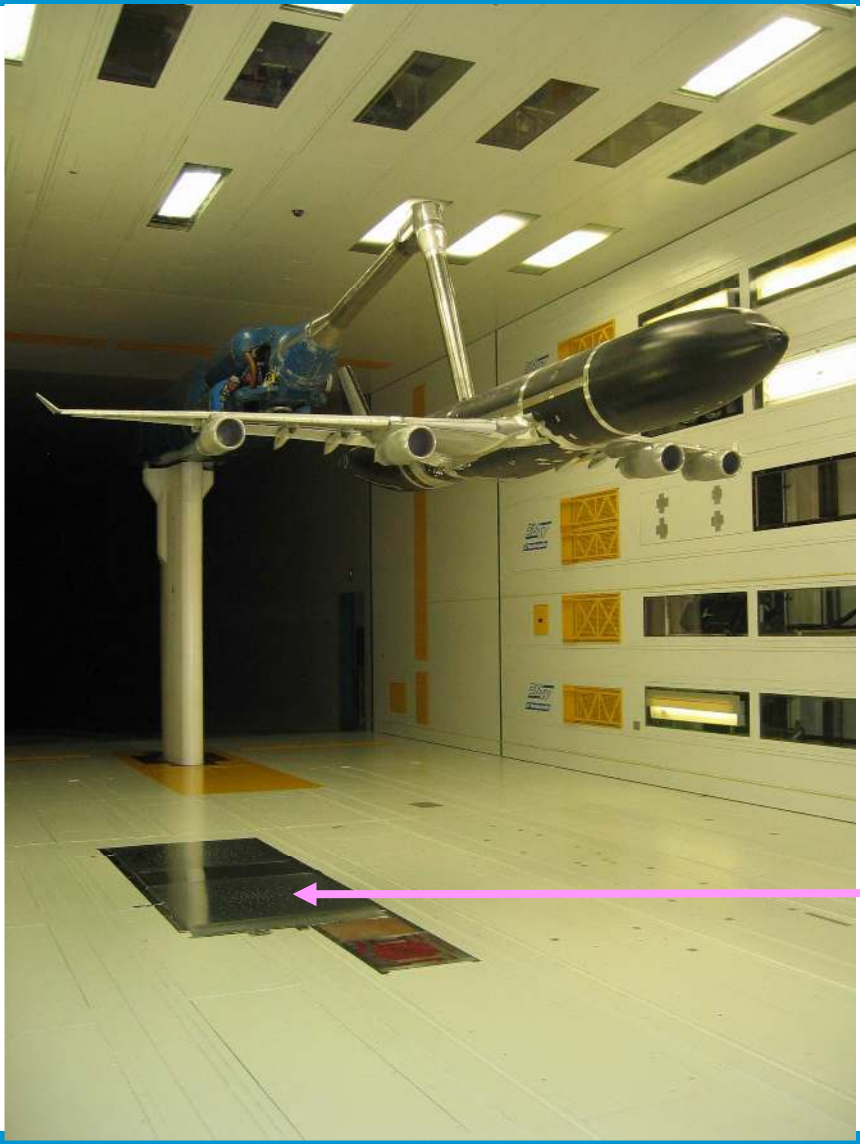
Dedicated to innovation in aerospace

CLEAN Based on Spatial Source Coherence

Pieter Sijtsma

Acoustic array measurements in closed wind tunnel test section

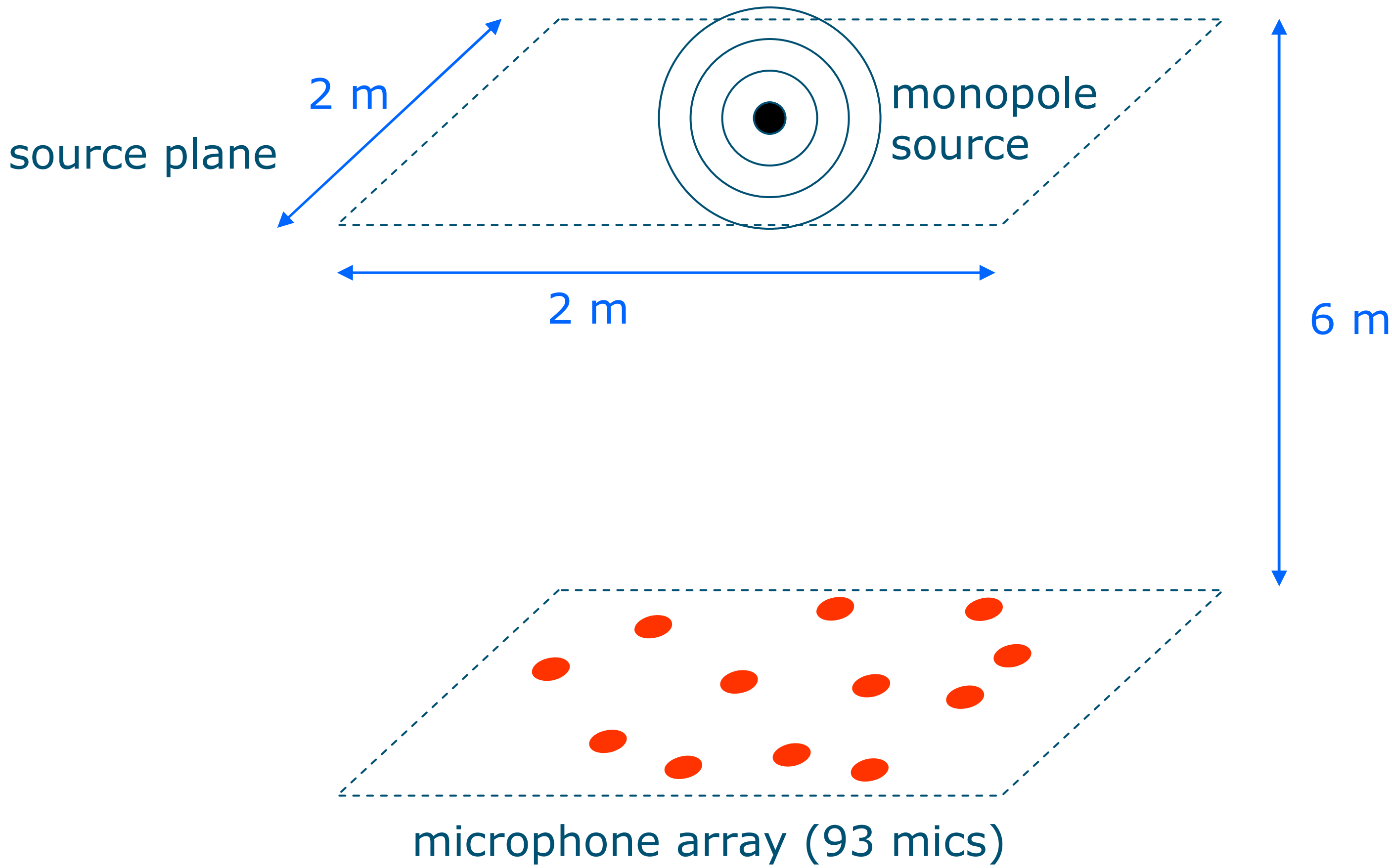


Airbus A340 model in DNW-LLF 8x6 m² closed test section (AWIATOR)

wall mounted array

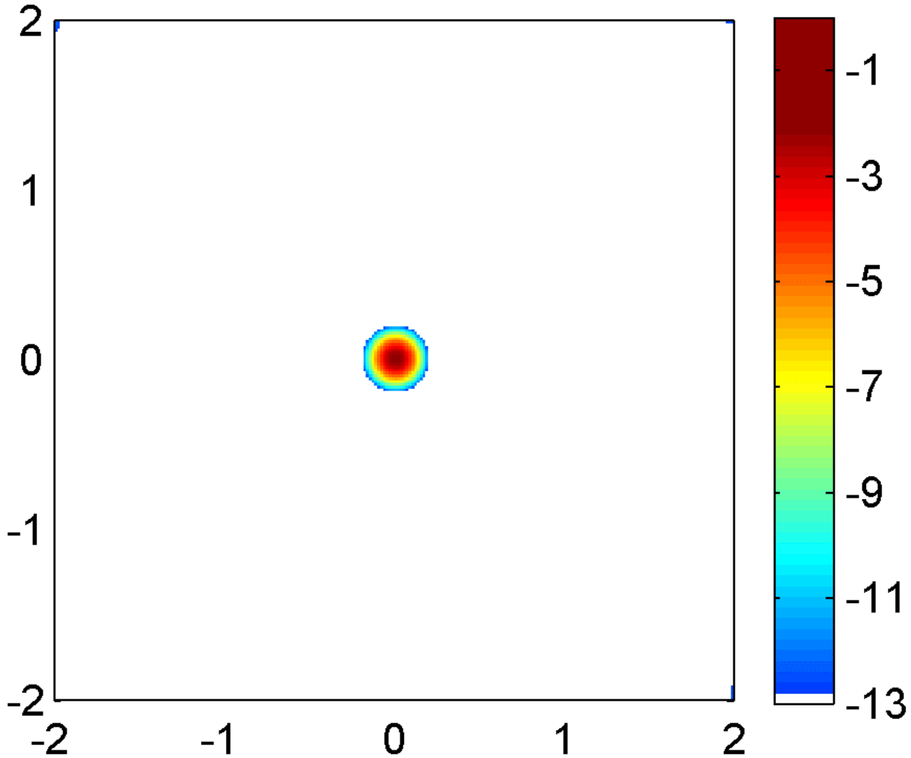
CLEAN-CLASSIC (1)

Array simulations

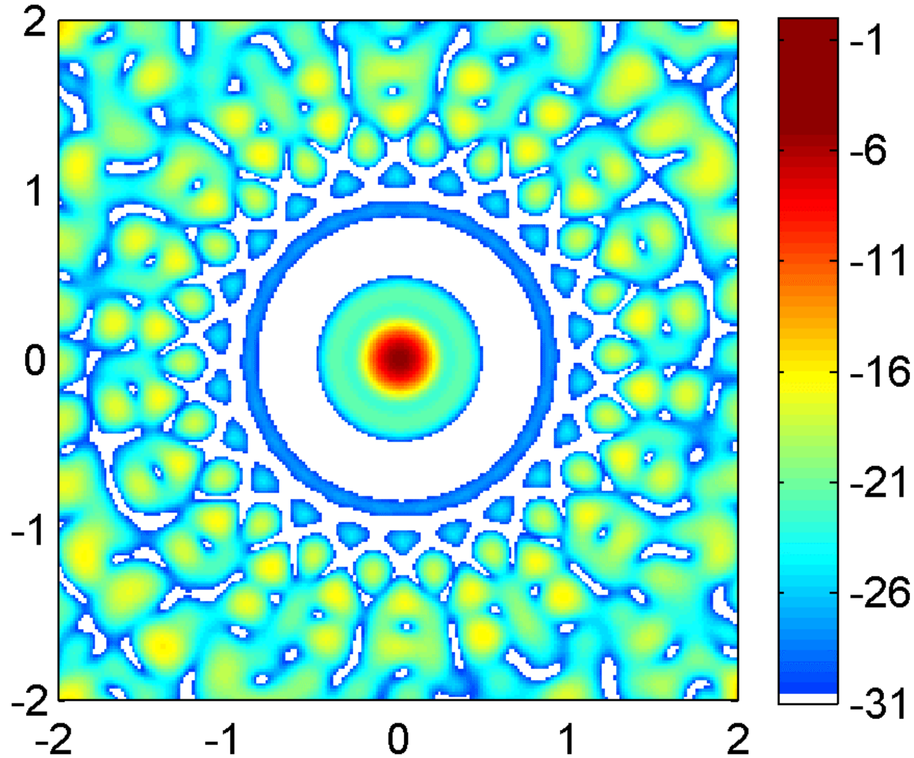


Conventional Beamforming

source plot at 13 dB range



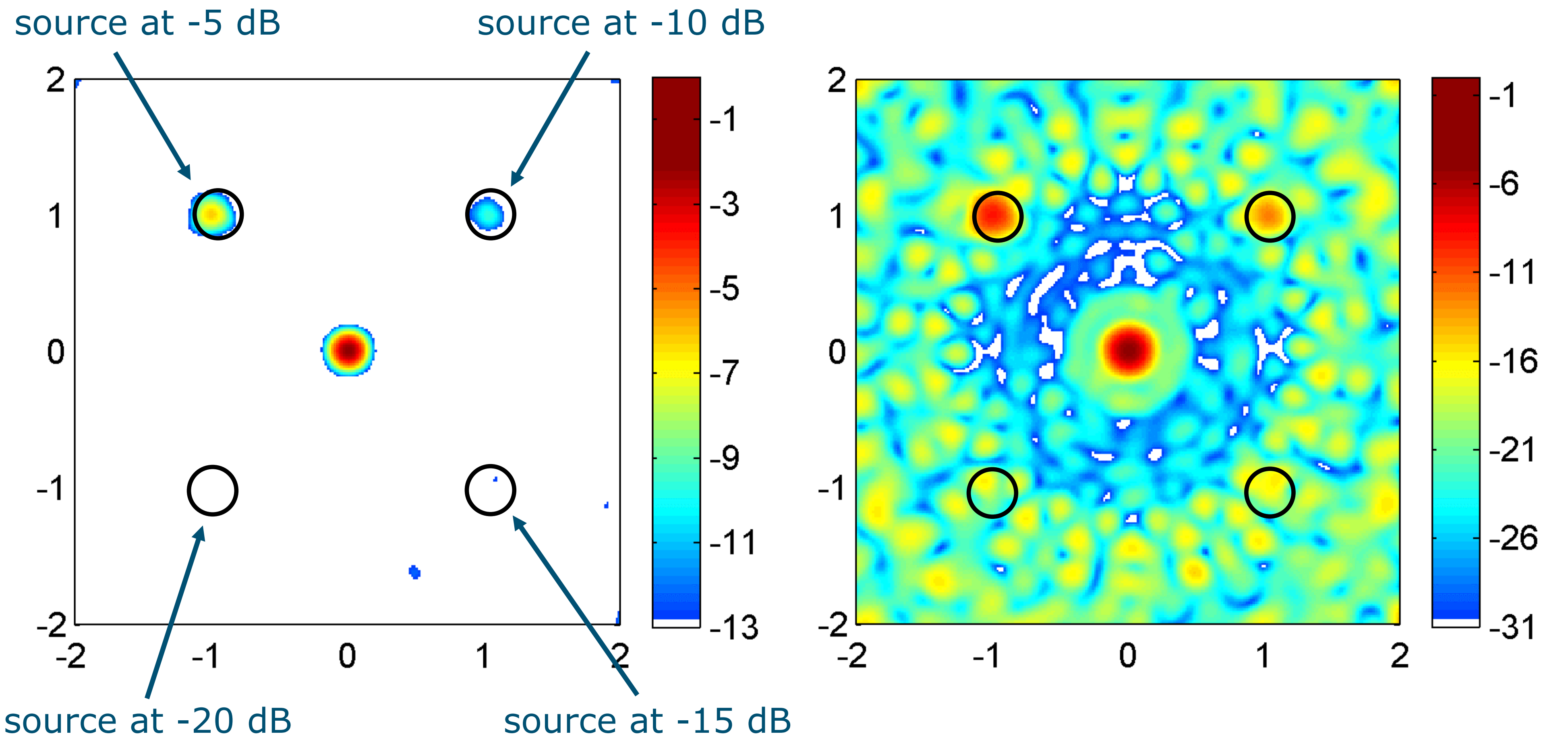
source plot at 31 dB range



“Point Spread Function”

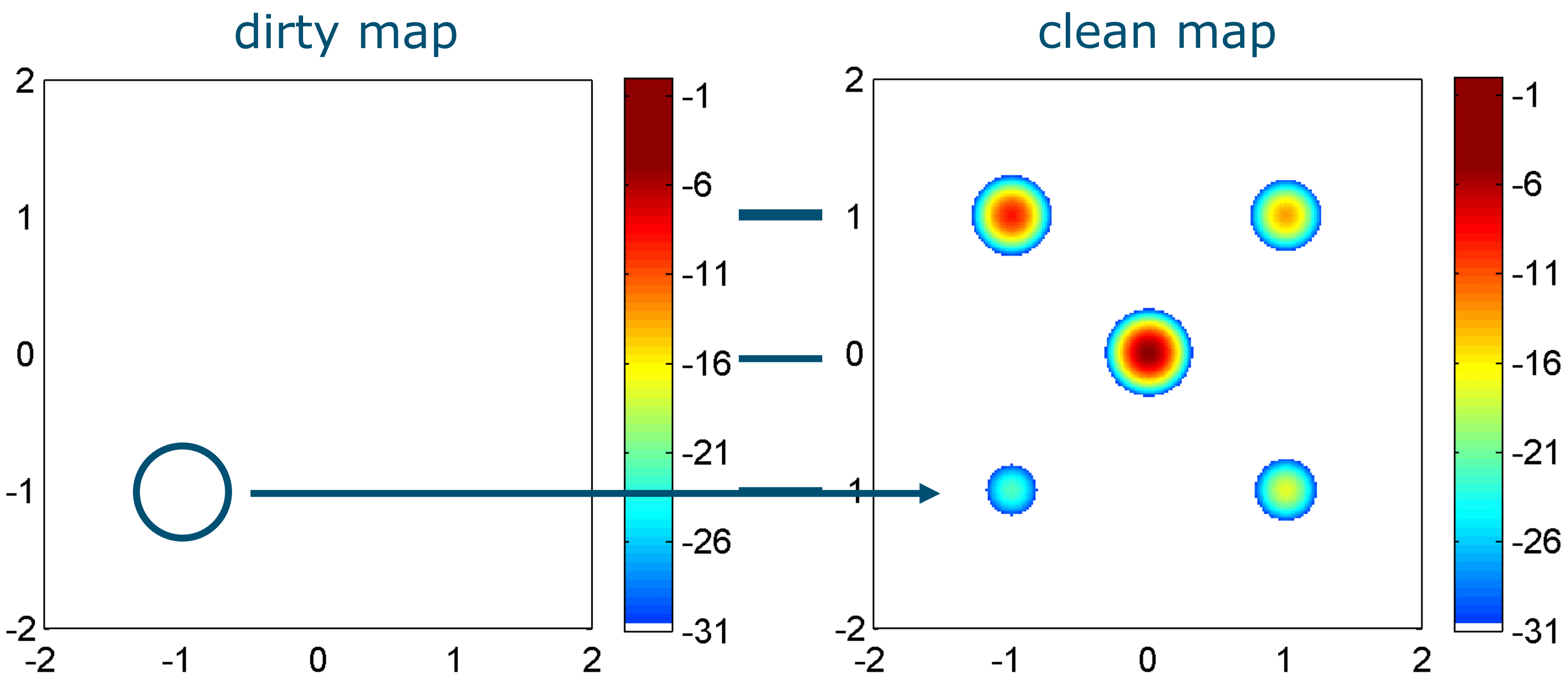
CLEAN-CLASSIC (3)

Secondary sources added



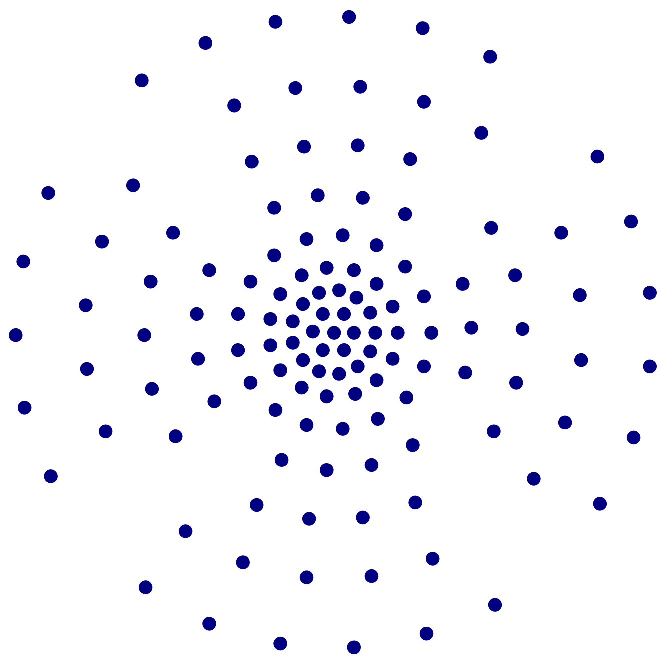
CLEAN-CLASSIC (4)

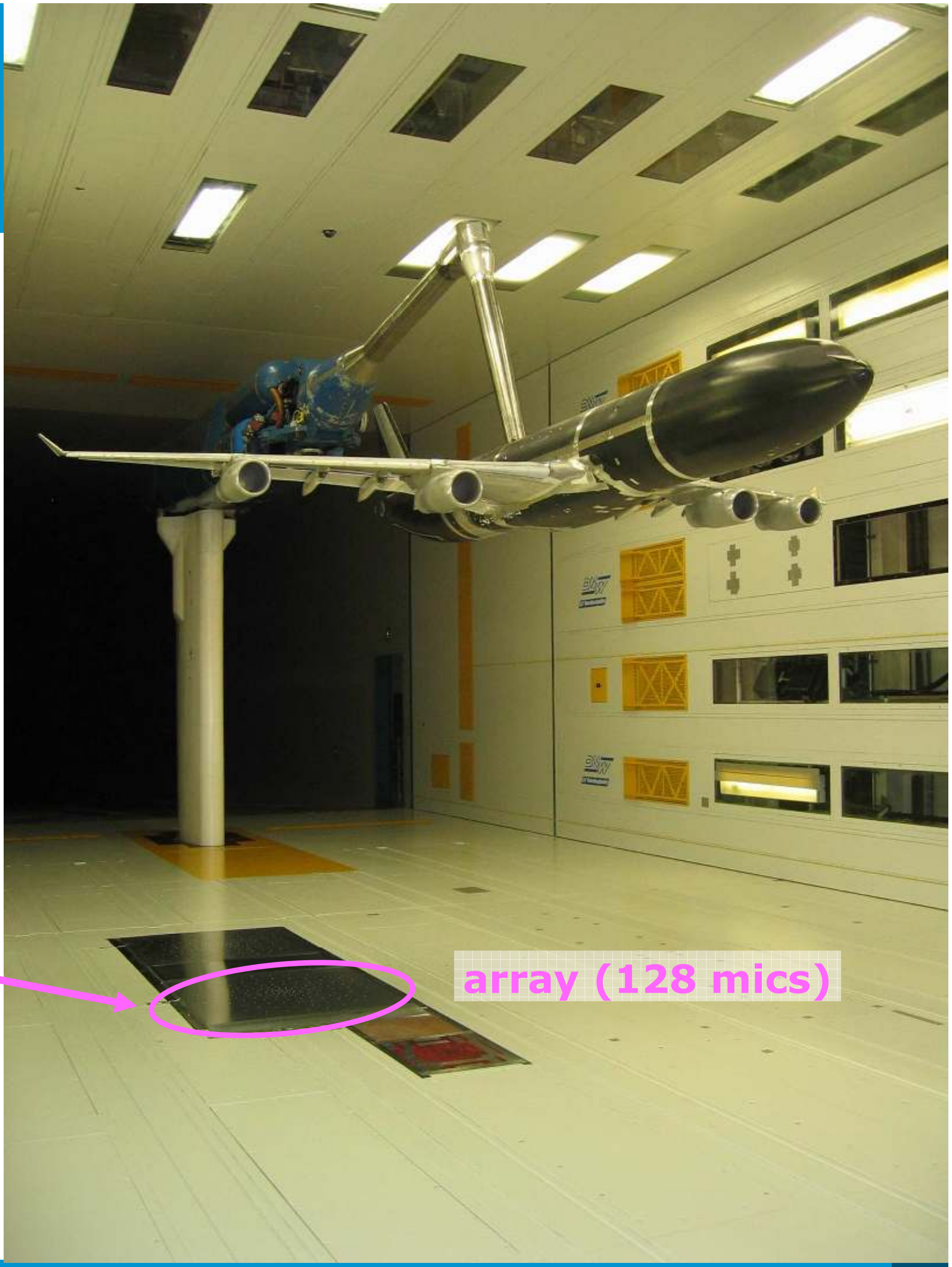
Successively remove Point Spread Functions



A340 scale model (1)

- DNW-LLF 8x6 m² closed test section
- Scale = 1:10.6
- Flush mounted floor array
- 128 microphones

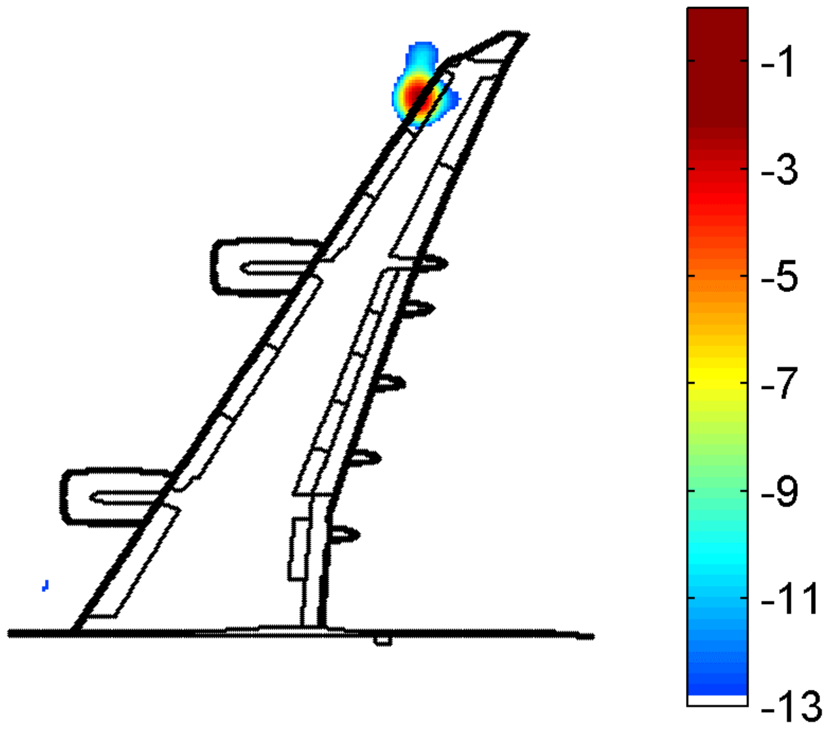




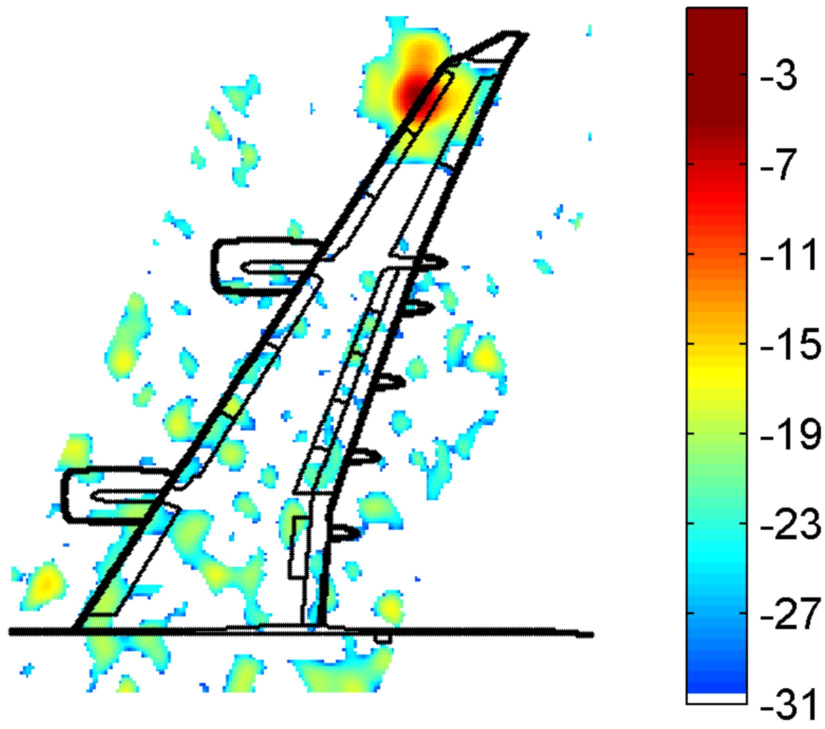
A340 scale model (2)

dominant slat noise source at 12360 Hz

source plot at 13 dB range

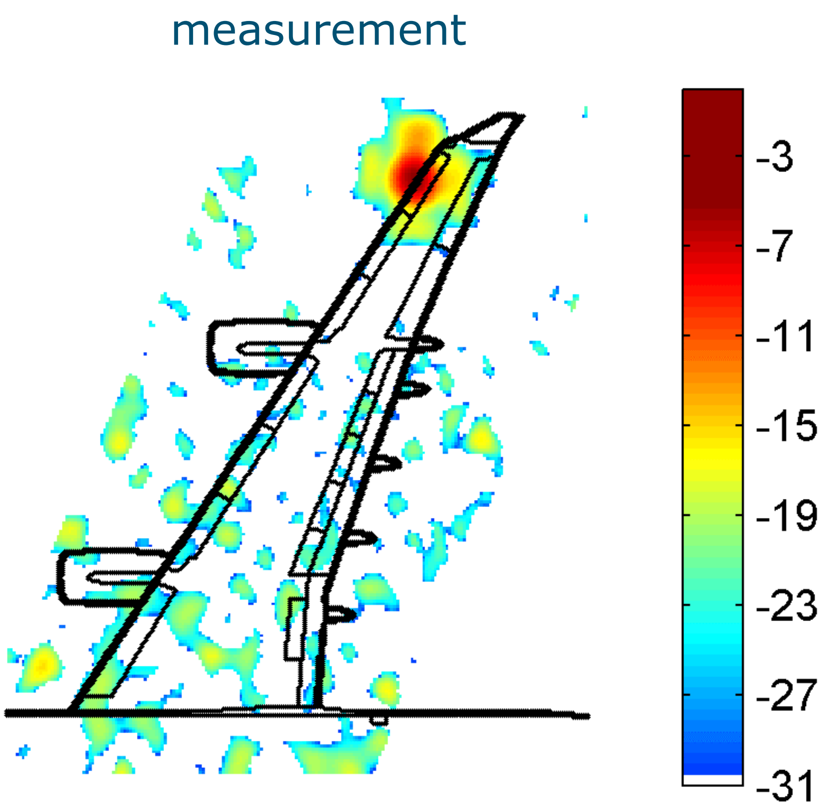


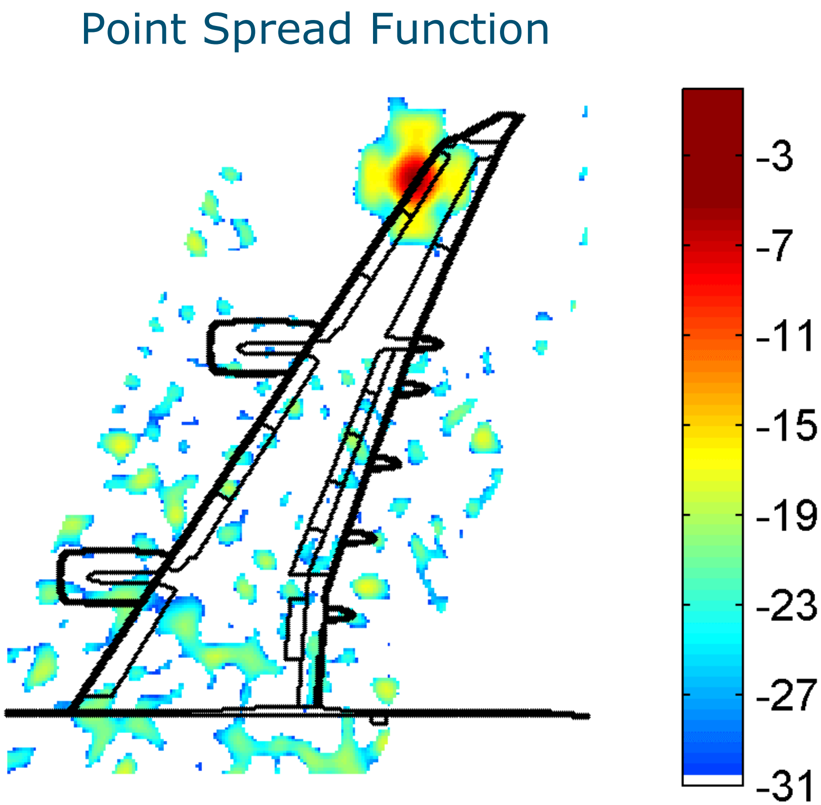
source plot at 31 dB range



A340 scale model (3)

Similarity with Point Spread Function

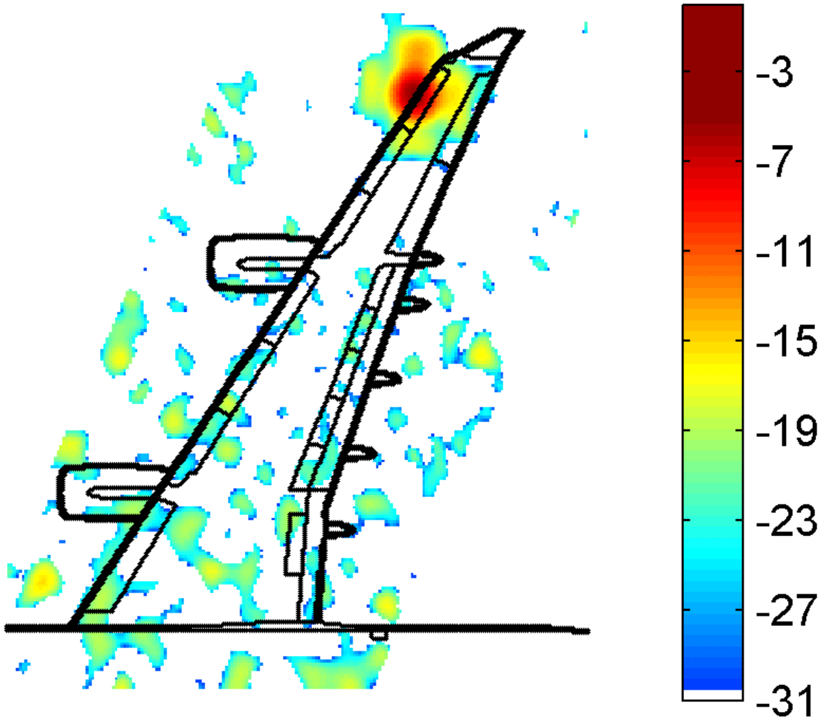




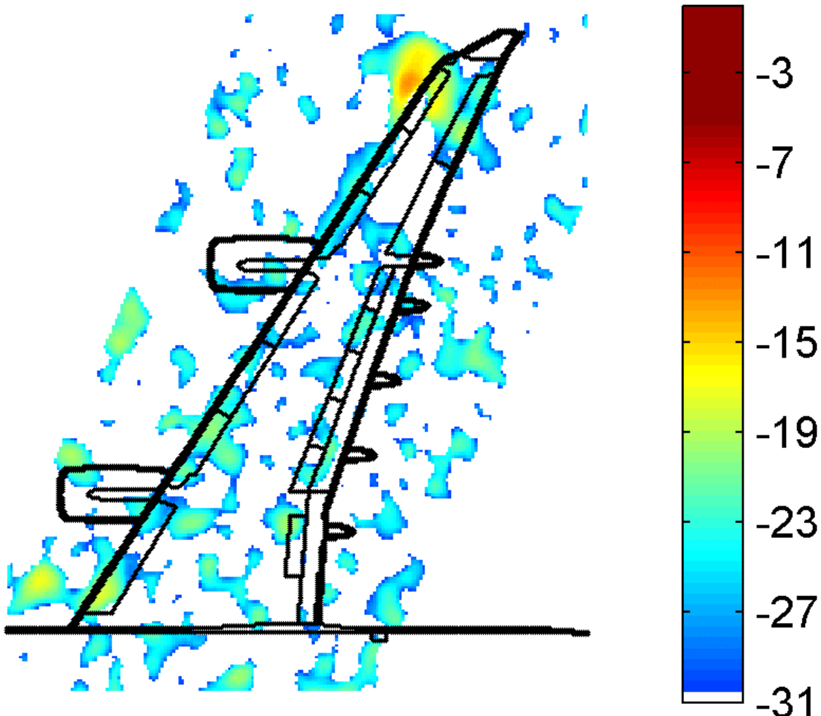
A340 scale model (4)

Application of CLEAN – first step

Conventional Beamforming



PSF subtracted



disappointing!

A340 scale model (5)

- **Beam pattern of dominant source ≠ Point Spread Function**
- **Possible reasons:**
  - source is not a point source
  - source does not have uniform directivity
  - error in height of source plane
  - error in flow Mach number
  - flow is not uniform
  - errors in microphone sensitivity
  - loss of coherence
- **Motivation for CLEAN based on spatial source coherence (CLEAN-SC)**

Spatial source coherence (1)

Conventional definition of coherence

Consider microphone signals p_n (frequency domain)

Auto-powers: $C_{nn} = \left\langle \left| p_n \right|^2 \right\rangle$

Cross-powers: $C_{mn} = \left\langle p_m p_n^* \right\rangle$

Coherence: $\gamma_{mn}^2 = \dfrac{\left| C_{mn} \right|^2}{C_{mm} C_{nn}}$

Suppose p_n and p_m have constant amplitude and fixed phase difference:

$\rightarrow \gamma_{mn}^2 = 1$

Otherwise: $0 \le \gamma_{mn}^2 < 1$

Spatial source coherence (2)

Beamforming summary

Source auto-power in scan point $\vec{\xi}_j$: $A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j$

$\mathbf{C}$ = matrix of cross-powers (Cross-Spectral Matrix)

$\mathbf{w}_j$ = weight vector associated with $\vec{\xi}_j$

property: $A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j = 1$, when $\mathbf{C} = \mathbf{g}_j \mathbf{g}_j^*$

$\mathbf{g}_j$ =steering vector

(microphone pressures due to theoretical point source in $\vec{\xi}_j$)

Spatial source coherence (3)

Source coherence

Source auto-power in $\vec{\xi}$: $A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j$

Source cross-power between $\vec{\xi}_j$ and $\vec{\xi}_k$: $A_{jk} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_k$

Source coherence: $\Gamma_{jk}^2 = \dfrac{\left|A_{jk}\right|^2}{A_{jj} A_{kk}} = \dfrac{\left|\mathbf{w}_j^* \mathbf{C} \mathbf{w}_k\right|^2}{\left(\mathbf{w}_j^* \mathbf{C} \mathbf{w}_j\right)\left(\mathbf{w}_k^* \mathbf{C} \mathbf{w}_k\right)}$

Spatial source coherence (4)

Single coherent source

Constant amplitude and fixed phase difference: $\mathbf{C} = \left\langle \mathbf{p}\mathbf{p}^* \right\rangle = \mathbf{p}\mathbf{p}^*$

$$\Rightarrow A_{jk} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_k = \mathbf{w}_j^* \mathbf{p}\mathbf{p}^* \mathbf{w}_k = \left(\mathbf{w}_j^* \mathbf{p}\right)\left(\mathbf{p}^* \mathbf{w}_k\right)$$

$$\Gamma_{jk}^2 = \frac{\left|A_{jk}\right|^2}{A_{jj} A_{kk}} = \frac{\left|\left(\mathbf{w}_j^* \mathbf{p}\right)\left(\mathbf{p}^* \mathbf{w}_k\right)\right|^2}{\left(\mathbf{w}_j^* \mathbf{p}\right)\left(\mathbf{p}^* \mathbf{w}_j\right)\left(\mathbf{w}_k^* \mathbf{p}\right)\left(\mathbf{p}^* \mathbf{w}_k\right)} = 1$$

Spatial source coherence (5)

Peaks & side lobes

At peaks and side lobes: array output dominated by single coherent source

$$\Rightarrow A_{jk} \approx \mathbf{w}_j^* \mathbf{p}\mathbf{p}^* \mathbf{w}_k = \left(\mathbf{w}_j^* \mathbf{p}\right)\left(\mathbf{p}^* \mathbf{w}_k\right) \text{ and } \Gamma_{jk}^2 \approx 1$$

Basics of CLEAN-SC (1)

General loop

1. Calculate source auto-powers in $\vec{\xi}_j$: $A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j$

2. Determine peak value A_{kk}

3. Subtract coherent part: $A_{jj}^{\text{updated}} = A_{jj}\left(1 - \Gamma_{jk}^2\right)$

$$= A_{jj}\left(1 - \frac{\left|A_{jk}\right|^2}{A_{jj} A_{kk}}\right) = A_{jj} - \frac{\left|A_{jk}\right|^2}{A_{kk}}$$

Basics of CLEAN-SC (2)

Small complication

Main diagonal is usually removed from **C**

$A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j$ can be negative

$\Gamma_{jk}^2 = \dfrac{\left|A_{jk}\right|^2}{A_{jj} A_{kk}}$ can have all sorts of values (not necessarily $0 \leq \Gamma_{jk}^2 \leq 1$)

$A_{jj}^{\text{updated}} = A_{jj} \left(1 - \Gamma_{jk}^2\right)$ unstable

Complication

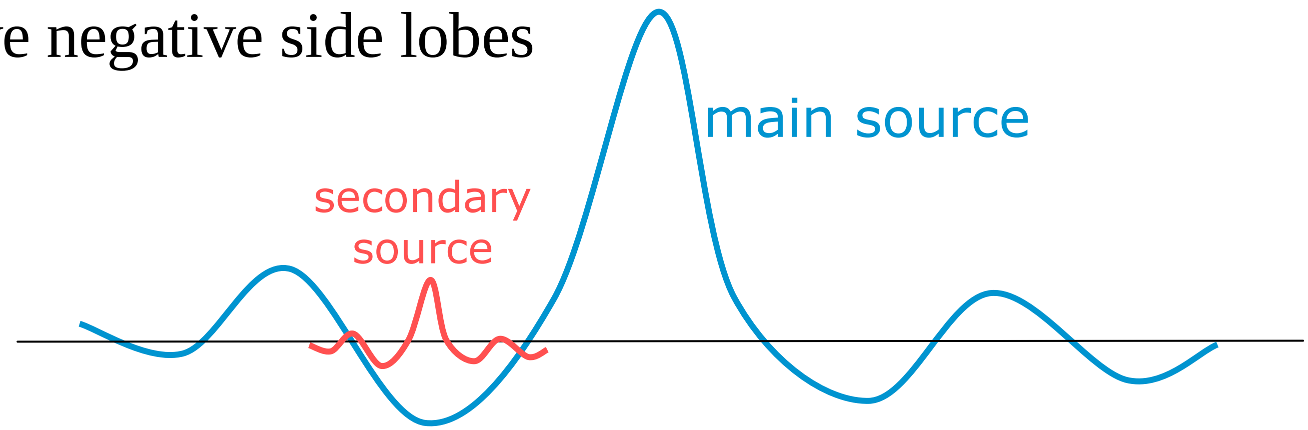
Main diagonal is usually removed from **C**

$A_{jj} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j$ can have negative values

But $A_{kk} > 0$ (being a maximum value)

$$A_{jj}^{\text{updated}} = A_{jj} - \frac{|A_{jk}|^2}{A_{kk}} < A_{jj}$$

This does not remove negative side lobes



CLEAN-SC: more general approach

More general approach required

Subtract "coherent" part: $A_{jj}^{\text{updated}} = A_{jj} - \mathbf{w}_j^* \mathbf{G} \mathbf{w}_j$

$$\begin{cases} \mathbf{G} \text{ is responsible for the cross-powers with peak source in } \vec{\xi}_k: \\ \qquad \mathbf{w}_j^* \mathbf{C} \mathbf{w}_k = \mathbf{w}_j^* \mathbf{G} \mathbf{w}_k \text{ for all } \vec{\xi}_j \\ \mathbf{G} \text{ is induced by a single coherent "source component" } \mathbf{h}: \\ \qquad \mathbf{G} = A_{kk} \left(\mathbf{h}\mathbf{h}^* - \mathbf{H} \right) \\ \qquad \left(\mathbf{H} \text{ contains the diagonal elements of } \mathbf{h}\mathbf{h}^* \right) \end{cases}$$

CLEAN-SC: analysis

$$\mathbf{w}_j^* \mathbf{C}\mathbf{w}_k = \mathbf{w}_j^* \mathbf{G}\mathbf{w}_k \quad \text{for all } \vec{\xi}_j$$

$$\mathbf{C}\mathbf{w}_k = \mathbf{G}\mathbf{w}_k$$

$$\mathbf{G} = A_{kk}\left(\mathbf{h}\mathbf{h}^* - \mathbf{H}\right)$$

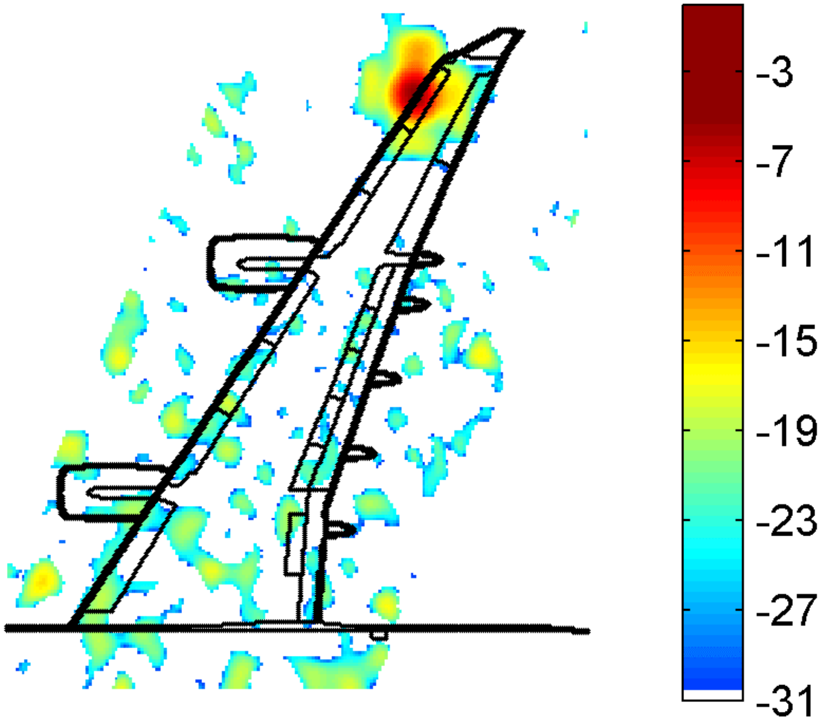
$$\mathbf{C}\mathbf{w}_k = A_{kk}\left(\mathbf{h}\mathbf{h}^* - \mathbf{H}\right)\mathbf{w}_k$$

$$\text{Solution: } \mathbf{h} = \frac{1}{\left(1 + \mathbf{w}_k^* \mathbf{H}\mathbf{w}_k\right)^{1/2}} \left(\frac{\mathbf{C}\mathbf{w}_k}{A_{kk}} + \mathbf{H}\mathbf{w}_k \right)$$

To be solved iteratively, starting with: $\mathbf{h} = \mathbf{g}_k$ (steering vector)

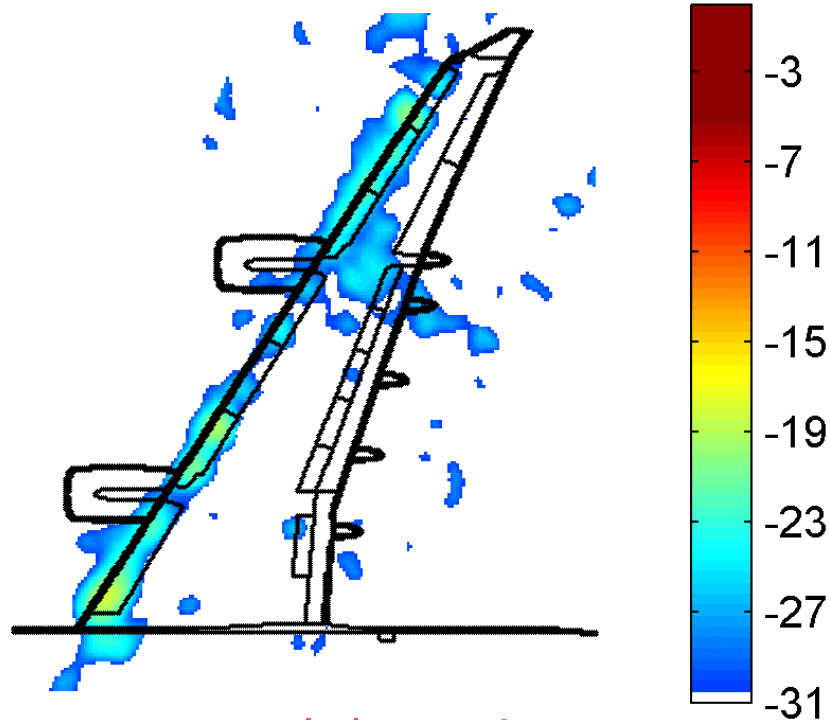
CLEAN-SC: main source removal

Conventional Beamforming



coherent component subtracted

PSF subtracted



much better!

disappointing!

CLEAN-SC: full deconvolution (1)

$$\mathbf{C} = \sum_{i=1}^{I} A_{\max}^{(i-1)} \mathbf{h}^{(i)} \mathbf{h}^{*(i)} + \mathbf{D}^{(I)}$$

- number of iterations (I)
- source component nr *i* ($A_{\max}^{(i-1)} \mathbf{h}^{(i)} \mathbf{h}^{*(i)}$)
- remainder of the CSM ($\mathbf{D}^{(I)}$)
- original CSM ($\mathbf{C}$)
- peak value after *i-1* subtractions ($A_{\max}^{(i-1)}$)
- Dirty map ($\mathbf{D}^{(I)}$)
- Clean map: each **h** replaced by clean beam (the source components)

CLEAN-SC: full deconvolution (2)

$$\mathbf{C} = \sum_{i=1}^{I} A_{\max}^{(i-1)} \mathbf{h}^{(i)} \mathbf{h}^{*(i)} + \mathbf{D}^{(I)}$$

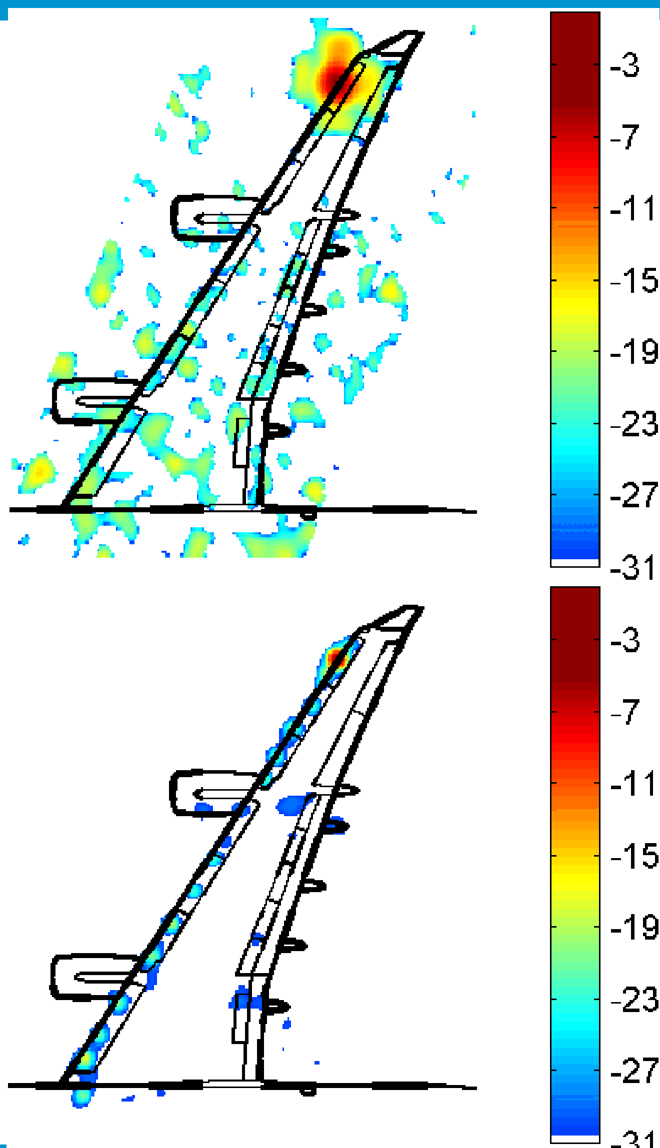
Stop criterion: $\left\| \mathbf{D}^{(I+1)} \right\| \geq \left\| \mathbf{D}^{(I)} \right\|$

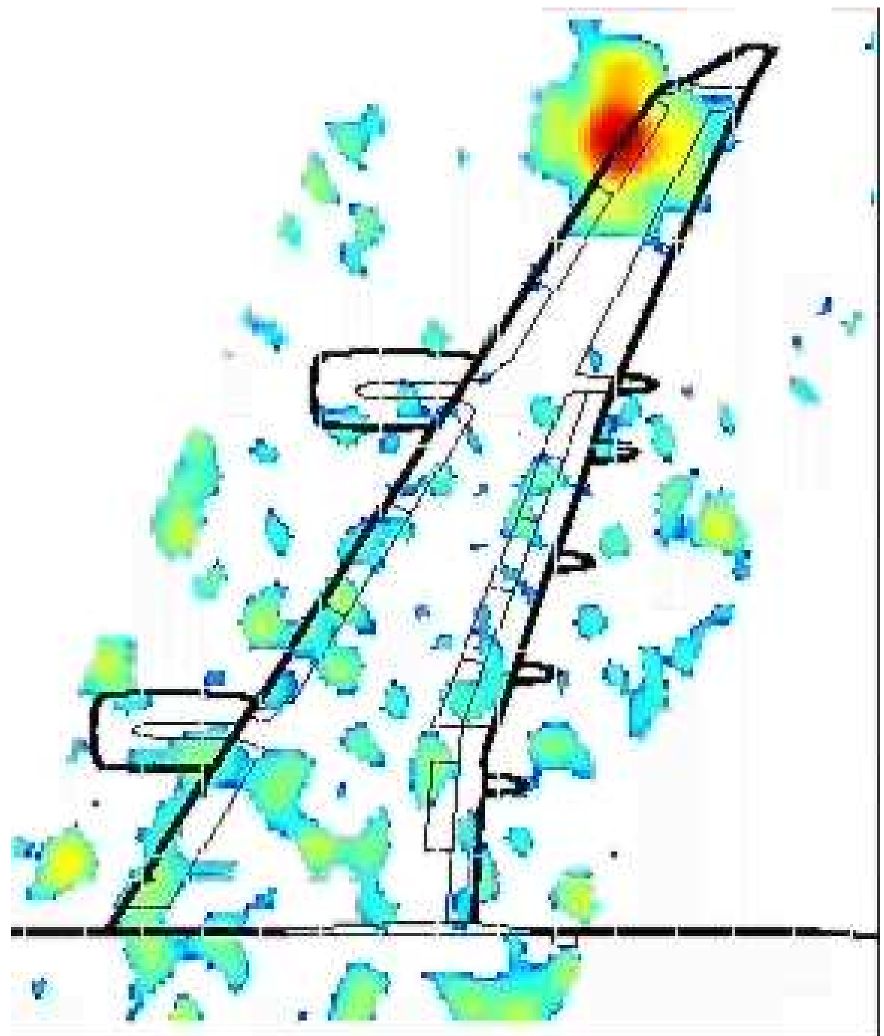
For example: $\left\| \mathbf{D} \right\| = \sum_{m,n} \left| D_{m,n} \right|$

CLEAN-SC: full deconvolution (3)

Conventional Beamforming

CLEAN-SC

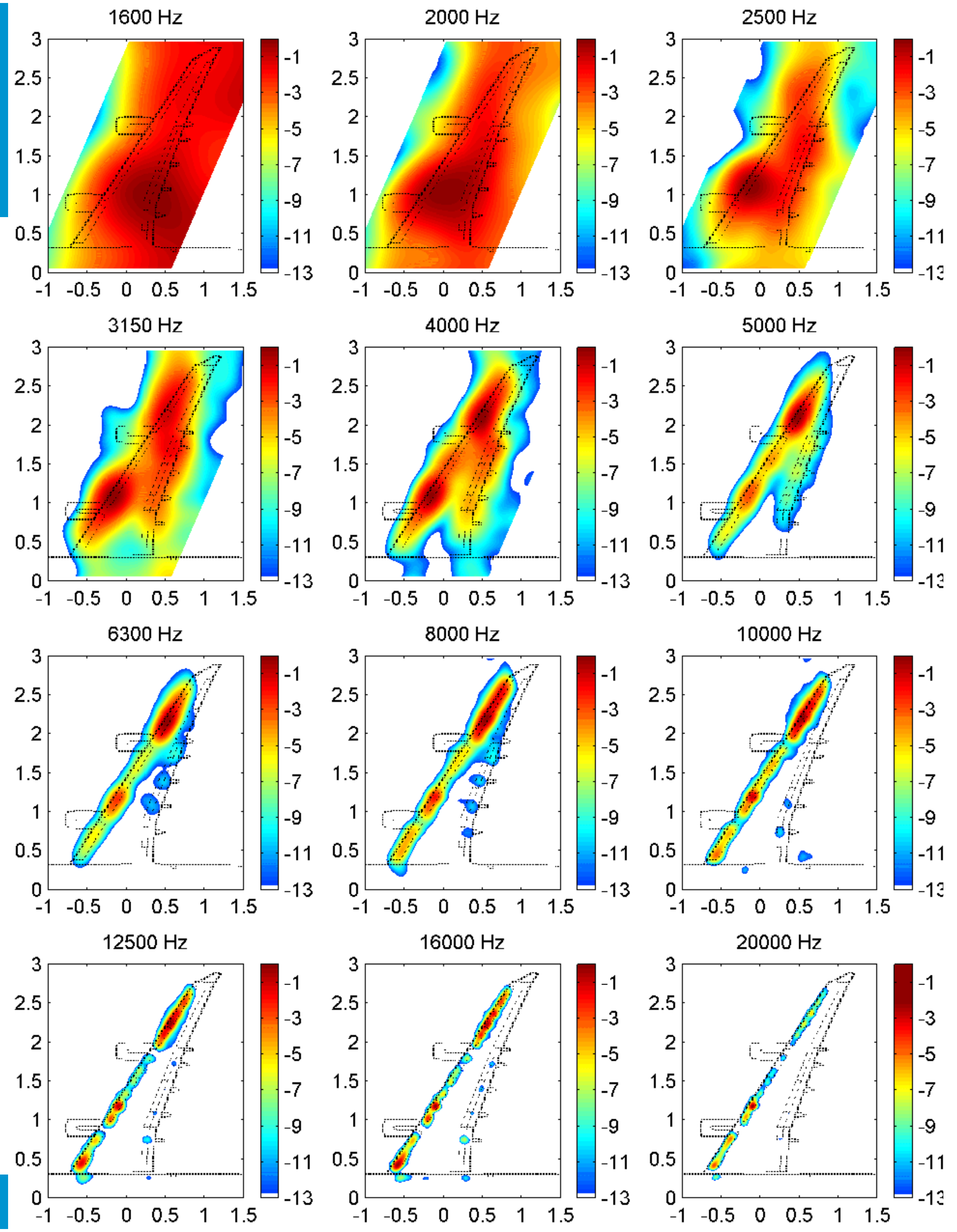




Example (1)

Typical result at 60 m/s

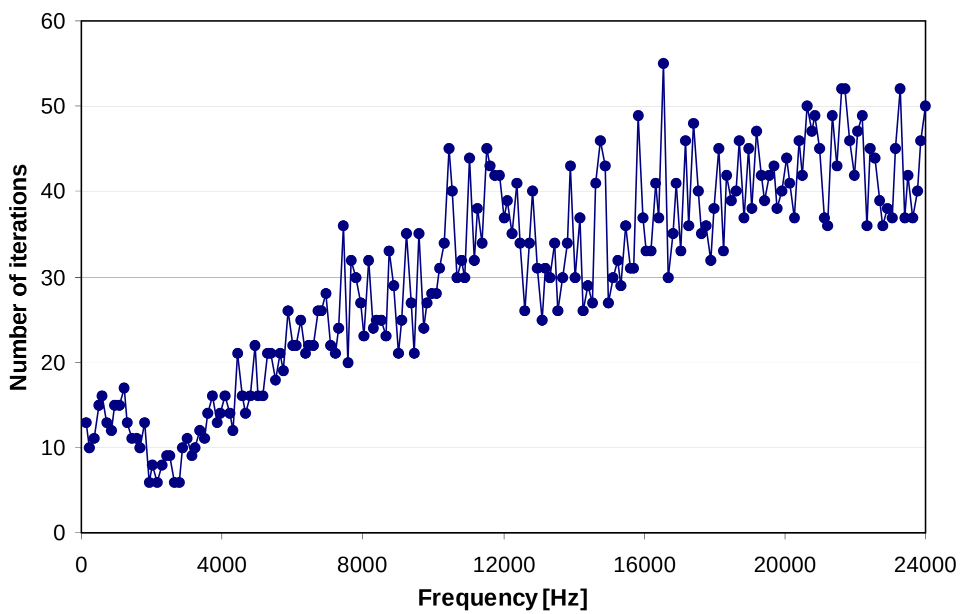
Conventional Beamforming

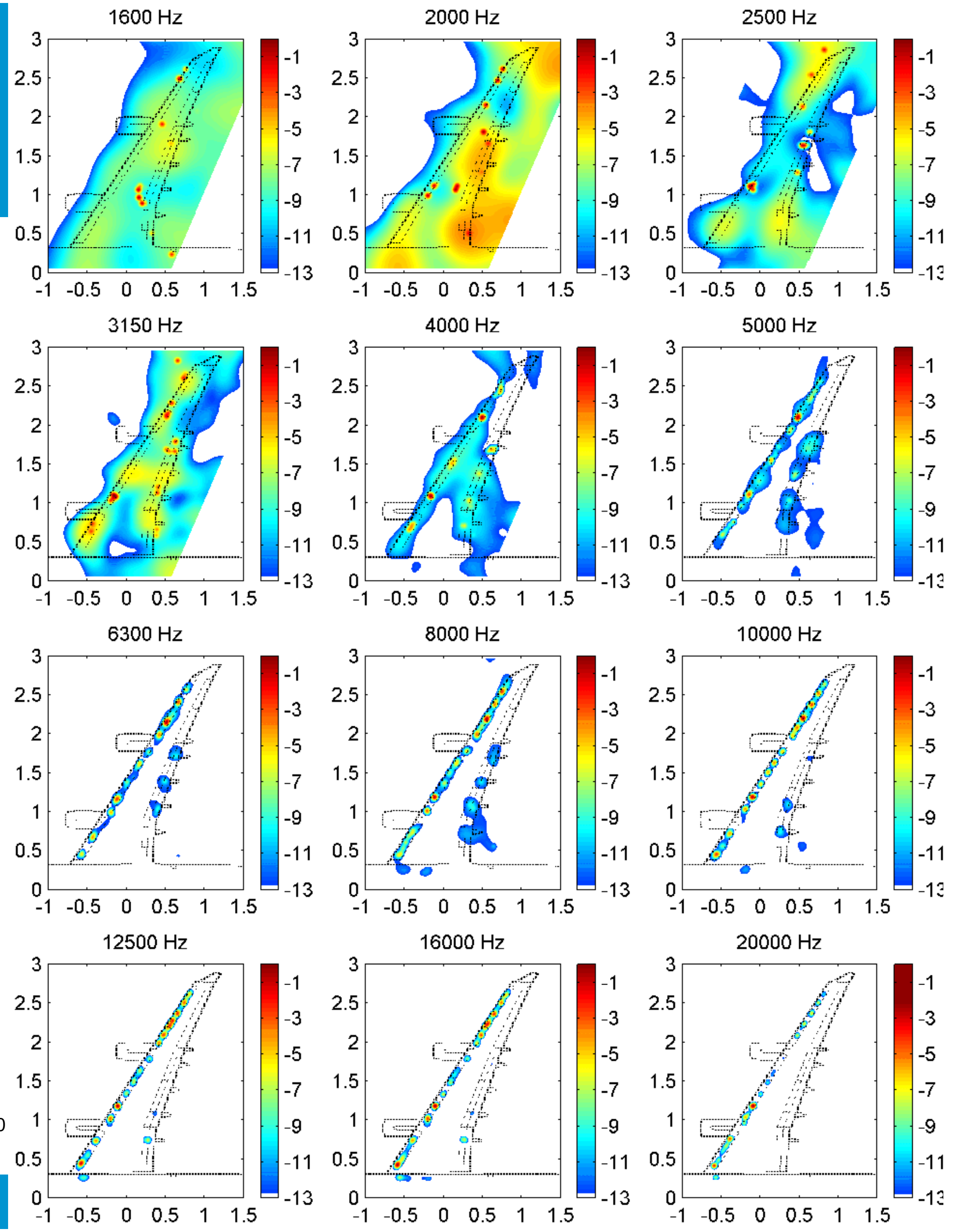


Example (2)

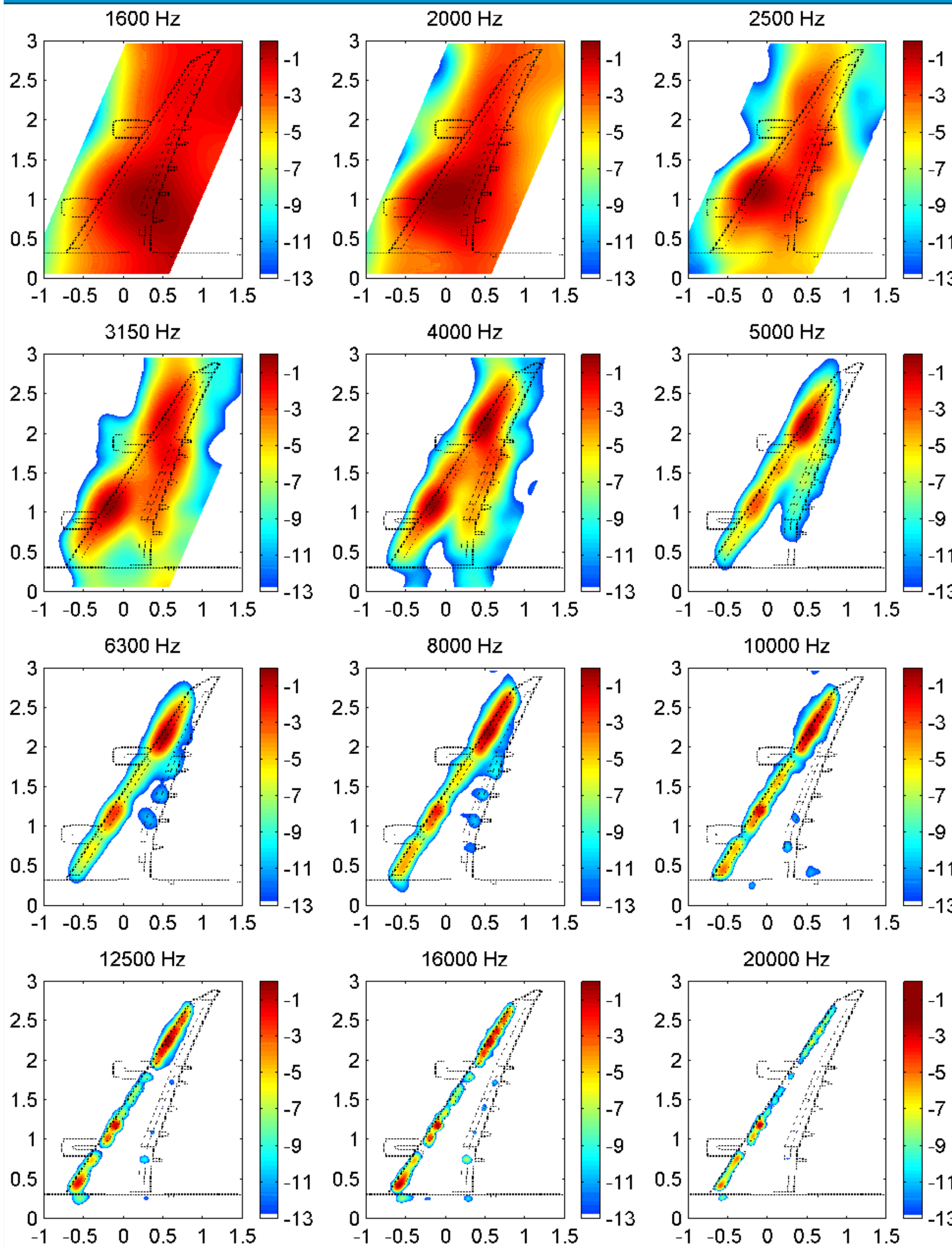
Typical result at 60 m/s

CLEAN-SC

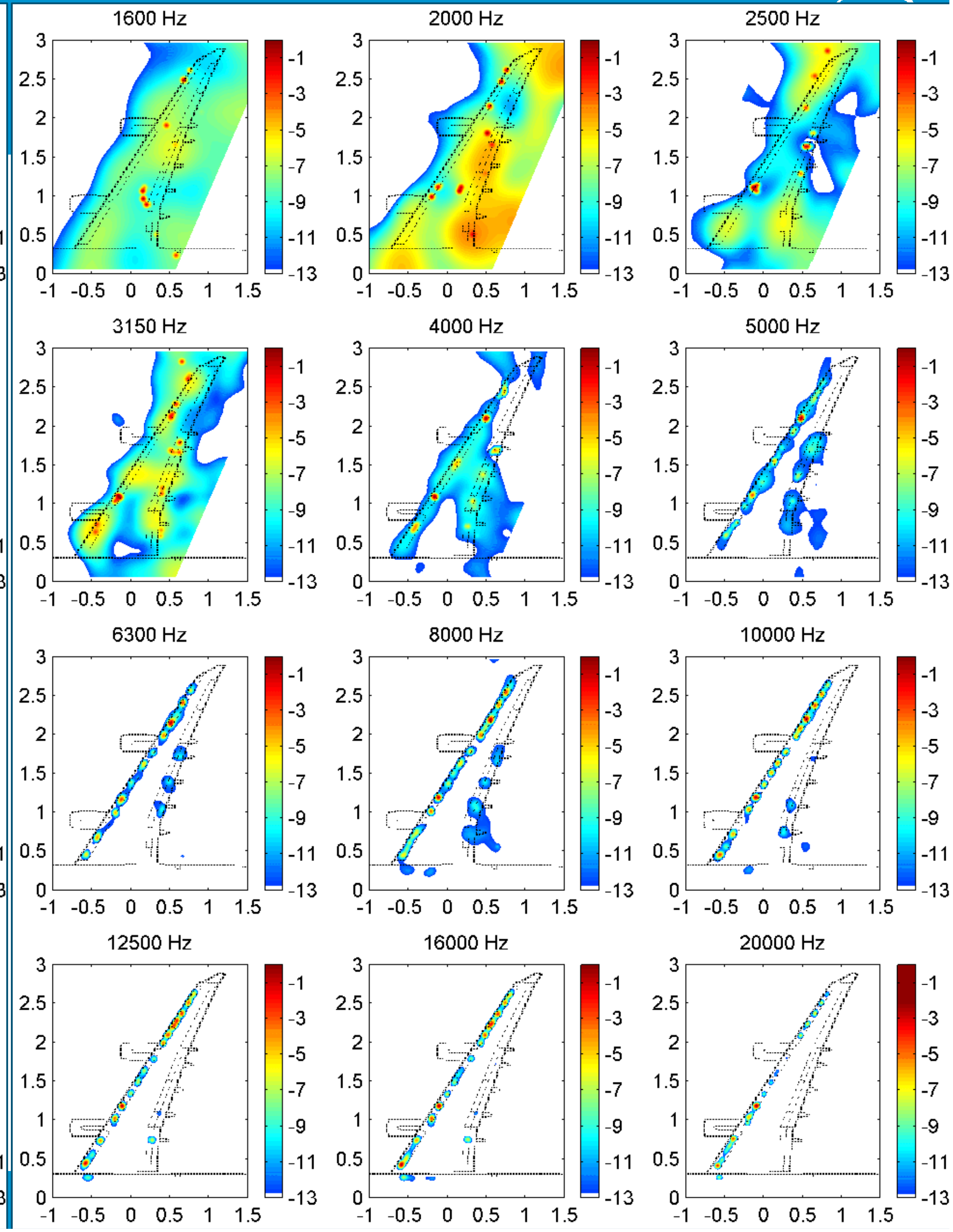




Conventional Beamforming



CLEAN-SC



Features of CLEAN-SC

- Determination of absolute source contributions
- Processing speed
- Filtering low frequency wind tunnel noise

Absolute source contributions (1)

diagonal removed (due to boundary layer noise)

contains diagonal elements without boundary layer noise

$$\mathbf{C} = \sum_{i=1}^{I} A_{\max}^{(i-1)} \mathbf{h}^{(i)} \mathbf{h}^{*(i)} + \mathbf{D}^{(I)}$$

trace: $$\sum_{n=1}^{N} C_{nn} = \sum_{i=1}^{I} A_{\max}^{(i-1)} \left\| \mathbf{h}^{(i)} \right\|^2$$

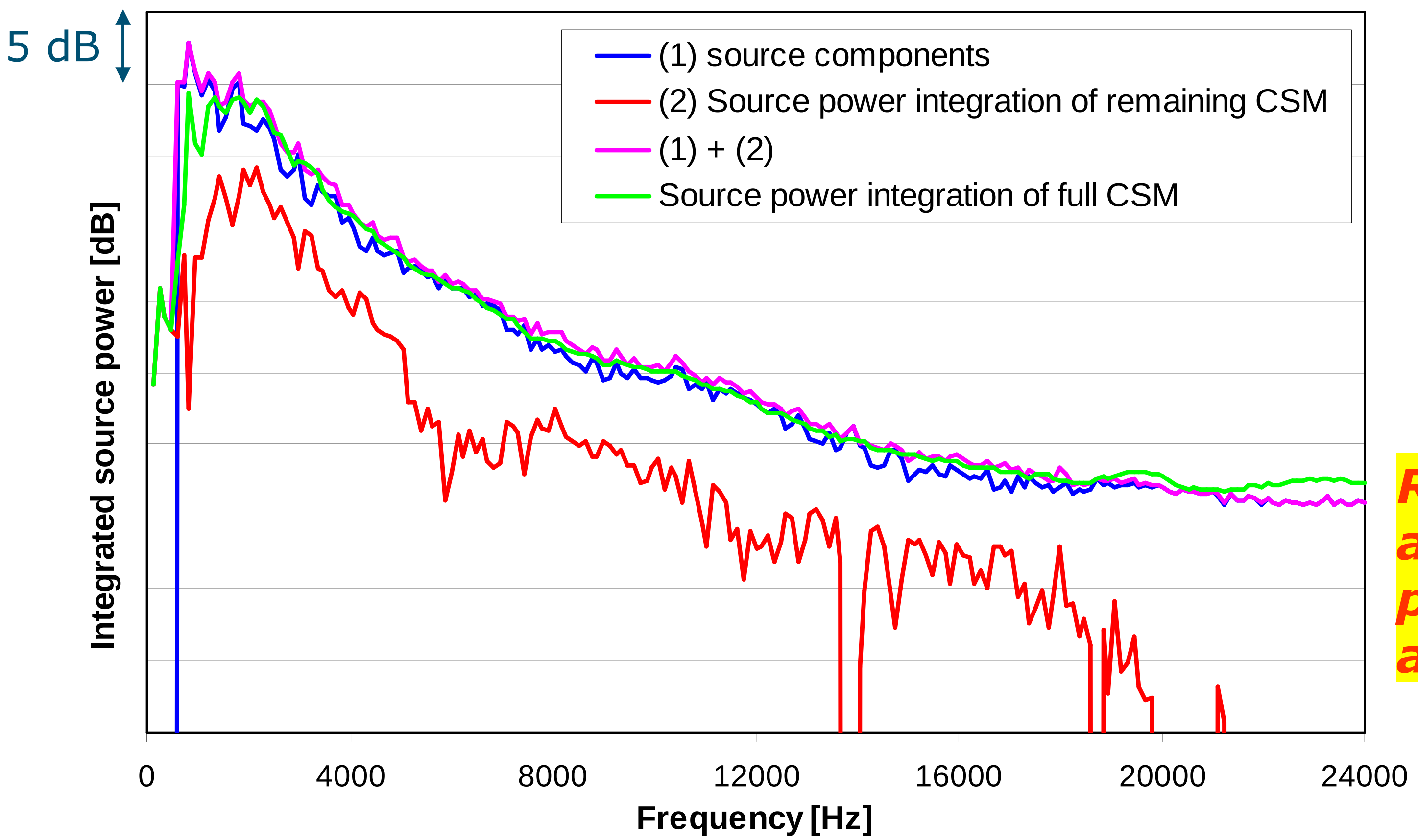
breakdown into source components

Absolute source contributions (2)

$$\mathbf{C} = \sum_{i=1}^{I} A_{\max}^{(i-1)} \mathbf{h}^{(i)} \mathbf{h}^{*(i)} + \mathbf{D}^{(I)}$$

(1) (2)

Integrated results



Results of CLEAN-SC and traditional source power integration are almost equal!

Absolute source contributions (3)

Differences with Source Power Integration:

- **Less grid dependence**
  - Grid needs to be such that sources are recognized
  - But no need to have grid points on the exact peaks
- **No integration threshold**
- **Pitfalls:**
  - Side lobes of sources outside the grid are also included
  - Coherent reflections are includes as well

Processing speed (1)

Summary of algorithm

start
$$\begin{cases} i = 0 \\ A_{jj}^{(0)} = \mathbf{w}_j^* \mathbf{C} \mathbf{w}_j \end{cases}$$

most time consuming parts

iteration
$$\begin{cases} i = i + 1 \\ A_{\max}^{(i-1)} = \max\left(A_{jj}^{(i-1)}\right) \\ \text{determine } \mathbf{h}^{(i)} \\ A_{jj}^{(i)} = A_{jj}^{(i-1)} - \mathbf{w}_j^* \mathbf{G} \mathbf{w}_j = A_{jj}^{(i-1)} - A_{\max}^{(i-1)} \mathbf{w}_j^* \left(\mathbf{h}^{(i)} \mathbf{h}^{(i)*} - \mathbf{H}^{(i)}\right) \mathbf{w}_j \end{cases}$$

Processing speed (2)

start:

$$\mathbf{w}_j^* \mathbf{C} \mathbf{w}_j = \sum_{n=1}^{N} \sum_{m=1}^{M} w_{j,n}^* C_{nm} w_{j,m}$$

double summation

iteration steps:

$$\mathbf{w}_j^* \left(\mathbf{h}^{(i)} \mathbf{h}^{(i)*} - \mathbf{H}^{(i)} \right) \mathbf{w}_j = \left| \sum_{n=1}^{N} w_{j,n}^* h_n^{(i)} \right|^2 - \sum_{n=1}^{N} \left| w_{j,n}^* h_n^{(i)} \right|^2$$

single summations

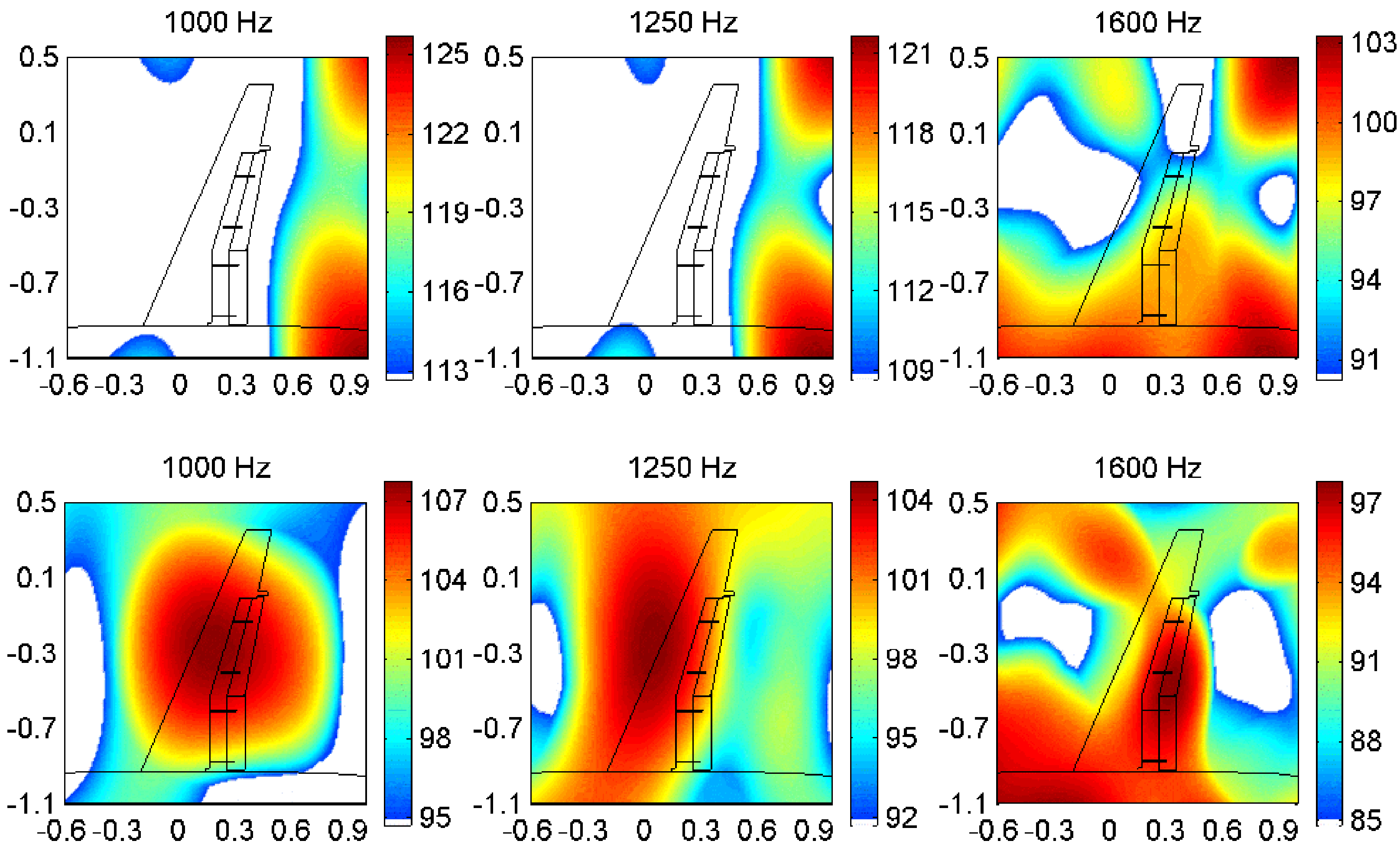
- One double summation + many single summations
  ≈ two double summations
- CLEAN-SC about twice as slow as Conventional Beamforming

Filtering low frequency wind tunnel noise

Fokker-100 half model in DNW-LST (3x2.25 m^2)

Conventional

'Cleaned'



Note

- **CLEAN-SC extracts coherent sources from the CSM**
- **When decorrelation occurs: CLEAN-SC will perform less well**
  - Outdoor measurements at large distances from the source
  - Open jet wind tunnel measurements (out-of-flow array)

Conclusions

- **New deconvolution method: CLEAN-SC**
  - No Point Spread Functions used
  - Works with removed CSM diagonal
  - Counteracts negative side lobes
- **Effective tool for removing dominant sources**
  - also when they are from outside the grid
- **Absolute source powers can be obtained**
  - Good agreement with Source Power Integration
  - No threshold, and not much grid dependence
  - Few pitfalls
- **Relatively short processing time**

Reference

International Journal of Aeroacoustics:

"CLEAN based on spatial source coherence"

volume 6, number 4, 2007, pages 357 – 374