Chromatic fiber dispersion in single-mode coherence multiplex systems

Arjan Meijerink, Niek Niëns, Geert Heideman and Wim van Etten
University of Twente, Department of Electrical Engineering, Telecommunication Engineering Group
P.O. Box 217, 7500 AE, Enschede, The Netherlands
Email: a.meijerink@ieee.org

***Abstract*— Coherence multiplexing (CM) is a relatively unknown form of optical CDMA, which is particularly suitable in medium bit rate, small-scale optical networks like access networks, LANs or optical interconnects. Despite the small bit rates and small distances, this technique is rather sensitive to chromatic fiber dispersion, since broadband sources have to be used. In this paper, the impact of chromatic dispersion on the performance of a digital transmission system using CM and binary PSK will be studied. Moreover, it will be proven that the performance of a system with significantly dispersive fiber links can be enhanced by performing QPSK modulation, rather than the so far more conventional binary PSK modulation.**

I. Introduction

A. Coherence Multiplexing (CM)

Coherence multiplexing (CM) is a simple form of optical code division multiplexing (OCDM), in which broadband sources and delay-lines are used to multiplex signals from distinct users over one optical fiber cable [1]–[6]. The difference between CM and the more well-known OCDM techniques using optical orthogonal codes (OOCs), is that the randomness of the broadband source is used as a code, rather than a (known) pseudorandom code. An example of a simple CM system, consisting of one transmitter and receiver, is shown in Fig. 1. Since the code is not known at the receiver, it has to be transmitted twice: once modulated and once unmodulated. These two signals are moreover delayed in time with respect to each other by a timeshift that exceeds the so-called coherence time τ_c [7]. The latter is a measure of the width of the coherence function of the light emitted by the source (autocorrelation function of the corresponding field amplitude), which indicates the time interval over which one can reasonably predict the phase of the lightwave. Hence, the modulation signal does not result in a visible intensity modulation in the output signal of the transmitter. In the receiver, the modulating signal can be recovered by mixing the received signal by a time-delayed version of the same received signal, such that the time-delays in the transmitter and receiver are the same. When these delays are not the same, however, and moreover, their difference is much larger than τ_c, then the mixing process will simply result in (zero-mean) noise. Hence, multiplexing can be achieved by launching the signals of several (say N transmitters) into the common fiber, each transmitter using a different time delay.

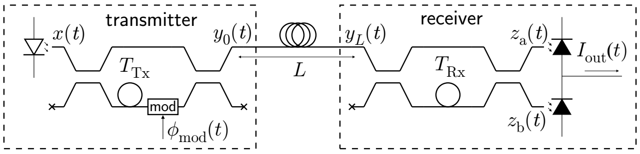


Fig. 1. A simple coherence multiplex (CM) system

B. Application area

CM has several advantages and disadvantages with respect to other optical multiplexing techniques, which makes it suitable for very specific applications.

The particular advantage with respect to wavelength division multiplexing (WDM) is that the conditions with respect to stability and optical bandwidth of both the sources and the receiver components are not so strict: broadband sources and simple couplers and delay lines can be used to perform the multiplexing, and small fluctuations in source wavelength or path delays do not introduce crosstalk between the CM channels.

A particular advantage with respect to optical time division multiplexing (OTDM) is that CM does not require a synchronization scheme to avoid crosstalk between the CM channels. This is a particular advantage in a multiple access scheme, where the transmitters are not localized to a single node, for example in the upstream of a distribution network.

And finally, the advantage of CM with respect to OCDM with OOCs is that the electrical signal processing can be performed at a rate that corresponds to the bit rate of the modulating signal, rather than at a (much higher) chip rate of the OOCs. Moreover, an advantage with respect to both OTDM and OCDM with OOCs is that CM is transparent to any signal format, which makes it suitable for analog modulation, and moreover, enables combination with subcarrier multiplexing (SCM).

A disadvantage of CM is the fact that the signals that mix incoherently result into optical beat interference (OBI) noise. The power spectral density of this noise increases inversely proportional to the bandwidth of the source, and approximately proportional to the square of the number of channels. This limits both the number of channels that can be multiplexed, and their bandwidth (or bit rate, in case of digital transmission). Another disadvantage of CM is the fact that the broadband light renders the multiplexed channels rather sensitive to dispersion, even when single-mode fiber (SMF) is used. This limits the span lengths of the fiber links that are to be used, as well as the bandwidth of the multiplexed channels.

As a result, CM is particularly suitable for applications where required bandwidth, number of users and link lengths are modest, and costs are to be kept low. Examples of such applications are optical LANs and access networks. Moreover, the signal transparency enables Radio over Fiber (RoF) distribution, as described elsewhere in these proceedings [2]. Actually, these functionalities could even be integrated into a single network, for example for an indoor network that provides both wireless and fixed access.

C. Scope of this paper

In this paper, the influence of chromatic dispersion in SMF on the output signal of a CM receiver is studied in general, and applied to digital transmission in particular. In Section II, an analysis will be performed in which the output signal of a CM receiver is related to the modulating signal in the transmitter, incorporating the effect of chromatic fiber dispersion. Using this relation, the performance of a digital transmission system using CM and binary phase shift keying (BPSK) modulation is studied in terms of the bit error rate (BER), in Section III. Moreover, a formula will be derived that gives the maximum bit rate that can be achieved at a BER of 10^{-9}, optimized with respect to τ_c. In Section IV, M-ary PSK will be considered as an alternative modulation format, and it will be shown that QPSK will result in higher bit rates than BPSK. This is illustrated in Section V, where a numerical example with 500 m of standard SMF is presented. Conclusions are formulated in Section VI.

II. Analysis of the coherence multiplex system

A. Light model

Consider the simple CM system in Fig. 1. If we assume that the light that is coupled into the Mach-Zehnder Interferometer (MZI) is linearly polarized, and that the optical circuit is single-mode, then we can describe the optical signal by the normalized pre-envelope $x(t)$ of the corresponding electrical field, which is defined such that its center frequency corresponds to the optical carrier frequency f_c, its phase corresponds to the optical phase, and the instantaneous power that is coupled into the MZI is given by

$$P_\mathrm{in}(t) = \tfrac{1}{2}\big|x(t)\big|^2 \tag{1}$$

This notation saves us from bothering about the actual spatial profile of the electrical field in the fundamental mode. If we assume that all the lightwaves in the system have the same polarization state, then we can describe interference effects by simply summing normalized electrical fields and then calculating the power of the sum.

Since the light is generated by means of spontaneous emission, we can approximately model $x(t)$ as thermal light [7], which implies that the corresponding complex envelope is a circular complex Gaussian process. Moreover, if the light source is a light emitting diode (LED) or superluminescent diode (SLD), the spectrum of $x(t)$ is also Gaussian. Hence, its autocorrelation function can be written as

$$\begin{aligned} R_{x^*x}(\tau) &\triangleq \mathrm{E}\big[x^*(t)x(t+\tau)\big] \\ &= 2P_\mathrm{in}\exp\left(-\frac{\pi}{2}\left(\frac{\tau}{\tau_\mathrm{c}}\right)^2\right)\exp(\mathrm{j}2\pi f_\mathrm{c}\tau) \end{aligned} \tag{2}$$

where P_in is the average power $\mathrm{E}\big[P_\mathrm{in}(t)\big]$ and τ_c is the coherence time. The spectrum of $x(t)$ follows by Fourier transformation:

$$S_{x^*x}(f) = 2\sqrt{2}P_\mathrm{in}\tau_\mathrm{c}\exp\Big(-2\pi(f-f_\mathrm{c})^2\tau_\mathrm{c}^2\Big) \tag{3}$$

B. Transmitted signal

If the 3-dB couplers are assumed to be ideal, their transfer matrix is given by

$$[H_2] = \tfrac{1}{\sqrt{2}}\begin{bmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \end{bmatrix} \tag{4}$$

Hence, the output signal of the transmitter's MZI can be written as

$$y_0(t) = \tfrac{1}{2}\Big[x(t) - x(t-T_\mathrm{Tx})s(t)\Big] \tag{5}$$

where $s(t)$ is a complex modulating signal which is related to the phase modulation $\phi_\mathrm{mod}(t)$ as

$$s(t) \triangleq \exp\Big(\mathrm{j}\,\phi_\mathrm{mod}(t)\Big) \tag{6}$$

Hence, when $s(t)$ is considered as a deterministic signal, $y_0(t)$ is a non-stationary random signal with autocorrelation function

$$\begin{aligned} R_{y_0^*y_0}(t_1,t_2) &\triangleq \mathrm{E}\big[y_0^*(t_1)y_0(t_2)\big] \\ &= \tfrac{1}{4}\Big[R_{x^*x}(t_2-t_1) \\ &\qquad - R_{x^*x}(t_2-t_1-T_\mathrm{Tx})s(t_2) \\ &\qquad - R_{x^*x}(t_2-t_1+T_\mathrm{Tx})s^*(t_1) \\ &\qquad + R_{x^*x}(t_2-t_1)s^*(t_1)s(t_2)\Big] \end{aligned} \tag{7}$$

C. Output signal of receiver

The lightwave signals at the upper and lower output ports of the receiver's MZI can be related to the received signal $y_L(t)$ as

$$z_\mathrm{a}(t) = \tfrac{1}{2}\Big[y_L(t) - y_L(t-T_\mathrm{Rx})\Big] \tag{8}$$

$$z_\mathrm{b}(t) = \tfrac{1}{2}\mathrm{j}\Big[y_L(t) + y_L(t-T_\mathrm{Rx})\Big] \tag{9}$$

For large received powers, shot noise in the photodiodes can be ignored. Hence, by assuming that the photodiodes are perfectly linear with identical responsivities R_pd, the instantaneous output current can be written as

$$\begin{aligned} I_\mathrm{out}(t) &= \tfrac{1}{2}R_\mathrm{pd}\Big[\big|z_\mathrm{a}(t)\big|^2 - \big|z_\mathrm{b}(t)\big|^2\Big] \\ &= -\tfrac{1}{2}R_\mathrm{pd}\mathrm{Re}\Big\{y_L(t)y_L^*(t-T_\mathrm{Rx})\Big\} \end{aligned} \tag{10}$$

where $\mathrm{Re}\{.\}$ denotes real part. The output signal consists of a desired term and noise. The desired term can be found by taking the expected value:

$$\mathrm{E}\big[I_\mathrm{out}(t)\big] = -\tfrac{1}{2}R_\mathrm{pd}\mathrm{Re}\Big\{R_{y_L^*y_L}(t-T_\mathrm{Rx},t)\Big\} \tag{11}$$

D. Output signal for short fiber lengths

In case of short fiber lengths, both attenuation and dispersion can be neglected, so that we can write

$$y_L(t) \approx y_0(t) \Rightarrow R_{y_L^* y_L}(t_1,t_2) \approx R_{y_0^* y_0}(t_1,t_2) \quad (12)$$

where an irrelevant propagation delay was omitted. Hence, the output signal of the receiver can be found by substituting (7) into (11). Assuming that both T_{Tx} and T_{Rx} are much larger than τ_{c}, this results in

$$\mathrm{E}\big[I_{\mathrm{out}}(t)\big] = \tfrac{1}{8} R_{\mathrm{pd}} \mathrm{Re}\Big\{R_{x^*x}(T_{\mathrm{Rx}}-T_{\mathrm{Tx}})s(t)\Big\} \quad (13)$$

Using (2) and (6), this can be written as

$$\mathrm{E}\big[I_{\mathrm{out}}(t)\big] = \tfrac{1}{4} R_{\mathrm{pd}} P_{\mathrm{in}} \exp\left(-\frac{\pi}{2}\left(\frac{T_{\mathrm{Rx}}-T_{\mathrm{Tx}}}{\tau_{\mathrm{c}}}\right)^2\right) \cdot \cos\Big(2\pi f_{\mathrm{c}}(T_{\mathrm{Rx}}-T_{\mathrm{Tx}}) + \phi_{\mathrm{mod}}(t)\Big) \quad (14)$$

This equation once more indicates how the output signals depend on the relation between T_{Rx} and T_{Tx}:

- for $\big|T_{\mathrm{Rx}} - T_{\mathrm{Tx}}\big| \gg \tau_{\mathrm{c}}$, the exponential becomes very small, resulting in a negligible output signal;
- for $T_{\mathrm{Rx}} = T_{\mathrm{Tx}}$, the exponential becomes 1, resulting in

$$\mathrm{E}\big[I_{\mathrm{out}}(t)\big] = \tfrac{1}{4} R_{\mathrm{pd}} P_{\mathrm{in}} \cos\Big(\phi_{\mathrm{mod}}(t)\Big) \quad (15)$$

This mathematically confirms that T_{Rx} can be used as a parameter to tune to a particular CM channel. Note that the cosine-term in (14) implies that T_{Rx} has to be tuned to T_{Tx} very precisely (in the order of $1/f_{\mathrm{c}}$). This can be done for example by means of a feedback loop [3]. This is a typical problem in self-homodyne detection, and can be avoided by performing self-heterodyne detection instead, by means of an optical frequency shifter [4], or by performing pseudo-self-homodyne detection by means of optical phase diversity [5], [6]. In this paper, however, we will assume that a balanced receiver is used, and that T_{Rx} and T_{Tx} match precisely.

E. Output signal for long fiber lengths

When a longer fiber cable is used, $y_0(t)$ and $y_L(t)$ are no longer the same. In that case, a simple expression for the output signal of the receiver can be found by means of a derivation that is similar to the procedure that Gimlett and Cheung performed in order to find the output signal of an IM/DD link with an LED and SMF [8]. Assuming a linear and time invariant relation between input and output signal of the fiber, we can write

$$y_L(t) = \int h_{\mathrm{f}}(\xi) y_0(t-\xi)\,\mathrm{d}\xi \quad (16)$$

where $h_{\mathrm{f}}(t)$ denotes the impulse response of the fiber, which can be written as

$$h_{\mathrm{f}}(t) = \int \Big|H_{\mathrm{f}}(f)\Big| \exp\Big(\mathrm{j}2\pi f t - \mathrm{j}\ \beta(f)L\Big)\,\mathrm{d}f \quad (17)$$

where $\Big|H_{\mathrm{f}}(f)\Big|$ denotes the magnitude of the optical field transfer function of the fiber, and $\beta(f)$ denotes the phase transfer per unit length of the fiber. Using (7), and assuming that

- the bandwidth of $s(t)$ is much smaller than the bandwidth of the source;
- $\Big|H_{\mathrm{f}}(f)\Big|$ can be considered constant in any frequency interval with bandwidth in the order of the bandwidth of the source;
- $\beta(f)$ can be considered linear in any frequency interval with bandwidth in the order of the bandwidth of $s(t)$;
- the bandwidth of $s(t)$ is much smaller than $\frac{1}{T_{\mathrm{Tx}}}$,

it can be proven (the proof is omitted for brevity) that the autocorrelation function of $y_L(t)$ can be approximated as

$$\begin{aligned} R_{y_L^* y_L}(t_1,t_2) \approx \tfrac{1}{4}\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 \Big[& 2R_{x^*x}(t_2-t_1) \\ & - \int s\Big(t_2 - L\,\tau_{\mathrm{g}}(f)\Big) S_{x^*x}(f) \\ & \quad \cdot \exp\Big(\mathrm{j}2\pi f(t_2-t_1-T_{\mathrm{Tx}})\Big)\,\mathrm{d}f \\ & - \int s^*\Big(t_1 - L\,\tau_{\mathrm{g}}(f)\Big) S_{x^*x}(f) \\ & \quad \cdot \exp\Big(\mathrm{j}2\pi f(t_2-t_1+T_{\mathrm{Tx}})\Big)\,\mathrm{d}f \Big] \end{aligned} \quad (18)$$

where the group delay per unit length $\tau_{\mathrm{g}}(f)$ is related to $\beta(f)$ by

$$\tau_{\mathrm{g}}(f) \triangleq \frac{1}{2\pi}\frac{\mathrm{d}\beta(f)}{\mathrm{d}f} \quad (19)$$

Using (11) and assuming that T_{Tx} and T_{Rx} are identical and much larger than τ_{c}, the expected value of the output signal can now be written as

$$\mathrm{E}\big[I_{\mathrm{out}}(t)\big] \approx \tfrac{1}{8}\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 R_{\mathrm{pd}} \int \mathrm{Re}\Big\{s\Big(t - L\,\tau_{\mathrm{g}}(f)\Big)\Big\} S_{x^*x}(f)\,\mathrm{d}f \quad (20)$$

(Note that this is an intuitively appealing result, and that for $L=0$, this result corresponds to (13).) By substituting (3) and approximating $\tau_{\mathrm{g}}(f)$ by its second order Taylor series about the optical carrier frequency f_{c}, we can write (20) as

$$\mathrm{E}\Big[I_{\mathrm{out}}(t)\Big] \approx \int h_{\mathrm{sys}}(\xi) \mathrm{Re}\Big\{s(t-\xi)\Big\}\,\mathrm{d}\xi \quad (21)$$

where

$$\begin{aligned} h_{\mathrm{sys}}(t) = \tfrac{1}{4}\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 R_{\mathrm{pd}} P_{\mathrm{in}} & \frac{\mathrm{u}\left(t - T_0 + \frac{T_1^2}{T_2}\right)}{\sqrt{(t-T_0)T_2 + T_1^2}} \\ \cdot \Bigg[& \exp\left(-\frac{4\pi}{T_2^2}\left(\sqrt{(t-T_0)T_2+T_1^2} - T_1\right)^2\right) \\ & + \exp\left(-\frac{4\pi}{T_2^2}\left(\sqrt{(t-T_0)T_2+T_1^2} + T_1\right)^2\right)\Bigg] \end{aligned} \quad (22)$$

in which $\mathrm{u}(.)$ is the unit step function and

$$T_0 = L\,\tau_{\mathrm{g}}(f_{\mathrm{c}}) \tag{23}$$

$$T_1 = \frac{L\,\tau'_{\mathrm{g}}(f_{\mathrm{c}})}{\sqrt{2}\tau_{\mathrm{c}}} \tag{24}$$

$$T_2 = \frac{L\,\tau''_{\mathrm{g}}(f_{\mathrm{c}})}{\tau_{\mathrm{c}}^2} \tag{25}$$

$h_{\mathrm{sys}}(t)$ can be looked upon as the impulse response of the CM system.

For most fibers, the dispersion coefficients are given as derivatives of the group delay τ_{g} with respect to wavelength rather than frequency. The derivatives with respect to frequency can be found using the following conversion formulas

$$\tau'_{\mathrm{g}}(f_{\mathrm{c}}) = -\frac{\lambda_{\mathrm{c}}^2}{c_0}\tau'_{\mathrm{g}}(\lambda_{\mathrm{c}}) \tag{26}$$

$$\tau''_{\mathrm{g}}(f_{\mathrm{c}}) = \frac{\lambda_{\mathrm{c}}^3}{c_0^2}\Big[2\tau'_{\mathrm{g}}(\lambda_{\mathrm{c}}) + \lambda_{\mathrm{c}}\tau''_{\mathrm{g}}(\lambda_{\mathrm{c}})\Big] \tag{27}$$

III. Impact of chromatic dispersion on digital transmission employing BPSK modulation

A. General derivation of output pulse shape

When ordinary binary phase shift keying (BPSK) modulation with symbol time T_{s} (without any pulse shaping) is applied, $s(t)$ (as defined in (6)) is a rectangular polar NRZ signal, which we can write as

$$s(t) = \sum_n a_n\, p_{\mathrm{in}}(t - n\,T_{\mathrm{s}}) \tag{28}$$

where $p_{\mathrm{in}}(t)$ is a rectangular pulse with length T_{s}

$$p_{\mathrm{in}}(t) = \begin{cases} 1 & \text{if } |t| < \frac{1}{2}T_{\mathrm{s}} \\ 0 & \text{if } |t| > \frac{1}{2}T_{\mathrm{s}} \end{cases} \tag{29}$$

The a_ns represent the information symbols, which take values $+1$ or -1. The expected output signal of the receiver will also be a polar NRZ signal, but the pulse shape will be distorted because of the dispersion of the fiber. The pulse shape can be found by convolving (29) with the impulse response of the fiber.

Of course, low pass filtering has to be applied prior to detection. Although several equalization techniques are known for optimizing detection, we will simply assume that the filter is matched to an undistorted pulse, resulting in a simple integrate and dump filter, which has an impulse response that is equal to $p_{\mathrm{in}}(t)$. Hence, using (21), the expected value of the detected signal can be written as

$$\begin{aligned} \mathrm{E}\big[I_{\mathrm{det}}(t)\big] &= \int \mathrm{E}\big[I_{\mathrm{out}}(t-\xi)\big]p_{\mathrm{in}}(\xi)\;\mathrm{d}\xi \\ &= \tfrac{1}{4}\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 R_{\mathrm{pd}}P_{\mathrm{in}}\sum_n a_n\, p_{\mathrm{det}}(t - n\,T_{\mathrm{s}}) \end{aligned} \tag{30}$$

with pulse shape

$$\begin{aligned} p_{\mathrm{det}}(t) &= \frac{4\iint p_{\mathrm{in}}(\xi_1)p_{\mathrm{in}}(\xi_2)h_{\mathrm{sys}}(t-\xi_1-\xi_2)\;\mathrm{d}\xi_1\;\mathrm{d}\xi_2}{\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 R_{\mathrm{pd}}P_{\mathrm{in}}} \\ &= \frac{4\int_{-T_{\mathrm{s}}}^{T_{\mathrm{s}}} h_{\mathrm{sys}}(t-\xi)\big(T_{\mathrm{s}} - |\xi|\big)\;\mathrm{d}\xi}{\Big|H_{\mathrm{f}}(f_{\mathrm{c}})\Big|^2 R_{\mathrm{pd}}P_{\mathrm{in}}} \end{aligned} \tag{31}$$

Note that this pulse shape is normalized such that its peak value is equal to T_{s} when $L = 0$. Substituting (22), it can be proven that (31) can be written as

$$\begin{aligned} p_{\mathrm{det}}(t) = {} & q\,(t + T_{\mathrm{s}} - T_0) \\ & - 2\,q\,(t - T_0) \\ & + q\,(t - T_{\mathrm{s}} - T_0) \end{aligned} \tag{32}$$

with

$$\begin{aligned} q(t) = {} & \mathrm{u}\left(t + \frac{T_1^2}{T_2}\right) \\ & \cdot \Bigg\{ \frac{1}{4\pi}\Bigg[\left(\sqrt{T_2 t + T_1^2} - T_1\right) \\ & \qquad \cdot \exp\left(-\frac{4\pi}{T_2^2}\left(\sqrt{T_2 t + T_1^2} + T_1\right)^2\right) \\ & \quad + \left(\sqrt{T_2 t + T_1^2} + T_1\right) \\ & \qquad \cdot \exp\left(-\frac{4\pi}{T_2^2}\left(\sqrt{T_2 t + T_1^2} - T_1\right)^2\right)\Bigg] \\ & + \left(t - \frac{T_2}{8\pi}\right)\Bigg[1 - \mathrm{Q}\left(\frac{2\sqrt{2\pi}}{T_2}\left(\sqrt{T_2 t + T_1^2} + T_1\right)\right) \\ & \qquad - \mathrm{Q}\left(\frac{2\sqrt{2\pi}}{T_2}\left(\sqrt{T_2 t + T_1^2} - T_1\right)\right)\Bigg]\Bigg\} \end{aligned} \tag{33}$$

where $\mathrm{Q}(.)$ is the Gaussian tail probability

$$\mathrm{Q}(z) \triangleq \frac{1}{\sqrt{2\pi}}\int_z^{\infty} \exp\left(-\frac{x^2}{2}\right)\;\mathrm{d}x \tag{34}$$

For most center wavelengths, the value of T_1 will be much larger than T_2. When the center wavelength λ_{c} is chosen close to the so-called zero-dispersion wavelength, however, T_2 will be much larger than T_1. Hence, for most practical applications, either T_1 or T_2 can be neglected. These cases will be treated separately in the following subsections.

B. Wavelengths far from zero-dispersion wavelength

For operating wavelengths far from the zero-dispersion wavelength, we have $T_1 \gg T_2$, so that (33) becomes

$$q(t) = \frac{T_1}{2\pi}\exp\left(-\pi\left(\frac{t}{T_1}\right)^2\right) + t\left[1 - \mathrm{Q}\left(\frac{t\sqrt{2\pi}}{T_1}\right)\right] \tag{35}$$

From symmetry considerations, one can easily see that the resulting pulse shape has maximum value

$$\begin{aligned} p_{\mathrm{det,max}} &= p_{\mathrm{det}}(T_0) = \\ &= \frac{T_1}{\pi}\left[\exp\left(-\pi\left(\frac{T_\mathrm{s}}{T_1}\right)^2\right)-1\right] \\ &\quad + T_\mathrm{s}\left[1-2\mathrm{Q}\left(\frac{T_\mathrm{s}\sqrt{2\pi}}{T_1}\right)\right] \end{aligned} \tag{36}$$

For large x, one can approximate

$$\mathrm{Q}(x) \approx \frac{\exp\left(-\frac{x^2}{2}\right)}{x\sqrt{2\pi}} \tag{37}$$

so for $T_1 \ll T_\mathrm{s}$, we have

$$p_{\mathrm{det,max}} \approx T_\mathrm{s} - \frac{T_1}{\pi} \tag{38}$$

Obviously, the maximum pulse amplitude decreases with increasing dispersion. Moreover, inter-symbol interference (ISI) occurs, as the absolute pulse width is larger than T_s. The amount of ISI can be quantified by calculating the pulse amplitudes at time instants $T_0 + n\,T_\mathrm{s}$ with n integer. One can show that for $T_1 \ll T_\mathrm{s}$, these are negligible for $|n| > 1$. For $n = \pm 1$, we have:

$$p_{\mathrm{det}}(T_0 \pm T_\mathrm{s}) \approx \frac{T_1}{2\pi} \tag{39}$$

The most ISI occurs when a particular bit is neighbored by bits with opposite sign. In that case, the amplitude of that bit is degraded with respect to the case without dispersion by a factor

$$\begin{aligned} P_\mathrm{d} &= \frac{p_{\mathrm{det}}(T_0) - p_{\mathrm{det}}(T_0 - T_\mathrm{s}) - p_{\mathrm{det}}(T_0 + T_\mathrm{s})}{T_\mathrm{s}} \\ &= 1 - \frac{2T_1}{\pi\,T_\mathrm{s}} \end{aligned} \tag{40}$$

where P_d stands for dispersion penalty. As the bandwidth of the noise is much larger than the bandwidth of the matched filter, the output signal can be considered as Gaussian distributed, resulting in a bit error probability

$$\mathrm{P_e} = \mathrm{Q}\left(\sqrt{2\gamma_\mathrm{b}}\right) \tag{41}$$

where γ_b is the SNR per bit. In the absence of dispersion it is known to be given by [1], [6]

$$\gamma_\mathrm{b} = \frac{1}{4N^2+2N+1}\frac{T_\mathrm{s}}{\tau_\mathrm{c}} \tag{42}$$

Hence, using (24) and (40), the worst case SNR per bit in case of first order dispersion follows as

$$\begin{aligned} \gamma_\mathrm{b} &= \frac{1}{4N^2+2N+1}\frac{T_\mathrm{s}}{\tau_\mathrm{c}}P_\mathrm{d}^2 \\ &= \frac{1}{4N^2+2N+1}\frac{T_\mathrm{s}}{\tau_\mathrm{c}}\left(1-\frac{\sqrt{2}L\,\tau_\mathrm{g}'(f_\mathrm{c})}{\pi\,T_\mathrm{s}\tau_\mathrm{c}}\right)^2 \end{aligned} \tag{43}$$

Hence, one can show that for a given SNR, fiber length, number of users and coherence time, the bit rate that can be achieved is given by

$$\begin{aligned} R_\mathrm{b} \triangleq \frac{1}{T_\mathrm{s}} &= \frac{\pi\tau_\mathrm{c}}{\sqrt{2}L\,\tau_\mathrm{g}'(f_\mathrm{c})} - \frac{\pi^2(4N^2+2N+1)\gamma_\mathrm{b}\tau_\mathrm{c}^3}{4L^2\left(\tau_\mathrm{g}'(f_\mathrm{c})\right)^2} \\ &\quad \cdot\left[\sqrt{1+\frac{4\sqrt{2}L\tau_\mathrm{g}'(f_\mathrm{c})}{\pi(4N^2+2N+1)\gamma_\mathrm{b}\tau_\mathrm{c}^2}}-1\right] \end{aligned} \tag{44}$$

It can be proven that this can be optimized with respect to τ_c by choosing τ_c as

$$\tau_{\mathrm{c,opt}} = \frac{2}{3}\sqrt{\frac{3\sqrt{2}L\tau_\mathrm{g}'(f_\mathrm{c})}{\pi\gamma_\mathrm{b}(4N^2+2N+1)}} \tag{45}$$

resulting in a maximum bit rate

$$R_{\mathrm{b,max}} = \frac{2}{9}\sqrt{\frac{3\pi}{\sqrt{2}L\tau_\mathrm{g}'(f_\mathrm{c})\gamma_\mathrm{b}(4N^2+2N+1)}} \tag{46}$$

This will be numerically illustrated in Section V.

C. Wavelengths close to zero-dispersion wavelength

For operating wavelengths close to the zero-dispersion wavelength, we have $T_1 \ll T_2$, so that (33) becomes

$$\begin{aligned} q(t) = \Bigg\{ &\frac{\sqrt{T_2 t}}{2\pi}\exp\left(-\frac{4\pi\,t}{T_2}\right) \\ &+ \left(t-\frac{T_2}{8\pi}\right)\left[1-2\mathrm{Q}\left(2\sqrt{\frac{2\pi\,t}{T_2}}\right)\right]\Bigg\}\mathrm{u}(t) \end{aligned} \tag{47}$$

By calculating its derivative with respect to t,

$$q'(t) = \left[1-2\mathrm{Q}\left(2\sqrt{\frac{2\pi\,t}{T_2}}\right)\right]\mathrm{u}(t) \tag{48}$$

it can be proven that the resulting pulse has its maximum for $t = t_0$ with

$$\mathrm{Q}\left(2\sqrt{\frac{2\pi(t_0-T_0)}{T_2}}\right) \approx \frac{1}{4} \Rightarrow t_0 \approx T_0 + 0.0181T_2 \tag{49}$$

so the maximum pulse amplitude is given by

$$p_{\mathrm{det,max}} = p_{\mathrm{det}}(t_0) \approx T_\mathrm{s} - 0.0341T_2 \tag{50}$$

where it is assumed that $T_2 \ll T_\mathrm{s}$. It can be proven that the ISI is dominated by the interference from the neighboring bits, which is quantified by

$$p_{\mathrm{det}}(t_0 + T_\mathrm{s}) \approx 0.0279T_2 \tag{51}$$

$$p_{\mathrm{det}}(t_0 - T_\mathrm{s}) \approx 0.0062T_2 \tag{52}$$

Hence, the dispersion penalty is given by

$$\begin{aligned} P_\mathrm{d} &= \frac{p_{\mathrm{det}}(t_0) - p_{\mathrm{det}}(t_0 - T_\mathrm{s}) - p_{\mathrm{det}}(t_0 + T_\mathrm{s})}{T_\mathrm{s}} \\ &\approx 1 - 0.0682\frac{T_2}{T_\mathrm{s}} \end{aligned} \tag{53}$$

Using (25), (42) and (53), we find a worst-case SNR per bit

$$\gamma_{\rm b} = \frac{1}{4N^2+2N+1}\frac{T_{\rm s}}{\tau_{\rm c}}P_{\rm d}^2 \approx \frac{1}{4N^2+2N+1}\frac{T_{\rm s}}{\tau_{\rm c}}\left(1-0.0682\frac{L\tau_{\rm g}''(f_{\rm c})}{\tau_{\rm c}^2 T_{\rm s}}\right)^2 \tag{54}$$

The bit rate that can be achieved at a fixed coherence time and SNR per bit follows as

$$R_{\rm b} \approx \frac{14.66\tau_{\rm c}^2}{L\,\tau_{\rm g}''(f_{\rm c})} - \frac{107.4(4N^2+2N+1)\gamma_{\rm b}\tau_{\rm c}^5}{L^2\Big(\tau_{\rm g}''(f_{\rm c})\Big)^2} \cdot \left[\sqrt{1+\frac{0.2729L\tau_{\rm g}''(f_{\rm c})}{(4N^2+2N+1)\gamma_{\rm b}\tau_{\rm c}^3}}-1\right] \tag{55}$$

The optimum coherence time can be proven to be

$$\tau_{\rm c,opt} \approx 0.6021\sqrt[3]{\frac{L\tau_{\rm g}''(f_{\rm c})}{\gamma_{\rm b}(4N^2+2N+1)}} \tag{56}$$

and the maximum bit rate is

$$R_{\rm b,max} \approx \frac{1.063}{\sqrt[3]{\gamma_{\rm b}^2(4N^2+2N+1)^2L\tau_{\rm g}''(f_{\rm c})}} \tag{57}$$

This will be numerically illustrated in Section V.

IV. M-ARY PSK MODULATION

As an alternative to BPSK modulation, M-ary PSK modulation is proposed [6]. In that case, each symbol represents k bits, and $\phi_{\rm mod}(t)$ can take $M=2^k$ different values rather than only 0 and π, so that $s(t)$ is not necessarily real anymore. The symbol time $T_{\rm s}$ is related to the bit rate by

$$R_{\rm b} = \frac{k}{T_{\rm s}} \tag{58}$$

Detection of M-ary PSK requires replacing the balanced mixer in Fig. 1 by a double balanced mixer, which consists of a 4×4 phase diversity coupler and two balanced photodiode pairs. This is illustrated in Fig. 2. If the 4×4 coupler is assumed to have an ideal transfer matrix

$$[H_4] = \tfrac{1}{2}\begin{bmatrix} 1 & \mathrm{j} \\ \mathrm{j} & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{59}$$

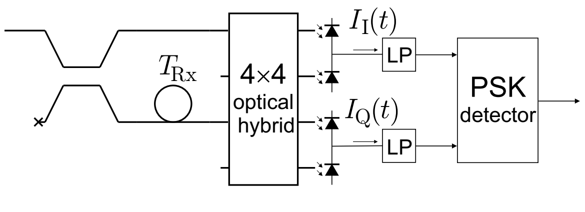


Fig. 2. An M-ary PSK receiver for coherence multiplexing

then one can find for the quadrature currents

$$\mathrm{E}\big[I_{\rm I}(t)\big] = -\tfrac{1}{4}R_{\rm pd}\mathrm{Re}\Big\{R_{y_L^* y_L}(t-T_{\rm Rx},t)\Big\} \tag{60}$$

$$\mathrm{E}\big[I_{\rm Q}(t)\big] = -\tfrac{1}{4}R_{\rm pd}\mathrm{Im}\Big\{R_{y_L^* y_L}(t-T_{\rm Rx},t)\Big\} \tag{61}$$

Hence, using a procedure that is similar to the one in Section II, one can easily prove that the output signals for short fiber lengths and $T_{\rm Rx}=T_{\rm Tx}$ are given by

$$\mathrm{E}\big[I_{\rm I}(t)\big] = \tfrac{1}{8}R_{\rm pd}P_{\rm in}\mathrm{Re}\Big\{s(t)\Big\} = \tfrac{1}{8}R_{\rm pd}P_{\rm in}\cos\Big(\phi_{\rm mod}(t)\Big) \tag{62}$$

$$\mathrm{E}\big[I_{\rm Q}(t)\big] = \tfrac{1}{8}R_{\rm pd}P_{\rm in}\mathrm{Im}\Big\{s(t)\Big\} = \tfrac{1}{8}R_{\rm pd}P_{\rm in}\sin\Big(\phi_{\rm mod}(t)\Big) \tag{63}$$

As a result, the transmitted symbols can be reconstructed using integrate and dump filters and a conventional PSK detector. The rest of the system remains unchanged.

In case of longer fiber lengths, however, we can again find relations between the quadrature currents and the modulating signal $s(t)$. Using a similar procedure as in Section II, one can prove that

$$\mathrm{E}\big[I_{\rm I}(t)\big] = \tfrac{1}{2}\int h_{\rm sys}(\xi)\mathrm{Re}\Big\{s(t-\xi)\Big\}\,\mathrm{d}\xi \tag{64}$$

$$\mathrm{E}\big[I_{\rm Q}(t)\big] = \tfrac{1}{2}\int h_{\rm sys}(\xi)\mathrm{Im}\Big\{s(t-\xi)\Big\}\,\mathrm{d}\xi \tag{65}$$

where $h_{\rm sys}(t)$ is given by (22). Consequently, when integrate and dump filters are used for the quadrature currents, the distortion of the output pulses is more or less the same as in the case of BPSK modulation, as analyzed in Section III.

The worst case BER does not follow directly from the dispersion penalty of each of the quadrature currents, however. This will be illustrated in the following two subsections, where the BERs will be derived for quadriphase shift keying (QPSK) and M-ary PSK with $M\geq 8$.

A. QPSK modulation

Consider the case that QPSK is employed, and that $\phi_{\rm mod}(t)$ is assigned values which are odd multiples of $\frac{\pi}{4}$ radians. Moreover, assume that Gray coding is used to assign the information bits to the transmitted symbols, such that the BER is minimized [9]. Then, detection of the transmitted bits can be found by simply tresholding the filtered quadrature currents $I_{\rm I}(t)$ and $I_{\rm Q}(t)$. This is illustrated in Fig. 3(a), where the possible values of $\mathrm{E}\big[I_{\rm I}(t)\big]$ versus $\mathrm{E}\big[I_{\rm Q}(t)\big]$ are plotted.

The effect of dispersion can be analyzed as follows. Without loss of generality, consider the case in which the two binary zeroes are transmitted, corresponding to $\phi_{\rm mod}=\frac{\pi}{4}$. Similarly as in the case of BPSK modulation, two effects occur:

- Pulse height reduction; this results in a shift towards the origin of Fig. 3(a), such that the distance between the received symbol and the origin is reduced by a factor $p_{\rm det}(t_0)/T_{\rm s}$.
- Inter symbol interference (ISI); this is dominated by the neighboring symbols, and results in shifts which

are proportional to $p_{\rm det}(t_0 \pm T_{\rm s})$, in the directions that correspond to the phase of the interfering symbols.

Now consider the probability that the first bit (0) is detected incorrectly. This equals the probability that the filtered $I_{\rm Q}(t)$ is negative. From Fig. 3(a), it should then be obvious that this probability is not affected by the second bit of the preceding or succeeding symbol, but that it does depend on their first bit: the worst ISI occurs when the first bit of both the preceding and succeeding symbol is 1. A similar story holds for the second bit: the worst ISI occurs when the second bit of both the preceding and succeeding symbol is 1. In general, we can say that the worst case ISI for QPSK occurs when the phase of the preceding and succeeding symbol have π radians difference with respect to the symbol that is to be detected. This results in the same relation between the dispersion penalties and the symbol time $T_{\rm s}$ as in Section III. (The difference is that for the same value of the bit rate, $T_{\rm s}$ is now twice as large.) When there is no dispersion, the probability of bit error is given by [6], [9]

$$\mathrm{P_e} = \mathrm{Q}\left(\sqrt{\gamma_{\rm s}}\right) \tag{66}$$

where, in absence of dispersion, the SNR per symbol $\gamma_{\rm s}$ is given by

$$\gamma_{\rm s} = \frac{1}{4N^2+2N+1}\frac{T_{\rm s}}{\tau_{\rm c}} = \frac{2}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}} \tag{67}$$

Applying the dispersion penalty, however, we find

$$\gamma_{\rm s} = \frac{2}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}}\left(1-\frac{L\,\tau_{\rm g}'(f_{\rm c})R_{\rm b}}{\pi\sqrt{2}\,\tau_{\rm c}}\right)^2 \tag{68}$$

for wavelengths far from the zero dispersion wavelength, and

$$\gamma_{\rm s} = \frac{2}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}}\left(1-0.0341\frac{L\,\tau_{\rm g}''(f_{\rm c})R_{\rm b}}{\tau_{\rm c}^2}\right)^2 \tag{69}$$

for wavelengths close to the zero dispersion wavelength. Using a similar approach as in Section III, the maximum bit rate that can be achieved for a given coherence time and SNR per symbol can be derived to be

$$R_{\rm b,max} = \frac{4}{9}\sqrt{\frac{3\pi}{\sqrt{2}L\tau_{\rm g}'(f_{\rm c})\gamma_{\rm s}(4N^2+2N+1)}} \tag{70}$$

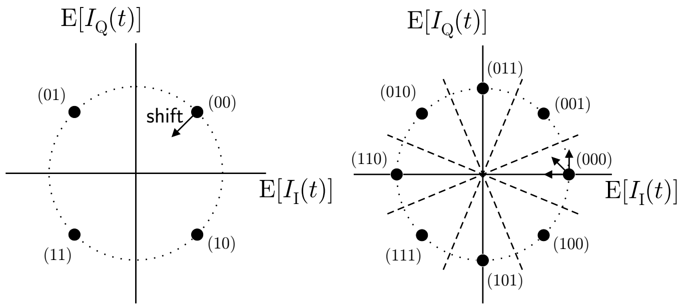
for wavelengths far from the zero dispersion wavelength, and

$$R_{\rm b,max} = \frac{2.126}{\sqrt[3]{\gamma_{\rm s}^2(4N^2+2N+1)^2L\tau_{\rm g}''(f_{\rm c})}} \tag{71}$$

for wavelengths close to the zero dispersion wavelength. This will be numerically illustrated in Section V.

B. M-ary PSK modulation with $M \geq 8$

Now consider the case that M-ary PSK with $M \geq 8$ is employed, and that $\phi_{\rm mod}(t)$ is assigned values which are multiples of $\frac{2\pi}{M}$. Again assume that Gray coding is used. The possible values that $\mathrm{E}\big[I_{\rm I}(t)\big]$ and $\mathrm{E}\big[I_{\rm Q}(t)\big]$ can take in that case are illustrated for $M = 8$ in Fig. 3(b). The straight dotted lines in this figure represent the decision boundaries. From the geometry of the figure, it should be obvious that when a symbol error is made, it is very unlikely that more than one bit error is made. Hence, the probability of bit error can be approximated by dividing the probability of a symbol error by k [9]. When there is no dispersion, this can be proven to result in (for high SNRs) [6], [9]



(a) QPSK (b) 8-PSK

Fig. 3. Output signals for M-ary PSK and the effect of dispersion

$$\mathrm{P_e} \approx \frac{2}{k}\mathrm{Q}\left(\sqrt{2\gamma_{\rm s}}\sin\left(\frac{\pi}{M}\right)\right) \tag{72}$$

where, in absence of dispersion, the SNR per symbol $\gamma_{\rm s}$ is given by

$$\gamma_{\rm s} = \frac{1}{4N^2+2N+1}\frac{T_{\rm s}}{\tau_{\rm c}} = \frac{k}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}} \tag{73}$$

In case of dispersion, however, pulse height reduction and ISI occurs. Without loss of generality, consider the case when the phase of a particular symbol is 0. The worst case ISI occurs when the contribution of the neighboring symbols causes a shift as close to one of the two closest decision lines as possible. This is the case when the phases of the neighboring bits are identical and equal to either $\frac{\pi}{2}$, $\frac{(M+1)\pi}{2M}$, $-\frac{\pi}{2}$ or $-\frac{(M+1)\pi}{2M}$ radians. The distance to one decision boundary is then reduced by a factor

$$P_{\rm d}^- = 1 - \frac{T_{\rm s}-p_{\rm det}(t_0)}{T_{\rm s}}\left[\cot\left(\frac{\pi}{M}\right)+1\right] \tag{74}$$

The distance to the other decision line is then increased. The minimum increase occurs when the phases of the neighboring bits are either $\frac{(M+1)\pi}{2M}$ or $-\frac{(M+1)\pi}{2M}$ radians and is given by a factor

$$P_{\rm d}^+ = 1 + \frac{T_{\rm s}-p_{\rm det}(t_0)}{T_{\rm s}}\left[\frac{\cos\left(\frac{3\pi}{M}\right)}{\sin\left(\frac{\pi}{M}\right)}-1\right] \tag{75}$$

Hence, one can prove that the worst case BER for M-ary PSK with $M \geq 8$ can be approximated as

$$\mathrm{P_e} = \frac{\mathrm{Q}\left(\sqrt{2\gamma_{\rm s}^-}\sin\left(\frac{\pi}{M}\right)\right)+\mathrm{Q}\left(\sqrt{2\gamma_{\rm s}^+}\sin\left(\frac{\pi}{M}\right)\right)}{k} \tag{76}$$

where

$$\gamma_{\rm s}^- \triangleq \frac{k}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}}\left(P_{\rm d}^-\right)^2 \tag{77}$$

$$\gamma_{\rm s}^+ \triangleq \frac{k}{4N^2+2N+1}\frac{1}{R_{\rm b}\tau_{\rm c}}\left(P_{\rm d}^+\right)^2 \tag{78}$$

Now, the aim is again to optimize the bit rate that can be achieved at a BER of 10^{-9} with respect to τ_c. This can be done as follows. For the time being, neglect the second term in the nominator of (76). Then, for each value of $M = 2^k$, one can numerically find the value of γ_s^- that is required to have $P_e \approx 10^{-9}$. Then, an analytical expression can be found for the achievable bit rate as a function of amongst others τ_c, which can be analytically optimized. This results in

$$R_{b,\max} = \frac{2k}{9}\sqrt{\frac{3\pi}{\sqrt{2}L\tau_g'(f_c)\gamma_s^-(4N^2+2N+1)\left[\cot\left(\frac{\pi}{M}\right)+1\right]}} \quad (79)$$

for wavelengths far from the zero dispersion wavelength, and

$$R_{b,\max} = \frac{0.844k}{\sqrt[3]{\left(\gamma_s^-\right)^2(4N^2+2N+1)^2L\tau_g''(f_c)\left[\cot\left(\frac{\pi}{M}\right)+1\right]}} \quad (80)$$

for wavelengths close to the zero dispersion wavelength. (One can verify that at these bit rates and for low BERs, the second term of (76) can indeed be neglected. Hence, the obtained expressions are indeed good approximations of the maximum achievable bit rate at $P_e = 10^{-9}$.)

V. Numerical example for standard single-mode fiber

In this subsection, we will give some numerical examples for applications in which standard SMF [10] is used. We will consider two operating wavelengths:

- $\lambda_c \approx 1550$ nm; in that case, second order dispersion can be neglected, and the first order dispersion coefficient as prescribed by the ITU is given by $\tau_g'(\lambda_c) \leq 20$ ps/(nm·km).
- λ_c is equal to the zero-dispersion wavelength, which is close to 1310 nm. In that case, the first order dispersion coefficient is zero (by definition) and the second order dispersion coefficient is $\tau_g''(\lambda_c) \leq 0.093$ps/(nm^2·km).

As a fiber length, we will assume a typical indoor length of $L = 500$ m. As an SNR requirement, we put $\gamma_b = 18.0$ for BPSK, $\gamma_s = 36.0$ for QPSK and $\gamma_s^- = 115.5$ for 8-PSK. (Using (41), (66) and ((76), one can verify that this corresponds to a BER of 10^{-9}.)

Using (26), (27), (46), (57), (70), (71), (79) and (80), the maximum achievable bit rates for both wavelengths can now be found as a function of the number of users N. The results are shown in Fig. 4.

VI. Conclusion

An expression has been derived that relates the output signal of a coherence multiplex (CM) receiver to the modulating signal in the transmitter, in which chromatic dispersion in single-mode fiber (SMF) is taken into account. The expression is used to evaluate the effect of dispersion on digital transmission using CM. Expressions are derived for the maximum bit rate

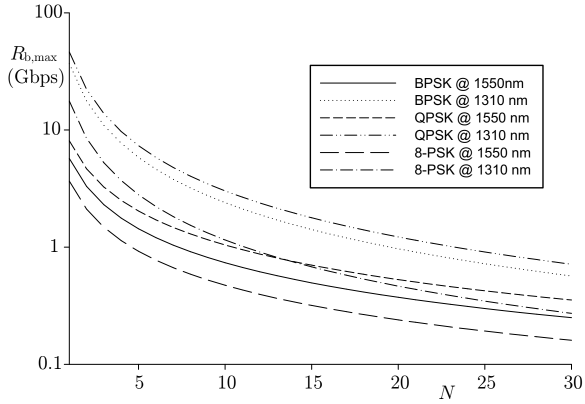


Fig. 4. Maximum bit rate at $P_e = 10^{-9}$ as a function of number of channels

that can be achieved at a BER of 10^{-9} for binary PSK, QPSK and M-ary PSK (with $M > 8$), both for wavelengths far from and close to the zero-dispersion wavelength. These bit rates were optimized with respect to source linewidth, showing that the highest bit rate can be achieved when QPSK modulation is used. Numerical results are given for a standard SMF of 500 m length, indicating a maximum bit rate of 3.02 Gbps per user for 10 simultaneous users and 1.22 Gbps per user for 20 simultaneous users.

Acknowledgment

A. Meijerink would like to thank Philips Research for funding his Ph.D. position.

References

[1] G.J. Pendock, D.D. Sampson, "Capacity of coherence-multiplexed CDMA networks", *Opt. Commun.*, vol. 143, pp. 109–117, 1997.

[2] R.O. Taniman, A. Meijerink, J.C. Haartsen, W. van Etten, "Indoor RF signal distribution using a coherence multiplexed/subcarrier multiplexed optical transmission system", *Proc. 10th Symposium on Communications and Vehicular Technology in the Benelux*, Eindhoven, The Netherlands, 2003, this issue.

[3] R.A. Griffin, D.D. Sampson, D.A. Jackson, "Demonstration of data-transmission using coherent correlation to reconstruct a coded pulse sequence", *IEEE Photonics Technol. Lett.*, Vol. 4, No. 5, 1992, pp. 513–515, 1992.

[4] D.A. Blair, G.D. Cormack., "Optimal source linewidth in a coherence multiplexed optical fiber communications system", *J. Lightwave Technol.*, Vol. 10, No. 6, pp. 804–810, 1992.

[5] W. van Etten, A. Meijerink, "Optical stabilization of coherence multiplex output signal by means of a phase diversity network", *Proc. 6th Ann. Symp. IEEE/LEOS Benelux Chapt.*, Brussels, Belgium, 2001, pp. 149–152.

[6] A. Meijerink, G.H.L.M. Heideman, W. van Etten, "M-ary (D)PSK modulation in coherence multiplex systems", *Proc. 7th Ann. Symp. IEEE/LEOS Benelux Chapt.*, Amsterdam, The Netherlands, 2002, pp. 207–210.

[7] J.W. Goodman, *Statistical Optics*, Wiley, New York, 1985.

[8] J.L. Gimlett, N.K. Cheung, "Dispersion penalty analysis for LED/Single-Mode Fiber Transmission Systems", *J. Lightwave Technol.*, Vol. 4, No. 9, pp. 1381-1392, 1986.

[9] J.G. Proakis, *Digital Communications*, Fourth Edition, New York: McGraw-Hill, 2001.

[10] *Characteristics of a single-mode optical fibre cable*, ITU-T Recommendation G.652, International Telecommunication Union, 1993.