Translation *Educ. Technol. Res.*, **34**, 75-83, 2011

Classification of Efficient Calculation Problems and the Effect of Instruction Using an Abstract Strategy*

Masayuki SUZUKI[*1, *2], Etsuko TANAKA[*1], Kou MURAYAMA[*3] and Shinichi ICHIKAWA[*1]

[*1] *The University of Tokyo, 7-3-1 Hongou, Bunkyo-ku, Tokyo, 113-0033 Japan*
[*2] *Japan Society for the Promotion of Science*
[*3] *University of Munich, Leopoldstraße 13, Munich, Germany*

Received for publication, April 30, 2011

Calculation problems such as 1＋2＋3＋97＋98＋99 can be solved rapidly and easily by using certain techniques; we call these problems "efficient calculation problems." However, many students do not always solve them efficiently. To address this problem among students, this study developed a new teaching method. The first study sought to examine underlying subcategories of efficient calculation problems. To accomplish this, multidimensional scaling and cluster analysis to a similarity matrix obtained from expert judgments were conducted. As a result, we classified 20 efficient calculation problems into 8 categories. The second study examined the effect of an intervention on 59 eighth grade students and 52 fifth grade students. The students were instructed to use an abstract strategy that stated, "Think carefully about the whole expression," and were then taught the solution to the problem. The results indicated that the eighth grade students solved similar problems efficiently after the intervention, while the fifth grade students did not. The results also suggested that the effect of the intervention was greater among students with sufficient basic calculation skills.

Key words : COMPASS, efficient calculation, structure generation-based approach, transfer, multidimensional scaling

1. INTRODUCTION

Skill in performing a calculation has both quantitative and qualitative aspects. When solving calculation problems, the quantitative aspect allows a person to calculate systematically and precisely, whereas the qualitative aspect allows a person to solve a problem rapidly and easily by using certain techniques. For example, 12×7÷3 can be solved more quickly and easily if one first solves 12÷3 and then calculates 4×7, rather than if one starts by calculating 12×7.

Although calculation skill is essential in mathematical problem solving, past efforts to improve calculation skills have focused on quantitative skills such as *hyakumasu keisan* (KAGEYAMA 2002) and mental calculation skills (KAWAYACHI 2002), whereas qualitative skills have been overlooked. As a result, most students do not seem to solve calculation problems efficiently even if they can use certain basic calculation techniques easily.

In fact, data shows that many students have problems with qualitative calculation skills. ICHIKAWA *et al.* (2009) have developed two calculation tasks using both quantitative and qualitative skills besides a basic calculation task as part of COMPASS[1)], an assessment test of mathematical ability. The first part is "a simple speed calculation task," which assesses quantitative calculation skills, and the second is "an efficient calculation task," which assesses qualitative calculation skills. The efficient calculation task is composed of 16 items at the junior high level and 10 items at the elementary level. COMPASS has been conducted in many elementary schools and junior high schools in Japan. While the results indicate that test scores on the efficient calculation task have a positive correlation with grades in mathematics, the scores are generally low and many students have a problem with qualitative calculation skills (ICHIKAWA *et al.* 2007).

As discussed above, efficient calculation skills are essential for improving calculation skills and, eventually, overall mathematical ability.

* This paper was originally published in *Jpn. J. Educ. Technol.*, Vol.34, No.1, pp.35-43 (2010)

Nevertheless, most students do not acquire the efficient calculation skills. Accordingly, it is of practical importance to have students acquire the skills. In the present article, we instruct the efficient calculation skills and examine its effects. There are a few practical reports as EBISAWA (2003) and OGATA *et al.* (2005), which focus on only one efficient calculation skill. However, the efficient calculation skills include a variety of skills. The study which instructed various skills includes MURAYAMA and ICHIKAWA (2006). MURAYAMA and ICHIKAWA (2006) instructed junior high students the solutions to 16 efficient calculation problems and showed that the socres on the efficient calculation task improved. However, this study did not address the facilitation of transfer to similar targets. The improvement of the calculation skills requires that students calculate efficiently not only the same problems that are taught in intervention but also the analogous problems. Thus, in the present study we develope a new teaching method to promote transfer and examine its effects.

Although the "example-based approach" is a common method to promote transfer, in which instructors teach multiple examples and their solutions, this approach requires much effort to memorize the solutions and to retrieve the specific examples when it is applied to similar problems. In contrast, recent research on transfer has showed that abstractions that are based on a few concrete examples promote transfer (GICK and HOLYOAK 1983). SUZUKI (1996) explained the nature of abstracted knowledge by using the concept of "quasi-abstraction" and pointed out that knowledge acquisition from analogous problems requires knowledge construction at a practicable level. Furthermore, research has shown that not only is the self-performed abstraction effective, but receiving abstract knowledge is also beneficial. For instance, SUZUKI (1995) showed that instruction on the solutions to word problems called "a problem of work" by using an abstraction "accomplish" promoted transfer.

Accordingly, in this article we attempt to promote transfer by instructing efficient calculation skills using an abstract strategy "think carefully about the whole expression." In contrast to approaches that teach specific solution structures, an approach that promotes transfer using an abstract strategy or knowledge for generalizing solution structures is known as a "structure generation-based approach." This approach is known as "knowledge about strategies" or "heuristics" (TERAO and KUSUMI 1998). In the present article, we call "think carefully about the whole expression" an abstract strategy, rather than concrete strategy.

For the problem 12×7÷3, advising students to "think carefully about the whole expression" helps learners to retrieve a concrete solution such as "perform 12÷3 as a first step." In general, most students do not calculate efficiently because they do not regularly consider the whole expression. Therefore, the instruction of this strategy is expected to be effective. If this strategy promotes transfer, the findings in this article will contribute to educational practice and be especially attractive because of the low cost of teaching the strategy.

However, identifying problems that students solve efficiently using the abstract strategy is difficult because there are a variety of efficient calculation problems. Therefore, in study 1 we examine the underlying subcategories of efficient calculation problems by conducting multidimensional scaling (MDS) based on the judgment of experts. MDS provides us with a map of efficient calculation problems, in which problems judged similar are located close together. The map can be interpreted in two ways: dimensional and configurational (KRUSKAL and WISH 1978). In the former, we can interpret how the similarities between the problems were judged on the basis of the meaning of the axes (dimensions) in the map. And in the latter, we can base the interpretation on the classification of the problems located close in the map. Therefore, we can classify the efficient calculation problems based on the structure of the participants' judgment and the similarity between problems. In study 2, we apply the strategy in lessons and identify problems that students solve efficiently using the abstract strategy.

2. STUDY 1

2.1. Methods

2.1.1. Efficient Calculation Problems Investigated

We investigated 16 items in COMPASS and 10 items created in this study. Participants judged similarity by pairwise comparison (based on a scale where very similar = 1 and very dissimilar = 9). We placed items in random order and created 2 separate sheets to display the items in reverse order from the other to counterbalance any order effects.

2.1.2 Participants

The similarity of the efficient calculation problems was judged by 6 graduate students and 1 professor.

2.1.3. Analytic Approaches

We conducted nonparametric MDS to the similarity matrix obtained from the expert judgments. In addition, because the combined use of cluster analysis is an effective method when interpreting the results of MDS (KRUSKAL and WISH 1978), we conducted cluster analysis to the same similarity matrix using the maximum method (JOHNSON 1967).

2.2. Results

Because interpreting the results is straightforward and increasing the number of dimensions does not improve the fit of the model (Young's S-stress = .41, .37, .36, .35, …), we adopted a two-dimensional solution. Based on the results of MDS and cluster analysis, we classified the problems into 8 categories by combining the two analyses, as shown in Figure 1. The number of each item in Figure 1 corresponds to the same item in Table 1. In Figure 1, the horizontal axis indicates the arithmetic operations used and the vertical axis indicates the type of solution to the efficient calculation problems.

2.3. Discussion

1st cluster In items 1 and 2, if we split up the number into separate units (10040 into 10000 and 40, and 708 into 700 and 8), we can solve these problems efficiently. Also, we can calculate 240÷6 more easily than 24000÷600.

2nd cluster If we utilize the distributive law in items 4 and 5, we can solve these problems efficiently. In the case of item 6, if we split up 32 into a multiple of 4 and then use 100 as the multiplier (i.e., 25×32 = 25×4×8 = 100×8), we can work out the calculation easily without writing.

3rd cluster If we transform the decimal fraction into a common fraction (in item 7, 0.5 becomes 1/2 and in item 8, 0.25 becomes 1/4), we can solve problems without having to write out the calculations or decimal point position.

4th cluster In these problems, we can change the order of the numerical terms in multiplication and division to solve the problem more efficiently. For example, rearranging the terms would give you 2×3×9, 36÷4×7, and 400÷4÷25 for items 9, 10, and 11, respectively. In case of item 12, this expression includes 0, so its solution can be known to be 0 without any calculation. When we change the order of terms, we can then solve these problems with smaller number of places.

5th cluster These types of problems involve changing the order or numerical terms in problems with addition and subtraction. In items 13 through 16 we can utilize complementary numbers to simplify the calculation; in items 18 through 20, we can see the same number in different places; in

Table 1. Subcategories of efficient calculation problems

1st cluster: Splitting up numbers in multiplication and division (mental calculation is easy)
1: 10040÷2
2: 708×9
3: 24000÷600
2nd cluster: Splitting up numbers in multiplication and division (mental calculation is difficult)
4: 19×4+19×6
5: 50÷7−1÷7
6: 25×32
3rd cluster: Transformation of decimal fraction into common fraction
7: 33÷0.5
8: 84×0.25
4th cluster: Changing order of numerical terms in multiplication and division
9: 2×9×3
10: 36×7÷4
11: 400÷25÷4
12: 12×9×0×8
5th cluster: Changing order of numerical terms in addition and subtraction
13: 29+27+25+23+21
14: 45−8−2−3−7
15: 1+2+3+97+98+99
16: 54−83+46−17
17: 63+69−30−1−33
18: 200−7−200+3+200+4
19: 45+19+8−19−45
20: 100+3+100+2+100+2
6th cluster: Splitting up numbers in addition and subtraction
21: 388+99
22: 400−299
23: 998×5
7th cluster: Transformation of addition or subtraction into multiplication
24: 9+9+9+9
25: −4−4−4−4
8th cluster: Addition and subtraction by counting
26: 2000−3

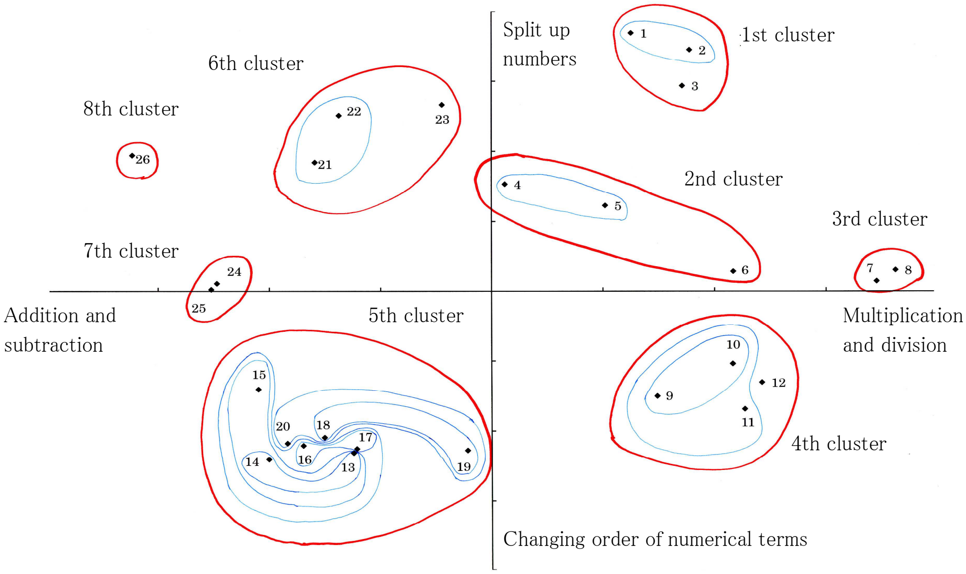

Fig 1. MDS and cluster analysis

case of item 17, we can represent 63＋69－30－1－33 as 63－(30＋33)＋69－1, which allows one to use both the skills for items 13 though 16 and for item 19. Although problems in the 5th cluster can be calculated efficiently by changing the order of numerical terms just as for problems in the 4th cluster, these two subcategories are different since the 5th cluster uses addition and subtraction and the 4th cluster involves multiplication and division, which require different skills and thought processes.

6th cluster The problems in this cluster are cumbersome, but we can calculate them efficiently by utilizing complementary numbers. For example, 99 can be thought of as 100－1 and 299 can be replaced mentally as 300－1. These problems are similar to the problems in 1st and 2nd clusters in that we split up numbers into components that increase calculation efficiency.

7th cluster For these problems, we can transform addition or subtraction into multiplication. For example, 9＋9＋9＋9＝9×4 and －4－4－4－4＝－4×4.

8th cluster Though the regular solution is straightforward, many students choose to compute these types of problems on paper. We can solve this problem without written calculation using a method such as "counting back three numbers from 2000."

To summarize the results, items 1 and 2 in the 1st cluster are similar to items in the 2nd cluster in that they require use of the distributive law.

Also, items 1 and 2 have efficiency skills in common with item 23, but those are classified into different categories. The experts on our panel seem to have judged the problems based not only on the solutions to the efficient calculation, but also on the solutions used in the basic calculations and the appearance of the problem. That is, while it is difficult to mentally solve the items in the 2nd cluster and item 23, it is much easier to solve items in 1st cluster by mental calculation. Furthermore, the number 998 in item 23 resembles the numbers 99 and 299 in the same cluster more than the numbers in 1st cluster.

3. STUDY 2

3.1. *Purpose*

In the second study, we examined the effects of instruction using an abstract strategy that advised students to "think carefully about the whole expression." MURAYAMA and ICHIKAWA (2006) examined the validity of the efficient calculation task and the results showed that the task was sensitive to the intervention that specifically focused promoting the efficient calculation skills. Therefore, improvements in test scores following the promotion of efficient calculation skills represents the degree to which participants actually came to use the efficient calculation skills.

3.2. *Methods*

3.2.1. *Participants*

Study participants included 59 eighth grade students (30 boys and 29 girls) from a Japanese public junior high school and 52 fifth grade students (32 boys and 20 girls) from a Japanese public elementary school. Both schools were located in Okinawa prefecture. All participants took the COMPASS in May 2008.

3.2.2. *Teachers*

The first and second authors served as the teachers in this study. The first author instructed the eighth grade students and the second author taught the fifth grade students. Because each grade at the schools was composed of two classes, the intervention was conducted for each class.

3.2.3. *Date of Intervention*

The intervention was conducted in October 2008.

3.2.4. *Materials*

We excluded items that can be solved in a single step so that the participants would realize the utility of the strategy. Participants are not likely to use a strategy if they do not realize its usefulness (SATO 1998). In addition, we excluded items that required knowledge of the distributive law because fifth grade students had not yet covered this topic. Thereby, intervention materials included items from the 4th, 5th, and 7th clusters. Also, we replaced subtraction signs with addition signs in materials for fifth grade students (e.g., −4−4−4−4 was replaced by 4 +4+4+4) because the fifth grade students had not yet learned negative number.

3.2.5. *Procedure*

The intervention was held during arithmetic or mathematic lessons. Each lesson was 50 minutes. The lesson was composed of pre-test, intervention, and post-test sessions. The pre- and post-test sessions included all 14 items in 4th, 5th, and 7th clusters. The items taught during the intervention included 2 items in 4th cluster, 4 items in 5th cluster, and 1 item in 7th cluster. Therefore, the post-test included 7 "isomorphic transfer problems" that had the same solution but different numbers, and 7 "similar transfer problems" that required a modification of the solution (for more detail, see Table 9). In this article, we define "similar transfer" as transferring use of a skill within a cluster, because we go over 3 of the 8 categories in the lesson and selected items from each category covered in the post-test.

Pre-test Session The pre-test session was conducted at the beginning of the lesson. Each item was printed on a separate sheet of A5-size paper to control the time spent on each item. Additionally, papers on which irrelevant numbers were written were placed between the question sheets. The experimenters instructed the participants to solve each item for 10 seconds after hearing "start" and then to turn the page after hearing "stop." The experimenters explained that the aim of the pre-test is to check the participants' calculation skills.

Intervention Session The experimenters distributed worksheets and began to conduct the intervention session after collecting the pre-test. Initially, to motivate the participants they were told that they can solve the problems in the pre-test within 10 seconds. And then, the experimenters instructed that it is important to think carefully about the whole expression before solving.

After instructing the strategy, the problem 4+4 +4+4 was presented as a concrete example and then practice exercises were shown. The exercises that were written on large pieces of paper were exhibited on the blackboard, and the participants were told to solve them within 10 seconds. After 10 seconds, the paper was removed and students wrote the answer in their worksheets. After all participants finished writing, the paper was placed on blackboard again, and the solution was given. This procedure was repeated 6 times. On this occasion, the experimenters emphasized to students that if you think carefully about the whole expression you can find a way to solve it efficiently, and did not articulate specific concrete strategies so that students did not attempt to memorize each solution (i.e., they were not given specific hints such as "utilize complementary numbers that equal 100", "if multiplying by 0, then the answer is 0, etc.,").

Post-test Session The post-test was conducted in the same procedure as the pre-test. The experimenters told students that the aim of the post-test is to check the participants' improvements.

3.3. *Results and Discussion*

At the end of the lesson, students indicated on a 7-point Likert scale their agreement with the statement, "The strategy is helpful for solving problems efficiently." The results showed students viewed the strategy as sufficiently useful ($M = 6.55$, $SD = 1.11$ for the fifth grade students; $M = 6.52$, $SD = 1.11$ for the eighth grade students).

Table 2 shows the average numbers of the correct answers in the pre-test, post-test, and intervention exercise. Also, we classify the change scores from the pre-test to the post-test as either "isomorphic transfer effect" or "similar transfer effect" based on the type of problem. Table 3 and 4 show the frequency distributions of each transfer effect. Because the format of the intervention exercise is different from that of the pre-test and post-test, we do not compare the score from the exercises to the scores from the pre-test and post-test.

The results of the t-test showed that the isomorphic transfer effects were statistically significant (for fifth grade students, $t(58) = 13.86$, $d = 3.64$, $p < .01$; for eighth grade students, $t(51) = 9.34$, $d = 2.62$, $p < .01$). Moreover, Table 3 indicated that few participants regressed and most participants improved. Therefore, we conclude that for isomorphic problems, students at both grade levels became more proficient in calculating the problems efficiently. On the other hand, the results of the t-test showed that there was a statistically significant effect in similar transfer problems for the eighth grade students ($t(51) = 6.90$, $d = 1.93$, $p < .01$), but not for fifth grade students ($t(58) = 1.12$, $d = 0.29$, $p = .267$), indicating that the eighth grade students solved similar problems efficiently by using the abstract strategy, whereas fifth grade students did not. Hence, the similar transfer effect was different between the fifth- and eighth-grade students.

Table 2. Average numbers of the correct answers and change scores (*SD*)

	Pre-test	Post-test	Isomorphic transfer	Similar transfer	Exercise
Fifth grade students	5.10	8.08	2.81	0.17	3.90
(*N* = 59)	(3.26)	(2.65)	(1.59)	(1.16)	(1.80)
Eighth grade students	6.13	9.65	2.06	1.46	4.75
(*N* = 52)	(2.95)	(2.34)	(1.59)	(1.53)	(1.52)

Table 3. Frequency distribution of isomorphic transfer effect

	Fifth grade students (*N* = 59)		Eighth grade students (*N* = 52)	
Score	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
−1	1	1.69	2	3.85
0	2	3.39	6	11.54
1	11	18.64	12	23.08
2	9	15.25	14	26.92
3	17	28.81	8	15.38
4	10	16.95	6	11.54
5	7	11.86	3	5.77
6	2	3.39	1	1.92

3.3.1. Differences in Similar Transfer Effect

Table 4 indicates that some fifth grade students performed worse after the intervention. We found that most of the fifth grade students did not improve their scores when applying the skills to similar problems. On the other hand, most eighth grade students showed improved scores on these problems. These differences probably are a result of differences in the capacity to understand abstract explanations and the students' proficiency and knowledge in handling mathematical expressions. Generally, mathematics is more logical and abstract than arithmetic and MIURA et al. (1996) argue that students improve the basic calculation skills during junior high school. Moreover, junior high students have greater proficiency in handling mathematical expression than elementary students because they have learned about negative numbers and expressions with variables in mathematics. The strategy "think carefully about the whole expression" appears to be so abstract that elementary students have difficulty comprehending how to do so. In addition, finding the solutions is likely difficult for the younger students because they have lower proficiency in handling mathematical expressions. Thus, the differences in the capacity to understand abstract explanations and basic calculation skills between students in the fifth and eighth grades probably explain the differences in proficiency in using the abstract strategy. Ability to use the abstract strategy also resulted in differences in the similar transfer effect.

Table 4. Frequency distribution of similar transfer effect

	Fifth grade students (*N* = 59)		Eighth grade students (*N* = 52)	
Score	Frequency	Relative frequency (%)	Frequency	Relative frequency (%)
−4	1	1.69	0	0.00
−3	1	1.69	1	1.92
−2	2	3.39	0	0.00
−1	9	15.25	3	5.77
0	20	33.90	10	19.23
1	22	37.29	11	21.15
2	4	6.78	16	20.77
3	0	0.00	6	11.54
4	0	0.00	4	7.69
5	0	0.00	1	1.92

However, these differences could possibly have resulted from having different teachers conduct the lesson and from the underlying differences in basic academic skills between the 2 groups. So, future research needs to explore these differences between elementary students and junior high students.

3.3.2. *Effects of Each Calculation Skill on the Similar Transfer Effect*

The differences in the similar transfer effect may result from the calculation skills each student had before the intervention took place. Therefore, we conducted a hierarchical regression analysis for each grade to determine the independent contribution of each of the 3 calculation skills on the scores from the similar problems in the post-test, after controlling for the scores from the similar problems in the pre-test. However, the analysis was conducted on students who had both scores from COMPASS and the experiment. COMPASS scores are shown in Table 5 and the correlations between variables are shown in Table 6. The first step included only the scores from similar problems on the pre-test, and the second step included the scores from all 3 calculation tasks as additional predictors (Table 7).

The result for the fifth grade students showed that efficient calculation skills were a significant predictor at the 10% level in the second step, which shows that the effects of the intervention were observed among fifth grade students who already had efficient calculation skills. Given that most students did not improve on the similar problems, it may be advantageous to memorize the solutions and apply them to isomorphic problems. In contrast, the result for eighth grade students showed that basic calculation skills were a significant predictor in the second step, which means that the effects of the intervention were greater among eighth grade students with sufficient basic calculation skills. Though most eighth grade students improved, the degree of improvement increased most prominently for students with sufficient basic calculation skills.

3.3.3. *Changes in Scores for Each Cluster and Every Problem*

We examined changes in scores for each cluster and every problem in each cluster to determine the effects of the strategy in more detail (see Tables 8 and 9). However, as for problems in the 7th cluster we examined only changes in scores for each problem because there were no similar problems given to the fifth grade students and only one similar problem given to the eighth grade students from this cluster.

The similar transfer effect in 4th cluster effect was significant at the 10% level for the fifth grade students (t (58) = 1.84, d = 0.48, p = .072), though the previous analysis showed that the similar transfer effect was not significant for these students. This result suggests that the ease of

Table 5. Average COMPASS scores

	Basic calculation		Simple speed calculation		Efficient calculation	
Fifth grade students	8.21	(10)	12.04	(20)	4.70	(10)
(N = 57)	1.52		3.53		2.20	
Eighth grade students	15.37	(20)	13.67	(20)	6.33	(16)
(N = 54)	5.59		3.51		3.06	

Figures in parentheses indicate the best possible score and figures in the second row show standard deviation

Table 6. Correlations between variables

	Post-test	Pre-test	Basic	Simple	Efficient
Post-transfer		.545 **	.661 **	.424 **	.414 **
Pre-transfer	.783 **		.581 **	.711 **	.566 **
Basic	.175	.289		.685 **	.643 **
Simple	.583 **	.687 **	.387 **		.633 **
Efficient	.677 **	.715 **	.315 *	.675 **	

Upper-right portion: eighth grade students, Lower-left portion: fifth grade students

$^{*}p < .05$ $^{**}p < .01$

Table 7. Hierarchical multiple regression analysis

	Fifth grade students (N = 54)		Eighth grade students (N = 51)	
	STEP1	STEP2	STEP1	STEP2
Pre-transfer	.783 **	.610 **	.545 **	.382 *
Basic calculation		−.090		.655 **
Simple speed calculation		.029		−.259
Efficient calculation		.250 †		−.059
Adj. R^2	.605	.619	.282	.464

$^{\dagger}p < .10$ $^{*}p < .05$ $^{**}p < .01$

Table 8. Changes in scores for each cluster (*SD*)

	4th cluster		5th cluster	
	Isomorphic transfer	Similar transfer	Isomorphic transfer	Similar transfer
Fifth grade students	1.14 **	0.15 †	1.59 **	−0.10
(N = 59)	(0.71)	(0.64)	(1.31)	(0.88)
Eighth grade students	0.52 **	0.69 **	1.48 **	0.67 **
(N = 52)	(0.64)	(0.81)	(1.15)	(1.13)

$^{\dagger}p < .10$ $^{*}p < .05$ $^{**}p < .01$

transfer is different among categories. However, we need to interpret the results carefully since substantial improvements were made on the problems that were taught in the lesson. Thus, whether fifth grade students calculate similar problems efficiently is a future issue. As for eighth grade students, the effects of each cluster were all significant and the effects for each problem were significant except for 2 items: 92−4+8−96 and −4−4−4−4. Inadequate proficiency in utilizing complementary numbers and negative numbers stunted the improvements in these items. The results suggest that difficulty in transfer is not due only to similarity of the problems. Thus, it is not necessarily the case that 92−4+8−96 and −4−4−4−4 are different from the other items. Additionally, the results for fifth grade students suggest that promoting transfer is easier for the problems in the 4th cluster compared with other problems, but this cannot be explained by similarity. Therefore, in future research we need to examine the mechanism by which the transfer occurs and which problems are most likely to facilitate transfer and to which problems. Finding these solutions may lead to the development of new methods to promote more transfer.

4. CONCLUSIONS AND LIMITATIONS

In this article, we examined the effects of teaching efficient calculation skills to students by using an abstract strategy "think carefully about the whole expression." In study 1, we conducted MDS and cluster analysis using a similarity matrix to determine the underlying subcategories of efficient calculation problems. As a result, we classified 20 efficient calculation problems into 8 categories. Evaluation by experts was used to judge the efficient calculation problems and we found which problems were similar to one another. In study 2, we examined the effects of an intervention that taught students to use an abstract strategy. Eighth grade students solved similar problems efficiently after the intervention, whereas fifth grade students did not. The results also showed that the effects of the intervention were greater among students with sufficient basic calculation skills. Basic calculation skills were also important in that lack of skills in this area lead to wrong solutions. But, we think it is important to get students to understand a computation rule while they are acquiring proficiency in efficient calculation, rather than to wait to teach efficient calculation after students have acquired basic calculation skills. In addition, the strategy may be too abstract for fifth grade students to use. Therefore, more concrete strategies may be more effective for elementary students.

Lastly, we discuss three limitations of this study. First, in study 2, because we were not able to establish controls on the groups involved in the intervention and cannot examine the strengths and uniqueness of the intervention. However, we can conclude that the intervention in this study is

Table 9. Changes in correct response rate

	Pre-test			Post-test			Lesson		
		Fifth	Eighth		Fifth	Eighth		Fifth	Eighth
9	2×7×4	.46	.71	2×8×3	.86 **	.88 *	2×9×3	.80	.85
10	35×7÷4	.10	.00	42×7÷6	.17	.33 **			
11	56÷4÷7	.31	.44	72÷3÷8	.39	.81 **			
12	29×7×0×9	.20	.65	17×8×0×9	.93 **	1.00 **	19×9×0×3	.68	.85
	4th cluster	.27	.45		.59 **	.75 **		.74	.85
13	40+25+9+20+25	.37	.50	25+30+7+25+20	.46	.75 **			
14	45−7−3−4−6	.36	.37	35−8−2−3−7	.63 **	.77 **	55−6−4−2−8	.49	.65
15	1+2+3+99+98+97	.22	.25	1+2+3+49+48+47	.56 **	.75 **	97+98+99+1+2+3	.41	.69
16	92−4+8−96	.25	.44	97−9+3−91	.19	.46			
17	65+69−30−1−35	.15	.00	83+88−40−1−43	.22	.15 **			
18	200+7−200−3+200−4	.27	.25	200+9−200−6+200−3	.19	.42 *			
19	32+46+9−32−46	.22	.42	34+29+8−34−29	.73 **	.90 **	49+31+4−49−31	.58	.71
20	100+1+100+3+100+2	.54	.77	100+4+100+3+100+1	.92 **	.94 *	1+100+3+100+2+100	.95	1.00
	5th cluster	.30	.38		.49 **	.64 **		.61	.76
24	7+7+7+7	.86	.88	8+8+8+8	.95	.94	4+4+4+4		
25	−4−4−4−4	.78	.44	−6−6−6−6	.90	.54	(the presented example)		
	7th cluster	.82	.66		.93 *	.74			

The number on the far left corresponds to the number in Figure 1 and Table 1. * $p < .05$ ** $p < .01$

effective because most students improved, especially for eighth grade students. Secondly, we cannot generalize the results to problems that were not included in the intervention. Therefore, we need to examine the facilitation of transfer for the other problems and to determine which strategies are the more effective. Finally, it is possible that the results in study 2 are the effects of the lesson-specific situation where experimenters teach students to solve problems efficiently. It is important for students to solve problems efficiently in regular situations for tasks such as basic calculation problems and word problems. Therefore, it is necessary to examine whether students calculate problems efficiently in these situation.

NOTES

1) COMPASS is an assessment test based on the cognitive model of mathematical problem solving. This test diagnoses components of mathematical ability that are required in the process of understanding and solving mathematical problems. COMPASS has been developed for students in the fifth through eighth grade and the time limitations are set for each task to measure the target component accurately. The problem contents for tasks at each grade level are defined by the previous curriculum guidelines. The number of problems in some tasks varies by grade. The basic calculation task assesses students' knowledge of calculation algorithms and includes the four arithmetic operations, decimals, fractions, negative numbers, and expressions with variables, depending on the grade. Because this task is intended to assess basic knowledge rather than the speed of computation, sufficient time was provided to perform the task (e.g., $3+4\times2$ and $3-9$). The simple speed calculation task does not require any efficient calculation skills. Rather the task requires speed in the four arithmetic operations. (e.g., $56\div7$ and $8+4-7+9$).

2) It seems that the ceiling effect in the basic calculation task resulted in insignificant correlations between scores on the basic calculation task and scores on the similar transfer task on both the pre-test and the post-test.

ACKNOWLEDGEMENTS

Authors express their appreciation to the teachers and students for their participation in the survey and implementation phases of this research.

This research was supported by a Grant-in-Aid for Scientific Research from The Ministry of Education, Culture, Sports, Science and Technology (No. 70134335) to Ichikawa.

REFERENCES

EBISAWA, T. (2003) Shougakkou sansuka ni okeru keisan no kufu ni kansuru kenkyu: shouhizei no keisan no kufu wo kangaeru jugyou jissen wo tooshite. Journal of Japan Society Mathematical Education: Research Journal of Mathematical Education, 79 : 41 (in Japanese)

GICK, M. L. and HOLYOAK, K. J. (1983) Schema induction and analogical transfer. Cognitive Psychology, 14 : 1-38

ICHIKAWA. S., HAEBARA, T., SUGISAWA, T., SEO, M., KIYOKAWA, S., INUZUKA, M., MURAYAMA, K., UESAKA, Y., KOBAYASHI, H. and SHINOGAYA, K (2009) Development of COMPASS: componential assessment for basic competence and study skills in mathematics. Cognitive Studies, 16 : 333-347 (in Japanese)

ICHIKAWA. S., KIYOKAWA, S., MURAYAMA, K., T., SEO, M. and UESAKA, Y. (2007). Suugaku no gakuryoku, gakushuuryoku asesumento tesuto COMPASS no kaihatsu: dai 2 han no sakusei, jisshi, kekka to kyouiku jissen he no riyou. 21st Century COE Program , Graduate School of Education, The University of Tokyo, Center for Research of Core Academic Competences, Working Paper, 26 : 1-29 (in Japanese)

JOHNSON, S. C. (1967) Hierarchical clustering schemes. Psychometrika, 32 : 241-254

KAGEYAMA, H. (2002) Kageyama mesoddo tettei hanpuku "hyakumasu keisan". Shougakukan, Tokyo (in Japanese)

KAWAYACHI, T. (2002) Keisanryoku wo tsukeru tameno kokoromi. Koutou Kyouiku Kenkyu, 54 : 29-47 (in Japanese)

KRUSKAL, J. B. and WISH, M. (1978) Multidimensional scaling. Sage Pubns; Expanded.

MIURA, K., SHIBUYA, M. and TSUCIYA, A. (1996) How well do secondary schoolers achieve basic arithmetics and how do they think about importance of their achievements? Bulletin of the Faculty of Education, Chiba University. I, Pedagogy, 44 : 29-44 (in Japanese)

MURAYAMA, K. and ICHIKAWA, S. (2006) COMPASS ni okeru "kufu keisan mondai" no datousei no kentou.Proceedings of the 48th Annual Meeting of the Japanese Association of Educational Psychology : 344 (in Japanese)

OGATA, H., OKAZAWA, J. and YAMADA, A (2005) Shiroyama shougakkou 2 nen 2 kumi tangenmei "kufu shite keisan shiyou". Journal of Japan Society Mathematical Education, 87 : 47-48 (in Japanese)

SATO, J. (1998) Effects of Learners' Perceptions of Utility and Costs, and Learning Strategy Preferences. Japanese Journal of Educational Psychology, 46 : 367-376 (in Japanese)

SUZUKI, H. (1995) Tenni ni okeru gutairei to chuushouka no yakuwari. Proceedings of the 37th Annual Meeting of the Japanese Association of Educational Psychology: 204

SUZUKI, H. (1996) Ruiji to shikou. Kyouritsu shuppan, Tokyo (in Japanese)

TERAO, A. and KUSUMI, T. (1998) A Review of Knowledge Acquisition Promoting in Mathematical Problem Solving Transfer. Japanese Journal of Educational Psychology, 46 : 461-472 (in Japanese)