Chirp scaling algorithm for processing SAR data with high squint angle and motion error

Alberto Moreira

German Aerospace Research Establishment (DLR)

Institute for Radio Frequency Technology
D-82230 Oberpfaffenhofen, Germany

Yonghong Huang
Beijing University of Aeronautics and Astronautics (BUAA)
Department of Electronic Engineering
Beijing 100083, P.R. China

ABSTRACT

This paper proposes a modified chirp scaling algorithm which is suitable for the processing of highly squinted data with motion errors. By means of azimuth subaperture processing with extended spectral length, the variations of the Doppler centroid in range and azimuth can be accommodated without using block processing with overlap. The new approach, denoted as extended chirp scaling (ECS), is considered to be a generalized algorithm suitable for the high resolution processing of most airborne SAR systems.

1. INTRODUCTION

The chirp scaling (CS) algorithm has recently been proposed for high quality SAR processing [2, 12]. This algorithm avoids any interpolation in the SAR processing chain and is suitable for the high quality processing of several spaceborne SAR systems. It consists basically of multiplying the SAR data in the range-Doppler domain with a quadratic phase function (chirp scaling) in order to equalize the range cell migration for a reference range, followed by an azimuth and range compression in the wave-number domain. After transforming the signal back to the range-Doppler domain, a residual phase correction is carried out. Finally, azimuth IFFTs are performed to generate the focussed image. It has been shown [10, 14] that the image quality and the phase accuracy of the chirp scaling algorithm is equal or superior to that of the range-Doppler algorithm.

For spaceborne SAR systems, the CS algorithm can be used to process data with up to approximately 20° squint angle in L-band without deterioration of the impulse response function IRF [3]. In order to accommodate higher squint angles, a non-linear phase term can be introduced into the processing or into the transmitted pulse [3]. In the former case, the non-linear phase term is added in the wave-number domain before applying the quadratic chirp scaling phase function. However, this cubic term requires additional range FFTs (fast Fourier transformation) for the transformation to the wavenumber domain and back to the range-Doppler domain. The image quality provided by the chirp scaling algorithm in the case of spaceborne SAR processing is excellent.

However, airborne SAR processing requires a motion error correction and also an update of the Doppler centroid as a function of the range distance and azimuth position. The above mentioned requirements cannot be included in the original chirp scaling algorithm, since it is basically a frequency domain focussing approach, whereby only one value of the Doppler centroid can be used in each block of the processing.

The new algorithm, denoted as the extended chirp scaling (ECS), has been developed for the processing of airborne data with strong motion errors (e.g. E-SAR system of DLR), high squint angles, and with variable Doppler centroid in range and/or azimuth.

The ECS algorithm including the accurate motion compensation and the high squinted data processing is described in section 2 of this paper. Section 3 presents the highly squinted data processing. Unlike other chirp scaling implementations, a deterministic, cubic phase term is added by the extended chirp scaling method during the equalization of the range cell migration, so that no deterioration of the geometric resolution is measured up to 30° squint angle in the C-band mode of the E-SAR system. Section 4 extends the ECS algorithm to include the update of the Doppler centroid with range and azimuth. In the case of the Doppler variation in the range direction, the proposed approach is based on the fact that this variation can be precisely accommodated if the chirp scaling phase function and also the subsequent processing steps are extended in the azimuth frequency by the amount of the Doppler centroid variation. For incorporating the variations of the Doppler centroid in azimuth, subaperture processing is used (i.e. short azimuth FFTs), whereby for each subaperture the value of the Doppler centroid is updated.

In section 5, the results of the image processing with 7.8° squint angle and strong motion errors are presented.

2. EXTENDED CHIRP SCALING ALGORITHM

A detailed formulation of the extended chirp scaling can be found in [7, 13]. In this section, the main processing steps including the accurate motion compensation will be summarized.

We consider in the following text an airborne side looking geometry with constant squint angle. The hyperbolic equation of the range history for a point target is expressed as:

$$R(t; r_o) = \sqrt{r_o^2 + v^2 \cdot (t - t_c)^2} \quad , \tag{1}$$

where r_o is the range to target at the closest approach, v is the aircraft velocity, t is the azimuth time measured in the flight direction, and t_c is the time at the center of the azimuth illumination path, which is directly related to the Doppler centroid f_{dc} .

After the transformation of the SAR signal to the range-Doppler domain, the range migration can be expressed as

$$R(f_a; r_a) = r_a \cdot (1 + a(f_a))$$
, (2)

where f_a is the azimuth frequency, λ is the radar wavelength and $a(f_a)$ is the linear chirp scaling factor given by

$$a(f_a) = \frac{1}{\sqrt{1 - \left(\frac{\lambda \cdot f_a}{2 \cdot \nu}\right)^2}} - 1 . \tag{3}$$

In the case of airborne SAR, the scaling factor $a(f_a)$ is not dependent on range, so that a linear scaling in range leads to a perfect equalization of the RCMC, if the so called secondary range compression (SRC) term can be neglected. The traditional chirp scaling method performs this equalization by means of a quadratic phase term in the range-Doppler domain. Actually, the scaling consists of changing the position of the phase minimum of each chirp signal. No explicit interpolation is carried out.

The quadratic phase term of the chirp scaling introduces a frequency offset in the range chirp signal, which can assume values as high as several MHz for high squint angle. If the frequency offset is high enough so that the signal bandwidth is aliased or shifted outside of the processed bandwidth, then the range IRF will deteri-

orate. In order to minimize this effect, the offset scaling factor due to the squint angle should be removed from the original chirp scaling term a(f), so that the new scaling factor for the processing is defined by:

$$a'(f_a) = a(f_a) - a(f_{dc}) \tag{4}$$

The scaling factor leads to an equalization of the range migration. However, the range positioning of the targets in the final image will be according to the slant range at the time t_c (center of the azimuth illumination path). The range positioning can be changed to that of the broadside geometry (range distances according to closest approach) during the slant to ground range transformation.

The scaling factor $a'(f_a)$ is now used for the range migration representation in frequency domain. The phase function for RCMC including a quadratic term (chirp scaling) and also a cubic term (for high squint processing) can be written as

$$H_1(\tau, f_a; r_{ref}) = \exp\left[-j \cdot \pi \cdot k(f_a; r_{ref}) \cdot a'(f_a) \cdot \left(\tau - \frac{2 \cdot R'(f_a; r_{ref})}{c}\right)^2\right]. \tag{5}$$

where $k(f_a; r_o)$ is the modified range frequency modulation in the range-Doppler domain, which consists basically of two terms:

$$\frac{1}{k(f_a; r_a)} \approx \frac{1}{k_r} + k_{src} \quad , \tag{6}$$

The term k_{src} in the range processing is called secondary range compression (SCR). After the phase correction H_1 , all targets have their range migration trajectories equalized to that of the reference range r_{ref} . The next step in the ECS algorithm is the transformation to the wavenumber domain. One basic characteristic of the extended chirp scaling algorithm is that no azimuth compression is performed in the wave-number domain. Only the range compression and the bulk range cell migration (for the reference range) are performed in this domain. The removal of the azimuth compression from this step is necessary for accurate motion compensation [13].

The phase correction term of the ECS algorithm in the wave-number domain consists of the range compression with the SRC for the reference range and an additional linear phase term with respect to the range frequency, which corresponds to a linear time shift in the range time. Using the wave-number formulation of the SAR signal, this phase correction can be calculated as

$$H_{21}(f_r, f_a; r_{ref}) = \exp\left[-j \cdot \pi \cdot \left(\frac{1}{k_r \cdot (1 + a'(f_a))} - r_{ref} \cdot \frac{2 \cdot \lambda \cdot (\beta^2 - 1)}{(1 + a'(f_a)) \cdot \beta^3 \cdot c^2}\right) \cdot f_r^2\right] \cdot \exp\left[-\frac{4 \cdot \pi \cdot r_{ref} \cdot a'(f_a)}{c} \cdot f_r\right].$$

$$(7)$$

where $\beta = 1/(a(f_a) + 1)$. When transforming the SAR signal back to the range-Doppler domain, a residual phase correction is applied as which compensates a slowly varying range dependent azimuth phase, and is given by

$$H_{22}(\tau, f_a; r_o) = \exp\left[j \cdot \pi \cdot k(f_a; r_{ref}) \cdot (1 + a'(f_a)) \cdot a'(f_a) \cdot \left(\frac{2}{c} \cdot (r_o - r_{ref})\right)^2\right]. \tag{8}$$

After the above correction, the SAR signal can be transformed to the signal domain (azimuth- and range-time) to perform accurate motion compensation.

The first order motion compensation is defined as being the phase correction for a reference range, and it can be directly carried out with the range uncompressed data (i.e. before the processing starts). The second order motion compensation includes the update of the phase correction as a function of the range distance. If the deviations of the aircraft trajectory are greater than one range bin, then the range delay must also be compensated on-line.

Let $\phi_{mc}(t; r_o)$ be the total phase for motion compensation and $\phi_{mc}(t; r_{ref})$ be the first order motion compensation, then the second order motion compensation, which is applied after transforming the range compressed SAR data to the signal domain, is formulated as

$$H_{mc}(t, r_o) = \exp\left[j \cdot (\phi_{mc}(t; r_o) - \phi_{mc}(t; r_{ref}))\right] . \tag{9}$$

After performing the second order motion compensation, only an one-dimensional azimuth compression must be performed. In the case that the squint angle is constant, the SAR signal is transformed again to the range-Doppler domain, and the azimuth modulation is corrected in this domain by an hyperbolic phase modulation:

$$H_3(\tau, f_a; r_o) = \exp\left[j \cdot \frac{4 \cdot \pi}{\lambda} \cdot r_o \cdot \sqrt{1 - \left(\frac{\lambda \cdot f_a}{2 \cdot \nu}\right)^2}\right]. \tag{10}$$

Fig.1 shows the block diagram of the ECS algorithm with the corresponding phase correction functions. The extra computation consists of the additional phase corrections and of the transformation to the signal domain before azimuth compression (additional azimuth IFFT's and FFT's). If the azimuth time-bandwidth product is low, then the azimuth compression is carried out more efficiently by time domain correlation approachs (e.g. subaperture processing [6]).

Using the ECS algorithm and the parameters of the E-SAR system in C-band, the simulation results up to 30 ° squint angle in C-band are almost perfect. The deterioration of the azimuth and range resolution are always lower than 1.7 % and 1.4 %, respectively. The PSLR is deteriorated by less than 0.6 dB and the measured phase accuracy is better than 2°.

3. PROCESSING HIGHLY SQUINTED DATA

The traditional chirp scaling algorithm assumes one reference range for the SRC and updates it with the azimuth frequency. The missing update of the SCR with range causes a phase error in the range compression for ranges different from the reference range. For high squint angles and low frequencies, this error will affect the image quality. This phase error can be compensated by the cubic term, which is introduced in phase function H_1 , The new chirp scaling phase is changed to:

$$H_{1_new}(\tau, f_a; r_{ref}) = \exp\left[-j \cdot \pi \cdot k(f_a; r_{ref}) \cdot a'(f_a) \cdot \left(\tau - \frac{2 \cdot R'(f_a; r_{ref})}{c}\right)^2\right]. \tag{11}$$

$$\exp\left[-j \cdot \pi \cdot k_r^2 \cdot \frac{\lambda \cdot (\beta^2 - 1)}{3 \cdot c \cdot \beta^3} \cdot \left(\tau - \frac{2 \cdot R'(f_a; r_{ref})}{c}\right)^3\right].$$

With this additional phase term, up to 30° squint angle can be processed without deterioration of the image quality. Due to the additional cubic term, the phase H_{22} becomes

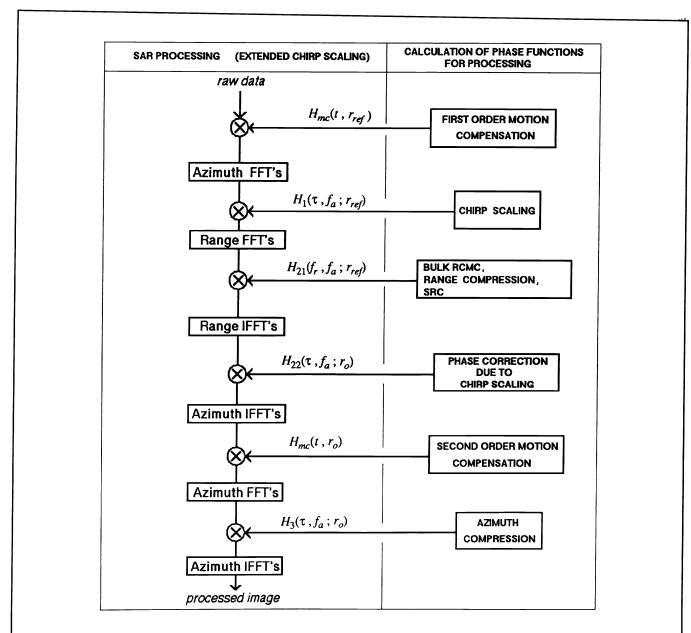


Figure 1. Block diagram of the extended chirp scaling (ECS) algorithm for high precision airborne SAR processing with integrated motion compensation

$$H_{22_new}(\tau, f_a; r_o) = \exp\left[j \cdot \pi \cdot k(f_a; r_{ref}) \cdot (1 + a'(f_a)) \cdot a'(f_a) \cdot \left(\frac{2}{c} \cdot (r_o - r_{ref})\right)^2\right].$$

$$\exp\left[-j \cdot \pi \cdot k_r^2 \cdot \frac{8 \cdot \lambda \cdot (\beta^2 - 1)}{c^4 \cdot \beta^3} \cdot (r_o - r_{ref})^3 \cdot (a'^2(f_a) - \frac{1}{3})\right].$$
(12)

The second term in the above equation corrects a residual phase error which was introduced by the cubic phase term in the scaling function of eq. 11.

4. ACCOMMODATION OF DOPPLER CENTROID VARIATIONS

Due to the large variation of the look angle in the airborne SAR geometry, the Doppler centroid can vary several hundred hertz from near to far range. The update of the Doppler centroid avoids a loss of signal-to-noise ratio, an azimuth defocussing and azimuth ambiguities. In the conventional chirp scaling algorithm, this variation could be accommodated in an inaccurate and inefficient way by a block processing in the range direction with an overlap between blocks equal to the length of the range reference function. A simple update of the Doppler centroid value in the chirp scaling phase would not work well, since the uncompressed range chirp signals from different range positions are overlapped in the range-Doppler domain.

Due to the variation of the look angle θ_i from near to far range, the Doppler centroid will vary according to

$$f_{dc}(r_o) = -\frac{2 \cdot \nu}{\lambda} \cdot \left[\sin \theta_l \cdot \sin \theta_d + \cos \theta_l \cdot \sin \theta_p \right] . \tag{13}$$

where θ_d is the drift angle (projected squint angle on ground, which is not dependent on the range distance) and θ_p is the pitch angle.

The proposed solution to this problem is very accurate. After transforming the SAR raw data to the range-Doppler domain, the azimuth frequency variation is artificially increased by means of an azimuth spectral length extension (see fig. 2), which accommodates the variation of the Doppler centroid with range. The azimuth spectral length extension should be at least as great as the variation of the Doppler centroid from near to far range. In a general case, the azimuth spectrum must be extended according to the following limits:

$$\min[f_{dc}(r)] - \frac{PRF}{2} < f_a < \max[f_{dc}(r)] + \frac{PRF}{2} , \qquad (14)$$

where $\min[f_{dc}]$ and $\max[f_{dc}]$ are the minimum and maximum Doppler centroid values used in the processing. Due to the azimuth spectral extension, all the phase functions $(H_{1_{new}}, H_{21} \text{ and } H_{22_{new}})$ applied in the range-Doppler and wavenumber domain are unambiguous, so that the measured quality of the IRF's are the same as in the case presented in the previous section with constant Doppler centroid. The only limitation of this approach is that the Doppler centroid variation within one range chirp length must be less than the PRF. Otherwise, the azimuth frequencies cannot be represented in an unambiguous way after the azimuth spectral extension.

B. Doppler centroid variation in azimuth

Due to the motion errors of the aircraft, the antenna must be steered in order to keep the squint angle constant. In the case of the E-SAR system, the antenna is fixed on the fuselage of the aircraft and a wide beam is used in azimuth, so that a constant squint angle could be adopted for the processing. The variations of the squint angle lead to a small signal loss, which could be are radiometrically corrected after the processing. Additionally, the ambiguity level is increased due to the use of a constant squint angle.

Then, for accurate processing, the squint angle value for the processing must be updated with azimuth. The basic assumption in the following approach is that the Doppler centroid for the processing is almost constant within the synthetic aperture length but it can vary several hundred hertz along the entire data acquision interval (data take), which is several minutes long. Since the processed azimuth bandwidth for the E-SAR system is less than one fourth of the total azimuth signal bandwidth, this assumption leads to very accurate results. With the update of the Doppler centroid with azimuth, the signal-to-noise ratio, the ambiguity level and the image resolution are optimized for the whole data take.

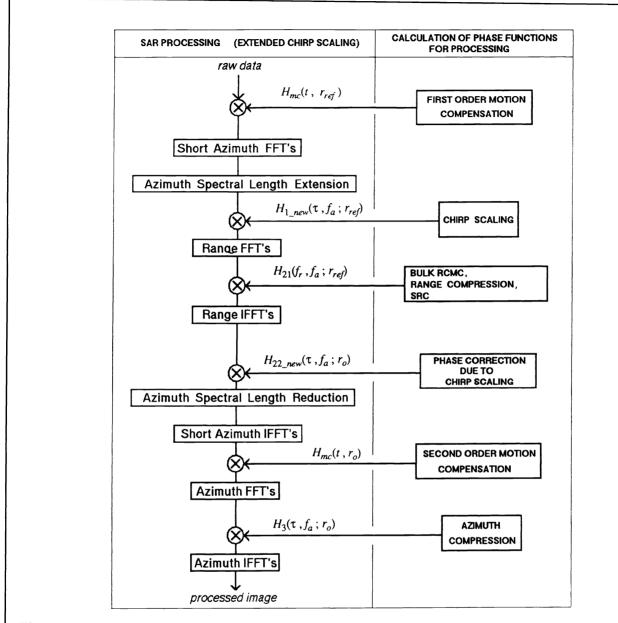


Figure 2. Block diagram of the extended chirp scaling (ECS) algorithm with integrated motion compensation and variable Doppler centroid.

The proposed approach for accommodating the Doppler centroid variations with azimuth consists of dividing the entire data take in small azimuth subapertures (e.g. 128 points), which are much smaller than one synthetic aperture length (see fig. 2). A small overlap between the subapertures is used in order to guarantee a phase continuity between the subapertures. Defining T_{sub} as the duration of each azimuth subaperture, T_{ovl} as the overlap between the azimuth subapertures and $T_{\text{data_take}}$ as the duration of the whole data take we obtain

$$s_i(\tau, t; r_o) = s(\tau, t + i \cdot (T_{\text{sub}} - T_{ovl}); r_o) \qquad -\frac{T_{data_take}}{2} \le t < T_{\text{sub}} - \frac{T_{data_take}}{2} , \qquad (15)$$

where s_i is the subaperture signal. For each subaperture, the azimuth frequency variation is centered around the actual Doppler centroid value for that subaperture, i.e.

$$-\frac{PRF}{2} + f_{dc_i} \le f_{a_i} \le f_{dc_i} + \frac{PRF}{2} \quad . \tag{16}$$

where i is the subaperture number. The above variation of the azimuth frequency is used for all the phase corrections in the range-Doppler and wave-number domain (i.e. H_{1_new} , H_{21} and H_{22_new}). However, the chirp scaling operation of each subaperture must be performed with respect to one Doppler centroid reference value in order to guarantee the same range scaling for the complete image. Then, the Doppler centroid value f_{dc} in the last term of eq. 4 is kept constant for the entire image. The reference Doppler centroid should be selected as the mean value of f_{dc_1} , so that the scaling amount is minimized. Let f_{dc_avg} be the mean value of f_{dc_1} , then eq. 4 is rewritten as

$$a'(f_a) = a(f_{ac}) - a(f_{dc \ avg}) \ . \tag{17}$$

It is important to note that the azimuth frequency variation f_{a_i} must be updated for each subaperture, although the reference Doppler centroid f_{dc_avg} is kept constant.

Fig.1 shows the block diagram of the ECS algorithm with integrated motion compensation and variable Doppler centroid. After the multiplication with the functions $H_{1_{new}}$, H_{21} and $H_{22_{new}}$, the subapertures are transformed to the signal domain. In this domain the time overlap is removed and the subaperture signals are joined in order to reconstruct the full data take. After applying the second order motion compensation (eq. 9), the data is compressed in azimuth using the reference value of the Doppler centroid (f_{dc_avg}) . The azimuth phase correction H_3 is then applied according to eq. 10 in the range-Doppler domain.

5. IMAGE PROCESSING

A flight of the E-SAR system over the airfield of Oberpfaffenhofen in Germany was used to test the new approach. The selected data set for processing has a squint angle of 7.8° and very strong motion errors, which were intentionally induced by the aircraft's pilot. The phase correction for motion compensation was obtained by the approach developed at DLR [1].

Fig. 3 shows the processed image (laser printer output) using the extended chirp scaling algorithm with motion compensation and variable Doppler centroid. The image sizes are $2895 \text{ m} \times 3766 \text{ m}$ (range \times azimuth). The evaluation of the image quality and the comparison with the results obtained by processing with the range-Doppler and hybrid algorithms shows that a better IRF and a higher signal-to-noise ratio is achieved by the ECS algorithm in the image areas of severes motion errors. No significant differences can be measured in the areas of small motion errors. The improved image quality of the ECS algorithm is justified by the accommodation of the Doppler centroid variation and also by the more accurate second order motion compensation, which is carried out after the complete range migration has been removed.

6. CONCLUSIONS

The ECS algorithm is able to accommodate the Doppler centroid variation in range by means of an azimuth spectral lenght extension in the range-Doppler domain. In the azimuth direction, the Doppler centroid variation is incorporated by means of azimuth subaperture processing. Due to the very accurate second order motion compensation and processing up to 30 ° squint angle, the proposed approach is considered to be a generalized algorithm suitable for the high resolution of most airborne SAR systems.

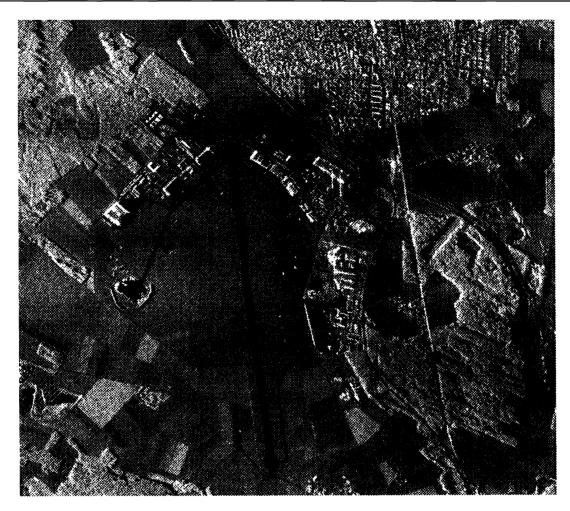


Figure 3. Processed E-SAR image of the runway at Oberpfaffenhofen using the extended chirp scaling algorithm. Main sensor and processing parameters are: 2205 m flight altitude, 74 m/s ground speed, 7.8° squint angle, C-band, 8 looks with 50% overlap, VV polarization, 2.5 m \times 4.0 m resolution (range \times azimuth).

7. REFERENCES

- 1. Buckreuss, S.: "Motion Errors in an Airborne Synthetic Aperture Radar System". European Trans. on Telecomm. and Related Technologies, Vol. 2, No. 6, 1991, pp. 55-64.
- 2. Cumming, I., Wong, F. and Raney, K.: " A SAR Processing Algorithm With No Interpolation". Proc. of IGARSS, 1992, pp. 376-379.
- 3. Davidson, G.W., Cumming, I.G. and Ito, M.R.: "A Chirp Scaling Approach for Processing Squint Mode SAR Data". Submitted to I.E.E.E. Trans. on Aerosp. and Electr. Syst., 1994.
- 4. Horn, R.: "E-SAR The Experimental Airborne L/C-Band SAR System of DFVLR". Proc. of IGARSS, Sept. 1988, pp.1025-1026.

- 5. Jin, M.Y. and Wu, C.: "A SAR Correlation Algorithm which Accommodates Large Range Migration". IEEE Trans. on Geosci. and Remote Sensing, Vol. 22, No. 6, 1984, pp. 592-597.
- 6. Moreira, Alberto: "Real-Time Synthetic Aperture Radar Processing with a New Suabperture Approach", IEEE Trans. on Geosci. and Remote Sensing, Vol. 30, No. 4, July 1992.
- 7. Moreira, Alberto and Yonghong, Huang: "Airborne SAR Processing of High Squinted Data Using a Chirp Scaling Approach with Integrated Motion Compensation", Submitted to IEEE Trans. Geosci. and Remote Sensing, 1994.
- 8. Moreira, J.: "A New Method of Aircraft Motion Error Extraction from Radar Raw Data for Real-Time Motion Compensation". IEEE Trans. Geosci. and Remote Sensing, Vol. 28, No. 4, 1990, pp. 620-626.
- 9. Prati, C. and Rocca, F.: "Focusing SAR Data with Time-Varying Doppler Centroid". IEEE Trans. on Geosci. and Remote Sensing, Vol. 30, No. 3, May, 1992, pp. 550-559.
- 10. Raney, R.K., Runge, H., Bamler, R. Cumming, I. and Wong, F.: "Precision SAR Processing without Interpolation for Range Cell Migration Correction". Submitted to I.E.E.E. Trans. on Geosci. and Remote Sensing, 1994.
- 11. Rocca, F.: "Synthetic Aperture Radar: A New Application for Wave Equation Techniques". In Stanford Exploration Project rep., SEP-56, 1987, pp. 167-189.
- 12. Runge, H. and Bamler, R.: " A Novel High Precision SAR Focussing Algorithm Based on Chirp Scaling". Proc. of IGARSS, Houston, 1992, pp. 372-375.
- 13. Yonghong, H. and Moreira, A.: "A Chirp Scaling Algorithm for Airborne SAR Processing". DLR Internal Report, Oct. 1993, pp. 1-54.
- 14. Wong, F., Cumming, I. and Raney, R.K.: "Processing Simulated RADARSAT SAR Data with Squint by a High Precision Algorithm". Proc. of IGARSS, Aug. 1993, pp. 1176-1178.