CERN-TH.3990/84
PM 84/22

CHIRAL SYMMETRY BREAKING AND THE LIGHT MESON SYSTEMS*)

S. Narison

CERN - Geneva
and
USTL, Physique Mathématique et Théorique**)
F-34100 Montpellier

ABSTRACT

Existing estimates from the QCD-Laplace transform sum rules of the dynamical parameters which control the Gell-Mann, Oakes, Renner and Nambu-Goldstone realizations of chiral symmetry are systematically improved. Various sources of the light quark masses lead to a particularly accurate weighted average value of the strange quark mass while the ones of the quark vacuum condensate ratio $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle$ give a weighted average value which deviates significantly from the $SU(3)_V$ expectation. This latter fact is due to the important role of the m_s^2 and M_K^2 corrections in the analysis. A violation of the four-quark factorization hypothesis in the strange quark channel has been also observed like in the case of the u, d quarks. An upper bound three times smaller than the present experimental one is expected for the $\tau \to \nu_\tau K^*$ Cabibbo suppressed branching ratio.

---

*) Some parts of this work were presented at the XXII International Conference on High Energy Physics, Leipzig, 1984.

**) Equipe de Recherche associée au CNRS.

CERN-TH.3990/84
PM 84/22
September 1984

CONTENTS : page

1. INTRODUCTION

There are now various reasons to believe that the SU(3) gauge theory of quarks and gluons, quantum chromodynamics (QCD)[1], is the best candidate theory of hadronic physics, though many essential properties that it is presumed to have, such as confinement, dynamical mass generation and chiral symmetry breaking are poorly understood. Because of the complexity of the strong interaction phenomena which this theory describes, including for example nuclear physics, we cannot even dream of solving the SU(3) gauge theory exactly, as the S-matrix of this theory is far more complicated than anything which we can control. Therefore, a sort of approximation scheme is needed. Due to the asymptotic freedom property of QCD at large momentum transfer[1], a parametrization of a hadronic Green's function by a series of the renormalization group improved coupling constant, $\bar{\alpha}_s \equiv g^2/4\pi$, appears to give a good approximation as we know from various hard processes and quark and gluon jet phenomena[2]. We also know that the quark masses are important parameters in describing heavy quark bound states, which we believe can be described by non-relativistic approaches. In the light quark sector, i.e. the up and down quarks and presumably the strange quark as well, we have less controlled quantitative features as we have to deal with the long-distance dynamics of QCD. From the perspective of chiral symmetry, we know that for massless quarks or in the limit of large number of colour N[3], the QCD Lagrangian possesses a global chiral symmetry $SU(n)_L \times SU(n)_R \times U(1)_A \times U(1)_V$. The $U(1)$ symmetry is associated to the baryonic current.
The $G \equiv SU(n)_L \times SU(n)_R$ global symmetry is spanned by the generators which are the charges associated to the Noether axial-vector and vector currents

$$A^{\mu}{}^{i}_{j}(x) = \bar{\psi}_i \gamma^{\mu} \gamma^5 \psi_j \tag{1.1a}$$

$$V^{\mu}{}^{i}_{j}(x) = \bar{\psi}_i \gamma^{\mu} \psi_j \qquad i,j \equiv u,d,s\ldots \tag{1.1b}$$

acting on the quark flavour components and which satisfy the current algebra of Gell-Mann[4]. The symmetry framework inherited from the successes of current algebra and the pion PCAC[5] is the one where the axial charge does not annihilate the vacuum, i.e. the chiral symmetry is realized à la Nambu-Goldstone[6]. In this scheme, the chiral flavour group $SU(n)_L \times SU(n)_R$ is broken spontaneously by the quark vacuum condensate down to a subgroup $H \equiv SU(n)_{L+R}$ with respect to which the vacuum-condensates are symmetric :

$$< \bar{\psi}_u \psi_u > = < \bar{\psi}_d \psi_d > = < \bar{\psi}_s \psi_s > . \tag{1.2}$$

These features are expected to follow from the long-distance dynamics of the QCD Lagrangian, and indeed, there appears some evidence of such spontaneous breaking of chiral symmetry from lattice Monte-Carlo simulations[7a)] and from some other dynamical calculations[7b, c)]. This spontaneous breaking mechanism is accompannied by n^2-1 massless Goldstone bosons P which are associated to each unbroken generator of the coset space G/H. On the other hand, the vector charges annihilate the vacuum and the corresponding symmetry is realized à la Wigner-Weyl[8)], so that the corresponding particles are classified in irreducible representations of $SU(n)_{L+R}$. The massless Goldstone bosons can acquire a tiny electromagnetic mass[9)] and mainly a mass induced by an explicit breaking of the $SU(n)_L \times SU(n)_R$ global symmetry due to the quark mass terms in the QCD Lagrangian. In this way, the divergence of the axial-vector current is non-zero :

$$\partial_\mu A^\mu(x)^i_j = (m_i + m_j) \quad \bar{\psi}_i \, (i \gamma_5) \, \psi_j \tag{1.3}$$

to which are associated the "quasi-Goldstone" parameters :

$$< 0 | \partial_\mu A^\mu(x)^i_j | P > = \sqrt{2} \, M_P^2 \, f_P \tag{1.4}$$

where, in the $SU(3)_{L+R}$ limit, $f_P \simeq f_\pi \simeq 93.28$ MeV is the decay amplitude of these bosons. We also know that the spectrum of the pseudoscalar boson octet (π, K, η) does not show a degeneracy in their masses and then a large explicit breaking of the $SU(3)_L \times SU(3)_R$ scheme à la Gell-Mann, Oakes and Renner[10)] is suggested :

$$\mathcal{L}_{GOR}(x) = - \epsilon_o \, U_o(x) - \epsilon_8 \, U_8(x) - \epsilon_3 \, U_3(x) \tag{1.5}$$

where the Hermitian scalar densities $U_a(x)$ can be expressed in terms of quark bilinears :

$$U_a(x) = \mathrm{Tr} \, \bar{\psi}(x) \, \lambda_a \, \psi(x) \qquad a = 0,8,3$$

with

$$\lambda_o = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -2 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & & 0 \\ & -1 & \\ 0 & & 0 \end{pmatrix} \tag{1.6}$$

and where the symmetry breaking parameters ϵ_a are combinations of the quark masses :

$$\epsilon_o = \frac{1}{\sqrt{6}} \, (m_u + m_d + m_s) \tag{1.7a}$$

$$\epsilon_8 = \frac{1}{\sqrt{3}} \frac{1}{2} \, (m_u + m_d - 2m_s) \tag{1.7b}$$

$$\epsilon_3 = \frac{1}{2} \, (m_u - m_d) \quad . \tag{1.7c}$$

So, among other things, it is essential to have a good control of the quark mass values and of the deviation of the SU(3) expectation in Eq. (1.2) for a quantitative understanding of the explicit and of the spontaneous breaking of chiral symmetry. There are convincing estimates of the quark mass ratios from the comparison of various current algebra Ward identities at zero momentum transfer with physical parameters like the masses and coupling constant of hadrons, which give the so-called current algebra result.
The success of the current algebra prediction is mainly due to the fact that the ratio of the quark masses is defined unambiguously, as it is scale-independent and (or) needs not be renormalized. On the contrary, it is more difficult to make estimates of the absolute values of the light quark masses. In this paper, it is proposed to look at the chiral symmetry breaking parameters from the analysis of the short-distance behaviour of the correlation functions :

$$\psi_{(5)}{}^i_j\,(q^2) \equiv i \int d^4x\, e^{iqx} < 0 \,\mathbb{T}\, \partial_\mu A^\mu{}^i_j(x)\, (\partial_\mu A^\mu{}^i_j)^+ \mid 0 > \quad (1.8a)$$

$$\psi(q^2)^i_j \equiv i \int d^4x\, e^{iqx} < 0 \,\mathbb{T}\, \partial_\mu V^\mu{}^i_j(x)\, (\partial_\mu V^\mu{}^i_j)^+ \mid 0 > \quad (1.8b)$$

$$\Pi_V^{\mu\nu}(q^2)^i_j \equiv i \int d^4x\, e^{iqx} < 0 \mid \mathbb{T}\, V^\mu{}^i_j(x)\, (V^\nu{}^i_j(o)\,)^+ \mid 0 > \quad (1.9a)$$

$$\Pi_A^{\mu\nu}(q^2)^i_j \equiv i \int d^4x\, e^{iqx} < 0 \mid \mathbb{T}\, A^\mu{}^i_j(x)\, (A^\nu(o)\,{}^i_j)^+ \mid 0 > \quad (1.9b)$$

built respectively from the divergence of the axial and vector currents and from the vector and axial-vector currents given in Eqs (1.1), (1.3) and the divergence :

$$\partial_\mu V^\mu{}^i_j = (m_i - m_j)\quad \bar{\psi}_i\,(i)\,\psi_j\,. \quad (1.10)$$

It has been known for a long time that the Weinberg sum rules[11a)] which are superconvergent in the chiral limit $(m_j = 0)$*):

$$\int_0^\infty dt\, [\mathrm{Im}(\Pi_V^{(1)} + \Pi_V^{(0)}) - \mathrm{Im}(\Pi_A^{(1)} + \Pi_A^{(0)})(t)] = 0 \quad 1^{\underline{st}} \text{ sum rule} \quad (1.11a)$$

$$\int_0^\infty dt\, t\,(\mathrm{Im}\, \Pi_V^{(1)} - \mathrm{Im}\, \Pi_A^{(1)})(t) = 0 \quad 2^{\underline{nd}} \text{ sum rule} \quad (1.11b)$$

$$\int_0^\infty dt\, t(\mathrm{Im}\, \Pi_V^{(0)} - \mathrm{Im}\, \Pi_A^{(0)})(t) = 0 \quad 3^{\underline{rd}} \text{ sum rule} \quad (1.11c)$$

---

*) The indices (1) and (0) correspond to the spin 1 and 0 parts of the spectral function, i.e. to the transverse part and to the longitudinal one.

can become a good place to measure the strength of the chiral symmetry breaking parameters once one has a good control of the spectral functions in the LHS of the sum rules, because it is known within QCD that the quark mass terms and the quark vacuum condensates break such a convergence property [11b,d]. Here, I shall concentrate on the analysis of the two-point functions Eq (1.9 a-c) as the latter in Eq (1.9 d) can be deduced from the others via the Weinberg sum rules. The analysis consists mainly of the improvement of previous estimates given in Refs. 12d) to 21). The short-distance behaviour of the two-point function can be studied along the line proposed by SVZ[22], using the Wilson operator product expansion (OPE) which provides a systematic framework for parametrizing non-perturbative effects. These appear as $\frac{1}{Q^2}$ $(Q^2 \equiv -q^2 > 0)$ power corrections to the usual asymptotic freedom perturbative behaviour of the two-point function with coefficients which are proportional to the vacuum expectation value of higher dimensional operators like the quark condensates $<\bar\psi\psi>$, $<\bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi>$ (Γ_1 is any Dirac matrix), the gluon condensates $\alpha_s<F^a_{\mu\nu}F^{\mu\nu}_a>$, $g\,f_{abc}<F^{\mu\nu}_{\lambda a}F_{\nu\rho,b}F^{\rho}_{\mu,c}>$ and the mixed quark-gluon condensate $<\bar\psi\sigma^{\mu\nu}\frac{\lambda^a}{2}\psi F^{\mu\nu}_a>$[*]. These low-dimension vacuum condensates can be interpreted actually as a manifestation of the large size instanton while a small size instanton manifests itself as a power of $1/Q$ larger than 11[**]. Small size instantons can, in principle, break the OPE but it is easy to notice that their effects are very sensitive to values of the QCD scale Λ and will be highly suppressed in the analysis of this paper. In addition, they are, for example, also proportionnal to the light quark mass values in the vector meson channels and so they can be completely neglected compared to the contributions of the large size instanton[***].

---

*) An example of two-point hadronic correlation functions exhibiting these different contributions is shown in Fig. 1.

**) Original works on the subject are reviewed in Ref 22).

***) If we use, for instance, the result of the analysis by J. Ellis et al given by Ref 22), one can realize that for the present values of $\Lambda_{\overline{MS}} \simeq 100$ MeV and of the light quark masses (see the table given in the next section), the contribution of the small size instanton is completely negligible for Q larger than 0.6 GeV.

Using the analyticity of the hadronic correlation function, one can relate the short distance expansion to the spectral function which is governed by the low energy data in writing the Källen-Lehmann or the Hilbert representation of such hadronic correlation functions.The various forms of QCD sum rules consist of the improvement of the naive Hilbert representation in order to obtain much more information both from the available low energy data and from the few terms retained in the OPE. Sum rules prior to the SVZ work can be seen in Refs [11 b)] and [23)] while the ones within the spirit of the SVZ OPE are reviewed in Ref [24)]. In this paper, I shall mainly be interested in the cases of the Laplace (Borel) [22, 13 b, 25)] transform sum rule which, for instance, is :

$$\mathcal{F}(\tau) = \tau^3 \int_0^\infty dt\, e^{-t\tau}\, \frac{1}{\pi}\, \mathrm{Im}\, \psi_{(5)}{}^{i}_{j}(t) \tag{1.12}$$

and which can be obtained by applying to the two-point function or to its first convergent derivative, the Laplace operator :

$$\hat{\mathbb{L}} \equiv \lim_{\substack{N \to \infty \\ Q^2 \to \infty}} \;\frac{N}{Q^2} \equiv \tau \quad (-1)^N \frac{1}{(N-1)!} \quad (Q^2)^N \quad \frac{\partial^N}{(\partial Q^2)^N} \quad . \tag{1.13}$$

It is clear that due to the exponential factor appearing in the RHS of Eq (1.12), the sum rule is very sensitive to the low energy behaviour of the spectral function, for moderate values of the sum rule scale variable τ which are of the typical hadronic mass value, i.e., the sum rule is sensitive to the small-t region which in most cases is the only one where some phenomenological information is available. Also, the contribution of the $n^{\underline{th}}$ higher dimensions condensates is suppressed by a $\frac{1}{n!}$ factor compared to the original $\psi_{(5)}(q^2)$ two-point function. In section 2, I shall discuss the determinations of the quark masses from a more careful analysis of the pseudoscalar two-point function following the line in Refs [12d - 16)]. In section 3, I extract the decay amplitudes of the scalar mesons in the aim of studying in section 4, the deviation from the $SU(3)_{L+R}$ expectations of the quark vacuum condensate values which serves to check the results obtained in Refs [18) to 21)]. I discuss in sections 5 and 6 the vector two-point function in connection with the recent results of the strange quark parameters obtained in Ref [17)] from this channel. I finally discuss in section 7 an improved form of the Gell-Mann-Okubo mass formula for vector mesons.

2. LIGHT QUARK MASSES FROM THE PSEUDOSCALAR SUM RULES TO THREE-LOOPS

Since the first attempt of Leutwyler[26] to estimate the absolute values of the light quark masses using a $U(6)$ symmetry in order to relate the π and ρ Bethe-Salpeter wave functions, there have been important efforts using the QCD sum rule approach for the estimate of the absolute values of the quark masses[12-17] and for the interpretation of the following Leutwyler formula :

$$\frac{1}{2}(m_u + m_d) \simeq \frac{2}{3} f_\pi \left(\frac{m_\pi}{M_\rho}\right)^2 \gamma_\rho \simeq 5.4 \text{ MeV} \tag{2.1}$$

within QCD[13b) ,

where $f_\pi \simeq 93.28$ MeV is the pion decay amplitude ; γ_ρ is the ρ-coupling to the electromagnetic current with :

$$\Gamma_{\rho\to e^+e^-} \simeq \frac{2}{3}\pi\alpha^2 \frac{M_\rho}{2\gamma_\rho^2} \quad . \tag{2.2}$$

The quark masses are the ones which appear in the QCD Lagrangian and which provide an explicit realization of the Gell-Mann-Oakes-Renner scheme of chiral symmetry breaking[10]. However, it is difficult to interpret the result in Eq (2.1) within QCD, as one does not really know what is the exact meaning of the current algebra mass value in terms of the so-called running mass of QCD[23b, 27] because one does not know at what scale this running mass should be evaluated, or even what type of renormalization scheme[28] should be used for defining such a running mass. Within the QCD sum rule approach, the battle-horses for hunting the quark masses are the two-point functions[13] $\psi_{(5)}(q^2)$ (see Eq 1.8) associated to the divergence of the axial and (or) vector currents[14] as they are sensitive to leading order to the absolute values of the quark masses. We have at present a good control of the two-point function from QCD and of its spectral function. $\psi_5(-q^2 \gg \Lambda^2)$ is known to two[13] and three[15]-loops from a QCD perturbative calculation. The non-perturbative contribution to $\psi_5(q^2)$ is known up to the dimension six vacuum condensate contributions[22]. The direct instanton contribution is known within the dilute gas approximation. It has been discussed[29] that such a contribution is proportional to the instanton density $d(\tau) \simeq (\Lambda_{eff}\sqrt{\tau})^{11}$, where τ is the "imaginary time" sum rule variable and Λ_{eff} is an effective scale which takes into account the pre-exponential factor and other coefficients in the relation between $d(\tau)$ and the $\overline{MS}$-scheme[28] scale Λ. It is difficult to state how accurate this way of parametrizing the instanton effect is, but within such an approximation it is easy to be convinced that the direct instanton effect can be safely neglected for τ smaller or equal to 1 GeV^{-2} for a value of Λ of the order of

100 MeV[30)] which corresponds to $\Lambda_{eff} \simeq 0.15\,\mathrm{GeV}$ [29)]. The spectral function $\mathrm{Im}\,\psi_5\,(t)$ is also under control as one has experimental data [31)] on the first excitations of the π and K mesons and also various consistent theoretical results for the coupling of these excitations to the pseudoscalar currents. It is clear enough that due to the quasi-Goldstone nature of the π and K mesons, one cannot have a naive copy of the ρ-meson picture in the pseudoscalar channel. We know from a Laplace (Borel) sum rule method that the ρ-meson alone can (roughly speaking) saturate the sum rule at the value of τ of the order of the hadronic scale M_ρ^{-2}, and one can already consider the ρ' (1.6) to belong to the continuum [22)]. In the pseudoscalar channel, a more consistent analogue of the ρ-meson case needs a saturation of the spectral function by the two first ground states but not by the lowest ground state alone. This particular situation can be understood by the too-small value of the quasi-Goldstone mass compared to the hadronic scale, say 1 GeV^{-2} *) and secondly by the fact that the contributions of the π and of its first excitation (the π') to the spectral function $\mathrm{Im}\,\psi_5(t)$ have the same dependence on m_π^2 (recall that $f_{\pi'}$ vanishes like m_π^2 from the current algebra analysis [5)]).

In my opinion, the most relevant effort in parametrizing the $\pi'(K')$ contribution to the spectral function is the one in Ref 16), where the π'-finite width effect to the sum rule is taken into account. The approach of Ref 16) is based on an inspired-Veneziano linear dual model[33)] where the spectral function is approximated by (in principle) infinite series of resonances. The parameters of the model have been tested and fixed from various low-energy processes which show a departure from the PCAC predictions. In this way, the Laplace-transformed spectral function reads[16)] in the pion channel :

$$\int_0^\infty dt\; e^{-t\tau}\,\frac{1}{\pi}\,\mathrm{Im}\,\psi_{5_d}^{u}\,(t) \simeq 2\, m_\pi^4\, f_\pi^2\; e^{-m_\pi^2\tau}\,\{1 + r_\pi(\tau)\} + \text{"QCD continuum"} \qquad (2.3)$$

with the phenomenological parametrization :

$$r_\pi(\tau) \simeq \frac{8}{3}\left(\frac{M_{\pi'}}{8\pi\, f_\pi}\right)^4 e^{m_\pi^2\tau}\,(1+\gamma^2)\int_0^\infty dt\; e^{-t\tau}\,\frac{t}{(M_{\pi'}^2 - t)^2 + \gamma^2 M_{\pi'}^4} \qquad (2.4)$$

---

*) If one insists on saturating the spectral function by the lowest ground state, one should go to a large value of τ where the instanton dominates the sum rule and a consistency of the theoretical and spectral parts of the sum rule is lost. This situation has been observed in Ref 32).

where $\gamma \simeq 0.1$ and $M_{\pi'} \simeq 1.1$ GeV within the model.*) The QCD continuum is the one obtained from the discontinuity of the two-point function in Fig. 1a-c and it averages the contributions of the higher excitations. Its threshold starts around the π''-mass, which is about 1.55 GeV within the model and which is of the order of 1.73 GeV if one uses the experimental mass of the π''[31]. The Laplace transform of the two-point function is known to three loops[15] and up to the dimension six vacuum condensates[22]. Combining the QCD information with the one in Eq (2.3), one can derive the constraints in the $\overline{MS}$-scheme for $SU(3)_C \times SU(3)_F$:

$$
\begin{aligned}
(\hat{m}_u + \hat{m}_d)^2 \simeq \frac{16\pi^2}{3} m_\pi^4 f_\pi^2 \, e^{-m_\pi^2 \tau} (1+r_\pi(\tau))\, \tau^2 \left(\frac{L}{2}\right)^{8/9} . \\
\cdot \Big\{ [1-(1+t_c\tau)\, e^{-t_c\tau}]\, [1 + \frac{2.94}{L} - 0.7\frac{\log L}{L} \\
+ \frac{3.21}{L^2} - 4.93\frac{\log L}{L^2} + 0.52\left(\frac{\log L}{L}\right)^2] \\
- 2\,(\bar{m}_u^2 + \bar{m}_d^2 - \bar{m}_u \bar{m}_d)\,\tau \\
+ \tau^2 [\frac{\pi}{3}\alpha_s \langle F^2\rangle - \frac{8\pi^2}{3}[(\bar{m}_d - \frac{\bar{m}_u}{2}) \langle \bar{u}u\rangle + (u \leftrightarrow d)] \\
+ C_6 \langle O_6\rangle \tau^3 \Big\}^{-1}
\end{aligned}
\qquad (2.5)
$$

where : $L \equiv -\log \tau \Lambda^2$; $\hat{m}_i$ is the renormalization group invariant mass[23b] which is related to the running mass $\bar{m}_i$ to three-loops as :

$$
\bar{m}(\tau) = \hat{m}\left(\frac{9}{2}\frac{\bar{\alpha}_s}{\pi}\right)^{4/9} \{1 + 0.895\frac{\bar{\alpha}_s}{\pi} + 2.707\left(\frac{\bar{\alpha}_s}{\pi}\right)^2\} \qquad (2.6)
$$

with the corresponding QCD coupling to three-loops :

$$
\left(\frac{\bar{\alpha}_s}{\pi}\right) = \left(\frac{4}{9L}\right) \{1 - 0.79\frac{\mathrm{Log}\, L}{L} + 0.62\frac{\log^2 L}{L^2} - 0.62\frac{\log L}{L^2}\} ; \qquad (2.7)
$$

$\alpha_s \langle F^2\rangle = (0.04 \pm 0.01$ GeV^4[34])[35] is the gluon vacuum condensate ;

---

*) If, instead, one uses $M_{\pi'} \simeq 1.24$ GeV from the data, one can check that the analysis is not significantly affected by this change. It is important to notice that the large value of the sum rule scale variable in the pseudo-scalar channel ($\tau^{-1} \geq 2$ GeV^2) is dual to a π'-mass larger than 1.7 GeV[23]. A value of $M_{\pi'} \simeq 1.24$ GeV is then consistent with a smaller value of the sum rule scale ! I thank also C.A. Dominquez for discussions on the parametrization of the spectral function using the "linear dual model".

$< \bar{u}\, u >$ is the quark vacuum condensate which is known through the PCAC relation[5) *)] :

$$(m_u + m_d) < \bar{u}u + \bar{d}d > \simeq 2\, m_\pi^2\, f_\pi^2 \quad . \tag{2.8}$$

$C_6 < O_6 >$ is the dimension-six vacuum condensate contributions which is :

$$C_6 < O_6 > \simeq 0.12 \text{ GeV}^6 \tag{2.9}$$

if one uses the phenomenological estimate in Ref. 35, which shows that the factorization hypothesis for the estimate of the $C_6 < O_6 >$ which yields to $\frac{896}{81} \pi^3\, \alpha_s < \bar{u}u >^2$ is an underestimate of a factor of the order four of the actual value of the operator (see also Ref 36). However, one should mention that the effects of the dimension-six operator are not relevant in the range of τ where we are working. I give in Fig. 2 the prediction of $\hat{m}_u + \hat{m}_d$ for the value of Λ which is 100 MeV. The previous bound of Ref 13b) using the pion pole plus the positivity of the spectral function is shown as well as the bound including the π'- contribution. One can notice that the optimal bound of Ref 13b) is almost saturated when one adds the excitation and the QCD continuum. One can see in the two continuous curves that the QCD continuum is necessary for the stabilization of the predictions. It starts to exceed the $\pi + \pi'$- contributions to the sum rule for $\tau^{-\frac{1}{2}}$ larger than 1.5 GeV as the curves start to increase. Comparing the two curves corresponding to the threshold of the QCD continuum at $t_c \simeq 2.4$ GeV^2, one can see that the finite width parametrization gives a much more stable result than the one with a δ-function for the π'. In Fig 3, I analyze the effect of the value of Λ on the predictions. The final result takes into account the changes of Λ between 100 to 150 MeV, the variation of the continuum threshold between 2.4 to 3 GeV^2. It corresponds to the "window" where the non-perturbative effects are irrelevant and the QCD continuum is less or equal to the lowest two ground states effect[**)]. Then, one gets :

$$\hat{m}_u + \hat{m}_d \simeq (27 \pm 5) \text{ MeV} \tag{2.8a}$$

where the central value corresponds to the smallest value of t_c where the

---

*) The mass m_i can either correspond to $\bar{m}_i$ or $\hat{m}_i$. As m_i enters into a renormalization group invariant quantity, one can refer to one or the other quantity.

**) It is interesting to notice from Eq(2.5) that the two loop effects are about 100% while the three-loop ones are only 10%, which can indicate that the convergence of the series in Eq (2.5) is quite good.

curve in Fig 2 starts to exhibit a "plateau of stability". The error bar in Eq (2.8) for a given value of Λ comes from the distance of the central curve and the ones corresponding to a narrow width for the π' or to the one where t_c moves around the experimental value of the π''-mass. The value of the running mass to three loops evaluated at 1 GeV is :

$$(\bar{m}_u + \bar{m}_d)\ (1\ \text{GeV}) \simeq (16 \pm 3)\ \text{MeV} \quad . \tag{2.8b}$$

It is interesting to compare the above results with the one obtained from a QCD finite energy sum rule (FESR)[37] analysis to three loops[15]. Let me recall that the FESR method gives a set of constraints among the operators of given dimension, the quark masses and the meson parameters. It has been demonstrated in Ref 37b) that the FESR constraints to leading order in the radiative QCD corrections can be derived from a Gaussian-like sum rule[37b] by using the orthogonality properties of the Hermite polynomials which are broken when radiative QCD corrections are included. The nice feature of the FESR constraints comes from the fact that the few first lowest dimension constraints used for the derivation of the quark mass values are unaffected by some eventual instanton contributions, as these latter would appear only in high dimension constraints (eleven dimensions [29]). These high dimension constraints should be affected by many other eventual uncontrolable contributions. The FESR result to three loop is [15]*) :

$$(\hat{m}_u + \hat{m}_d) \simeq (25 \pm 5)\ \text{MeV} \quad \text{for} \quad 100 \leq \Lambda \leq 150\ \text{MeV} \tag{2.9}$$

I present in the Table the range of values corresponding to Eqs(2.8) and (2.9). The intersection of the two ranges is taken as the final estimate coming from the pseudo-scalar channel which is :

$$(\hat{m}_u + \hat{m}_d) \simeq (26 \pm 4)\ \text{MeV}$$
$$(\bar{m}_u + \bar{m}_d)\ (1\ \text{GeV}) \simeq (15.4 \pm 2.4)\ \text{MeV} \tag{2.10}$$

for $100 \leq \Lambda \leq 150$ MeV and where I have used the definition of the running mass to three loops in Eq (2.6). The above results agree of course with the one obtained from the Laplace sum rule [12, 13, 16] and the FESR[38] to two loops. One can extend the analysis to the strange quark channel. The parametrization of the spectral function is very similar to the pion channel one

---

*) We learn from Prof E. de Rafael that he is deriving similar results with the Gaussian-like sum rule and using the "linear dual model" parametrization of the spectral function[16].

shown is Eq (2.3). The analogue of Eq (2.5) can be obtained by interchanging the d quark with the s-quark. The estimate of the quark vacuum condensate $m_{s,u} < \bar{s}\, s >$ is done by taking the conservative attitude that the kaon PCAC is uncertain within 50%. In this way, one solves iteratively the analogue of Eq (2.5). The result is summarized in Fig. 4.

In the "window" where one has a negligible effect of the non-perturbative terms and a contribution of the QCD continuum less than the two first resonances, we obtain the value :

$$\hat{m}_u + \hat{m}_s \simeq (350 \pm 86)\ \text{MeV} \tag{2.11}$$

where I have used $f_K \simeq 1.2\, f_\pi$ from recent estimates[39]. As in the pion case, the central value corresponds to the smallest value of t_c where one has a stability (Fig 4). The error bar takes into account the distance of this central value to the extremal ones obtained by moving t_c around the K"-mass obtained from the model and from the data. The error induced by the change of Λ between 100 and 150 MeV is also taken into account. The FESR results for $100 \leq \Lambda \leq 150$ MeV and for $f_K \simeq 1.2\, f_\pi$ can be deduced from the ones of Ref 15) which are :

$$\hat{m}_u + \hat{m}_s \simeq (302 \pm 60)\ \text{MeV}\ . \tag{2.12}$$

Showing such results in the Table, and taking the common range of values as a final estimate, one gets :

$$\hat{m}_s + \hat{m}_u \simeq (313 \pm 49)\ \text{MeV}$$

$$(\bar{m}_s + \bar{m}_u)\ (1\ \text{GeV}) \simeq (185.5 \pm 30)\ \text{MeV} \tag{2.13a}$$

and then :

$$r_s \equiv \frac{(\hat{m}_s + \hat{m}_u)}{(\hat{m}_u + \hat{m}_d)} \simeq (12.0 \pm 2.6)\ , \tag{2.13b}$$

which is well controlled from a current algebra pre-QCD analysis because the ratio of the quark masses is known to be scale independent and needs not be renormalized. The current algebra results are [12] :

$$r_s = (12.3 \pm 1.7)\ , \tag{2.14a}$$

$$\frac{m_d}{m_u} = \frac{\hat{m}_d}{\hat{m}_u} = (1.8 \pm 0.3) \tag{2.14b}$$

Using Eq (2.14b), one can deduce the final estimate from the pseudoscalar channel (see also Table)

$$\hat{m}_u \simeq (9.3 \pm 2.)\ \text{MeV} \qquad \bar{m}_u(1\ \text{GeV}) \simeq (5.5 \pm 1.2)\ \text{MeV}$$
$$\hat{m}_d \simeq (16.7 \pm 3.6)\ \text{MeV} \qquad \bar{m}_d(1\ \text{GeV}) \simeq (9.9 \pm 2.1)\ \text{MeV}$$
$$\hat{m}_s \simeq (303.7 \pm 49.2)\text{MeV} \qquad \bar{m}_s(1\ \text{GeV}) \simeq (180 \pm 29)\ \text{MeV} \qquad (2.15)$$

for $100 \leq \Lambda \leq 150$ MeV. The results in Eq (2.15) can be considered as an improvement of various results from the pseudoscalar channel[12d),13),15),16)]. In particular, one can also notice that the value of the strange quark mass agrees with the lower bound obtained from the analysis of the scalar two-point function $\psi(q^2)$ (Eq 1.9 b) to two loops[14)], where a parametrization of its spectral function by the $K\pi$ phase shift data and where a positivity of the higher states lying above 2 GeV have been used. The bound obtained in Ref 14) is :

$$(\hat{m}_s - \hat{m}_u) \gtrsim (210 \sim 240)\ \text{MeV} \quad \text{for} \quad 100 \leq \Lambda \leq 150\ \text{MeV}. \qquad (2.16)$$

One could also compare the results from the one issued from a QCD sum rule analysis in the baryon sector[24)]. In my opinion, the most serious attempt for the estimate of the chiral symmetry breaking parameters in the baryon sector is the one in Ref 36) where the effects of the radiative corrections, the higher dimension condensates, the factorization hypothesis of the four-quark operator and the choice of the nucleon interpolating fields have been taken into account in the analysis. In this way, the predictions from the baryon are in good agreement with the ones from the pseudoscalar sum rules. However, due to the complexity of the QCD calculation in the baryon sector, an independent check of the analysis should be useful and a much more accurate result is still needed.

3. THE SCALAR MESON DECAY AMPLITUDES

In this section, I shall study the two-point function $\psi(q^2)$ built from the divergence of the vector current (see Eqs 1.1b,2b). One can analyze the corresponding sum rule in order to extract the quark mass differences provided one introduces some phenomenological input for the spectral function[14)]. In my opinion, the analysis of the strange quark channel done in Ref 14),where the $K\pi$ phase shift data [40)] have been used for the estimate of the spectral function gives a rigorous bound for the $m_s - m_u$ quark mass difference. However, the analysis performed for the extraction of the $m_d - m_u$ quark mass difference uses a theoretical input for the δ-meson decay amplitude because of the lack of experimental data, and so the bound on $m_d - m_u$ becomes less rigorous. In the following, I extract the value of the decay amplitude of the δ and κ mesons using as input in the sum rule analysis the quark mass values obtained in section 2. So, I use the "duality ansatz" :

$$\frac{1}{\pi} \mathrm{Im}\ \psi_d^u(t) = 2 f_\delta^2 M_\delta^2\, \delta(t-M_\delta^2) + A(t)\, \Theta(t-t_c) \tag{3.1}$$

for the parametrization of the spectral function. $A(t)$ is the coefficient coming from the discontinuity of the diagram in Fig 1a-c ; f_δ is the δ-meson decay amplitude normalized as $f_\pi \simeq 93$ MeV and t_c is the continuum threshold. One expects that the above parametrization gives a good description of the spectral function as the lowest ground state mass is of the order of the typical hadronic mass of 1 GeV and so , due to the exponential factor of the Laplace sum rule, the contribution of the lowest mass resonance is optimized at the typical optimization scale of the sum rule which is of the order of the hadronic mass. One can fix the value of the continuum threshold using the linear dual Veneziano-type model[33]. In this way :

$$M_{\delta'}^2 \simeq t_c \simeq M_\delta^2 + 1/\alpha' \tag{3.2}$$

where $1/\alpha' \simeq 2 M_\rho^2$ is the (almost) Regge universal slope. One can also fix the value of the continuum threshold by assuming an asymptotic $SU(2)_L \times SU(2)_R$ symmetry for the scalar and pseudoscalar channel, i.e. one expects to have the same scaling behaviour in the two channels. So, I take the range of t_c :

$$t_c \simeq (2.4 \sim 3)\ \mathrm{GeV}^2 \tag{3.3}$$

One can use the analogue of Eq (2.3) in the scalar channel*) in order to give a prediction for the δ-decay amplitude. The results versus the sum rule

---

*) The modifications of Eq(2.3) correspond to the change $(m_u + m_d)^2$ into $(m_u - m_d)^2$ and to the change of the $m\langle\bar\psi\psi\rangle$ and $C_6\langle O_6\rangle$ contributions (see e.g Ref 14)).

scale τ are given in Fig. 5. Then, I deduce :

$$f_\delta \simeq (1.28 \pm 0.17)\ \mathrm{MeV} \tag{3.4}$$

One can compare this sum rule-prediction of f_δ with the one obtained using a δ-dominance of the $\delta K\bar{K}$ form factor which is normalized at $t = 0$ to the kaon-tadpole mass difference of the hadronic origin [14] :

$$f_\delta^{pole} \simeq \sqrt{6}\ \frac{1}{M_\delta}\ (M^2_{K^o} - M^2_{K^+})_{Tad} \tag{3.5}$$

where the kaon tadpole mass-difference was estimated several years ago using a pole dominance estimate of the $(K^+ - K^o)$ electromagnetic mass difference[41] and more recently in Ref[42]. Using the average of various estimates :

$$(M^2_{K^o} - M^2_{K^+})_{Tad} \simeq 6\ 10^{-3}\ \mathrm{GeV}^2 \tag{3.6a}$$

one deduces :

$$f_\delta^{pole} \simeq 1.8\ \ \mathrm{MeV}\ . \tag{3.6b}$$

The apparent discrepancy between Eqs (3.4) and (3.6) can be interpreted as being due to the non-negligible role of the continuum contribution in the analysis, making the pole dominance assumption a crude approximation. Actually, the result in Eq (3.6) should be interpreted as an upper bound rather than an estimate of the δ-decay amplitude because if one neglects the continuum contribution in the sum rule analysis, one can realize that the estimate of f_δ from the sum rule tends to the one in Eq (3.6b). One can extend the above analysis to the strange quark sector. I take an effective κ-resonance having a mass of the order of 1.35 GeV, as is suggested by the Rosenfeld table and a QCD continuum starting from the threshold $t_c \simeq 3 \sim 3.4\ \mathrm{GeV}^2$. One expects that such a model reproduces grossly the features of the $K\pi$ phase shift data[40] plus a QCD continuum. The analysis is summarized in Fig. 6. One can deduce :

$$f_\kappa \simeq (37.8 \pm 3.4)\ \mathrm{MeV} \tag{3.8}$$

It is clear that a good determination of f_δ and f_κ implies a better control of the breaking of the chiral symmetry as we shall see in the next section.

4. RATIO OF THE QUARK VACUUM CONDENSATE

It has been noticed earlier[18] that a sum rule analysis of the subtracted two-point function $(\psi_{(5)}(q^2) - \psi_{(5)}(0))/q^2$ allows a prediction of the non-perturbative quantity $\psi_{(5)}(0)$ which is known from the current algebra Ward identity[5] :

$$\psi_5(0)^i_j = -(m_i + m_j) < \bar{\psi}_i\psi_i + \bar{\psi}_j\psi_j > , \qquad (4.1)$$

$$\psi(0)^i_j = -(m_i - m_j) < \bar{\psi}_i\psi_i - \bar{\psi}_j\psi_j > , \qquad (4.2)$$

where ψ_i is the quark field. The interesting information for the sum rule analysis is the fact that it permits us to test the validity of, for instance, the pion and kaon PCAC relations which have been obtained from the Dashen's formula of current algebras [5),10)] :

$$\psi_5(0)^u_d \simeq 2\, m_\pi^2\, f_\pi^2 \qquad (4.3)$$

$$\psi_5(0)^u_s \simeq 2\, M_K^2\, f_K^2 \quad . \qquad (4.4)$$

The success of Eq (4.3) can be attributed to the small corrections due to the chiral symmetry breaking parameters m_π^2 and $m_{u,d}^2$, as in the sum rule analysis they contribute as[18)] $m_\pi^2\,\tau$ or $m_{u,d}^2\,\tau$, where τ is the sum rule variable. In contradiction, the corrections to Eq(4.4) have been shown to be important[18,19)] due presumably to the fact that the quark mass value is larger than the $\overline{\text{MS}}$-scheme scale Λ and making relevant the corrections of the order $\bar{m}_s^2\,\tau$ and $M_K^2\,\tau$ in the sum rule analysis. Within the chiral perturbation theory framework [43)], these large corrections due to the $\bar{m}_s^2$ term indicate that the chiral perturbation expansion of the strange quark sector is not a naive extension of the one made for the u,d quarks. In this paper, I shall be concerned by the improvement of the results[19)] from the scalar two-point channel using the new information obtained in sections 2 and 3. As I have mentioned earlier, the relevant quantity for the analysis is the Laplace transform of $(\psi^i_j\,(q^2) - \psi^i(0))/q^2$. Following Refs 18, 19), it is easy to express $\psi^i_j(0)$ in terms of the spectral integral :[*)]

$$\psi^i_j(0) = \int_0^\infty \frac{dt}{t}\; e^{-t\tau}\, \{1 - t\tau[1 + 2\,\frac{\bar{\alpha}_s}{\pi} + 2\,\bar{m}_j^2\,\tau\,(\log \tau\,\bar{m}_j^2 + \gamma_E)]\}\, \frac{1}{\pi}\, \mathrm{Im}\, \psi(t) \; , \qquad (4.5)$$

where the information coming from the Laplace transform of $\psi^i_j(q^2)$ has also been used. The Euler constant γ_E has been induced by the Laplace transform of $\bar{m}_j^2/q^2 \;\log - q^2/\bar{m}_j^2$. One can notice that the leading contributions of the perturbative and of the non-perturbative terms are absent in Eq (4.5) which indicates that Eq(4.5) is less sensitive to the non-perturbative effects and more convergent than $\psi^i_j(q^2)$. I use the "duality" anzatz in Eq(3.1) for the description of the spectral function. The QCD continuum contribution can be

*) The mass correction in Eq (4.5) corresponds to the case $m_j^2 \gg m_i^2$.

described by :

$$-\frac{3}{8\pi^2}(\bar{m}_i - \bar{m}_j)^2 e^{-t_c\tau}\{t_c + 2(\frac{\bar{\alpha}_s}{\pi})(t_c + \tau^{-1})\} \tag{4.6}$$

which comes from the usual Feynman diagram discontinuity.
The analysis in the $\bar{u}d$ channel is summarized in Fig.7 for two sets of the extremal values of the parameters coming from Fig. 5. The stability of the curves is obtained for $1/\sqrt{\tau}$ of the order of 0.8 GeV. The arrow at 0.9 GeV indicates that the continuum contribution is still less than the resonance one up to this value of τ. If the unknown higher dimension condensates do not spoil the OPE at the above range of τ, like, for instance, in the ρ-meson channel, then one can deduce the estimate :

$$\psi(0)^u_d \simeq -(0.48 \pm 0.15)\ 10^{-6}\ \mathrm{GeV}^4 . \tag{4.7}$$

Let me introduce the parameter x_2 which controls the breaking of the SU(2) condensate *) :

$$\frac{<\bar{d}\,d>}{<\bar{u}\,u>} \equiv 1 - x_2 . \tag{4.8}$$

Then, using Eq(4.7) together with the pion PCAC relation in Eq(4.3) and the quark mass values obtained in Eq(2.15), one can deduce :

$$x_2 \simeq (1 \pm 0.3)\ 10^{-2}. \tag{4.9}$$

The value in Eq (4.9) is slightly lower than the one in Ref 19b), as in the latter we have used a larger value of the δ-decay amplitude and a slightly higher value of the quark mass **). One can notice that the result in Eq (4.9) agrees with the one from the ω-ρ mixing analysis [22)]:

$$x_2 \simeq 1.5\ 10^{-2}, \tag{4.10}$$

which can indicate that the OPE still works at the value of τ where x_2 in Eq (4.9) has been determined. However, the result is still larger than the "crude estimate" from the baryon sector[44)] which needs still to be improved following the lines in Ref. 36).

One can apply the same method in the $\bar{u}s$ channel. The analysis corresponding to the two sets of parameters from Fig. 6 is summarized in Fig. 8. For

---

*) Here and in the following : $<\bar{\psi}_i\psi_i> \equiv <\bar{i}\ i>$.

**) The change in x_2 does not affect the other results in Ref. 19).

$\Lambda \simeq 0.15$ GeV, we show the curves without the continuum and with the continuum contributions. The "window" where the continuum contribution is less than the resonance one and where the non-perturbative effects are expected to be small corresponds to $(\tau)^{-\frac{1}{2}}$ of the order of 0.9 GeV. The stability for $\Lambda \simeq 0.1$ GeV correspond to $\tau^{-\frac{1}{2}}$ of the order of 1.05 GeV. For smaller values of τ, the continuum contribution is larger than the resonance one. The prediction for $\psi(0)^u_s$ corresponds to the range of values between the minimum of the one-resonance contribution and the one corresponding to the largest value of f_κ where the QCD continuum contribution has been taken into account but does not exceed the resonance by 50%. Then :

$$\psi(0)^u_s \simeq -(8 \pm 2)\ 10^{-4}\ \mathrm{GeV}^4 \ . \tag{4.11}$$

If one uses the values of $\hat{m}_s$, $\hat{m}_u$, $\hat{m}_d$ given in the table and of $\langle \bar{u} u \rangle / \langle \bar{d} d \rangle$ in Eq (4.9), it is easy to deduce with the help of pion PCAC :

$$\frac{\langle \bar{s} s \rangle}{\langle \bar{u} u \rangle} \simeq (0.53 \pm 0.16) \tag{4.12}$$

which confirms the previous results of Refs 18,19). It is also interesting to notice that Ref 20) have obtained a very similar conclusion from the analysis of the $I = 0$, $J^{PC} = 0^{++}$ channel using a FESR method. Following their analysis, one can see that the authors interpret the $\epsilon(1300)$ as a $I = 0$ $\bar{s}s$ state which is actually supported by the Gell-Mann-Okubo mass formula for scalar mesons which implies a crude estimate of the order of 1.45 GeV for the mass of the lowest $\bar{s}s$ isoscalar meson[45]. Then, the authors express the quantity in Eq (4.12) in terms of the first derivative of the quark condensate with respect to the strange quark mass which have been estimated to be 0.044 GeV^2 within a typical accuracy of 20 % which we expect from the sum rule analysis. In this way :

$$\gamma \equiv 1 - \frac{\langle \bar{s} s \rangle}{\langle \bar{u} u \rangle} \simeq (44 \pm 9)\ 10^{-3}\ \mathrm{GeV}^2 \left. \frac{\bar{m}_s}{-\langle \bar{u} u \rangle} \right|_{t_c \simeq 2.5\ \mathrm{GeV}^2} \tag{4.13}$$

which for the values of the quark masses in the table and with the help of pion PCAC gives :

$$\frac{\langle \bar{s} s \rangle}{\langle \bar{u} u \rangle} \simeq (0.40 \pm 0.16) \ . \tag{4.13}$$

One can still have another source of information on γ from the analysis of $\psi_5(0)^u_s$ (Eq 4.1)[18], which has been improved[19,21]. A Laplace transform sum rule of this quantity gives :

$$\psi_5(0)^u_s \simeq (3.2 \pm 0.1)\ 10^{-3}\ \mathrm{GeV}^4 \tag{4.14}$$

which is an average of the results from Refs 19) and 21).
From Eq (4.1), the table and pion PCAC, we deduce :

$$\frac{<\bar{s}\,s>}{<\bar{u}\,u>} \simeq (0.76 \pm 0.61)\ . \qquad (4.15)$$

Considering the above estimates as independent, we can deduce the weighted average with a minimum variance :

$$\frac{<\bar{s}\,s>}{<\bar{u}\,u>} \simeq (0.47 \pm 0.11)\,. \qquad (4.16)$$

The final estimate in Eq (4.16) shows a large violation of the $SU(3)_V$ symmetry for the quark vacuum condensate and suggests a much more careful reanalysis of the strange quark phenomenology. One should also mention that the leading order baryon analysis [24b,d] gives a larger value of the ratio in Eq (4.16) but as I have noticed already earlier, a better and reliable result from the baryon systems needs a careful control of the factorization hypothesis of the four quark condensate, of the choice of the nucleon interpolating fields and of the radiative corrections. The improvement of the baryon result is beyond the spirit of this paper.

Now, defining the renormalization group invariant spontaneous mass μ_i as :

$$<\bar{\psi}_i\psi_i>(Q^2) = -\mu_i^3\left(\log\frac{Q}{\Lambda}\right)^{4/9}\left\{1 + O\left(\frac{\log\log Q/\Lambda}{\log Q/\Lambda}\right)\right\} \qquad (4.17)$$

for $SU(3)_C \times SU(3)_F$, we can deduce from Eqs (4.8, 4.15), the pion PCAC and the value of the u,d quark masses in the table :

$$\mu_u \simeq \mu_d \simeq (192.4 \pm 9.9)\ \mathrm{MeV}\ ;\ -<\bar{u}\,u>^{1/3}(1\ \mathrm{GeV}) \simeq (229.1 \pm 13.6)\mathrm{MeV}$$
$$\mu_s \simeq (149.6 \pm 12.9)\ \mathrm{MeV}\ ;\ -<\bar{s}\,s>^{1/3}(1\ \mathrm{GeV}) \simeq (178.1 \pm 15.4)\mathrm{MeV}\,. \qquad (4.18)$$

Using the value in Eq (4.16) and the weighted value of $\bar{m}_s$ in the table, we can give an improved estimate of $\psi_5(0)^u_s$ normalized to the kaon PCAC estimate in Eq (4.4) :

$$\psi_5(0)^u_s \simeq (0.4 \pm 0.1)\ 2\,M_K^2 f_K^2 \qquad (4.19)$$

for $f_K \simeq 1.2\, f_\pi$ [39]. It seems surprising to get the large violation of $SU(3)_V$ (Eq 4.16) and of the kaon PCAC (Eq 4.19) and still to have an agreement of the quark mass ratio (see the table) with the crude estimate of "strong PCAC" where the $SU(3)_V$ symmetry for the vacuum condensate (and consistently $f_\pi = f_K$) and the kaon PCAC have been used *) :

$$\frac{\langle \bar{u}u + \bar{s}s \rangle (m_s + m_u)}{\langle \bar{u}u + \bar{d}d \rangle (m_d + m_u)} \simeq \frac{M_K^2}{m_\pi^2} \frac{f_K^2}{f_\pi^2} \Bigg|_{PCAC} . \tag{4.20}$$

Actually, one can understand the result because the two sides of Eq (4.20) decrease at the same time when the SU(3) corrections are taken into account so that these effects almost compensate for the quark mass ratio.

5. DOES THE Φ-MESON GIVE A GOOD SIGNATURE OF THE CHIRAL SYMMETRY BREAKING ?

It has been known for a long time within the quark model of Gell-Mann[46] that what is responsible for the mass splitting of different vector mesons belonging to the same octet is the U(3) breaking effect on the Gell-Okubo meson mass formula [46]. It is not, a priori, clear to me that such a breaking is due mainly to the linear quark mass term as would have suggested a naive extension to the strange quark sector of the ordinary chiral perturbation theory, which works nicely for the u and d quarks[43]. Actually, the situation is much more complicated here, as if one uses a spectral function sum rule approach for the analysis of the GMO mass formula, one can realize that the effect of the m_s^2 - term can be as large as that of the linear term $m_s \langle \bar{s} s \rangle$ [45], i.e. due to the fact that m_s is larger than the $\overline{MS}$-scale Λ, one should be more careful in using the chiral perturbation scheme. Because of the almost equal contributions of the linear and quadratic mass terms in the GMO mass formula for vector mesons, one cannot naively extract from the vector meson splitting a firm estimate of one of the above chiral parameters. However, it has been suggested recently by the authors in Ref 17) (hereafter denoted by R^2) that the analysis of the spectral function sum rules associated respectively to the Φ, K* and the tensor mesons f' and E can provide an estimate of the chiral symmetry breaking parameters within an "unexpected" high degree of accuracy for the strange quark mass value :

$$\bar{m}_s \text{ (1 GeV)} \simeq (110 \pm 10) \text{ MeV} \tag{5.1 a}$$

$$\langle \bar{s} s \rangle / \langle \bar{u} u \rangle \simeq 0.8 \pm 0.1 \quad . \tag{5.1 b}$$

*) I thank G. Veneziano for this important remark.

The fact which is more intringuing to me is that the above "precise" estimate of the strange quark mass disagrees with most of the available estimates [12-16]. Then, it becomes urgent to re-examine carefully the analysis of R^2 by emphasizing the effects of the continuum threshold, of the factorization hypothesis of the four-quark operator, of the next to leading radiative and mass corrections not taken into account in their analysis and by taking care of the value of $\chi^2/_{NDF}$ which serves for the selection of the set of the output parameters which are m_s and $< \bar{s}\, s >$. One can inspect from the analysis of R^2 that the most stringent information on the chiral symmetry breaking parameters comes from the Φ channel, as expected, because their relative contributions compared to be leading QCD term in the OPE are more important than in the other channels. So, I shall mainly focus the analysis on the Φ channel and later on for the K^* one, but I shall disregard completely the less controlled states like the E and f' mesons, which in any case do not give many more constraints than the Φ and K^* mesons on the size of the chiral symmetry breaking terms [17) *)]. In order to see more clearly the effects of the input parameters, I shall work for the analysis with the sum rules used by R^2 and, in addition, I improve their QCD expression by using the two loop mass corrections obtained within the $\overline{MS}$-scheme [47]. Then, the expression of the two-point function :

$$\Pi^{\mu\nu}_{\Phi}(q^2) \equiv - (g^{\mu\nu} q^2 - q^{\mu} q^{\nu}) \quad \Pi_{\Phi}(q^2)$$

$$= i \int d^4 x \, e^{iqx} < 0 \mid \mathbb{T} \; J^{\mu}_{\Phi}(x) \quad (J^{\nu}_{\Phi}(o))^{+} \mid 0 > \quad (5.2)$$

associated to the current :

$$J^{\mu}_{\Phi} = \frac{1}{3} \; \bar{s} \, \gamma^{\mu} \, s \tag{5.3}$$

of the Φ-meson is in the $\overline{MS}$-scheme [**)] :

$$\Pi_{\Phi}(q^2) = \frac{1}{36\pi^2} (-) \{ \mathrm{Log} \; \frac{Q^2}{Q_o^2} (1 + \frac{\bar{\alpha}_s}{\pi}) + $$

$$+ 5 \frac{\bar{m}_s^2}{Q^2} (1 + \frac{8}{3} \; \frac{\bar{\alpha}_s}{\pi}) - \frac{24}{7} \; \frac{\bar{m}_s^4}{Q^4} (1 - \frac{\pi}{\bar{\alpha}_s})$$

$$- (8 \, \pi^2 (1 + \frac{\bar{\alpha}_s}{3\pi}) \, m_s < \bar{s} \, s > + \frac{\pi}{3} \, \alpha_s < F^2 >) \frac{1}{Q^4} + C_6 < O_6 > | Q^6 \} \quad (5.4)$$

---

*) It seems more appropriate to use the values of the strange quark parameters in order to predict the masses and couplings of the E and f-mesons as the E meson is not well established.

**) We use the radiative corrections to the quark vacuum condensate evaluated in Ref. 48).

with $-q^2 \equiv Q^2 > 0$. The non-perturbative terms have been evaluated originally in Ref. 22). The contribution of the dimension six operators is [22] (see also Ref 35) :

$$C_6 \langle O_6 \rangle = 4\pi^2 \{ 2\pi \alpha_s < \bar{s} \gamma^\mu \gamma_5 \frac{\lambda_a}{2} s \bar{s} \gamma_\mu \gamma_5 \frac{\lambda_a}{2} s > + \frac{4\pi \alpha_s}{9} \bar{s} \gamma^\mu \frac{\lambda_a}{2} s \sum \bar{q} \gamma^\mu \frac{\lambda_a}{2} q \} \tag{5.5}$$

which would become :

$$C_6 \langle O_6 \rangle \Big|_{fact} \simeq \frac{896}{81} \pi^3 \alpha_s < \bar{s} s >^2 \tag{5.6}$$

if the factorization hypothesis has been used for the estimate of the operator. However, it is known that such an assumption can give an underestimate of the exact value of the operators by a factor of the order of four[35] (see also Ref. 36), so one should be careful in using Eq (5.6). In the first part of the analysis, I follow closely the strategy of R[2]. Then, I introduce the two parameters

$$< O_4 > \text{ and } \bar{m}_s , \tag{5.7}$$

where $\langle O_4 \rangle \equiv - m_s < \bar{s} s >$ is renormalization group invariant. Eq. (5.7) allows us to eliminate the $< \bar{s} s >$ vacuum condensate, so that the sum rule reads, to two loops and including the m_s^4 -corrections,

$$\begin{aligned} \mathcal{F}_\Phi(\tau) \equiv \int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \operatorname{Im} \pi_\Phi(t) = \frac{\tau^{-1}}{36\pi^2} \{ 1 &+ \frac{4}{9L} - 0.35 \frac{\log L}{L} \\ &- 6 \hat{m}_s^2 \left(\frac{2}{L}\right)^{8/9} \tau [1 + \frac{4}{9L} (\frac{8}{3} + 2\gamma_E)] \\ &+ \frac{6}{7} \hat{m}_s^4 \left(\frac{2}{L}\right)^{16/9} \tau^2 (11 - 7\gamma_E - 9L) + \tau^2 (0.042 - 79 < O_4 > \cdot \\ &\cdot (1 + \frac{4}{27L})) - C_6 < O_6 > \frac{\tau^3}{2} \} \end{aligned} \tag{5.8a}$$

and

$$\begin{aligned} -\frac{d}{d\tau} \mathcal{F}_\Phi(\tau) \equiv \int_0^\infty dt\, e^{-\tau t}\, t \frac{1}{\pi} \operatorname{Im} \pi_\Phi(t) = \frac{\tau^{-2}}{36\pi^2} \{ 1 &+ \frac{4}{9L} - 0.35 \frac{\log L}{L} \\ &+ \frac{16}{3} \hat{m}_s^2 \left(\frac{2}{L}\right)^{8/9} \frac{\tau}{L} - \frac{6}{7} \hat{m}_s^4 \left(\frac{2}{L}\right)^{16/9} \tau^2 (4 - 7\gamma_E - 9L) \\ &- \tau^2 (0.042 - 79 < O_4 > (1 + \frac{4}{27L})) + C_6 < O_6 > \tau^3 \} , \end{aligned} \tag{5.8b}$$

where $$L \equiv -\log \tau\Lambda^2 \;,\; C_6 < O_6 > \simeq 258.6 \left(\frac{<O_4>}{\hat{m}_s}\right)^2 (L)^{-1/9}$$

if one uses the factorization hypothesis in Eq (5.6) and the parametrization in Eq (5.7). I have introduced the expression of the running QCD coupling and quark masses to two loops which can be deduced from Eqs (2.6) and (2.7). I have used the value of $\alpha_s < F^2 > \simeq 0.04\ \mathrm{GeV}^4$ from Refs 34) and 35). The spectral function appearing in Eq (5.8) can be parametrized by the duality ansatz :

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi_\Phi(t) = \frac{M_\Phi^2}{4\gamma_\Phi^2}\,\delta(t - M_\Phi^2) + A_\Phi(t)\,\Theta(t - t_c) \tag{5.9}$$

where the coupling of the Φ-meson to the photon is normalized as in Eq (2.2) ; $A_\Phi(t)$ takes into account the usual discontinuity of the perturbative graphs in Fig. 1. The main unknown input in the analysis is the value of the continuum threshold. The common way for estimating its value is its identification with the value of mass of the first radial excitation on the Φ-trajectory. Then, from the analogue of Eq (3.2), one can deduce :

$$t_c \simeq M^2_{\Phi'} \simeq M_\Phi^2 + 2\,M_\rho^2 \simeq 2.24\ \mathrm{GeV}^2 \tag{5.10}$$

which is slightly lower than the experimental value $M^2_{\Phi'} \simeq 2.82\ \mathrm{GeV}^2$. One can also estimate the value of t_c using the duality-like FESR analysis [37]. In this way, one gets the constraint *) :

$$\frac{M_\Phi^2}{\gamma_\Phi^2} \simeq \frac{1}{9\pi^2}\left\{t_c\left(1 + \frac{4}{9L} - 0.35\,\frac{\log L}{L}\right) - 6\,\hat{m}_s^2\left(\frac{2}{L}\right)^{8/9}\left(1 + \frac{1.7}{L}\right)\right\} \tag{5.11}$$

Using the experimental value $\gamma_\Phi \simeq (6.60 \pm 0.14)$, $M_\Phi \simeq 1.02$ GeV and using the positivity of the m_s^2-contribution, one deduces *) :

$$t_c \gtrsim (2.1 \pm 0.1)\ \mathrm{GeV}^2\ . \tag{5.12}$$

---

*) This constraint is also the one obtained from the asymptotic consistency condition of the moment ratio [47].

**) It is difficult to estimate the $\hat{m}_s$-contribution in Eq (5.11) because one does not exactly know at what scale the log-term should be evaluated. In any case, one expects that Eq (5.11) should be valid for $\tau^{-1} \gg t_c$.

I have given in Eqs (5.10) and (5.12) the range of values of t_c, but one will see later on that the sum rule itself will select the most relevant value of t_c. In order to be as close as the R^2 analysis, I shall also work with the moment ratio [22]:

$$M_\Phi^2 \simeq \frac{\left(-\frac{d\mathcal{F}_\Phi}{d\tau}\right) - (\text{QCD continuum})_1}{\mathcal{F}_\Phi(\tau) - (\text{QCD continuum})_0} \tag{5.13}$$

where the $(\text{QCD continuum})_{0,1}$ in Eq (5.13) corresponds rspectively to the contribution to the zeroth order moment $\int_0^\infty dt\, e^{-t\tau} \frac{1}{\pi} \text{Im}\, \Pi_\Phi(t)$ and to its first derivative $\int_0^\infty dt\, t\, e^{-t\tau} \frac{1}{\pi} \text{Im}\, \Pi_\Phi(t)$. It is known from R^2 that the optimal information from Eq (5.13) corresponds to the values of τ between 0.5 and 0.95 GeV^{-2}, where the RHS of Eq (5.13) can present a stability. Then, I do a two-parameter fit of Eq (5.13) with the parameters in Eq (5.7) for the values of t_c in the range of Eqs (5.10) and (5.12) and by assuming for the moment the validity of Eq (5.6) like R^2. In order to have a strong constraint on the parameters, I have assumed an uncertainty of 1 % in the LHS of Eq (5.13)*). The analysis is summarized in Fig. 9. First of all, one can notice that the choice of the set of parameters used by R^2 in Eq (5.1) is not the one which comes consistently from a two arbitrary parameter fit analysis. The R^2 procedure for extracting the parameters is not clearly explained but, as one can see in their figure captions, it seems that at least one of the parameters in Eq (5.7) is introduced by hand. Actually, if one tries to perform a two-parameter fit analysis with the leading QCD expression, one can realize that the χ^2 is very bad and the output parameters are quite unrealistic (too large value of $\hat{m}_s \simeq 450$ MeV and too small value of $-m_s \langle \bar{s} s \rangle$). It is difficult to choose the value of t_c from Fig. 9. However, from the smallest $\chi^2/_{\text{NDF}}$ criterion and from the constraint in Eqs (5.11) and (5.12), it is likely that the value of t_c is around 2 GeV^2. It is amusing to notice that the value of $\langle O_4 \rangle$ at such a value of t_c is the one obtained from the results of Eq (4.16). This consistency of the vector and (pseudo)scalar results allows us to take definitely the value 2 GeV^2 for t_c.

---

*) Actually, M_Φ^2 is known within an accuracy of 1 ‰ from the data.

The set of parameters giving the best fit is *):

$$\begin{aligned}
&\hat{m}_s \simeq (205 \pm 20 \text{ MeV}) \\
&\bar{m}_s\ (1 \text{ GeV}) \simeq (138 \pm 28) \text{ MeV} \\
-\ &m_s < \bar{s}\, s > \simeq (1. \pm 0.1)\ 10^{-3} \text{ GeV}^4 \\
&\chi^2/_{NDF} \simeq 0.045 \qquad ,
\end{aligned} \tag{5.14}$$

where one has included, into the error bars, the effects induced by the change of Λ. The best fit is given in Fig. 10 (continuous line) together with the best fit of R^2 from Fig. 2 of their paper. One can also notice from Fig. 9 that the value of $< O_4 >$ satisfies :

$$< O_4 > \equiv - m_s < \bar{s}\, s > \leq 1.5\ 10^{-3} \text{ GeV}^4 \tag{5.15}$$

if the factorization hypothesis of the four-quark condensate holds. $< O_4 >$ is of the order of a factor two smaller than the one from a kaon PCAC analysis.

Now, let us analyze the effect of the factorization hypothesis on the output parameters, by letting $C_6 < O_6 >$ be a free parameter moving in the range :

$$5.10^{-3} \lesssim C_6 < O_6 > \lesssim 30\ 10^{-3} \text{ GeV}^6 , \tag{5.16}$$

where the lower bound corresponds to the factorization hypothesis within the values of parameters in Eq (5.15), and the upper value corresponds to the claim that the factorization hypothesis of the four-quark operator can give an underestimate of a factor of the order four [35, 36] of the exact value of the operator. The two-parameter fit for $\hat{m}_s$ and the spontaneous mass μ_s versus $C_6 < O_6 >$ and for a given value of t_c and Λ is in Fig. 11. One can see that $\hat{m}_s$ increases by about 20 % while μ_s decreases by about 47 %. In the absence of any factorization hypothesis and with the range of values of $C_6 < O_6 >$ given by Eq (5.16), Fig. 11 tells us that :

$$\begin{aligned}
170 \leq \hat{m}_s \leq 260 \text{ MeV} \\
85 \leq \mu_s \leq 188 \text{ MeV} \quad ,
\end{aligned} \tag{5.17}$$

---

*) One can notice that the QCD corrections decrease the value of $\hat{m}_s$. The same situation has been noticed in Ref 48 where the authors include QCD corrections to the GMO mass formula[47]. We shall come back to the discussion of GMO mass formula in section 7.

i.e., one has a wide range of values. If, in addition, one uses the values of μ_s from the scalar sum rule (Eq 4.16), one can deduce from Fig. 11, the set of parameters :

$$6\ 10^{-3} \lesssim C_6 < O_6 > \lesssim 20\ 10^{-3}\ \mathrm{GeV}^6$$
$$180 \lesssim \hat{m}_s \lesssim 250\ \mathrm{MeV}\ . \tag{5.18}$$

The above value of $C_6 < O_6 >$ must be compared to the one from Eq (5.6), within the value of μ_s in Eq (4.16), which is $3.2\ 10^{-3}\ \mathrm{GeV}^6$. Then, one can conclude from the ϕ-meson analysis that the value of the strange quark mass cannot be known with a high degree of accuracy. Secondly, a deviation of the strange quark vacuum condensate from the $SU(3)_{L+R}$ expectation implies a violation of the factorization hypothesis of the four quark operator by a factor more than 2, which is a conclusion very similar to the one obtained in the ud channel of mesons[35)] and baryons[36)]. The value of $\hat{m}_s$ in Eq (5.18) is given in the Table. One can notice that it agrees with various previous estimates coming from the vector meson channels[49, 50)] but it is slightly higher than the leading order analysis of R^2 [17)].

6. WHAT CAN WE LEARN FROM THE K* CHANNEL ?

The extension of the analysis to the K*-meson channel can be done provided one works with the two-point function $\Pi^{(1+0)}(q^2)$ used in Ref 46) for the analysis of the GMO mass formula and defined as :

$$\Pi^{\mu\nu}(q^2) = -(g^{\mu\nu} q^2 - q^\mu q^\nu)\ \Pi_V^{(1+0)}(q^2) + g^{\mu\nu}\ \psi(q^2)$$
$$\equiv i \int d^4x\ e^{iqx} < 0 \mid T\ (\bar{u}\ \gamma^\mu s)(x)\ ((\bar{u}\ \gamma^\mu s)(0))^+ \mid 0 > \tag{6.1}$$

as it is the one which is free of mass singularities[47) *)]. The two point function involves the contribution of spin 1 (the K*) ans spin 0 (The $\varkappa$-meson discussed in section 3) cases. The QCD expression of $\Pi^{(1+0)\,u}_{(q^2)\quad s}$ for $m_s \gg m_u$ can be obtained from Ref 47). It reads :

*) The strength of the mass corrections is not the same for $\Pi_V^{(1)}$ and $\Pi_V^{(1+0)}$.

$$\Pi_{V\ \ s}^{(1+0)u} = (\frac{-1}{4\pi^2}) \{ \log \frac{Q^2}{Q_o^{\ 2}} + 0\ (1) + \frac{\bar{\alpha}_s}{\pi} \log \frac{Q^2}{Q_o^{\ 2}} + \frac{3\bar{m}_s^2}{Q^2} (1 + \frac{7}{3}\ \frac{\bar{\alpha}_s}{\pi}) - \frac{\bar{m}_s^4}{Q^4} (1 - \frac{12}{7}\ \frac{\pi}{\bar{\alpha}_s}) - \{4\pi^2\ m_s < \bar{s}\ s > (1 - \frac{\bar{\alpha}_s}{\pi}) + \frac{16}{3} \pi^2\ (\frac{\bar{\alpha}_s}{\pi})\ m_s < \bar{u}\ u > + \frac{\pi}{3} \alpha_s\ < F^2 > \}\ \frac{1}{Q^4} + C_6 < 0_6 > \frac{1}{Q^6}\ \} \qquad (6.2)$$

for $SU(3)_C \times SU(3)_F$. The radiative corrections to the quark vacuum condensate come from Ref 48). $C_6 < 0_6 >$ would be equal to Eq (5.6) if the factorization of the four-quark condensate is used. The Laplace transform sum rule reads :

$$\mathcal{F}_V^{(1+0)}(\tau) \equiv \int_0^\infty dte^{-t\tau}\ \frac{1}{\pi}\ \mathrm{Im}\ \Pi_V^{(1+0)}(t) = \frac{\tau^{-1}}{4\pi^2} \{ 1 + \frac{4}{9L} - 0.35\ \frac{\log L}{L} - 3\ \hat{m}_s^2\ (\frac{2}{L})^{8/9} \tau\ [1 + \frac{4}{9L}\ (\frac{7}{3} + 2\gamma_E)] + \hat{m}_s^4\ (\frac{2}{L})^{16/9}\ \tau^2\ [4 - 3\gamma_E - \frac{12}{7} \cdot \frac{\pi}{\alpha_s}] + \tau^2\ [0.042 - 39.5 < 0_4 > (1 - \frac{\alpha_s}{\pi}) - 0.153\ \mathrm{GeV}^3\ \hat{m}_s\ \frac{1}{L}] - \frac{1}{2}\ C_6 < 0_6 > \tau^3\} \qquad (6.3)$$

and

$$-\frac{d\mathcal{F}}{d\tau} \equiv \int_0^\infty dt\ e^{-t\tau}\ t\ \frac{1}{\pi}\ \mathrm{Im}\ \Pi_V^{(1+0)}(t) = \frac{\tau^{-2}}{4\pi^2} \{1 + \frac{4}{9L} - 0.35\ \frac{\log L}{L} + \frac{8}{3}\ \hat{m}_s^2\ (\frac{2}{L})^{8/9}\ \frac{\tau}{L} - (1 - 3\gamma_E - \frac{12}{7}\ \frac{\pi}{\alpha_s})\ \hat{m}_s^4\ (\frac{2}{L})^{16/9}\ \tau^2 - \tau^2\ [0.042 - 39.5 < 0_4 > (1 - \frac{\bar{\alpha}_s}{\pi}) - 0.153\ \frac{\hat{m}_s}{L}] + C_6 < 0_6 > \tau^3 \}. \qquad (6.4)$$

One saturates the spectral function by the K*, the κ mesons and a QCD continuum. Then :

$$\frac{1}{\pi}\ I_m\ \Pi_V^{(1+0)}(t) = \frac{M_{K^*}^2}{2\gamma_{K^*}^2}\ \delta(t - M_{K^*}^2) + 2\ f_\kappa^2\ \delta(t - M_\kappa^2) + A_V(t)\ \Theta(t-t_c)\ . \qquad (6.5)$$

γ_{K^*} is the coupling of the K^* to the gauge field which is the W-boson and controls, for instance, the decay rate $\tau \to \nu_\tau K^*$ of the τ (1.8) lepton . $A_V(t)$ takes into account the discontinuity coming from the perturbative QCD diagram. γ_{K^*} can be related to t_c from the analogue of Eq (5.11). For a better comparison with the R^2 result, we work with the moment ratio analogous to the one in Eq (5.13) *), i.e. :

$$M^2_{K^*} \simeq f(\tau, \; t_c, \; f^2_\kappa, \; M^2_\kappa) \; . \tag{6.6}$$

We use the value of f_κ and M_κ obtained in section 2. The value of t_c will be taken in the range of the one of the Φ-meson channel (asymptotic $SU(3)_F$ symmetry for the QCD continuum). If one starts the analysis of Eq (6.6) by a two-parameter fit :

$$\mu_s \quad , \quad \hat{m}_s \tag{6.7}$$

and for different values of $C_6 \langle O_6 \rangle$, one can realize that the fit is very bad and the output values in Eq (6.7) are very inaccurate. We fix, instead, μ_s to be the one in Eq (4.14) and do a one-parameter fit. The resulting value of $\hat{m}_s$ is given in Figs 12 and 13. One can see that the smallest χ^2/NDF favours a value of t_c less than 1.7 GeV^2 while Fig 13 does not give any useful constraint. If one uses literally the above value of t_c as an upper bound within a 20 % accuracy, then we deduce from the analogue of Eq (5.11):

$$\gamma_{K^*} \simeq \pi M_{K^*} \sqrt{\frac{2}{t_c}} \gtrsim (4.3 \pm 0.4) \tag{6.8}$$

which is a factor almost two stronger than the one from the observed rate :

$$B_{\tau K^*} \equiv \frac{\Gamma(\tau \to \nu_\tau K^*)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} \simeq \frac{12\pi}{M^2} \sin^2\theta_c \int_0^{M^2} dt \left(1 - \frac{3t^2}{M^4} + 2\frac{t^3}{M^6}\right) \cdot$$

$$\cdot \, \mathrm{Im}\, \Pi_{K^*}(t) \lesssim (0.1 \pm 0.04) \tag{6.9}$$

---

*) Notice that R^2 and SVZ work with the $\Pi^{(1)u}_{\;\;\;\;\;\;s}{}^{V}(q^2)$ two-point function which has mass singularities for $m_u \to 0$ [28, 47]. So, they have no spin zero contribution in their sum rule and the contribution of the m_s^2-term in their analysis is about half of the one in Eq (6.3).

where :

$$\frac{1}{\pi} \mathrm{Im} \ \Pi_{K^*} \simeq \frac{M^2_{K^*}}{2\gamma^2_{K^*}} \ \delta(t - M^2_{K^*}) \ . \tag{6.10}$$

The result in Eq (6.8) is however consistent with the one from Weinberg-like sum rules [11] or from other sum rule-results [22,49]. Comparing the result in Eq (6.8) with the one from $SU(3)_V$, which is :

$$\gamma_{K^*}\Big|_{SU(3)_V} \simeq \frac{\sqrt{2}}{3} \ \gamma_\Phi \simeq 3.1 \ , \tag{6.11}$$

one then expects a large SU(3) violation of the value of the K*-coupling to the current. Eq (6.8) leads to the prediction :

$$B_{\tau K^*} \lesssim (3.3 \pm 0.7) \ 10^{-2} \tag{6.12}$$

which is three times smaller than the present experimental upper limit and which stimulates a further experimental improvment of the upper bound on this branching ratio.

7. IMPROVED GELL-MANN-OKUBO (GMO) MASS FORMULA FOR VECTOR MESONS.

The Gell-Mann-Okubo (GMO) mass formula has been reconsidered in Ref 45) using the moment sum rules ratio [25] and taking into account the contributions of the leading quark masses and vacuum condensates. Recently, the authors in Ref 47 have improved the leading QCD expression for vector mesons in Ref 45) by including radiative corrections to the $\bar{m}_s^2$ and $\bar{m}_s^4$-terms and to the quark vacuum condensate $m_s < \bar{s} \ s >$ where the latter radiative correction has been obtained earlier in Ref 48). Once these new QCD effects have been under control, it seems necessary to reexamine more carefully the analysis in Refs 45) and 47). The GMO-like relation for the ρ and Φ mesons is :

$$\begin{aligned} R^\Phi(\tau) - R^\rho(\tau) \simeq 6 \ \bar{m}_s^2 \ \{1 &+ (2\gamma_E + \frac{11}{3}) \ (\frac{\bar{\alpha}_s}{\pi}) \\ &+ (\frac{8\pi}{7\bar{\alpha}_s} + 2\gamma_E + \frac{19}{7}) \ \bar{m}_s^2 \ \tau\} \\ &- 16\pi^2 \ (1 - \frac{2\bar{\alpha}_s}{3\pi}) \ m_s < \bar{s} \ s > \tau \\ &+ \frac{3}{2} \ (C_6 < O_6 >_\Phi - C_6 < O_6 >_\rho) \ \tau^2 \end{aligned} \tag{7.1}$$

with $C_6 < O_6 >_V$ normalized as in Eq (5.6) and where[25)] :

$$R^V(\tau) \equiv - \frac{d}{d\tau} \log \int_0^{\infty} dt\, e^{-t\tau} \frac{1}{\pi} \mathrm{Im}\, \Pi_V(t) \; ; \tag{7.2}$$

is a slightly modified form of the moment used in sections 5 and 6. $\Pi_V(q^2)$ is the two-point correlation function associated to the vector mesons $V \equiv \Phi, \rho$. $R^V(\tau)$ reproduces the meson mass squared in the limit $\tau \to \infty$ but due to our limitation of the control of the non-perturbative QCD terms, we have to take a finite value of τ where in principle the continuum also plays a non negligible role in the sum rule given by Eq (7.2). So, one parametrizes the spectral function sum rule by the "duality ansatz" (see e.g Eqs (3.1) and (5.9)) which leads to the Eq (2.7) of Ref 45) and which becomes after the use of the FESR-like constraint (Eq (5.11)) or of the asymptotic consistency of the QCD and of the phenomenological parts of the sum rule for $\tau \to 0$:

$$R^V_{exp}(\tau) \simeq M_V^2 \left\{ \frac{1 + \left(\frac{1}{t_c \tau}\right)\left(\frac{1}{\tau M_V^2}\right) e^{-(t_c - M_V^2)\tau} (1 + t_c \tau)}{1 + \frac{e^{-(t_c - M_V^2)\tau}}{t_c \tau}} \right\} \tag{7.3}$$

where the value of t_c is of the order of 2 GeV^2 as we learn from Eq (5.11) and from the analysis in section 5 and from Ref. 45). Using Eq (7.1) for the estimate of the running strange quark mass at 1 GeV, we evaluate the moment at $\tau = 1\ \mathrm{GeV}^{-2}$. Then, one gets :

$$R^{\Phi}_{exp}\,(1\ \mathrm{GeV}^{-2}) \simeq 1.3\, M_\Phi^2 \tag{7.4a}$$

$$R^{\rho}_{exp}\,(1\ \mathrm{GeV}^{-2}) \simeq 1.44\, M_\rho^2 \; . \tag{7.4b}$$

In order to control the accuracy of the result in Eq (7.4), one can use, for instance, the value of R^ρ_{exp} obtained using the $e^+e^- \to I = 1$ Hadrons data from Ref 35). One can see that the data imply a value of R^ρ_{exp} of the order of 1.7 M_ρ^2, which led us to conclude that Eq (7.4) should be considered within a 20 % accuracy. Then :

$$(R^{\Phi}_{exp} - R^{\rho}_{exp})(1\ \mathrm{GeV}^{-2}) \simeq (0.49 \pm 0.32)\ \mathrm{GeV}^2 \tag{7.5}$$

One also knows the value of $C_6 \langle O_6 \rangle_\rho \simeq 0.12\ \mathrm{GeV}^6$ from Ref 35). From the analysis in section 5, we have obtained the range of values in Eq (5.18). The value of $\langle \bar{s} s \rangle (1\ \mathrm{GeV}) \simeq -5.8\ 10^{-3}\ \mathrm{GeV}^3$ can be derived from Eq (4.14). Collecting the above informations, Eq (7.1) becomes at $\tau = 1\ \mathrm{GeV}^{-2}$ after transferring the contribution of the dimension-six operator into the LHS of Eq (7.1) :

$$y \equiv (0.64 \pm 0.32)\ \mathrm{GeV}^2 \simeq 6\ \bar{m}_s^2 \left\{1 + 4.8 \left(\frac{\bar{\alpha}_s}{\pi}\right) + \left(\frac{8}{7}\ \frac{\pi}{\bar{\alpha}_s} + 3.87\right) \bar{m}_s^2 / \mathrm{GeV}^{-2}\right\}$$
$$+ 0.92\ \mathrm{GeV} \left(1 - \frac{2}{3}\ \frac{\bar{\alpha}_s}{\pi}\right) . \bar{m}_s \qquad (7.6)$$

which one solves in Fig 14 for $100 \leq \Lambda \leq 150$ MeV.
The resulting value of the running strange quark mass at $\tau = 1\ \mathrm{GeV}^{-2}$ is :

$$120 \leq \bar{m}_s\ (1\ \mathrm{GeV}) \leq 230\ \mathrm{MeV} \qquad (7.7)$$

which is shown in the table *). One may also extend the analysis to the GMO mass formula involving the K*-meson. However, due to the uncertainty on the value of the K*-coupling to the gauged current, one cannot hope to have much more useful information than the one obtained previously from the Φ-meson. One can limit oneself at this stage to a very qualitative analysis like done in Ref.45) for the mass formula involving the Φ, K^* and ρ mesons.

8. CONCLUSIONS

I have discussed in detail various determinations of the chiral symmetry breaking parameters from the light meson-systems via the SVZ-Laplace transform QCD sum rules and I have done a weighted average of various estimates coming from other methods.

1) Concerning the light quark u,d masses, the only available sources of information from Laplace sum rules of the meson-systems are the two-point correlation functions associated to the axial[13]) and vector[14]) current divergences

*) This result is consistent either with the leading QCD analysis in Ref 45) or with the one including next-to-leading QCD term in Ref 47). Contrary to the naive result of Ref 47) one cannot claim a net effect on the value of the strange quark mass from the improved form of the GMO mass formula.

which are known to three-loops[15] and up to the contribution of dimension-six vacuum condensates. Due to the Goldstone nature of the pion and the kaon and due to the fact that the strength of the $\pi'(K')$ contribution to the spectral function has the same dependence on $m_{\pi(K)}$ than the contribution of the $\pi(K)$, it is essential to consider these first excitations at the same level as the $\pi(K)$ but not as the continuum smeared by the QCD-theta function. Once this has been done, one can make a more consistent comparison with the ρ-meson channel and the apparent inconsistency noticed for instance in Ref.32) is absent as we do not need to go at a too small value of τ^{-1} in order to see the consistency of the two sides of the sum rule. In the same way, the large scale of the pseudoscalar channel ($\tau^{-1} \gtrsim 2$ GeV^2) advocated by the authors of Ref 29) is dual to a π' mass of the order of 1.7 GeV and the observed value $M_{\pi'} \simeq 1.24$ GeV[31] should actually correspond to a smaller value of the sum rule scale which should be inside the "sum rule window" shown in Figs 2 to 4 where the small-size instanton effect can be neglected if one follows the dilute gas instanton estimate of Ref 29) for the value of $\Lambda_{\overline{MS}}$ which is by now known to be of the order of 100 MeV. According to the above remarks, we expect that the results in Eq (2.15) are a "good estimate" of the u,d light quark masses. These results agree perfectly with the ones of Gasser and Leutwyler [12d] obtained using a SU(4) symmetry for the 16-plet meson wave functions with $\bar{m}_c$ (1 GeV) $\simeq (1.35 \pm 0.05)$ GeV as input coming from various analyses of the charmonium systems. Using a weighted average of the pseudoscalar sum rule and of the SU(4) results, we get the final estimate for $100 \leq \Lambda \leq 150$ MeV :

$$\begin{aligned} \hat{m}_u &\simeq (8.6 \pm 1.5) \text{ MeV} \quad ; \quad \bar{m}_u(1 \text{ GeV}) \simeq (5.1 \pm 0.9) \text{ MeV} \\ \hat{m}_d &\simeq (15.2 \pm 2.7) \text{ MeV} \quad ; \quad \bar{m}_d(1 \text{ GeV}) \simeq (9.0 \pm 1.6) \text{ MeV} \end{aligned} \tag{8.1}$$

where, I recall that $\hat{m}_i$ and $\bar{m}_i$ refer respectively to the invariant and running quark masses. One can notice that these values in Eq (8.1) are slightly lower than the ones obtained using an analytical continuation of the pseudoscalar two-point function at $Q^2 = 0$ [51]. However, one should keep in mind that the result in Ref 51) is very sensitive to the value of the dimension six operators which can be actually higher than the SVZ-value used by the authors as has been noticed in Ref 35) from the analysis of the $e^+e^- \to I = 1$ Hadrons data. Actually a higher value of the value of the dimension-six condensates can favour a slightly smaller value of the u,d quark masses from the analysis of Ref 51).

Using the Laplace transform or FESR methods, it is easy to realize that the results in Eq (8.1) is not sensitive to the value of the vacuum condensates which are already small in the "sum rule-window".

2) The strange quark mass result comes from various sources as one can see in the table. The weighted average*) of these various estimates leads to a very accurate value of the strange quark mass :

$$\hat{m}_s \simeq (250.6 \pm 25.8) \text{ MeV} \; ; \; \bar{m}_s(1 \text{ GeV}) \simeq (148.4 \pm 15.3) \text{ MeV}. \quad (8.2)$$

3) We have scanned various light meson-sources of estimate of the ratios of the quark vacuum condensates $\langle \bar{d} d \rangle / \langle \bar{u} u \rangle$ and $\langle \bar{s} s \rangle / \langle \bar{u} u \rangle$. We have shown that the vector mesons Φ and K^* are not very sensitive to the above ratios. That is mainly due to the fact that the role of the $\bar{m}_s^2$-corrections is much more important in these channels than the one of the $\langle \bar{s} s \rangle$ condensate. The "good" places for the extraction of the quark condensate are the scalar and pseudoscalar channels via the estimate of the subtraction constant $\psi_{(5)}(0)^i_j$ introduced in Ref 18) and followed later on by various authors 19)21)(5) and used again in this paper**) where a better value of the strength of the scalar meson-decay amplitudes [Eqs (3.4) and (3.8)] has been used. Then, we have obtained [Eqs (4.8), (4.9)] :

$$\frac{\langle \bar{d} d \rangle}{\langle \bar{u} u \rangle} = 1 - (1 \pm 0.3)\, 10^{-2} \quad (8.3)$$

and the result for the strange quark condensate is in Eq (4.2). Combining this result with the one from various sources [Eqs (4.13)-(4.15)], we deduce the weighted average [Eq (4.16)] :

$$\frac{\langle \bar{s} s \rangle}{\langle \bar{u} u \rangle} \simeq (0.47 \pm 0.11) \quad , \quad (8.4)$$

*) I thank N. Paver for a stimulating remark on this point.

**) One could also work with a chirality-odd two-point correlation [as done by the authors in Ref 52)] which is sensitive to leading order to the vacuum condensate $\langle \bar{\psi} \psi \rangle$. However, the result of the analysis is rather inaccurate due to the less controlled value of the meson coupling to the tensor current as well as to the less controlled contribution of the higher excitations in the sum rule.

which clearly indicates a large deviation from the $SU(3)_V$ symmetry and consistently implies a large violation of the kaon PCAC. The result in Eq (8.4) needs a more careful reanalysis of the kaon phenomenology where the role of the $\bar{m}_s^2$-term should be important in contrast to the pion case where one can safely neglect the $m_{u,d}^2$ term compared to the linear $m_{u,d} \langle \bar{u} u \rangle$ one. We have encountered such a situation in various examples discussed in this paper as well as in the case of the $U(1)_A$-channel where the m_s^2-term affects the estimate of the topological susceptibility of the $U(1)_A$ current[53] and the η'-mass value [53,54].

4) We have observed from the analysis of the Φ-meson channel, that the factorization of the four-quark condensate is violated by a factor more than two (see Eq (5.18)) which is a very similar conclusion than the one obtained from the ρ-meson channel[35] and from the u,d baryon systems[36]. We hope that other method like, e.g, the lattice Monte-Carlo simulations, checks our results. One can also notice from Eq (5.18) and from the u,d channel result in Eq (2.9) from Ref 35) that the dimension-six vacuum condensates also exhibit a large violation of the SU(3) expectation, i.e. the value of the condensate for the strange quark is much smaller than the one for the u,d quarks.

5) Finally, we conclude that the K*-meson channel does not provide any useful information on the chiral symmetry breaking parameters mainly due to the fact that the K*- coupling to the gauged current is not known experimentally. For this reason, it is more useful to deduce this less controlled coupling of the K* to the current. We find the result in Eq (6.8) which implies the branching ratio :

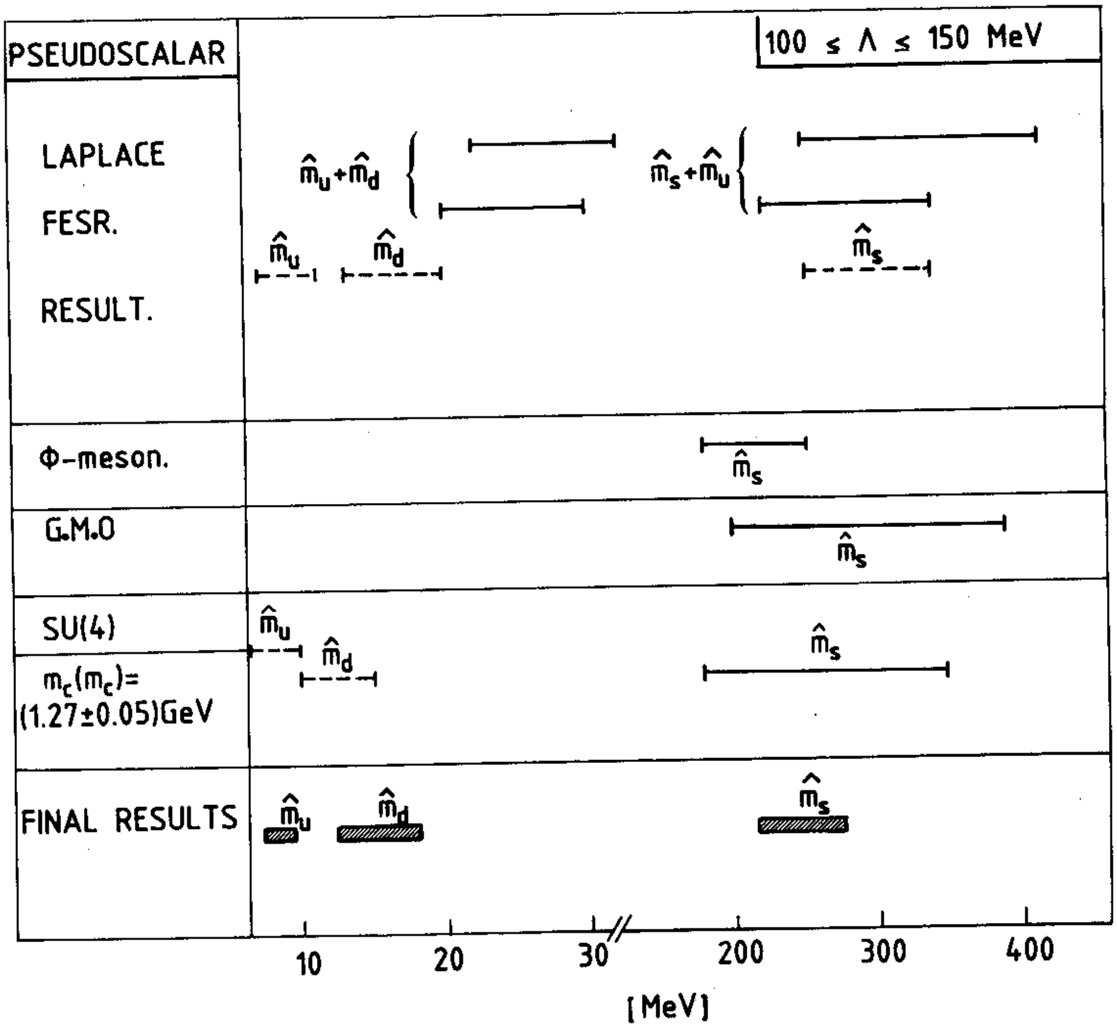
$$\frac{\Gamma(\tau \to \nu_\tau K^*)}{\Gamma(\tau \to \nu_\tau e \bar{\nu}_e)} \lesssim (3.3 \pm 0.7)\, 10^{-2} \tag{8.5}$$

which is three times smaller than the present experimental upper limit and which motivates a further improvement of the data.

ACKNOWLEDGEMENTS

I thank J.S. Bell for carefully reading the manuscript.
Without the fruitful experience which I got from the collaborations and discussions with my colleagues, this paper would never have been written !

Table: Light quark invariant masses



REFERENCES

1a) M. Gell-Mann, Acta Phys. Austriaca, Suppl. IX, 733 (1972).

1b) H. Fritzsch and M. Gell-Mann, Proceedings of the XVI International Conf. on High Energy Physics, Chicago (1972) Vol. 2 p. 135.

1c) H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. 47B (1973) 365.

1d) H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346.

1e) D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.

2) For a recent review, see e.g. M. Jacob, talk given at the XXII International Conference in High Energy Physics, Leipzig, RDA (1984).

3a) G. 't Hooft, Nucl. Phys. B72 (1974) 461, B75 (1974) 461.

3b) E. Witten, Nucl. Phys. B 160 (1979) 57 and references therein.

4) M. Gell-Mann, Physics 1 (1964) 63.

5) For reviews on current algebras, see e.g.

a) S.L. Adler and R.F. Dashen, Current algebra and applications to particle physics (Benjamin, New York 1968).

b) H. Pagels, Phys. Reports 16 (1975) 221.

c) V.De Alfaro, S. Fubini and C. Rossetti, Currents in Hadron Physics, North-Holland (Amsterdam 1973).

d) E. Reya, Rev. Mod. Phys. 46 (1974) 545.

6a) Y. Nambu, Phys. Rev. 117 (1960) 648.

6b) Y. Nambu and G. Jona - Lasinio, Phys. Rev. 122 (1961) 345.

6c) J. Goldstone, Nuovo Cim. 19 (1961) 154.

7a) With apologies to the original works, I recommend a recent review by : J. Kripfganz, Talk given at the XXII international conference in High Energy Physics, Leipzig RDA (1984).

7b) E.G. Floratos, E. Papantonopoulos and G. Zoupanos, CERN preprint TH.4020.

7c) M. Peskin, SLAC - PUB - 3021 (1982) and references therein.

8a) E. Wigner, Group Theory(Princeton University, 1939).

8b) H. Weyl, The classical groups (Princeton University Press, 1939).

9) A classic derivation of for instance, the $\pi^+-\pi^0$ mass difference is due to :
T. Das, G. Guralnik, V.S. Mathur and S. Okubo, Phys. Rev. Lett. 18 (1967) 759. Among other modern derivations, see e.g. :
B. Machet and A. Natale, LBL, Berkley Preprint 16829 (1983) and references therein.

10) M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175 (1968) 2195.

11a) S. Weinberg, Phys. Rev. Letters 18 (1967) 507.

11b) E.G. Floratos, S. Narison and E de Rafael, Nucl. Phys. B155 (1979) 115.

11c) P. Pascual and E de Rafael, Z. Phys. C12 (1982) 127.

11d) S. Narison, Z Phys. C14 (1982) 263.

12a) P. Langacker and H. Pagels, Phys. Rev. D19 (1979) 2070.

12b) P. Langacker, Phys. Rev D20 (1979) 2983.

12c) J. Gasser, Ann. of Phys. 136 (1981) 62.

12d) For a recent review of the determination of quark masses, see e.g. :
J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 79.

13a) C. Becchi, S. Narison, E de Rafael and F.J. Yndurain, Z Phys. C8 (1981) 335.

13b) S. Narison and E de Rafael, Phys. Lett. 103B (1981) 57.

14) S. Narison, N. Paver, E de Rafael and D. Treleani, Nucl. Phys. B212 (1983) 365.

15) S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys. Lett. 135B (1984) 457.

16) C.A. Dominguez, Valparaiso preprint (Chile) (1984) (to appear as Z.für Phys.C).

17) L.J. Reinders and H.R. Rubinstein, CERN preprint TH 3912 (1984).

18) S. Narison, Phys. Lett. 104B (1981) 485.

19a) S. Narison, N. Paver and D. Treleani, Nuovo Cim. 74A (1983) 347.

19b) E. Bagan, A. Bramon, S. Narison and N. Paver, Phys. Lett. 135B (1984) 463.

20) N.V. Krasnikov and A.A. Pivovarov, Nuovo Cim. 81A (1984) 680.

21) C.A. Dominguez, M. Kremer, N.A. Papadopoulos and K. Schilcher, Mainz preprint MZ-TH/84-10 (1984) and private communication from C.A. Dominguez.

22) M.A. Shifman, A.I. Vainshtein and V.I. Zakharov , Nucl. Phys. B 147 (1979) 385, 448.

23) V.A. Novikov, L.B. Okun, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Phys. Reports 41c (1978) 1.

24) For recent reviews, see e.g. :

a) E de Rafael, talk given at the Dubrovnik summer school, Marseille preprint CPT 84/P.1587 (1984).

b) L.J. Reinders, H.R. Rubinstein and S. Yazaki, CERN preprint TH 3767 (1983).

c) S. Narison, talk given at the XXII International Conference on High Energy Physics, Leipzig (1984) CERN preprint TH 3989 (1984).

d) B.L. Ioffe, talk given at the XXII International Conference on High Energy Physics, Leipzig (1984).

25a) J.S. Bell and R.A. Bertlmann, Nucl. Phys. B 177 (1981) 218.

b) R.A. Bertlmann, Acta. Phys. Austr. 53 (1983) 305.

26) H. Leutwyler, Nucl. Phys. B 76 (1974) 413.

27) H.D. Politzer, Nucl. Phys. B117 (1976) 397.

28) For a recent review of different renormalization schemes, see e.g. : S. Narison, Phys. Rep. 84 (1982) 263.

29) V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 191 (1981) 301.

30) J.J. Aubert et al, Phys. Lett. 114B (1982) 291.

31a) M. Bonesini et al, Phys. Lett. 103B (1981) 75

31b) G. Bellini et al, Phys. Rev. Lett. 48 (1982) 1697.

31c) C. Daum et al, Nucl. Phys. 187B (1981) 1.

31d) G.W. Brandenburg et al, Phys. Rev. Lett. 36 (1976) 706.

32) B.V. Geshkenbein and B.L. Ioffe, Nucl. Phys. B 166 (1980) 340. I thank B.L. Ioffe for bringing this reference to my attention.

33a) G. Veneziano, Nuov. Cim. 57A (1968) 190.

33b) R.A. Bryan, C.A. Dominguez and B.J. Ver West, Phys. Rev. C22 (1980) 160 and references therein.

34a) L.J. Reinders, H.R. Rubinstein and S. Yazaki, CERN preprint TH 3776 (1983) and references therein.

34b) K. Ishikawa, G. Schierholz, H. Schneider and M. Teper, Desy report 82-087(1982).

34c) A. Di Giacomo and G. Paffuti, Phys. Lett. 108 B (1982) 327.

35) G. Launer, S. Narison and R. Tarrach, CERN preprint TH 3712 (1983) (to appear in Z für Phys. C) and references therein.

36) Y.Chung, H.D.Dosch, M. Kremer and D. Schall, Heidelberg preprint HD-THEP-84-1 (1984).

37a) N.V. Krasnikov, A.A. Pivovarov and N.N. Tavkhelidze, Z. Phys. C19 (1983) 301 and references therein.

37b) R.A. Bertlmann, G. Launer and E de Rafael, Marseille preprint CPT 84/P 1586 (1984) and references therein.

37c) J.J. Sakurai, K. Schilcher and M.D. Tran, Phys. Lett. 102B (1981) 55.

38) A.L. Kataev, N.V. Krasnikov and A.A. Pivovarov, Phys. Lett. 123B (1983) 93 and Nuovo Cim. 76A (1983) 723.

39) H. Leutwyler and M. Roos, CERN preprint TH 3830 (1984).

40a) P. Estabrooks et al, Nucl. Phys. B76 (1974) 413.

40b) D. Aston et al, Phys. Lett. 106B (1981) 235.

41) R.H. Socolow, Phys. Rev. 137 (1965) 221.

42) N. Fuchs and M.D. Scadron, University of Tucson preprint (1981).

43) J. Gasser and H. Leutwyler, CERN preprint TH 3798, 3829 (1984).

44) P. Pascual and R. Tarrach, Phys. Lett. 116B (1982) 443.

45) S. Narison, Z. Phys. C 22 (1984) 161.

46) For a review, see e.g. M. Gell-Mann "The eightfold way" (Benjamin, New York 1966).

47) D.J. Broadhurst and S.C. Generalis, Open university preprint OUT-4102-12 (August 1984). I thank the authors for discussions and for correspondence on their perturbative calculations.

48) P. Pascual and E de Rafael, Z Phys. C12 (1982) 127.

49) S. Narison, Nucl. Phys. B 182 (1981) 59.

50) S. Narison and E de Rafael, Nucl Phys. B 169 (1980) 253.

51) M. Kremer, N.A. Papadopoulos and K. Schilcher, Phys. Lett. 143B (1984)476.

52) N.S. Craigie and J. Stern, Phys. Rev. D26 (1982) 2430.

53) S. Narison, CERN preprint TH 3796 (1984) to appear as Z.für Phys. C.

54) S. Narison, N. Pak and N. Paver, CERN preprint TH 3857 (1984) to appear as Phys. Lett. B.

FIGURE CAPTIONS

Fig 1 : QCD contributions to the hadronic two-point function where ⊗ denotes the hadronic current insertion, — the internal quark line, ∿∿ internal gluon line :

a - c) : perturbative QCD

d) : $m < \bar{\psi} \psi >$, e) : $\alpha_s < F_a^{\mu\nu} F_{\mu\nu}^a >$

f) : $m < \bar{\psi} \sigma^{\mu\nu} \frac{\lambda_a}{2} \psi F_a^{\mu\nu} >$, g) : $< \bar{\psi} \gamma^\mu \frac{\lambda_a}{2} \psi \sum_{q=u,d..} \bar{q} \gamma^\mu \frac{\lambda_a}{2} q >$

h) : $\alpha_s < \bar{\psi} \Gamma_1 \psi \; \bar{\psi} \Gamma_2 \psi >$, i): $g f_{abc} < F_a^{\mu\nu} F_{\nu\rho b} F_c^{\rho\mu} >$

Fig 2 : Estimate of $(\hat{m}_u + \hat{m}_d)$ from the pseudoscalar two-point function versus the sum rule scale $1/\sqrt{\tau}$ for different values of t_c and for $\Lambda \simeq 100$ MeV :

- ////// represent the lower bound using the positivity of the spectral function.
- —— Finite width parametrization of the π'
- —•—• Narrow width parametrization of the π' with $M_{\pi'}^4 f_{\pi'}^2 / m_\pi^4 f_\pi^2 \simeq 6$ [Refs 12d, 15, 19a)].

The arrow A indicates the region where the OPE is expected to be a good approximation, while the arrow B corresponds to the one, where the QCD continuum does not exceed the resonances contributions, i.e. the so-called "sum rule window" is inside A and B.

Fig 3 : Behaviour of $(\hat{m}_u + m_d)$ for two different values of Λ and for a given value of t_c.

Fig 4 : Strange quark analogue of Fig. 2.

Fig 5 : Estimate of f_δ for a given value of $\hat{m}_d - \hat{m}_u$ and taking the set of (Λ, t_c) which gives the lowest and the highest estimates of f_δ. The arrows A and B delimit the "sum rule window".

Fig 6 : Strange quark analogue of f_δ.

Fig 7 : Estimate of the non-perturbative two-point function subtraction constant $\psi(0)_d^u$ corresponding to the set of parameters in Fig 5. The arrow indicates the point where the continuum contribution does not exceed the resonance one.

Fig 8 : Strange quark analogue of Fig 7 using the set of parameters in Fig 6.

Fig 9 : Correlated set of $\hat{m}_s$ and $m_s < \bar{s} s >$ from a two-parameter fit of the Φ-meson mass squared as function of t_c.

Fig 10 : "Microscopic figure" showing the prediction of M_Φ^2 for different sets of the QCD parameters.

Fig 11 : Correlated set of $\hat{m}_s$ and μ_s versus different values of the dimension-six operators for a given value of t_c from the Φ-meson channel.

Fig 12 : Value of $\hat{m}_s$ versus t_c for given values of Λ, μ_s and $C_6 < O_6 >$ from the K^*-channel.

Fig 13 : Values of $\hat{m}_s$ versus $C_6 < O_6 >$ for given values of t_c, Λ and μ_s.

Fig 14 : Determination of $\bar{m}_s$ (1 GeV) from the G.M.O mass formula.

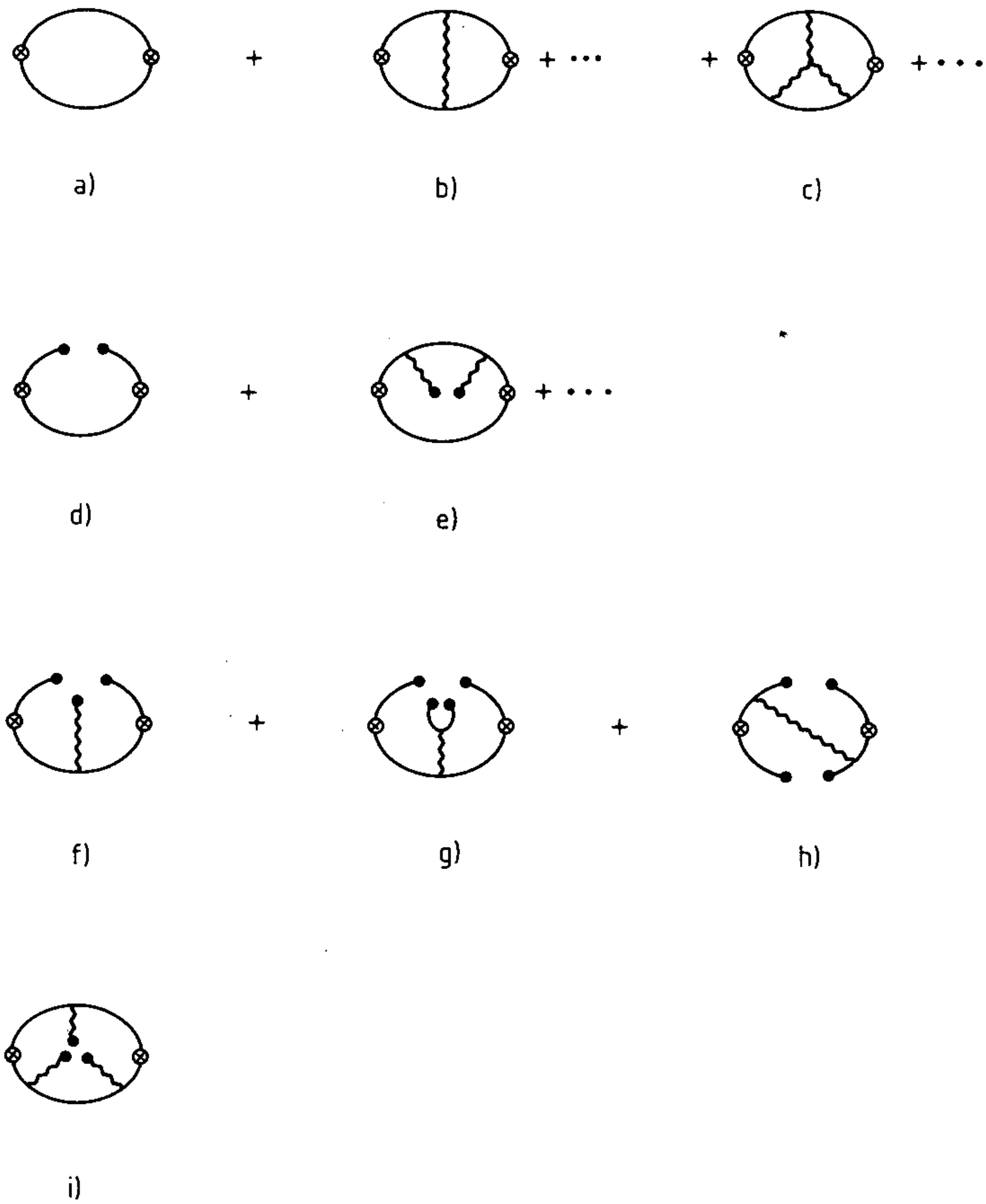
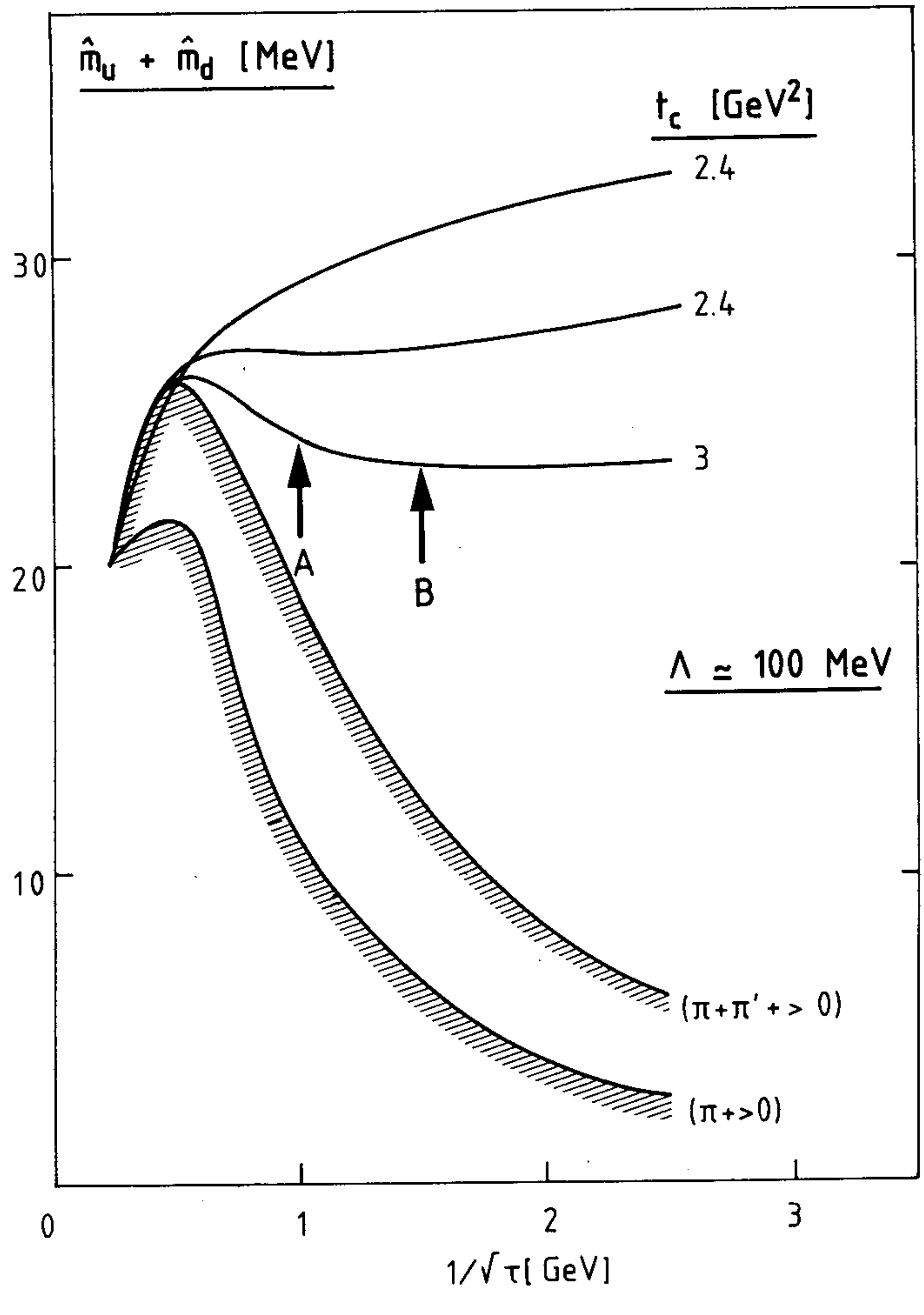


Fig. 1



$\hat{m}_u + \hat{m}_d$ [MeV]
t_c [GeV^2]
2.4
2.4
3
A
B
$\Lambda \simeq 100$ MeV
$(\pi+\pi'+>0)$
$(\pi+>0)$
30
20
10
0
1
2
3
$1/\sqrt{\tau}$ [GeV]

Fig. 2

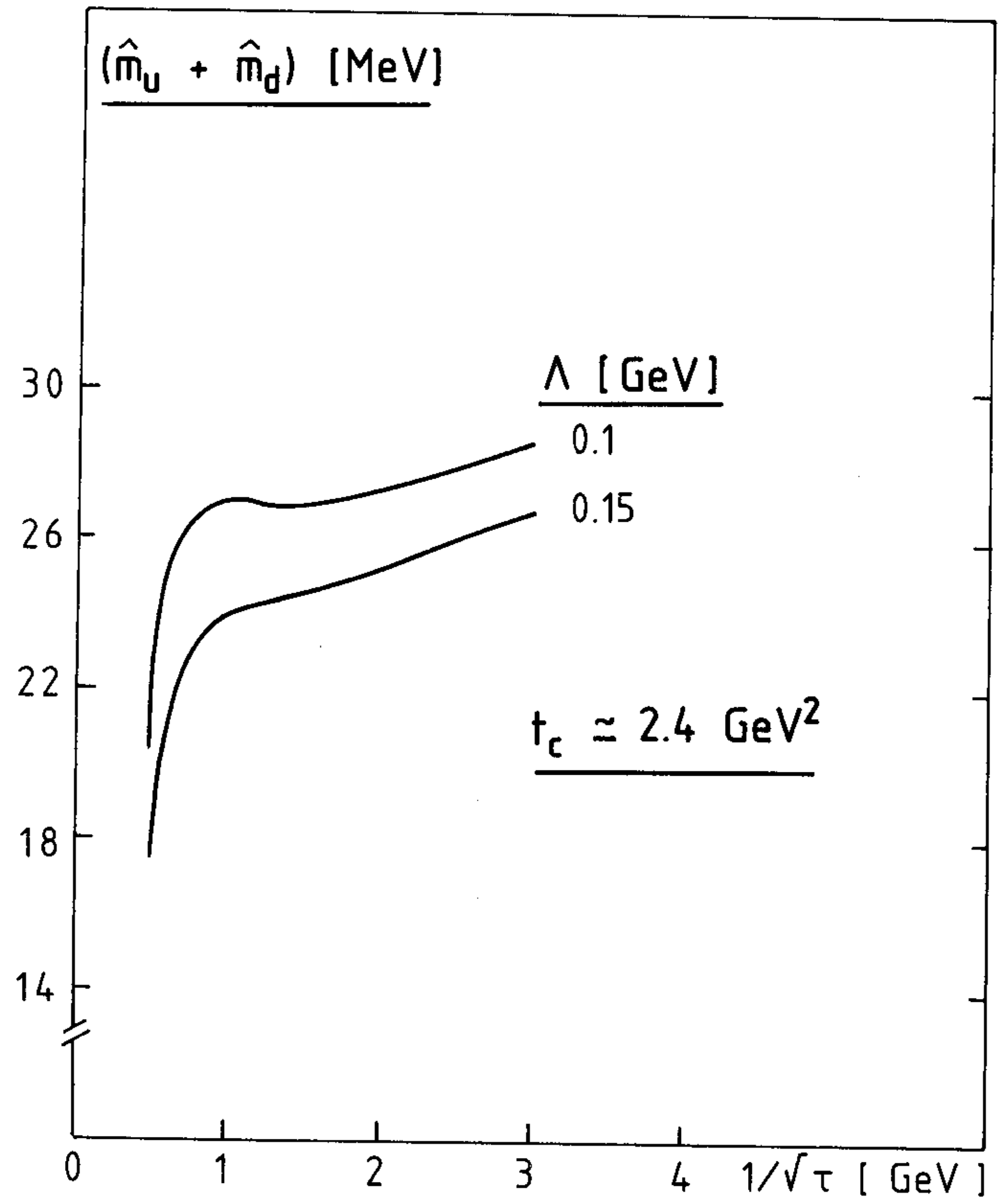
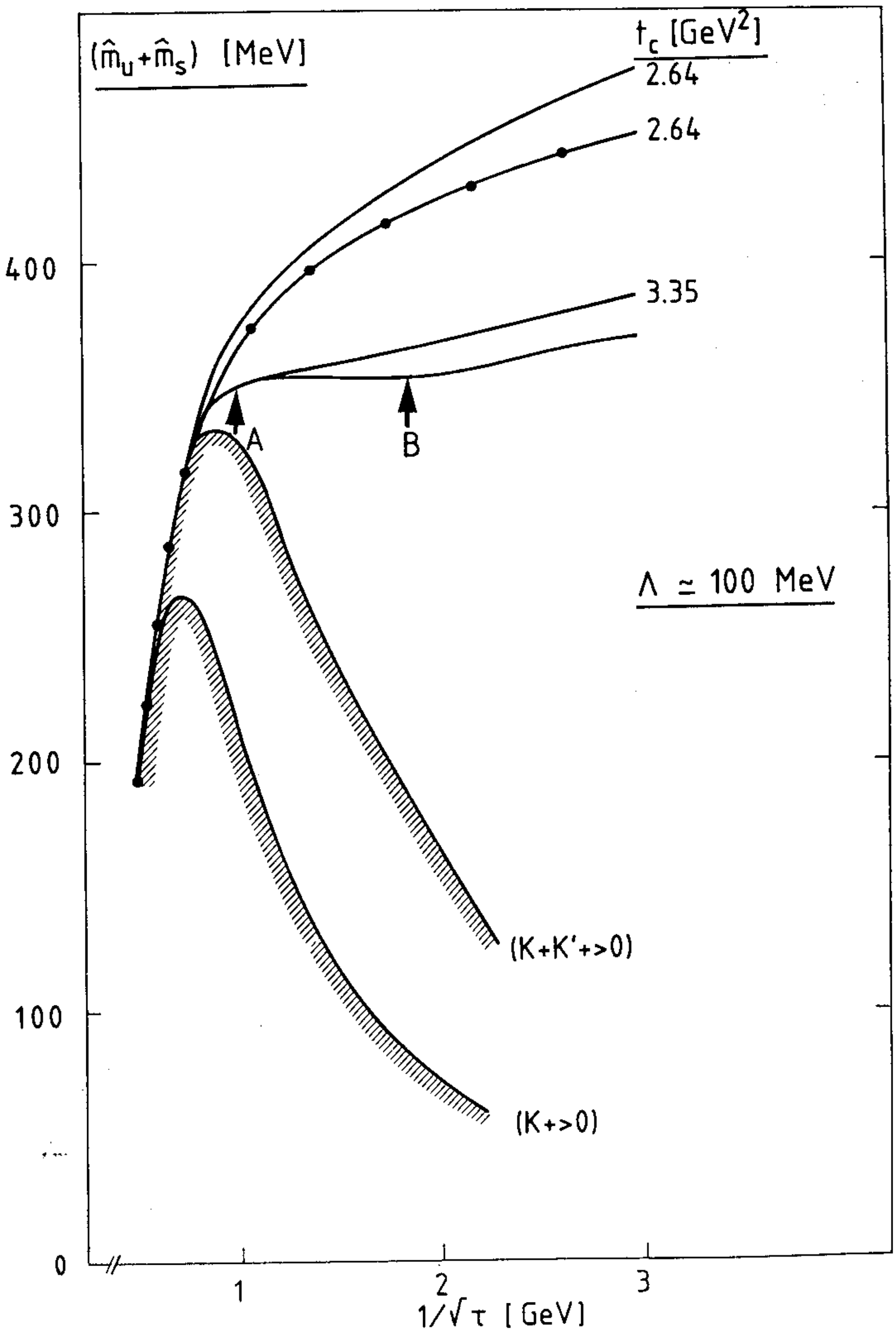
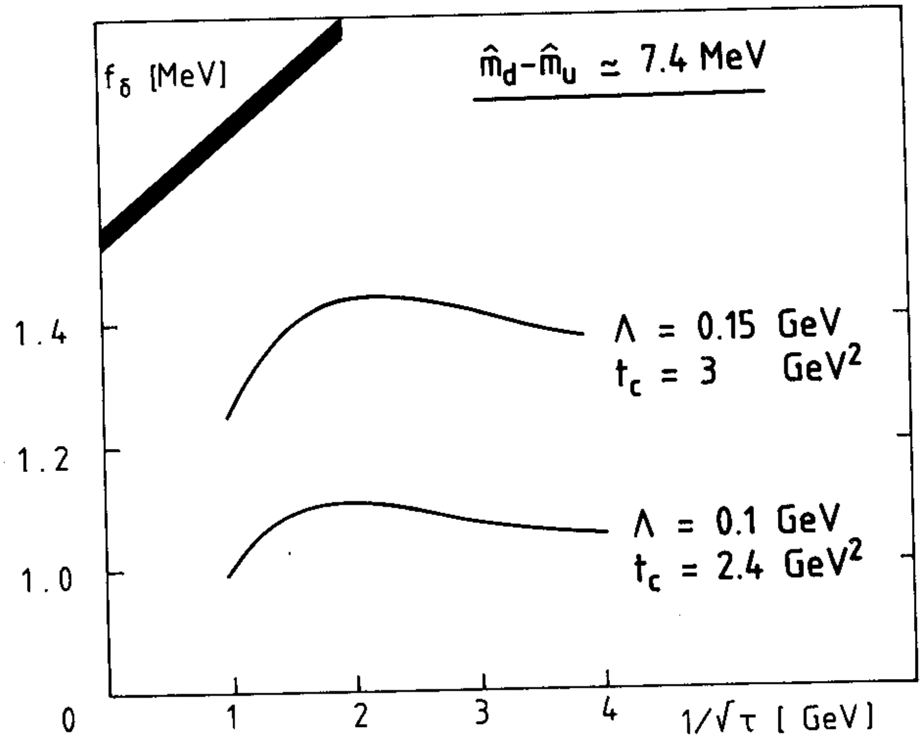

$(\hat{m}_u + \hat{m}_d)$ [MeV]
30
26
22
18
14
0
1
2
3
4
$1/\sqrt{\tau}$ [GeV]
Λ [GeV]
0.1
0.15
$t_c \simeq 2.4$ GeV^2

Fig. 3



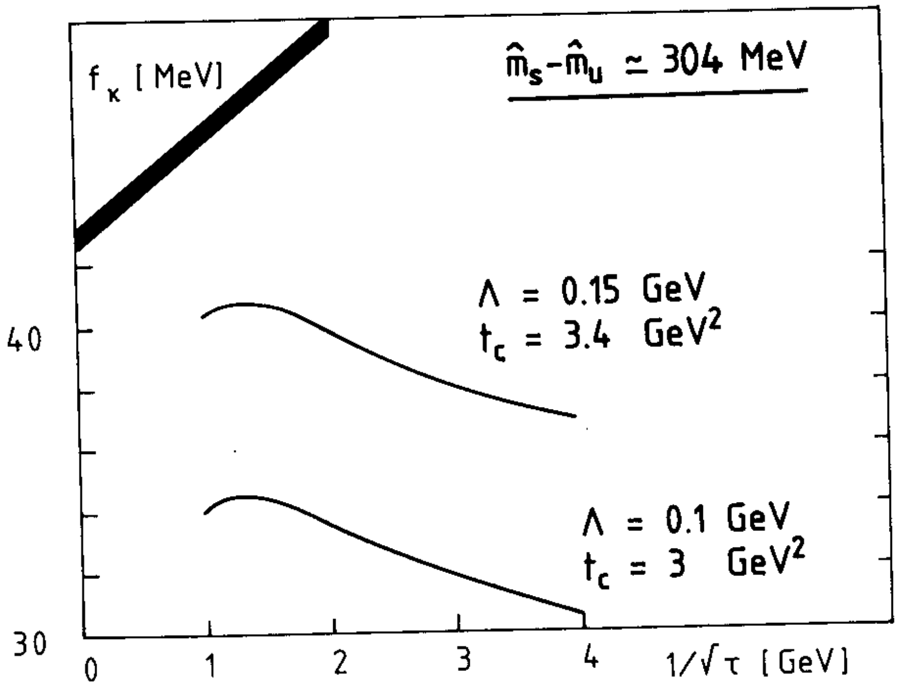
$(\hat{m}_u+\hat{m}_s)$ [MeV]
t_c [GeV^2]
2.64
2.64
3.35
A
B
$\Lambda \simeq 100$ MeV
(K+K'+>0)
(K+>0)
400
300
200
100
0
1
2
3
$1/\sqrt{\tau}$ [GeV]

Fig. 4



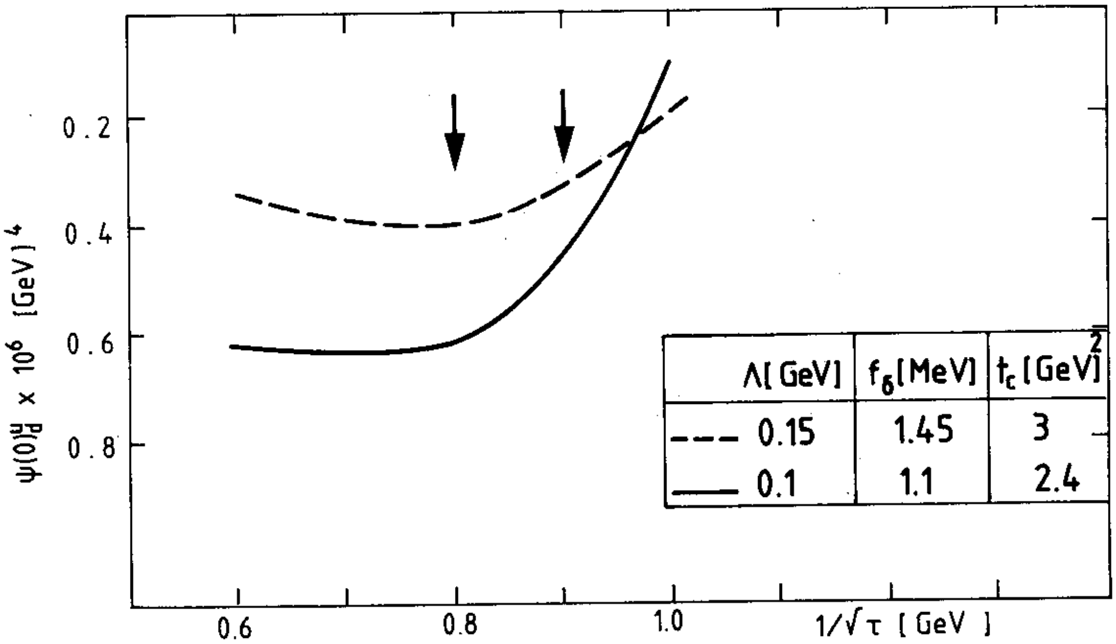
f_δ [MeV]
$\hat{m}_d - \hat{m}_u \simeq 7.4$ MeV
1.4
1.2
1.0
0
1
2
3
4
$1/\sqrt{\tau}$ [GeV]
Λ = 0.15 GeV
t_c = 3 GeV^2
Λ = 0.1 GeV
t_c = 2.4 GeV^2

Fig. 5



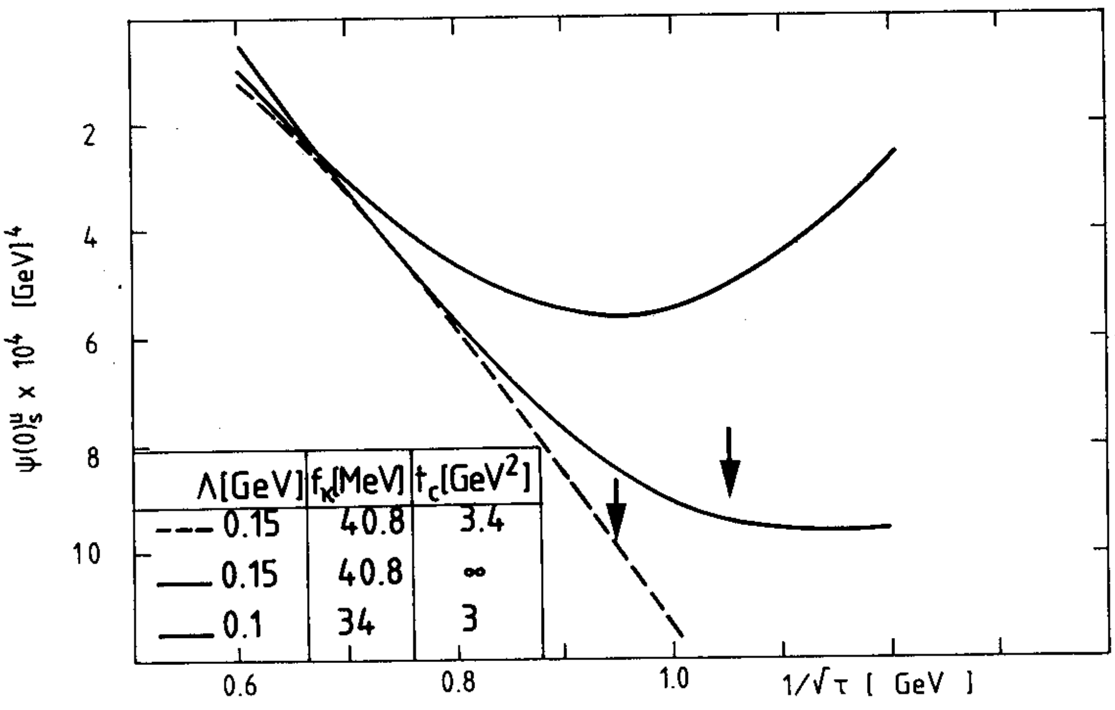
f_κ [MeV]
$\hat{m}_s - \hat{m}_u \simeq 304$ MeV
40
30
0
1
2
3
4
$1/\sqrt{\tau}$ [GeV]
Λ = 0.15 GeV
t_c = 3.4 GeV^2
Λ = 0.1 GeV
t_c = 3 GeV^2

Fig. 6



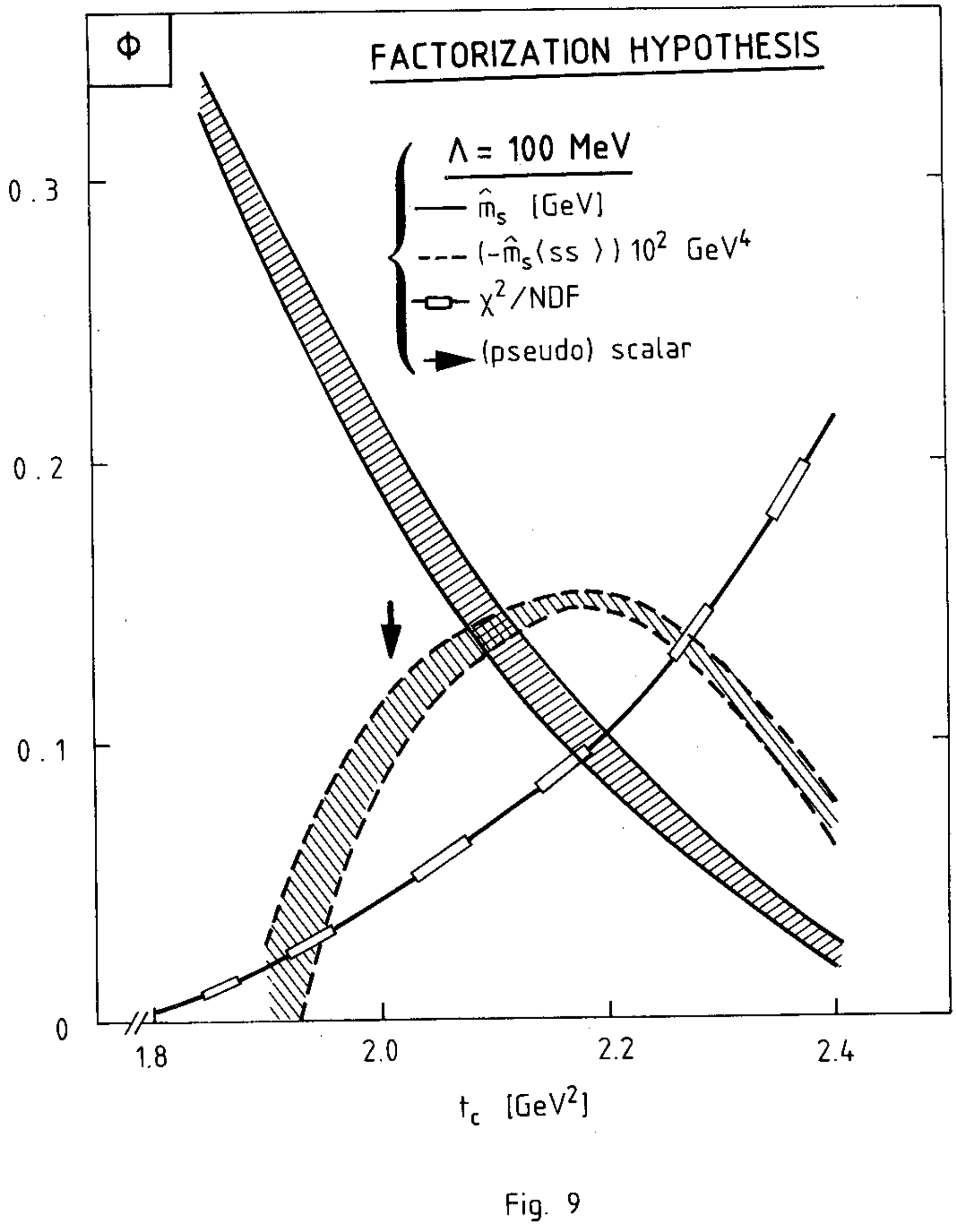
0.2
0.4
0.6
0.8
$\psi(0)_R^u \times 10^6$ [GeV]^4
Λ[GeV]
f_δ[MeV]
t_c[GeV^2]
0.15
1.45
3
0.1
1.1
2.4
0.6
0.8
1.0
$1/\sqrt{\tau}$ [GeV]

Fig. 7



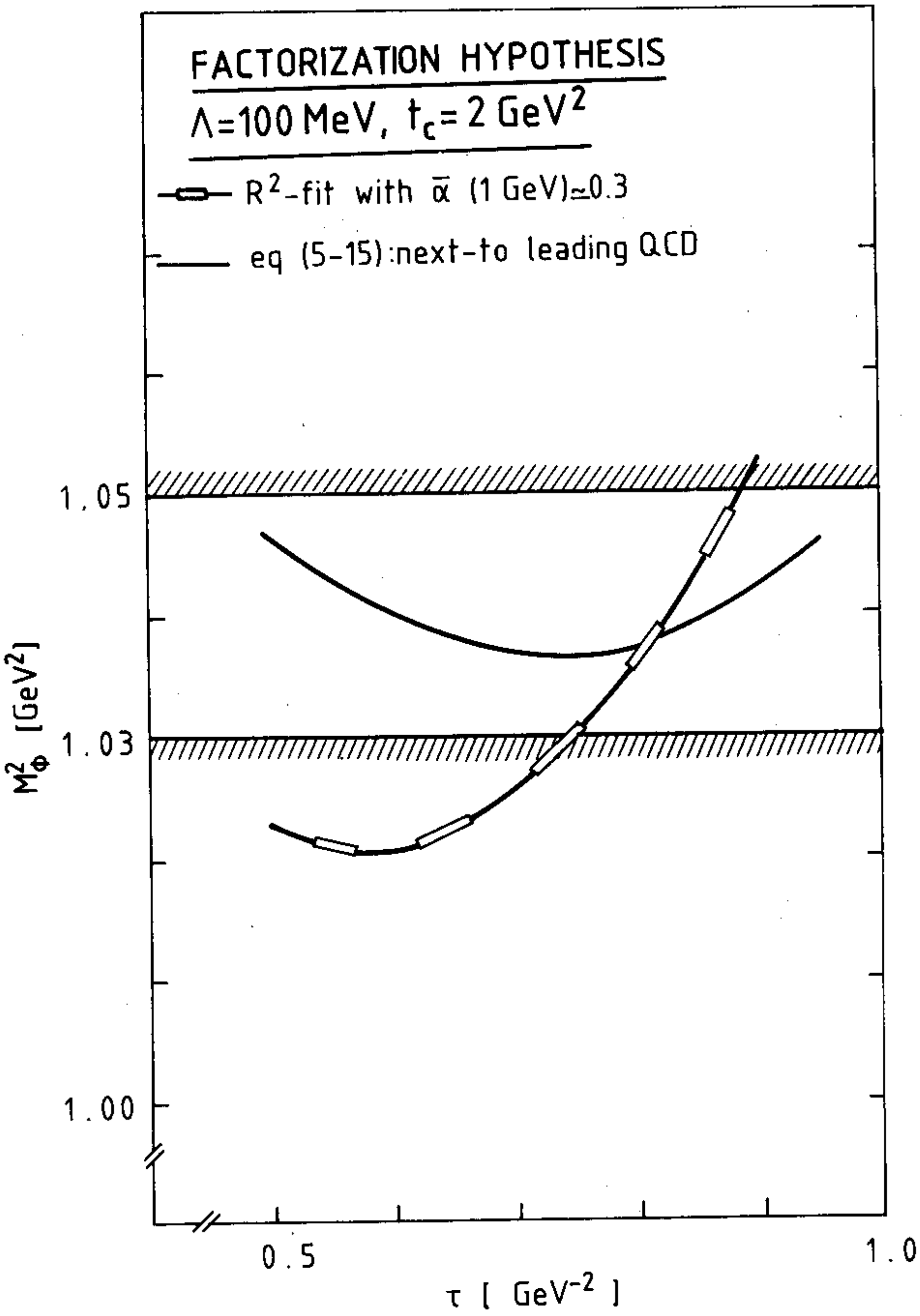
2
4
6
8
10
$\psi(0)_5^u \times 10^4$ [GeV]^4
Λ[GeV]
f_K[MeV]
t_c[GeV^2]
0.15
40.8
3.4
0.15
40.8
∞
0.1
34
3
0.6
0.8
1.0
$1/\sqrt{\tau}$ [GeV]

Fig. 8



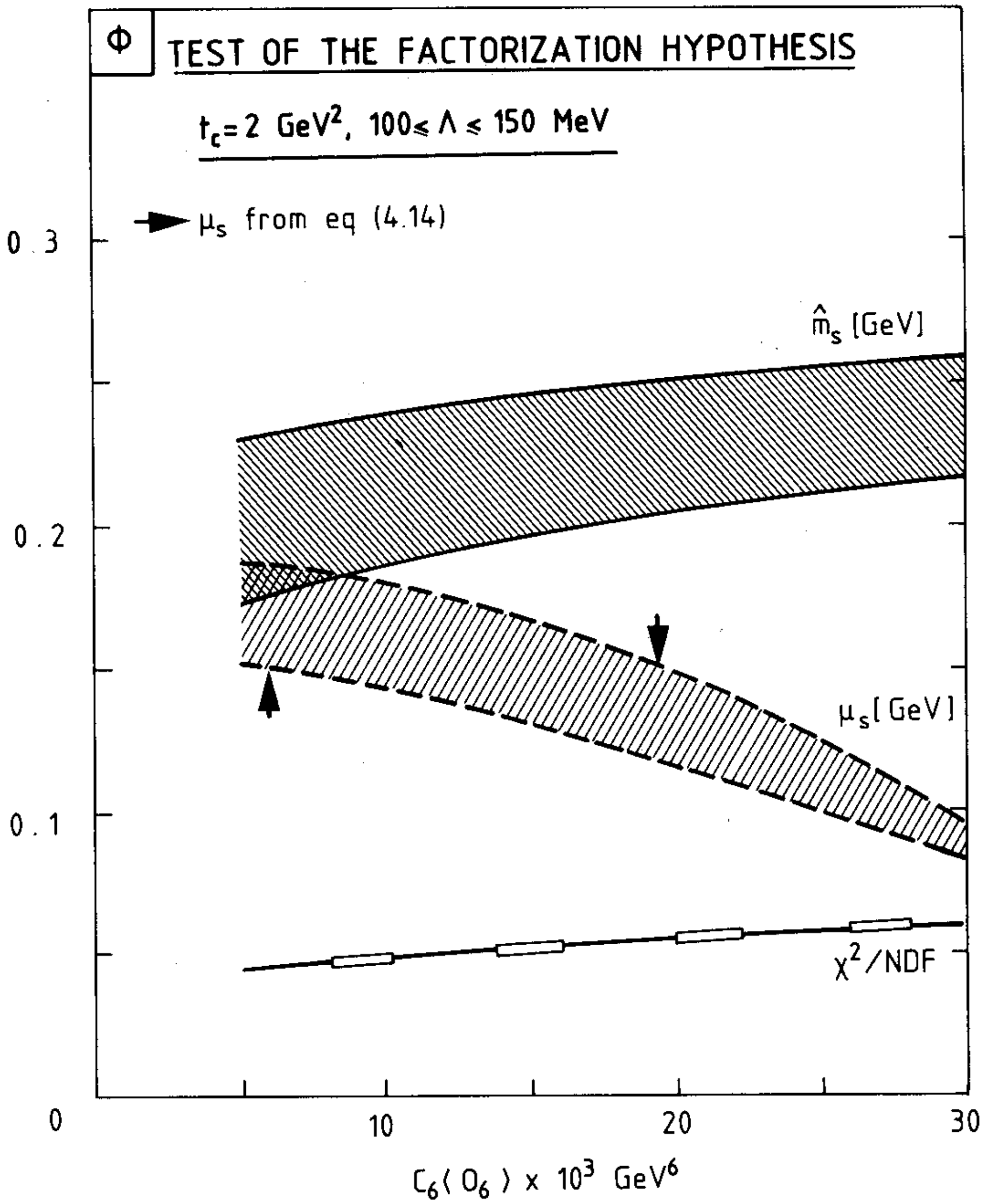
Φ
FACTORIZATION HYPOTHESIS
Λ = 100 MeV
m̂s [GeV]
(-m̂s⟨ss⟩) 10² GeV⁴
χ²/NDF
(pseudo) scalar
0.3
0.2
0.1
0
1.8
2.0
2.2
2.4
tc [GeV²]

Fig. 9



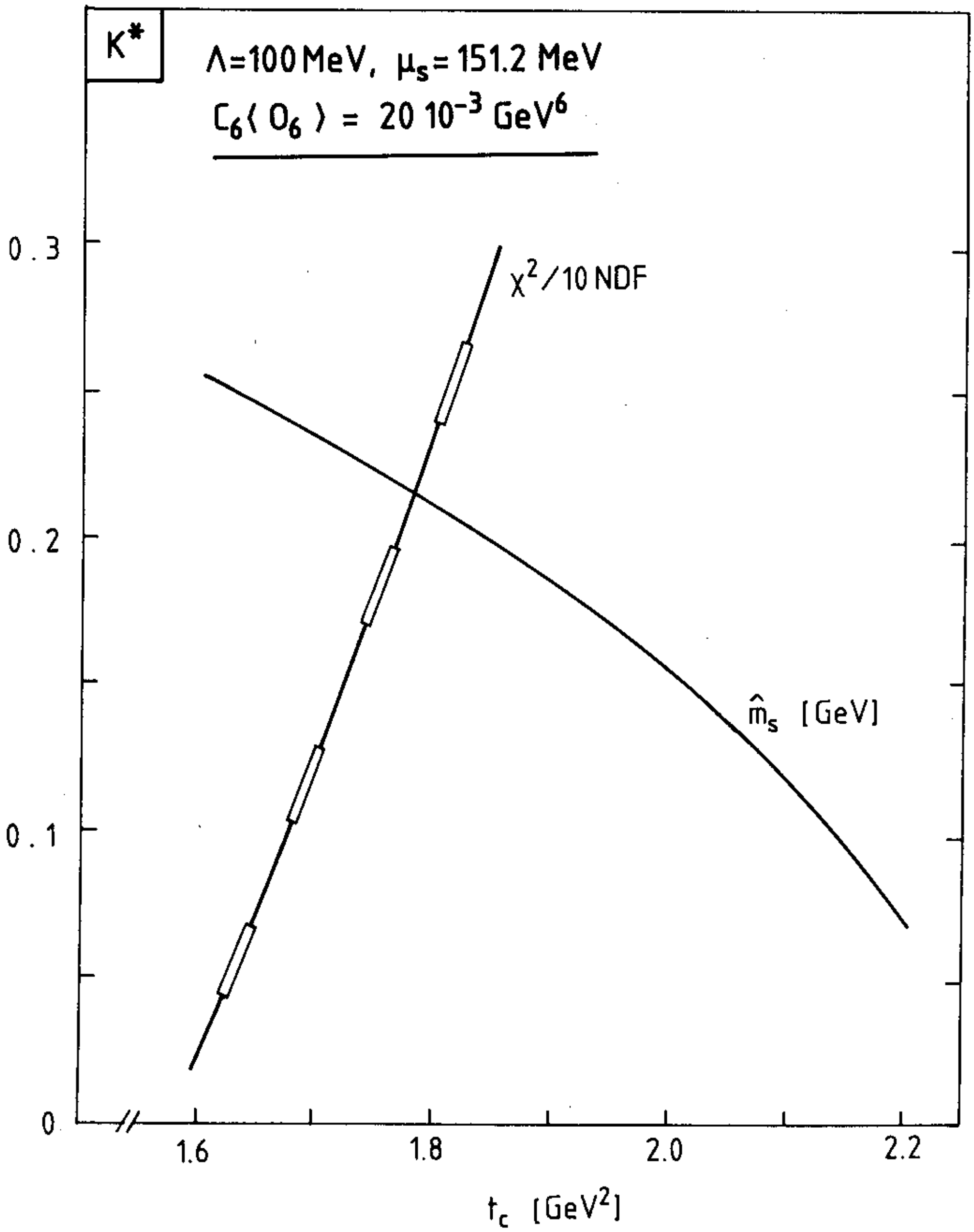
FACTORIZATION HYPOTHESIS
Λ=100 MeV, t_c= 2 GeV^2
R^2-fit with $\bar{\alpha}$ (1 GeV)≃0.3
eq (5-15):next-to leading QCD
1.05
1.03
1.00
M^2_Φ [GeV^2]
0.5
1.0
τ [GeV^{-2}]

Fig. 10



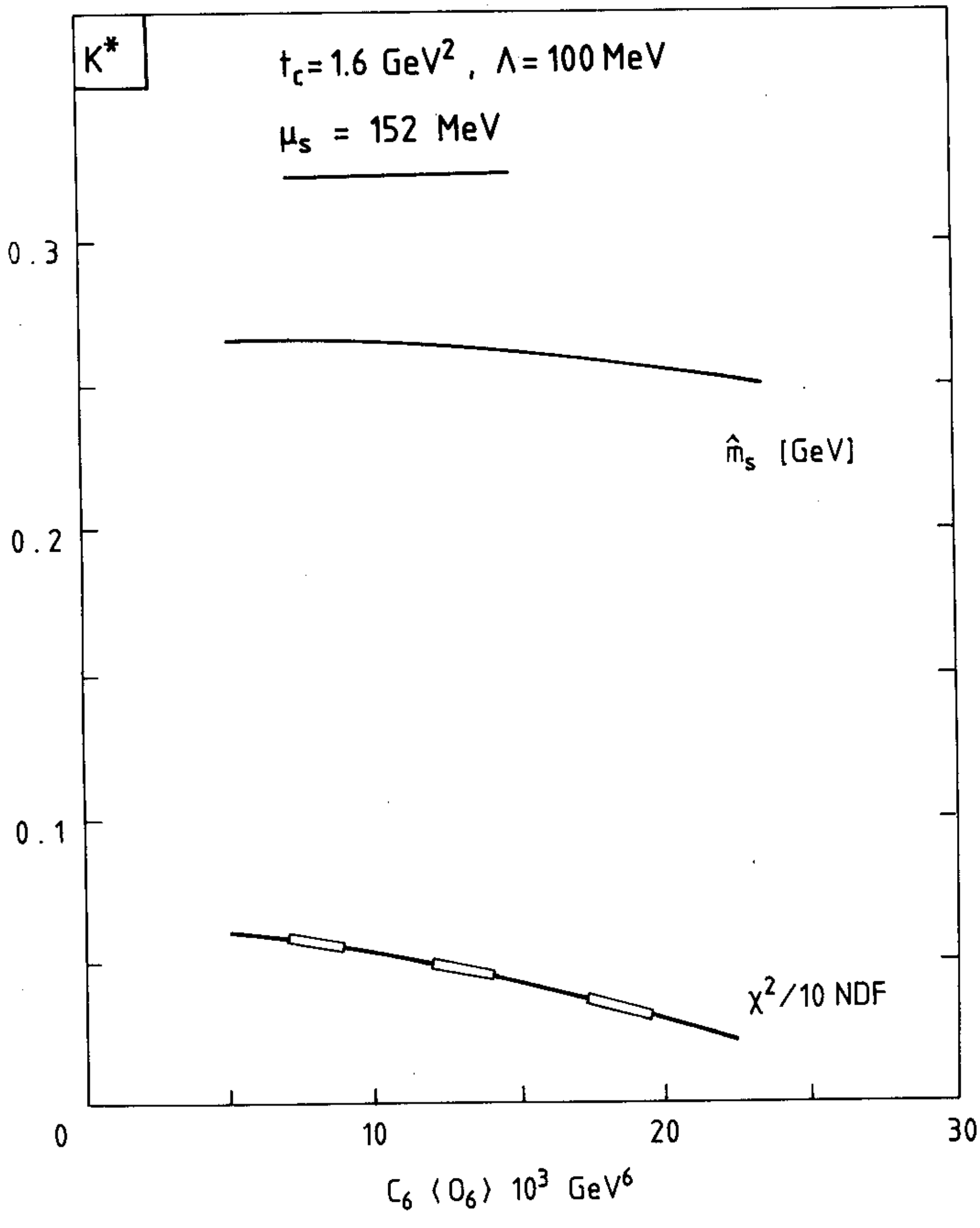
Φ
TEST OF THE FACTORIZATION HYPOTHESIS
$t_c = 2$ GeV^2, $100 \leq \Lambda \leq 150$ MeV
μ_s from eq (4.14)
$\hat{m}_s$ [GeV]
μ_s [GeV]
χ^2/NDF
0.3
0.2
0.1
0
10
20
30
$C_6 \langle O_6 \rangle \times 10^3$ GeV^6

Fig. 11



K*
Λ=100 MeV, μs = 151.2 MeV
C6⟨ O6 ⟩ = 20 10^-3 GeV^6
0.3
0.2
0.1
0
χ²/10 NDF
m̂s [GeV]
1.6
1.8
2.0
2.2
tc [GeV²]

Fig. 12



K*
$t_c = 1.6\ GeV^2$, $\Lambda = 100\ MeV$
μ_s = 152 MeV
0.3
0.2
0.1
$\hat{m}_s$ [GeV]
$\chi^2/10$ NDF
0
10
20
30
$C_6\ \langle O_6 \rangle\ 10^3\ GeV^6$

Fig. 13

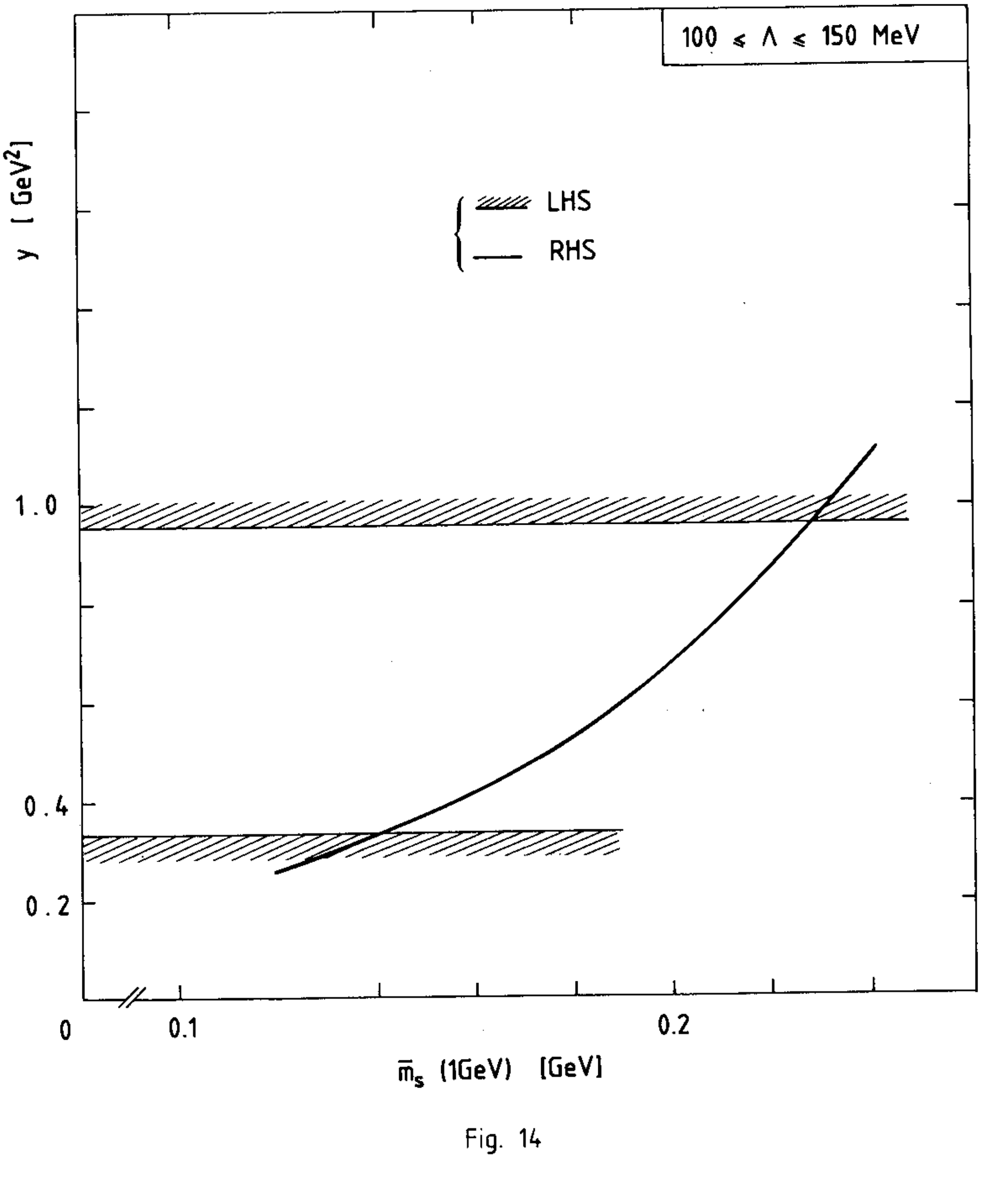


Fig. 14