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Chiral Symmetry Breaking and $K_S^0 \to 2\pi$ Decay

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The suppression of the decay amplitude for $K_S{}^0 \to 2\pi$ in the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model of chiral symmetry breaking is examined. Pole model calculations show that the suppression factor is not ε^2 as Dashen stated. It is pointed out that the number of emerging pseudoscalar Goldstone bosons is less than eight, and that degenerate perturbation calculations become meaningless in this case.

§ 1.

In the chiral invariant scheme of strong interaction, the important problem is to determine the symmetry breaking term, which is not yet fixed in a satisfactory way.

Among the various models of chiral symmetry breaking, the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model[1] is the simplest and promising one. The smallness of the pion mass compared to the K and η masses can be naturally explained in this model. In a wide class of gauge theories of the weak, electromagnetic and strong interactions, the $(3, \bar{3}) \oplus (\bar{3}, 3)$ breaking seems to be a natural scheme which can readily be incorporated.[2] For these reasons and others, it is interesting to know whether the chiral symmetry breaking part of strong interaction Hamiltonian transforms like pure $(3, \bar{3}) \oplus (\bar{3}, 3)$ or not.

As for the phenomenological analyses on symmetry breaking, it can be said that almost no explicit disagreement against the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model is found, though present experimental and theoretical situations are still rather ambiguous. For some time the K_{l3} decay[3] and σ-term[4] in π-N scattering seemed to show difficulties for $(3, \bar{3}) \oplus (\bar{3}, 3)$ model. However, the experimental data are still fluttering in the case of K_{l3} decay[5] and there are some ambiguities especially in phase shift analyses in the case of π-N scattering.[6] Thus we may expect that these will become consistent with $(3, \bar{3}) \oplus (\bar{3}, 3)$ model in future.

On the other hand Dashen claimed that the decay amplitude for $K_S{}^0 \to 2\pi$ should be suppressed by a factor of ε^2 in the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model, where ε is a measure of chiral $SU(3) \otimes SU(3)$ symmetry breaking in the strong interaction.[7] This predicts the value of about 10^{-7} sec, which is several orders larger than the experimental value, for the $K_S{}^0$ lifetime. This might be thought as the only example that contradicts $(3, \bar{3}) \oplus (\bar{3}, 3)$ model explicitly at the present stage.

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In this paper we reinvestigate this problem and show that this difficulty is not that of specific model, but is caused by an inadequate treatment.

§ 2.

It has been known that, neglecting the CP violation, the decay amplitude for $K_S^0 \to 2\pi$ is of order $G\lambda$ where λ is a parameter which measures $SU(3)$ breaking in the strong interaction.[8] But Dashen showed that it is of order $G\varepsilon^2$ in the $(3,\bar{3})\oplus(\bar{3},3)$ model of chiral symmetry breaking. These results seem to contradict each other, since ε and λ are essentially the same. So we first calculate the decay amplitude for $K_S^0 \to 2\pi$ in the pole model assuming a simple model Lagrangian, and see whether the amplitude is of order $G\varepsilon$ or not in the $(3,\bar{3})\oplus(\bar{3},3)$ model.

We introduce basic fields σ_i, $\phi_i (i=0, 1, \cdots, 8)$ which transform according to the representation $(3,\bar{3})\oplus(\bar{3},3)$ of chiral $SU(3)\otimes SU(3)$ group. The following notation is used hereafter:

$$M \equiv \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i (\sigma_i + i\phi_i) \equiv \sigma + i\phi \,,$$

$$M^\dagger \equiv \frac{1}{\sqrt{2}} \sum_{i=0}^{8} \lambda_i (\sigma_i - i\phi_i) \equiv \sigma - i\phi \,. \tag{1}$$

The strong interaction Lagrangian is assumed to have the form

$$\mathcal{L}_{st} = \frac{1}{2} \operatorname{Tr} \partial_\mu M \partial^\mu M^\dagger - \frac{1}{2}\mu^2 \operatorname{Tr} MM^\dagger - \frac{g}{4} \operatorname{Tr} MM^\dagger MM^\dagger + \varepsilon_0 \sigma_0 + \varepsilon_8 \sigma_8 \,. \tag{2}$$

This form of the Lagrangian is not to be thought as a realistic one, but might be sufficient for our purpose. The strong interaction Lagrangian (2) can be treated in a conventional way if we reexpress the Lagrangian in terms of new variables

$$\sigma' \equiv \sigma - \Sigma \,, \tag{3}$$

where

$$\Sigma = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix} \tag{4}$$

is determined from the condition that terms linear in σ' disappear from the Lagrangian (2). We choose Σ such that $a = b = \sqrt{-\mu^2}/g$ in the chiral invarant limit $(\varepsilon_0 = \varepsilon_8 = 0)$, then the fields ϕ_i $(i=1, 2, \cdots, 8)$ appear as octet of Goldstone particles. In this case, a and b are of zeroth order in ε, and

$$a - b = \frac{1}{m_{\sigma_7}^2} \sqrt{\frac{3}{2}} \varepsilon_8 \,, \tag{5}$$

where m_{σ_7} is the mass of the scalar K meson.

The weak interaction Lagrangian which satisfies CP conservation and $\Delta I = 1/2$

rule is chosen to be

$$\mathcal{L}_W = -G\,\mathrm{Tr}\,\lambda_6 M^\dagger M. \tag{6}$$

Using the Lagrangians (2) and (6), we calculate the amplitude for $K_S^0 \to 2\pi$ in the pole model approximation. The result is

$$A(K_S^0 \to \pi^0 + \pi^0) = i\sqrt{2}G\frac{a-b}{b(a+b)}\left\{1 - \frac{2}{3}g\frac{1}{m_{\phi_7}^2}(2a-b)(a-b)\right\}. \tag{7}$$

Equation (7) indicates that the amplitude is of order $G\varepsilon$. This gives the counter-example against Dashen's proof that the decay amplitude for $K_S^0 \to 2\pi$ is of order $G\varepsilon^2$ in the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model.

We shall show a more direct counterexample which leads to contradiction with the Callan-Treiman relation between the amplitude for $K_S^0 \to \pi^+ + \pi^-$ and that for $K_L^0 \to \pi^+ + \pi^- + \pi^0$.[9] The relation

$$A(K_S^0 \to \pi^+ + \pi^-) = \frac{1}{f_\pi} A[K_L^0 \to \pi^+ + \pi^- + \pi^0;\ q(\pi^0) = 0] \tag{8}$$

indicates that $A(K_S \to 2\pi)$ and $A(K_L \to 3\pi;\ q(\pi^0) = 0)$ should be of the same order in ε, since the pion decay constant $1/2f_\pi$ remains finite when $\varepsilon = 0$. This relation holds in the $(3, \bar{3}) \oplus (\bar{3}, 3)$ model. To keep consistency with Eq. (8), $A(K_L^0 \to 3\pi;\ q(\pi^0) = 0)$ must be of order $G\varepsilon^2$ in Dashen's scheme, but this is not the case. To show this we define the following quantity

$$\begin{aligned} F(q_a^2, q_b^2, q_c^2; q_d = 0) &= 16 f_a f_b f_c f_d \frac{m_a^2 - q_a^2}{m_a^2}\frac{m_b^2 - q_b^2}{m_b^2}\frac{m_c^2 - q_c^2}{m_c^2}\frac{m_d^2}{m_d^2} \\ &\times \int d^4x\, d^4y\, d^4z \exp\{i(q_a x + q_b y + q_c z)\} \\ &\times \langle 0|T(\partial_\mu j_a^\mu(x)\partial_\nu j_b^\nu(y)\partial_\lambda j_c^\lambda(z)\partial_\sigma j_d^\sigma(0))|0\rangle. \end{aligned} \tag{9}$$

Under the conditions $q_a + q_b + q_c = 0$, $q_d = 0$, it becomes

$$F(m_a^2, m_b^2, m_c^2;\ q_d = 0) = i\, g_{abcd}(q_d = 0), \tag{10}$$

where $g_{abcd}(q_d = 0)$ is the off-shell quardrilinear coupling of Goldstone bosons. With the perturbation $\varepsilon\mathcal{H}'$ belonging to $(3, \bar{3}) \oplus (\bar{3}, 3)$, one can show that

$$\begin{aligned} g_{abcd}(q_d = 0) &= i f_a f_b f_c f_d \{\sum_e c_{abcde}\langle 0|[\boldsymbol{Q}_e^+, \varepsilon\mathcal{H}''(0)]|0\rangle \\ &+ d_{abcd}\langle 0|\varepsilon\mathcal{H}'(0)|0\rangle\} + O(\varepsilon^2), \end{aligned} \tag{11}$$

where c_{abcde} and d_{abcd} are defined by the following relation:

$$\sum_{\text{permutation}} \lambda_a\lambda_b\lambda_c\lambda_d = 8ic_{abcde}\lambda_e + 16 d_{abcd},$$

and $\mathcal{H}''$ is obtained from $\mathcal{H}'$ by replacing σ_0 and σ_8 by ϕ_0 and ϕ_8, respectively. In contrast to the case of $K_S \to 2\pi$, $g_{abcd}(q_d = 0)$ remains to be nonvanishing and, therefore, a first-order quantity in ε, since the dependence of the c's and the d's on suffices is different.

§ 3.

From the two examples in § 2, it is obvious that Dashen's proof is inconsistent with the correct treatment of the degenerate perturbation theory. In the following we consider the reason why the misleading result was derived in his scheme.

If the nonleptonic part of the weak interaction Hamiltonian GH_W is built of the V-A currents, the following relation holds:

$$[\boldsymbol{Q}_a{}^+, H_0+GH_w]=0, \tag{12}$$

where $\boldsymbol{Q}_a{}^+ = (\boldsymbol{Q}_a+\boldsymbol{Q}_a{}^5)/2$ $(a=1, 2, \cdots, 8)$.

The crucial point of Dashen's proof[7] is that H_0+GH_w has right-handed $SU(3)$ symmetry due to Eq. (12), and that this $SU_R(3)$ symmetry breaks down spontaneously. He claimed that adding GH_w to H_0 leaves π, K, η as exactly massless Goldstone particles. If this is the case, the conclusion that the amplitude for $K_S \to 2\pi$ is suppressed by a factor of ε^2 necessarily follows taking $\varepsilon H'$ as a perturbation term.

However, adding GH_w to H_0 does not leave all of π, K, η exactly massless Goldstone particles. We show this by using a classical model satisfying Eq. (12).

The potential V_0 of chiral invariant strong interaction is taken as some function of four invariants I_i,[10] which are formed of M, $M^\dagger$,

$$V_0=f(I_2, I_3, I_4). \tag{13}$$

The potential V_w of weak interaction is assumed to have the specific form

$$V_w=G\,\mathrm{Tr}\,\lambda_6 M^\dagger M, \tag{14}$$

where G is assumed to be a positive constant for simplicity. Then the potential V, sum of V_0 and V_w, satisfies the condition (12).

We diagonalize the matrix λ_6 by performing a suitable transformation:

$$V=f(\tilde{I}_2, \tilde{I}_3{}^\pm, \tilde{I}_4)+G\,\mathrm{Tr}\begin{bmatrix}0&&\\&1&\\&&-1\end{bmatrix}\widetilde{M}^\dagger\widetilde{M}, \tag{15}$$

where

$$U\lambda_6 U^{-1}=\begin{bmatrix}0&&\\&1&\\&&-1\end{bmatrix}, \quad UMU^{-1}=\widetilde{M},\; UI_iU^{-1}=\tilde{I}_i. \tag{16}$$

We introduce vectors $\tilde{\phi}_i$ in the following way:

$$M=[\tilde{\phi}_1, \tilde{\phi}_2, \tilde{\phi}_3]. \tag{17}$$

When V has equilibrium solutions at nonvanishing σ_i's and ϕ_i's, the symmetry of H_0+GH_w breaks down spontaneously.

When V_0 has a form

$$V_0=-\frac{1}{2}\mu_0{}^2 I_2+\frac{1}{4}\lambda^2 I_4, \tag{18}$$

the equilibrium solution is given by

$$\tilde{\phi}_i^\dagger \tilde{\phi}_i = \alpha_i ,$$

$$\alpha_1 = \frac{\mu_0^2}{\lambda^2}, \quad \alpha_2 = \frac{\mu_0^2 - 2G}{\lambda^2}, \quad \alpha_3 = \frac{\mu_0^2 + 2G}{\lambda^2} . \tag{19}$$

From Eq. (19), we can define the vacuum state of $H_0 + GH_w$ which satisfies the condition

$$\langle \tilde{M} \rangle_0 = \begin{pmatrix} \sqrt{\alpha_1} & 0 & 0 \\ 0 & \sqrt{\alpha_2} & 0 \\ 0 & 0 & \sqrt{\alpha_3} \end{pmatrix} . \tag{20}$$

Here we note that all of the square roots should have the same sign to ensure the nonnegative property of the second order variation of the potential.

The number of Goldstone bosons is obtained in a following way.[11] We consider the 18 dimentional representation space of the σ_i's and ϕ_i's. In this space we obtain an orbit $\mathcal{O}$ by the transitive action of the symmetry group, which the Lagrangian initially has, to the equilibrium solution (19). Then the dimension of the orbit $\mathcal{O}$ in this space equals the number of emerging Goldstone bosons. In this case the symmetry group is $[U(1)_L]^2 \otimes SU(3)$, and the dimension of orbit, just the number of emerging Goldstone bosons, is eight. Except for two pseudoscalar bosons P_6, $P_3 + P_8/\sqrt{3}$, they have no definite parity.

Although the above argument depends on the special solution of specific model, some of the results remain valid in the general case. The potential (15) has the symmetry $U(1)_L \times U(1)_L \times SU(3)_R$, where the symmetry groups $U(1)_L$'s are generated by $\boldsymbol{Q}_3^- + \boldsymbol{Q}_8^-/\sqrt{3}$ and $\boldsymbol{Q}_6^-$, respectively. This symmetry property is due to the specific form of GH_w. If the weak interaction has a general form except that it is $SU_R(3)$ invariant and conserves electromagnetic charge, the symmetry of the total Hamiltonian is $U(1)_L \times SU(3)_R$. Here the generators of the group are $\boldsymbol{Q}_3^- + \boldsymbol{Q}_8^-/\sqrt{3}$ and $\boldsymbol{Q}_\alpha^+$ $(\alpha = 1, \cdots, 8)$. The electromagnetic charge should be conserved even after the spontaneous breaking down of the symmetry. Then the maximal set of the possible Goldstone bosons will be $P_3 + P_8/\sqrt{3}$ and the seven spin 0 bosons with indefinite parity. We can even construct a model which gives Goldstone bosons less than eight. In any case, they cannot be identified with pseudoscalar octet.

It should be noted that the addition of GH_w breaks the $SU(3)$ symmetry of H_0 which remains after the spontaneous breaking down of chiral symmetry. Then it is obvious that the pseudoscalar octet cannot be left as exact Goldstone bosons by the inclusion of GH_w.

We are now in a position to clarify the reason why in Dashen's scheme the decay amplitude for $K_S \to 2\pi$ appeared of order $G\varepsilon^2$ in contrast with the correct perturbation calculations. We have shown that some of π, K, η cannot remain massless exact Goldstone bosons after the addition of GH_w to H_0. It makes the degenerate vacuum state of H_0 split, and the level difference is the order of typical matrix element of the weak interaction Hamiltonian. Since the chiral symmetry breaking part $\varepsilon H'$ of strong interaction, which is treated as the perturbation term in Dashen's calculation, is fairly large compared with the level difference of unperturbed Hamiltonian H_0+GH_w, it is clear that Dashen has made a meaningless calculation in this problem. It has already been pointed out by Hori and Chiba[12] in the case of hyperon decay that no physically reasonable answer is obtained by a calculation taking H_0+GH_w as an unperturbed Hamiltonian.

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