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CIRCULAR POLARIZATION OF LIGHT SCATTERED FROM ROUGH SURFACES

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SUMMARY

It is suggested that double reflection of light on rough and absorbing surfaces (complex index of refraction) is the principal cause of the circular polarization observed on Mars, Mercury and the Moon as well as on rock samples studied in the laboratory by Pospergelis. Two simple models of rough surfaces are presented. The fractional circular polarization obtained with such models, using geometrical optics and Fresnel's laws, and its dependence on the scattering angles is compared with observations. Generally good agreement is obtained within the limitations of the models.

1. INTRODUCTION

Kemp and his colleagues have measured the small circular polarization of sunlight scattered by the planets and by the Moon (Kemp *et al.* 1971a; Kemp & Wolstencroft 1971b; Kemp, Wolstencroft & Swedlund 1971c). For planets with little or no atmosphere the polarizing agent must be the solid surface, and the observed fractional circular polarization, Q, its sign and magnitude and its dependence on scattering angles and on wavelength, are a measure of surface properties such as roughness and refractive index. Measurements of Q for planets, satellites and asteroids may become an important tool in planetary research, and a sufficiently general theory can hopefully be developed which describes the relationships between Q and the surface properties. We have already made some progress in this direction and shall report here some results obtained with particularly simple models of surface scattering involving double reflection and using geometrical optics plus Fresnel's laws. These models account for some of the important properties of the planetary Q's as well as of the fractional circular polarization of light scattered by volcanic rock as observed by Pospergelis (1969). We take the partial success of these very simple models to be convincing although not conclusive evidence that multiple scattering on rough surfaces is the principle mechanism causing the observed circular polarization.

A successful model must explain the following observed features:

(1) For small phase angles φ (Sun–Planet–Earth angle), i.e. at least for $|\varphi| \lesssim 30°$, Q must have the opposite sign from the so-called 'gas-surface' sign characteristic of, for example, Venus and Jupiter (Kemp *et al.* 1971c). This was the most striking result of the first observations on planets.

(2) A sign change apparently not explained by symmetry considerations occurs at some moderate phase angle ($|\varphi| \lesssim 90°$). That angle may depend on the surface coordinate of the scattering region (latitude γ, and longitude ψ, in the notation of Pospergelis (1969); these angles will be defined more precisely in Section 3(ii) and in Fig. 6) and, presumably, on the character of the surface—but this sign

change seems to be a universal feature and occurs in addition to the fundamental antisymmetries $Q(\gamma) = -Q(-\gamma)$ and $Q(\varphi) = -Q(-\varphi)$.*

(3) For constant values of φ and ψ, $|Q|$ increases, from a value of zero at $\gamma = 0°$, monotonically with increasing γ. Furthermore, for constant values of φ and γ, Q decreases with increasing value of ψ; it is positive for large negative ψ, and vice versa; however, for small phase angles, Q is negative even for large negative ψ. These conclusions are drawn from the laboratory data on volcanic rock, published by Pospergelis, in particular, from his Figs 9 and 10. The planetary data are in general not conclusive concerning the detailed dependence of Q on γ and ψ since one integrates over entire hemispheres (north versus south).†

2. CIRCULAR POLARIZATION BY DOUBLE REFLECTION

When initially unpolarized light with electric field amplitude E_0 in each component is reflected by a plane surface it becomes partially linearly polarized. This is because the two components of the reflected electric field **E** are generally different. The components are (see, e.g. J. D. Jackson, *Classical Electrodynamics*, Chap. 7, J. Wiley & Sons, New York, 1962)

$$E_{\parallel} = E_0\Psi = E_0 \frac{n^2 \cos\alpha - \sqrt{n^2 - \sin^2\alpha}}{n^2 \cos\alpha + \sqrt{n^2 - \sin^2\alpha}} \tag{1a}$$

parallel to the plane of incidence, and

$$E_{\perp} = E_0\chi = E_0 \frac{\cos\alpha - \sqrt{n^2 - \sin^2\alpha}}{\cos\alpha + \sqrt{n^2 - \sin^2\alpha}}. \tag{1b}$$

perpendicular to the plane of incidence. Ψ and χ are the Fresnel coefficients, α is the angle of incidence and is equal to the angle of reflection, by Snell's law, and n is the complex refractive index of the surface,

$$n = \nu + i\kappa, \quad \kappa \geqslant 0. \tag{2}$$

Since the components of the incident electric field are uncorrelated in phase, there is no phase relation between the components of **E** and hence no circular polarization. If now the reflected radiation is incident upon another surface, then each component of **E** is independently scattered. The component $E_{\parallel}$ gives rise to a reflected field with components π_a parallel, and σ_a perpendicular to the plane of incidence, and π_a and σ_a have a constant phase difference. Similarly, $E_{\perp}$ gives rise to π_b and σ_b; and the circularly polarized intensity of the total emergent radiation is

$$QI = \frac{c}{4\pi} Im[\sigma_a \pi_a^* + \sigma_b \pi_b^*] \tag{3}$$

where π_a^* is the complex conjugate of π_a, etc., and c is the speed of light. The radiation has positive helicity $[QI > 0]$ if the electric vector rotates clockwise when one looks along the direction of propagation. By Pospergelis' definition of the fourth

* These antisymmetries are suggested by the planetary data although observations at exactly corresponding sets of the angles (i.e. equal magnitude but opposite sign) are not yet available.

† This is not necessary for the Moon. The integration is probably one of the reasons why the Q-values for the planets are much smaller than for the Moon.

Stokes parameter, S, we have $QI = -S$. It is clear from equation (3) that the radiation has no ellipticity if the surface is loss-free, i.e. if $\kappa = 0$. In this paper we shall assume that n is constant over the surface, i.e. n is the same for both reflections, but this may not be so in an actual case. For the emergent radiation to be partially circularly polarized, a non-zero value of κ is required only for the second reflection (see also equation (6)).

The situation for a general rough surface is illustrated in Fig. 1: Plane-parallel unpolarized radiation, incident from a direction $-\hat{e}_1$, is reflected by a surface element, ΔS_1, in a direction $\hat{e}$ toward another surface element, ΔS_2, which in turn

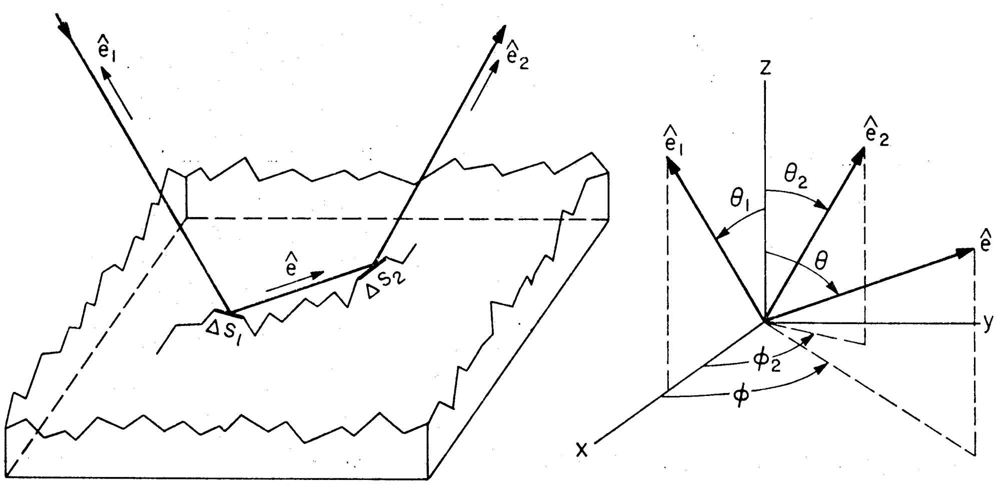

FIG. 1. *Double reflection on a rough surface. The radiation is incident from a direction* $-\hat{e}_1$ *and emerges from the surface in a direction* $\hat{e}_2$ *where the observer is located. The accompanying diagram defines the coordinate system used in the analysis.*

reflects the radiation in a direction $\hat{e}_2$ toward the observer. We choose a rectangular coordinate system such that the z-axis is normal to the rough surface and $\hat{e}_1$ lies in the (x, z)-plane. The co-latitudes of the unit vectors $\hat{e}_1$, $\hat{e}$ and $\hat{e}_2$ are θ_1, θ, and θ_2, respectively, and the azimuths are zero, ϕ (not to be confused with the phase angle φ between $\hat{e}_2$ and $\hat{e}_1$) and ϕ_2, respectively. Depending on the topology of the surface, there may be several pairs of surface elements ΔS_1 and ΔS_2 which for a given direction of incidence scatter radiation into the direction $\hat{e}_2$, i.e. there may be several intermediate directions $\hat{e}$, and one must calculate the circularly polarized radiation scattered toward the observer by the entire surface. We shall disregard contributions to this radiation which have been scattered more than twice, and we shall also neglect contributions by diffracted and refracted rays. Single scattering does not contribute to QI but it affects the total intensity of the emergent radiation and therefore also the magnitude of the fractional circular polarization but not its sign with which this paper is primarily concerned.

Details of the two successive reflections are shown in Fig. 2. For the first reflection the angle of incidence is α_1; $\hat{s}_1 = \hat{e}_1 \times \hat{e}/|\hat{e}_1 \times \hat{e}|$ and $\hat{p}_1 = \hat{e}_1 \times \hat{s}_1$ are unit vectors perpendicular and parallel, respectively, to the plane of incidence defined by $\hat{e}_1$ and $\hat{e}$. For the second reflection, the angle of incidence is α_2, and the corresponding unit vectors are $\hat{s}_2 = \hat{e}_2 \times \hat{e}/|\hat{e}_2 \times \hat{e}|$ and $\hat{p}_2 = \hat{s}_2 \times \hat{e}$. One easily shows that the components of the electric field amplitude of the emergent radiation are, except for phase factors which disappear from the expression (3) for QI and

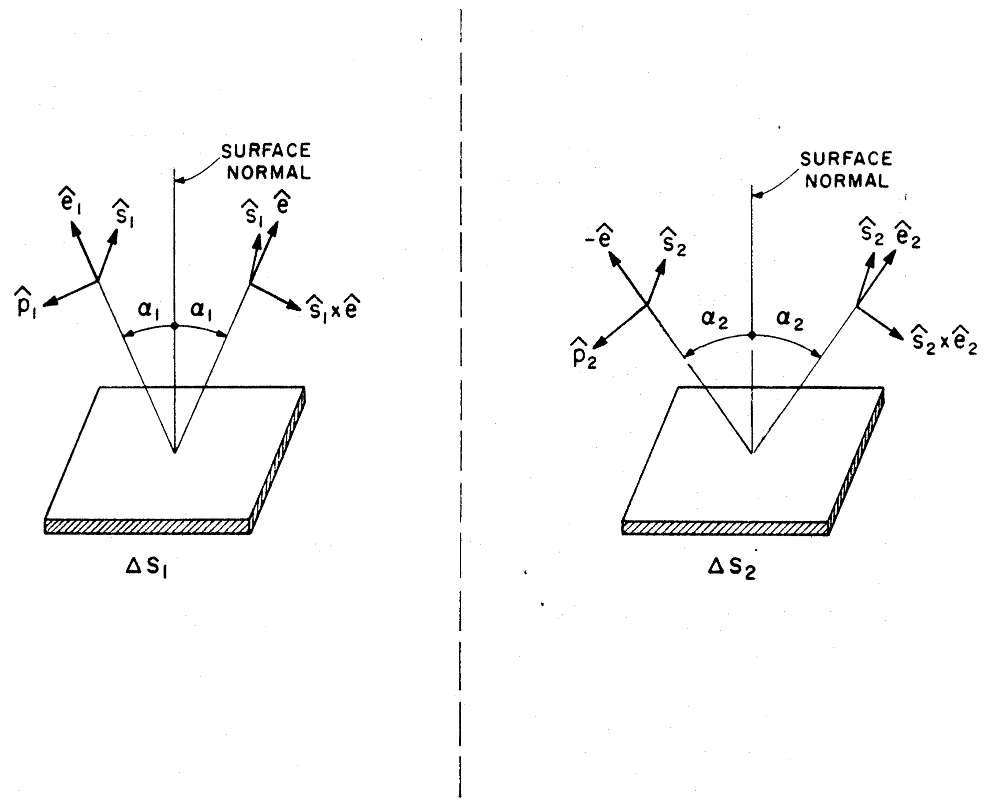

FIG. 2. *Detail of the first and second reflection, showing the orientations of the unit vectors:* $\hat{s}_1$ *and* $\hat{s}_2$ *are perpendicular to the respective planes of incidence whereas* $\hat{p}_1$, $\hat{p}_2$, $\hat{s}_1 \times \hat{e}$, *and* $\hat{s}_2 \times \hat{e}_2$ *lie in those planes.*

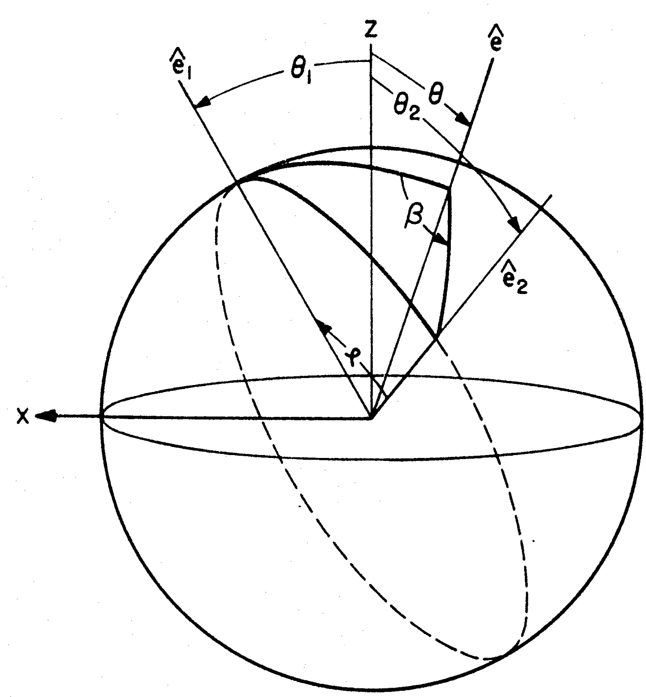

FIG. 3. *Definition of the angle* β *where* $\cos\beta = \hat{s}_1 \cdot \hat{s}_2$, *these unit vectors being defined in Fig. 2. The arcs drawn on the unit sphere between the unit vectors* $\hat{e}_1$, $\hat{e}$ *and* $\hat{e}_2$ *all lie on great circles.*

therefore need not be determined,

$$\left.\begin{aligned}\sigma_a &= E_0\Psi_1\chi_2\hat{s}_1\times\hat{e}\cdot\hat{s}_2\\ \pi_a &= E_0\Psi_1\Psi_2\hat{s}_1\times\hat{e}\cdot\hat{p}_2\\ \sigma_b &= E_0\chi_1\chi_2\hat{s}_1\cdot\hat{s}_2\\ \pi_b &= E_0\chi_1\Psi_2\hat{s}_1\cdot\hat{p}_2\end{aligned}\right\} \quad (4)$$

where Ψ_1 and χ_1 are the Fresnel coefficients (equation (1)) for the first reflection, and Ψ_2 and χ_2 for the second reflection. From the definitions of the unit vectors we find $\hat{s}_1\cdot\hat{s}_2 = \hat{s}_1\times\hat{e}\cdot\hat{s}_2$ and $\hat{s}_1\cdot\hat{p}_2 = -\hat{s}_1\times\hat{e}\cdot\hat{p}_2$. If we write

$$\hat{s}_1\cdot\hat{s}_2 = \cos\beta \quad (5)$$

then $\hat{s}_1\cdot\hat{p}_2 = \sin\beta$, and β is the angle between the projections of $\hat{e}_1$ and $\hat{e}_2$ onto a plane perpendicular to $\hat{e}$. It is measured clockwise, when looking along $\hat{e}$, from the projection of $\hat{e}_1$ toward the projection of $\hat{e}_2$ (Fig. 3). These relations and equations (4) and (5) give

$$QI = E_0^2\frac{c}{8\pi}[|\chi_1|^2-|\Psi_1|^2]\,Im[\chi_2\Psi_2^*]\sin(2\beta). \quad (6)$$

If we further write

$$(\nu+i\kappa)^2-\sin^2\alpha_j = r_j\exp[i\rho_j] \quad j = 1, 2 \quad (7)$$

so that $r_j = [(\nu^2-\kappa^2-\sin^2\alpha_j)^2+4\nu^2\kappa^2]^{1/2}$ and $\tan\rho_j = 2\nu\kappa/(\nu^2-\kappa^2-\sin^2\rho_j)$ then from equation (1a) and (1b) we obtain

$$|\Psi_j|^2 = \frac{(\nu^2+\kappa^2)^2\cos^2\alpha_j+r_j-2\sqrt{r_j}[\nu^2-\kappa^2)\cos(\rho_j/2)+2\nu\kappa\sin(\rho_j/2)]\cos\alpha_j}{(\nu^2+\kappa^2)^2\cos^2\alpha_j+r_j+2\sqrt{r_j}[(\nu^2-\kappa^2)\cos(\rho_j/2)+2\nu\kappa\sin(\rho_1/2)]\cos\alpha_j} \quad (8a)$$

and

$$|\chi_j|^2 = \frac{\cos^2\alpha_j+r_j-2\sqrt{r_j}\cos\alpha_j\cos(\rho_j/2)}{\cos^2\alpha_j+r_j+2\sqrt{r_j}\cos\alpha_j\cos(\rho_j/2)} \quad [j = 1, 2]. \quad (8b)$$

If we let $D[|\Psi_j|^2]$ be the denominator of $|\Psi_j|^2$ given by equation (8a), and $D[|\chi_j|^2]$ the denominator of $|\chi_j|^2$ given by equation (8b), then after some algebra we find

$$|\chi_1|^2-|\Psi_1|^2 = \frac{4[(\nu^2-\kappa^2-1)^2+4\nu^2\kappa^2]\sqrt{r_1}\cos\alpha_1\sin^2\alpha_1\cos(\rho_1/2)}{D[|\Psi_1|^2]\,D[|\chi_1|^2]} \quad (9a)$$

and

$$Im[\chi_2\Psi_2^*] = \frac{2[(\nu^2-\kappa^2-1)^2+4\nu^2\kappa^2]\sqrt{r_2}\cos\alpha_2\sin^2\alpha_2\sin(\rho_2/2)}{D[|\Psi_2|^2]\,D[|\chi_2|^2]} \quad (9b)$$

so that QI can be brought into the useful form

$$QI = E_0^2\frac{c}{\pi}\frac{[(\nu^2-\kappa^2-1)^2+4\nu^2\kappa^2]^2\sqrt{r_1r_2}\cos\alpha_1\cos\alpha_2\sin^2\alpha_1\sin^2\alpha_2\cos(\rho_1/2)\sin(\rho_2/2)\sin(2\beta)}{D[|\chi_1|^2]\,D[|\Psi_1|^2]\,D[|\chi_2|^2]\,D[|\Psi_2|^2]}. \quad (10)$$

The denominator of QI is a relatively slowly varying function of α_1 and α_2. QI depends most strongly on these angles through the cosine and sine factors in the numerator, and on the refractive index through the factors $[(\nu^2-\kappa^2-1)^2+4\nu^2\kappa^2]^2$

and sin $(\rho_2/2)$ in equation (10). For $\kappa \ll 1$ as is appropriate for most rocks, QI depends on κ significantly only through the factor sin $(\rho_2/2)$, and it is, in fact, proportional to κ. The sign of QI, i.e. the helicity, is determined exclusively by the factor sin (2β) which does not depend on ν or κ.

3. TWO SIMPLE MODELS

Two simple models of a rough surface are planar arrays of (i) cylindrical wells for which we shall use the word puka (pronounced ' pookah ') which is Hawaiian for ' hole ', and (ii) widely dispersed spheres. A rough surface does, of course, not look like such arrays, but these models contain important similarities to rough surfaces: The puka model mimics the existence of edges and corners, rapid slope changes and indentations, and the sphere–sphere model mimics the non-correlation between the orientation of widely separated surface elements as well as the random orientation of surface elements anywhere on the surface. We shall calculate QI for these models and compare the sign dependence of QI on the scattering angles with the observations.

(i) *The puka model*

Consult Fig. 4. For a set of directions of incidence, $-\hat{e}_1$, and emergence, $\hat{e}_2$, two intermediate rays are possible (all other types of rays involve more than two reflections): Type I ray strikes the bottom of the puka first, is reflected to the wall

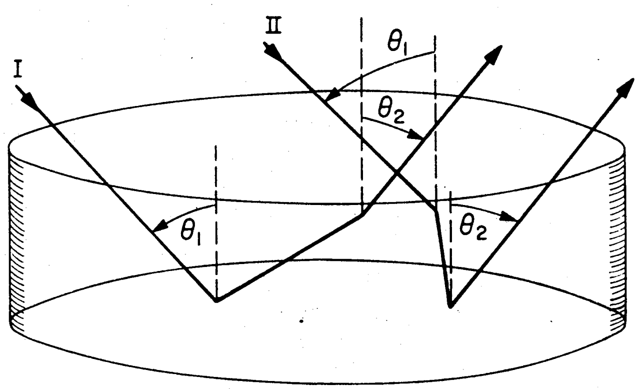

FIG. 4. *Double reflection in a cylindrical well (puka-model): the directions of the incident and emergent rays for type I ray are the same as for type II ray. Note that the inclinations of the intermediate rays are not the same.*

and from there to the observer. Type II ray strikes the wall first, then the bottom, and is then reflected to the observer. We shall assume that for a given set of $\hat{e}_1$ and $\hat{e}_2$ the puka is always flat enough such that the two types of rays are possible, i.e. we neglect shadowing effects. One easily sees from Fig. 4 and Fig. 1, that $\alpha_1 = \theta_1 = \theta = \theta_2$ and $\phi = \pi$ for type I rays whereas $\alpha_2 = \theta_2 = \theta_1 = \pi - \theta$ and $\phi = \phi_2$ for type II rays. The restriction $\theta_2 = \theta_1$ on the emergent ray direction has special relevance to the planetary observations. As Fig. 5 illustrates, rays with $\theta_2 = \theta_1$ originate on the mean meridian NM(S) of that part of the illuminated surface which is visible from the Earth. It is reasonable to assume that the dominant contributions to the polarized flux from either hemisphere come from regions on the planet near this meridian and that therefore the sign of the polarization for an

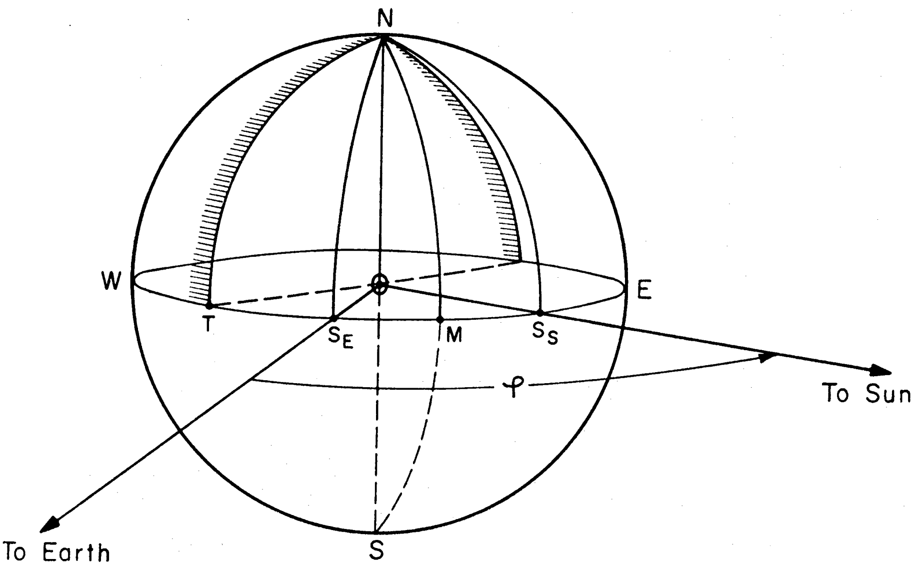

FIG. 5. *$NM(S)$ is the mean meridian of the part of the illuminated surface of a planet which is visible from the Earth. The subsolar and sub-Earth points are S_S and S_E, respectively. $\measuredangle S_EOE = 90° = \measuredangle S_EOW = \measuredangle TOS_S$. One easily shows $\measuredangle S_EOM = \varphi/2 = \measuredangle MOE_S$, and $\measuredangle TOM = \measuredangle MOE = (180° - \varphi)/2$. On the meridian $NM(S)$ we have $\theta_2 = \theta_1$.*

entire hemisphere, north versus south, is the same as that for rays with $\theta_2 = \theta_1$. The observations are summarized in Table I, and they are most simply interpreted as follows. For small and moderate values of $|\varphi|$ the polarization is negative in the Northern Hemisphere (NH) and positive in the Southern Hemisphere (SH) if $\varphi > 0$; the reverse is true if $\varphi < 0$. As the magnitude of the phase increases toward 90°, Q changes sign. This behaviour of the polarization is entirely consistent with observations done in the laboratory on rock samples, as has already been pointed out (Kemp *et al.* 1971c).

TABLE I

Sign of observed planetary polarization

Planet	Observation period	Phase angle φ	Sign of Q NH	Sign of Q SH
Mars*	5/11/71–6/2/71	−40°	positive	negative
Mars*	9/19/71–9/22/71	+31°	negative	positive
Mercury	6/2/71	−67°	negative	positive
Moon	5/30/71	+90°	positive	negative
Moon	11/1/71	+15°	negative	positive

* At a wavelength $\gtrsim$7000 Å where effects due to scattering in Mars' atmosphere are presumably negligible.

We shall now show that the behaviour of the sign of QI for the puka model is consistent with the results of Table I. The two types of rays give a total polarized intensity $QI = (QI)_I + (QI)_{II} = (QI)_I[1 + (QI)_{II}/(QI)_I]$; the Roman subscripts refer to the ray type in question. The sign of $(QI)_I$ is that of $\sin(2\beta)_I$. Furthermore, a little reflection shows that the angles α_1 and α_2 are interchanged between the two types of rays; consequently, we have

$$(QI)_{II}/(QI)_I = [\tan(\rho_1/2)/\tan(\rho_2/2)]_I \sin(2\beta)_{II}/\sin(2\beta)_I.$$

From the definitions of the unit vectors for the two types of rays we find $\sin(2\beta)_{II} = -\sin(2\beta)_{I}$. For $\kappa \ll 1$ we can write

$$\tan(\rho_j/2) \cong (1/2)\tan\rho_j \cong \nu\kappa/(\nu^2 - \sin^2\alpha_j); \quad j = 1, 2.$$

This is true for both rays, of course. Furthermore, for type I rays, we have $\alpha_1 = \theta_1$ and $\sin^2\alpha_2 = 1 - \frac{1}{2}(1+\cos\phi_2)\sin^2\theta_1$; the last relation is obtained from $\cos^2\alpha_2 = -\hat{e}\cdot\hat{e}_2$ (consult Fig. 1). With these results we obtain

$$QI = (QI)_I \frac{\frac{1}{2}(1+\cos\phi_2)\sin^2\theta_1 - \cos^2\theta_1}{\nu^2 - 1 + \frac{1}{2}(1+\cos\phi_2)\sin^2\theta_1}. \tag{11}$$

Furthermore, we find $\mathrm{sign}[\sin(2\beta)_I] = -\mathrm{sign}[\sin\phi_2]$, whence

$$\mathrm{sign}[(QI)_I] = -\mathrm{sign}[\sin\phi_2].$$

This, together with equation (11) gives

$$\mathrm{sign}[QI] = \mathrm{sign}[(2\cos^2\theta_1 - (1+\cos\phi_2)\sin^2\theta_1)\sin\phi_2]. \tag{12}$$

According to equation (12), we have (i) $QI \gtrless 0$ for $\sin\phi_2 \gtrless 0$ if $\theta_1 \ll 1$ which corresponds to small phase angles $\varphi \gtrless 0$; (ii) QI vanishes at zero phase since then $\phi_2 = 0$. For small θ_1 one can obtain a simple formula for QI by expanding QI in an infinite series involving powers of θ_1 and retaining only the first non-vanishing term; the result is

$$QI \cong E_0^2 \frac{c}{\pi\sqrt{2}} \left(\frac{\nu-1}{\nu+1}\right)^2 \frac{\kappa}{\nu^2\sqrt{\nu^2-1}} \theta_1^3 \sqrt{1+\cos\phi_2}\sin\phi_2. \tag{13}$$

Notice the linear dependence of QI on κ. (iii) QI changes sign if θ_1 is increased from a small value and ϕ_2 is kept constant, i.e. if φ is increased, and the change occurs at a phase angle $\varphi = \cos^{-1}[(1+3\cos\phi_2)/(3+\cos\phi_2)]$. The planetary data indicate that a sign change occurs at a phase angle less than 90°, but the precise value of the phase in still uncertain. Furthermore, the data involve an integration over entire hemispheres and therefore a range of values, rather than a single one, of ϕ_2. We therefore conclude that although the puka model does correctly predict a sign change to occur, it is uncertain if the change occurs at the correct phase angle. We shall therefore consult the laboratory results of Pospergelis. In these, the scattering surface has a fixed orientation with respect to the incident and emergent directions, and therefore a set of angles (θ_1, θ_2, ϕ_2) is specified for each data point (actually, Pospergelis uses the angles φ, γ, and ψ). Unfortunately, the laboratory data do not generally satisfy the condition $\theta_1 = \theta_2$ required by the Puka model. In some of the cases where it is nearly satisfied (for instance in the left-hand branch of the data curve in his Fig. 8) the sign of Q does not consistently agree with that predicted by the puka model. (iv) Finally, the polarization as predicted by the puka model has opposite signs in the two planetary hemispheres: this follows from equation (12) and from the fact that corresponding areas in the two hemispheres have equal θ_1 but opposite sign of $\sin\phi_2$.

(ii) *The sphere–sphere model*

This model is not restricted to rays for which $\theta_2 = \theta_1$, and therefore a more detailed comparison with Pospergelis' data is possible. The model allows all directions of incident and emergent rays but restricts the intermediate rays (in a direction $\hat{e}$; consult Fig. 1) to lie in the (x, y)-plane. The situation is illustrated in

Fig. 6. The restriction $\theta = \pi/2$ results from the assumption that the scattering elements ΔS_1 and ΔS_2 are far apart, or, in terms of this model, that the sphere diameter is much smaller than the separation distance. The remaining small inclination of intermediate rays to the (x, y)-plane will be neglected.

Consult Fig. 6. Radiation incident in the (x, z)-plane is reflected by sphere 1 located at the origin to another sphere located in the (x, y)-plane at the point $(x = \cos\phi, y = \sin\phi)$. Sphere 2 reflects the radiation toward the observer. Let

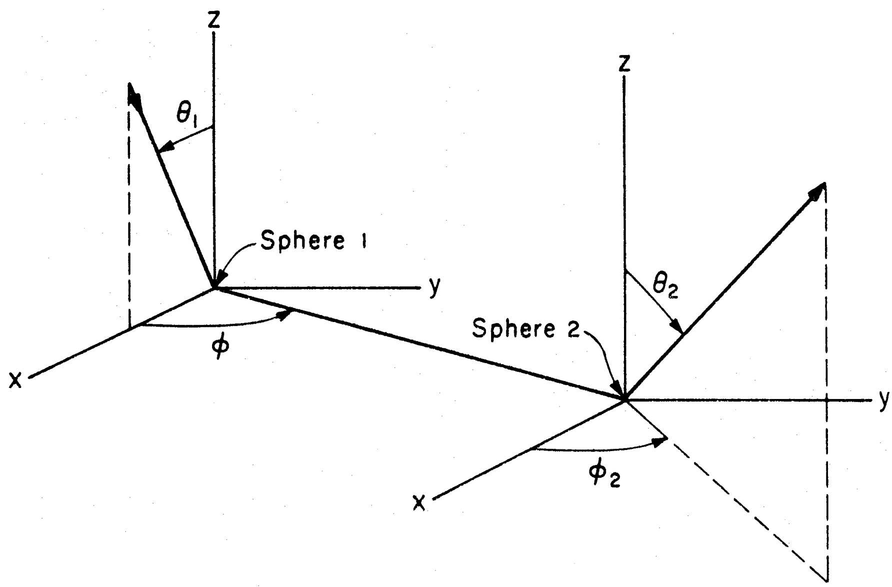

FIG. 6. *Reflection between spheres in a plane (sphere–sphere model). The ray from sphere* 1 *to sphere* 2 *is assumed to have negligible inclination to the* (x, y)*-plane.*

the contribution of the circularly polarized radiation by these two spheres be equal to $(QI)_\phi$ obtained by setting $\theta = \pi/2$ in equations (10). As seen from the 'central' sphere l located at the origin (each sphere in turn can be regarded as 'central') there are secondary spheres located at all azimuths ϕ, and if we assume them to be a common mean distance from sphere 1, they all receive an equal amount of radiation from sphere 1. The total polarized intensity emerging from an array of spheres is therefore proportional to

$$QI = \frac{1}{2\pi}\int_0^{2\pi} (QI)_\phi \, d\phi.$$

We shall first investigate the properties of this quantity at small θ_1 and θ_2 which implies also small phase angles. We expand $(QI)_\phi$ in an infinite series involving powers of θ_1 and θ_2 and integrate over ϕ. The resulting series is rapidly converging, and the first non-vanishing term gives a simple formula for QI

$$QI \cong -4E_0{}^2c\left(\frac{\nu^2-1}{t+1}\right)^4 \frac{\nu\kappa}{t^2(t+\nu^2)^2}\,\theta_1\theta_2\sin\phi_2 \tag{14}$$

where $t = \sqrt{2\nu^2-1}$. The sign of QI disagrees with observations, and the model does not fare better at other values of θ_1 and θ_2 as long as the phase angle is small. Notice however that it does correctly predict QI to vanish at zero phase (for which $\phi_2 = 0$) and a sign change to occur as the phase goes from positive values to negative values, and vice versa. This is true regardless of the values of θ_1 and θ_2 because for $\phi_1 = 0$ (as well as for $\phi_2 = \pi$) all quantities in equation (10) except $\sin(2\beta)$ are even functions of ϕ; therefore the integral over ϕ vanishes for $\phi_2 = 0$.

For a comparison with Pospergelis' data on volcanic rocks we present values of QI computed from this model for an assumed value of the refractive index $n = 1{\cdot}5+0{\cdot}1i$. Numerous calculations with other values of n have shown that the general behaviour, particularly the sign dependence, of QI shown for $n = 1{\cdot}5+0{\cdot}1i$ is typical. The reader may be reminded again that for $\kappa \ll 1$, $QI \propto \kappa$. The relationship between Pospergelis' angles φ, γ, and ψ and the angles θ_1, θ_2, and ϕ_2 (which are much more convenient for analysis) is (see Fig. 7)

$$\left.\begin{aligned} \cos\theta_1 &= \cos\gamma\cos(\varphi-\psi) \\ \cos\theta_2 &= \cos\gamma\cos\psi \\ \tan\phi_2 &= \frac{-\sin\varphi}{\cos\psi\cos(\varphi-\psi)\sin\gamma} \\ \mathrm{sign}[\sin\phi_2] &= -\mathrm{sign}[\sin\gamma]. \end{aligned}\right\} \tag{15}$$

The results are shown in Figs 8 and 9; the experimental data are taken from the Figs 9 and 10 of Pospergelis (1969); they are the observed fractional circular polarization in per cent, as a function of φ, γ, and ψ. The theoretical data are values of QI (computed from the sphere–sphere model) multiplied by an arbitrary constant factor which was chosen such as to match the curve for $\varphi = 80°$ in Fig. 9 reasonably well with the laboratory data. Since for the assumed planar array of spheres the singly scattered radiation is the same in any direction and is much greater than the multiply scattered radiation, the fractional circular polarization is equal to QI multiplied by a constant. This justifies the direct comparison between QI and the

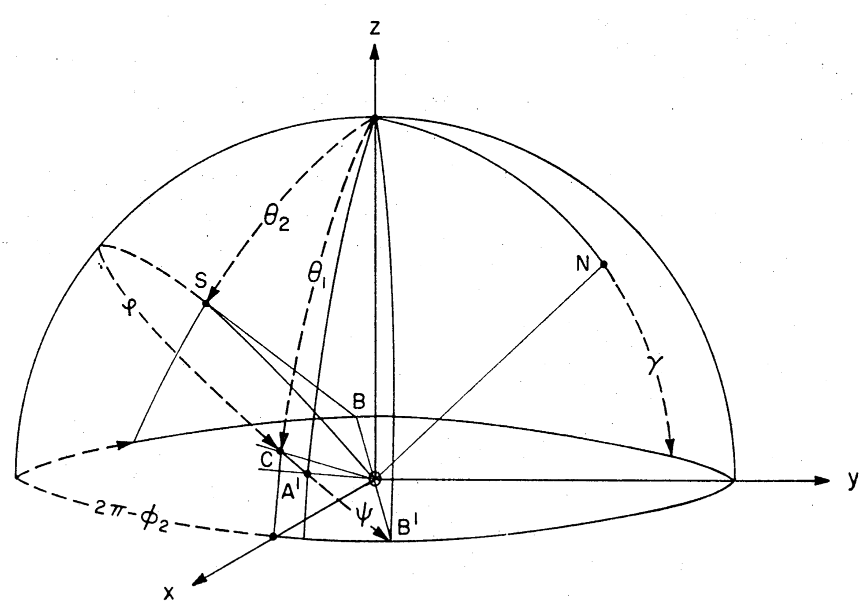

FIG. 7. *Relationship between Pospergelis' angles φ, γ, and ψ and the angles θ_1, θ_2 and ϕ_2. The direction of incidence is from C to O, that of emergence from O to S, and the phase angle, φ, is measured in the scattering plane which contains OS and OC. The normal to this plane is ON and it makes an angle γ with the* (x, y)*-plane. The scattering plane intersects the* (x, y)*-plane on the line BB′. The angle $\measuredangle SOA'$ is a right angle so that OS, OA′ and ON define a right-handed coordinate system. The angle ψ is measured in the scattering plane, from A′ to B′.*

polarization data of Pospergelis and the use of an arbitrary constant. As the reader will determine for himself, the model is on the whole successful in predicting the sign of QI and its dependence on the various angles—except for small phase angles as is already known for small θ_1 and θ_2. For instance, Q decreases from large positive values at $\psi = -45°$ to negative values at $\psi = 45°$ (Fig. 8). The larger values of $|Q|$ generally belong to larger values of φ. Furthermore, $|Q|$ increases with increasing value of γ, and $Q = 0$ for $\gamma = 0$ (Fig. 9). However,

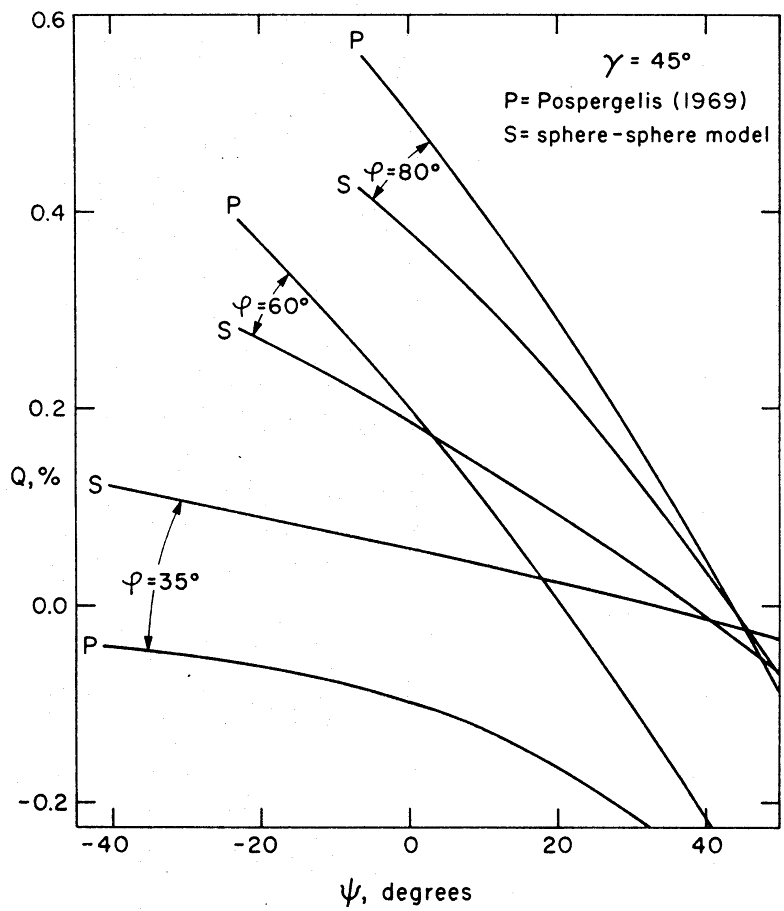

FIG. 8. *Comparison between results for the fractional circular polarization* Q *obtained from the sphere–sphere model, and Pospergelis' laboratory data on volcanic rock. The comparison concerns only the sign dependence of* Q, *and the theoretical values have an arbitrary scale factor.*

in conflict with the observations, this model predicts zero polarization for $\gamma = 90°$ (as does the puka model; in both cases this occurs because $\sin(2\beta) = 0$). Observations on the Moon seem to confirm the laboratory result (Kemp 1971, private communication). This seems a serious deficiency in the model; however, for this extreme case of illumination of the scattering surface shadowing effects surely become important, and these have not been incorporated into our model.

(iii) *Basic anti-symmetry properties of the models*

We have seen that for both models the *sign* of the polarization has the anti-symmetric properties listed in the Introduction. We must still show that the magnitude is unaffected by the changes $\gamma \rightarrow -\gamma$, or $\varphi \rightarrow -\varphi$, i.e. we must show that $QI(\gamma) = -QI(-\gamma)$ and $QI(\varphi) = -QI(-\varphi)$. This is most simply done by consulting Fig. 3. Let the great circle on which $\hat{e}_1$ and $\hat{e}_2$ lie be the equator of the unit sphere whose radii are $\hat{e}_1$, $\hat{e}$, and $\hat{e}_2$. Each $\hat{e}$ (for a given set $\hat{e}_1$ and $\hat{e}_2$) defines a set of values of α_1, α_2, and β, and consequently a value of QI. Between corresponding points in the two hemispheres then only the sign of β is different and, consequently,

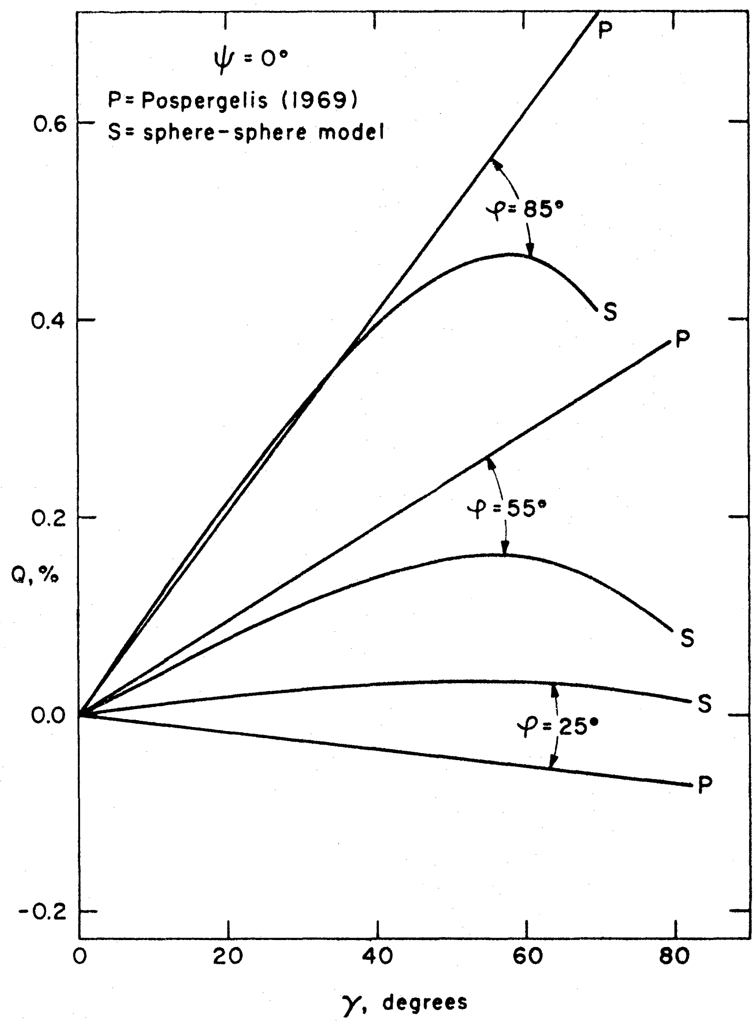

FIG. 9. *Comparison between results for the fractional circular polarization* Q *obtained from the sphere–sphere model, and Pospergelis' laboratory data on volcanic rock. The comparison concerns only the sign dependence of* Q, *and the theoretical values contain an arbitrary scale factor.* (*The factor is the same for all curves and the same factor used in Fig.* 8).

they have polarization of equal magnitude but opposite sign. Now in case of the planets, corresponding points in the two hemispheres have $\gamma_{NH} = -\gamma_{SH}$ which, by equation (15), implies $(\phi_2)_{NH} = -(\phi_2)_{SH}$. For the puka model, the change $\phi_2 \to -\phi_2$ also implies $\phi \to -\phi$. But, as Fig. 3 shows, such a simultaneous change affects only the sign of β but not the angles α_1 and α_2; therefore, it affects only the sign of QI. This proves the assertion $QI(\gamma) = -QI(-\gamma)$ for this model. For the sphere–sphere model we use the antisymmetry property

$$QI(-\phi, -\phi_2) = -QI(\phi, \phi_2)$$

as follows:

$$\int_0^{2\pi} QI(\phi, \phi_2)\, d\phi = -\int_0^{2\pi} QI(-\phi, -\phi_2)\, d\phi = \int_0^{-2\pi} QI(-\phi, -\phi_2)\, d(-\phi)$$

$$= \int_0^{-2\pi} QI(f, -\phi_2)\, df = -\int_0^{2\pi} QI(f, -\phi_2)\, df = -\int_0^{2\pi} QI(\phi, -\phi_2)\, d\phi.$$

To prove that the planetary polarization changes sign but not magnitude with a sign change in the phase angle we note that reversing the position of Earth and Sun (E and S in Fig. 5) has the same effect as interchanging N and S which in turn has the effect of substituting $\gamma \rightarrow -\gamma$ for each surface element, and we have already shown that for the puka model and the sphere–sphere model such a change affects only the sign of QI.

We would like to point out that not all models have these antisymmetry properties though $QI(-\phi, -\phi_2) = -QI(\phi, \phi_2)$ is always satisfied, where QI is given by equation (10). For instance, changing the sign of φ implies interchanging $\hat{e}_1$ and $\hat{e}_2$ which has the effect of changing the sign of $\hat{s}_1 \cdot \hat{p}_2$; consequently, $\sin(2\beta)$ changes sign. It also implies interchanging α_1 and α_2, but QI given by equation (10) is not symmetric with respect to these angles and consequently changes in magnitude. Finally, it needs to be said that the basic anti-symmetries are not implied by Pospergelis' data although they are not in conflict with them.

4. DISCUSSION

In the Introduction we listed several requirements which a successful model of a mechanism causing the observed circular polarization must satisfy. We assumed this mechanism to be double reflection on rough, absorbing surfaces. Each of the two models presented in this paper satisfies some of these requirements. Both have the basic anti-symmetric properties $Q(\gamma) = -Q(-\gamma)$ and $Q(\varphi) = -Q(-\varphi)$ implied by the planetary observations. In addition, the puka model predicts the sign of the polarization at small phase angles and a sign change with increasing phase. The sphere–sphere model predicts the sign of the polarization at moderate and large phase angles ($\varphi \gtrsim 40°$) and it gives the qualitatively correct dependence of Q on the angles γ and ψ, except for small φ and large γ. The complementary nature of the models is not unexpected since they also complement each other in their description of a rough surface, as we have indicated at the beginning of the last section. E.g. the sphere–sphere model assumes that the successive reflections occur between widely separated surface elements, ΔS_1 and ΔS_2, whereas in the puka model the elements are neighbouring. In the case of near-normal incidence and emergence $[\theta_1 + \theta_2 \ll 1]$ the puka model is clearly more realistic because the rays essentially see a cavity-like geometry. For larger angles, on the other hand, they see protrusions. It comes as no surprise, therefore, that the puka model is successful for near-normal incidence, and the other model is not. Finally, when incidence and emergence are nearly grazing to the surface $[\gamma \approx 90°]$, much of the surface is inaccessible because of mutual shadowing. We did not consider shadowing, and, indeed, both models fail for these conditions.

In summary, we feel that the extent of success of the two very simple models and their complementary nature indicates that the observations can be explained in terms of double (or multiple) reflection within an absorbing rough surface, assuming only geometrical optics and Fresnel's laws and using a model for the rough surface which incorporates elements from the puka and sphere–sphere model. Several ways of modifying the models suggest themselves: for instance, to relieve the constraint $\theta_1 = \theta_2$ of the puka model the wells can be tilted with respect to the surface normal (the z-axis), or the wall or bottom can be made to be diffusely reflecting, or the shape of the well can be changed. Similarly, the sphere–sphere model can be modified by assuming a distribution of distances perhaps not much larger than the sphere diameter. A wholly different approach to the problem is to design, on the basis of known obvious physical features of surfaces, distribution functions describing the orientations of individual surface elements. From these one then derives the probabilities of intermediate rays, for specified directions of incidence, $\hat{e}_1$, and emergence, $\hat{e}_2$. The probabilities enter as weighting factors multiplying QI (as given by equation (10)) and the resulting quantity is integrated over all angles. Although this approach is perhaps more elegant than constructing simple models as was done in this paper, one may gain little insight into the problem. A third approach is to trace rays within a specified geometry representing a rough surface; from a mathematical as well as physical standpoint, this is perhaps the least satisfying method.

We would like to emphasize that although the most important features of the observed polarization seem to be explained in terms of double reflection as interpreted by simple models of rough surfaces, and using geometrical optics, we recognize the possibility that refracted and diffracted rays may play an important role in some cases and that consideration of these rays may remove some of the difficulties with our models.

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