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Research Article

Classification of Multiply Travelling Wave Solutions for Coupled Burgers, Combined KdV-Modified KdV, and Schrödinger-KdV Equations

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Some explicit travelling wave solutions to constructing exact solutions of nonlinear partial differential equations of mathematical physics are presented. By applying a theory of Frobenius decompositions and, more precisely, by using a transformation method to the coupled Burgers, combined Korteweg-de Vries- (KdV-) modified KdV and Schrödinger-KdV equation is written as bilinear ordinary differential equations and two solutions to describing nonlinear interaction of travelling waves are generated. The properties of the multiple travelling wave solutions are shown by some figures. All solutions are stable and have applications in physics.

1. Introduction

The investigation of traveling wave solutions of nonlinear evolution equations (NLEEs) plays a vital role in different branches of mathematical physics, engineering sciences, and other technical arenas, such as plasma physics, nonlinear optics, solid state physics, fluid mechanics, chemical physics, and chemistry.

The Burgers' equation has been found to describe various kinds of phenomena such as a mathematical model of turbulence [1] and the approximate theory of flow through a shock wave traveling in viscous fluid [2]. Fletcher using the Hopf-Cole transformation [3] gave an analytic solution for the system of two-dimensional Burgers' equations.

The Korteweg-de Vries (KdV) equation which models shallow-water phenomena has been analyzed extensively using the invariance properties that occur from the Lie point symmetry generator that admits it. In particular, travelling wave solutions arise from the combination of translations in space and time. Also, Galilean invariants and scale-invariant solutions are dependent on first and second Painleve transcendent [4]. Further, the modified KdV (mKdV) has attracted interest in a similar way and its Lie point symmetry generators are known [5]. Recently, the combined KdV (cKdV) and mKdV equation has been studied using various methods with a special reference to soliton-type solutions. For example, simple soliton solutions to cKdV-mKdV used in plasma and fluid physics are obtained in [6]. Here, the particular form uses the fact that the equation admits a scaling symmetry which is nonexistent for the general cKdV [7, 8].

The topic of solitons produced by nonlinear interactions is a very fundamental topic in various fields, including optical solitons in fibers [9]. The one-dimensional soliton can be considered as a localized wave pulse that propagates along one space direction undeformed; that is, dispersion is completely compensated by the nonlinear effects. There is an enormous amount of literature about the integrability of nonlinear equations related to scattering equations, including especially inverse scattering theories, in relation to solitons [10]. In particular, analysis related to NLS and KdV equations has been studied [10–12].

In recent years, various methods have been established to obtain exact traveling solutions of nonlinear partial differential equations, for example, the Jacobi elliptic function expansion method [13], the generalized Riccati equation method [14], the Backlund transformation method [15], the Hirota's bilinear transformation method [16], the variational iteration method [17–20], the tanh-coth method [21, 22], the direct algebraic method [23, 24], the Cole-Hopf transformation method [25, 26], the Exp-function method [27–29], and others [30]. Recently, Wang et al. [31] introduced a method to obtain traveling wave solutions of the nonlinear partial differential equations, called the (G'/G)- expansion method.

This paper is organized as follows. An introduction is given in Section 1. In Section 2, an analysis and theory of method theory of transformation the partial differential equations to a bilinear ordinary differential equations. In Section 3, the multiple travelling wave solutions of coupled Burgers' equations are obtained. Two cases of traveling wave solutions of the combined KdV-modified KdV equation are given in Section 4. In Section 5, multiple travelling wave solutions of the coupled Schrödinger-KdV equations are given.

2. An Analysis of the Method and Applications

Now, we simply describe the generalized extended tanh-function method. Consider a given system of NEEs, say, in two variables, x and t:

$$H(u, v, u_t, v_t, u_x, v_x, u_{xt}, v_{xt}, \ldots) = 0, \tag{1}$$

$$G(u, v, u_t, v_t, u_x, v_x, u_{xt}, v_{xt}, \ldots) = 0. \tag{2}$$

We consider the following formal traveling wave solutions $u(x,t) = U(\xi)$, $v(x,t) = V(\xi)$, and $\xi = x - ct$, where c is a constant to be determined later. Then, (1) and (2) become system of nonlinear ordinary differential equations as

$$H\left(U, V, U', V', U'', V'', \ldots\right) = 0, \tag{3}$$

$$G\left(U, V, U', V', U'', V'', \ldots\right) = 0. \tag{4}$$

In order to seek the traveling wave solutions of (3) and (4), we introduce the following new ansätz:

$$U(\xi) = a_{10} + \sum_{j=1}^{m_1} \left\{ a_{1j}\phi^j + b_{1j}\phi^{-j} + c_{1j}\phi^{j-1}\sqrt{b+\phi^2} + d_{1j}\frac{\sqrt{b+\phi^2}}{\phi^j} \right\}, \tag{5}$$

$$V(\xi) = a_{20} + \sum_{j=1}^{m_2} \left\{ a_{2j}\phi^j + b_{2j}\phi^{-j} + c_{2j}\phi^{j-1}\sqrt{b+\phi^2} + d_{2j}\frac{\sqrt{b+\phi^2}}{\phi^j} \right\}, \tag{6}$$

where a_{i0}, a_{ij}, b_{ij}, c_{ij}, d_{ij} $(j = 1, 2, \ldots, m_i; i = 1, 2)$, and b are constants to be determined later. The value of m_i in (5) and (6) can be determined by balancing the highest-order derivative term with the nonlinear term in (5) and (6). The new variable $\phi = \phi(\xi)$ satisfies $\phi' = b + \phi^2$.

There exist the following steps to be considered further.

Step 1. Determine the m_1 and m_2 of (5) and (6) by, respectively, balancing the highest order partial differential terms and the nonlinear terms in (3) and (4).

Step 2. Substituting (5) and (6) into (3) and (4), the corresponding ODEs, then let all coefficients of $\phi^p(\sqrt{b+\phi^2})^q$ $(q = 0, 1;\ p = 0, 1, 2, \ldots)$ be zero to get an overdetermined system of nonlinear algebraic equations with respect to a_{i0}, a_{ij}, b_{ij}, c_{ij}, d_{ij}, b, c $(j = 1, 2, \ldots, m_i;\ i = 1, 2)$.

Step 3. By solving the system, we may determine the above parameters.

Step 4. Substituting the parameters a_{i0}, a_{ij}, b_{ij}, c_{ij}, d_{ij}, b, c $(j = 1, 2, \ldots, m_i;\ i = 1, 2)$ obtained in Step 3 into (2), we can derive the solutions of equation.

3. Example I

The coupled Burgers' equations [32] have applications in the quantum field theory, plasma physics, fluid mechanics, and solid state physics. The usual system of equation is as follows:

$$u_t - u_{xx} + 2uu_x + \alpha(uv)_x = 0, \tag{7}$$

$$v_t - v_{xx} + 2vv_x + \beta(uv)_x = 0. \tag{8}$$

Let us consider the traveling wave solutions $u(x,t) = U(\xi)$, $v(x,t) = V(\xi)$, and $\xi = x - ct$, and then (4) becomes

$$-cU - U' + U^2 + \alpha UV = 0, \tag{9}$$

$$-cV - V' + V^2 + \beta UV = 0. \tag{10}$$

Balancing the nonlinear term U^2 and the highest order derivative U' gives $m = 2$. We suppose the solutions of (5) and (6) are of the forms

$$U(\xi) = a_0 + a_1\varphi + a_2\varphi^{-1} + a_3\sqrt{b+\varphi^2} + a_4\frac{\sqrt{b+\varphi^2}}{\varphi}, \tag{11}$$

$$V(\xi) = b_0 + b_1\varphi + b_2\varphi^{-1} + b_3\sqrt{b+\varphi^2} + b_4\frac{\sqrt{b+\varphi^2}}{\varphi} + b_5\varphi^2 + b_6\varphi^{-2} + b_7\varphi\sqrt{b+\varphi^2} + b_8\frac{\sqrt{b+\varphi^2}}{\varphi^2}. \tag{12}$$

Substituting (11) and (12) into (9) and (10) with $\phi' = b + \phi^2$ yields a set of algebraic equations for $a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8$.

By the solution of the above system of equations, we can find

$$a_0 = \frac{2c}{\beta}, \qquad a_1 = 0, \qquad a_2 = \frac{2c^2}{\beta-1}, \qquad a_3 = -\frac{3c\sqrt{-b-2c^2-2c^2\beta}}{\beta(b+2c^2+2c^2\beta)}, \qquad a_4 = 0, \tag{13}$$

$$b_0 = -2c, \qquad b_1 = -1, \qquad b_2 = \frac{2\beta^2c^2}{1-\beta}, \qquad b_3 = \frac{3c\sqrt{-b-2c^2-2c^2\beta}}{b+2c^2+2c^2\beta}, \qquad b_4 = \pm\sqrt{-b-2c^2-2c^2\beta}, \qquad b_5 = b_6 = b_7 = b_8 = 0. \tag{14}$$

Then, combining (11) and (12) with (13) and (14), we obtain the traveling wave solutions of (7) and (8) as

$$u(x,t) = \frac{2c}{\beta} + \frac{2c^2}{(\beta-1)\sqrt{b}}\cot\sqrt{b}(x+ct) - \frac{3c\sqrt{-b-2c^2-2c^2\beta}}{\beta(b+2c^2+2c^2\beta)}\sqrt{b}\sec\sqrt{b}(x+ct), \tag{15}$$

$$v(x,t) = -2c - \sqrt{b}\tan\sqrt{b}(x+ct) + \frac{2\beta^2c^2}{(1-\beta)\sqrt{b}}\cot\sqrt{b}(x+ct) + \frac{3c\sqrt{-b-2c^2-2c^2\beta}}{b+2c^2+2c^2\beta}\sqrt{b}\sec\sqrt{b}(x+ct) \pm \sqrt{-b-2c^2-2c^2\beta}\csc\sqrt{b}(x+ct). \tag{16}$$

The shape solutions (15) and (16) are shown in Figures 1(a), 1(b), 1(c), and 1(d), with $\beta = -2$, $b = 2$, and $c = \pm 2$.

4. Example II

Consider the combined KdV-modified KdV equation:

$$u_t - \alpha u u_x + \beta u^2 u_x + \delta u_{xxx} = 0, \qquad \beta \neq 0, \tag{17}$$

where α, β, and δ are constants. Let us consider the traveling wave solutions $u(x,t) = U(\xi)$ and $\xi = x + ct$, and then (17) becomes

$$cU + \frac{\alpha}{2}U^2 + \frac{\beta}{3}U^3 + U'' = 0. \tag{18}$$

We suppose the solution of (18) is of the form (11) and substituting into (18) yields a set of algebraic equations for a_0, a_1, a_2, a_3, a_4. We have two cases for these equations that are found.

Case 1. By the solution of (11), we can find

$$a_0 = -\frac{\alpha}{2\beta}, \qquad a_1 = 0, \qquad a_2 = \pm i\sqrt{\frac{3}{2\beta}}b, \qquad a_3 = 0, \qquad a_4 = \pm i\sqrt{\frac{3b}{2\beta}}. \tag{19}$$

Substituting (11) into (19) we have obtained the following solution of (17):

$$u(x,t) = -\frac{\alpha}{2\beta} \pm i\sqrt{\frac{3b}{2\beta}}\cot\left[\sqrt{b}(x+ct)\right] \pm i\sqrt{\frac{3b}{2\beta}}\csc\left[\sqrt{b}(x+ct)\right]. \tag{20}$$

Case 2. By the solution of (11), we can find

$$a_0 = -130, \qquad a_1 = \pm 65\sqrt{\frac{2}{b}}, \qquad a_2 = \pm 65\sqrt{2b}, \qquad a_3 = a_4 = 0. \tag{21}$$

Substituting from (11) and (21) we have obtained the following solution of (17):

$$u(x,t) = -130 \pm 65\sqrt{2}\tan\left[\sqrt{b}(x+ct)\right] \pm 65\sqrt{2}\cot\left[\sqrt{b}(x+ct)\right]. \tag{22}$$

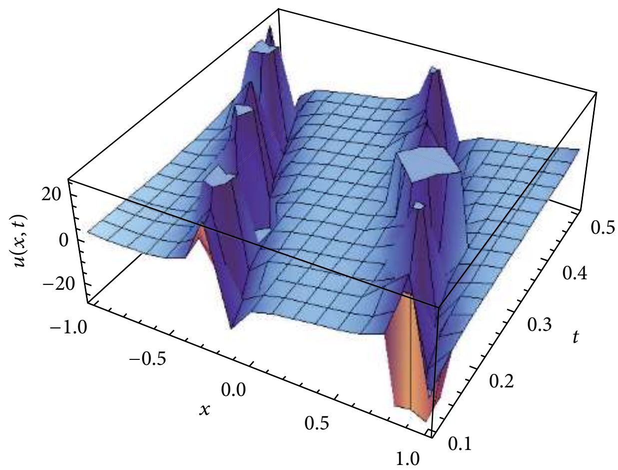
The shape solution (22) is shown in Figure 2(b), with $b = 2$ and $c = -16$.

5. Example III

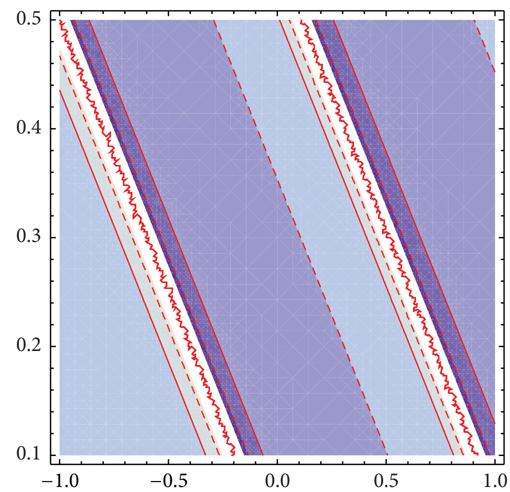
Consider the coupled Schrödinger-KdV equations (Davey-Stewartson)

$$iu_t + u_{xx} - u_{yy} - 2|u|^2 u - 2uv = 0, \tag{23}$$

$$v_{xx} - v_{yy} - 2\left(|u|^2\right)_{xx} = 0. \tag{24}$$



(a)



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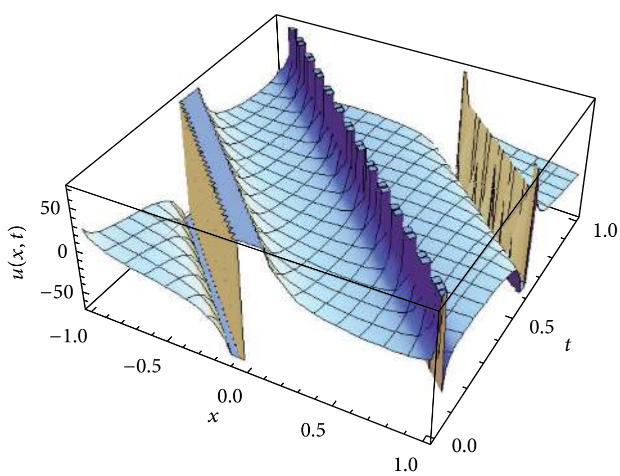

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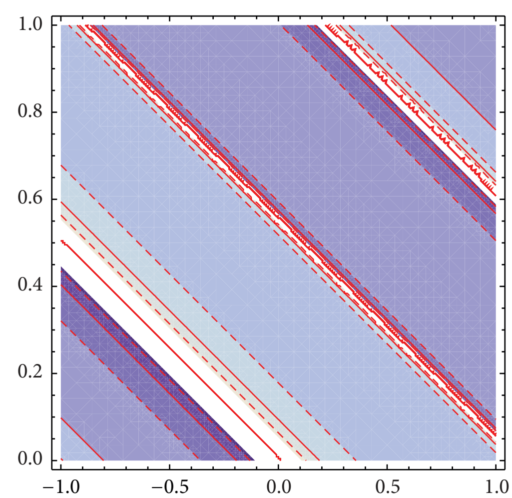

(d)

FIGURE 1: Travelling waves solutions (15) with various different shapes are plotted: periodic solitary waves in (a) and contour plot in (b). Travelling waves solutions (16) with various different shapes are plotted: periodic solitary waves in (c) and contour plot in (d).

Let us consider the traveling wave solutions $u(x,t) = U(\xi)$ and $\xi = (kx + cy + dt)/(x + y - dt)$, and then (23) and (24) become

$$\left(q^2 - p^2 - r\right)U + \left(k^2 - c^2\right)U'' + k^3 - 2U^{(3)} - 2UV = 0, \tag{25}$$

$$\left(k^2 + c^2\right)V'' + \left(U^2\right)'' = 0. \tag{26}$$

We suppose the solutions of (25) and (26) are of the forms (11) and (12). Substituting (11) and (12) into (25) and (26) yields a set of algebraic equations for $a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8$. The solution of the system of equations has two cases.

Case 1. Consider the following: $k^2 = 1/4$, $c^2 = 3/4$, and $b = 5/9$,

$$\begin{gathered} a_0 = 0, \qquad a_1 = \pm\frac{10\sqrt{2}}{9}, \qquad a_2 = -1, \\ a_3 = 2, \qquad a_4 = \frac{3}{2}, \\ b_0 = \frac{1}{36}\left(-17 \pm 80\sqrt{2} - 18p^2 + 18q^2 - 18r\right), \\ b_1 = -\frac{112}{27}, \qquad b_2 = -\frac{10}{3}, \end{gathered} \tag{27}$$

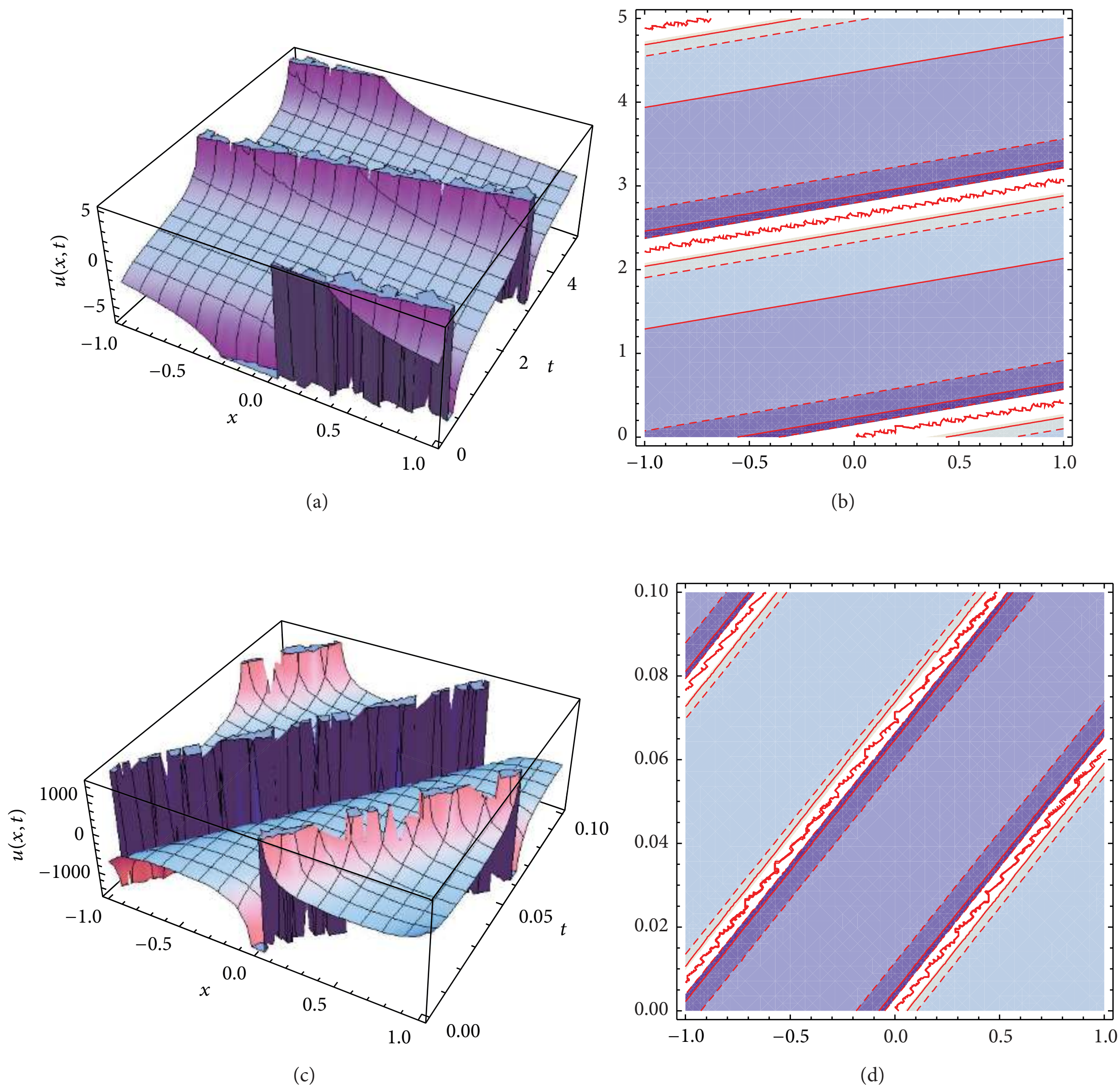

FIGURE 2: Travelling waves solutions (20) with various different shapes are plotted: periodic solitary waves in (a) and contour plot in (b). Travelling waves solutions (22) with various different shapes are plotted: periodic solitary waves in (c) and contour plot in (d).

$$b_3 = \mp\frac{10\sqrt{2}}{3}, \qquad b_4 = 4, \qquad b_5 = -16,$$
$$b_6 = -\frac{9}{4}, \qquad b_7 = \frac{20}{9}\left(1 \pm 2\sqrt{2}\right), \qquad b_8 = 3. \tag{28}$$

Substituting from (25), (26), (27), and (28), we have obtained the following solutions of (23) and (24):

$$u(x,t) = \pm\frac{10\sqrt{2b}}{9}\tan\xi - \frac{1}{\sqrt{b}}\cot\xi + 2\sqrt{b}\sec\xi + \frac{3}{2}\csc\xi, \tag{29}$$

$$\begin{aligned} v(x,t) &= \frac{1}{36}\left(-17 \pm 80\sqrt{2} - 18p^2 + 18q^2 - 18r\right) \\ &\quad - \frac{112\sqrt{b}}{27}\tan\xi - \frac{10}{3\sqrt{b}}\cot\xi \mp \frac{10\sqrt{2b}}{3}\sec\xi + 4\csc\xi \\ &\quad - 16b\tan^2\xi - \frac{9}{4b}\cot^2\xi + \frac{20b}{9}\left(1 \pm 2\sqrt{2}\right)\tan\xi\sec\xi \\ &\quad + \frac{3}{\sqrt{b}}\cot\xi\csc\xi, \end{aligned} \tag{30}$$

with $\xi = \sqrt{b}((x + \sqrt{3}y + \sqrt{4}t)/\sqrt{4}(x + y - t))$.

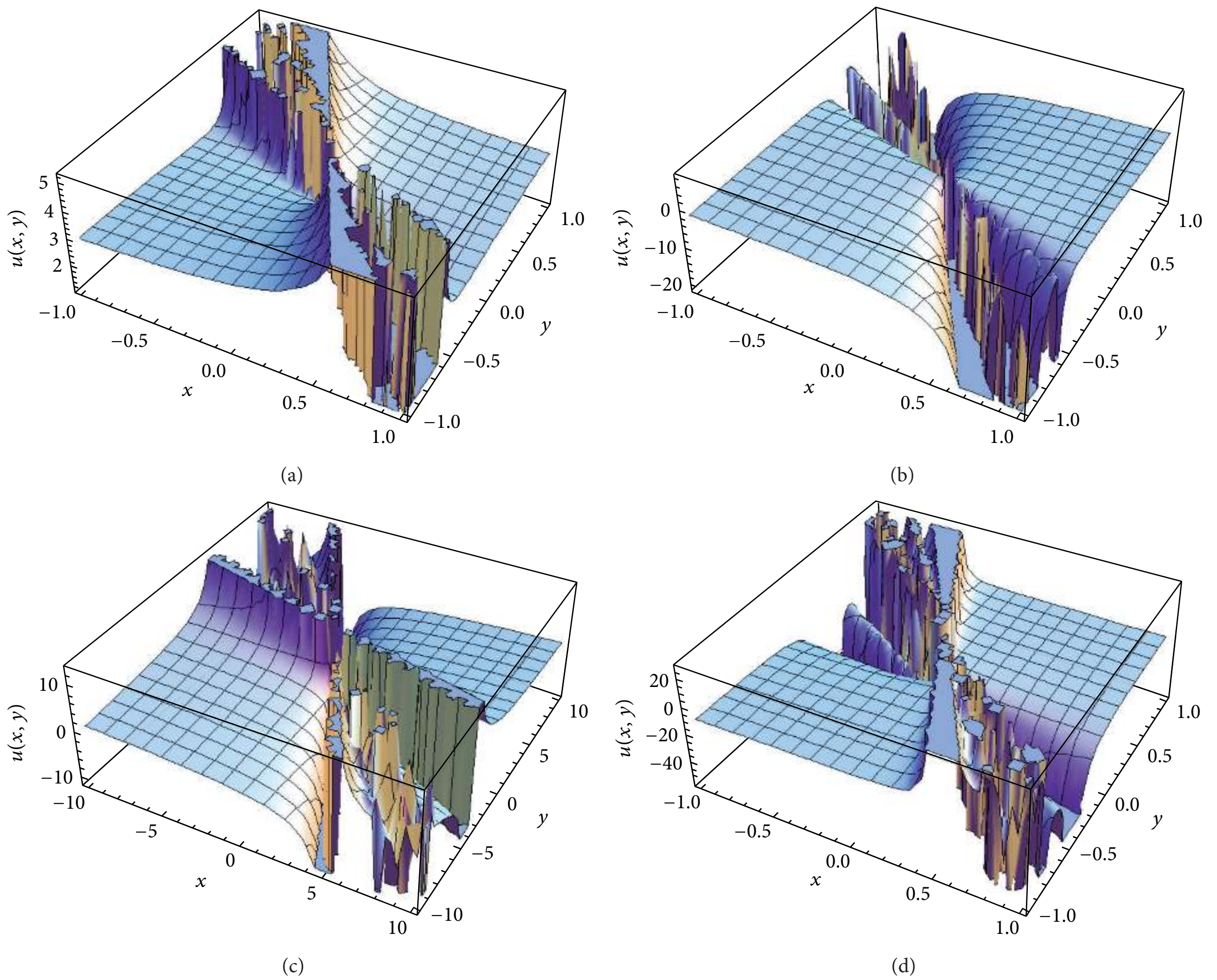

FIGURE 3: Travelling waves solutions (29) and (30) with various different shapes are plotted: multiple travelling wave solutions (a and b). Travelling waves solutions (33) and (34) with various different shapes are plotted: multiple travelling wave solutions (c and d).

Case 2. Consider the following: $k^2 = 1/9$, $c^2 = 8/9$, and $b = 1$,

$$a_0 = a_4 = 1, \qquad a_1 = a_3 = \mp\frac{\sqrt{14}}{3}, \qquad a_2 = \pm\frac{1}{3}\sqrt{\frac{7}{2}}, \tag{31}$$

$$\begin{gathered} b_0 = -2, \qquad b_1 = \pm 2\sqrt{\frac{14}{3}}, \qquad b_2 = \pm\frac{\sqrt{14}}{3}, \\ b_3 = \mp\frac{8\sqrt{14}}{3}, \qquad b_4 = \frac{1}{54}\left(-10 + 9p^2 - 9q^2 + 9r\right), \\ b_5 = -\frac{14}{9}, \qquad b_6 = \frac{1}{108}\left(-36 + 9p^2 - 9q^2 + 9r\right), \\ b_7 = \frac{28}{9}, \qquad b_8 = \mp\frac{\sqrt{14}}{3}. \end{gathered} \tag{32}$$

Substituting (31) and (32) into (25) and (26), we have obtained the following solutions of (23) and (24):

$$u(x,t) = 1 \mp \frac{\sqrt{14b}}{3}\tan\xi \pm \frac{1}{3}\sqrt{\frac{7}{2b}}\cot\xi \mp \frac{\sqrt{14b}}{3}\sec + \csc\xi, \tag{33}$$

$$\begin{aligned} v(x,t) = {} & -2 \pm 2\sqrt{\frac{14b}{3}}\tan\xi \pm \frac{\sqrt{14}}{3b}\cot\xi \mp \frac{8\sqrt{14b}}{3}\sec\xi \\ & + \frac{1}{54}\left(-10 + 9p^2 - 9q^2 + 9r\right)\csc\xi \\ & - \frac{14b}{9}\tan^2\xi \\ & + \frac{1}{108b}\left(-36 + 9p^2 - 9q^2 + 9r\right)\cot^2\xi \\ & + \frac{28b}{9}\tan\xi\sec\xi \mp \frac{\sqrt{14}}{3\sqrt{b}}\cot\xi\csc\xi, \end{aligned} \tag{34}$$

with $\xi = \sqrt{b}((x + \sqrt{8}y + \sqrt{9}t)/\sqrt{9}(x + y - t))$.

The shape solutions (29) and (30) are shown in Figures 3(a)and 3(b), with $p = q = r = 1$ and $t = 0.01$. The shape solutions (33) and (34) are shown in Figures 3(c)and 3(d), with $p = q = r = 1$ and $t = 0.01$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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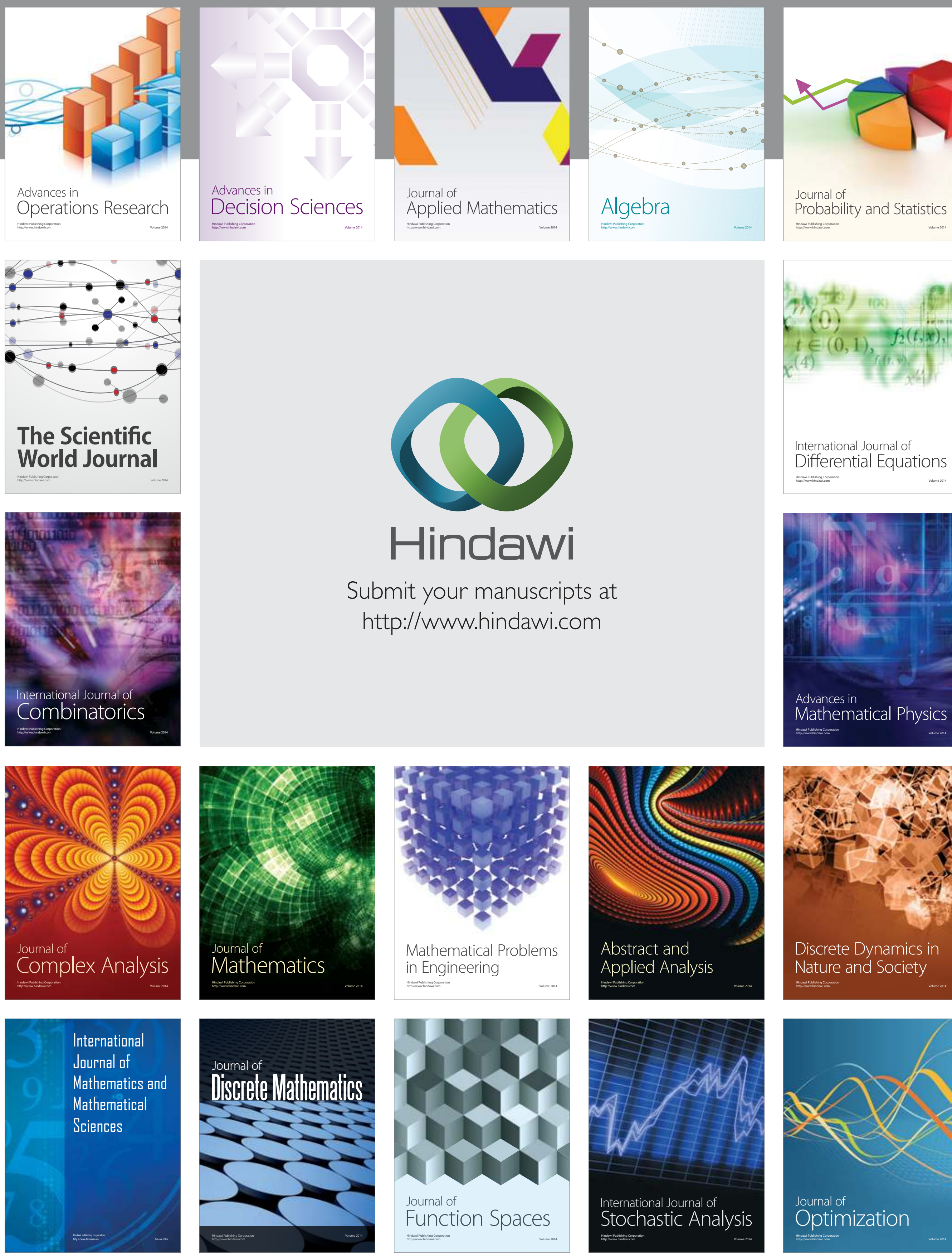
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International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
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