CIRCUIT AND SCATTERING MATRIX ANALYSIS OF THE WIRE MEASUREMENT METHOD OF BEAM IMPEDANCE IN ACCELERATING STRUCTURES*

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Abstract

In order to measure the wakefield left behind multiple bunches of energetic electrons we have previously used the ASSET facility in the SLC [1]. However, in order to produce a more rapid and cost-effective determination of the wakefields we have designed a wire experimental method to measure the beam impedance and from the Fourier transform thereof, the wakefields. In this paper we present studies of the wire effect on the properties of X-band structures in study for the JLC/NLC (Japanese Linear Collider/Next Linear Collider) project. Simulations are made on infinite and finite periodical structures. The results are discussed.

1 INTRODUCTION

The progress of multiple bunches of electrons down several thousand accelerating structures can, at worst, lead to a Beam Break Up instability or at the very least can give rise to a dilution in the emittance of the beam. These affects are driven by both intra-bunch (or short range) wakefields and long-range wakefields. In order to ensure that these effects do not occur it is important to be able to predict and measure the associated beam impedance, loss factor and wakefield for a given accelerator structure in a routine manner. Measurements already made on X-band structures inserted in the 2-mile SLAC linac using the ASSET facility [1] have indicated that theoretical models, give good predictions as to how well the wakefield is damped [2]. However, such measurements are so time consuming and expensive that it is only practical to make this measurement on a very limited number of accelerating structures. The wire measurement technique [3,4,5,6], once perfected, will enable a routine measurement of the impedance of dipole frequency bands. We would envisage having a number of wire measurement setups being available according to the frequency band of interest [7].

The wire measurement method essentially replaces the electron beam with a pulse propagating along a wire placed in the DUT (Device Under Test). In practice, the loss factor is determined through the perturbation of the current due to the DUT [3]. In the frequency domain the transmission factor leads to the impedance [4], while the area under the resonances allows the loss factor to be determined [8]. In this paper we investigate the frequency domain behavior of multiple cell structures. In section 2 the dispersion properties are calculated with the aid of a circuit model. The circuit is developed to model the monopole and first dipole mode regions. In the third section we utilize a matrix cascading technique in order to calculate the overall transmission coefficient of multi-cell structures.

2 CIRCUIT MODEL OF LOWER BAND MODES

We model the interaction between the TEM coaxial mode of a wire inserted on-axis into a cavity with the TM01 mode of the cavity. The cavity consists of two half irises separated from each other by a section of waveguide. This cavity will later be considered to be infinitely repeating in order to obtain the dispersion properties of the system.

The circuit model chosen to model the electromagnetic behavior of a single traveling wave cavity is shown in Fig. 1. The structure is represented by two sections of coax with a parallel resonant circuit connecting the upper conductors of the two sections. The lower conductors of the two sections are connected together directly.
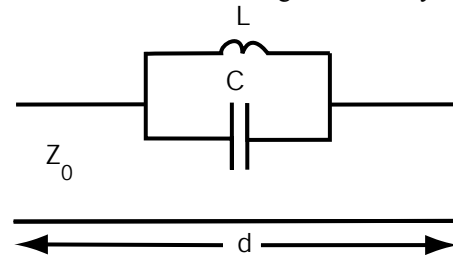

Fig. 1 Circuit model of wire inserted into accelerating cell. The period of the structure is d and the characteristic impedance of the transmission line is Z_0.

For an accelerator made up of an infinite sequence of cavities with identical parameters, the dispersion relation for a structure modeled by the circuit in Fig. 1 is:

$$\cos\psi = \cos\left[\psi_{acc} f / \sqrt{1+b}\right] - \frac{\psi_{acc} b f \sin\left[\psi_{acc} f / \sqrt{1+b}\right]}{2\sqrt{1+b}(1+a^2 f^2)}$$

Here ψ refers to a particular phase advance, f is the frequency normalized with respect to the accelerating frequency (11.424 GHz), ψ_{acc} is the characteristic phase advance per cell at the accelerating frequency. The parameters a and b are the accelerating frequency normalized with respect to the cell resonance frequency and, the parallel resonant circuit inductance divided by the coax inductance per unit length multiplied by the cell period, respectively. In practice, these two parameters are obtained such that the dispersion equation identically fits the second 0 mode and the first π mode.

In the limit of small frequencies this dispersion relation yields:

$$1 - \frac{\psi^2}{2} = 1 - \frac{\psi_{acc}^2 f^2}{2}$$

and thus there is a linear relation between frequency and phase. For frequencies close to zero, circuit theory predicts that the mode follows the light line in this region as there is little or no coupling to the monopole or indeed any other modes.

We apply this circuit method to a single cell of the SLAC detuned accelerating structure known as DS2S. Fig. 2 shows the dispersion points for this structure computed with HFSS (High Frequency Structure Simulator) together with the curves obtained from the dispersion equation. The points are seen to be well simulated with the circuit model for all points apart from those in the upper branch close to the π phase advance point. Replacing the L-C circuit with a transmission line may improve this representation. Work is in progress to improve this model in the π phase advance region and also to include the higher order frequency bands.
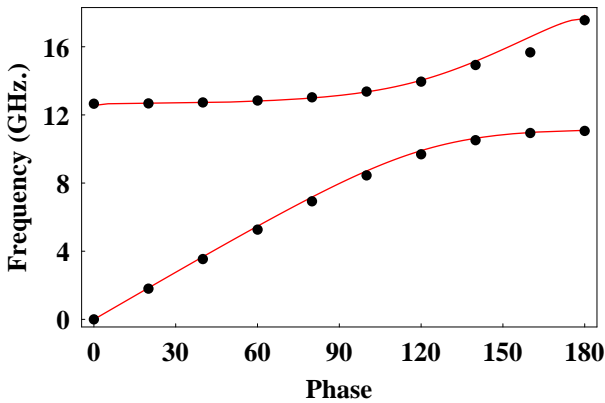

Fig. 2 Dispersion curves for the detuned accelerating structure DS2S. The circuit model is denoted by the curves and the HFSS simulation is given by the points.

3 MATRIX CASCADE SIMULATIONS

In order to determine the impedance or loss factors of various passbands, in the wire method the transmission S_{21} has to be determined. Here we calculate the transmission coefficient of structures consisting of multiple cells in order to facilitate comparison with experimental measurements. We considers the

dimensions of a 11.4 GHz standing wave structure built for the JLC/NLC studies. This accelerator structure, known as SW20PI, is 20cm in length and consists of 15 cells. The cells have an iris radius of 4.75 mm and are all identical prior to tuning the cells (in order to obtain a flat field). The calculations were made with the HFSS code. The walls and the wire are considered to be perfect conductors except where otherwise noted. All curves in the geometry were approximated by straight lines subscribing angles of 10-15 degrees..

Performing simulations of full 3D accelerating structures is computationally expensive in terms of both memory and time. Therefore, we used the technique of cascading scattering matrices to obtain the overall scattering matrix from the individual multi-mode matrices. The scattering matrix of an object is given by:

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix},$$

where S_{12} is in general a matrix relating modes of port 1 to the modes of port 2. The individual elements of the
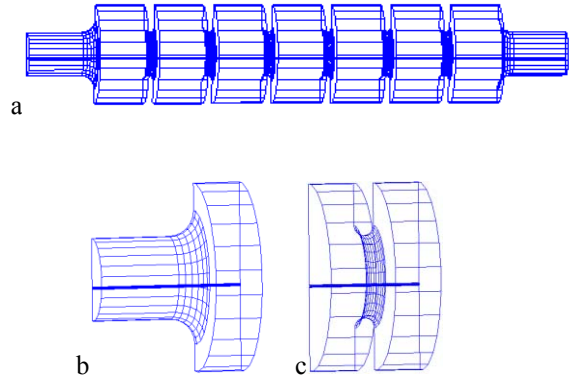

Fig. 3 Structures used for simulation of S parameters: a. full structure; b. and c. geometries used in simulations for cascading

2×2 matrix are themselves matrices as they represent the scattering into higher order modes. However, in calculating the impedance, only the first component of the S_{21} matrix is necessary once the overall scattering matrix has been evaluated.

In order to obtain the scattering parameters for a structure similar to the one illustrated in Fig. 3a, we calculated the S matrix for the end section 3b and a middle section 3c. Six modes were considered in each port. The conductors are assumed to have an infinite conductivity. The wire has a diameter of 300 μm. The global scattering matrix is obtained by cascading two-sections-at a-time using the generalized scattering matrix method [9,10]:

$$S_{11} = S_{11}^A + S_{12}^A \left(I - S_{11}^B S_{22}^A \right)^{-1} S_{11}^B S_{21}^A$$

$$S_{12} = S_{12}^A \left(I - S_{11}^B S_{22}^A \right)^{-1} S_{12}^B$$

$$S_{21} = S_{21}^B \left(I - S_{22}^A S_{11}^B \right)^{-1} S_{21}^A$$

$$S_{22} = S_{22}^B + S_{21}^B \left(I - S_{22}^A S_{11}^B \right)^{-1} S_{22}^A S_{12}^B$$

Here, S^A and S^B represent the matrices of the individual sections. This allows the overall matrix to be determined quite rapidly as the calculation of a single section is very efficient.

In order to verify the accuracy of our cascading, a structure consisting of a limited number of cells (seven for the calculation herein) has been studied. The detailed results are presented in the following subsections. Two cases have been considered: a centered and an offset wire.

Centered wire

For the centered wire we sliced a 10 degrees section of each simulated object in order to reduce the calculation time. An initial comparison of the cascading results to the ones obtained directly by simulating the full length of the structure showed good agreement, except in the region of the cutoff frequencies of higher band modes. The agreement is improved considerably by performing a simulation in which the actual conductivity of copper ($\sigma \sim 5.8 \cdot 10^7$ $\Omega^{-1} \cdot m^{-1}$) is used for the walls of the structure and for the wire. This prevents numerical instabilities occurring as the modes now start to propagate gradually. Excellent agreement is now obtained between the two simulations as seen in Fig. 4a.
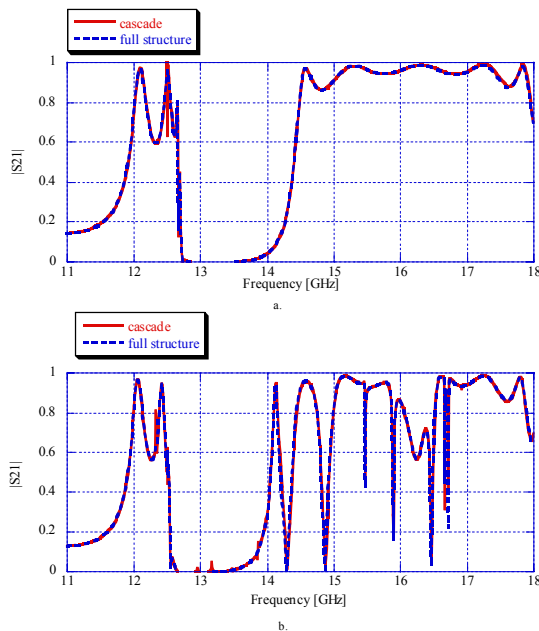

a.

b.

Fig. 4 S_{21} obtained with the cascading technique compared to the direct simulation of a complete 7-cell structure, having a wire of 300 μm in diameter placed on-axis (a) and at 1 mm offset (b)

Offset wire

In order to excite dipole modes, the wire is displaced from the axis by 1 mm. Symmetry considerations dictate that half of the geometry is required to be modeled. The walls and the wire are again simulated with the conductivity of copper. The results are compared in Fig. 4b. The discrepancy between the two methods is negligible.

We also note that in Fig. 4 the fundamental monopole band can be distinguished between about 12 and 12.6 GHz. Moving the wire hardly perturbs it. The frequency range above about 13 GHz contains modes of the first and second dipole bands interacting with the coaxial TEM mode. This region is perturbed significantly by moving the wire off-axis.

4 DISCUSSION

The dispersion properties of the wire-loaded accelerating structure have been seen to be quite well modeled with a transmission line and an L-C circuit. Using this circuit for the complete 15-cell structure will also enable the impedance of the structure to be obtained and enable a thorough understanding of the nature of the resonances obtained in the experimental measurement of the transmission coefficient. Work is already ongoing in this area. Further research is in progress on developing the circuit model to include higher order frequency bands.

Application of the generalized scattering matrix method has allowed the overall transmission coefficient to be obtained accurately and efficiently. The monopole frequency band is, as expected, little affected by the movement of the wire whilst dipole resonances are seen to appear as the wire is moved off axis.

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