

Classical EHL Versus Quantitative EHL: A Perspective Part I—Real Viscosity-Pressure Dependence and the Viscosity-Pressure Coefficient for Predicting Film Thickness

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Abstract That classical elastohydrodynamic lubrication (EHL) is not a quantitative field can be illustrated by its failure to provide a consistent and rigorous definition of the viscosity-pressure coefficient. Indeed, if the pressure dependence of viscosity cannot be accurately described, then the viscosity-pressure coefficient cannot be defined. Classical EHL has employed fictional narratives to justify the pressure dependences that have been utilized. In this context, the purpose of this perspective article is to review specific and real needs from EHL and to show that data and models describing the viscosity-pressure dependence are already available and how they can properly be used. The final aim is to encourage researchers to change their philosophy of classical EHL to a quantitative approach, in which every hypothesis and every result, whether experimental or numerical, would be justified on the basis of acceptable physics.

Keywords Elastohydrodynamic lubrication · Rheology of lubricants · Viscosity-pressure dependence · Viscosity-pressure coefficient · Quantitative approach · Primary laboratory data

List of Symbols

a, b, c, d Parameters of the Dowson and Higginson density–pressure empirical relationship

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E', E	Equivalent Young's modulus, solid body Young's modulus
G	Hamrock and Dowson dimensionless parameter
h	Film thickness (m)
H	Hamrock and Dowson dimensionless film thickness
L	Moes dimensionless parameter $L = G\sqrt{4}U$
M	Moes dimensionless parameter $M = \frac{w}{U^{3/4}}$
Md	Mean relative deviation for film thickness measurements normalized by the Hamrock and Dowson predictions
Mad	Mean of the absolute values of the relative deviation for film thickness measurements normalized by the Hamrock and Dowson predictions
p	Pressure (Pa)
R	Reduced radius of curvature of the two surfaces (m)
$SdMd$	Standard deviation on Md
$SdMad$	Standard deviation on Mad
u	Solid surface velocity (m/s)
\bar{u}	Mean entrainment velocity (m/s)
U	Hamrock and Dowson dimensionless parameter
w	Normal load (N)
W	Hamrock and Dowson dimensionless parameter
α	Viscosity-pressure coefficient (GPa^{-1})
α^*	Reciprocal asymptotic isoviscous pressure coefficient (GPa^{-1})
μ	Viscosity (Pa.s)
ν	Poisson's coefficient
ρ	Density (kg/m^3)

Subscripts

0.1	Refers to ambient pressure (0.1 MPa)
1, 2	Refers to solids 1 or 2
exp	Means an experimental value
c	Indicates central film thickness
HD	Refers to Hamrock and Dowson parameter
inf	Indicates the occurrence of an inflexion
LMS	Means obtained through a least mean square regression
m	Indicates minimum film thickness
secant	Calculated for a given pressure increase from the ambient value
tangent	Calculated for a pressure increment

1 Introduction

The effective lubrication of machine elements such as rolling element bearings or gears is essential, in our personal everyday life activity as well as in manufacturing, transportation and energy production. New demands have emerged from society related to energy issues, conserving natural resources, preserving the environment and worldwide economic competition. Combined with the continuous evolution of technology, the margin for safety in the operation of highly loaded lubricated mechanisms has drastically been reduced.

As reported by Dowson [1], at the earlier stage of elastohydrodynamic lubrication (EHL), the typical film thicknesses were of the order of one micrometer or a little less; nowadays, they can be as low as some nanometers. This has made EHD contact operation much more sensitive to many factors, such as surface roughness, additives and contaminants, or to thermal effects, shear thinning behavior of the lubricant, inlet starvation and others.

In this context, predicting film thickness or friction with good confidence has become much more relevant and important. However, in spite of the progress achieved in the field of modeling and simulation and in the improvement of the experimental techniques for investigating the contact response, it must be stated that EHL has not matured into a physics-based science. The literature clearly shows that some properties of lubricants have been assumed without regard to sound theory or measurement, making the ensuing interpretations of EHL behavior inexact: the pressure dependence of viscosity is regularly misinterpreted or adjusted, interchanging of thermal and non-Newtonian effects sometimes occurs, density variations that have a direct influence on film thickness are often ignored, and finally, the set of physical properties necessary for accurate film thickness prediction seem to be unknown to the majority in the classical EHL community.

This situation should no longer be accepted because the EHL researchers have at this time access to all the required

devices, models, data and knowledge to change their practices from the classical EHL view to a quantitative (physics-based) approach. The present authors have decades of experience [2–7] in the rheological and physical characterization of lubricants under high pressure. They are more motivated and concerned than ever for the relevant use of realistic properties in EHL [8–11], especially for predicting film thickness and friction from data independent of a contact measurement, also sometimes called primary laboratory data.

Thus, the purpose of this article is neither to incriminate publications from the past years nor to redefine the expression “elastohydrodynamic lubrication”; it is rather to recount the specific and real needs and to show that data and models describing the viscosity-pressure dependence are already available for many lubricants and how these data can be properly used in the EHL context. Finally, our aim is to encourage people to change their philosophy of EHL to a quantitative approach, in which every hypothesis and every result, whether experimental or numerical, would be justified on the basis of acceptable physics and not based upon fictional narratives.

As a first step, in this perspective paper, we focus only on the pressure dependence of the lubricant viscosity, one of the major effects influencing EHL. Other features such as compressibility and shear thinning will be addressed in later articles. Thus, the objectives are simply (1) to recount the basic goals from the EHL point of view, (2) to report facts from the related literature and (3) to invite the EHL community to change their approach to make this branch of lubrication evolve toward a quantitative science.

2 EHL Theory, Equations and Requirements for the Lubricant

Regarding equations and models, a very brief literature survey is given. The reader is invited to refer to general textbooks on lubrication to find more details and to revisit the underlying theory. EHL combines at least three major and interrelated mechanisms: the elastic deformation of the solids as the contact pressure is large enough to cause significant deformation, the hydrodynamic effect in the fluid and the pressure dependence of the lubricant properties. The first mechanism concerns exclusively the solids, and therefore, it will be ignored in the following. The Reynolds equation is classically used to model the hydrodynamic effect. It was derived by Osborne Reynolds by simplifying the Navier–Stokes equations using the thin-film (or Reynolds) assumption, neglecting inertia and external forces, considering laminar and isothermal flow, no-slip boundary conditions and smooth surfaces. The viscosity was also assumed to be constant. This equation

relates hydrodynamic pressure p to film thickness h and has been used as the basis of all lubrication theories. Using the mass continuity equation to integrate the velocity through the film thickness, considering that the surface velocities are parallel to the x -axis and do not vary in space, the Reynolds equation reads for steady-state conditions:

$$\frac{\partial}{\partial x} \left(\frac{\rho h^3}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{\mu} \frac{\partial p}{\partial y} \right) = 12\bar{u} \frac{\partial}{\partial x} (\rho h) \quad (1)$$

where $\bar{u} = \frac{u_1 + u_2}{2}$

Here, \bar{u} , u_1 and u_2 are the mean entrainment speed, and the velocity of solids 1 and 2 in the x -direction, respectively.

At this point, a first statement should be expressed.

In the Reynolds Eq. (1) one finds two lubricant properties, μ and ρ , respectively the dynamic viscosity and the density of the lubricant.

For the sake of argument, ignore the fact that Eq. (1) is not strictly valid for variable viscosity. Solving an EHL problem numerically requires accounting for at least two supplementary equations, the solid elasticity (or film thickness) equation and the normal load (or force balance) equation. As for the pressure dependence of the lubricant properties, two complementary relationships must be introduced, the viscosity-pressure and the density-pressure laws. The nature and the physical basis of these relationships will be discussed in the following.

From this set of three governing equations and two relations for the pressure dependence of the fluid properties, it becomes possible to solve the fully flooded Newtonian isothermal EHD contact problem. Although Reynolds published his equation in 1886, the first full EHD numerical solution appeared by Dowson and Higginson [12] in 1959 for line contacts. Numerous other solutions for circular and then elliptical contacts were later published, and some of them accounted for thermal effects due to inlet shear-heating, non-Newtonian behavior of the lubricant, starved conditions, solid surface features, etc. With the further development of computing capabilities, the solution technique persists today in spite of the difficulty in solving this highly nonlinear problem. However, due to the large number of parameters required to characterize the lubricant behavior and the operating conditions, an alternative emerged and met with tremendous success among the EHL community. It is mostly inspired by the pioneer work of Hamrock and Dowson [13], who proposed analytical formulas for predicting both the central and minimum film thicknesses in EHD circular contacts working under isothermal and fully flooded conditions and lubricated by a Newtonian fluid. They also introduced a set of three dimensionless parameters to characterize an EHD contact with a reduced number of variables. These are the load

parameter W , the material properties parameter G and the speed parameter U defined as:

$$W = \frac{w}{E'R^2} \quad G = \alpha E' \quad U = \frac{\mu(u_1 + u_2)}{E'R} \quad (2)$$

where w , E' , R and α are the normal load, the equivalent Young's modulus (function of E and ν for each solid), the reduced radius of curvature of the two surfaces and the viscosity-pressure coefficient (VPC) of the lubricant, respectively.

The expressions for h_c and h_m , respectively, the central and minimal film thicknesses for circular contacts, read:

$$H_c = \frac{h_c}{R} = 1.916U^{0.67}G^{0.53}W^{-0.067} \quad (3)$$

$$H_m = \frac{h_m}{R} = 1.828U^{0.68}G^{0.49}W^{-0.073} \quad (4)$$

where H_c and H_m are the dimensionless central and minimum film thicknesses, respectively.

Three major statements emerge from these analytical relationships.

Hamrock and Dowson [13] defined the VPC as the reciprocal asymptotic isoviscous pressure coefficient (first proposed by Blok [14], often denoted α^* and defined in the following) and not the Barus coefficient, in contrast to common belief.

The lubricant density, ρ , no longer appears in expressions (2) to (4), which is inconsistent with the Reynolds Eq. (1).

The Hamrock and Dowson formulas and other classical formulas have not been thoroughly validated as this would have required the pressure dependence of the liquids used for validation.

The absence of density from these solutions may be surprising, not only because density is explicitly shown in Eq. (1) but because straightforward evidence of film thickness dependence on compressibility was clearly demonstrated for some time (see [15] (1994) for instance). In their conclusion, Venner and Bos [15] wrote the following sentence:

It was shown that, although compressibility is not one of the predominant effects accounting for film formation, it does determine to a great extent the shape of the lubricant film in the central region of the contact.

They quantified numerically the importance of the lubricant compressibility on H_c and H_m . Their comparisons with incompressible EHL solutions revealed that central film thickness reductions of approximately 15 to even 25 % could be obtained, according to the compressibility model and the normal load, for medium ($M = 50$, $L = 10$) to highly loaded ($M = 1,000$, $L = 10$) EHD contacts, respectively (M and L being the Moes parameters [16]).

Note that the Hamrock and Dowson film thickness Eqs. (3) and (4) were derived from full numerical solutions [17] established using the classical Dowson and Higginson density-pressure relationship, the most employed empirical density equation used in EHL. The latter can be expressed as follows:

$$\frac{\rho(p)}{\rho_{0.1}} = 1 + \frac{ap}{1 + bp} \quad \text{or} \quad \frac{\rho(p)}{\rho_{0.1}} = \frac{c + dp}{c + p} \quad (5)$$

where $\rho_{0.1}$, a , b , c and d are, respectively, the density at ambient pressure and two sets of two constants.

At this point, it is important to emphasize another misinterpretation of previous work.

Hamrock and Dowson [17] specified that the two constants a and b of the density-pressure Eq. (5) are dependent on the specific fluid.

This comment was forgotten afterward and thus never taken into account by the EHL community as a and b were considered as empirical universal constants with $a = 0.6$ and $b = 1.7$ (or alternatively $c = 0.59$ and $d = 1.34$), with p expressed in GPa. It is also critical to point out that temperature must influence compressibility and thus a , b (or c , d) should vary accordingly with temperature.

Many elements have been in place for an improper use of analytical EHL film thickness expressions. On the one hand, generating a full numerical EHL solution is a complex and difficult task; it was not necessarily easy to determine the values of and incorporate the necessary physical properties. On the other hand, rather simple analytical expressions, dealing with a limited number of parameters, were available. The choice of a significant part of the lubrication community has been to ignore these. Analytical film thickness formulas have been extensively used (1) to assess EHD film thickness in an engineering approach (they are only estimates), which is sensible, but also (2) to deduce the properties of lubricants, which is not sensible considering that measured values of these properties have been reported. This will be illustrated in the following.

3 Viscosity-Pressure Dependence and Viscosity-Pressure Coefficients

Until recently, the EHL community has not made use of high-pressure viscometers or even shown an awareness of their capabilities and results, leading to confusion about the actual viscosity-pressure dependence of lubricants and thus to multiple definitions of the VPC. There is no doubt that the more straightforward manner to define the latter is to rely on experimental data directly provided from high-pressure viscometry. Since Bridgman [18] in 1926, it has been known that

lubricant viscosity increases dramatically with pressure and that this increase begins as slower than exponential and, at high pressures, becomes faster than exponential. In other words, at constant temperature, the slope of $\log(\text{viscosity})$ versus pressure is not constant but varies with pressure. However, in the EHL field a pure exponential relationship has been widely used for its limited number of parameters and for its easy introduction in analytical problems. The general expression, mistakenly attributed to Barus, takes the form:

$$\mu(p) = \mu_{0.1} \exp(\alpha p) \quad (6)$$

where $\mu_{0.1}$ and α are the viscosity at ambient pressure and the viscosity-pressure coefficient both at constant temperature, respectively.

From a set of experimental results, it is possible to derive the following viscosity-pressure coefficients, namely the tangent VPC α_{tangent} (for a pressure increment), the secant VPC α_{secant} (for a given pressure increase from the ambient value), α_{LMS} the VPC corresponding to a least mean square regression of the experimental data, α^* the reciprocal asymptotic isoviscous pressure coefficient proposed by Blok [14] and finally α_{film} recently defined by Bair et al. [19] to satisfy the requirement that Newtonian liquids with the same ambient viscosity and the same VPC should generate the same central film thickness:

$$\alpha_{\text{tangent}}(p) = \frac{\partial \ln(p)}{\partial p} \quad (7)$$

$$\alpha_{\text{secant}}(p) = \frac{\ln(\mu(p)) - \ln(\mu(0.1))}{p} \quad (8)$$

α_{LMS} obtained from a least square fit to Eq. (6) assuming $\mu = \mu_{0.1}$ at $p = 0.1$ MPa

$$\alpha^* = \frac{1}{\int_{0.1}^{\infty} \frac{\mu_{0.1}}{\mu(p)} dp} \quad (9)$$

$$\alpha_{\text{film}} = \frac{1 - \exp(3)}{3/\alpha^* \int_0^{\infty} \frac{\mu_{0.1}}{\mu(p)} dp} \quad (10)$$

Considering the nonlinear increase in $\log(\text{viscosity})$ as a function of pressure, it is obvious that α_{tangent} and α_{secant} should vary with pressure. This is shown in Fig. 1, where viscosity measurements obtained at 75 °C for hydrocracked mineral base oil [20] are reported. In this figure, several VPCs are highlighted with arrows at different pressure conditions. The corresponding values are summarized in Table 1 together with α^* and the corresponding viscosity increases estimated for three representative pressures: 100, 400 and 800 MPa that represent the typical inlet pressure (fundamental for the film thickness generation), the mean central pressure in most of steel/glass ball-on-disk contacts

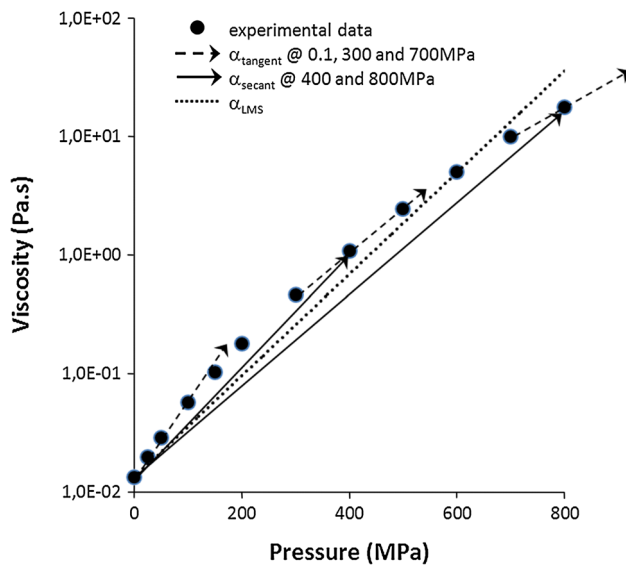


Fig. 1 Viscosity-pressure coefficients derived from a single set of experimental data from [20]

Table 1 Different viscosity-pressure coefficients (VPC) derived at different conditions from a single set of measurements plotted in Fig. 1, and corresponding viscosity increases at three representative pressures

Parameter	VPC (GPa^{-1})	$\mu_{100}/\mu_{0.1}$	$\mu_{400}/\mu_{0.1}$	$\mu_{800}/\mu_{0.1}$
α^*	13.62	3.90	232.5	$5.40 \cdot 10^4$
α_{LMS}	9.88	2.69	52.0	$2.71 \cdot 10^3$
α_{tangent}				
@ 0.1 MPa	16.04	4.97	611.6	$3.74 \cdot 10^5$
@ 250 MPa	8.42	2.32	29.0	$8.42 \cdot 10^2$
@ 700 MPa	5.80	1.79	10.2	$1.04 \cdot 10^2$
α_{secant}				
@ 400 MPa	10.99	3.00	81.1	$6.58 \cdot 10^3$
@ 800 MPa	9.00	2.46	36.6	$1.34 \cdot 10^3$

(from which central film thickness is measured) and the maximum pressure investigated experimentally (and characteristic of highly loaded EHD contacts), respectively. The differences between the VPC values reported in Table 1 clearly illustrate the difficulty in representing viscosity-pressure variations by a simple exponential relationship. More important also is the use made of such expressions in numerical models: the viscosity increases reported in Table 1 vary within a factor of 2.8, 60 and 3,600 at pressures of 100, 400 and 800 MPa, respectively. This would lead to inaccurate predictions of, for instance, film thickness that is directly dependent upon the inlet viscosity (i.e., the viscosity at low pressure) or of traction that also results from many physical interactions and, among them, the temperature dependence of the physical properties [10] of the lubricant due to shear heating.

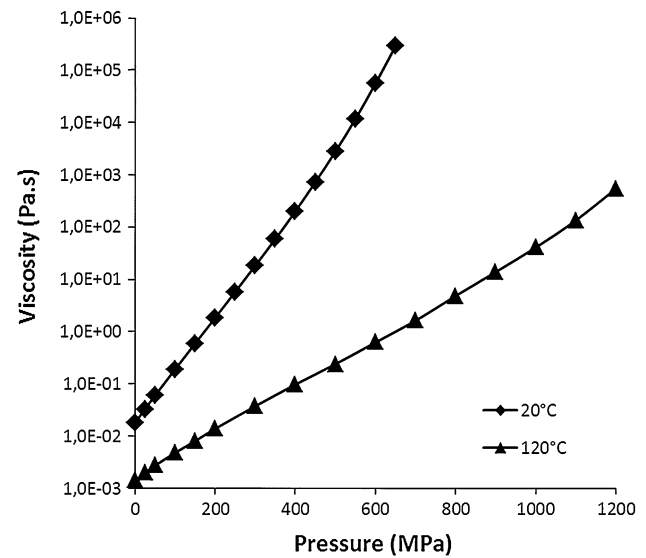


Fig. 2 Viscosity-pressure measurements for a mineral base oil at two temperatures, from [10]

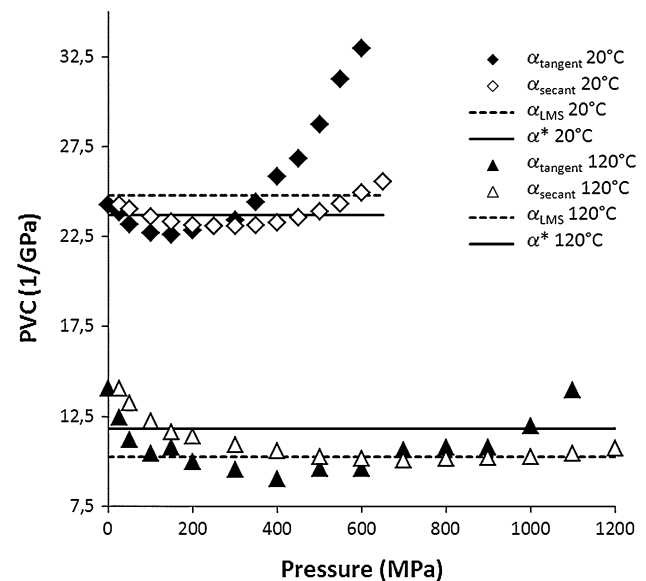


Fig. 3 Viscosity-pressure coefficients derived from the experimental results of Fig. 2

Moreover, for the temperature dependence of the VPCs, the situation may become even more complex. As an illustration, Fig. 2 reports experimental viscosity results published in [10] for a mineral base oil (Shell T9) at two temperatures, 20 and 120 °C. Apart from the temperature influence, one observes that the viscosity domain covered in Fig. 2 is much larger than in the previous case of Fig. 1. From these measurements, the viscosity-pressure coefficients defined by Eqs. (7–9) were calculated at the two temperatures and reported in Fig. 3, where α^* and α_{LMS} are plotted as horizontal lines for the eyes, as by definition they are constant for a given temperature. Together with the

examination of two temperatures over a large range of viscosity, two additional features related to the viscosity-pressure dependence appear in Figs. 2 and 3.

- First, following a decrease to a minimum value, α_{secant} values increase from an inflection pressure, p_{inf} , approximately equal to 300 and 700 MPa, at 20 and 120 °C, respectively. This behavior is the general response of organic liquids under pressure and is described in detail in [7]. Below p_{inf} the viscosity-pressure dependence is slower than an exponential form, while above p_{inf} the relationship is faster than exponential.
- Secondly, the comparison of the respective values of α^* and α_{LMS} indicates an inversion: at 20 °C α^* is lower than α_{LMS} , whereas at 120 °C α^* is larger than α_{LMS} . Contrarily to the belief or the hope that prevails among the EHL community, there is poor probability that a given rule of thumb would specify the proper VPC value from a limited set of data.

The alternative solution for considering adequate VPCs, described here as a quantitative or physics-based approach, utilizes relationships with a physical basis, among them those dealing with the free-volume concept. These not only link viscosity with pressure but also give an insight into the density-pressure and the density-temperature dependence, density being an implicit and important parameter [15] of the Reynolds Eq. (1). A brief summary of the authors' experience in the matter shows that this dependence can be successfully described with one of two equations of state, namely the Tait [21, 22] or the Murnaghan [23] expressions for the density variations, and one of the three viscosity–pressure–temperature relationships, the extended Doolittle [24], the Bair and Casalini [25] or the modified WLF equations [3, 26]. Comparisons quantitatively consistent with experiments conducted under various EHD operating conditions and lubricants proved the combinations of these equations to be relevant for accurately predicting film thickness and traction. Tait and Doolittle relationships were used for a high-viscosity (and highly non-Newtonian) PAO [8, 28], glycerol and a polymer solution [28]; Murnaghan and Bair and Casalini expressions were utilized for a mineral base oil under highly loaded TEHD conditions [10, 11] and in the simulation of the spin effect in the roller-end flange contacts [29]; finally, the Murnaghan and the modified WLF equations were introduced to model TEHL spinning skewing circular contacts [30].

Several statements can be drawn from this section.

There can be no doubt that high-pressure viscometry provides suitable data for representing the pressure (and temperature) dependence of the lubricant viscosity.

At constant temperature, the slope of $\log(\text{viscosity})$ versus pressure is not constant but varies with pressure.

Therefore, most of the VPCs derived from a simple exponential relationship are unable to properly represent the viscosity-pressure dependence.

As a consequence, this approach might lead to erroneous calculations, because any numerical solver only reflects the influence of the models that it utilizes.

The alternative solution lies in the use of physics-based relationships applied to precise measurements, suitable by their nature to describe both density and viscosity dependence on pressure and temperature.

4 The Link Between EHD Film Thickness and VPCs

4.1 Material and Operational Parameters Controlling EHD Film Thickness

With easy access to optical EHL machines, the EHL community has believed that it would be possible to derive viscosity-pressure coefficients from EHD analytical models fitted to film thickness measurements. Contrary to what is commonly thought, the cost and complexity of a high-pressure viscometer is less than that of the optical EHL machine. Its operation requires less time and certainly less specific skill. Nevertheless, it is instructive to conduct an objective analysis of known and unknown parameters involved:

- 1) The classical operating parameters U_1 , U_2 , w , R and E' for the two solid bodies,
- 2) The characteristics of the lubricant at the inlet temperature, α^* and μ , according to Eqs. (2–4).

Some experimental parameters are seldom considered but are of primary importance and among them are:

- 3) The specimen roughness, the surface cleanliness and their reactivity with the lubricant components (this concerns not only the specimen but all parts of the device in contact with the lubricant),
- 4) The delivery of liquid to the contact, to prevent lubricant starvation that could impact the film thickness,
- 5) The lubricant behavior, it must be purely Newtonian in the contact inlet region for the application of the analytical formulas,
- 6) The inlet and specimen temperatures (a deviation of 1 °C at room temperature implies a typical change of 3–7 % in ambient pressure viscosity).

Thus, it should be possible to use analytical formulas to estimate a viscosity-pressure coefficient only if all these parameters and conditions were precisely controlled or known. With what accuracy are the elastic quantities E and ν for each solid, the applied normal load, the actual inlet or surface temperature and the ambient viscosity known? How

are the operational parameters (3) to (6) controlled or how does the researcher ensure they do not influence the results? Furthermore, there is a missing parameter important for the film thickness determination in this analysis, the compressibility that varies with the nature of the fluid and pressure and temperature, as mentioned earlier. Finally, two other open questions that can be the cause of discrepancies between experiments and analytical predictions are (1) the domain for which the relationships like Eqs. (3–4) were derived and (2) the uncertainties as they are approximations and cannot provide an exact solution of the full EHL problem.

4.2 Film Thickness Prediction Using Appropriate VPC

If all the parameters listed in the previous section are accurately known and/or well controlled, an acceptable prediction of film thickness of Newtonian liquids can be achieved using α^* , the reciprocal asymptotic isoviscous pressure coefficient proposed by Blok [14] and explicitly used by Hamrock and Dowson [13]. This was clearly demonstrated by Bair [31] in 1993 on the basis of seven lubricants that are Newtonian in the inlet, whose viscosity-pressure dependence departed from the pure exponential relationship, and for which both rheological and film thickness measurements were carried out. In that work, several types of esters, mineral oils and mixtures were studied.

Another strong evidence of the relevance in using α^* to predict the film-forming capacity of lubricants was reported much later in 2007 by Chaomleffel et al. [32], not only for h_c but also for h_m , the minimum film thickness that is the key parameter for estimating the lubrication regimes. Compared with [31], a good correlation between experiments and predictions based on measured α^* values was successfully obtained not only for a wider range of operating parameters ($M = 3$ –10,000 and $L = 1$ –40) but also from a broader variety of lubricants, including a diester, several mineral and synthetic hydrocarbons, a polyphenyl ether and a traction fluid. Note that in Ref. [32] a particular emphasis was placed on the identification of shear heating and/or shear thinning effects, in accordance with points (5) and (6).

Nevertheless, any experimental result includes some degree of uncertainty even if it is our primary objective to reduce it. As an illustration, Table 2 reports comparisons

between experimental and the predicted central film thicknesses from Eq. (3) in which α^* was employed, based on measurements of α^* published in [32, 33]. Four parameters expressed as follows were derived:

Mean relative deviation:

$$Md = \frac{\sum_1^{n_{\text{exp}}} \frac{(h_{\text{cHD}} - h_{\text{exp}})}{h_{\text{cHD}}}}{n_{\text{exp}}} \quad (11)$$

Standard deviation on Md :

$$SdMd = \sqrt{\frac{1}{n_{\text{exp}}} \sum_1^{n_{\text{exp}}} \left(\frac{(h_{\text{cHD}} - h_{\text{exp}})}{h_{\text{cHD}}} - Md \right)^2} \quad (12)$$

Mean of the absolute values of the relative deviation:

$$Mad = \frac{\sum_1^{n_{\text{exp}}} \frac{|h_{\text{cHD}} - h_{\text{exp}}|}{h_{\text{cHD}}}}{n_{\text{exp}}} \quad (13)$$

Standard deviation on Mad :

$$SdMad = \sqrt{\frac{1}{n_{\text{exp}}} \sum_1^{n_{\text{exp}}} \left(\frac{|h_{\text{cHD}} - h_{\text{exp}}|}{h_{\text{cHD}}} - Mad \right)^2} \quad (14)$$

where n_{exp} and h_{cHD} are the number of experimental values and the central film thickness predicted by the Hamrock and Dowson Eq. (3), respectively.

Approximately 20–50 entrainment velocities were considered for each lubricant, and the film thicknesses covered a typical range from 400 nm down to 20 nm. In cases where shear heating or shear thinning effects were suspected, the results were discarded. Although the tests were conducted in recognition of the assumptions of the Hamrock and Dowson theory and the lubricants' properties properly characterized, the comparison with the Hamrock and Dowson Eq. (3) calls for comment. In particular, the combination of Md and $SdMd$ (or Mad and $SdMad$) values shows an overall deviation between h_{cHD} and h_{exp} of approximately 10 % on the whole set of data. Thus, Table 2 points out that even if experiments and predictions visually appeared in good agreement in [32, 33]—this can easily be the case when data are plotted in log–log scales

Table 2 Analysis of experimental–numerical correlations for central film thicknesses obtained with five lubricants, which are Newtonian in the inlet, in the absence of thermal and boundary film effects

Lubricants and data origin	DOP di-(2-ethylhexyl) phthalate [33]	Mineral base oil [32]	Squalane hexamethyl-tetracosane [32]	Penzanne synthetic hydrocarbon [32]	Santotrac 40 traction fluid [32]
Md (%)	−0.8	−0.8	−3.6	−7.4	7.8
$SdMd$ (%)	7.3	7.1	5.8	5.6	3.2
Mad (%)	6.5	6.0	5.5	8.0	7.8
$SdMad$ (%)	3.4	3.8	4.1	4.7	3.2

The definition of the parameters in the first column refers to Eqs. (11–14)

with at least three decades on each axis—a deviation is found and might have some consequences if the results would be curve fitted to deduce an unknown parameter characterizing the operating conditions or a material property.

Finally, one might speculate on the accuracy in the determination of α^* , the reciprocal asymptotic isoviscous pressure coefficient. Table 3 provides evidence that when correctly performed, the high-pressure measurements of viscosity converge to very close values. In this table, α^* values are reported for two lubricants (Fluid 84A in [31] or TIN in [34], an ester base oil; Fomblin Z25 [35, 36], a linear perfluoropolyalkylether) tested using two high-pressure falling-body viscometers located at Georgia Tech (USA) and LaMCoS (France). For both fluids, the authors employed the Yasutomi-modified WLF expression [3] to model the viscosity–pressure–temperature dependence, and they published the parameters' values, making their results generally available and independent of the applied operating conditions, which differed among the groups. It is also important to note that these experiments were carried out at different times and very likely on different batches of the fluids. This is certainly true for the Z25 lubricant as the sample studied at Georgia Tech was of lower viscosity (typically –15 % from 0 to 60 °C) than the one tested at LaMCoS. In spite of different devices, operating conditions and different batches, the comparison of the α^* values (see Table 3) provides evidence that with these experimental methods the chosen parameter can be precisely and unambiguously determined.

4.3 Can Film Thickness be Used for Deriving VPCs?

The term “estimating” used several times in the previous section was intentionally employed. Indeed, if the interval of confidence is of the order of 10 % in predicting film thickness compared to experimental values, making an inverse calculation to derive a viscosity–pressure coefficient will give a value within 20 %, the α^* exponent in Eqs. (3–4) being approximately 0.5. As was quoted in Sect. 2

and illustrated in Table 1, such a discrepancy leads to totally erroneous values that cannot match the actual behavior of lubricants under high pressure. Using a reference fluid (or calibration fluid) whose properties are known only to a certain precision contributes to increased uncertainty and may degrade the accuracy by a factor of two. Moreover, the misinterpretation of experimental material properties can in turn lead to large numerical errors when incorporating an inaccurate value in a full EHD model solver. Indeed, when the literature on film-derived VPC is surveyed and compared to viscometer measurements, the film-derived values are from 60 to 150 % of the viscometer-derived values.

Hartl et al. [33] tried for pure liquids and lubricants of high purity to conduct accurate film thickness tests under extreme care of cleanliness and precision with the aim to directly derive suitable VPC values. Some of their results were considered acceptable and certainly closer to VPCs generated from rheological measurements than those published by other groups who also derived VPCs from film thickness measurements. But, on the one hand, the work required to obtain the VPC values was more difficult than the direct determination from a high-pressure viscometer, and on the other hand, the reference values published later [37] showed that some of the deduced results for squalene were not accurate.

When it comes to a lubricant for which the actual rheological behavior is ignored by the researchers, the consequences can be even more disastrous. A fairly symbolic illustration was given in 1995 by Jones [38], who reported enormous deviations between viscosity–pressure coefficients obtained for perfluoropolyalkylether fluids. Among them, the case of Fomblin Z25, widely used for space applications, deserves special attention. Jones compared effective VPCs (deduced from experimental film thickness) with values obtained from high-pressure devices. Even for the latter, some deviations were visible, mainly caused by an insufficient pressure range (for instance in [39] cited by Jones, the maximum pressure was limited to only 100 MPa) or by the use of different VPC definitions. But, the errors resulting from film thickness data were much larger. Film-derived VPCs were close to 10 and 8 GPa^{−1} at

Table 3 Values of the reciprocal asymptotic isoviscous pressure coefficient, α^* (in GPa^{−1}), obtained for two lubricants at several temperatures and inferred from two independent measurements performed on two high-pressure falling-body viscometers

	From Ref. [31], 1993				From Ref. [34], 2002			
	50 °C	100 °C	150 °C		50 °C	100 °C	150 °C	
Fluid 84A or TIN	12.2	9.4	7.7		12.6	9.3	7.4	
	From Ref. [35], 1996				From Ref. [36], 2002			
	13 °C	40 °C	63 °C	100 °C	13 °C	40 °C	63 °C	100 °C
Fomblin Z25	22.7	19.3	17.1	14.2	22.2	19.3	17.3	14.7

25 and 60 °C, respectively, when high-pressure measurements gave for the same conditions values around 21 and 19 GPa⁻¹, more than double. The explanation came some years later when:

- the shear thinning behavior of Z25 was clearly proven [40] from flow curves established at high pressure and described by a Carreau model,
- good agreement was found between experimental film thickness values and those predicted after incorporating the shear thinning behavior [40].

In Sect. 3, the compressibility problem was deliberately not included, given what was presented before in this matter, but it remains an unresolved issue.

Summary of important statements from Sect. 3.

An objective analysis of the parameters involved in the EHD film thickness problem shows that numerous factors both computational and experimental are involved. Contrary to the general feeling, they are not easy to control, and this is not discussed in the relevant literature. The reciprocal asymptotic isoviscous pressure coefficient α^* must be considered as the proper viscosity-pressure coefficient for estimating EHD film thickness when using the existing formulas. Determining α^* is rather straightforward from high-pressure viscometers. This is a parameter that has good reproducibility when a viscometer is employed.

Given the uncertainties related to the large number of the involved parameters, and given the fact that compressibility and shear dependence are not taken into account in the EHL formulas, it is not possible to infer viscosity-pressure coefficients from regressions based on existing formulas and film thickness measurements.

5 Conclusion

This perspective paper is aimed at (1) clarifying the quantification of the viscosity-pressure dependence of lubricants and (2) addressing the possibility of deriving viscosity-pressure coefficients from experimental film thickness. A simple methodology has existed for many decades. It breaks with the usual practices of the field and is based on the following fundamental arguments:

- For the quantification of VPCs, viscosity testing under high pressure is much easier and straightforward compared with film thickness measurements where numerous factors may intervene.
- VPCs derived from a simple exponential relationship are unable to properly represent the actual viscosity-pressure dependence of lubricants.

- The reciprocal asymptotic isoviscous pressure coefficient α^* should be used for estimating EHD film thickness from the existing formulas, and it is easily deduced from high-pressure viscosity measurements.
- However, a single parameter cannot accurately represent the actual viscosity-pressure relationship of lubricants, and a model based on physical concepts, such as the modified Yasutomi-WLF [26] or the Tait and Doolittle expressions, is highly preferable.
- It is not reasonable to infer accurate values of VPCs from regressions based on existing formulas and on measured film thickness.
- Any EHD film thickness model should include a term that properly reflects the lubricant compressibility.

Regarding the last argument, it is clear there is still progress required for realistic modeling of lubricant compressibility.

A further point concerns the publication of high-pressure results for lubricants in the form of the parameters of one of the modified WLF models [3, 26], keeping in mind that α^* can be easily calculated from these data. The numerical values reported in the appendix are a first attempt; the authors believe it is of general interest to share these results in order to contribute to the transition from classical EHL toward a quantitative, physics-based EHL.

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Appendix

A correlation was initially proposed in the field of polymer physics by William, Landel and Ferry (WLF) [41], based on the time–temperature superposition principle or the method of reduced variables. The latter stipulates that it is possible to represent a rheological property on one single master curve, scaled from a reference temperature corresponding to the glass transition temperature of the fluid, T_g . Later, Yasutomi et al. [3] provided an extended version of the WLF model to the pressure dependence. And very

Table 4 Parameters of the modified Yasutomi-WLF [26] model Eq. (A1) for different lubricants, $\mu_{0,1}$ and α^* values were computed from the model

Name	Nissan CVT Fluid ^a 50	Santotrac	HMB	HMB P1	HMB P2	PGB	T9 ^a	80 W-90 ^a	85 W-140 ^a
Chemical nature/origin	Commercial traction fluid	Traction Fluid	Hydrocracked mineral base oil	HMB + 1.2 %w/w PMA polymer [26]	HMB + 1.2 %w/w OCP + PISH polymers [26]	Polyalkylene glycol base oil [26]	Shell T9	Formulated gear oil	Formulated gear oil
Reference	[26]	[43]	[26]	[26]	[26]	[26]	[26]	–	–
$\mu_{0,1}$ at 40 °C (Pa.s)	0.0304	0.0256	0.0189	0.0193	0.0392	0.0205	0.0080	0.1243	0.3500
$\mu_{0,1}$ at 100 °C (Pa.s)	0.0048	0.0044	0.0039	0.0046	0.0075	0.0044	0.0020	0.0116	0.0241
α^* at 40 °C (GPa ⁻¹)	27.03	28.34	16.84	17.33	17.51	15.55	20.24	22.61	24.30
α^* at 100 °C (GPa ⁻¹)	17.54	17.83	11.70	11.66	12.16	10.45	13.14	15.05	16.11
C_1	11.18	15.820	16.10	15.78	15.880	15.93	16.0858	16.372	16.461
C_2 (°C)	33.40	16.130	21.96	18.21	22.860	19.87	17.377	29.99	35.74
A_1 (°C)	317.4	166.000	232.61	271.49	505.460	225.46	188.86	210.58	172.4
A_2 (GPa ⁻¹)	0.7704	0.846	0.1780	0.1581	0.0676	0.1989	0.719	0.455	0.6787
B_1 (GPa ⁻¹)	4.29	5.96	7.595	9.0843	8.356	7.8907	8.200	7.606	8.081
B_2	-0.7704	-0.8392	-0.5258	-0.5232	-0.5032	-0.4917	–	-0.4193	-0.3537
$T_g(0)$ (°C)	-66.90	-58.46	-86.80	-80.94	-84.00	-81.36	-83.21	-71.70	-71.20
μ_g (Pa.s)	10^{+7}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}

Name	Royco 555 ^a	Squalane ^a	CPRI	SAE 5 W-30	SAE 5 W-40	Krytox 143AZ ^a	Z25	Mobil Jet II ^a
Chemical nature/origin	Helicopter transmission oil	Hexamethyl-tetracosane	Paraffinic mineral base oil	Low SAPS formulated engine oil	Formulated engine oil	Branched perfluorinated polyether	Linear perfluorinated polyether	L 23699 aircraftturbine oil
Reference	–	[26]	[34]	[44]	–	[42]	[42]	[43]
$\mu_{0,1}$ at 40 °C (Pa.s)	0.0249	0.0149	0.0160	0.0504	0.0597	0.0289	0.2448	0.0216
$\mu_{0,1}$ at 100 °C (Pa.s)	0.0051	0.0030	0.0032	0.0085	0.0103	0.0061	0.0780	0.0043
α^* at 40 °C (GPa ⁻¹)	14.25	18.15	18.20	18.09	18.19	39.51	18.58	15.58
α^* at 100 °C (GPa ⁻¹)	9.66	12.60	12.22	12.81	12.84	26.56	13.57	10.63
C_1	16.189	16.38	16.10	16.37	16.01	16.074	14.997	16.28
C_2 (°C)	26.17	25.22	19.88	33.34	27.84	25.645	36.300	26.15
A_1 (°C)	655.6	270.5	138.97	138.43	113.299	528.5	157.7	2285
A_2 (GPa ⁻¹)	0.0975	0.2377	0.3957	0.6064	0.5342	0.3322	0.7469	0.0263
B_1 (GPa ⁻¹)	9.586	9.83	8.178	8.993	7.543	31.87	19.48	10.47
B_2	-0.3354	-0.4151	-0.5200	-0.3650	-0.4787	-0.3061	-0.2788	-0.3435
$T_g(0)$ (°C)	-97.71	-96.60	-78.85	-104.45	-91.94	-96.26	-152.0	-96.72
μ_g (Pa.s)	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}	10^{+12}

^a Indicates that the viscosity-pressure inflection was present in the measured data

recently, an improvement has been proposed to better reflect the behavior of fluids that present an inflection. This new modified Yasutomi-WLF model also proved [26] to give a more accurate representation of viscosity at even low pressures, which is an important feature for developing full numerical EHL solutions. It reads:

$$\mu(p, T) = \mu_g \times 10^{\frac{-C_1 \cdot (T - T_g(p)) \cdot F(p)}{C_2 + (T - T_g(p)) \cdot F(p)}}$$

$$\text{with: } T_g(p) = T_g(0) + A_1 \ln(1 + A_2 p) \quad (A1))$$

$$F'(p) = (1 + B_1 p)^{B_2}$$

where A_1 , A_2 , B_1 , B_2 , C_1 and C_2 are constants characterizing each fluid, and μ_g the viscosity at the glass transition temperature T_g and ambient pressure.

The function $T_g(p)$ represents the variation of the glass transition temperature with respect to pressure based on experimental data, whereas $F(p)$ represents the dimensionless variation of the thermal expansion coefficient of the relative free volume with pressure.

Table 4 aims at providing the modified Yasutomi-WLF parameters [26], for a set of representative lubricants. Typical viscosity and reciprocal asymptotic isoviscous pressure values are also given.

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