Knuthian Drawings of Series-Parallel Flowcharts

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(Full paper)

Overview

Inspired by a classic paper by Knuth [7], we investigate the problem of drawing flowcharts of loop-free algorithms.

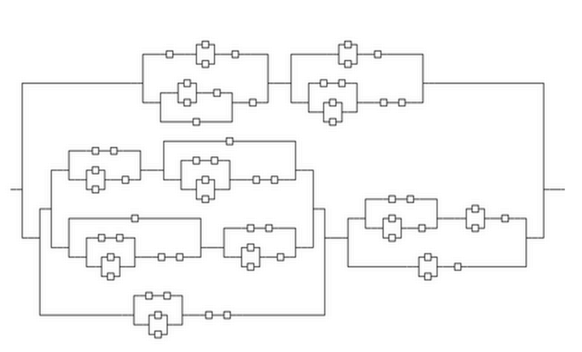


Figure 1: A degree-three series-parallel flowchart

Degree-3 Series-parallel digraph flowcharts [1,2,6]:

- Branching factor of two.
- Orthogonal edges.

Our algorithm improves both the area and aspect ratio, compared to two previous drawing algorithms.

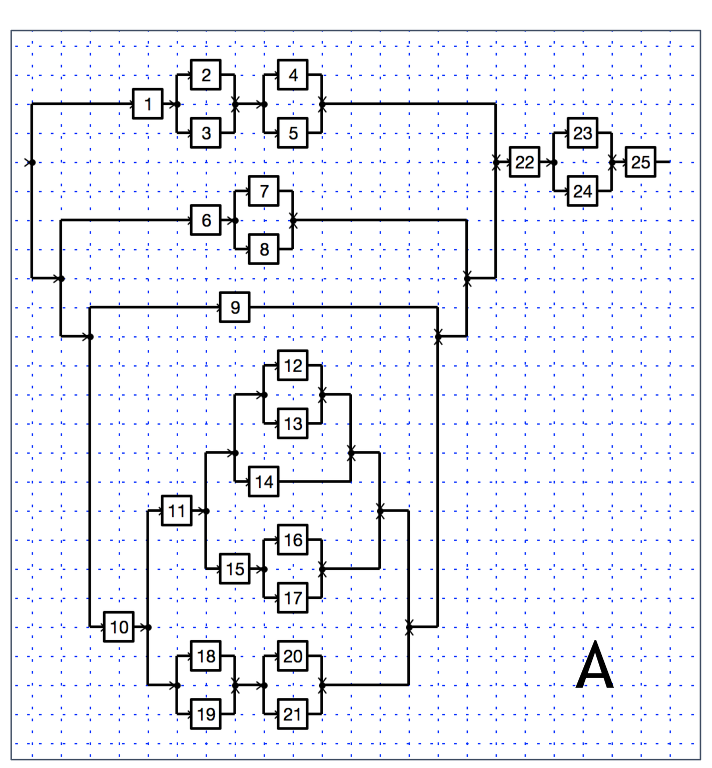
Based on Knuth's framework [7], we say that a directed orthogonal drawing is **Knuthian** if no node has an incident edge locally pointing up unless that node is a junction having in-degree 2 and out-degree 1. That is, no non-junction node has an in-coming edge into its bottom or an out-going edge from its top. Knuthian drawings are related to upward and quasi-upward orthogonal drawings [3,4,5].

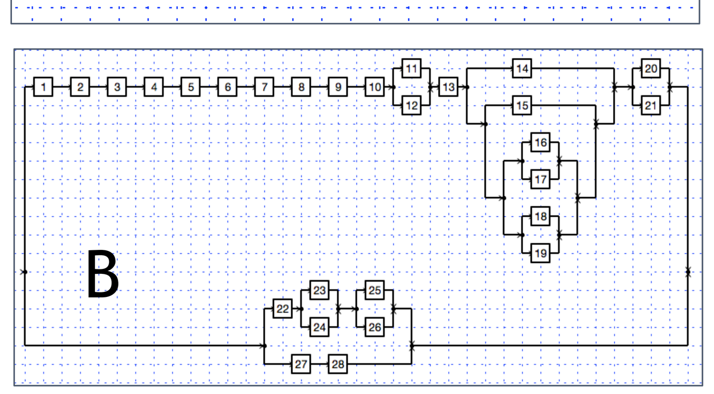
Two classic styles for series-parallel graphs (see also [1,2,6]):

1. Knuth's drawing style [7]: place all nodes on the same vertical line and route edges on the right side of this line.
2. Recursively draw parallel components side-by-side and series components one over the other.

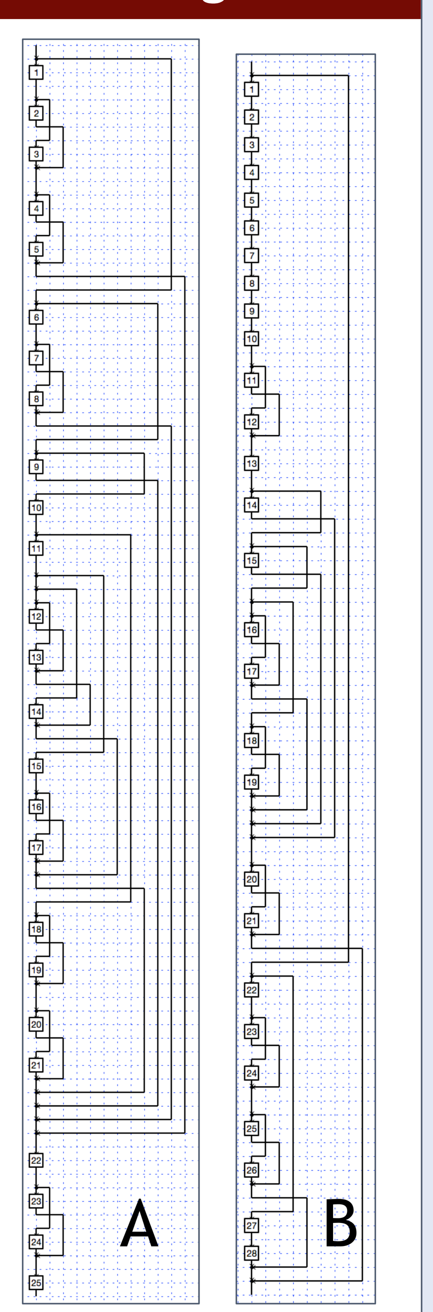
Our drawing algorithms show that it is possible to produce Knuthian drawings of degree-3 series-parallel digraphs with good aspect ratios and small numbers of bends per edge.

Classic-Style Series-Parallel Drawings





Standard drawings (the vertices are labeled with numbers; the standard drawings are Knuthian if the drawing is rotated 90° clockwise)



Drawings from Knuth's algorithm

Single-Row Theorems/Figures

Theorem 1: A degree-three series-parallel graph with n vertices has a Knuthian drawing with width $O(n)$ and height $O(\log n)$, such that each edge has at most two bends and the total number of bends is at most $1.25n$.

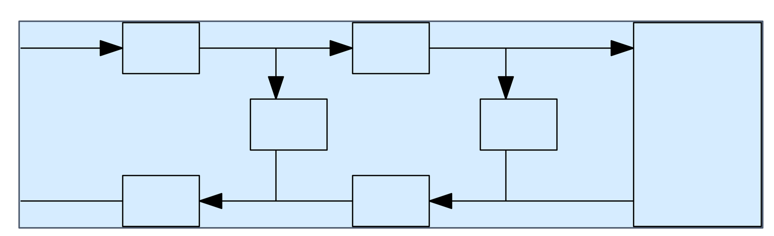


Figure 2: Our main drawing pattern, flowing from upper left to lower left.

For each of the drawings below, s_1 and s_2 represent subgraphs of greater than or equal to zero nodes in series and p_1, p_2, p_3, p_4, and p_5 represent any type of series-parallel subgraph (either broad, pinched [see Figure 3], or nodes only in series)

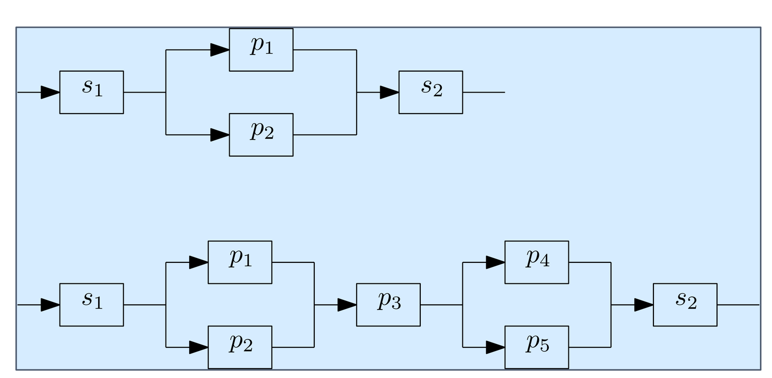


Figure 3: Two types of series-parallel graphs: *broad* (top) and *pinched* (bottom). The branches from the first decision point either recombine at the final junction point, or they meet sooner.

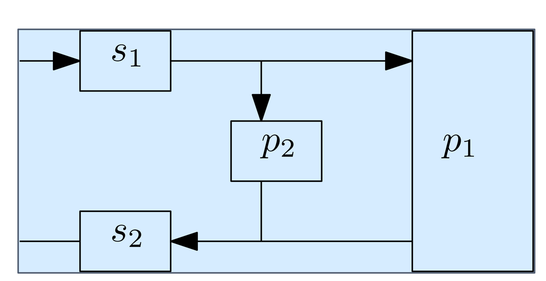


Figure 4: Recursive drawing for broad series-parallel graphs with width $O(n)$ and height $O(\log n)$.

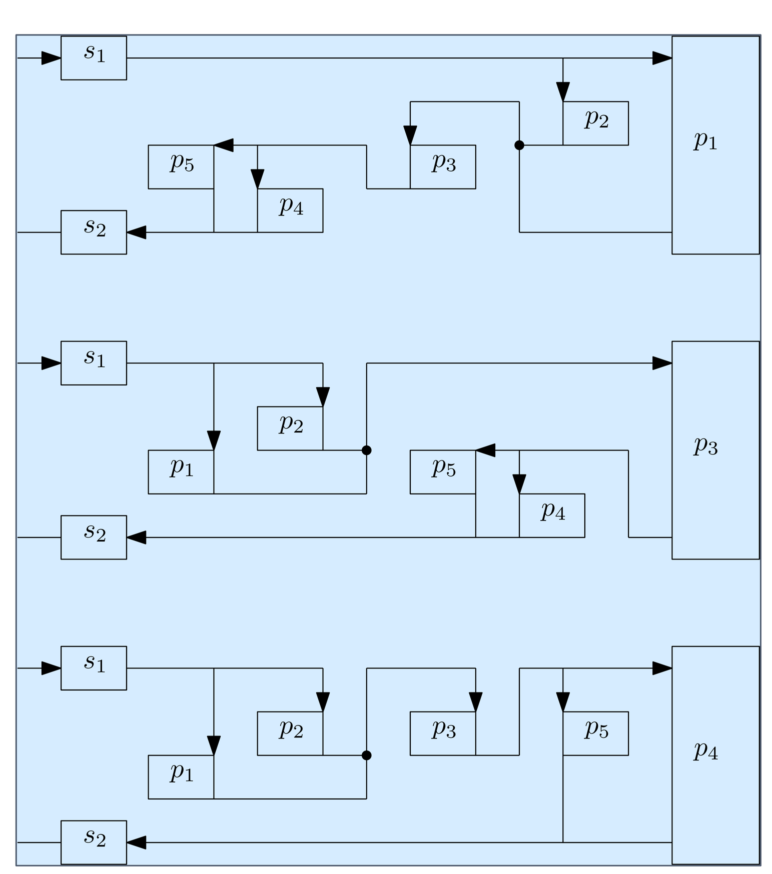


Figure 5: Recursive drawings of pinched series-parallel graphs. We always choose the largest parallel section to go on the right. The bold node is an example of a junction node that takes advantage of Knuthian-style drawings.

The total width is obviously O(n), since each node takes up a constant width.

Since the inner recursive sections are smaller than the one on the right, we can show that they can take up at most a constant fraction of the graph. Each new level of recursion adds a constant height to our graph, so the total height is O($\log n$).

Fixed-Width Theorems/Figures

Theorem 2: A degree-three series-parallel graph with n nodes has a Knuthian drawing that can be produced in linear time to have width $O(A + \log n)$ and height $O((n/A) \log n)$, for any given $A \geq \log n$; hence, the area is $O(n \log n)$. The total number of bends is at most $4.5n + o(n)$.

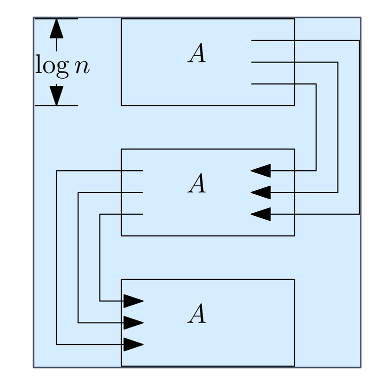


Figure 6: We wrap our single-row drawings to satisfy a fixed-width of A nodes by ordering our nodes by their x-coordinate, breaking ties by their y-coordinate, then dividing the list into slabs with exactly A nodes.

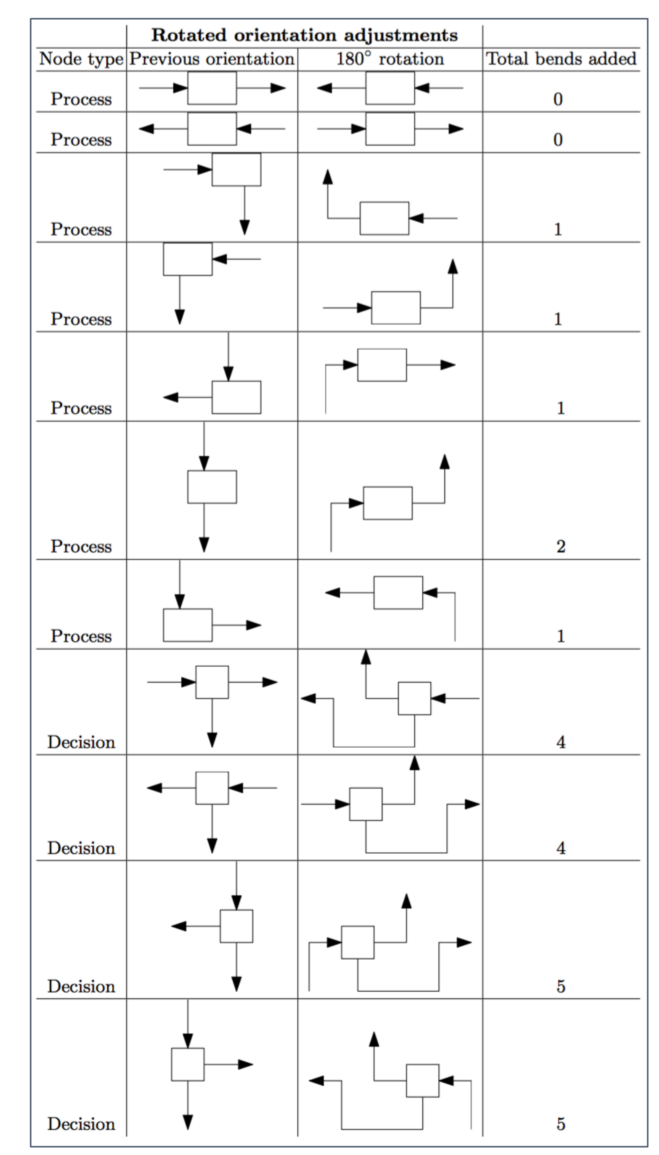


Figure 7: When we wrap our single-row drawing as shown in Figure 6, every other section is rotated 180°. To maintain the Knuthian-drawing property, we must add bends to the incoming and outgoing edges at some of our decision and process nodes. These transformations only increase the width of our graph by a small constant factor. The width of each slab's transformed section is $O(A)$ plus $O(\log n)$ for the extended edges to maintain the overall $O(A + \log n)$ width.

Single-Row Drawings

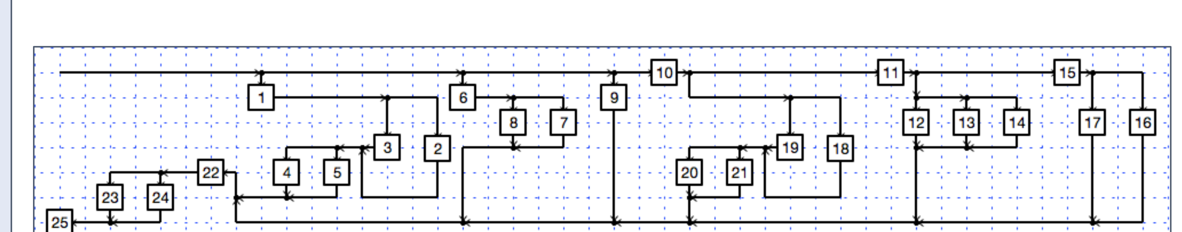


Figure 8: A Knuthian drawing using our $O(n \log n)$-area drawing algorithm of the same series-parallel graph as drawing A in the classic-style series-parallel drawings section.

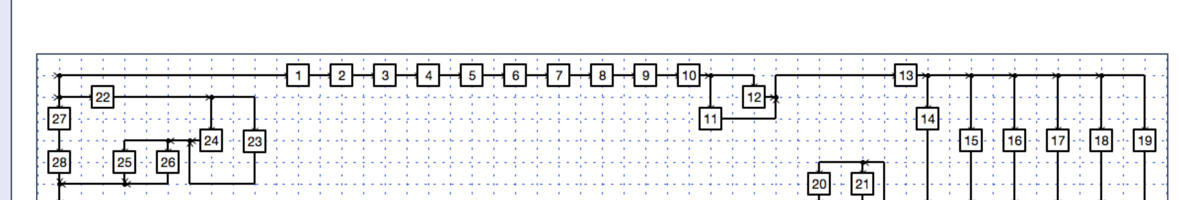
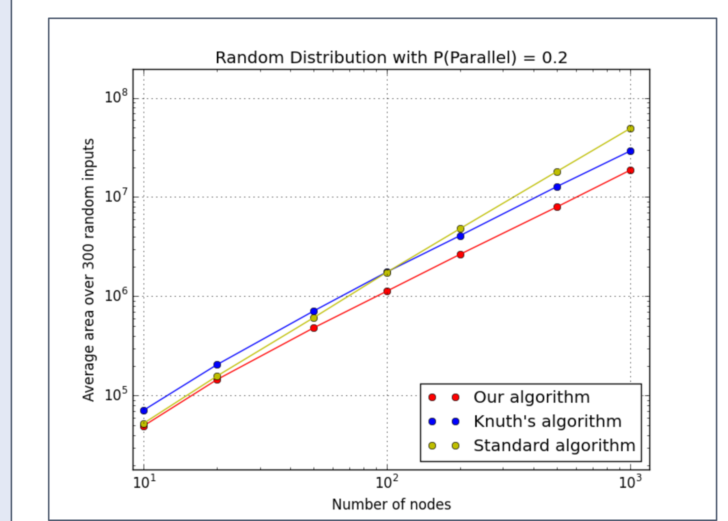
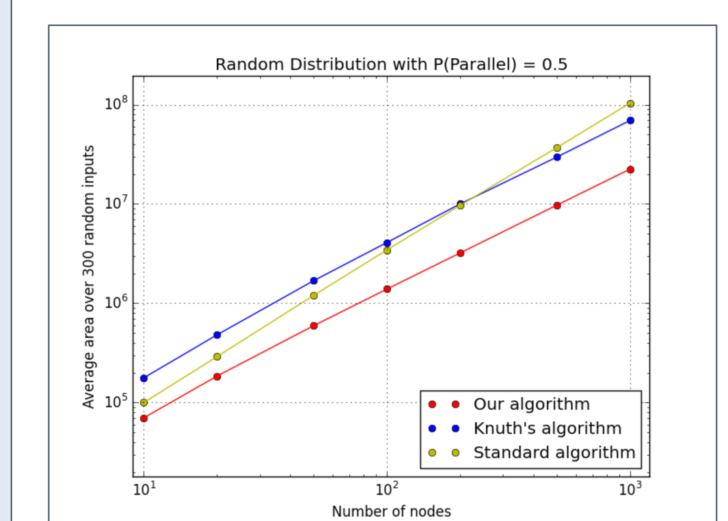
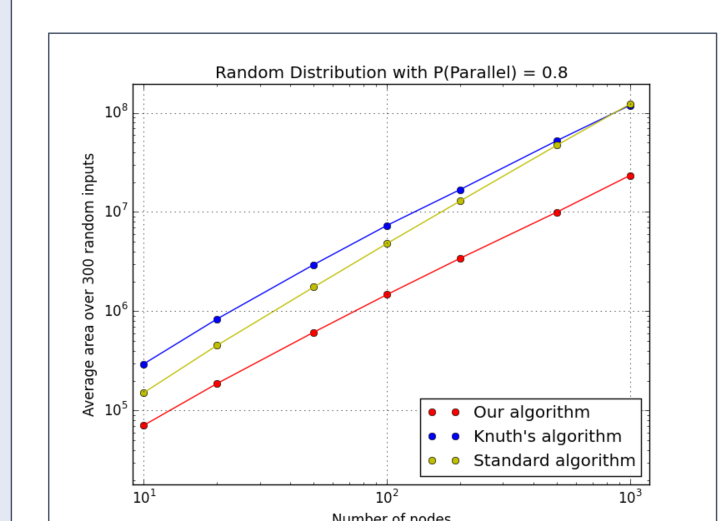


Figure 9: A Knuthian drawing using our $O(n \log n)$-area drawing algorithm of the same series-parallel graph as drawing B in the classic-style series-parallel drawings section.

Experiments







Figures 10, 11, 12:
To test our algorithm, Knuth's algorithm, and the standard algorithm, we used a random distribution to create random binary series-parallel decomposition trees.

The distribution is defined by two parameters: n, the number of nodes, and p, the probability an internal node is a parallel composition. We choose a root node and set it to a parallel composition with probability p and series composition with probability $1 - p$. The size of the left subtree has size nx, where x is sampled from a uniform distribution from 0 to 1 and the right subtree has size $n - nx$. This process is recursively repeated for the left and right subtrees.

We performed experiments by running 300 random inputs through all three algorithms for graphs of 10, 20, 50, 100, 200, 500, and 1000 nodes.

References

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Acknowledgments

This research was supported in part by the NSF under grants 1011840 and 1228639, and DARPA under agreement no. AFRL FA8750-15-2-0092. The views expressed are those of the authors and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

Full paper is available at http://arxiv.org/abs/1508.03931.

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