## CIRCULAR INTERLACING WITH RECIPROCAL POLYNOMIALS

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Abstract. The purpose of this paper is to show that all zeros of the reciprocal polynomial

$$
P_{m}(z)=\sum_{k=0}^{m} A_{k} z^{k} \quad(z \in \mathbb{C})
$$

of degree $m \geqslant 2$ with real coefficients $A_{k} \in \mathbb{R}$ (i.e. $A_{m} \neq 0$ and $A_{k}=A_{m-k}$ for all $\left.k=0, \ldots,\left[\frac{m}{2}\right]\right)$ are on the unit circle, if there is a $B \in \mathbb{R}$ such that $A_{m} B \geqslant 0,\left|A_{m}\right| \geqslant|B|$ and
holds.

$$
\left|A_{m}+B\right| \geqslant \sum_{k=1}^{m-1}\left|A_{k}+B-A_{m}\right|
$$

If the inequality is strict then the zeros of $P_{m}$ have the form $e^{ \pm u_{j}}\left(j=1, \ldots,\left[\frac{m}{2}\right]\right)$ where

$$
\frac{2(j-1) \pi}{m}<u_{j}<\frac{2 j \pi}{m} \quad\left(j=1, \ldots,\left[\frac{m}{2}\right]\right)
$$

and they are simple (for odd $m$, in addition to these zeros, $-1=e^{-i \pi}$ is a zero too).
This implies that the polynomial $P_{m}$ (with $A_{m}>0$ ) and $z^{2 m}-1$ satisfy the circular interlacing condition.

If in the inequality (for the coefficients) equality holds, then double zeros may arise, we discuss how this can happen.

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