Classical Dynamic Consensus and Opinion Dynamics Models: A Survey of Recent Trends and Methodologies

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Abstract

Consensus reaching is an iterative and dynamic process that supports group decision-making models by guiding decision-makers towards modifying their opinions through a feedback mechanism. Many attempts have been recently devoted to the design of efficient consensus reaching processes, especially when the dynamism is dependent on time, which aims to deal with opinion dynamics models. The emergence of novel methodologies in this field has been accelerated over recent years. In this regard, the present work is concerned with a systematic review of classical dynamic consensus and opinion dynamics models. The most recent trends of both models are identified and the developed methodologies are described in detail. Challenges of each model and open problems are discussed and worthwhile directions for future research are given. Our findings denote that due to technological advancements, a majority of recent literature works are concerned with the large-scale group decision-making models, where the interactions of decision-makers are enabled via social networks. Manag-

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ing the behavior of decision-makers and consensus reaching with the minimum adjustment cost under social network analysis have been the top priorities for researchers in the design of classical consensus and opinion dynamics models.
Key words: Opinion Dynamics, Group Decision Making, Consensus Reaching, Feedback Mechanism.

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1. Introduction

The design of intelligent systems is witnessing rapid development due to the advances in information technology for making automatic and effective decisions based on the collected information from different sources. In this regard, group decision-making (GDM), as a core part of intelligent decision-making, has gained much attention in recent years [1]. GDM refers to a decision problem, in which a group of experts is designated to assess a set of alternatives w.r.t. a set of attributes through a communication regime, who aim at giving orders to the available set of alternatives [2]. However, decision-makers (DMs) have different backgrounds and levels of knowledge, which result in potential conflicts in the expressed opinions, and, therefore, there is a need to design mechanisms for consensus achievement in the group [3]. Such a mechanism is called the consensus-reaching process (CRP) in GDM problems [4].

Ideally, the hope is to reach a total agreement, i.e., a unanimous decision, even though, this is neither practical nor necessary in many real-life decision problems [5]. Instead, the goal could be making decisions that are agreed on by most of the involved DMs, so-called consensual decisions. This has consequently paved the way for a softer consensus methodology that could quantify the level of consensus from absence to the total agreement [6]. To this end, the CRP could be considered as a convergent and multi-stage procedure, where the opinions of DMs are initially assessed, and in case the level of consensus among them is lower than a given threshold, they are encouraged to negotiate in order to bring their opinions closer for the sake of consensus reaching. This negotiation process, however, is required to be equipped with an efficient feedback (or adjustment or recommendation) mechanism in order to guide DMs towards consensus reaching, which is the topic of the present work.

The feedback mechanism could be treated as a dynamic procedure, by which the initial opinions of DMs are modified through multiple discussion rounds. This has been studied by classical dynamic consensus methodologies, where DMs' opinions are modified through a dynamic and iterative mechanism. Many attempts have been devoted to addressing the consensus-reaching process by considering the environment conditions and characteristics of the decision problems. In this category of methods, the feedback mechanism is influenced by two major components of decision problems, i.e., the representation structure of opinions and the environment of the decision problem [7]. Representation structures are diverse and could be of different formats such as utility values, preference ordering, multiplicative preference relations, fuzzy preference relations, and linguistic preferences [8]. As for the decision environment, it could be either static, i.e., the environment parameters are not subject to changes from one discussion round to another, or dynamic, where the set of DMs, alternatives, and attributes could be subject to changes in the CRP [5].

In another category of methods, time is used as a parameter to model dynamism in the CRP. Opinion dynamics models fall into this category of methods, in which the evolution of DMs' opinions from time instant (t) to $(t+1)$ is characterized through weighted arithmetic mean of the provided opinions in the previous time steps [9]. The most important component of opinion dynamics models could be considered as the fusion rule used for updating the opinions of DMs. The fusion process under opinion dynamics models has also been extensively studied and various models have been devised for continuous time (e.g., DeGroot model [10] and bounded confidence model [11]) and discrete-time (e.g., Ising model [12], Voter model [13], Sznajd model [14]) environments [15]. Regardless of the environment of models, recently-developed opinion dynamics models are mostly concerned with managing the behavior of DMs, e.g., non-cooperative DMs, the impact of interactive social networks on developed models, developing models to deal with linguistic opinions, and the minimum required adjustment for such models [16].

Following the emerging topics in the CRP, this work is devoted to the review of developed methods for modeling the dynamism in consensus reaching. In this regard, developed methods are categorized into two general categories including classical dynamic models, where time does not play a role in modeling the dynamism, and opinion dynamic models, for which time is used as a parameter to model the dynamism. To this end, we initially provide the required background materials for the analysis of both models by introducing the concept of GDM and the basic opinion dynamics models, which are categorized into two categories of discrete-time and continuous-time models. We then comprehensively review the three main components of classical dynamic consensus models including preference representation structure, decision environment, and feedback mechanism. Finally, we delve into the developed fusion mechanisms based upon opinion dynamics models. Therefore, the major features of the present study can be listed as follows:

1. A detailed study of different preference representation structures and their properties along with multiple illustrative examples is provided that are missing in recent surveys.
2. We have identified the main trends of the most recent feedback mechanisms developed for classical dynamic consensus models. These trends include the novel preference representation structures along with the developed operators, recent attempts towards managing the behavior of DMs, developed mechanisms for large-scale GDM, and techniques that are built upon the concept of minimum adjustment cost.
3. We have also identified and comprehensively studied the most recent trends of the developed opinion dynamics models for consensus reaching. These models are divided into multiple categories by considering the impact of DMs' behavior and their interactions through social networks, models that consider optimization schemes for opinion evolution, and developed techniques that have enabled linguistic opinion dynamics

mechanisms. Finally, we survey the application of learning algorithms such as reinforcement learning (RL) and game-theoretic mechanisms for the development of opinion dynamics models, which are less considered in recent surveys.

The rest of the paper is organized as follows. In Section 2, an introduction to GDM along with discrete-time and continuous-time opinion dynamics models is given. A review on the properties of the CRP for classical dynamic consensus models is provided in Section 3. We review the most recent advances in opinion dynamics models in Section 4. Section 5 includes the challenges of literature works and provides some future trends and concluding remarks are given in Section 6.

2. Background

In GDM, it is usually assumed that a set of DMs $\mathcal{D} = \{d_1, \ldots, d_n\}$, with n being the total number of decision makers, aim at giving orders to a set of alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$ w.r.t. a set of attributes $\mathcal{A} = \{a_1, \ldots, a_m\}$ based on the opinions of the group. The framework for the consensus-based solution to this decision problem through the soft methodology is depicted in Figure 1. Depending on the decision problem and the available set of alternatives and attributes, the initial opinions of DMs are passed into the consensus process block. In case the consensus level among DMs satisfies a given threshold, the consensus process finishes and the selection process gets started. Otherwise, a feedback mechanism gets activated and inconsistent DMs will be provided by recommendations on how to change their opinions for the sake of consensus reaching. As it can be observed, *time* does not play a role in this classical dynamic consensus model. However, in the *opinion dynamics models*, time does play an important role in the modeling of dynamism.

In opinion dynamics models, DMs are usually referred to as agents, however, in order to unify this term for both classical dynamic consensus and opinion dynamics models, the term DM is used for both models in the present work. In

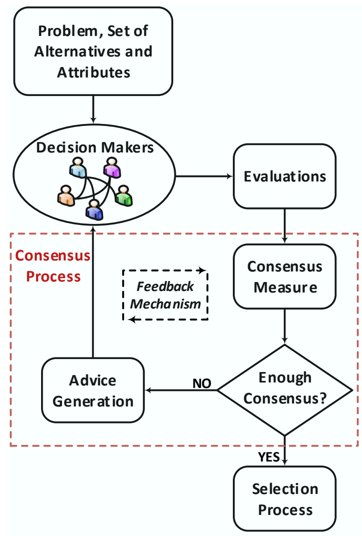

Figure 1: The general framework of the consensus reaching process.

opinion dynamics models, it is assumed that each DM d_i ($i = 1, \ldots, n$) expresses an opinion of the form $\sigma_i(t)$ at time t ($t = 0, 1, 2, \ldots$). It is also assumed that the ith DM gives a weight to the jth DM as w_{ij} satisfying $w_{ij} \geq 0$ and $\sum_{j=1}^{n} w_{ij} = 1$. Then, the opinion evolution of the ith DM is modeled as follows:

$$\sigma_i(t+1) = \sum_{j=1}^{n} w_{ij}\sigma_j(t) = w_{i1}\sigma_1(t) + \ldots + w_{in}\sigma_n(t), \tag{1}$$

or equivalently,

$$\Sigma(t+1) = \mathcal{W} \times \Sigma(t), \tag{2}$$

where $\Sigma \in \mathbb{R}^n$ and $\mathcal{W} \in \mathbb{R}^{n \times n}$. This fusion process can lead to a consensus among DMs in case $\lim_{t\to\infty} \sigma_i(t) = \mathcal{C}$, where $i = 1, \ldots, n$, and $\mathcal{C}$ is a constant and it is called the consensus opinion.

Definition 1. *[17] DMs $d_1, \ldots, d_n$ will form a consensus if for any initial set of opinions $\Sigma(0) \in \mathbb{R}^n$, there exists a constant value $\mathcal{C} \in \mathbb{R}$ for which $\lim_{t\to\infty} \sigma_i(t) = \mathcal{C}$, with $i = 1, \ldots, n$.*

However, when the fusion process ends up with two or more than two different consensus opinions, a polarization or fragmentation happens, respectively. In what follows, we introduce some basic opinion dynamics models that will be widely referred to throughout this paper. These models are divided into two categories, including *discrete opinion models* and *continuous opinion models* [18].

2.1. Discrete opinion models

In discrete opinion dynamics models, opinions of DMs can take only discrete values such as in the Ising, Voter, majority rule, and Sznajd models.

2.1.1. Ising model

The Ising model is known as the earliest opinion dynamics model that has been widely employed in describing the evolution of binary opinions [19]. This model is originated from a mathematical model of ferromagnetism in statistical mechanics. It assumes a binary opinion for each DM, where $+1$ is used to represent the support opinion and -1 represents the opposition. The evolutionary mechanism is towards the state, for which the lowest value of a pre-defined energy function, e.g., Hamiltonian energy function, can be resulted.

2.1.2. Voter model

The voter model, that borrows the name from its applications in electoral competitions, could be refer to as one of the simplest models of opinion dynamics. According to this model, each DM d_i can either hold the opinion $\sigma_i(t) = +1$ or $\sigma_i(t) = -1$. Then, a random DM d_i and one of its neighbors d_j are selected at each time step t and the random DM d_i takes the opinion of its neighbor d_j.

2.1.3. Sznajd model

In the original formulation of this model, the ith DM has an opinion σ_i ($i = 1, \ldots, n$) that can only take $\sigma_i = +1$ and $\sigma_i = -1$ values. This means that each DM can either agree or disagree to the given decision problem. In the original one-dimensional formulation, the following updating rules can be constructed by resorting to a random sequential updating scheme:

1) Randomly select a DM; denote it with d_i.

2) In case d_i and d_{i+1} have the same opinion at time t, i.e., $\sigma_i(t) = \sigma_{i+1}(t) = \sigma$, these two DMs will impose this opinion on their neighbors, i.e., $\sigma_{i-1}(t+\Delta t) = \sigma_{i+2}(t+\Delta t) = \sigma$.
3) In case $\sigma_i(t) \neq \sigma_{i+1}(t)$, the neighbors d_{i-1} and d_{i+2} will take the opposite opinion to their first neighbors, i.e., $\sigma_{i-1}(t+\Delta t) = -\sigma_i(t)$ and $\sigma_{i+2}(t+\Delta t) = -\sigma_{i+1}(t)$.

It is worth noting that the unit time in this formulation contains n elementary updates, i.e., $n\Delta t = 1$.

2.2. Continuous opinion models

With the continuous models, we mean models those are continuous in opinion and not in the time. This refers to the cases, where the opinion of a DM can take real numbers, i.e., $\sigma_i(t) \in \mathbb{R}$, with $i = 1, \ldots, n$. In this regard, it is worth noting that discrete models with binary opinions can be extended to more than two ordered values to get closer to continuous models. In this section, we review the basics of some well-known continuous models such as DeGroot model and two representative bounded confidence models including Defuant-Weisbuch (DW) [20, 21] and Hegselmann-Krause (HK) [17] models.

2.2.1. DeGroot model

This model is known as the classical model of opinion dynamics. When the matrix $\mathcal{W}$ in (2) is independent of time or opinions, the evolution mechanism given in (1) is known as the DeGroot model. Following Definition 1, the sufficient and necessary condition in order to form consensus in the DeGroot model is firstly studied by Berger [22], which is given in the following lemma.

Lemma 1. *[22] DMs $d_1, \ldots, d_n$ will form a consensus in the DeGroot model if and only if there exists $t' \in \{1, 2, \ldots\}$ for which the matrix $\mathcal{W}(t')$ contains at least one strictly positive column.*

Matrix $\mathcal{W}$ is a Markov or stochastic matrix since its elements are all nonnegative and each row sums to one. A Markov matrix has an eigenvalue 1 and the remaining eigenvalues are in absolute value and are always smaller or equal to 1. In this regard, DeGroot showed that the consensus opinion in his model can be formed as given in the following lemma.

Lemma 2. *[10] For $\lambda_1, \ldots, \lambda_n$ with $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda_i \geq 0$, in case DMs $d_1, \ldots, d_n$ form a consensus in the DeGroot model, the consensus opinion $\mathcal{C}$ can be obtained as $\mathcal{C} = \sum_{i=1}^{n} \lambda_i \sigma_i(0)$, where $\lambda \mathcal{W} = \lambda$.*

Lemma 2 states that in case DMs form a consensus in the DeGroot model, this consensual opinion is a linear combination of the initial opinions and the combinational coefficients $\lambda_i (i = 1, \ldots, n)$ that can be computed based on the eigenvalue 1 of the weight matrix $\mathcal{W}$.

This classical model of opinion dynamics has several variants depending on the adjustment mechanisms of $\mathcal{W}$, where the Friedkin and Johnson model [23], time-variant model [17], and bounded confidence model [11] are three popular variants. In particular, the difference between Friedkin and Johnson model and DeGroot model goes back to the nature of the weights $\mathcal{W}$, where the set of weights are time-independent, however, each DM sticks to its initial opinion to a certain degree. In the time-variant model, the weights are assumed to be time-dependent, while in the bounded confidence models, the weights are opinion-dependent.

2.2.2. Bounded confidence models

In this model, opinions of DMs are dependent to their social interactions. In particular, two DMs, whose absolute difference of opinions is smaller than a given threshold, can interact and influence each other's opinions. This threshold value is called the bounded confidence level and is shown by ϵ in this work. Based upon a generic idea of repeated averaging, DW and HK models have been developed under the bounded confidence.

In the DW model, two DMs d_i and d_j, with $i \neq j$, are randomly selected. In case their opinions at time step t, i.e., $\sigma_i(t)$ and $\sigma_j(t)$, follow the bounded confidence rule, i.e., $|\sigma_i(t) - \sigma_j(t)| < \epsilon$, they update their opinions according to

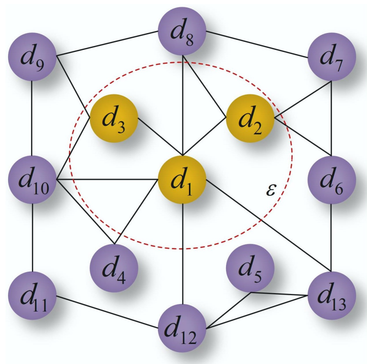

Figure 2: The concept of the HK model. DM d_1 and neighbors in its confidence set (shown by a circle with radius ϵ) are highlighted in yellow. The opinion of DM d_1 will be influenced by those of its confidence set.

the following update rule:

$$\sigma_i(t+1) = (1-\alpha)\sigma_i(t) + \alpha\sigma_j(t), \tag{3}$$

$$\sigma_j(t+1) = (1-\alpha)\sigma_j(t) + \alpha\sigma_i(t), \tag{4}$$

where $\sigma_i(t+1)$ is the updated opinion of the ith DM with $\alpha \in [0, 0.5]$ being a convergence parameter.

In the HK model, as shown in Figure 2, it is needed to first construct the confidence set of a DM d_i as follows:

$$\mathcal{I}\left(d_i, \Sigma(t)\right) = \{d_j |\ |\sigma_i(t) - \sigma_j(t)| < \epsilon\}, \tag{5}$$

where $\mathcal{I}\left(d_i, \Sigma(t)\right)$ is the confidence set of the DM d_i at time step t. Then, the opinion of DM d_i will be formed by resorting to the average opinion of its neighbors as follows:

$$\sigma_i(t+1) = \frac{1}{|\mathcal{I}(d_i, \Sigma(t))|} \sum_{j \in \mathcal{I}(d_i, \Sigma(t))} \sigma_j(t), \tag{6}$$

where $t = 0, 1, \ldots,\ i = 1, \ldots, n$, and $|.|$ is used to denote the cardinality of the enclosed set.

3. Consensus Reaching Process

A hard consensus process offers only two states of either absence or total agreement among a set of interactive DMs on a feasible set of alternatives. This classical logic that relies on the unanimous and total agreement, however, might neither be realistic nor necessary in practical decision problems. A more realistic scheme could be quantifying the level of consensus into some interval (e.g., $[0, 1]$) to not only encompass the crisp hard consensus values (i.e., 0 and 1 for the absence or total agreement, respectively), but also to consider any partial levels of consensus among DMs. The latter scheme is known as the soft consensus process, where its general structure has been illustrated in Figure 1. In this section, we aim to review the influence of the preference representation, decision environment, and feedback mechanism on the CRP.

3.1. Preference representation structures

In a GDM problem, the set of DMs $\{d_1, \ldots, d_n\}$ provide their evaluations or preferences for a set of alternatives $\{x_1, \ldots, x_q\}$ w.r.t. to the available attributes $\{a_1, \ldots, a_m\}$. The provided preferences could be of different formats, which are arguably dividable into five categories including utility values, preference ordering, multiplicative preference relations, fuzzy preference relations, and linguistic preferences. In what follows, each category of representation structures are reviewed and illustrative examples are provided to discuss their applications.

3.1.1. Utility values

Preferences of a DM can be provided through a utility function, where the DM provides a real evaluation w.r.t. each alternative by associating a real number to the corresponding alternative. This real number is indeed used to quantify the opinion of DM towards the given alternative. A DM d_k can express its opinions on a set of alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$ under a utility valued preference vector as $\mathcal{U}^k = \{u_i^k | i = 1, \ldots, q\}$ with $u_i^k \in [0, 1]$, where the ith element of $\mathcal{U}^k$ is the quantitative utility evaluation of the kth DM to the ith alternative [24]. By obtaining the utility vectors of n alternatives from q DMs, an overall preference matrix of dimension $q \times n$ can be constructed as follows:

$$\mathcal{U} = \begin{bmatrix} u_1^1 & \dots & u_i^1 & \dots & u_n^1 \\ \vdots & \ddots & & & \vdots \\ u_1^k & \dots & u_i^k & \dots & u_n^k \\ \vdots & & & \ddots & \vdots \\ u_1^q & \dots & u_i^q & \dots & u_n^q \end{bmatrix}. \tag{7}$$

In realistic decision environments, it is often difficult for a DM to provide its exact preferences due to the lack of expertise, or lack of information, or the time pressure. In this regard, there might be uncertainties in evaluations of a DM that can be captured by resorting to the uncertain preference structures. The interval utility-valued preference structure is a variant of the utility-valued preference structure to deal with uncertainties in preferences by replacing the crisp utility values with the value ranges in the form of an interval of numerical values. In this regard, for the set of alternatives $\mathcal{X}$, a set of n utility intervals $\tilde{\mathcal{U}} = \{\tilde{u}_i, \dots, \tilde{u}_n\}$ can be constructed, where $\tilde{u}_i = [\underline{\tilde{u}}_i, \overline{\tilde{u}}_i]$ is the interval utility-valued preference w.r.t. the ith alternative with $0 \leq \underline{\tilde{u}}_i \leq \overline{\tilde{u}}_i \leq 1$. The upper and lower limits of $\tilde{u}_i$ are represented by $\overline{\tilde{u}}_i$ and $\underline{\tilde{u}}_i$, respectively. To this end, the crisp utility value preference is a special case of interval utility values with $\underline{\tilde{u}}_i = \overline{\tilde{u}}_i$.

3.1.2. Preference ordering

This representation format can be used to provide orders for a set of alternatives from the best to the worst [25]. In particular, for a set of alternatives $\mathcal{X} = \{x_1, \dots, x_q\}$, a DM can provide its evaluations in terms of preference ordering as $\mathcal{O}^k = \{o^k(1), \dots, o^k(n)\}$, with $o^k(.)$ being a permutation of $\{1, 2, \dots, q\}$ from the viewpoint of the kth DM. For instance, suppose that four alternatives $\mathcal{X} = \{x_1, x_2, x_3, x_4\}$ are put into discussion and the first DM d_1 provides its evaluations in terms of preference ordering as $\{3, 2, 4, 1\}$. This means that from the viewpoint of d_1, the best alternative is x_4 and x_3 is the worst.

3.1.3. Multiplicative preference relations

The multiplicative representation format leads to numerical preference relations that interpret the ratio of the preference degree of an alternative over other alternatives in a given scale. Specifically, for the DM d_k, the multiplicative preference relation over the set of alternatives $\mathcal{X} \in \mathbb{R}^q$ could be of the form of a matrix as $\mathcal{P} = [p_{ij}]_{q\times q}$, being p_{ij} belonged exactly to a designated scale to indicate the preference intensity of alternative x_i over the alternative x_j. One of the most-widely used scales is the Saaty 1-9 scale [26]. In this regard, a preference value of $p_{ij} = 1$ denotes no difference between alternatives x_i and x_j from the viewpoint of a DM, while $p_{ij} = 9$ indicates that x_i is absolutely preferred to x_j.

3.1.4. Fuzzy preference relations

Fuzzy preference relations could be referred to as the most-widely used representation structure. It is a numerical representation and could be defined as a fuzzy set on the product set $\mathcal{X} \times \mathcal{X}$ [27]. It is often characterized by means of a membership function $\mu_P : \mathcal{X} \times \mathcal{X} \to [0, 1]$. When the cardinality of the set of feasible solutions $\mathcal{X}$ is small, a fuzzy preference relation can be expressed via a matrix $P = [p_{ij}]_{q\times q}$, where $p_{ij} = \mu_P(x_i, x_j)$ with $i, j \in \{1, \ldots, q\}$, and indicates the preference intensity of alternative x_i over the alternative x_j. For instance, $p_{ij} = 0.5$ shows the indifference evaluation between alternatives x_i and x_j, or $p_{ij} = 1$ denotes that x_i is absolutely preferred to x_j. In this representation, it is required to set $p_{ii} = 0.5$, with $i = 1, \ldots, n$. In case $p_{ij} + p_{ji} = 1$ ($\forall i, j \in \{1, \ldots, n\}$), it is said that the evaluation matrix P is additive reciprocal and the fuzzy preference relation is often called additive preference relation.

3.1.5. Linguistic preferences

The linguistic assessment of DMs can be enabled by resorting to the linguistic term sets (LTSs) and computing with word (CWW) methodologies [28]. A balanced LTS $\mathcal{S} = \{s_i | i = 0, 1, \ldots, 2r\}$, is a completely ordered and finite set with odd cardinality, where r is a nonnegative integer value. In this LTS, s_i represents a linguistic variable, where for two arbitrary linguistic values s_i and

s_j, the following criteria hold: 1) it is an ordered set, i.e., $s_i \leq s_j$ if and only if $i \leq j$, and, 2) there is a negation operator for which $neg(s_i) = s_j$ if $i+j = 2r$. An example of an LTS can be $\mathcal{S} = \{s_0 =$ 'very poor', $s_1 =$ 'poor', $s_2 =$ 'slightly poor', $s_3 =$ 'fair', $s_4 =$ 'slightly good', $s_5 =$ 'good', $s_6 =$ 'very good'$\}$.

The semantics of linguistic terms in an LTS can be extracted by means of type-1 and interval type-2 fuzzy sets, however, to employ LTSs in GDM problems, CWW tools are required to be developed. This has been initiated in [29] by introducing the concept of 2-tuple linguistic modeling.

Definition 2. *[29] Given an LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$, suppose that $\beta \in [0, 2r]$ is resulted by means of a symbolic aggregation operation on $\mathcal{S}$. Then, the equivalent information to β can be expressed in terms of a 2-tuple as follows:*

$$\Delta(\beta) = (s_i, \alpha),\ with \begin{cases} s_i, & i = round(\beta), \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{8}$$

where $\Delta : [0, 2r] \to \mathcal{S} \times [-0.5, 0.5)$ and 'round' is used to denote the round operation.

In this regard, the following definition represents how to evoke the numerical information designated to a 2-tuple linguistic assessment.

Definition 3. *[29] Given an LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$ and a 2-tuple (s_i, α), the numerical value $\beta \in [0, 2r]$ of this 2-tuple can be evoked by means of function Δ^{-1} as follows:*

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta, \tag{9}$$

where $\Delta^{-1} : \mathcal{S} \times [-0.5, 0.5) \to [0, 2r]$.

Following the above definitions, it is straightforward that a linguistic term $s_i \in \mathcal{S}$ can be represented by means of a 2-tuple as $s_i = \Delta(s_i, 0)$. To this end, in what follows, we review some of the commonly-used linguistic representation structures in terms of fuzzy Z-numbers, hesitant fuzzy linguistic preferences, intuitionistic linguistic preference relations, and interval linguistic preference relations.

Z-numbers contain two different components to describe an uncertain variable and have been extensively used in different applications including decision analysis.

Definition 4. *[30] A Z-number, denoted by $\mathcal{Z} = (A, B)$, contains two components, where the first component, i.e., A, is a constraint on the values that a real-valued uncertain variable can take. The second component, i.e., B, denotes the certainty of the first component.*

As it can be observed from Definition 4, Z-numbers rely on two LTSs to describe an assessment on a given variable. As an example, the first component of a Z-number can be taken from the LTS $\mathcal{S} = \{s_0 = \text{'very poor'}, \ldots, s_6 = \text{'very good'}\}$ as before, and, the certainty about the first component can be chosen from another LTS defined by $\mathcal{S}' = \{s_0 = \text{'very uncertain'}, s_1 = \text{'uncertain'}, s_2 = \text{'slightly uncertain'}, s_3 = \text{'neutral'}, s_4 = \text{'slightly certain'}, s_5 = \text{'certain'}, s_6 = \text{'very certain'}\}$. To this end, a Z-number can be represented by an ordered pair of fuzzy numbers as $\mathcal{Z} = (s_i, s_i')$ with $s_i \in \mathcal{S}$ and $s_i' \in \mathcal{S}'$, such as $\mathcal{Z} = (s_6, s_6') = (\text{'very good'}, \text{'very certain'})$. Thanks to their advantages in dealing with uncertainties, Z-numbers have been extensively studied and new extensions such as z^*-numbers [31], Z-Advanced numbers [32], and uncertain Z-numbers [33] along with different tools for acquiring the information [34], measuring the uncertainty [35], arithmetic operations [36], and ranking methods for Z-numbers [37] are recently developed.

Hesitant fuzzy linguistic term sets (HFLTSs) are also useful tools for DMs to express their opinions by making use of several LTSs simultaneously. This is to overcome the limitations of granularity of DMs' knowledge that might not be concurrent with the granularity of a given single LTS.

Definition 5. *[38] For a given LTS $\mathcal{S} = \{s_0, s_1, \ldots, s_{2r}\}$, an HFLTS, denoted by $\mathcal{H}$, is an ordered and finite subset of consecutive linguistic terms of $\mathcal{S}$.*

From the above definition, it is evident that different HFLTSs extracted from a given LTS may contain different number of linguistic elements. In this regard, several schemes have been developed to normalize HFLTSs. For instance, authors in [39] proposed two normalization principles, called α-normalization and β-normalization, that rely on the risk preferences of DMs to remove some elements from the given HFLTSs (α-normalization) or add elements (β-normalization) to maintain the same number of elements in each HFLTS. To this end, the definition of hesitant fuzzy linguistic preference relations (HFLPRs) can be given as follows.

Definition 6. *[39] Assume that $M_{\mathcal{S}}$ is a set of HFLTSs constructed based on the LTS $\mathcal{S}$. An HFLPR can then be represented by a matrix $\mathcal{P} = (p_{ij})_{n\times n}$, where $p_{ij} \in M_{\mathcal{S}}$ and the negation operator holds for p_{ij}, i.e., $neg(p_{ij}) = p_{ji}$.*

As an example, let $\mathcal{S} = \{s_0 = \text{'very poor'}, \dots, s_6 = \text{'very good'}\}$ be an LTS as before. An HFLPR can the be constructed as follows:

$$\mathcal{P} = \begin{bmatrix} \{s_3\} & \{s_2, s_6\} & \{s_1, s_3, s_4\} \\ \{s_4, s_0\} & \{s_3\} & \{s_4, s_5, s_6\} \\ \{s_5, s_3, s_2\} & \{s_2, s_1, s_0\} & \{s_3\} \end{bmatrix}.$$

Another linguistic representation structure that we review is the intuitionistic linguistic fuzzy preference relations (ILFPRs). The above-mentioned representation structures are mainly used to express the preferred assessments of DMs through either numerical or linguistic preference relations. ILFPRs, however, enable DMs to provide not only their preferred assessments, but also their non-preferred assessments. This representation structure is built upon the intuitionistic fuzzy sets (IFSs) that first introduced by Atanassov [40]. Szmidt and Kacprzyk [41] made use of IFSs to propose IFPRs that are constructed based upon numerical values. Then, Yager [42] extended the operations on IFSs to linguistic intuitionistic fuzzy sets, which then led to the introduction of linguistic intuitionistic fuzzy variables [43] to qualitatively represent the preferred and non-preferred assessments of DMs.

Definition 7. *[44] An intuitionistic linguistic set $\tilde{A}$ on the set of alternatives $\mathcal{X}$ can be defined as $\tilde{A} = \{\langle x_i| < s_{\theta(x_i)}, (u(x_i), v(x_i)) >\rangle\}$, where $s_{\theta(x_i)} \in \mathcal{S}$*

is a linguistic term, $u(x_i)$ and $v(x_i)$ are used to denote the preferred and non-preferred degrees of alternative $x_i \in \mathcal{X}$ to the designated linguistic variable $s_{\theta(x_i)}$, with $u(x_i), v(x_i) \in [0,1]$ and $u(x_i) + v(x_i) = 1, \forall x_i \in \mathcal{X}$.

With the characteristics of intuitionistic linguistic sets given in Definition 7, an intuitionistic linguistic variable can then be represented by $\tilde{a} = (s_{\theta(a)}, < u(a), v(a) >)$. Then, an ILFPR can be defined as follows.

Definition 8. *[45] An ILFPR on a set of given alternatives $\mathcal{X}$ can be represented by a matrix of the form $\mathcal{P} = (p_{ij})_{n\times n}$, where $p_{ij} =< s_{\theta_{ij}}, (u_{ij}, v_{ij}) >$ for $i, j = 1, \ldots, n$, $s_{\theta_{ij}} \in \mathcal{S}$, u_{ij} and v_{ij} being the preferred and non-preferred degrees of alternative x_i over x_j w.r.t. the designated linguistic term θ_{ij}.*

As an example, having the LTS $\mathcal{S} = \{s_0, \ldots, s_6\}$ as before, an element of an ILFPR $\mathcal{P}$ can be represented by $p_{12} =< s_2, (1, 0) >$.

3.1.6. Interval valued preference relations

This representation structure is a general case that can contain any of the above-mentioned numerical and linguistic representations, however, through the concept of interval valued preference relations.

Definition 9. *[46] An interval multiplicative preference relation $\mathcal{P}$ on a given set of alternatives $\mathcal{X}$ can be represented by a matrix of the form $\mathcal{P} = (p_{ij})_{n\times n}$ satisfying $p_{ij} = [p_{ij}^L, p_{ij}^U]$, $p_{ij}^U \geq p_{ij}^L > 0$, $p_{ij}^L p_{ji}^U = p_{ij}^U p_{ji}^L = 1$, and $p_{ii}^L = p_{ii}^U = 1$ for $i, j = 1, \ldots, n$.*

In Definition 9, p_{ij} is used to denote the preference degree of the alternative x_i over x_j through an interval-valued representation that can be interpreted as alternative x_i is p_{ij} times as good as alternative x_j. This representation structure is then extended to cope with other representation structures such as interval-valued (IV) fuzzy preference relations [47], IV-HFLPR [48], and IV-ILFPR [49].

3.2. Consensus measures

A common practice in assessment of the level of consensus among DMs is the use of similarity functions for quantifying the closeness of DMs' preferences. These functions are typically built upon the distance between DMs' evaluations, where notable attempts have been devoted to the construction of efficient and meaningful distance measures. The most commonly-used distance measures in the literature of GDM are the Manhattan [50], Euclidean [51], Cosine [52], Dice [53], and Jaccard [54] distance functions. The authors in [55] provided a detailed study on the effect of the aforementioned distance functions on the level of achieved consensus by DMs and the speed of CRP by employing the non-parametric Wilcoxon significant test. The results are then extended to the case, in which various aggregation mechanisms are coupled with the aforementioned distance functions in order to identify a set of rules for the speed control of CRP in GDM problems [56].

3.3. Decision environment

In addition to the representation structure of opinions, the decision environment could also be referred to as a major module of the CRP. This process is dynamic by nature and the dynamism of the decision environment can make it a more complicated task. With the decision environment, it is usually referred to the set of DMs, alternatives, attributes, and the adjustment mechanism of the DMs' importance weights. Any of these characteristics of the decision problem can either be static or dynamic during the CRP. In a static environment, it is assumed that from one discussion round to another, the set of DMs, alternatives, and attributes are constant and are not subject to any changes in any discussion round. In contrast to static environments, these characteristics of the decision problem can undergo some changes during the consensus process in dynamic environments [57].

The dynamic set of DMs refers to the case, in which the number of DMs can dynamically change from one discussion round to another. This actually reflects a realistic situation, where a DM might leave or might be incorporated to the negotiation process at any discussion round in the consensus process. In this regard, the set of DMs $\mathcal{D} = \{d_1, \ldots, d_n\}$ will be replaced by $\mathcal{D}^{(t)} = \{d_1^{(t)}, \ldots, d_{n(t)}^{(t)}\}$, with t being the index of the discussion round in the consensus process and $n(t)$ is used to indicate the number of DMs at discussion round t.

The set of alternatives $\mathcal{X} = \{x_1, \ldots, x_q\}$ could also be subject to changes during the consensus process. This could happen due to the introduction of new alternatives to or the removal of the worst alternative from the decision problem. A typical way to deal with such changes is to resort to the assessment of DMs to see if they agree or disagree with these changes. Same as the representation scheme for the dynamic set of DMs, a dynamic set of alternatives can be represented as $\mathcal{X}^{(t)} = \{x_1^{(t)}, \ldots, x_{q(t)}^{(t)}\}$, with $q(t)$ indicating the number of available alternatives at the discussion round t.

Finally, the set of attributes $\mathcal{A} = \{a_1, \ldots, a_m\}$ might undergo some changes during the consensus process, too. It is a common practice to evaluate a given set of alternatives w.r.t. a set of correlated attributes with the decision problem, leading to the multi-attribute GDM (MAGDM). Deciding on evaluating the given alternatives from new viewpoints, i.e., new attributes, or discarding those attributes that are hardly understandable for some DMs, could be considered as situations where the set of attributes could be subject to changes. In these cases, the set of attributes can be represented by $\mathcal{A}^{(t)} = \{a_1^{(t)}, \ldots, a_m^{(t)}\}$.

3.4. Feedback mechanism

A feedback mechanism, as shown in Figure 1, is typically referred to a recommendation mechanism that aims to help inconsistent DMs with modifying their opinions and to guide them towards the collective opinion of the group through either a couple of discussion rounds or in one step. The former scheme is usually employed by means of identification and direction rules, while the latter scheme can be realized in the context of optimization models. In this regard, the efficiency of developed feedback mechanisms for consensus reaching can be measured based on the number of DMs who adjust their opinions, the number of alternatives that are required to be adjusted, the number of discussion rounds, the adjustment cost, and the required number of preference values to be adjusted [58]. In this section, we review the most-recent advances in the design of feedback mechanisms for novel preference structures and operators by considering the behavior of DMs, size of the group, and employed optimization schemes.

3.4.1. Developed preference structures

Design of a feedback mechanism is highly dependent to the preference structure and requires the development of proper tools for consensus reaching. For instance, linguistic preference relations with self-confidence (LPRs-SC) is proposed in [59], where a two-step feedback mechanism is suggested in [60] to not only modify the opinions of DMs, but also to modify their corresponding level of self-confidence. This is enabled by proposing an aggregation operator and a self-confidence score function to meaningfully adjust the weights of DMs. The authors in [61] introduced the new concept of Pythagorean fuzzy linguistic preference relations (PFLPRs) along with the Pythagorean fuzzy linguistic values (PFLVs) that account for the linguistic membership and non-membership degrees, which are driven from the Pythagorean fuzzy sets theory proposed by Yager et al. in 2013 [62]. Based upon the definition of consistency, individual consensus degree (CD), and group CD for PFLPRs, a multi-step feedback mechanism is then proposed to adjust only the individual CD of the worst DM at each iteration. The interesting feature of the proposed mechanism is that the consistency level of evaluations is retained even after the employed adjustments. In [63], the authors proposed a novel preference structure, called flexible linguistic expressions (FLEs), where DMs are allowed to express their opinions by utilizing different subsets of a given linguistic term set along with the distribution information over the expressed subsets. This structure could be referred to as an extension to the linguistic distribution (LD) structure, where not only the LDs, but also incomplete LDs, possibility distribution for HFLTSs and proportional HFLTSs can be extracted from this representation. To deal with uncertainties, an aggregation operator with accuracy and minimum pref-

Table 1: Developed preference representation structures in the recent research works.

Reference	Representation Structure
[61, 73]	Pythagorean linguistic preference relations
[63]	Flexible Linguistic Expressions
[64]	Double hierarchy linguistic preference relations
[74]	Comparative linguistic expressions
[65–67]	Z-numbers and their extensions
[68]	Nonlinear preference relations
[60]	Self-confident linguistic preference relations
[70, 71]	q-rung orthopair fuzzy preference relations
[72]	Complex intuitionistic fuzzy preference relations
[69, 75, 76]	Probabilistic linguistic preference relations
[77]	Heterogeneous preference relations

erence loss is then proposed for FLEs to construct the collective evaluation and the feedback mechanism benefits from consensus rules with minimum preference loss to adapt inconsistent opinions. More recently, a preference structure is proposed in [64] based upon augmenting the concepts of self-confidence degree and double hierarchy linguistic preference relation (DHLPR). The authors proposed to construct the consensus model based on the individual and collective priority vectors, where a feedback mechanism based on the identification and direction rules is finally proposed to adjust inconsistent DHLPRs. Other representation structures based on the extended versions of Z-numbers such as Z^E-numbers [65] and Z probabilistic LTSs [66], Atanassov's interval valued intuitionistic fuzzy sets and trapezium clouds [67], nonlinear preferences [68], unbalanced probabilistic LTSs [69], incomplete q-rung and interval valued q-rung orthopair FPRs [70, 71], and complex LTSs [72] are also developed for the sake of decision making. Table 1 summarizes the developed preference representation structures in the recent research works.

3.4.2. Developed operators

New preference structures typically require the introduction of novel operational tools for the sake of consensus reaching in GDM. In this section, we review some recent efforts towards the development of useful operators to enable consensus reaching through feedback mechanisms under different preference structures.

Interval type-2 fuzzy sets (IT2FSs) have attracted the attention of researchers due to their efficiency in modeling uncertainties. The authors in [78] proposed the conversion of classical linguistic terms into triangular IT2FSs, where they developed weighted mean and weighted semi-absolute deviation operators for IT2FSs to construct a consensus model for portfolio allocation. The developed feedback mechanism considered the acceptable tolerance level of DMs in adjusting their preferences and maximum return and minimum risk models are then suggested for preference adjustment. An improved version of the Euclidean and Hamming distance measures for ILFPRs are proposed in [79], and, accordingly, a feedback mechanism is built upon adjusting the preference elements based on their closeness to a collective one. Various operational laws for probabilistic linguistic q-rung orthopair fuzzy sets (PLq-ROFS) are presented in [80], where the authors proposed to extract the semantics of PLq-ROFS by means of novel linguistic scale functions. The comparison between PLq-ROFSs is enabled by introducing new score and accuracy functions, where the aggregation of PLq-ROFSs is performed by means of PLq-ROF weighted averaging and PLq-ROF ordered weighted averaging. The designed feedback mechanism adjusts DMs' preferences by basic operations on PLq-ROFSs and by involving the correlation measures of each DM. Later in [81], the authors proposed the integration of neutrality aggregation into the q-ROFSs to construct a power aggregation operator for the sake of GDM. For dual hesitant q-ROFSs [82] and dual probabilistic linguistic environments [83], required operational laws are developed based on the Dombi and Bonferroni mean operators for aggregating preferences and ordering alternatives in the selection process. Furthermore, some attempts have been recently devoted to the design of operators for Z-numbers based on the Archimedean t-norms and t-conorms [84], distance operators for HFLTSs [85] and pair-wise preference relations [86]. Table 2 summarizes the developed operators in the recent research works.

Table 2: Developed operators in the recent research works.

Reference	Developed Operators
[82, 83]	Dombi operators and Bonferroni mean operators
[84]	Archimedean t-norms and t-conorms
[81]	Power neutrality aggregation operator
[86]	Distance operator for evidential preferences
[85]	Distance operator for hesitant information
[79]	Intuitionistic multiplicative distance measures
[80]	q-rung orthopair fuzzy weighted averaging operator
[78]	Information measures for IT2FSs

3.4.3. Behavioral mechanisms

We generally refer to feedback mechanisms that reflect DMs' interests, trust relations, attitude, and cooperative or non-cooperative behavior in the consensus process as the behavioral mechanisms. In what follows, we review the most-recent advances in this type of feedback mechanisms.

Due to differences in the nature of decision environments or knowledge and experience of DMs, it is a common practice to take into account the interest of DMs in selecting an attribute or a set of attributes to evaluate a predefined set of alternatives [87]. Following this and for a diverse set of DMs, the construction of a heterogeneous decision environment is beneficial due to providing an opportunity for DMs to express their opinions in terms of their preferred preference structures. Developed techniques for heterogeneous decision environments are usually relying on proposing and performing proper transformations to augment different structures into a homogeneous structure, while ensuring the consistency among preference relations [88]. In this regard, the most-recent techniques have focused on the unbalanced LTSs to address the nonlinearities in DMs' cognition [69], case-based reasoning for emergency decision making [89], criteria interactions [90], and to deal with dynamic contexts [91]. Another consideration in behavior modeling for consensus reaching is the trust relationships between a set of anonymous DMs, which is usually realized through a social network-based mechanism. We categorize these techniques under the large-scale decision making model, which will be given in the next section. However, it is worth mentioning that in contrast to conventional trust or distrust models, recently-developed techniques treat the trust among DMs as a matter of degree and novel trust functions and trust scores are proposed to model relationships among DMs [92, 93]. The attitude of DMs could also be considered in the behavioral category, where the aim is to reflect the attitude of DMs towards consensus reaching. To quantify the attitude of DMs in a continuous ranging scale to reflect the pessimistic attitudes to indifferent attitudes in construction of the trust relationships, the authors in [94] proposed an attitudinal trust degree, which makes use of an ordered weighted average operator guided by a unit-monotonic function. Considering the risk attitude of DMs in alternative ranking through an evidential reasoning methodology [95] and construction of linguistic quantifiers based upon the attitude of DMs [96] are of recent trends in the design of attitude-based feedback mechanisms.

As the last category of behavioral mechanisms, we review some recent advances on managing the non-cooperative behavior of DMs towards consensus reaching by means of a feedback mechanism. The non-cooperative behavior refers to the case, in which the inconsistent DMs are reluctant to modify their opinions according to the provided recommendations through the feedback mechanism. In particular cases, even some DMs intentionally take opposite actions to the recommended adjustments. Therefore, identifying and managing the non-cooperative DMs are of paramount importance for consensus reaching due to their negative impacts on the CRP in terms of the adjustment cost and consensus time. Weight punishment and exit-delegation are two commonly used approaches to manage non-cooperative DMs. The former aims to penalize non-cooperative DMs by reducing their designated weights so as to make them have less impact on the decision made by the group. In the latter, the non-cooperative DMs are removed from the group. One way to do this is presented in [97], where the authors proposed to use the degree of conflict of DMs to identify non-cooperative DMs. Then, they considered a weight penalty based on the triangular fuzzy numbers for internal DMs, while external non-cooperative

DMs were removed from the group. For uncertain decision making during the COVID-19 outbreak, a co-operation degree is devised in [98] to assign DMs into multiple clusters, where clusters with low co-operation degree are penalized with a low weight. By resorting to the number of adjustments of each DM, a co-operation index is introduced in [99] and it is proposed to take different actions for semi-cooperative and fully non-cooperative DMs in terms of weight penalties. An anti-biased statistical mechanism based upon a Biasedness index is proposed in [100] to manage non-cooperative DMs through extreme, moderate, and soft weight punishment schemes. Besides, the willingness of DMs in accepting the suggested modifications could also be considered in the design of feedback mechanism. In this regard, by considering the willingness of DMs, two simultaneous optimization problems are designed in [101] to maximize the consensus level among DMs and to minimize the adjustment cost.

3.4.4. Large-scale GDM

LSGDM is usually referred to a decision problem that involves at least twenty DMs [102]. Other than the size of involved DMs, LSGDM approaches need to deal with heterogeneous information due to diversity of DMs in terms of their background and level of knowledge. Furthermore, the management of non-cooperative DMs who interact through a designated social network platform could also be referred to as another challenge that LSGDM are facing with. In this regard, the most-recent works in LSGDM have focused on managing the non-cooperative DMs by considering their trust relationships in an interactive social network framework.

Management of non-cooperative DMs is an inevitable part of LSGDM for the sake of dimension reduction. This is usually performed by means of assigning DMs into multiple clusters based upon some constructed similarity indexes, where DMs with a lower value of the designated similarity index compared with other members of a cluster can be excluded. Therefore, there is a trend of works on attempting towards the design of efficient clustering-based mechanisms to deal with non-cooperative DMs w.r.t. preference representation structures. By resorting to the consensus evolution network of a large group of DMs, authors in [103] made use of the Louvain two-phase clustering algorithm [104] to extract communities. For heterogeneous representation structures, an extended version of the k-Means clustering algorithm could be employed based on the Euclidean distance between the normalized preference relations and the cluster centers. In this regard, cooperative and non-cooperative indexes of DMs and clusters can be constructed based on the enlargement of the deviation between the original and modified preferences to manage non-cooperative DMs [105]. The same structure could be implemented based on the weight punishment mechanism for the k-Means [52] or grey clustering [106] algorithms.

Another trend of LSGDM works follow the trust-based feedback mechanisms, which are realized through social networks. This study is important owing to the fact that trust relationships not only have impacts on the clustering process for dimension reduction of LSGDM, but also can influence the CRP. Trust relationships are usually modeled via directed and weighted trust graphs, where the nodes are assumed to be DMs, edges of the graph denote trust relationships, and the designated weights show the trust score from one DM to another. This modeling of trust relationships, which is enabled by means of social network analysis, has a significant impact on reducing the complexity of aggregation process by identifying the leadership behavior of DMs. This could also help with managing the non-cooperative DMs. The idea is to divide DMs by means of clustering algorithms such as the one proposed in [107], where a leader will be assigned to each cluster. In the feedback mechanism, followers (ordinary members of a cluster) are suggested to follow the behavior of the leader of the cluster so as to adjust their opinions, while non-cooperative members will be assigned a lower weight in the consensus process [108]. Opinion similarity could also be augmented with trust relationships in construction of clustering algorithms for LSGDM in order to involve the level of difference among opinions of DMs [109]. Other than building consensus based upon the opinion of trusted peers for a DM, recent studies show that the opinions of distrusted peers could also help with consensus reaching [110]. This could be employed for social networks with high or medium density because for low density social networks, the collective intelligence level will be diminished when the scope of distrust increases [111]. Another technique in management of non-cooperative DMs is to prevent individual manipulation behavior through assigning attitudinal weight-adjustment mechanisms, which is presented in [112] and it is realized through a minimum adjustment cost framework under social network GDM.

A worthwhile research field in social network-based GDM is the trust propagation in trust networks. A recent review on trust propagation in social networks can be found in [113]. As it was mentioned earlier, trust relationships can be modeled via directed and weighted graphs, where DMs are connected via either a direct or indirect path. In case of indirect paths, there is a need to estimate the value of trust among DMs, which can be done by means of trust propagation techniques. The most-recent research works in this field of study are devoted to multi-path trust propagation [114], linguistic trust propagation [115], and DMs' weight adjustment through trust propagation [116]. Managing the minority opinions [117], optimization schemes for consensus reaching [118, 119], minimizing the information loss [120], and dealing with incomplete preferences [121] are some interesting and open problems in social network-based GDM. Furthermore, due to the fact that words mean different things for different people, a linguistic GDM model is proposed in [122] to model the personalized individual semantics (PIS) of DMs and to manage their non-cooperative behavior under social network analysis.

3.4.5. Minimum adjustment cost

As it was mentioned earlier, the feedback mechanism can be realized through either identification and direction rules or minimum adjustment cost mechanisms. The former relies on an iterative approach to modify the opinion of inconsistent DMs during multiple discussion rounds. This can in turn have some disadvantages such as deviation of modified opinions from original ones in a great context, imposing high computational cost, and delaying the CRP. In this regard, in the last decade, we have witnessed the emergence of the minimum

Table 3: Developed feedback mechanisms in the recent research works.

Feedback Mechanism	Description	Reference
Behavioral Mechanisms	Mechanisms based on the trust relationships among DMs	[87, 92, 93]
	Developed mechanisms by considering the attitude of DMs	[95, 96]
	Management of the non-cooperative behavior of DMs	[97–99, 123]
	Management of the biased DMs	[100]
	Willingness of DMs	[101]
Large-Scale GDM Models	Trust-based mechanisms	[109, 116, 118, 124, 125]
	Trust propagation under social network	[111, 114, 115]
	Leadership and non-cooperative behaviors	[52, 103, 105, 106, 108, 112, 126, 127]
	PIS-based social network	[122]
Minimum Adjustment Cost	Behavioral mechanisms	[112, 123]
	Developed mechanisms under social network analysis	[115, 128, 129]
	PIS-based linguistic models	[130]
	bi-level optimization	[131, 132]

adjustment cost notion, where the aim is to adjust the opinions of inconsistent DMs in one step through optimization problems that are subject to different constraints. The reader is referred to a detailed review on these techniques given in [133]. In what follows, we review the most-recent feedback mechanisms constructed based upon the minimum adjustment cost notion.

The basic minimum adjustment cost model that is realized by means of an aggregation operator could be constructed as follows [134]:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i |\sigma_i - \sigma_i'| \\ \text{s.t.} \quad & |\sigma_i' - \sigma_c'| \leq \epsilon, \quad i = 1, \ldots, n, \\ & \sigma_c' = \sum_{i=1}^{n} w_i \sigma_i', \end{aligned} \tag{10}$$

where c_i denotes the unit adjustment cost of DM d_i, σ_i and σ_i' show the initial and modified opinions of DM d_i, respectively, n is the total number of DMs, and σ_c' is the collective opinion.

The unit adjustment cost c_i is usually assumed to be constant, however, in realistic decision making, this value is uncertain and the uncertainty has been realized by means of interval values or distribution uncertainty. To this end, the authors in [129] proposed an estimation mechanism in order to estimate c_i by augmenting three different constraints for giving higher costs to DMs who change their preferences frequently, to model its uncertainty by means of an ellipsoidal set, and to force the sum of total adjustments costs to be lower than the compensation cost of the moderator. The minimum adjustment feedback mechanism developed in [128] is subject to a maximum compromise limit, i.e., the adjusted preferences are required to be within a pre-defined compromise interval, which is rarely studied in social network GDM. Furthermore, a two-stage feedback mechanism is proposed in [131], where in the first stage, the aim is to determine reference points and to adjust the individual positional ordering of DMs, which are then fed into the second stage for the recommendation generation by minimizing the absolute values of the required adjustment for each DM. In addition, a novel framework under minimum adjustment cost method is developed in [130] for linguistic GDM, where the PIS of DMs are considered to not only improve the consensus among DMs, but also to improve the individual consistency under linguistic preference relations. Finally, a bi-level optimization model called consensus mechanism with maximum-return modifications and minimum-cost feedback (MRMCCM) is developed in [132] that is built upon the interactions between the moderator and DMs. In the MRMCCM, DMs are guided by the moderator to modify their opinions for the sake of consensus reaching with minimum cost, while DMs aim to modify their opinions towards maximization of individual return. Table 3 summarizes the developed feedback mechanisms in the recent research works.

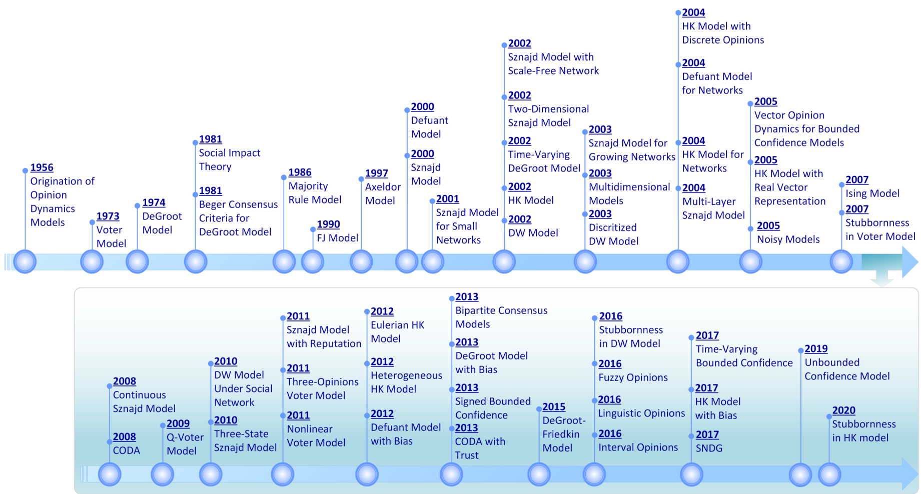

Figure 3: The timeline of some of the most important milestones in opinion dynamics models.

4. Opinion Dynamics for Consensus Reaching

Opinion dynamics can be categorized into the time-modeling category of dynamic consensus approaches [5]. This means that time is involved in opinion evolution of DMs and is an important parameter to model dynamism in the consensus process. A recent review on opinion dynamics models can be found in [135], where it is mainly focused on the application of opinion dynamics models in finance and business and only reviewed the developed DG and bounded confidence models in GDM problems. In contrast, the present work comprehensively reviews the consensus reaching problem under opinion dynamics models, where the developed models based on the management of DMs' behavior, DMs' interactions through social networks, optimization strategies, linguistic opinions, and RL are discussed in detail.

The timeline of some of the most important milestones in opinion dynamics models is represented in Figure 3. In Section 2, we reviewed the basic discrete and continuous opinion dynamics models. Following that, the general structure of opinion dynamics models w.r.t. the fusion process is depicted in Figure 4. In what follows, we review the most-recent developed models to tackle the men-

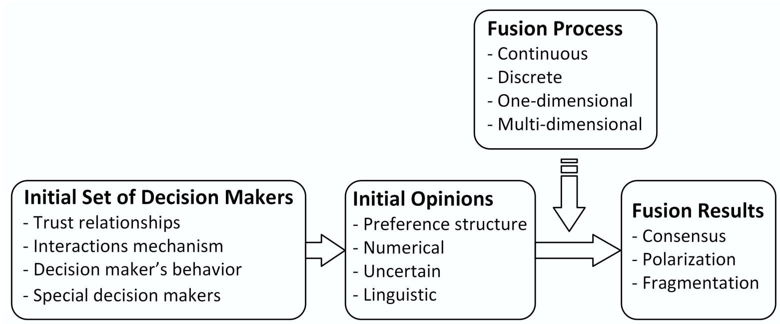

Figure 4: The general framework of opinion dynamics with the fusion process.

tioned issues associated with each block in Figure 4. In this regard, we arguably categorize these models into multiple categories by considering DMs' behavior, developed models based on the social network analysis, minimum adjustment cost or optimization models, and linguistic models. We then provide a detailed description of the new insights that have been brought by means of RL algorithms in classical dynamic consensus and opinion dynamics models.

4.1. Decision makers' behavior

To consider the willingness of DMs in accepting the provided recommendations through feedback mechanisms, bounded confidence models provide the opportunity for DMs to only consider preferences that do not exceed their designated confidence levels. The bounded confidence level could be either known or unknown, where the unknown levels are required to be estimated. Figure 5 depicts a general framework of bounded confidence models with known or unknown confidence levels. The general idea is that for $P_k = (p_{ij}^k)_{n\times n}$ being the original opinion of DM d_k, and $P_f = (p_{ij}^f)_{n\times n}$ being the recommended advice generated through the feedback mechanism, DM d_k accepts this recommendation if $D_{kf} \leq \epsilon_k$, where D_{kf} is some distance function and $\epsilon \in [0, 1]$ is the confidence bound. One way to deal with unknown bound of confidence is to estimate it via an interval $[\underline{b}^k, \overline{b}^k]$ and by setting a bounded confidence threshold τ. The estimation would be assumed accurate in case that $\overline{b}^k - \underline{b}^k \geq \tau$ [136]. Then, based upon D_{kc}, i.e., the distance between opinion of d_k and the collec-

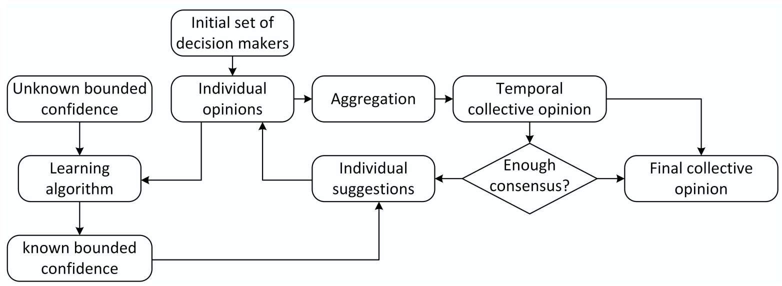

Figure 5: The general framework of recommendation mechanisms with unknown bounded confidence.

tive opinion, feedback rules can be generated. For instance, when $\overline{b}^k - \underline{b}^k \geq \tau$ and $D_{kc} > \underline{b}^k$, the generated advice can be $P_f = P_k + \underline{b}^k / D_{kc} \times (P_c - P_k)$.

Self-persistence behavior refers to the DMs' adherence to their opinions, which should be considered in the weight-adjustment phase of opinion dynamics models [23]. One way to realize this behavior is through a trust network, where the self-persistence degree of DM d_i, i.e., α_i, can form the diagonal elements of the weight matrix $\mathcal{W}$ as follows [137]:

$$w_{ii} = \begin{cases} \alpha_i, & deg_i^- > 0, \\ 1, & deg_i^- = 0, \end{cases} \tag{11}$$

where deg_i^- denotes the sum of the incoming edges to node d_i in the constructed trust network of DMs. Other non-diagonal elements could also be shaped based on α through an influence index,

$$\digamma_i = \frac{\alpha_i + \varkappa + \varpi}{3}, \tag{12}$$

where $\varkappa = deg_i^+/(n-1)$, $\varpi = deg_i^-/3$, deg_i^+ denotes the sum of outgoing edges, and n is the total number of DMs. The self-persistence guided weight assignment could then be as follows [137]:

$$w_{ij} = \frac{\digamma_j}{\sum_k \digamma_k}(1 - \alpha_i) a_{ij}, \quad i \neq j, \tag{13}$$

where $\sum_k \digamma_k$ denotes the sum of influence of one-step neighbors of d_i and

a_{ij} denote the adjacency elements. This mechanism is extended the case that considers the influence of two-step neighbors [137].

The cognitive dissonance of DMs could also shape their communications and updating rule of the opinion dynamics models [138]. One case of the cognitive dissonance is the situation, in which a DM aims to eliminate the uncomfortable feelings, meaning that when $D_{ij}(t)$ (the distance between opinions of d_i and a trusted peer d_j at time-step t) is larger than some confidence threshold ϵ, i.e., $D_{ij}(t) > \epsilon$, DM d_i feels uncomfortable and breaks the connection with DM d_j. Another case refers to a realistic situation that DMs aim to build more connections so they feel the support of more DMs. Let $\mathcal{I}(d_i, \Sigma(t)) = \{d_j | D_{ij} \leq \epsilon, a_{ij} = 1\}$ be the confidence set of d_i. Then, in case DMs d_i and d_j have a common trusted peer, shown by d_k, where $d_k \in \mathcal{I}(d_i, \Sigma(t)) \cap \mathcal{I}(d_j, \Sigma(t))$, and $D_{ij}(t) \leq \epsilon$, DMs d_i and d_j can make a connection. Once the connections and eliminations are done at time-step t, the weight-adjustment can be simply fulfilled as follows [138]:

$$w_{ij} = \begin{cases} \frac{1}{|\mathcal{I}(d_i, \Sigma(t))|}, & d_j \in \mathcal{I}(d_i, \Sigma(t)), \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

The concept of leadership behavior has also been used to guide feedback mechanisms in opinion dynamics model [139, 140]. The leader is usually referred to DMs with high influence in the trust network, where different approaches are proposed for identifying the set of leaders. One common way is to divide the complex network of DMs into multiple sub-networks [141], construct the accessibility matrix [142], and perform iterative searches in each sub-network to identify DMs with more influential connections [143]. Other than leadership in a group of DMs, the pressure imposed by the group could also be categorized into the behavioral category of opinion dynamics models [144]. This is proposed to model the situation, in which a DM feels pressure to give away an opinion which is similar to the collective opinion of the group. The authors in [145] addressed this issue by forming an update rule as given below:

$$\sigma_i(t+1) = \frac{(1-\rho_i)\sum_{j\in\mathcal{I}(d_i,\Sigma(t))}\sigma_j(t)}{|\mathcal{I}(d_i,\Sigma(t))|} + \rho_i\sigma_c(t), \tag{15}$$

where $\sigma_c(t)$ is the weighted average of DMs' opinions and ρ_i is used to account for the group pressure. Other than the group pressure, a DM may also suffer from the peer pressure [146]. Other behavioral actions such as stubbornness [147] and prejudice [148] could also affect the opinion dynamics models. To model these all behavioral actions, a stress function of the following form is proposed in [149]:

$$\begin{aligned}\Theta_i(\sigma_i(t),\sigma_i(t-1),t) &= \zeta_i\left(\sigma_i(t)-\sigma_i^+(t)\right)^2\\ &+ \upsilon(t)\sum_{j=1}^{n}|a_{ij}|\left(\sigma_i(t)-\mathrm{sign}(a_{ij})\sigma_j(t-1)\right)^2,\end{aligned} \tag{16}$$

where ζ_i is used to model the prejudice of DM d_i, $\sigma_i^+(t)$ shows the constant prejudice of the DM d_i, and $\upsilon(t)$ denotes the peer pressure. Following this structure, the aim is to minimize the stress function so as to find the update rule of the fusion process. It is resulted that the following update rule will minimize the stress function given in (16):

$$\sigma_i(t) = \frac{\zeta_i\sigma_i^+ + \upsilon(t)\sum_{j=1}^{n}a_{ij}\sigma_j(t-1)}{\zeta_i+\upsilon(t)\gamma_i}, \tag{17}$$

where $\Gamma = diag[\gamma_1,\ldots,\gamma_n] = L + A$, $\gamma_i = \sum_{j=1}^{n}a_{ij}$, and L and A are the Laplacian and signed adjacency matrix of the DMs' signed network. By resorting to the graph theory, the willingness of DMs [9], the problem of unilateral DMs [150] and antagonistic and indifference DMs [151] have also been recently addressed through opinion dynamics models.

4.2. Social networks

Most of the recent research works fall into this category of methods for opinion dynamics models. One of the most-recent advances rely on the continuous opinion and discrete action (CODA) model [152], which can be categorized into continuous opinion dynamics model. Developing opinion dynamics models with the simultaneous evolution of opinions and actions under social network analysis is an interesting research topic. This is addressed in [153] under the assumption that DMs' opinions are private [154] and cannot be obtained by others unless they are directly connected in the social network. The actions, however, are public and DMs are aware of others' actions. The relationship between actions and opinions is modeled as follows [153]:

$$A_i(t) = \begin{cases} 0, & \sigma_i(t) \in [0, h_i) \\ 1, & \sigma_i(t) \in [h_i, 1], \end{cases} \tag{18}$$

where h_i is a threshold for action selection of DM d_i. Then, based upon the relationships among DMs, the update rule given in (19) is constructed, where $\mu \in (0, 0.5]$ is a convergence parameter [153].

$$\sigma_i(t+1) = \begin{cases} \sigma_i(t), & a_{ij} = 1 \wedge |\sigma_i(t) - \sigma_j(t)| > \epsilon, \\ \sigma_i(t) + \mu\left(\sigma_j(t) - \sigma_i(t)\right), & a_{ij} = 1 \wedge |\sigma_i(t) - \sigma_j(t)| \leq \epsilon, \\ \sigma_i(t), & a_{ij} = 0 \wedge |\sigma_i(t) - A_j(t)| > \epsilon, \\ \sigma_i(t) + \mu\left(A_j(t) - \sigma_i(t)\right), & a_{ij} = 0 \wedge |\sigma_i(t) - A_j(t)| \leq \epsilon. \end{cases} \tag{19}$$

Recently, a novel model under the structure of a social graph is proposed in [155], where the DMs' interactions do not rely on the proximity of their opinions, but on the influence of their opinions on one topic to other topics. The continuous opinion evolution of DMs is modeled as follows [155]:

$$\sigma_i(t + dt) = \sigma_i(t) + \mathcal{C} d_{\sigma_i}(t), \tag{20}$$

where $\mathcal{C}$ is used to denote the influence of opinions and d_{σ_i} is as follows [155]:

$$\begin{aligned} d_{\sigma_i} = {} & \frac{1 - \beta(P_i)}{n-1} \sum_{j \neq i} \zeta(P_i, P_j)[\sigma_j(t) - \sigma_i(t)]dt \\ & + \beta(P_i)[\mathbf{u}(P_i) - \sigma_i(t)]dt + \gamma \mathbf{w}_i(t), \end{aligned} \tag{21}$$

where $\beta(P_i) \in (0, 1]$ is the insensitivity of DM d_i that holds the P_i personality [156], $\zeta(P_i, P_j)$ is used to model intensity of interactions among DMs, $\mathbf{u}(P_i)$

accounts for the prejudice of a DM, and $\mathbf{w}_i(t)$ denotes the endogenous process of opinion evolution for each DM [157]. Other models are also developed for different types of interactions in opinion evolution of DMs. For a set of homogeneous DMs, the effect of interaction intensity is investigated in [158] for biased (opinion-dependent) and unbiased (opinion-independent) intensity, where the results are then extended to heterogeneous DMs in [159]. Furthermore, by considering the dependency of DMs' interactions to their current and past opinions, a memory-based connectivity mechanism for opinion dynamics models under social network is proposed in [160]. In addition, for social networks with switching topology, an opinion dynamics model is proposed in [161], where under an arbitrary switching signal, the system bipartite (polarization) consensus or consensus is guaranteed. The evolution of the network over time is studied in [162] by resorting to constructing a rule-base by means of a distance matrix, which contains the proximity of opinions of paired DMs. The network could also evolve w.r.t. temporal activity patterns such as contact strength of DMs and daily patterns, where the impact of these temporal activities on the speed of consensus is investigated in [163]. In order to improve the CRP in GDM under opinion dynamics models and to fully benefit from the evolution of social networks, the concept of local world opinion, which is extracted from individuals' common friends is proposed in [164], where the evolution of the network is realized through the distance between individual opinions and network structure similarity.

Another interesting research trend in social network-based opinion dynamics models is the attempt towards handling uncertainties in DMs' opinions [165, 166]. One way to consider uncertainties is to introduce novel preference structures for DMs to express their opinions. Recently, the concept of interval-valued opinions by considering the uncertainty tolerance of DMs is proposed in [167]. It is proposed to model opinions by numerical intervals $\sigma_i(t) = [\underline{\sigma}_i(t), \overline{\sigma}_i(t)] \subseteq [0,1]$, with $\underline{\sigma}_i(t) \leq \overline{\sigma}_i(t)$. Then, for the DMs with uncertainty tolerances, the opinion evolution follows an updating rule as given below

[167]:

$$\underline{\sigma}_i(t+1) = T_i\underline{\sigma}_i(t) + \sum_{j\neq i} w_{ij}\underline{\sigma}_j(t), \tag{22}$$

$$\overline{\sigma}_i(t+1) = T_i\overline{\sigma}_i(t) + \sum_{j\neq i} w_{ij}\overline{\sigma}_j(t), \tag{23}$$

where T_i is the trust of DM d_i. As for DMs without uncertainty tolerances, the update rules are the same as above, however, the terms $\underline{\sigma}_j(t)$ and $\overline{\sigma}_j(t)$ in the summations are replaced with $f_{ij}(t)$, which is an accurate estimation of opinion d_j from d_i. Linguistic models have also been proposed to deal with associated uncertainties, which will be reviewed in next sections.

4.3. Social Network DeGroot Models

A recent milestone in opinion dynamics models under social networks is the social network DeGroot model (SNDG). In the SNDG, DMs' interactions through a social network is modeled via a directed graph $G(\mathcal{D}, \mathcal{V})$, where $\mathcal{D}$ is the set of DMs as before and $\mathcal{V}$ is a set of two-tuples $(d_i, d_j) \in \mathcal{V}$ that defines DM d_i directly trusts DM d_j. Suppose that $\mathcal{A} = (a_i, a_j)_{n\times n}$ is the associated adjacency matrix of G and DM d_k assigns a trust degree of β_k to his own opinion and gives $(1-\beta_k)$ to other DMs' opinions. In this regard, the weight that DM d_k assigns to his peer d_l is given as follows [141]:

$$w_{kl} = \frac{(1-\beta_k)a_{kl}}{\sum_{l=1,l\neq k}^{n} a_{kl}}. \tag{24}$$

Then, the opinion evolution of DM d_k in the SNDG is represented as [141]:

$$\sigma_k(t+1) = \beta_k\sigma_k(t) + \sum_{l=1,l\neq k}^{n} w_{kl}\sigma_l(t), \tag{25}$$

which can also be represented in the matrix format as given below:

$$\Sigma(t+1) = \mathcal{W}\Sigma(t), \tag{26}$$

where

$$\mathcal{W}=\begin{bmatrix} \beta_1 & w_{12} & \dots & w_{1n} \\ w_{21} & \beta_2 & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & \beta_n \end{bmatrix}, \tag{27}$$

and $\Sigma(t)=[\sigma_1(t),\dots,\sigma_n(t)]^T$. In the SNDG, it is well-studied that leadership and trust relationships improvements are key elements in the evolution of individual's opinions. To this end, a model is presented in [168] that builds a bridge between opinion dynamics models and GDM by resorting to the concept of leadership, where clique-based strategies are proposed to improve trust relationships by manipulation. For hybrid opinion dynamics models, i.e., models that involves two types of DMs including leaders and followers, an SNDG is proposed in [169] that initially divides the network into multiple sub-networks to construct the set of leaders and followers. Then, based on the constructed sub-networks and the shortest-path concept in graph theory, the level of trusts among DMs are determined to be ultimately used for the weight adjustment of DMs. Finally, a minimum adjustment cost model is proposed for the sake of consensus reaching. However, the level of trust among DMs could also evolve due to the changes in the opinion similarities. In this regard, an SNDG is proposed in [170] that simultaneously makes use of the historical trust degrees and opinion similarities of DMs to construct an opinion dynamics-based endogenous feedback mechanism and a trust evolution-based exogenous feedback mechanism for consensus reaching. Other than the trust, self-confidence of DMs and node degree of network play an important role in SNDG, where their impacts have been discussed in detail in [171].

4.4. Optimization models

The change of opinion means cost and the sources for doing so are limited. To this end, the concept of minimum adjustment cost mechanism has been devised for opinion dynamics models to minimize the cost of feedback mechanism. In this section, we review the most-recent minimum adjustment cost techniques along with developed optimization frameworks under opinion dynamics models.

Under the DeGroot structure, a minimum adjustment mechanism is proposed in [172] in order to minimize the required adjustments of the initial opinions for the sake of consensus building. This idea is realized through social networks, where the network with initial and adjusted initial opinions are modeled via graphs $G(V, E, \Sigma^0)$ and $\overline{G}(V, E, \overline{\Sigma}^0)$, where V and E denote nodes and edges of the graph, respectively. For $\overline{\sigma}_{ij}(t)$ being the d_i's adjusted opinion w.r.t. the jth alternative, with $i = 1, \dots, n$ and $j = 1, \dots, q$, and c_j^* being the consensus opinion of the jth alternative, which is obtained as a weighted average of initial opinions, then, the optimization model of the following form is constructed to obtain optimal adjusted initial opinions [172]:

$$\begin{aligned} \min_{\overline{\Sigma}^0} \quad & \sum_{i=1}^{n}\sum_{j=1}^{q} |\overline{\sigma}_{ij}^0 - \sigma_{ij}^0| \\ \text{s.t.} \quad & |\lim_{t\to\infty} \overline{\sigma}_{ij}(t) - c_j^*| \le \gamma, \;\; i = 1, \dots, n, j = 1, \dots, q \\ & 0 \le \overline{\sigma}_{ij}^0 \le 1, \;\; i = 1, \dots, n, j = 1, \dots, q \\ & 0 \le c_j^* \le 1, \;\; j = 1, \dots, q \end{aligned} \tag{28}$$

where γ is an acceptable level of consensus, collective opinions $(c_1^*, \dots, c_q^*)$ are the decision variables and the aim is to obtain the optimal collective opinions. For large-scale decision problems, a feedback mechanism based on the bounded confidence model is proposed in [173], where DMs are initially divided into multiple clusters and a specific advice is generated for each cluster by resorting to an optimization scheme. For multidimensional opinions $\Sigma^k = (\sigma_{ij}^k)_{m\times q}$ and $\Sigma^s = (\sigma_{ij}^s)_{m\times q}$, a distance function of the following form is presented in [174]:

$$D(\Sigma^k, \Sigma^s) = \frac{1}{m\times q}\sum_{i=1}^{m}\sum_{j=1}^{q} |\sigma_{ij}^k - \sigma_{ij}^s|. \tag{29}$$

Then, under the bounded confidence model, an optimization model of the following form is proposed [174]:

$$
\begin{aligned}
\min \quad & \sum_{k=1}^{n} D(\overline{\Sigma}^k, \overline{\Sigma}^c) \\
\text{s.t.} \quad & -\epsilon^k \leq \overline{\sigma}_{ij}^k - \sigma_{ij}^k \leq \epsilon^k, \;\; i = 1, \ldots, q, j = 1, \ldots, m, \\
& \overline{\sigma}_{ij}^c = \sum_{k=1}^{n} w_k \overline{\sigma}_{ij}^k, \\
& 0 \leq \overline{\sigma}_{ij}^k \leq 1,
\end{aligned}
\tag{30}
$$

where $\overline{\Sigma}^c$ is the adjusted collective opinion, $\overline{\Sigma}^k$ $(k = 1, \ldots, n)$ are the decision variables, and the aim is to obtain the optimal solution $\overline{\Sigma}^{k,*}$ for $\overline{\Sigma}^k$. The first constraint is given to produce acceptable recommendations by taking into account the bounded confidence of each DM. A consensus reaching mechanism for hybrid opinion dynamics models under social network is proposed in [169]. The authors proposed to initially divide the network into multiple sub-networks and employ the Floyd algorithm for finding the shortest path between each pair of nodes for the sake of weight adjustment. Then, an optimization model is suggested for consensus reaching, where it augments two optimization models for minimizing the opinion adjustment cost of leaders and the weights that each leader assigns to others. Consensus reaching with minimum adjustment cost under dynamic evolution of opinions and weights [175], change of topology for maximizing the influence on the network [176], optimizing the trade-off between group and individual interactions [177], and consensus reaching in finite-time by means of distributed optimization over digraphs [178], are some recent developments in optimization schemes realized through opinion dynamics models.

4.5. Linguistic models

As it was mentioned earlier in Section 4.2, uncertainty in opinions can also be modeled through linguistic models. This is a new concept in opinion dynamics models and some efforts have been devoted to the design of linguistic models based on the 2-tuple and fuzzy linguistic preference structures for the opinion evolution [179]. In what follows, the most-recent research works concerned with linguistic models are reviewed.

In [180], the authors proposed a personalized individual semantic (PIS) linguistic opinion dynamics model under the bounded confidence framework. Following Definition 3 and the idea of numerical scale models for LTSs [181], the numerical scale of an LTS $\mathcal{S} = \{s_0, \ldots, s_{2r}\}$ for (s_i, α) is defined as follows:

$$NS(s_i, \alpha) = \begin{cases} NS(s_i) + \alpha \left(NS(s_{i+1}) - NS(s_i) \right), & \alpha \geq 0, \\ NS(s_i) + \alpha \left(NS(s_i) - NS(s_{i+1}) \right), & \alpha < 0. \end{cases} \tag{31}$$

Then, the process of a linguistic model with PIS consists of three steps; (1) semantics translation, in which a linguistic term is translated into a semantic in the interval $[0, 1]$; (2) numerical computation, which takes semantics as input and outputs a numerical value in interval $[0, 1]$; (3) semantic retranslation, in which the output of step 2 will be retranslated into a 2-tuple. In this regard, the proposed model can be constructed by following three main steps.

The first step for DMs is to estimate the semantics of other peers as given below:

$$e_{ij}(t) = \kappa NS_j(\sigma_j(t)) + (1 - \kappa) NS_i(\sigma_j(t)), \tag{32}$$

where $e_{ij}(t)$ with $i, j = 1, \ldots, n$ and $i \neq j$ denotes the estimated semantic of DM d_j by DM d_i based on their familiarity modeled by κ. In the second step, the confidence set of DM d_i can be constructed as follows:

$$\mathcal{I}(d_i, \sigma_i(t)) = \{d_j | \; \| NS_i(\sigma_i(t) - e_{ij}(t)) \| \leq \epsilon\}, \tag{33}$$

and, then, the weights of DMs can be adjusted in the same way as discussed in Eq. (14). The update rule of semantics is proposed to be as follows:

$$NS_i(\sigma_i(t+1)) = w_{i1}(t) e_{i1}(t) + \ldots + w_{in}(t) e_{in}(t). \tag{34}$$

Finally, in the third step, the evolution of opinions can be modeled as given below:

$$\sigma_i(t+1) = NS_i^{-1}(NS_i(\sigma_i(t+1))), \tag{35}$$

where NS_i^{-1} is given in Definition 2 in [180]. This scheme has enabled the emergence of other opinion dynamics models under multi-granular [182] and probabilistic linguistic models [183].

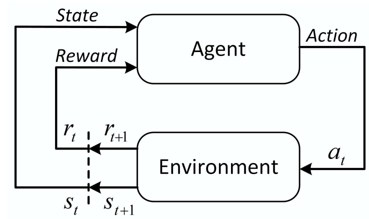

Figure 6: Interactions of an RL agent with its environment.

4.6. Reinforcement learning-based models

The essence of RL is learning by interacting with an environment by taking actions. As it can be seen in Figure 6, an RL agent takes an action a_t in its environment and based upon the consequences of its actions, which is the received reward r_t from the environment, it can learn how to alter its behavior towards collecting more rewards. For each state transition ($s_t \rightarrow s_{t+1}$) in the environment, the agent receives a feedback through a scalar reward r_{t+1}. The agent aims at learning a policy that maximizes the expected return (also known as discounted reward). In brief, in case the environment satisfies the Markov property, that is the current state is only dependent to the previous state, RL can be realized through a Markov decision process (MDP). The consensus process in GDM models and the fusion process in opinion dynamics models (despite of the memory-based mechanisms discussed earlier [160]), can be treated as MDPs and the solutions can be achieved by means of RL algorithms. A very limited number of opinion dynamics and GDM models have considered the application of RL, which are reviewed in this section.

For consensus boost and recommendations to guide DMs in opinion dynamics models, a framework based on RL is presented in [184]. The authors proposed a state space to contain opinions as $S = \{s_i | s_i \in [0,1], i = 1, \dots, n\}$, and, each agent can take an action from the constructed action space $A = \{a_i | a_i \in [-1,1], i = 1, \dots, n\}$. Then, a reward signal is constructed as follows:

$$r_t = w_1 r_{ac}(t) + (1 - w_1) r_{cd}(t), \tag{36}$$

where r_{ac} and r_{cd} account for the adjustment cost and consensus boost, respectively, and w_1 is used to model the trade-off between them. The adjustment cost is modeled as given below:

$$r_{ac}(t) = -\sum_{i=1}^{n} |a_i(t)|, \tag{37}$$

where it is the negative sum of actions taken by agents. For the consensus boost part, it is required to find the state transition rule, which is realized by means of HK model. In this regard, for those agents who do not adopt adjustment actions, the following transition rule is adopted:

$$s'_i(t+1) = \frac{1}{|\mathcal{I}(s'_i(t))|} \sum_j s'_j(t), \tag{38}$$

where $\mathcal{I}(s'_i) = \{s'_j(t)| \, s'_j(t) - s'_i(t)| \leq \epsilon\}$, with ϵ being the bounded confidence threshold. Then, for other agents, the transition law is as follows:

$$s_i(t+1) = \frac{1}{|\mathcal{I}(s_i(t) + a_i(t))|} \sum_j s_j(t) + a_j(t), \tag{39}$$

where $\mathcal{I}(s_i(t) + a_i(t)) = \{s_j(t) + a_j(t)| \, (s_j(t) + a_j(t)) - (s_i(t) + a_i(t))| \leq \epsilon\}$. Finally, $r_{cd}(t)$ is constructed as follows:

$$r_{cd}(t) = n[\mathrm{cd}(t) - \mathrm{cd}'(t)], \tag{40}$$

with $\mathrm{cd}(t) = 1 - \frac{\sum_{i=1}^{n} |s_i(t+1) - \frac{\sum_{i=1}^{n} s_i(t+1)}{n}|}{n}$ and $\mathrm{cd}'(t) = 1 - \frac{\sum_{i=1}^{n} |s'_i(t+1) - \frac{\sum_{i=1}^{n} s'_i(t+1)}{n}|}{n}$. Once the set of actions, rewards, and transition laws are constructed, any RL algorithm (depending on the nature of actions and states) can be employed in the learning process of the agent, where an actor-critic learning algorithm is used in [184] for the sake of learning. By considering the effect of stubborn, controlled, and uncontrolled agents, an RL-based mechanism is proposed in [185] for opinion shaping in opinion dynamics models by moderating the behavior of influential DMs. The opinion evolution is modeled via a value iteration mechanism, where the policy evaluation is then converted into a shortest path problem. Furthermore, a model based on the Q-learning algorithm for RL agents in presented in [186], where agents' opinions are assumed to be binary, i.e., $\sigma_i(t) \in \{-1, +1\}$, and at each time instant, an agent is randomly selected and expresses its opinion to a randomly selected neighbor. By considering an internal evaluation function $\mathcal{Q}$ based on the social response of other peers, an update rule of the following form is constructed:

$$\mathcal{Q}_i(\sigma_i(t+1)) = (1-\alpha)\mathcal{Q}_i(\sigma_i(t)) + \alpha r_i(t), \tag{41}$$

where $r_i(t) = \sigma_i(t)\sigma_j(t)$ is the reward signal. This is treated as Q-values required in training of an agent based on the Q-learning algorithm. For the same opinion dynamics structure, a game theoretic-based mechanism is employed in [187] to model agents' interactions, where the Q-learning algorithm is used for each agent to learn the optimal policy, which is gaining more rewards in their interactions with other peers. In case a neighbor of an agent has the same opinion, the agent will receive a reward of $+1$, otherwise, -1. Agents opinions are also supposed to be binary and to be selected from $\{-1, +1\}$. This framework is extended in [188] to the case, in which agents can take more than two actions. Another game theoretic-based opinion dynamics model is proposed in [189], where agents communications are random, however, each agent who decides to express its expression is penalized with a cost, and it will be penalized more in case the neighboring agent decides not to reply to its opinions or express disagreeing opinions. Without considering the exploration and exploitation [190] in taking actions, a framework based on RL is developed in [191], where agents are assumed to express their opinions randomly from a continuous set of actions to communicate in a social network towards maximizing the number of their followers in mainstream media. RL has also been used for conventional GDM models for DMs' weight adjustment in context-aware heterogeneous decision environments [192, 193]. Table 4 summarizes the developed opinion dynamics models.

5. Challenges and Future Trends

A considerable number of research works have been recently devoted to the design of CRP for GDM as reviewed in the present work. Based on the reviewed

Table 4: Developed opinion dynamics models in the recent literature works.

Category	Model	Characteristics	Reference
DMs' behavior	Bounded confidence	Willingness of DMs, known and unknown confidence bound	[136]
		Cognitive dissonance behaviors	[138]
		Opinion natural reversals dynamics	[139]
		Leadership (opinion leaders and opinion followers)	[140, 143]
		Group and peer pressure	[144, 145]
		Antagonistic and indifference behaviors between individuals	[151]
	DeGroot	Self-persistence of DMs	[137]
		Leadership with minimum number of interactions	[141]
		Peer pressure and stubbornness of DMs	[146, 149]
		Willingness and self-confidence of DMs	[9]
Social networks	Bounded confidence	Opinion and action evolution, modified expressed private opinions	[153, 154]
		Individual and local world opinion	[164]
		Stochastic interactions	[157]
		Dynamic interactions among DMs	[162]
		Fuzzy inference approach to describe bounded confidence	[166]
	Stochastic models	Repulsive interactions between DM's opinions	[155]
		Modulation of the interaction intensity	[159]
		Centralized tuning of the strength of interactions between DMs	[158]
	Hybrid model	Interactions depend on current and past opinions	[160]
	DeGroot	Competition between DMs and switching topology	[161]
		SNDG	[141, 168–171]
		Failure mode and effect analysis	[165]
		Numerical interval opinions and uncertainty tolerances	[167]
	Deffuant	Temporal networks with ordering of interactions	[163]
Optimization models	Bounded confidence	Willingness of DMs	[173, 174]
		Self-trust and fuzzy trust sets	[175]
		Network rewiring for maximizing influence on overall opinion	[176]
	DeGroot	Network partitioning algorithm	[172]
	Hybrid model	Network partitioning algorithm	[169]
		Combining pairwise and group interactions for DMs	[177]
	Interconnected dynamics	Distributed optimization problems over an unbalanced digraph	[178]
Linguistic models	Bounded confidence	Two-tuples linguistic model with numerical scale	[179]
		Personalized individual semantics model	[180]
		Multi-granular unbalanced linguistic term sets	[182]
		Opinion similarity, DMs' credibility and bounded rationality	[18]
RL-based models	bounded confidence	Consensus boost and recommendation mechanism	[184]
	Stochastic	Stubbornness of DMs	[185]
	Binary opinions	Internal evaluation function based on the social responses	[186]
	Game-theoretic model	Reward shaping through interactions with peers	[187–189]
	Gossiper-Media model	Maximizing the number of followers in mainstream media	[191]
	Fuzzy consensus model	Context-aware heterogeneous decision environment	[192, 193]

papers, we have found some challenges that need to be addressed in future works concerning with the design of feedback mechanisms for the sake of consensus reaching.

1. The first issue is regarding the recently-developed representation structures for opinion expression. As it was mentioned in Section 3.4.1, new representation structures such as Z^E-numbers are recently developed, where, on one hand, the development of operational tools such as aggregation and similarity-checking measures, could be an important research attempt towards evoking the information of such representation structures as much as possible. On the other hand, these newly-developed representation structures pave the way for the design of novel and efficient CRPs. For instance, the problem of minimum adjustment cost, social network-based analysis of GDM, linguistic opinion dynamics models, and managing the behavior of DMs could all be addressed for these new representation structures.
2. RL has been recently deployed in many control and learning applications. Throughout our review on CRPs for GDM, we witnessed the lack of applications of this powerful tool in research works. The CRP is a dynamic mechanism by its nature, because it is modeling the evolution of the consensus among DMs. What makes the application of RL in GDM possible is the fact that regardless of other involved parameters such as the weights of DMs or attributes, the consensus among DMs at each discussion round is dependent to only the consensus of the previous discussion round. This conducts and satisfies the Markov property in MDPs, and, therefore, RL is applicable in modeling the CRP in conventional GDM models. RL can be implemented for the adjustment of the weights of DMs, attributes, and alternatives, and even in adjustment of the feedback parameter for consensus reaching through feedback mechanisms. In this regard, the environment would be discrete and depending on the purpose of the RL agent, its actions could be either discrete or continuous. The same is true in the design of feedback mechanisms based on the opinion dynamics models,

where an RL agent can be assigned to the fusion process for managing the evolution of opinions. The old problem of the trade-off between the consensus speed and harmony degree of DMs (which states that DMs aim to keep their original opinions as much as possible) can be realized by means of RL by modeling the consensus process through game-theoretic mechanisms.

3. Even though some advancements have been made to the linguistic opinion dynamics models, however, the results are required to be extended to other linguistic representation structures as well. This is of paramount importance due to the fact that different DMs might need to express their opinions using different preference structures due to their level of knowledge or background. Following this, the design of novel heterogeneous GDM models under opinion dynamics could be another challenge and future trend towards paving the way of the application of the developed linguistic opinion dynamics models.

4. A common assumption in the reviewed research works is that agents with similar opinions which are less than a given threshold, i.e., the bound of confidence of agents, are able to communicate in order to modify their opinions. In this mechanism, other neighboring agents who do not fall into the confidence bound of agents are ignored. However, it is quite possible in the real life situations where agents might have friends with quite different opinions. Taking the opinions of these long-range neighbors who are out of the confidence bound could also help with the consensus reaching. This idea is missing in the most-recent research works.

In an opposite situation to what stated in item 4, another way to treat neighboring agents is when not all the neighbors of an agent participate in updating an agent's opinion. Instead, some of them could be selected through a similarity-based probability rule. In this case, the convergence problem and its properties could be an interesting research study. Furthermore, the integration of LTSs into opinion dynamics models has been recently addressed, however, inclusion of more complex representation structures such as Z-numbers could also be considered. Besides, the problem of unbalanced LTSs could also be addressed under the opinion dynamics models.

6. Concluding Remarks

In this paper, we surveyed the most recent research works that are concerned with the CRP in GDM problems. We followed two different research directions including the classical dynamic consensus processes and opinion dynamics models. For the classical models, a detailed description of their major components including preference representation structures and decision environment was provided. We then surveyed the most productive research works that aimed at designing a feedback mechanism for such classical models. By introducing the most popular opinion dynamics models, a very detailed review of such models was provided by identifying the mainstream and trends of the research works. The challenges that both classical consensus and opinion dynamics models are facing were discussed and new research trends were introduced for future studies. Throughout our study, we realized that due to the technological advancements, there is a strong tendency among researchers towards proposing consensus reaching or opinion dynamics models that involve a large number of DMs who are assumed to interact through a social network platform. Managing the behavior of the involved DMs in such models was another important research trend in recent works. Furthermore, it was noticed that a considerable number of research works are focused on the conversion of the decision problem into an optimization problem for guaranteeing the minimum adjustment cost through the feedback mechanism. What is missing and is paid less attention to, is the integration of learning algorithms into the GDM and opinion dynamics models. Learning algorithms such as RL are powerful tools that despite the size of the decision problem can lead to optimal or near-optimal solutions.

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