Circles on surfaces

Niels Lubbes

King Abdullah University of Science and Technology

Thuwal, Saudi Arabia

niels.lubbes@gmail.com

Abstract

The sphere in 3-space has an infinite number of circles through any closed point. The torus has 4 circles through any closed point. Two of these circles are known as Villarceau circles ([0]). We define a "celestial" to be a real surface with at least 2 real circles through a generic closed point. Equivalently, a celestial is a surface with at least 2 families of real circles.

In 1980 Blum [1] conjectured that a real surface has either at most 6 families of circles or an infinite number. For compact surfaces this conjecture has been proven by Takeuchi [2] in 1987 using topological methods. In 2001 Schicho [3] classified complex surfaces with at least 2 families of conics. This result together with Moebius geometry led to a classification of celestials in 3-space [4].

In 2012 Pottmann et al. [5] conjectured that a surface in 3-space with exactly 3 circles through a closed point is a Darboux Cyclide. We confirm this conjecture as a corollary from our classification in [4].

We recall that a translation is an isometry where every point moves with the same distance. In this talk we consider celestials in 3-space that are obtained from translating a circle along a circle, in either Euclidean or elliptic space. This is a natural extension of classical work by William Kingdon Clifford and Felix Klein on the Clifford torus.

Krasauskas, Pottmann and Skopenkov conjectured, that celestials in 3-space of Moebius degree 8 are Moebius equivalent to an Euclidean or Elliptic translational celestial. This conjecture is true if its Moebius model has a family of great circles ([6]). Moreover, its real singular locus consist of a great circle. As a corollary we obtain a classically flavored theorem in elliptic geometry: if we translate a line along a circle but not along a line then exactly 2 translated lines will coincide ([6]).

Keywords

families of circles, cyclides, Moebius geometry, elliptic geometry, weak Del Pezzo surfaces, translational surfaces

[0] M. Berger, *Geometry revealed, A Jacob's Ladder to Modern Higher Geometry*, Springer, 2010

[1] R. Blum, *Circles on surfaces in the Euclidean 3-space*, Geometry and differential geometry, 1979

[2] N. Takeuchi, *A closed surface of genus one in E^3 cannot contain seven circles through each point*, Proc. Amer. Math. Soc., 1987

[3] J. Schicho, *The multiple conical surfaces*, Contributions to Algebra and Geometry, 2001

[4] N. Lubbes, *Families of circles on surfaces*, Contributions to Algebra and Geometry, 2013

[5] H. Pottmann, L. Shi, M. Skopenkov, *Darboux cyclides and webs from circles*, Comput. Aided Geom. Design, 2012

[6] N. Lubbes, *Translations of circles in Euclidean and elliptic space*, Submitted, arXiv:1302.6710, 2013