Classical non-Gaussian state preparation through squeezing in an opto-electromechanical resonator

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We demonstrate squeezing of a strongly interacting opto-electromechanical system using a parametric drive. By employing real-time feedback on the phase of the pump at twice the resonance frequency the thermo-mechanical noise is squeezed beyond the 3 dB instability limit. Surprisingly, this method can also be used to generate highly nonlinear states. We show that using the parametric drive with feedback on, classical number-like and cat-like states can be prepared. This presents a valuable electro-optomechanical state-preparation protocol that is extendable to quantum regime.

INTRODUCTION

The field of nano- and optomechanics has recently moved into the domain of quantum mechanics [1] by cooling resonators to the ground state [2–4], achieving strong coupling between photons and phonons [5–7], and, most recently, by entangling them [8]. Also, squeezing of light mediated through optomechanical backaction has been demonstrated [9–11]. Along the same lines, experiments with quantum protocols are currently being explored in the classical regime [12–14], but first nonclassical states of the harmonic oscillator [15–18] will need to be prepared. As we will demonstrate here experimentally, parametric squeezing is well suited for this purpose when combined with real-time control.

Squeezing is a powerful technique where the stochastic motion of a resonator is reduced in one quadrature at the expense of an increase in the other one. It is an important concept in the context of backaction evading [19] and pulsed measurements [16]. However, for the most often used squeezing method where the resonance frequency ω_0 is modulated at twice that frequency [20] (so-called parametric squeezing) the maximum squeezing that can be achieved is only 3 dB; this happens when the modulation amplitude of ω_0 equals the damping rate γ_0 . Beyond this limit the other quadrature of the resonator becomes unstable leading to regenerative parameteric oscillations. Currently a number of ways to circumvent this problem are being studied [21, 22]. Here we demonstrate a feedback technique where the phase of the parametric modulation is adjusted in real time and the 3 dB limit is overcome. The method also naturally leads to non-Gaussian states of the mechanical resonator.

THE OPTO-ELECTROMECHANCIAL SYSTEM AND PARAMETRIC SQUEEZING

To achieve a high degree of squeezing in an optomechanical system, one wants to efficiently modulate the spring constant k. Although optical readout can give unsurpassed displacement sensitivity [1], typically opti-



FIG. 1: (a) Colorized scanning electron micrograph showing an overview of a device taken under an angle; zooms of typical H-resonators are shown in (b) and (c). The waveguide is shown in red, the electrodes in yellow, and the part of the resonator not covered by the moveable electrode in blue. The arrows indicate the direction of motion for the fundamental in-plane mode. (d) The measurement setup. (e) Power spectral density of the displacement noise (symbols) with fit (line) measured at $V_{\rm DC} = 0.5$ V. (f) Driven response at $V_{\rm DC} = 0.5$ V and $V_d = 11 \,\mu$ V. The fit yields a quality factor of 62,000 ± 1,000.

cal backaction effects are weak. Therefore, we combine optical readout with strong electrostatic forces in an integrated opto-electromechanical device (Fig. 1(a)-(c)). Recently we used similar nanofabricated devices as optomechanical phase-shifters [23] and showed that these display very strong electrostatic interactions [24]. The movable part of the device consists of four thin arms that connect to a rectangular block containing a photonic crystal. An electrode runs over one pair of arms and is close to another, fixed, electrode. The other side of the resonator runs close to the waveguide of an on-chip Mach-Zehnder interferometer (Fig. 1(a)), enabling sensitive displacement detection using the measurement setup shown in Fig. 1(d). The fundamental in-plane eigenmode of this "H-resonator" has a resonance frequency around 723 kHz and the thermo-mechanical noise at room-temperature is resolved with a signal-to-background ratio of 29 dB as shown by the spectrum in Fig. 1(e). By applying a static $(V_{\rm DC})$ and an a.c. voltage (V_d) between the electrodes, the resonator is actuated; Fig. 1(f) shows the driven response. The small driving voltage that is used is a manifestation of the large electrostatic interactions in the H-resonator.

In such strongly interacting electromechanical systems, the resonance frequency can be tuned via the electrostatic spring effect [24–26]. By recording driven measurements (cf. Fig. 1(f)) while sweeping $V_{\rm DC}$, a curvature of $f_0'' = -2.0 \,\mathrm{kHz}/\mathrm{V}^2$ is found. Hence, by applying a pump voltage $V_P \sin(2\omega_F t + \theta)$ the resonance frequency is modulated at twice the reference frequency $\omega_F \approx \omega_0$ with an amplitude $\chi = f_0'' V_{\rm DC} V_P$. Such a signal is called a "2f" parametric pump and its effect is most easily analyzed using the complex amplitude of the resonator A. In the frame rotating at ω_F it is defined as [27]:

$$A \equiv \left(u + \frac{\dot{u}}{i\omega_F}\right) \exp(-i\omega_F t). \tag{1}$$

This means that an oscillating displacement $u(t) = A_0 \cos(\omega_F t)$ has a complex amplitude $A = A_0$ and likewise $u(t) = A_0 \sin(\omega_F t)$ becomes $A = iA_0$. By taking the time derivative of Eq. (1) and inserting the equation of motion $m\ddot{u} = -k(t)u - m\omega_0\gamma_0\dot{u} + F$ into that expression, the differential equation for the dynamics of the complex amplitude is obtained. In the rotating wave approximation (RWA) it is [20]:

$$\dot{A} \approx i\Delta_0 A - \frac{\gamma_0}{2} A + \frac{\chi}{2} e^{i\theta} A^* - \frac{i}{2} f_F, \qquad (2)$$

where $\Delta_0 = (\omega_0^2 - \omega_F^2)/2\omega_F \approx \omega_0 - \omega_F$ is the detuning, $mf_F = \langle 2Fe^{-i\omega_F t} \rangle_{\text{RWA}}$ is the force acting on the resonator in the RWA, and *m* is its mass. The term with A^* in Eq. (2) indicates that the parametric pump breaks the time invariance and leads to squeezing [20].

The rotating frame is not only a convenient representation for the equation of motion, but is also directly accessible in the experiment: a lockin amplifier (Zurich Instruments HF2; Fig 1(d)) demodulates the photodetector signal at ω_F and, after scaling by the transduction factor, its two quadratures X_1 and X_2 are the real and imaginary part of A. The inset of Fig. 2(e) shows a representative phase-space trajectory of such a demodulated timetrace. Note that when the signal-to-noise ratio between the mechanical signal and the imprecision noise is



FIG. 2: Parametric squeezing of the thermal motion. (a)-(c) Colorplots of the measured pdf at the indicated pump power. (d),(e) Evolution of the angle (d) and variance (e) of the squeezing for increasing pump power. The dashed line shows the 3 dB limit for parametric squeezing. (f) Squeezing angle versus the applied phase shift of the 2f drive signal. The insets show the pdf near $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ respectively. All markers and error bars in this figure are the mean value and standard deviation of 10 individual measurements, respectively. The pdfs are reconstructed from these 10 traces combined, and $\Delta_0 \approx 0$ everywhere.

high (see Fig. 1(e)) the trajectories accurately represent the mechanical motion. By repeatedly measuring these trajectories, the probability density function (pdf) of the resonator, which is the classical analogue of the quantum Wigner function, can be reconstructed [28]. As shown in Fig. 2(a) the Brownian motion of the resonator appears as a circular Gaussian in the pdf in the absence of a pump voltage, i.e. at $V_P = 0$.

By increasing V_P the pdfs becomes ellipsoidal as fluctuation in one quadrature decrease, whereas they are enhanced in the orthogonal direction [Fig. 2(b,c)] thereby squeezing the thermal motion. This can be quantified by extracting the variances in these two directions (i.e., the squared lengths of the minor and major axes of the ellipses). As shown in Fig. 2(e) the squeezing increases with increasing V_P but when the normalized variance of the squeezed quadrature reaches $\frac{1}{2}$ the fluctuations in the anti-squeezed quadrature grow exponentially until limited by nonlinearities in the resonator [29–31]. Above this threshold $\chi > \gamma_0$ and parametric oscillations result, which are characterized by a large mean value of A. In our strongly coupled high-quality electromechanical resonator [24] this happens at $V_P = 4.1 \text{ mV}_{\text{rms}}$ which is orders of magnitude lower than previously reported thresholds for top-down devices [20, 32] and is even below that of extremely floppy bottom-up devices [33]. Interestingly, the low threshold could be used to efficiently encode information in the phase of the oscillations [32], but here we focus on the sub-threshold behavior. In that regime the maximum attainable squeezing is thus limited to 3 dB.

OVERCOMING THE 3D LIMIT USING REALTIME PUMP-PHASE FEEDBACK

The angle of the squeezing ellipse is set by the phase between the 2f pump and the reference frame. Figure 2(d) indicates that the squeezing angle remains constant for fixed θ . Changing the phase of the reference frame rotates the apparent squeezing angle [20], but nothing happens to the actual motion. However, when the *pump phase* is varied, the squeezing angle of the actual motion rotates. This is an important distinction that enables squeezing beyond 3 dB as we will show later. Figure 2(f) shows that by adjusting θ the ellipsoidal pdf's rotate proportionally to $\frac{1}{2}\theta$.

In principle, squeezing exceeding 3 dB is possible when the pump exceeds the threshold (i.e. $\chi > \gamma_0$), but in this case the resonator will ring up to large oscillation amplitudes and the squeezing cannot be stationary [34]. By using a pump that is not exactly at twice the resonance frequency (i.e. when $\Delta_0 \neq 0$) the threshold pump power can be increased, but still the maximum amount of squeezing that can be obtained is limited to 3 dB, although estimation schemes can further reduce the uncertainty in the position [35]. Here we use a different method, where the phase of the pump is adjusted in real time [36], based on the measured location of the resonator in phase-space as illustrated in Fig. 3(a-c). In particular, we estimate $\varphi = \angle A$ using the phase $\hat{\varphi}$ measured by the lockin amplifier and use its programmable digital-signal processor to update the 2f phase θ every 70 μ s. When this adjustment is chosen carefully, the squeezing direction can be optimized in real time as illustrated in Fig. 3(a-c). Equation. (2) shows that when $\theta \to \theta_0 + 2\varphi(t)$, the dynamics of A become independent of A^* . However, in reality the actual phase φ is not known and only the estimated phase $\hat{\varphi}$ can be used. Inserting this into Eq. (2) yields:

$$\dot{A} \approx \left(i\Delta_0 - \frac{\gamma_0}{2} + \frac{\chi}{2}e^{i\theta_0 + 2i(\hat{\varphi} - \varphi)}\right)A - \frac{i}{2}f_F, \quad (3)$$

which no longer contains A^* . Equation (3) also shows that when the phase estimate is accurate (i.e. when $\hat{\varphi} = \varphi$) the resonance frequency and damping rate can be adjusted through the phase offset θ_0 as $\Delta = \Delta_0 + \chi \sin(\theta_0)/2$ and $\gamma = \gamma_0 - \chi \cos(\theta_0)$ respectively. Figure 3(d) shows the dependence of the linewidth γ on θ_0 for three different pump powers. Without a pump the linewidth remains constant at 12.8 Hz; with the pump on the linewidth shows the sinusoidal dependence $\gamma = \gamma_0 + \chi \cos(\theta_0 - \alpha)$ expected from Eq. (3) (α is an offset due to delays in the system). The maximum reduction in the thermal noise coincides with the largest damping [37], i.e. at $\theta_0 = \alpha \approx -0.25\pi$. This value is set and the pump power is increased. This squeezes the thermal motion of the resonator as shown in Fig. 3(e). For low V_P the amount of squeezing increases with increasing pump power, just as in the case without real-time feedback (cf. Fig. 2(e)). The squeezing approaches 3 dB around $V_P = 6.5 \,\mathrm{mV_p}$, but now when increasing the power further no instability is encountered and the motion is squeezed beyond the 3 dB limit (dashed line). At $V_P = 42 \text{ mV}_p$ the maximum squeezing of 6.7 ± 0.3 dB is obtained. Further increasing the power reduces the squeezing again.

This degradation is analogous to the process in active feedback cooling where imprecision noise reheats the resonator at strong feedback [1, 37–39]. In our case the imprecision noise makes that the pump phase is not exactly at the optimal value $\alpha + 2\varphi - \pi/2$ as $\hat{\varphi}$ is not an accurate estimation of φ . By improving the measurement sensitivity the phase estimation will become better and the maximum squeezing is enhanced. However, ultimately the squeezing must be bounded by the zero-point motion u_{zpm} due to the Heisenberg uncertainty principle which requires $\Delta X_1 \cdot \Delta X_2 \ge u_{\text{zpm}}^2$. Since the feedback tracks φ the pdfs remain circular as shown in Fig. 3(e). This means that $\Delta X_1 = \Delta X_2$, thus ultimately limiting the squeezing to $\Delta X_1 = \Delta X_2 = u_{\text{zpm}}$. However, the current device is still far from this limit (which requires $\sim 72 \,\mathrm{dB}$ of thermomechanical noise squeezing at room temperature) but by cooling the device with a dilution refrigerator this number can be reduced by more than 40 dB.

NONLINEAR FEEDBACK: CLASSICAL NON-GAUSSIAN STATE GENERATION

Squeezing from all directions simultaneously (i.e. isotropically) can be viewed as cooling, but we emphasize that the mechanism that causes the reduction of the occupied phase space in our parametric pump scheme with phase estimation is very different from other, linear, cooling techniques. Interestingly, our technique is inherently nonlinear and naturally generates non-Gaussian classical states that do not arise when simply cooling or parametrically driving the resonator. Figures 4(b,e) show two examples: a number-like and a cat-like state; the Wigner functions of the corresponding quantum states are shown in (a) and (d). The Wigner function of the number state $|n = 1\rangle$ consists of a donut-shaped density function with



FIG. 3: (a-c) Schematic illustrations of the real-time squeezing feedback process. The complex amplitude of the resonator follows at trajectory in the (X_1, X_2) -plane (gray). The red stick indicates the position at a certain time (a) and the ellipse indicates the squeezing direction. The maximum squeezing and anti-squeezing occurs along the minor and major axis respectively. (b) Some time later the complex amplitude has changed and no longer points along the minor axis where the optimal squeezing occurs. By measuring the phase of the complex amplitude, the squeezing angle can be corrected in real-time (c). (d) Dependence of the linewidth on the pump phase and pump power. Fits (solid lines) give $\chi = 0.03 \pm 0.06$, 3.01 ± 0.06 , and 7.06 ± 0.13 Hz for $V_P = 0, 2.5$, and 5.0 mV respectively. (e) Squeezing below (blue) and beyond (red) the 3 dB limit. The inset shows the stationary pdf at the maximum squeezing on the same scale as Fig. 2(a). All errorbars indicate fit uncertainties.

a negative region in the center (gray), a signature of the state's quantumness [40]. Experimentally, the resonator can be prepared in a similar, albeit classical, state by further increasing the pump strength [Fig. 4(c)]. This creates a pdf where the resonator is preferentially located on a circular region with an amplitude of about 30 pm. There is a clear reduction of the probability of finding the resonator near the origin. This can be understood as follows: When the resonator has a small amplitude, comparable to the imprecision noise, the phase estimation is not very reliable and the squeezing is ineffective, pushing the resonator away from the origin. However, as the amplitude grows due the error $\hat{\varphi} - \varphi$, the phase estimation gets more accurate and the resonator gets squeezed back toward the origin until a dynamic equilibrium is reached. It is thus less likely to find the resonator at small amplitudes, reducing the pdf near the origin. A dip in the probability density is clearly visible in the data. Note

that self-sustained and parametric oscillations would also show ring-like pdfs. However, this is not the case here. First of all, the amplitude is much smaller than for parametric oscillations; for 100 mV pump strength the most likely amplitude 16 pm as indicated by the cuts in Fig. 4(c). This is two orders of magnitude smaller than the much larger parametric oscillation amplitude (typical a few nm) that result above threshold without feedback. Furthermore, free oscillations are also ruled out theoretically since Eq. (3) shows that there is no fixed point except for the zero-amplitude solution A = 0. Both the pdf and the cuts clearly show the deviation from the Gaussian shape that one would obtain for regular parametric squeezing or with linear active feedback cooling. The nonlinearity of the feedback thus naturally leads to non-Gaussian pdfs and this bring interesting prospects for generating non-Gaussian quantum states.

The symmetry of the pdf in Fig. 4(b) indicates that the state of the resonator is phase-insensitive. This is because the feedback squeezes isotropically as explained above. However, it is also possible to generate phasedependent states where the circular symmetry is broken. By applying a separate 2f pump signal with a *fixed* phase and yet keeping the co-rotating feedback on, a time dependence is introduced and non-cylindrical pdfs should emerge. Figure 4(e) shows that this is indeed the case. For this combination of fixed and feedback pumps the resonator is in an equal probability of two displaced coherent states, analogous to the cat state shown in (d). This pdf is the result of the feedback that forces the resonator to the donut-like pdf of Fig. 4(b) and the fixed pump that pushes the resonator to the sigar-like shapes shown in Fig. 2. The resonator can thus be in two different positions in phase space separated by a region of low probability near the origin. The resonator dynamically switches between them [41] resulting in the pdf that resembles the cat state in Fig. 4(d). Figure 4(f) shows this switching, which is characterized by telegraph-noiselike transitions from one displaced state to the other. Although the feedback reduces the phase space of the resonator to one of the two states for most of the time, sometimes the thermal noise brings the resonator close to the origin where the phase estimation is inaccurate. This way the pump phase might become misaligned allowing the resonator to switch over to the state with the opposite phase. Then it will stay in that region of phase space until the next jump occurs.

CONCLUSIONS AND OUTLOOK

We have demonstrated that our parametric feedback cannot only be used to squeeze the thermomechanical motion by more than 3 dB but will also be an indispensable tool to prepare mechanical resonators in highly nonlinear classical states. We emphasize that everything



FIG. 4: The calculated Wigner functions (top panels) for a quantum mechanical number state $|n = 1\rangle$ (a) and a cat state $|\beta\rangle + |-\beta\rangle$ that consists of a superposition of two coherent states with amplitude $\beta = 3 \exp(i\pi/4)$ (d). The gray areas indicate regions where the Wigner function is negative. In the classical regime these areas are washed out and the distribution is always positive. (b),(e) experimentally observed pdf's as prepared using our parametric feedback scheme. (e) horizontal cuts through pdfs measured for different strengths of the 2f feedback. (f) measured dynamics of the quadrature along the long axis X_{II} of the cat-like state shown in (e).

demonstrated here can in principle be extended to the quantum regime [1]: The parametric actuation can be done using quantum-backaction-limited optical fields [42] through the optical spring effect, and also the optical readout is in principle shot-noise limited. The nonlinear feedback will thus create correlations between the quantum imprecision noise and the resonator motion. Currently we have demonstrated our feedback scheme using classical resonators but when employing cooling GHz resonators close to their groundstate (either using a dilution refrigerator [2] or using optical backaction cooling [3]) the thermal motion vanishes and is replaced by the zero-point motion allowing to employ our scheme with a true quantum system. Then, in principle only quantum noise will be fed back to the system and hence it seems feasible that not only highly nonlinear states, but even true non-classical mechanical states can be generated.

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- M. Poot and H. S. J. van der Zant, Mechanical systems in the quantum regime, Phys. Rep. 511, 273 (2012).
- [2] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, et al., *Quantum ground state and single-phonon control of a mechanical resonator*, Nature 464, 697 (2010).
- [3] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Groblacher, M. Aspelmeyer, and O. Painter, *Laser cooling of a nanomechanical oscillator into its quantum ground state*, Nature **478**, 89 (2011).
- [4] J. D. Teufel, T. Donner, D. Li, J. W. Harlow, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, K. W. Lehnert, and R. W. Simmonds, *Sideband cooling of micromechanical motion to the quantum ground state*, Nature 475, 359 (2011).
- [5] S. Gröblacher, K. Hammerer, M. R. Vanner, and M. Aspelmeyer, Observation of strong coupling between a micromechanical resonator and an optical cavity field, Nature 460, 724 (2009).
- [6] S. Weis, R. Rivire, S. Delglise, E. Gavartin, O. Arcizet, A. Schliesser, and T. J. Kippenberg, *Optomechanically Induced Transparency*, Science **330**, 1520 (2010).
- [7] J. D. Teufel, D. Li, M. S. Allman, K. Cicak, A. J. Sirois, J. D. Whittaker, and R. W. Simmonds, *Circuit cavity* electromechanics in the strong-coupling regime, Nature 471, 204 (2011).
- [8] T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, *Entangling Mechanical Motion with Mi*crowave Fields, Science **342**, 710 (2013).
- [9] D. W. Brooks, T. Botter, S. Schreppler, T. P. Purdy, N. Brahms, and D. M. Stamper-Kurn, Non-classical light generated by quantum-noise-driven cavity optomechanics, Nature 488, 476 (2012).
- [10] A. H. Safavi-Naeini, S. Groblacher, J. T. Hill, J. Chan, M. Aspelmeyer, and O. Painter, *Squeezed light from a silicon micromechanical resonator*, Nature **500**, 185 (2013).
- [11] T. P. Purdy, P.-L. Yu, R. W. Peterson, N. S. Kampel, and C. A. Regal, *Strong Optomechanical Squeezing of Light*, Phys. Rev. X 3, 031012 (2013).
- [12] T. A. Palomaki, J. W. Harlow, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, *Coherent state transfer between itinerant microwave fields and a mechanical oscillator*, Nature **495**, 210 (2013).
- [13] H. Okamoto, A. Gourgout, C.-Y. Chang, K. Onomitsu, I. Mahboob, E. Y. Chang, and H. Yamaguchi, *Coherent phonon manipulation in coupled mechanical resonators*, Nat Phys **9**, 480 (2013).

- [14] T. Faust, J. Rieger, M. J. Seitner, J. P. Kotthaus, and E. M. Weig, *Coherent control of a classical nanomechanical two-level system*, Nat Phys 9, 485 (2013).
- [15] S. Bose, K. Jacobs, and P. L. Knight, Preparation of nonclassical states in cavities with a moving mirror, Phys. Rev. A 56, 4175 (1997).
- [16] M. R. Vanner, I. Pikovski, G. D. Cole, M. S. Kim, u. Brukner, K. Hammerer, G. J. Milburn, and M. Aspelmeyer, *Pulsed quantum optomechanics*, Proceedings of the National Academy of Sciences **108**, 16182 (2011).
- [17] M. R. Vanner, Selective Linear or Quadratic Optomechanical Coupling via Measurement, Phys. Rev. X 1, 021011 (2011).
- [18] O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Large Quantum Superpositions and Interference of Massive Nanometer-Sized Objects, Phys. Rev. Lett. 107, 020405 (2011).
- [19] A. A. Clerk, F. Marquardt, and K. Jacobs, Back-action evasion and squeezing of a mechanical resonator using a cavity detector, New Journal of Physics 10, 095010 (2008).
- [20] D. Rugar and P. Grütter, Mechanical parametric amplification and thermomechanical noise squeezing, Phys. Rev. Lett. 67, 699 (1991).
- [21] A. Vinante and P. Falferi, Feedback-Enhanced Parametric Squeezing of Mechanical Motion, Phys. Rev. Lett. 111, 207203 (2013).
- [22] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, *Mechanical Squeezing via Parametric Amplifi*cation and Weak Measurement, Phys. Rev. Lett. **107**, 213603 (2011).
- [23] M. Poot and H. X. Tang, Broadband nanoelectromechanical phase shifting of light on a chip, Appl. Phys. Lett. 104, 061101 (2014).
- [24] M. Poot and H. Tang, Strong electrostatic coupling in an optomechanical resonator, In preparation.
- [25] Q. P. Unterreithmeier, E. M. Weig, and J. P. Kotthaus, Universal transduction scheme for nanomechanical systems based on dielectric forces, Nature 458, 1001 (2009).
- [26] I. Kozinsky, H. W. C. Postma, I. Bargatin, and M. L. Roukes, *Tuning nonlinearity, dynamic range, and frequency of nanomechanical resonators*, Appl. Phys. Lett. 88, 253101 (pages 3) (2006).
- [27] M. Poot, K. Y. Fong, M. Bagheri, W. H. P. Pernice, and H. X. Tang, *Backaction limits on self-sustained optomechanical oscillations*, Phys. Rev. A 86, 053826 (2012).
- [28] The demodulated voltages are converted back to displacements using the transduction factor obtained from the area under the thermal noise spectra without the pump (cf. Fig. 1(e)). The variances in Fig. 2 are the eigenvalues of the covariance matrix of $X_1(t)$ and $X_2(t)$ and thus include some (unsqueezed) imprecision noise.

The amount of squeezing of the motion reported is thus a lower bound. The pdfs are kernel densities where a Gaussian kernel is added on every sampled point in the $X_1 - X_2$ space, thus creating a smoothened estimator of the pdf.

- [29] R. Lifshitz and M. C. Cross, Reviews of Nonlinear Dynamics and Complexity: Volume 1 (Annual Reviews of Nonlinear Dynamics and Complexity) (Wiley-VCH, 2008), chap. 1, ISBN 3527407294.
- [30] R. B. Karabalin, S. C. Masmanidis, and M. L. Roukes, Efficient parametric amplification in high and very high frequency piezoelectric nanoelectromechanical systems, Appl. Phys. Lett. 97, 183101 (2010).
- [31] C.-C. Wu and Z. Zhong, Parametric amplification in single-walled carbon nanotube nanoelectromechanical resonators, Appl. Phys. Lett. 99, 083110 (2011).
- [32] I. Mahboob and H. Yamaguchi, Bit storage and bit flip operations in an electromechanical oscillator, Nat Nano 3, 275 (2008).
- [33] A. Eichler, J. Chaste, J. Moser, and A. Bachtold, Parametric Amplification and Self-Oscillation in a Nanotube Mechanical Resonator, Nano Lett. 11, 2699 (2011).
- [34] M. Blencowe and M. Wybourne, Quantum squeezing of mechanical motion for micron-sized cantilevers, Physica B: Condensed Matter 280, 555 (2000).
- [35] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, *Position estimation of a parametrically driven* optomechanical system, New J. Phys. 14, 095026 (2012).
- [36] J. Gieseler, B. Deutsch, R. Quidant, and L. Novotny, Subkelvin Parametric Feedback Cooling of a Laser-Trapped Nanoparticle, Phys. Rev. Lett. 109, 103603 (2012).
- [37] M. Poot, S. Etaki, H. Yamaguchi, and H. S. J. van der Zant, Discrete-time quadrature feedback cooling of a radio-frequency mechanical resonator, Appl. Phys. Lett. 99, 013113 (pages 3) (2011).
- [38] D. Kleckner and D. Bouwmeester, Sub-kelvin optical cooling of a micromechanical resonator, Nature 444, 75 (2006).
- [39] M. Poggio, C. L. Degen, H. J. Mamin, and D. Rugar, Feedback Cooling of a Cantilever's Fundamental Mode below 5 mK, Phys. Rev. Lett. 99, 017201 (pages 4) (2007).
- [40] A. Kenfack and K. Życzkowski, Negativity of the Wigner function as an indicator of non-classicality, Journal of Optics B: Quantum and Semiclassical Optics 6, 396 (2004).
- [41] H. B. Chan, M. I. Dykman, and C. Stambaugh, Switching-path distribution in multidimensional systems, Phys. Rev. E 78, 051109 (2008).
- [42] T. P. Purdy, R. W. Peterson, and C. A. Regal, Observation of Radiation Pressure Shot Noise on a Macroscopic Object, Science 339, 801 (2013).