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CHOOSING THE TECHNICAL EFFICIENCY ORIENTATION TO ANALYZE FIRMS' TECHNOLOGY: A MODEL SELECTION TEST APPROACH

Luis Orea[*], David Roibás[*♣] and Alan Wall[*]

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Abstract
We focus on the importance of the assumptions regarding how inefficiency should be incorporated into the specification of the data generating process in an examination of a sector's production or efficiency. Drawing on the literature on non-nested hypothesis testing, we find that the model selection approach of Vuong (1989) is a potentially useful tool for identifying the best specification before carrying out such studies. We include an empirical application using panel data on Spanish dairy farms where we estimate cost frontiers under different specifications of how inefficiency enters the data generating process (in particular, efficiency is introduced as an input oriented, output oriented and hyperbolic parameter). Our results show that the different models yield very different pictures of the technology and the efficiency levels of the sector, illustrating the importance of choosing the most correct model before carrying out production and efficiency analyses. The Vuong test shows that the input oriented model is the best, whereas the output oriented model is the worst. This is consistent with the fact that the input and output oriented models provide the most and least credible estimates of scale economies given the structure of the sector.

Keywords: Cost frontier, technical efficiency, non-nested hypothesis testing, Vuong test.

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1. Introduction

A correctly estimated production technology is an enormously useful tool for managers and policymakers and provides a valuable source of information with which to address issues such as the efficiency, productivity and competitiveness of firms and/ or sectors. Whereas in the early literature it was typical to estimate a production function under the assumption that producers operated on these functions[1], with any observed deviation being due simply to random statistical noise, in recent decades a new body of literature, stochastic frontier analysis, has challenged this assumption. The stochastic frontier literature stems from the recognition that firms may not be behaving efficiently and hence may be operating away from their production functions for reasons other than random noise. Thus, instead of estimating, say, production *functions*, analysis has shifted towards the estimation of production *frontiers*.

The issue facing the applied researcher, therefore, is that the observed data have not been generated simply by the firms' technology and random noise but that inefficiency has also played a role. As the nature of inefficiency is unknown, assumptions must be made about the way in which inefficiency has entered the Data Generating Process (hereafter DGP) in order for the technology to be estimated. This in turn gives rise to a problem which has received relatively little attention in the literature[2] and which forms the basis of this paper, namely that different assumptions about the nature of the inefficiency give rise to different estimates of the technologies. Given that there is only one true technology, however, making the wrong assumption about the nature of inefficiency will leave us with a misspecified model with the result that any calculations made on the basis of such an estimation (for example, efficiency indices, calculations of economies of scale, etc) may lose certain validity.

Given the variety of competing assumptions regarding the nature of inefficiency which have been prominent in the literature[3], our argument is that the applied researcher should first identify which of these most closely represents the true DGP as a preliminary step

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[1] Classic references for various types of functions include Nerlove (1963), Berndt and Christensen (1973) and Christensen, Jorgenson and Lau (1973).

[2] Atkinson and Cornwell (1994) is perhaps the first of the relatively few papers to address the implications of using alternative specifications of inefficiency in estimating technologies. See also Arias and Kumbhakar (2001).

towards estimating the true technology. Once this has been done, analyses of efficiency etc. can be carried out. Atkinson and Cornwell (1994) briefly address the issue of choosing between competing specifications of inefficiency, suggesting that the appropriate model may be selected on the basis of economic justifications (a thorough knowledge of the sector may provide guidance) or through statistical methods. Our approach is to use econometric tools from the model selection literature (the Vuong, J and JA tests) as a basis for identifying the best assumption on the nature of efficiency and hence the most accurate specification of the technology. In an empirical study of cost frontiers using panel data from Spanish dairy farms, we focus on the choice between three specifications of how inefficiency may have affected the data – input-oriented inefficiency, output-oriented inefficiency, and a hyperbolic measure. Having identified the specification closest to the true DGP, we compare results on efficiency indices and aspects of the technology such as scale economies under the different specifications in order to illustrate how they are affected by choosing the wrong assumption.

We proceed as follows. In Section 2 we discuss in more detail the issue that different assumptions on efficiency give rise to different technology estimates. In Section 3 we outline the three competing models which we study, where inefficiency is introduced into the model through input-oriented, output-oriented and hyperbolic measures of efficiency. Section 4 provides a brief overview of the procedures used to evaluate these models and discusses how they can be used to identify the correct technology (or DGP). The data used in our empirical application is discussed in Section 5. In Section 6 we present our results, and Section 7 concludes.

2. Efficiency and the data generating process

When estimating a technology frontier the applied researcher is faced with the problem that the DGP comprises three unobservable parts: the production technology, the nature of inefficiency, and random statistical noise. Leaving aside random noise for the moment, the researcher is thus obliged to make some assumption about the nature of inefficiency in order to be able to estimate the true technology from the observed data.

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[3] That is, input and output oriented indices, hyperbolic indices and so on.

When we estimate a production function without taking inefficiency into account, the *observed* production function (that which exactly fits our observed input and output data) will lie beneath the "true" technology frontier, which is that we would observe if all the firms were efficient. Our task therefore is to adjust the observed data (that is, eliminate the effect of inefficiency) in such a way as to shift the observed function upwards so that it coincides with the true technology. How we actually shift the observed function upwards depends on our assumptions on the nature of inefficiency, with each assumption generating a different specification, better or worse, of the technology. This problem is illustrated in Figure 1 for the simple one output-one input case.

Taking any two observations on inefficient firms, such as A and B in the Figure, we may for example assume that the nature of inefficiency is such that these firms could be producing more output from the inputs ($x_A$and x_B respectively) at their disposal. In this case we believe that inefficiency has entered the DGP in such a way as to have displaced the true data point (that corresponding to the firm were it on the frontier) vertically downwards. That is, we believe that if they had been efficient they would be producing at A_O and B_O. If the degree of inefficiency is the same for all observations, the true technology would be represented by the *output-oriented* frontier, which is a parallel vertical upward shift of the observed production function[4]. On the other hand, we could have assumed that the observed data embody *input-oriented* inefficiency, in that the observed level of output is being produced using more inputs than necessary. If this were correct, inefficiency will have displaced the data point on the technology frontier horizontally to the right. Hence, firms A and B, if efficient, would be producing at points A_I and B_I. Again, doing the same for all observations allows us to trace the *input-oriented* frontier, where the true technology is represented by a shift to the left of the observed function. Yet another possibility is that inefficiency could have simultaneously reduced output levels and increased input levels. Then, firms A and B would be producing at points A_H and B_H if they were efficient. The technology would be recovered by a simultaneous (radial) expansion of outputs and reduction in inputs, and doing the same for all data points we would have a parallel shift north-west of the observed function. This is the Hyperbolic frontier proposed by Färe, Grosskopf and Lovell (1985).

[4] Note that as the variables in the figure are expressed in logarithms, this is a proportional shift of the raw production function.

Although other possibilities exist, these constitute the main adjustments which are made in applied papers to take inefficiency into account when estimating frontiers[5]. Note that in the presence of global constant returns to scale each of the assumptions on the nature of inefficiency referred to above will yield the same technologies. In the absence of constant returns to scale, however, each of these assumptions gives rise to a *different* technology and consequently may provide a more or less accurate representation of the true DGP. As such, the closer or further our assumption on inefficiency takes us to or from the true frontier, the more or less valid any results based on our estimation will be.

To illustrate this, take the following example. Suppose we were interested in estimating the relative inefficiency of the firms, which is a common objective when estimating frontiers, and we assume that the DGP contains output-oriented inefficiency. That is, we assume that the technology is represented by the output-oriented frontier. Taking this as our reference, we would then calculate efficiency indices based on the vertical distance of the observed data from the frontier. Now suppose that, unknown to us, inefficiency is in fact input-oriented so that the correct reference should have been the input-oriented frontier. If returns to scale are not constant, as is the case in Figure 1, then the frontiers do not coincide: we have different technologies and hence different efficiency indices. The result of this is that the output efficiency indices are calculated with respect to the *wrong* frontier. In the representation in Figure 1, erroneously choosing the output-oriented frontier means that we will *overstate* the inefficiency of firm A (note that the vertical distance from point A to the output-oriented frontier is greater than that to the correct, input-oriented frontier) and *understate* the inefficiency of firm B. In general, choosing the wrong specification may lead us to greater understate or overstate average efficiency in a sector as a whole. If we desire a fuller picture of efficiency in a sector, then we can of course calculate output efficiency measures but these should be calculated with reference to the *input-oriented* frontier.

Apart from changes in the values of efficiency indices, it is easy to imagine siuations where the efficiency rankings of firms could also change. This was highlighted by Atkinson and Cornwell (1994) in their seminal paper, where technical efficiency indices were calculated using two different models of cost frontiers.[6] These authors found important differences between firm-level relative output and input technical efficiencies. Given these findings,

[5] However, the input and output oriented measures of efficiency are by far the most popular to date due to the simplicity with which they can be estimated and their intuitive appeal.

they go on to address the issue of selecting the appropriate model, stating that as "…neither model is nested in the other, it is natural to ask how one might choose between the measures" (p.254). They suggest some statistical method could be used[7] or else that there may be economic reasons to justify preference for one model over the other.

Given that there are many (indeed, an infinite) number of ways to shift the observed function upwards in order to approximate the DGP, there are an equivalent number of competing model specifications. In general, however, none of the common specifications will provide an exact representation of the technology so the issue we face is to choose the "best" model in the sense that it best represents the DGP. Given this, we advocate a model selection approach in order to represent the DGP as closely as possible and therefore provide a better estimation of the technology. We will focus on model specifications which incorporate input oriented, output oriented and hyperbolic efficiency measures and make use of tools from the model selection literature in order to identify the specification closest to the true technology. First, however, let us outline the models to be estimated.

3. Specification of the cost frontiers

To apply the concepts outlined in the previous section, we estimate cost frontiers for a set of Spanish dairy farms. In particular, we estimate three translog cost frontier models where technical inefficiency enters as a parameter, where the first two were proposed by Atkinson and Cornwell (1994). In the first model, the parameter measures the proportion to which actual output is less than frontier output (output-oriented model), whereas in the second it captures the proportion to which actual input usage is greater than frontier input usage (input-oriented model). The third and final model has a parameter which measures the degree to which firms can reduce all variable inputs and *simultaneously* increase their output (hyperbolic model). We now outline each of these in turn.

[6] One model captured input-oriented inefficiency, the other output-oriented inefficiency.
[7] In particular, they suggest using the log-likelihood as a criteria.

We begin with the output inefficiency cost frontier. Output technical efficiency implies that a firm's production may be below the maximum efficient output as given by the *frontier* production function. That is,

$$y_i = a_i \cdot f(x_i) \quad , \quad 0 < a_i \leq 1 \tag{1}$$

where y_i is the quantity produced by the i-th firm, $x_i = (x_1,...,x_n)$ is the vector of inputs employed by the firm, $f(\cdot)$ is a standard neoclassical production function and a_i is an output-oriented parameter of (relative) technical efficiency which represents the ratio of observed output to potential output.

The cost frontier associated with the specification of the production function in (1) can be expressed as

$$C(y_i/a_i\, , w_i) = \min_{x_i} \left[w'_i\, x_i :\ f(x_i) = y_i/a_i\right] \tag{2}$$

where $w_i = (w_1,...,w_n)$ is the vector of input prices which the i-th firm pays and y_i/a_i is the quantity it would produce if it were technically efficient. The translog specification of the cost frontier (2) is

$$\begin{aligned} \ln C_i &= \beta_1 + \beta_y \ln(y_i/a_i) + \frac{1}{2}\beta_{yy}\ln(y_i/a_i)^2 + \sum_{k=1}^{n}\beta_{w_k}\ln w_{ki} \\ &+ \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{ki}\ln w_{hi} + \sum_{k=1}^{n}\beta_{y_j w_k}\ln(y_i/a_i)\ln w_{ki} \end{aligned} \tag{3}$$

and applying Shephard´s Lemma we have that

$$S_{ki} = \beta_{w_k} + \sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{hi} + \beta_{yw_k}\ln(y_i/a_i) \quad , \quad k = 1,...,n \tag{4}$$

where S_{ki} is the share of the k-th input of costs.

Turning now to the input-oriented model, we begin with a production function which recognises that a firm producing a given level of output may be using more inputs than the minimum necessary. That is,

$$y_i = f(b_i x_i) \quad , \quad 0 < b_i \leq 1 \tag{5}$$

where b_i measures the extent to which actual input usage differs from the input usage an efficient firm would use to produce the observed output y_i. Denoting the efficient input quantity as $x_i^*=b_i x_i$, the input-oriented cost frontier can be expressed as

$$C^*(y_i, w_i) = \min_{x_i^*}\left[w'_i\, x_i^* \;\middle|\; f(x_i^*) = y_i\right] \tag{6}$$

Taking into account that $C_i=C^*(y_i,w_i)\cdot(1/b_i)$, the translog specification of (6) is

$$\begin{aligned} \ln C_i &= \beta_1 + \beta_y \ln(y_i) + \frac{1}{2}\beta_{yy}\ln(y_i)^2 + \sum_{k=1}^{n}\beta_{w_k}\ln w_{ki} \\ &+ \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{ki}\ln w_{hi} + \sum_{k=1}^{n}\beta_{y_j w_k}\ln(y_i)\ln w_{ki} + \ln(1/b_i) \end{aligned} \tag{7}$$

where $\ln(1/b_i)$ represents the distance of firm *i* from the cost frontier. The corresponding share equations are

$$S_{ki} = \beta_{w_k} + \sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{hi} + \beta_{yw_k}\ln(y_i) \quad , \quad k = 1,\dots,n \tag{8}$$

Finally, using a hyperbolic measure of efficiency we have a production function which recognises that a firm may be able to reduce all variable inputs and *simultaneously* increase its output at the same rate. That is,

$$y_i = h_i \cdot f(h_i x_i) \quad , \quad 0 < h_i \le 1 \tag{9}$$

Denoting the efficient input quantity as $x_i^* = x_i \cdot h_i$, the hyperbolic cost frontier can be written as:

$$C^*(y_i/h_i, w_i) = \min_{x_i^*}\left[w'_i\, x_i^* : f(x_i^*) = y_i/h_i\right] \tag{10}$$

Taking into account that $C_i=C^*(y_i/h_i,w_i)\cdot(1/h_i)$, the translog specification is

$$\begin{aligned} \ln C_i &= \beta_1 + \beta_y \ln(y_i/h_i) + \frac{1}{2}\beta_{yy}\ln(y_i/h_i)^2 + \sum_{k=1}^{n}\beta_{w_k}\ln w_{ki} \\ &+ \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{ki}\ln w_{hi} + \sum_{k=1}^{n}\beta_{y_j w_k}\ln(y_i/h_i)\ln w_{ki} + \ln(1/h_i) \end{aligned} \tag{11}$$

From this it is clear that the corresponding share equations have a similar structure to those of the output-oriented efficiency model (4), with the efficiency parameter a_i replaced by h_i:

$$S_{ki} = \beta_{w_k} + \sum_{h=1}^{n}\beta_{w_k w_h}\ln w_{hi} + \beta_{yw_k}\ln(y_i/h_i) \quad , \quad k = 1,\dots,n \tag{12}$$

The models to be compared are the systems formed by equations (3) and (4), (7) and (8), and (11) and (12)[8]. Note that in the presence of globally constant returns to scale, we will have $a_i=b_i=h_i^2$ and the three systems will be identical[9]. When this is not the case, however, it is clear that three models differ, each providing a different specification of the DGP.

As far as estimation of these cost systems is concerned, ML can be used once random disturbances are added to the cost function and share equations. In general terms, the econometric specification of the three systems using panel data can be written as[10]

Output oriented:
$$\begin{aligned} \ln C_{it} &= C^O(w_{it}, y_{it}, \beta, a_i) + u_{it} \\ S_{kit} &= S^O(w_{it}, y_{it}, \beta, a_i) + \varepsilon_{kit} \end{aligned} \tag{13a}$$

Input oriented:
$$\begin{aligned} \ln C_{it} &= C^I(w_{it}, y_{it}, \beta, b_i) + u_{it} \\ S_{kit} &= S^I(w_{it}, y_{it}, \beta) + \varepsilon_{kit} \end{aligned} \tag{13b}$$

Hyperbolic:
$$\begin{aligned} \ln C_{it} &= C^H(w_{it}, y_{it}, \beta, h_i) + u_{it} \\ S_{kit} &= S^H(w_{it}, y_{it}, \beta, h_i) + \varepsilon_{kit} \end{aligned} \tag{13c}$$

where the technical efficiency parameters (a_i, b_i, h_i) are assumed to be time-invariant, and β is the parameter vector of the cost function (and, by extension, of the share equations). The disturbances added to each of the cost functions belong to the same family of distributions but have different parameters, with the same applying to those added to the input share equations. In particular, they are assumed to follow a multivariate normal distribution with zero means and covariance matrix Ω.

Given that efficiency enters the models as a parameters to be estimated, the first model (13a) involves the estimation of a parameter vector $\theta = (\beta,\Omega,a)$, where $a = (a_1,a_2,\ldots,a_N)$ is a vector of N output-oriented technical efficiency levels. In contrast, the second model (13b) involves the estimation of the parameter vector given by $\gamma = (\beta,\Omega,b)$, where b =

[8] The systems comprised of the cost functions and the share equations are estimated instead of just the cost functions as the estimates will be more precise given the extra information provided.
[9] Färe and Lovell (1978) show that input and output oriented measures of efficiency are equivalent only under constant returs to scale. It is easily seen that the hyperbolic measure is the square root of both the input and output oriented measures under constant returns.
[10] We are implicitly assuming away allocative inefficiency. Its incorporation would considerably complicate the expression for the cost system (see, for instance, Lovell and Kumbhakar, 2000) and thus make it difficult to successfully carry out the model selection tests.

$(b_1,b_2,\ldots,b_N)$ is a vector of N input-oriented technical efficiency levels. Similarly, the third model (13c) requires estimation of the vector $\delta = (\beta,\Omega,h)$ where $h = (h_1,h_2,\ldots h_N)$. It can be seen that the models are non-nested (in particular, they are overlapping) since the set of parameters of any model includes some that are not in the others.

Below, we estimate the three models specified in (13a-c) using panel data from the dairy farm sector in a Spanish region. Before discussing the application, in the section that follows we briefly discuss the procedure used to select the model which best represents the data.

4. Choosing between models

From the variety of possibilities in the literature, we adopt the model selection procedure developed by Vuong (1989), which is a test based on the likelihood-ratio principle to select among non-nested or overlapping models such as those in (13a-c). The Vuong test is a symmetric and directional test designed to test the null hypothesis that two competing models adjust equally well the data versus the alternative that one model fits better. The hypotheses are therefore

H_0: Model A and Model B are equally close to the true model
H_1: One model is closer to the true model than the other

Specifically, for any pair of models in (13a-c), we calculate the likelihood-ratio statistic which is normalized by the so-called sample variance. Say, for example, we want to compare the output oriented (13a) and input oriented (13b) models. Let θ^{ML} and γ^{ML} be the ML estimators of the parameter vectors θ and γ. The Voung test can be written as:

$$VT(\theta^{ML},\gamma^{ML}) = \frac{LR(\theta^{ML},\gamma^{ML})}{NT^{1/2}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\ln V1_{it}(\theta^{ML}) - \ln V2_{it}(\gamma^{ML})\right)^2 - \left(\frac{1}{NT}LR(\theta^{ML},\gamma^{ML})\right)^2\right]^{1/2}} \tag{14}$$

where

$$LR(\theta^{ML}, \gamma^{ML}) = \sum_{i=1}^{N}\sum_{t=1}^{T} \ln V1_{it}(\theta^{ML}) - \sum_{i=1}^{N}\sum_{t=1}^{T} \ln V2_{it}(\gamma^{ML})$$

is the well-known likelihood-ratio statistic, and $V1_{it}$ and $V2_{it}$ are the respective individual values of the likelihood function evaluated at the estimated parameters.

The normalized LR statistic is normally distributed under the null hypothesis of equal fit. Once a critical value *c* from the standard normal distribution for some significance is chosen, then we cannot reject the null hypothesis that the models are equivalent if the normalized LR statistic is smaller than *c* in absolute value. We therefore conclude that the data do not enable us to discriminate between two models. Otherwise, we conclude that the competing models can be statistically discriminated, and the sign of the LR statistic indicates which of them dominates the other in the sense of being closer to the true model.

As mentioned above, many other methods exist with which non-nested models can be analysed. Broadly speaking, two main approaches have been followed in the literature analysing non-nested models: model selection criteria and hypothesis testing[11]. In the model selection approach, one begins with a set of competing models and then chooses one of them, typically on the basis of some statistical measure of fit such as the adjusted R^2 or the Akaike information criterion. While such procedures will always provide us with a "best" model, they have been criticized from the perspective that the deterministic nature of these criteria means that no information is provided as to "how much" better the chosen model is (i.e they do not allow probabilistic statements to be made regarding model selection). This does not apply in Vuong's framework, however, as a probabilistic decision rule is used to select the most adequate model and consequently "we do not have to choose a 'best' model if the competing models are statistically equivalent" (Vuong, 1989, p.319).

The hypothesis testing approach, on the other hand, basically applies the classical testing approach to non-nested models (examples include Cox-type tests and tests using artificial

[11] See Pesaran and Weeks (2000) for a recent survey of non-nested hypothesis testing. Gourieroux and Monfort (1995) also provides a good discussion on this topic.

nesting procedures such as the J-test and JA test[12]). The J and JA tests are well-known and relatively easy to implement. Consider for example the following two hypotheses:

$$\begin{aligned} H_0: &\quad y = X_0\beta_0 + u_0 \\ H_1: &\quad y = X_1\beta_1 + u_1 \end{aligned} \tag{15}$$

Both the J and JA tests involve the construction of a composite model which artificially nests the two non-nested models:

$$H_C: \quad y = (1-\alpha)\cdot X_0\beta_0 + \alpha \cdot X_1\beta_1 + u \tag{16}$$

If $\alpha = 0$, then the null hypothesis is accepted. However, as this parameter is unidentified, in practice $X_1\beta_1$ is replaced with a variable (y^*) based on parameter estimates predicted according to the alternative model H_1[13]. Then, standard t-tests for the coefficient of y^* (i.e. α) are carried out.

However, if this coefficient turns out to be significant (and H_0 is therefore rejected), this does not mean that we can accept H_1. Unlike the model selection approach, therefore, there may not be a definite outcome in that rejection of the null hypothesis does not allow us to conclude that the alternative hypothesis can be accepted. Generally, the hypotheses in these tests have to be reversed and the tests repeated so that four possible outcomes will arise: either hypothesis (model) may be accepted or rejected, or both (models) may be accepted or rejected[14]. Given that we would like to identify the model which most closely specifies the technology, the fact that we may not be able to discriminate between models is an undesirable aspect of the hypothesis testing approaches just outlined as far as our purposes are concerned and illustrates that they will generally not serve as selection criteria.

[12] The J test and JA test are due to Davidson and MacKinnon (1981) and Fisher and McAleer (1981) respectively.
[13] See the appendix for more details about the J and JA tests and their adaptation to the estimation of a cost-share system model.
[14] As such, these tests only provide an indirect method of model selection.

In summary, the Vuong test provides certain advantages from our viewpoint, but we will complement it with two of the more familiar, and easily implementable, non-nested hypothesis tests. The next section describes the data used in our empirical application.

5. Data

We use annual data covering a group of 89 farms that participated in a Dairy Cattle Management Program developed by the Agriculture and Fisheries Board of the Principality of Asturias, Spain. Observations for the farms cover the period 1987-91.

We consider that farms produce a single output (y), liters of milk. Although this is a simplification, the data that are available are consistent with this assumption. The reason for this is that the consumption of inputs destined towards other types of production is discounted in the accounts of the farms. In other words, the quantities of inputs that appear in the data set are those which are genuinely assigned to milk production. Also available is the average annual price which each farm receives for milk sold over the period under consideration.

The milk is produced using three variable inputs, namely Feed (x_1), Land (x_2) and Livestock (x_3), and one fixed input, Labor (z). Feed is represented in the sample as kilograms of foodstuff acquired by the producer over the year, the size of the herd is measured by the number of cattle, land is measured in hectares, and the quantity of labor employed in the holding is measured in Human Labor Units.[15] The average annual price which each producer pays for foodstuffs is available in our data set, and that of land has been calculated from the expenditure on this factor. The price of the cattle factor is more problematic in that the majority of livestock are bred on the farms, so that their price does not appear in the accounts of the holding. Instead, the valuation of the herds was carried out by the specialists who elaborated the Dairy Cattle Management Program, using the market price of the cows and the characteristics of the cattle on the farm as references[16].

[15] A Human Labor Unit is defined as the work carried out by a person employed full-time during one year.

[16] As cows produce for more than one year, the annual cost of each animal was obtained by a process which involves dividing the price of the cows by the number of years that it would normally be on the farm, which is four (see Chang and Stefanou, 1988).

Whereas the aforementioned inputs are treated as variable, labor is treated as a quasi-fixed factor since most of the farms are family-run.[17] Accordingly, we estimate a variable cost function, comprised of Feed, Land and Livestock costs.

6. Empirical application

Assuming that technical efficiency is time-invariant and interpreting a_i, b_i and h_i as farm-specific parameters, the three competing models (i.e. input-oriented, output-oriented and hyperbolic) were estimated by maximum likelihood where the usual restrictions of symmetry, equality and homogeneity of degree one in prices were imposed. Homogeneity of degree one in prices is imposed by normalizing cost and input prices using the price of cattle as a numeraire, and to avoid singularity, the share equation corresponding to cattle is dropped from all systems. The variables are normalized by dividing through by the sample geometric mean and the first order coefficients can therefore be interpreted as the elasticities evaluated at that point. The estimates of the system's parameters, except for the coefficients of farm-specific parameters, are presented in Table 1.

At the geometric mean, the three cost functions are increasing in both output and input prices and decreasing in the quasi-fixed input. The remaining regularity condition is concavity in input prices. We cannot reject that the principal minors from the Hessian matrix of second derivatives change their signs. Overall, these results confirm monotonicity of the cost frontiers and indicate that they are concave.

The J, JA and Voung tests values for the various pairwise non-nested hypothesis tests are presented in Table 2. The Voung test values clearly indicate that the output-oriented model is rejected in favor of the hyperbolic model, and the latter is rejected in favor of the input-oriented model. In general, the values of the J and JA tests do not allow us to discriminate between models, except for the case when the best model is compared with the worst one. This indicates that these latter two tests have problems in discriminating between models when they are not sufficiently different. This casts doubts on the power of the J and JA tests to discriminate between models in a frontier analysis framework. Indeed, it is well

[17] More than 50% of the observations show zero investment (i.e. the difference between the stock of the factor in the present period and that of the previous period) in labor. This would seem to

known that these tests work when the model being tested is very close to the true model, but that they tend to reject models when they are relatively simple (see, for example, Gasmi et al,1990). If our aim is to choose between models, therefore, the results in Table 2 provide evidence in favour of carrying out the Voung test as opposed to the comprehensive J and JA tests.

The Vuong test thus shows that the input oriented model is the closest to the “true” model, followed by the hyperbolic and ouput oriented model. This in turn implies that estimating the technology using the latter two models will lead to more biased estimates, especially in the case of the output oriented model. As such, any analyses of production or efficiency based on the estimation of these models will be less credible, or less reliable, than those based on the input oriented model.

We now analyse the results of the estimations in more detail in order to get a clearer idea of the implications of choosing the incorrect specifications. We begin with returns to scale, which can be estimated as one minus the output cost elasticity. At the sample mean, the scale elasticity is a function of the first-order output parameter only. This parameters is smaller than one in all estimations, indicating the existence of increasing returns to scale in line with past analyses of Spanish dairy farms[18].

It is worth noting, however, that the scale elasticity values differ across our three competing models, leading to quite different conclusions with regard to scale efficiency. To illustrate this, we use the parameters in Table 1 to simulate the evolution of Balk’s (2001) scale efficiency index evaluated at the sample mean values of prices and labor[19]. The result appears in Figure 2.

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indicate the existence of adjustment costs associated with this factor which make it quasi-fixed.

[18] See, for example, Cuesta (2000) and Alvarez and González (1999).

[19] Balk (2001) showed that when the technology is characterized by a translog cost function (with only one output) a dual measure of scale efficiency (DSE) can be expressed in natural logs as:

$$\ln DSE(y,w) = -\frac{[\partial \ln C(y,w)/\partial \ln y - 1]^2}{\alpha_{yy}}$$

where α_{yy} is the output second-order coefficient. This equation relates the scale efficiency of a particular point with the value of the local returns to scale measure at that point.

Scale efficiency compares the farm's average cost with the minimum average cost at the most efficient scale. The scale efficiency evaluated at the sample mean (size = 0) is 65%, 21% and 44% for the input-oriented, the output-oriented and the hyperbolic models respectively. These values indicate the existence of strong average cost reductions if farm size increases. Important from the point of view of our study is the fact that the estimated efficient scale size varies widely according to how technical efficiency is incorporated into the specification of the DGP. For instance, a technically efficient farm which is also scale efficient in accordance with the input-oriented model should use 36 cows, a result which is similar to the average dairy farms in Denmark (39.8), Luxembourg (32.9) or the Netherlands (41.8) in 1993. An analysis based on the output-oriented model, on the other hand, implies that scale-efficient farms should use up to 136 cows, a somewhat implausible figure when we take into account that the average farm size in European Union Countries in 1993 ranges from 5.6 cows (Greece) to 69.4 cows (United Kingdom).

The results from using the output-oriented model would cast doubts on the validity of policy measures that promote attainment of the efficient scale size, since this objective is far from feasible for the majority of dairy farms. This lends support to the outcome of the Vuong test, which points to the output oriented model as being the furthest from the "correct" model.

Also of note is the fact that the underlying production function is not homothetic (i.e. the coefficients δ_{yk} k=1,2 are statistically different from zero) regardless of the selected specification. Under a specification of the technology where technical efficiency is introduced as either an output-oriented or hyperbolic parameter, this would mean that the level of technical efficiency of the farms influences the input mix[20]. However, no such relationship between input mix and efficiency exists under the input-oriented specification, illustrating the point that results based on different specifications may give rise to very different policy implications. For example, given the estimated parameters in the output oriented and hyperbolic models, the share of feed increases with efficiency whereas those of livestock and land decrease, implying that the more efficient producers use livestock less intensively. Since using livestock pollutes (see Innes 2000), this result would indicate that any policy measure that increases the average efficiency of the sector (e.g. farmer training, voluntary abandonment schemes, etc.) would generate additional environmental benefits. Such a policy recommendation would be ill-advised given that the hyperbolic and output oriented models are rejected in favour of the input-oriented specification.

The choice of model will also influence results on efficiency *per se*. Individual indices of technical efficiency can be derived from the estimated farm-specific parameters [21] and the descriptive statistics of these efficiency indices are displayed in Table 3. It can be seen that the different models yield quite different results. The average technical efficiency levels in our input-oriented model (71%) and hyperbolic model (77%) are, on average, similar to those found by Álvarez and González (1999) and Cuesta (2000) for Spanish dairy farms using a model with an output oriented index of technical efficiency. However, our average output-oriented efficiency index (49%) is quite lower than those obtained in the previous dairy farm literature. The estimated IOE indicates that farms can reduce (radially) their costs (inputs) by 29% on average, holding the output and the quasi-fixed input constant. In addition, the results in our hyperbolic model seem to indicate that farms can reduce all variable inputs and simultaneously increase their output by some 21%.

A quick glance at the Spearman rank correlation coefficients in Table 3 suggests a high correlation (over 96%) between the hyperbolic efficiency and the other efficiency indices. The correlation between input and output oriented efficiency indices is, as expected according to the results of the Vuong test, less important (around 91%). In Figure 3 we graph the individual efficiency indices ordered by the size of each farm. This figure suggests not only a high correlation between indices but also a negative correlation with size, which seems to indicate that the aforementioned improvements in average cost when farm size increases would be tempered by a reduction in technical efficiency.

Collectively, these results show that technological features (such as returns to scale, input demands, etc.) and their policy implications are quite sensitive to the assumptions imposed regarding the nature of technical efficiency. In general, fitting a cost system under a wrong assumption about how data is “contaminated” by technical efficiency may result in a notably distorted estimate of firm technology. This in turn will cast doubts on the validity of the estimated efficiency indices as they are sensitive to the specification of the technology (in the sense that they are measured relative to the cost frontier).

[20] See equations (4) and (12), or (13a) and (13c).

In order to analyse the biases in the efficiency estimation we compute the "indirect" OOE and HYE indices, which are the OOE and HYE indices computed using the technology estimated according to the "best" available model, i.e. the IO model.

Figure 4 illustrates the difference between the direct[22] and the indirect output-oriented and hyperbolic efficiency indices. It can be seen that the indirect OOE index is constructed as the ratio between the actual output level and the maximum available output level that can be reached *holding the cost fixed*. Since this restriction is weaker than holding the input quantities fixed, the firms will be further from this frontier and the indirect OOE values will therefore be lower than the direct OOE values. The indirect HYE index measures the maximum increase in output which is feasible with a reduction of the same proportion in farm's costs. Again, since indirect efficiency is defined in terms of costs as opposed to input quantities, the indirect HYE indices will again be lower than the direct indices.

Going back to Table 3, it can be seen that the indirect efficiency values are, on average, higher than those found using the original output-oriented and hyperbolic models. An analysis based on these latter models will therefore lead us to significantly understate the average efficiency in the sector. Note, in addition, that the correlation between computed and original OOE indices is less than 88%, suggesting that choosing (incorrectly) an output-oriented or a hyperbolic model not only understates average efficiency but would also bias the ranking order. Again, choosing the wrong model leads to an altered picture of the sector.

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[21] To make sure that the efficiency indices take values between 0 and 1, we have estimated the models using as a reference the farm which was the most efficient in an initial estimation.

[22] That is, the OOE index calculated using the output oriented specification and the HYE index calculated according to the hyperbolic specification.

7. Conclusions

In this paper we highlight the importance of choosing the correct specification of the data generating process (DGP) before embarking on analyses of production or efficiency. We advocate a model selection approach based on the non-nested hypothesis testing literature to choose the correct way of introducing inefficiency into the DGP. Using panel data from dairy farms in a Spanish region covering the period 1987-1991, we have compared the results from three cost frontier models which correspond to three different ways of controlling for technical inefficiency: introducing inefficiency as an input oriented, output oriented and hyperbolic parameter.

Our empirical study highlights the importance of making the best assumptions about how data is “contaminated” by technical efficiency and the Vuong (1989) test would appear to be a very useful tool in this context. In particular, the estimates show that technological features and efficiency levels are quite sensitive to the assumptions imposed regarding the nature of technical efficiency. The Vuong test shows a greater potential for discriminating between these models than the better known J and JA tests. We therefore find that the test proposed by Vuong can serve as a useful tool for discriminatng between models in the context of frontier analysis. This test clerarly indicates that the input-oriented model can be accepted as better than the hyperbolic and output oriented models, and that the hyperbolic model is better than the output-oriented model. Support for this outcome is provided by the fact that the input and output oriented models provide the most and least credible estimates of scale economies when the sector is compared with that in other European countries.

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Appendix: Formulation of the J and JA tests

Taking any pair of models from the three systems proposed in (13a-c), we consider the following composite model which artificially nests the two models being compared:

$$\begin{aligned} \ln C_{it} &= (1-\alpha)\cdot C^{H_0}(\cdot) + \alpha\cdot \hat{C}(\cdot) + u_{it} \\ S_{ki} &= (1-\alpha)\cdot \frac{\partial C^{H_0}(\cdot)}{\partial \ln w_k} + \alpha\cdot \frac{\partial \hat{C}(\cdot)}{\partial \ln w_k} + \varepsilon_{kit} \end{aligned} \tag{A.1}$$

where $C^{H_0}(\cdot)$ is the cost function of the model corresponding with the null hypothesis H_0 in (15), and $\hat{C}(\cdot)$ and its derivative are predicted cost and shares values, based on the parameter estimates of the alternative model H_1.

While the predicted values in the J-test are directly obtained from the estimation of the alternative model, the JA-test is slightly more involved as it requires two regressions. We first estimate the cost and shares values according to H_0. This yields the estimates $\check{C}^{H_0}$ and $\partial\check{C}^{H_0}/\partial \ln w_k$. Then, we find their predictions according to H_1. These predictions are obtained by carrying out the estimation of the alternative model but using $\check{C}^{H_0}$ and $\partial\check{C}^{H_0}/\partial \ln w_k$ as dependent variables.

Once $\hat{C}(\cdot)$ and its derivative are obtained, the J and JA tests are simply a t-test for α=0. If this coefficient turns out to be significant (and H_0 is therefore rejected), this does not mean that we can accept H_1. The procedure then is to reverse the hypotheses and repeat the test, which means that four different outcomes may arise: either hypothesis may be accepted or rejected, or both may be accepted or rejected.

Table 1. Estimated coefficients

	Input-oriented		Output-oriented		Hyperbolic	
Independent variables	Estimated coefficients	t-statistics	Estimated coefficients	t-statistics	Estimated coefficients	t-statistics
Intercept	14.035	247.76	13.910	128.96	13.962	181.58
$Ln(y)$	0.739	28.24	0.596	14.69	0.659	24.16
$Ln(w_1)$	0.488	118.34	0.410	21.14	0.456	63.73
$Ln(w_2)$	0.092	52.71	0.122	15.95	0.104	35.97
$Ln(z)$	-0.159	-3.70	-0.119	-1.90	-0.149	-3.03
$½·Ln(y)^2$	0.158	4.11	0.105	4.03	0.140	4.29
$½·Ln(w_1)^2$	0.247	12.18	0.248	9.09	0.248	11.15
$½·Ln(w_2)^2$	0.090	12.10	0.082	9.47	0.090	11.17
$½·Ln(z)^2$	0.062	0.47	-0.010	-0.06	0.044	0.33
$Ln(w_1)·Ln(w_2)$	-0.061	-6.28	-0.063	-5.38	-0.064	-6.20
$Ln(y)·Ln(w_1)$	0.152	18.56	0.104	14.01	0.129	16.33
$Ln(y)·Ln(w_2)$	-0.060	-15.28	-0.041	-12.85	-0.050	-13.67
$Ln(y)·Ln(z)$	0.031	0.49	0.041	0.90	0.039	0.71
$Ln(w_1)·Ln(z)$	-0.091	-5.06	-0.097	-4.97	-0.097	-5.21
$Ln(w_2)·Ln(z)$	0.041	5.51	0.043	5.39	0.042	5.52
LnV	2306.62		2244.57		2276.89	
R^2 Cost	90.9		95.1		92.45	
R^2 Feed	48.5		44.3		44.5	
R^2 Land	51.6		47.5		46.6	

Table 2. Model selection tests

TEST	IO vs. OO			OO vs. HY			IO vs. HY		
	Null Hypothesis	Value	Model Accepted	Null Hypothesis	Value	Model Accepted	Null Hypothesis	Value	Model Accepted
Voung (1)	IO=OO	4.42*	IO	OO=HY	-3.85*	HY	IO=HY	4.97*	IO
J	IO	0.004	IO	OO	2.824*	Neither	IO	2.672*	Neither (2)
	OO	1.002*		HY	2.027*		HY	-0.871*	
JA	IO	1.277*	Neither (2)	OO	-0.980*	Neither (2)	IO	2.499*	Neither (2)
	OO	-0.527*		HY	1.455*		HY	-1.015*	

Notes: An asterisk (*) means significantly different from zero at the 1% level.
(1) A value above (below) the critical value (minus the critical value) means that the left-hand side model can be accepted as better (worse) than the right-hand side model.
(2) We have forced to carry out these tests by SURE due to the convergence problems we found when testing one of the null hypotheses. In the cases where both tests were carried out, the results had not changed when going from MLE to SURE.

Table 3. Efficiency indices: descriptive statistics

	IO	OO	HY	Indirect OO	Indirect HY
Average	71.1	49.0	77.5	65.1	82.3
Standard Deviation	11.1	13.6	7.9	11.7	6.9
Min	48.9	28.1	61.7	42.8	68.4
Max	100.0	100.0	100.0	100.0	100.0
Spearman rank correlation coefficients					
IO	100				
OO	91.5	100			
HY	98.7	96.1	100		
Indirect OO	97.2	87.4	97.4	100	
Indirect HY	99.3	90.2	97.8	99.1	100

Figure 1. Estimated technology and the nature of technical efficiency

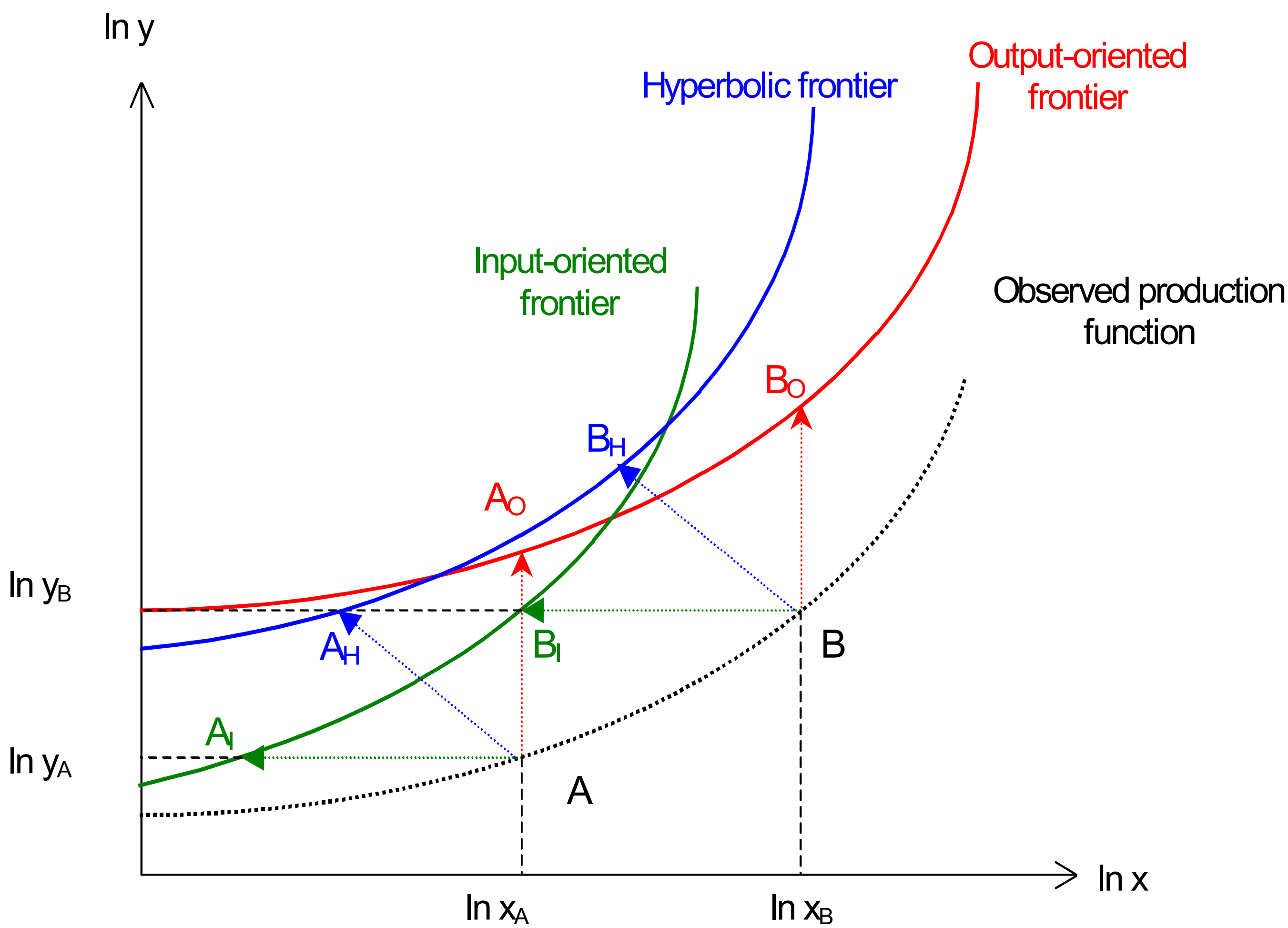


Figure 2. Scale efficiency and efficient scale size

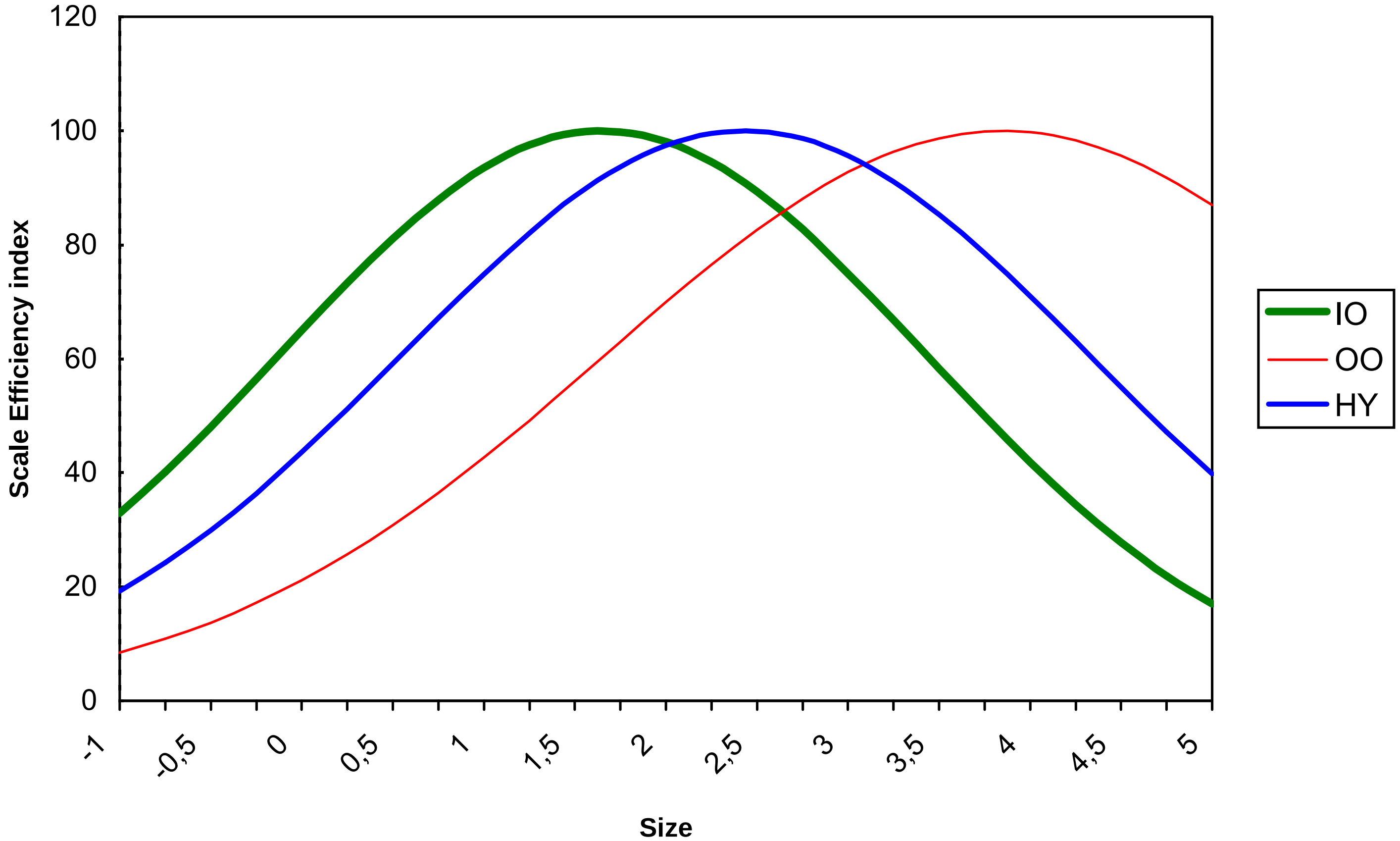


Figure 3. Input, Output and Hyperbolic efficiency indices

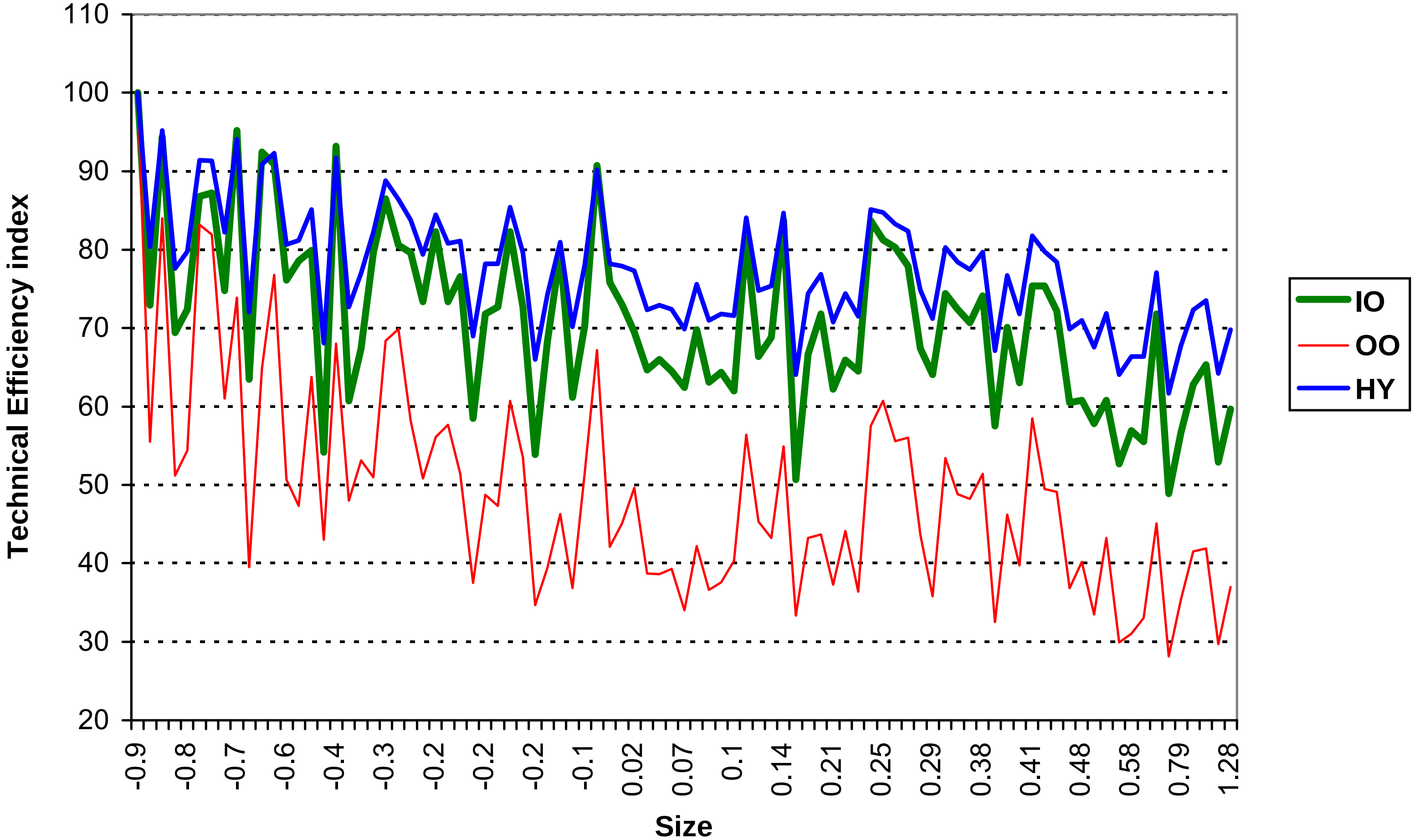
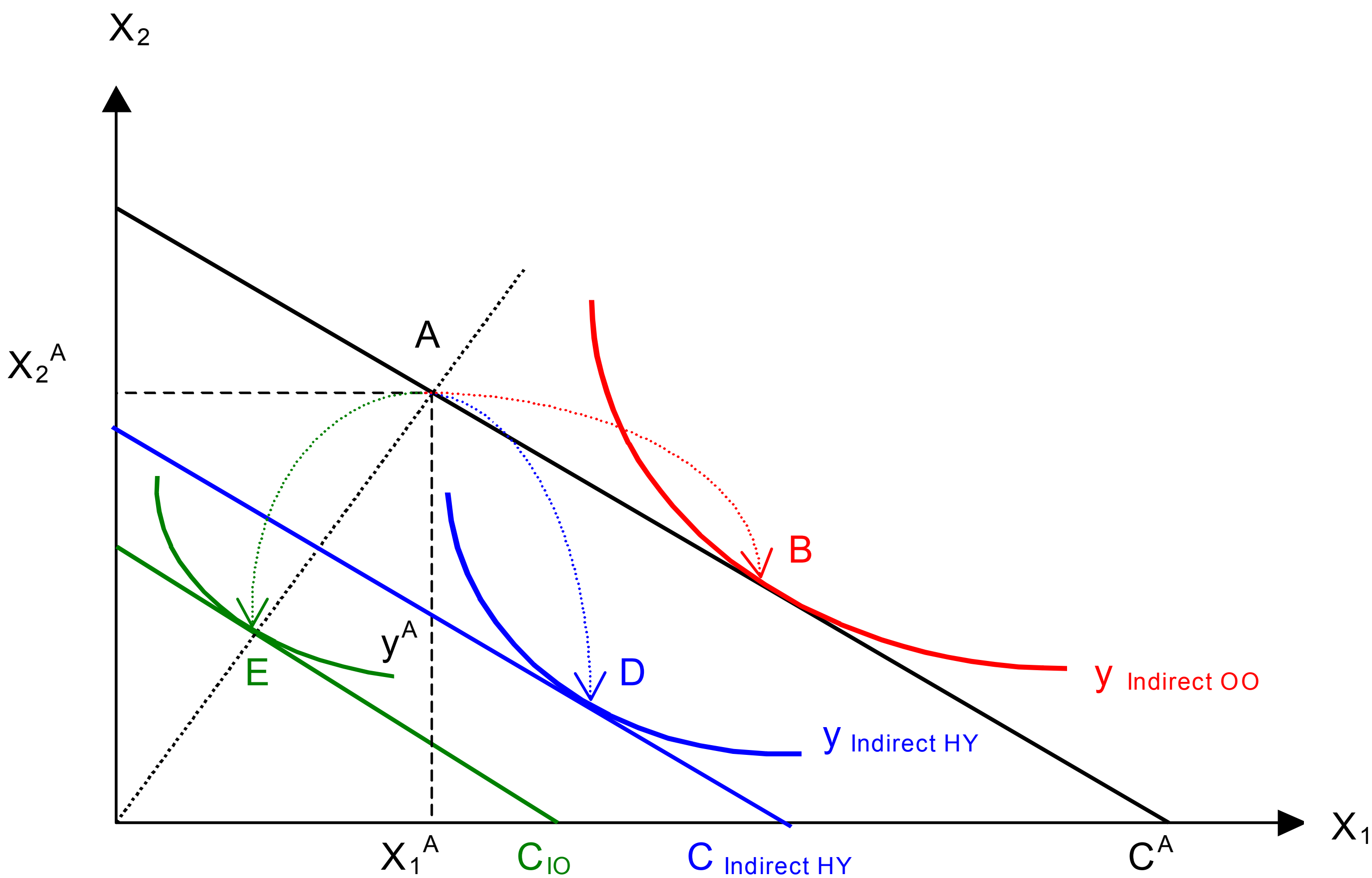


Figure 4. **Indirect efficiency indices**



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