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Classical Boson Wave Excitation Model and Multiplicity Distribution in High-Energy Hadron Collisions

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The multiplicity distribution is obtained on the assumption that the final boson state in the high-energy multiple production is described by a mixed state corresponding to the presence of a certain chaotic field (like thermal fluctuation) around a coherent state. It is shown that the multiplicity distribution derived from the model fairly well reproduces recent experimental data and gives the KNO scaling in the high-energy limit.

§ 1. Introduction

Many authors have presented rather mathematical models or formulas for the multiplicity distribution in multiple production processes on the basis of purely phenomenological analyses of recent experiments. At the next stage of the research work, it seems that we ought to find physical models favourable to the production mechanism. In this paper we propose, as one possible trial, a classical boson wave excitation model in which the final boson state in multiple production processes is described by a mixed state corresponding to the presence of a chaotic field around a coherent state. The model gives us a multiplicity distribution consistent with experiments, as will be seen later.

Here we must briefly mention a preliminary work given by one of the present authors (M.N.) ten years ago.[1] His motivation was first to search for a suitable representation to describe the final boson state in a compact form, because the large observed dispersion of the multiplicity distribution suggests us that no ordinary representation with a definite particle number is a nice candidate to describe the final boson state as mentioned above. He thought that bosons would be produced through emission of a sort of classical boson wave in the high-energy collision just like emission of classical electromagnetic waves, and that dynamics governing the multiple production process would be well described in terms of the classical boson wave. There was a naive anticipation that the wave or field nature of meson would be able to be first recognized in the multiple production phenomena. From this point of view it was also emphasized that we can discuss the characteristic nature of the boson wave through the dependence of the ratio $D/\langle n\rangle$ on $\langle n\rangle$. As an extreme case in which there is no correlation among bosons, the boson wave state is described by the so-called "coherent state"—the simplest form of the classical boson wave excitation model.[1),2)] As is well known, the coherent state gives us the multiplicity distribution of the Poisson type and vanishing correlation parameters $f_2=0, f_3=0, \cdots$. In this sense the coherent state may be used as a reference, but it is, of course, not consistent with recent experiments showing the presence of nonzero correlation parameters. Therefore, our task in this paper is to modify the coherent state so as to get nonzero correlation parameters along the basic line of the classical boson wave excitation model.

To do this we must first remark that the coherent state is directly connected to a c-number source relevant to the boson field. In fact, the coherent state is the eigenstate of the boson annihilation operator belonging to its eigenvalue which is proportional to a Fourier component of the c-number source function. Since the c-number source function is to be regarded as an approximation of the original source operator corresponding to its systematic or coarse-grained variation, we can proceed to the next step of approximation by taking account of derivations of the source operator from the c-number source function. As an extreme approximation we may replace such deviations with a c-number random source function. The c-number random source function ought to describe thermal fluctuations of the hot matter produced by high-energy collision. Thus we are led to the classical boson wave excitation model in which the final boson state is described by a mixed state corresponding to the presence of a chaotic field around a coherent state. A general theory along this line of thought has already been formulated for electromagnetic waves in the LASER problem.[3)]

In § 2 we give general formulas for the multiplicity distribution function and other quantities in the one-mode case on the basis of the present model. In § 3 we introduce a phenomenological assumption about free parameters on the basis of recent experiments and show the KNO scaling in the high energy limit. Section 4 is devoted to a many-mode case and a comparison with experiment. Concluding remarks are given in § 5.

§ 2. General formulas in the one-mode case

For a while we restrict ourselves to a case in which one kind of bosons in one mode are produced in multiple production processes. Extension to a many-mode case is straightforward as will be seen later.

If there is no correlation among produced bosons, the final boson state is described by the coherent state:[1),2)]

$$|\alpha\rangle = \exp(\alpha a^\dagger - \alpha^* a)|0\rangle$$
$$= \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \qquad (1)$$

where a and $a^\dagger$ are, respectively, annihilation and creation operators of the mode concerned and α stands for the eigenvalue of a —we have $a|\alpha\rangle = \alpha|\alpha\rangle$. Here

$|n\rangle$ is the eigenstate of a boson number belonging to n. The probability of finding n bosons in $|\alpha\rangle$ is given by the Poisson distribution

$$P_c(n, \alpha) = |\langle n|\alpha\rangle|^2 = \frac{|\alpha|^{2n}}{n!} \exp(-|\alpha|^2), \tag{2}$$

where $|\alpha|^2 = \langle n\rangle$ is the average multiplicity in $|\alpha\rangle$.

The mixed state proposed at the end of the preceding section is described by the density matrix

$$\rho = \int |\alpha\rangle w(\alpha) \langle\alpha| d^2\alpha\,, \tag{3}$$

where $d^2\alpha = d(\mathrm{Re}\,\alpha)\,d(\mathrm{Im}\,\alpha)$ and $\int w(\alpha)\,d^2\alpha = 1$. The multiplicity distribution function in the mixed state is given by

$$P(n) = \langle n|\rho|n\rangle = \int w(\alpha)\, \frac{|\alpha|^{2n}}{n!} \exp(-|\alpha|^2)\, d^2\alpha\,. \tag{4}$$

If ρ describes the presence of a chaotic field (like thermal fluctuation) around a coherent state $|\zeta\rangle$, then we can naturally put

$$w(\alpha) = \frac{1}{A\pi} \exp\left(-\frac{1}{A}|\alpha - \zeta|^2\right) \tag{5}$$

on the basis of the central limit theorem, where A and ζ are free parameters to be adjusted later. Substituting (5) into (4) we get

$$\begin{aligned} P(n) &= \frac{1}{An!} \exp\left(-\frac{|\zeta|^2}{A}\right) \int |\alpha|^{2n} \exp\left(-\left(1+\frac{1}{A}\right)|\alpha|^2\right) I_0\left(\frac{2|\zeta||\alpha|}{A}\right) d|\alpha|^2 \\ &= \frac{A^n}{(1+A)^{n+1}} \exp\left(-\frac{|\zeta|^2}{A}\right) F\left(n+1, 1, \frac{|\zeta|^2}{A(1+A)}\right), \end{aligned} \tag{6}$$

where F is the confluent hypergeometric function. The usual generating function $Q(\lambda)$ is defined by

$$Q(\lambda) \equiv \sum_{n=0}^{\infty} (1-\lambda)^n P(n) \tag{7}$$

or

$$P(n) \equiv (-1)^n \frac{1}{n!} \left[\frac{\partial^n}{\partial \lambda^n} Q(\lambda)\right]_{\lambda=1}. \tag{8}$$

Equation (7) gives

$$F^{(k)} \equiv \langle n(n-1)\cdots(n-k+1)\rangle = (-1)^k \left[\frac{\partial^k}{\partial \lambda^k} Q(\lambda)\right]_{\lambda=0}. \tag{9}$$

Putting (4) and (5) into (7), we get

$$Q(\lambda) = \int \exp(-\lambda|\alpha|^2)\, w(\alpha)\, d^2\alpha = \frac{1}{1+\lambda A} \exp\left(-\frac{\lambda|\zeta|^2}{1+\lambda A}\right), \tag{10}$$

which yields

$$F^{(1)}=f_1=\langle n\rangle=|\zeta|^2+A\,, \tag{11}$$

$$F^{(2)}=\langle n(n-1)\rangle=|\zeta|^4+4A|\zeta|^2+2A^2, \tag{12}$$

$$\cdots$$

$$F^{(k)}=(|\zeta|^2+(2k-1)A)F^{(k-1)}-(k-1)^2A^2F^{(k-2)}$$
$$=k!\,A^kL_k\left(-\frac{|\zeta|^2}{A}\right), \tag{13}$$ *)

where L_k denotes the normalized Laguerre polynomial defined by

$$L_k(x)=\frac{1}{k!}e^x\frac{d^k}{dx^k}(x^ke^{-x})\,.$$

For the sake of later convenience, it is the best to choose a new set of free parameters $\langle n\rangle$ and $|\zeta|^2$ instead of $|\zeta|^2$ and A. From (11), (12) and (13) we easily obtain a dispersion squared D^2 and the second and higher correlation parameters $f_2, f_3, \cdots$ as follows:

$$D^2=\langle(n-\langle n\rangle)^2\rangle=\langle n\rangle^2-|\zeta|^4+\langle n\rangle\,, \tag{14}$$

$$f_2=F^{(2)}-f_1^2=\langle n\rangle^2-|\zeta|^4, \tag{15}$$

$$f_3=F^{(3)}-3f_1f_2-f_1^3=2\langle n\rangle^3-6\langle n\rangle|\zeta|^4+4|\zeta|^6, \tag{16}$$

$$f_4=F^{(4)}-4f_1f_3-6f_1^2f_2-6f_2^2-f_1^4$$
$$=3\langle n\rangle^4-30\langle n\rangle^2|\zeta|^4+20\langle n\rangle|\zeta|^6-21|\zeta|^8 \tag{17}$$
$$\cdots.$$

Equations (11) and (14) tell us that $|\zeta|^2$ increases from D to $\langle n\rangle$ and D^2 decreases from $\langle n\rangle^2+\langle n\rangle$ to $\langle n\rangle$ for the fixed $\langle n\rangle$. In one limit $|\zeta|^2=\langle n\rangle$ and $D^2=\langle n\rangle$, the mixed state is reduced to a pure coherent state $|\zeta\rangle$ in which we have the Poisson distribution again. In the other limit $|\zeta|^2=0$ or $\langle n\rangle=A$ and $D^2=\langle n\rangle^2+\langle n\rangle$, we have a geometric distribution[4)]

$$P(n)=\frac{A^n}{(1+A)^{n+1}}\,. \tag{18}$$

Therefore, the distribution function (6) smoothly varies between the geometric and Poisson distributions, according to a continuous change in $|\zeta|^2$ at the interval $(0, \langle n\rangle)$.

§ 3. Phenomenological assumption and KNO scaling

We cannot determine the dependence of $|\zeta|^2$ on energy or $\langle n\rangle$ without re-

*) Note that $k\geq 2$ and $F^{(0)}\equiv 1$.

sort to further theoretical assumptions. However, we can proceed with our arguments on the basis of a phenomenological assumption suggested by an experimental fact that $R=\langle n\rangle/D$ becomes nearly constant and $(1-R^{-2})\langle n\rangle\gg1$ at higher energies. Equation (14) gives

$$|\zeta|^2=(1-R^{-2})^{1/2}\langle n\rangle\left(1+\frac{1}{(1-R^{-2})\langle n\rangle}\right)^{1/2}$$

$$\simeq(1-R^{-2})^{1/2}\langle n\rangle+\frac{1}{2}(1-R^{-2})^{1/2}. \tag{19}$$

Thus, we can safely assume that

$$|\zeta|^2=(1-p)\langle n\rangle+q \tag{20}$$

and correspondingly, from (11),

$$A=p\langle n\rangle-q\,, \tag{21}$$

where p and q are constants independent of energy or $\langle n\rangle$.

Substituting (20) and (21) into (6), we obtain

$$P(n)=\frac{(p\langle n\rangle-q)^n}{(1+p\langle n\rangle-q)^{n+1}}\cdot\exp\left(-\frac{(1-p)\langle n\rangle+q}{p\langle n\rangle-q}\right)$$

$$\times\sum_{k=0}^{\infty}\frac{(n+k)!}{n!(k!)^2}\cdot\left\{\frac{(1-p)\langle n\rangle+q}{(p\langle n\rangle-q)(1+p\langle n\rangle-q)}\right\}.$$

If we take the limit n and $\langle n\rangle\to\infty$ for fixed $z=n/\langle n\rangle$, then we have

$$\langle n\rangle P(n)\xrightarrow[n,\langle n\rangle\to\infty]{}\psi(z), \tag{22}$$

where

$$\psi(z)=\frac{1}{p}\exp\left(-\left(\frac{1-p}{p}+\frac{z}{p}\right)\right)I_0\left(2\sqrt{\frac{1-p}{p}}\sqrt{\frac{z}{p}}\right). \tag{23}$$

Here I_0 is the modified Bessel function. It may be useful to write down the following approximation formulas:

$$\psi(z)\simeq\frac{1}{p}\left\{1+\left(\frac{1-p}{p}\right)\left(\frac{z}{p}\right)+\frac{1}{4}\left(\frac{1-p}{p}\right)^2\left(\frac{z}{p}\right)^2\right\}\cdot\exp\left(-\left(\frac{1-p}{p}+\frac{z}{p}\right)\right) \tag{24}$$

for $((1-p)/p)(z/p)\ll1$ and

$$\psi(z)\simeq\frac{1}{p}\left\{1+\frac{1}{16}\left(\frac{1-p}{p}\cdot\frac{z}{p}\right)^{-1/2}+\frac{9}{512}\left(\frac{1-p}{p}\cdot\frac{z}{p}\right)^{-1}\right\}\exp\left(-\left(\sqrt{\frac{z}{p}}-\sqrt{\frac{1-p}{p}}\right)^2\right) \tag{25}$$

for $((1-p)/p)(z/p)\gg1$. From (23) we can easily get the high energy limit of the k-th order moment as follows:

$$C^k=\frac{\langle n^k\rangle}{\langle n\rangle^k}\xrightarrow[n,\langle n\rangle\to\infty]{}\int z^k\psi(z)\,dz=k!\,p^kL_k\left(-\frac{1-p}{p}\right). \tag{26}$$

Equations (22), (23) and (24) show that our distribution function satisfies the KNO scaling. It is also to be noted that all quantities in the high-energy limit do not depend on q but only on p.

§ 4. Many-mode case and comparison with experiment

Extension to the many-mode case is quite straightforward as was shown by Lachs in the LASER problem. He proved that a generating function $Q(\lambda)$ can be obtained from (10) by replacement

$$|\zeta|^2 \to \boldsymbol{\zeta} \mathscr{C} \boldsymbol{\zeta}^\dagger, \tag{27}$$

$$A \to \mathrm{Tr}(\mathscr{C}\mathscr{A}). \tag{28}$$

Here $\boldsymbol{\zeta}$ is the row vector with component amplitude ζ_i in the i-th mode. $\mathscr{C}$ and $\mathscr{A}$ stand for the matrices defined by Lachs:[3] The former has a matrix element $\mathscr{C}_{ij}$ representing the phase correlation between the i-th and j-th modes and the latter characterizes the fluctuation of the amplitude around $\boldsymbol{\zeta}$. Even in the many-mode case, therefore, we keep just the same generating function $Q(\lambda)$ as in the one-mode case, as far as we are concerned with its dependence on λ and free parameters (corresponding to $|\zeta|^2$ and A). Thus, we can use the very formulas for $P(n), f_k, C^k$ and others obtained in the previous sections. It is, however, to be noted that we must introduce a little modification (including additional parameters) into the theory given by Lachs, because his theory contains a certain condition not directly applicable to our problem. A detailed discussion will be reported in a forthcoming paper, together with the momentum distribution and other properties.

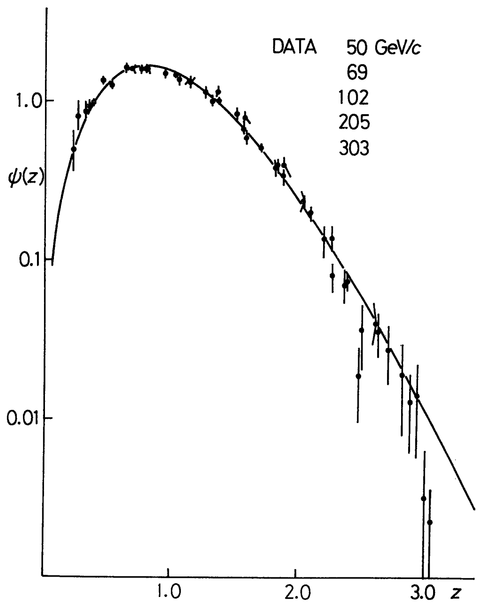

Fig. 1. Calculated curve of KNO scaling function $\psi(z)$ with $p=0.138$. Experimental data taken from Ref. 5).

Here let us compare our theoretical predictions with experiments, by identifying $P(n)$ with the multiplicity distribution function of charged particles. The best fit to recent experimental data is given by

$$p=0.138 \quad \text{and} \quad q=0.570\,, \tag{29}$$

which yields

$$R=\frac{\langle n\rangle}{D} \xrightarrow[\langle n\rangle\to\infty]{} (p(2-p))^{-1/2}=1.973 \tag{30}$$

—consistent with experiments. In Fig. 1 $\psi(z)$ given by (23) is plotted together with the experimental data.[5]

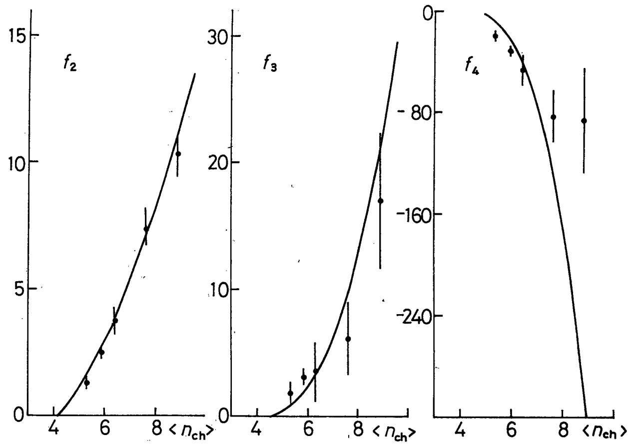

Fig. 2. Correlation parameters f_2, f_3 and f_4 vs $\langle n_{ch}\rangle$, together with experimental data.[5)]

Table I. The calculated values of C^k together with experimental data.[5)]

p_{lab} (GeV/c)		C^2	C^3	C^4	C^5	C^6	C^7	C^8	C^9	C^{10}
50	data	1.236 ±0.014	1.784 ±0.054	2.89 ±0.15	5.16 ±0.41	9.9 ±1.1	20.5 ±2.9	44.6 ±7.9	102 ±22	241 ±61
$\langle n\rangle=5.32\pm0.11$	theory	1.249	1.821	3.007	5.511	11.06	24.02	56.11	139.9	370.7
69	data	1.2415 ±0.0084	1.806 ±0.030	2.959 ±0.084	5.32 ±0.22	10.32 ±0.57	21.3 ±1.5	46.2 ±4.0	105 ±11	245 ±30
$\langle n\rangle=5.888\pm0.066$	theory	1.251	1.831	3.044	5.627	11.41	25.11	59.49	150.7	406.2
102	data	1.249 ±0.014	1.828 ±0.052	3.01 ±0.15	5.43 ±0.42	10.6 ±1.1	21.8 ±3.1	47.3 ±8.4	107 ±23	252 ±64
$\langle n\rangle=6.38\pm0.12$	theory	1.252	1.838	3.068	5.704	11.65	25.84	61.79	158.2	431.0
205	data	1.258 ±0.019	1.856 ±0.065	3.08 ±0.18	5.60 ±0.46	11.0 ±1.2	22.8 ±3.1	50.1 ±8.3	115 ±22	271 ±61
$\langle n\rangle=7.65\pm0.16$	theory	1.254	1.849	3.110	5.840	12.07	27.16	65.98	171.9	477.4
303	data	1.245 ±0.015	1.816 ±0.051	2.99 ±0.14	5.43 ±0.39	10.7 ±1.0	22.4 ±2.8	49.6 ±7.9	115 ±22	278 ±63
$\langle n\rangle=8.86\pm0.15$	theory	1.255	1.856	3.134	5.920	12.32	27.96	68.55	180.4	506.5
high energy limit	theory	1.257	1.875	3.211	6.186	13.19	30.80	78.00	212.7	620.6

Figure 2 shows the correlation parameters f_2, f_3 and f_4 together with the experimental values.[5)] The calculated values of C^k are given in Table I together with the experimental data.[5)] In these figures and the table we can see that the theoretical predictions given by our model are in agreement with experiments.

Finally we must pay attention to the fact $|\zeta|^2 \gg A$ which is a direct result of the choice (29). The fact means that bosons are produced through emission of the classical boson wave having a definite amplitude very near to ζ. Hence, our anticipation settled at the beginning of this paper has been shown to be consistent with experiments.

§ 5. Concluding remarks

In this paper we have obtained the multiplicity distribution, the correlation parameters and other quantities within the framework of the classical boson wave excitation model. The theoretical predictions are shown to be consistent with recent experiments. It is repeatedly remarked that there is no essential difference between the one-mode formulas and the many-mode ones as far as we are concerned with the dependence of the above quantities on free parameters which are to be adjusted to the best fit. The difference should appear in the momentum distribution and the related quantities as the two- or many-particle correlation functions, if we introduce a little modification into the one-mode formulas. Further investigations of the momentum distribution by our model will be required. It is also to be noted that we have discarded the conservation laws of energy, momentum and charge in the present calculation. We will discuss them in a forthcoming paper.

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