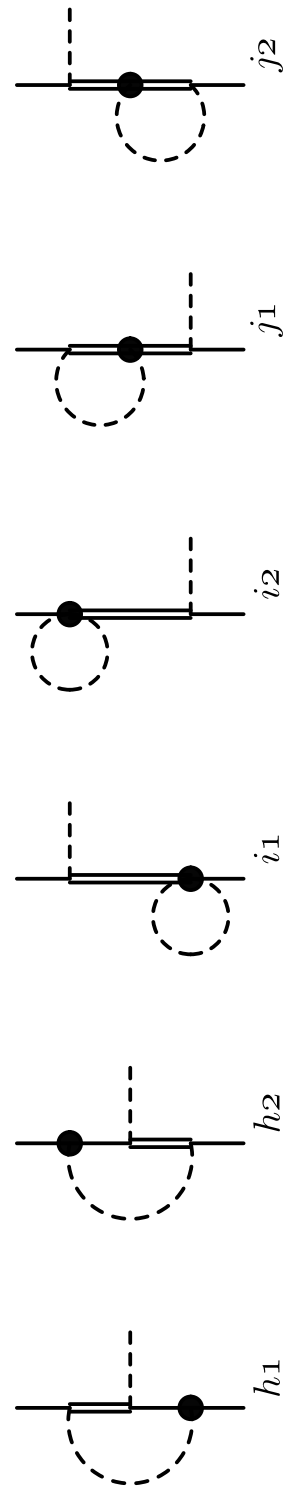
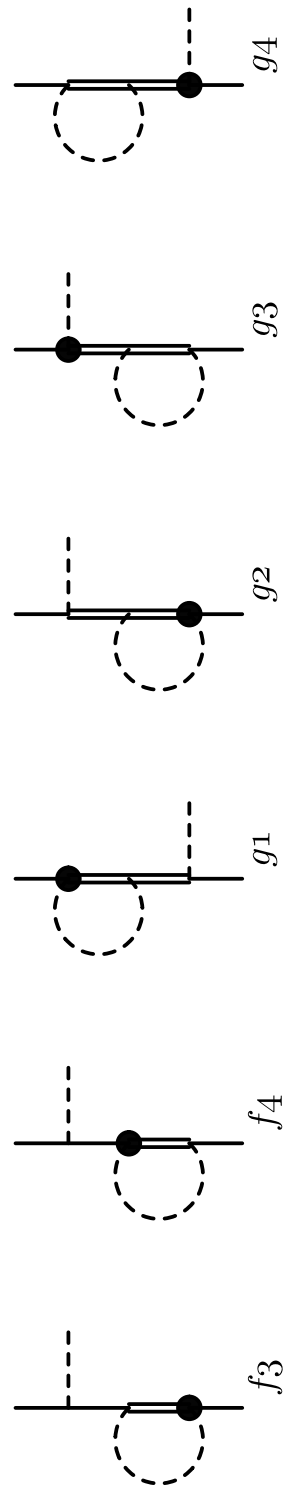
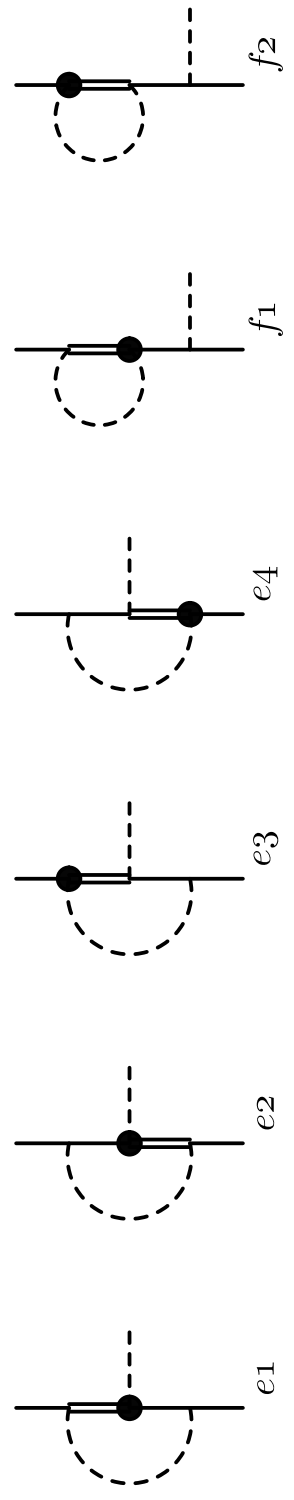
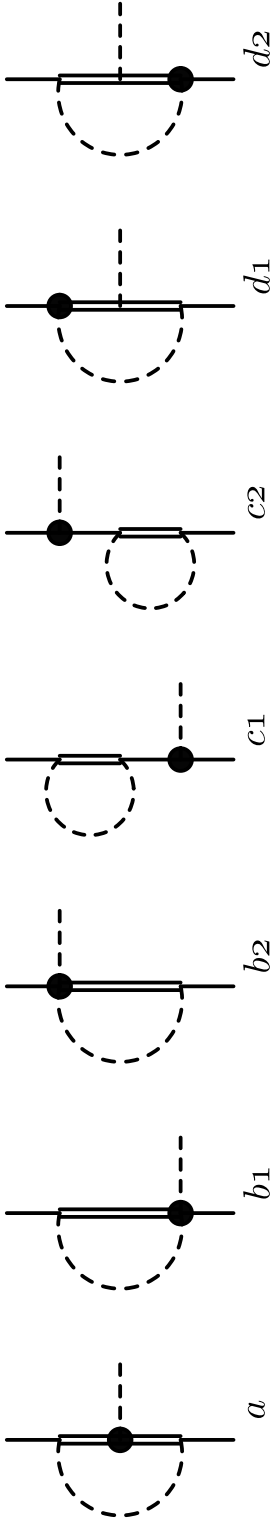
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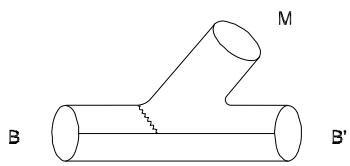


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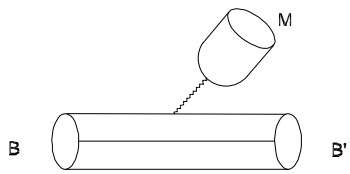


*k*

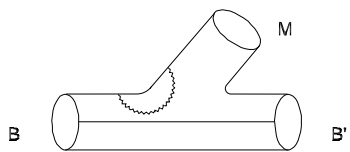




(a)



(b)



(c)

