Climate Change, Nuclear Power and Nuclear Proliferation: Magnitude Matters

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March, 2010

Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.
Princeton Plasma Physics Laboratory
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Climate Change, Nuclear Power and Nuclear Proliferation: Magnitude Matters

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Abstract
Integrated energy, environment and economics modeling suggests electrical energy use will increase from 2.4 TWe today to 12 TWe in 2100. It will be challenging to provide 40% of this electrical power from combustion with carbon sequestration, as it will be challenging to provide 30% from renewable energy sources. Thus nuclear power may be needed to provide ~30% by 2100. Calculations of the associated stocks and flows of uranium, plutonium and minor actinides indicate that the proliferation risks at mid-century, using current light-water reactor technology, are daunting. There are institutional arrangements that may be able to provide an acceptable level of risk mitigation, but they will be difficult to implement. If a transition is begun to fast-spectrum reactors at mid-century, without a dramatic change in the proliferation risks of such systems, at the end of the century proliferation risks are much greater, and more resistant to mitigation. The risks of nuclear power should be compared with the risks of the estimated 0.64°C long-term global surface-average temperature rise predicted if nuclear power were replaced with coal-fired power plants without carbon sequestration. Fusion energy, if developed, would provide a source of nuclear power with much lower proliferation risks than fission.

climate change, nuclear power, nuclear proliferation

1. Introduction
Nuclear power has the potential to produce energy with minimal atmospheric emission of carbon dioxide. It also has the potential to facilitate the proliferation of nuclear weapons. The damage to humanity and the world environment from either climate change or nuclear war would be very severe. Both could have devastating impacts on the heritage passed on to future generations. This paper uses recent energy, environment and economics modeling for the period up to 2100 to estimate the scale of a meaningful role for nuclear energy in mitigating climate change, and then uses calculations of stocks and flows of fissile materials based on recent technological studies to assess the key characteristics of such an undertaking. A quantitative time-dependent perspective is provided on the nuclear proliferation risks that would result, for comparison with the climate change risks that would be mitigated by nuclear power. This supplements earlier work by Williams and Feiveson (1990), Feiveson (2004), Feiveson et al. (2008), Socolow and Glaser (2009) and Feiveson (2010).

2. Integrated Energy, Environment and Economics Modeling
Nuclear energy is viewed primarily as a source of electrical power, although the high temperature process heat that may be producible in some designs could facilitate production of hydrogen or biofuels. Here we focus on the electricity market. The dominant contribution of
nuclear power to the transportation sector may in any event be through plug-in hybrid and electric vehicles.

Projections of future electricity use, while subject to the large uncertainties of any long-term projections, are relatively robust against variations in the projected requirement for limitation of CO$_2$ emission. In the study of electrification by Edmonds et al. (2006), as CO$_2$ emissions are more severely restricted, overall energy use is depressed. However at the same time the ratio of electrical power production to total final energy use in 2100 increases from 32% to 60%. These effects very nearly balance each other, providing a stable projection for future electricity production.

Figure 1. Electrical power production from EMF 22 models. “GWe-yr/yr” is used to indicate electrical power production, as opposed to production capacity, often denoted “GWe”.

It is valuable, however, to look beyond Edmonds’s results of 2006 to the most recent analyses, and to a wider range of models. The database from the Energy Modeling Forum 22 (EMF 22) study (Clarke et al., 2009) is a source of such information. Published in late 2009, it includes modeling results from a large number of different groups around the world, taking into account multiple energy sources and opportunities for improvements in efficiency. The study examined a wide range of cases. CO$_2$ constraints were varied from business-as-usual (no constraint) to atmospheric concentration as low as 450 ppm equivalent. Overshoot of CO$_2$ concentration compared with the ultimate goal was allowed or disallowed, and early participation in emissions
constraints was assumed only for developed countries, or full early participation was assumed. The projection for electrical energy production, across a wide range of models with this wide range of constraints, was surprisingly stable. The variation between models was greater than the variation vs. CO₂ and other constraints, and the direction of variation of electrical energy production as a function of the severity of the CO₂ constraint was not consistent. The median projection of electrical energy production from the EMF 22 database is shown in Figure 1. The 20’th and 80’th percentiles refer to the range of results over all models and all constraints. The average logarithmic growth in the median case from 2010 to 2100 is somewhat less than was experienced between 1980 and 2006.

While figure 1 provides a basis for considering future electrical energy needs, it does not provide a basis for estimating how much nuclear power will be needed. For perspective, according to online DOE Energy Information Agency data, world electrical power production in 2007, was 2142 GWe-yr total: 1482 GWe-yr (69.2%) from conventional thermal sources plus biomass and waste, 296 GWe-yr (13.8%) from nuclear, 342 GWe-yr (15.9%) from hydroelectric and 26 GWe-yr (1.2%) from wind, geothermal, solar, tide and wave.

The calculated mix of electrical energy sources for the various model runs was not provided to the EMF 22 study database, and the published descriptions of the EMF 22 model results (Blanford et al., 2009, Calvin et al., 2009a, Calvin et al., 2009b, Gurney et al., 2009, Krey et al., 2009, Loulou et al., 2009, Russ et al., 2009, van Vliet et al., 2009) indicate a great deal of variation in the mix. There is, however, a clear trend towards higher nuclear power, greater carbon sequestration, and more renewable energy as CO₂ concentration limits become more stringent. Overall it appears that combustion with carbon capture and storage is the largest contributor to electricity production in the carbon-constrained model runs, with renewables such as hydropower, wind and solar generally contributing somewhat less. Nuclear in different reported model runs contributes more or less than the renewables.

In order to provide a definite case for consideration, the assumption is made here that of the 12,000 GWe-yr/yr projected in 2100, 40% will be provided by combustion, including of biomass, with a large fraction of sequestration; 30% will be provided by nuclear energy, fission and potentially fusion; and 30% will be provided by tapping natural energy flows: hydro, wind, solar, geothermal, etc. These are very similar to detailed recent results from the MiniCam model (Kim et al., 2008) for a case constrained to 550 ppm CO₂. In order to model simply a possible
evolution of the energy system, the fraction of each electrical energy source is assumed to vary linearly from its current value to its assumed value in 2100. The resulting time profiles are shown in figure 2. The integrated electrical energy production from combustion is 320 TWe-years, from nuclear energy is 150 TWe-years, and from natural energy flows, 160 TWe-years. It is informative to note that this constitutes a 12x increase in nuclear power from 2010 to 2100.

Figure 2. Assumed electrical energy production time profiles.

The fractions and time profiles shown in figure 2 cannot be viewed as predictions, but they can be used to illustrate the scale of the problem at hand and its consequences. For example, from the climate perspective, one can estimate the impact of 150 TWe-years of electricity production from coal without sequestration, as a substitute for the nuclear power shown in figure 2. A typical pulverized coal plant emits (IPCC, 2005) 0.762 kgCO₂/kWh or 6.68 MtCO₂/GWe-yr. (Note that “Mt” in this paper denotes millions of metric tonnes, and “Gt” denotes billions, 10⁹ s, of metric tonnes.) The total emission of 1000 GtCO₂ would result in an increase of about 80 ppm in atmospheric concentration³ of CO₂ in 2100. The Intergovernmental Panel on Climate Change (2007a) estimates that an increase from 500 ppm-eq to 580ppm-eq CO₂ corresponds to an additional increase of long-term surface-average temperature of 0.64°C with an uncertainty range of a factor of 1.5 in “likely” (>2/3 probability) prediction. This could represent an overestimate of the climate impact, in the sense that in the absence of nuclear power there would be less total electrical power produced and not all substituted power would come from high-carbon-emitting
sources such as pulverized coal. On the other hand, it is in a sense an underestimate, in that the non-sequestering coal-fired plants operating in 2100, unless they were decommissioned before end-of-life or retrofit with carbon capture and storage, would represent a commitment to emission of a further 768 GtCO₂ post-2100 (see Appendix 1).

As noted above, here we will focus on the proliferation risks associated with nuclear power at this scale. First, however, we briefly overview the challenges for non-nuclear electrical energy sources at the specified scale.

3. Combustion and Sequestration: 320 TWe-yrs by 2100, 4800 GWe in 2100

As summarized in recent reports (IPCC, 2005, MIT, 2007, Socolow, 2005) subsurface injection of carbon dioxide is a well-developed technology, although not at the scale required for power generation in the GWe range. A single 1 GWe-yr/yr coal-fired power plant with a lifetime of 60 years would need to sequester about 450 MtCO₂ under an area of about 150 km². Substantial R&D is needed to determine the potential of various geological formations for retention of CO₂ at this scale, without significant leakage over hundreds of years. There will certainly be licensing issues associated with the safety of such a large undertaking and “Not Under My Back Yard”, so-called NUMBY, will be a significant constraint. Furthermore important areas of the world, such as Japan, appear to have very limited capacity for CO₂ storage. International export of waste CO₂ could be problematic.

The total world’s technical potential for CO₂ storage in geological formations is estimated by the Intergovernmental Panel on Climate Change (2005) at over 2000 GtCO₂, not considering economic feasibility. The upper limit of technically potential storage may be as much as an order of magnitude higher. Indeed the range of published projections is quite varied (MIT, 2007). The scenario shown in figure 2 would require 2300 GtCO₂ of storage. If the storage commitment associated with the remaining lifetime of the plants existing in 2100 is included (Appendix 1), with no sequestration beyond their lifetimes, this increases to about 3200 GtCO₂.

Carbon capture and storage (CCS) reduces the net efficiency of extracting electrical energy from coal by ~25%. CCS also is currently in the range of 90% efficient at capturing CO₂ produced, so that CO₂ emissions per net kWhe are reduced by about 87%, not 100%. [CO₂ emissions per kWhe from natural energy flows and from nuclear power are estimated by the IPCC (2007b) to be much less.] 4800 GWe generated using coal and CCS would emit 4 Gt CO₂/year, which is
beyond the total allowed world CO₂ emissions from energy and industrial processes, for
constrained scenarios. Taking into account areas of the world where CCS is impractical, coal
would need to be co-fired with a very substantial amount of biomass for acceptable net emission.

In sum, 40% of world electricity production in 2100 from combustion with CCS is a challenging
goal.

4. Natural Energy Flows: 160 TWe-yrs by 2100, 3600 GWe in 2100
The dominant non-carbon-emitting electrical energy source today is hydropower, providing
about 16% of world electrical production in 2007. While hydropower has potential for growth in
the future, it is not likely to be able to track the factor of five increase projected for 2100. If it
grows by a factor of two, to its realistic limit (IPCC, 2007b), large-scale hydropower will provide
about 6% of world electricity in 2100. Other sources based on hydrological flows such as tides
and wave power are not projected to be major contributors. The low thermal conductivity of
rock, the difficulty of drilling in igneous and metamorphic rock, and induced seismicity have
been raised as concerns for deep geothermal power, although some studies indicate a large
potential total capacity (MIT, 2006).

In the scenario of figure 2, this could leave about 24% of world electricity in 2100 to be provided
by tapping intermittent energy sources such as wind and solar. It is frequently reported that such
intermittent sources are difficult to incorporate into grid systems at over 20%. Large-scale energy
storage to smooth out the natural time variability of these sources is speculative. Given that
significant portions of the world do not have enough wind and solar resources to provide as
much as 20% of their needs, a world average fraction of 24% (starting from 1.2% in 2007) is a
challenging goal, as is the 30% overall total for tapping natural energy flows assumed in figure 2.

5. Nuclear Power: Fission and Fusion: 150 TWe-yrs by 2100, 3600 GWe in 2100
The above discussion illustrates the challenges associated with producing 70% of the projected
world’s electrical energy needs in 2100, with low CO₂ emissions, from a combination of
combustion with CCS and tapping of natural energy flows. This provides support to evaluate a
nuclear power scenario that produces 30% of the world’s projected electrical energy needs in
2100, up from 14% in 2007, while total electrical energy production increases by a factor of five.
Here we will discuss the leading fission technologies, light water reactors and fast-spectrum
reactors, followed by a discussion of fusion energy. In all cases we will focus on proliferation
risks. The reader is referred to Feiveson (2004) and Feiveson et al. (2008), which consider the proliferation risks associated with some other fission technologies.

5.1 Light Water Reactors

The far-dominant current fission reactor technology is light-water reactors (LWRs). In these systems conventional water is used both to remove fission-produced heat from the reactor and to slow down the fission-produced neutrons to thermal energies, where they have a high probability of maintaining the chain reaction rather than being absorbed without producing subsequent fission. This technology is mostly employed using a once-through fuel cycle, in which uranium is first mined from the earth and then enriched from its natural concentration of 0.7% $^{235}\text{U}$ to about 4.5%. As discussed in Appendix 2, about 200t of U is needed to provide 1 GWe-yr, with 0.25% $^{235}\text{U}$ concentration in uranium tails, a relatively aggressive level to maximize uranium utilization. If all of the nuclear power in the scenario of figure 2 were provided by LWRs, this would require mining of 33.4 Mt, comparing well with the estimate for a similar scenario by Feiveson et al. (2008) of 35 Mt (again, “Mt” here denotes millions of metric tonnes, not be confused with the symbol “MT”, frequently used in the U.S. nuclear power literature to denote metric tonnes). If the uranium required to complete operation of the LWRs in use in 2100 is included (see Appendix 1), with no further LWR construction, this increases to 59 Mt.

![Figure 3. Total discovered + undiscovered uranium reported in IAEA/NEA Red Books, since estimates of undiscovered resources have been included. During this time period 1.1 Mt of U was mined (OECD 2006, 2008).](image-url)
The Nuclear Energy Agency (NEA) of the Organization for Economic Co-operation and Development (OECD), together with the International Atomic Energy Association (IAEA) have estimated world Uranium resources in a broadly referenced series of “Red Books”, whose most recent edition (OECD, 2008) summarized data from 2007, and whose history has recently been summarized (OECD, 2006). These documents are based on national self-reporting of highly uneven geological studies. If one sums all categories of conventional uranium resources irrespective of price, including speculative, undiscovered resources (which have only been reported since 1982), the total uranium projection has been relatively stable over the last 25 years, as shown in figure 3.

This NEA/IAEA estimate of uranium resources would represent a significant limitation on using LWRs with the once-through fuel cycle to meet the nuclear energy requirements of our scenario. However there is considerable disagreement in the literature (Deffeyes et al., 1980, Schneider and Sailor, 2008) on future conventional uranium reserves, particularly because the price of electricity from LWRs is very weakly dependent on the price of mined uranium. Furthermore, unconventional uranium sources such as seawater (Seko et al., 2003) may become available at an acceptable price. Nonetheless, 59 Mt for the full scenario is a factor of 3.7 above NEA/IAEA estimates of total world resources and could be difficult to provide. By 2050 only 6.6 Mt will have been consumed, with a further 10 Mt committed, roughly consistent with the total NEA/IAEA estimate. It should be recognized, however, that there is considerable variation from country to country in uranium resources relative to potential consumption. Since many nations perceive a strong need for adequate domestic energy supplies, concerns remain about early depletion of uranium resources.

A second factor which could limit the ability of LWRs with once-through fueling to support the scenario of figure 2 is the production of nuclear waste. If all of the specified nuclear power were provided by LWRs, the nuclear waste created worldwide by 2100 would correspond to the equivalent of about 48x the statutory limit for Yucca Mtn., with an additional 38x associated with the estimated remaining lifetime of the installed LWR systems in 2100.

Calculations such as these lead to the consideration of different nuclear fission technologies, such as employing a fast (as opposed to thermal) spectrum of neutrons, which has the potential to
use uranium more efficiently and to reduce the longest-lived nuclear waste. A second alternative is fusion energy, powered by the fusion of light nuclei into helium, which is not limited by uranium resources and does not produce waste requiring geological burial. Before turning to these technologies, let us consider the proliferation risks associated with LWRs at this scale.

Table 1 summarizes some of the parameters relevant to proliferation risks of an LWR system designed to provide the full nuclear power specified in our scenario. Parameters for the years 2050 and 2100 are listed, anticipating the possibility that other technologies, such as fast-spectrum fission or fusion power plants, could provide a significant fraction of the nuclear power beyond 2050. In Table 1, “Pu+MA” denotes plutonium plus minor actinides, such as neptunium and americium, which can also be used to produce nuclear weapons (Albright and Barbour, 1999). Sometimes in this context Pu + MA are indicated as “TRU”, transuranics. Minor actinides typically represent less than 10% of the total TRU in used nuclear fuel.

<table>
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<th>2010</th>
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<th>2100</th>
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<td>Pu+MA Production (t/yr)</td>
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<td>Pu+MA in Waste (t)</td>
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<td>11,200</td>
<td>49,000</td>
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</tbody>
</table>

Table 1: Proliferation-relevant parameters of LWR systems to provide the nuclear power profile specified in figure 2.

Proliferation risks are conventionally divided into three categories (GIF, 2006):

1) Clandestine production of weapons materials in undeclared facilities
2) Covert diversion of weapons materials from declared facilities
3) Breakout from non-proliferation obligations and subsequent production of weapons materials.

There are also risks associated with the theft by sub-national groups of weapons material from nuclear facilities, with or without insider cooperation. This is generally classified under the rubric of physical security.
In nations that are signatory to the Non-Proliferation Treaty (NPT) and in particular to its Additional Protocol that allows inspection of non-declared facilities, there is little risk of clandestine production of weapons materials in small fission reactors, because these can be detected, for example, by their emissions. There is also relatively little risk of covert diversion of materials from declared LWR facilities, because fuel rods can be counted and monitored by the International Atomic Energy Agency (IAEA). With an increase of an order of magnitude in nuclear power, however, maintaining the same absolute level of error in accounting would be more challenging. The risk of theft of nuclear materials by sub-national groups for the production of nuclear weapons is relatively small, since the incoming fuel for LWRs is low-enriched uranium (LEU), not easily converted by a sub-national group to the highly enriched uranium (HEU) needed to produce nuclear weapons, and the Pu and minor actinides in the used nuclear fuel are mixed with highly radioactive fission products. The used fuel is deemed “self-protecting” against theft and subsequent use for nuclear explosives by sub-national groups for a period of order 100 years. Even after this period used fuel is bulky and radioactive, and can be well accounted. With adequate resources, it should be possible to detect rapidly a deficit of used nuclear fuel from cooling ponds, dry casks or even repositories, either due to diversion by a host nation or due to theft. It should be recognized, however, that the IAEA’s current budget of 122 M €/yr to verify 908 facilities under safeguards or containing safeguarded materials is far overstretched. Furthermore national resources committed to deterrence of theft are often characterized as inadequate to the challenge.

The largest risks for future LWR systems are associated with 1) clandestine production of highly enriched uranium using advanced technologies such as centrifuges, 2) breakout and use of declared enrichment facilities to produce weapons materials, and 3) breakout and use of Pu and possibly minor actinides from used nuclear fuel. The concerns about Iran’s development of centrifuges for uranium enrichment center on risks 1) and 2), while North Korea’s development of nuclear explosives is an example of risk 3).

Taking the year 2050 as an example, 1250 t/yr of $^{235}\text{U}$ would be provided to LWRs in the form of LEU, assumed here at 4.5% enrichment (Appendix 2). The IAEA (2001a) defines a significant quantity (SQ) of fissile material as “the approximate amount of nuclear material for which the possibility of manufacturing a nuclear explosive device cannot be excluded. Significant quantities take into account unavoidable losses due to conversion and manufacturing processes and should not be confused with critical masses.” For highly enriched uranium, HEU ($>20\%$
$^{235}\text{U}$, an SQ is defined as a quantity containing 25 kg of $^{235}\text{U}$. For Pu, an SQ is 8 kg, practically irrespective of its isotopic composition. It is widely reported that sophisticated experts can (and do) make weapons with significantly smaller quantities of these materials.

The fuel for LWRs is in the form of LEU rather than HEU. Thus the quantity needed to evaluate the level of success required to safeguard against clandestine production or breakout, is really the amount of $^{235}\text{U}$ in HEU that could be produced with the anticipated enrichment plants. Enrichment capability is measured in kg Separative Work Units (Benedict et al., 1981). About 5550 kgSWU are required for 1 SQ of HEU (Gilinksy et al., 2004), and 153,000 kgSWU for 1 GWe-yr of LWR power, at 4.5% enrichment with 0.25% $^{235}\text{U}$ concentration in uranium tails. World enrichment capability in 2050 would thus correspond to the capability to produce about 34,500 SQ of HEU per year. A single large centrifuge-based enrichment facility that could produce LEU for 50 GWe-yr/yr of LWR power, 4% of the anticipated world market in 2050, can be reconfigured to produce 1380 SQ/yr, of $^{235}\text{U}$ at 90% enrichment. It is relatively straightforward to verify that a commercial enrichment facility is not producing HEU, but breakout into HEU production can be rapid (Glaser, 2008).

Even more problematic, a clandestine enrichment facility using the P-2 centrifuge technology developed in Pakistan, with a footprint of about 550m$^2$ and drawing about 100 kWe, can produce 1 SQ of 90% enriched HEU per year (Gilinksy et al., 2004) starting with natural uranium, and over 5 SQ/yr starting with LEU. Current commercial centrifuge technologies are even more compact and efficient. Facilities based on either technology would be hard to detect, even with the Additional Protocol in place. Thus the broad dissemination of this and other advanced technologies for uranium enrichment is a major concern, and should be controlled to the degree possible by the use of “black-box” systems in carefully safeguarded multi-national facilities, in order to minimize the risk of technology leakage (Socolow and Glaser, 2009, Glaser, 2010).

The second major concern with LWR technology is the presence of significant quantities of plutonium and minor actinides (Pu + MA, or transuranics, TRU) in used fuel. At 50 MWD/kg burnup, 1 GWe-yr of LWR operation produces approximately 321 kg of TRU (Appendix 2), including about 295 kg of Pu. The 11,200t of TRU available in used fuel in 2050 corresponds to 1.3 million SQ of Pu. The production rate of 400t/year corresponds to 46,000 SQ of Pu/year. To address climate change, nuclear energy will need to become much more widespread than currently, so many new nations will need to join the nuclear “club”. A new nuclear nation that
had produced only 1 GWe of nuclear power for a decade would have in its possession 370 SQ of Pu. The IAEA (2001b) estimates that the time for a host state to produce nuclear weapons, starting with used nuclear fuel, is 1 – 3 months. While the expected nuclear explosive yield of “reactor-grade” Pu is more variable than that of weapons-grade, it is nonetheless highly destructive even in the worst-case “fizzle” (Mark, 1993, Gilinsky et al., 2004). Use of partially irradiated fuel or the ends of fuel rods that are less exposed to neutron irradiation provides higher-grade plutonium.

Clearly, used fuel would need to be carefully monitored in order to insure rapid detection of any violation of treaty obligations. On the other hand, short of military invasion, it is not practically possible to prevent a sovereign nation, in its own perceived supreme national interest, from breaking out of its non-proliferation agreements and accessing its own existing used fuel to produce nuclear weapons. Reprocessing plants prepared for operation can be hidden underground, and destroying a repository of used nuclear fuel could spread radioactivity over civilian populations, including those in neighboring countries. Such breakout could constitute a strong temptation for a state that perceived itself to be under existential threat, even by conventional weapons only. In some cases the attacking nation could respond by providing itself with nuclear arms, but even so, the rapid acquisition of nuclear weapons by both sides would turn an impending strategic defeat for the threatened state into a stalemate, a considerable benefit. The analyses of the motivations and behavior of North Korea by Sigal and Wit (2009) and of Iran by Ehteshami (2009) illustrate the attraction of nuclear weapons for states that perceive themselves to be seriously threatened.

Recently the United Arab Emirates, as part of its proposal to build a first nuclear power plant, has indicated that it is willing to return used nuclear fuel to its supplier. Arrangements such as this would help provide proliferation resistance at the so-called “back end” of the nuclear fuel cycle, although the need for fuel cooling before shipment would still leave a significant amount of material on site. It should be recognized, moreover, that the Non-Proliferation Treaty is interpreted by its signatories to allow enrichment and reprocessing by all states, including non-weapons states, so major changes would be needed in international agreements to prevent nations from acquiring and applying these technologies. The difficulty faced by the U.S. Global Nuclear Energy Partnership (GNEP) (USDOE, 2007) and IAEA “fuel bank” (IAEA, 2005) initiatives in attracting significant numbers of states willing to forgo enrichment and reprocessing for access to external fuel services is worrisome in this regard. The GNEP initiative would have defined states
with the right to enrich and reprocess fuel, and others that would relinquish such rights. By contrast the IAEA initiative did not define such distinctions, but proposed that all enrichment and reprocessing activities be placed exclusively under international control. However even this proposal encountered strong resistance from developing countries (Glaser, 2010).

Resistance to the needed strengthening of the non-proliferation regime stems in part from the slow rate of implementation of the disarmament clause of the existing NPT. However a large expansion and spread of nuclear power would make the disarmament process that much more difficult. The cooperative process of stepping away from nuclear weapons in a world with so much raw material for their production, widely dispersed, would be very difficult, because the magnitude and breadth of the system requiring control would be so daunting.

Since we will next consider “fast-spectrum” fission and then fusion scenarios, in which these new technologies begin to be commercialized around mid-century, it is valuable to consider, as an example, the climate impact of an LWR case which peaks in mid-century and uses all of the IAEA/NEA discovered + undiscovered uranium by 2100. Replacing that much nuclear power with pulverized coal plants without CCS would increase CO$_2$ concentration in 2100 by 44 ppm, with a predicted long-term surface-average temperature rise of 0.34°C (subject to the caveats discussed above).

Even with a much stronger nonproliferation regime in place, thoughtful people will want to weigh the risks associated with this temperature rise against the increased proliferation risks discussed so far.

5.2 Fast Spectrum Fission Reactors

Limitations of uranium supply and/or of the ability to store used nuclear fuel are perceived as potential drivers for adopting nuclear fission reactors that operate with a fast spectrum of fission-produced neutrons, sometimes called “fast reactors”, FRs. This generally requires the use of heavy metallic coolants, such as sodium or lead to limit the slowing-down of neutrons through collisions. Alternatively, a neutron-transparent coolant such as He may be able to be used. Such systems take as a design goal converting $^{238}$U to Pu isotopes and minor actinides (TRU) while burning only TRU, not $^{235}$U. The conversion ratio (CR) of such systems is defined as the production rate of TRU divided by the TRU burn rate. For example CR = 1 denotes a system which consumes no net TRU. The range of CR that is likely to be accessible is from 0.5 to 1.5,
although the limits are under study; TRU-based fuel is still in development. The high end is limited by neutron economy (Piet et al., 2009), since about 2.9 neutrons are produced per fission in TRU, and one of these neutrons is necessarily consumed in further fission, in order to sustain the chain reaction. The theoretical upper limit of \( CR \sim 1.9 \), which would result from capture of all of the remaining neutrons by \(^{238}\text{U}\) producing \(^{239}\text{Pu}\), is inevitably reduced by the loss of neutrons from the reactor core or by their absorption through parasitic capture in fuel or fission products, and in internal reactor structures (including control rods). The lower end of the CR range may be limited by the practical lifetime of TRU fuel cladding, or by safety issues that stem, for example, from the large swing in reactivity during burn at low CR and the small delayed neutron fraction of TRU (Hoffman et al., 2006).

The U.S. Advanced Fuel Cycle Initiative has as one of its goals, “Develop and make available the fuel cycle technology needed for commercial deployment by 2040 of fast spectrum reactors operating either exclusively as transuranics transmuters or as combined fuel breeders and transmuters.” (USDOE, 2005) Thus we consider scenarios in which fast spectrum fission reactors burning TRU come on line commercially in 2040. Other nations may be driven by different considerations than the U.S. to move more quickly than this. For example China and India have rapidly growing energy supply needs and limited domestic uranium supplies. The development of fast reactor fuels based on TRU, and of the full associated fuel cycle, are significant technological challenges, but many nations are pursuing research and development on fast-spectrum systems, both through the Generation IV International Forum (GIF, 2009) and through national deployment of prototype fast-spectrum systems.

The world will have a large resource of used nuclear fuel by 2040, so fast reactors can be started up as this used fuel is reprocessed to extract Pu and MA’s and is then fabricated into TRU fuel for the fast reactors. As shown in the dynamical equations of Appendix 3, the time evolution of the implementation of these reactors is controlled by the source of TRU, the conversion ratio (CR) of the fast reactors, and, very importantly, the residence time of fuel in the reactor, in cooling, and then in reprocessing and fabrication.

In these analyses we neglect country-to-country variations of uranium supplies and of access to used fuel, considering the world’s uranium and used fuel as world-wide resources. In figures 4a - 4d we consider both breeder (CR > 1) and burner (CR < 1) FRs: first, FR “breeder” cases with the ratio CR ≤ 1.5 that allows fast reactors to take over maximally from LWRs by the end of the
century (but not sooner), also pulling essentially all LWR used fuel into the FR system by that date, and second, FR “burner” cases with CR = 0.5, which employ fast reactors to reduce TRU waste as LWRs continue to operate past 2100. We consider for each of these classes two residence times for used TRU fuel in the cooling/reprocessing/fabrication stages (τF): a minimum time of 2 years, which might be achievable with reprocessing facilities collocated at fast reactors, and an estimated time of 11 years, which might be required to provide adequate cooling to allow transportation of used TRU fuel to centrally located reprocessing centers. These analyses extend the insightful work by Dixon et al. (2007), analyzing U.S. - only scenarios, which used these values for τF.
Figure 4a. Power production and stocks and flows of Pu+MA and $^{235}\text{U}$ for CR = 1.21, $\tau_F$ = 2 years.
Figure 4b. Power production and stocks and flows of Pu+MA and $^{235}$U for CR = 1.5, $\tau_F = 11$ years.
Figure 4c. Power production and stocks and flows of Pu+MA and $^{235}$U for CR = 0.5, $\tau_F = 2$ years.
Figure 4d. Power production and stocks and flows of Pu+MA and $^{235}$U for CR = 0.5, $\tau_F$ = 11 years.
Figures 4a - 4d show results for these cases, using the evolution equations for stocks and flows described in Appendices 2 and 3. In general one observes that longer fuel residence times result in slower growth rates for fast reactors. This stems from the fact that a fast reactor typically contains four ($\tau_R$) years worth of fuel, but the additional residence time in cooling ponds, transport, reprocessing and fabrication ($\tau_F$) requires substantial additional commitment of TRU for a smoothly operating system. No further allowance is made here for a reserve supply of fuel, which would likely be required by reactor operators. With $\tau_F = 2$ years, as assumed in Case A, all LWR nuclear power can be replaced with FRs having CR = 1.21. At CR = 1.5, to achieve this goal requires $\tau_F \leq 6$ years. With the assumed time for reprocessing and fabrication of only one year, the remaining five years for cooling and two-way transportation is likely to be inadequate for the use of international fuel recycling centers. In Case B, with $\tau_F = 11$ years, it is not possible to replace all LWRs by 2100.

It is also the case that lower CR results in fewer fast reactors. This is in part because in a “balanced” steady-state system in which the fast reactors steadily consume the TRU from LWRs (Appendix 3) fast reactors with CR = 0.5 would only account for ~39% of the total power production. However it is also the case that the world’s reserve of used nuclear fuel limits the total number of CR=0.5 reactors that can be constructed by 2100. The greater $\tau_F$ in Case D therefore reduces the number of fast reactors.

Table 2 provides some key results relevant both to the goals of fast reactors to reduce waste and extend the resources for fission, and also to proliferation risks. We start by analyzing the degree of success towards the goals considered for fast spectrum reactors, extension of uranium resources and reduction of waste.

From the point of view of extending uranium resources, clearly Case A is successful, requiring less total mined uranium than the IAEA/NEA total (discovered plus undiscovered) resource of 16 Mt, unlike the LWR-only scenario which required 59 Mt even if no further reactors were constructed after 2100. Case B is somewhat successful, and Cases C and D, because they are not designed to replace LWRs with FRs, not only far exceed the IAEA/NEA total, but are understated in Table 2. The “+” is meant to indicate that the “committed” resource associated with the existing reactors in 2100 far understates the very long-term commitment of a steady-state “balanced” system.
From the point of view of reducing TRU waste all four cases are successful. This essentially stems from the fact, discussed in Appendices 2 and 3, that 1 GWe-yr of LWR operation produces about 0.32t of TRU waste, while 1 GWe-yr of FR operation requires fueling roughly in the range of ~2t of TRU (picked up from the LWR waste or created by the FRs), but the only TRU waste that needs to be disposed is the 1%, or ~0.02t, that is anticipated to be lost in the reprocessing and fabrication steps. This results in a factor of ~16 reduction in waste TRU per GWe-yr produced in a fast reactor as compared with an LWR. As the fast reactors begin by loading TRU from the LWRs, in this model the LWR power produces no waste of its own. [Note that LWR TRU does not go to zero in 2100 in figures 4a - 4d, because it must be cooled for ~6 years (Dixon et al., 2007) before reprocessing.]

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Case D</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR</td>
<td>1.21</td>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau_F$ (yr)</td>
<td>2</td>
<td>11</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>Total U mined + Committed (Mt)</td>
<td>12.3</td>
<td>29.5</td>
<td>42.0 +</td>
<td>47.5 +</td>
</tr>
<tr>
<td>Pu+MA in Waste, 2100 (t)</td>
<td>2,220</td>
<td>4,210</td>
<td>6,030</td>
<td>6,550</td>
</tr>
<tr>
<td>Pu+MA in FR System, 2100 (t)</td>
<td>38,000</td>
<td>53,800</td>
<td>17,300</td>
<td>24,500</td>
</tr>
<tr>
<td>$^{235}$U Fueling, 2100 (t/yr)</td>
<td>0</td>
<td>1,470</td>
<td>2,390</td>
<td>2,770</td>
</tr>
<tr>
<td>Pu+MA Fueling, 2100 (t/yr)</td>
<td>6,510</td>
<td>3,420</td>
<td>3,190</td>
<td>2,170</td>
</tr>
</tbody>
</table>

Table 2: Parameters relevant both to fast reactor goals to extend resources and reduce waste, as well as to proliferation risks.

One should be cautious, however, because this waste assessment is not complete. The mass of fission products produced per GWe-yr is about the same for LWRs and FRs, except for the modest anticipated increase in efficiency at the higher temperatures of fast reactor coolants. To gain a large advantage with respect to the thermal capacity of waste storage, Cs and Sr must be
partitioned and stored for ~300 years, outside of the repository (Wigeland et al., 2006). It also appears that in an oxidizing environment such as predominates at Yucca Mtn., as opposed to the reducing environment now recommended by the IAEA for geological repositories (IAEA, 2003), the mobility of the long-lived 99Tc and 129I fission products relative to Pu and minor actinides could make them a significant radiological safety concern (Piet et al., 2007).

Now we consider the proliferation risks of the FR cases in figures 4a - 4d. What stands out most strongly in these figures is the rising line denoting the inventory of TRU in the FR system, including its storage and reprocessing facilities. Since FRs with CR > 1 create net TRU, and FRs with CR < 1 burn it, but slowly, the quantity of TRU in process is comparable to the quantity that would have been stored in dry casks or buried in geological repositories in the case of LWRs alone (see row 4 of Table 1). Thus in the FR cases one has traded TRU casks and geological repositories for TRU pools – of similarly large magnitude, but now being used and manipulated, and so requiring much more extensive safeguards. The pool size ranges from about 2 to 6 million SQ, from about 100 to about 600 times the estimated total number of nuclear weapons in the world today. This is an example where magnitude certainly matters. It should be recognized as well that in all four cases one is committed to continuing growth of the active pool of TRU. In particular, stopping abruptly for any reason would result in a very large amount of waste to dispose. Stated epigrammatically (Piet et al., 2009), “...one must put TRU ‘in play’ in order to reduce waste burdens. Use it to lose it.” and “Don’t stop!”

It is important to consider proliferation risks in terms of flows as well as stocks. Table 2 shows that Case A eliminates the need for uranium enrichment, because the only fissile fuels for the fast reactors in that scenario are the TRU from used LWR nuclear fuel and from the fast reactors themselves. 238U from natural or even depleted uranium provides the material to be converted to Pu. This is a very favorable result. Case B has some effect, and presumably in the very long run would allow elimination of uranium enrichment. Cases C and D, by construction, do not qualitatively affect this risk.

The largest concern in these cases is the flow of Pu and minor actinides indicated in Table 2. Case A, the most attractive from the point of view of resolving other issues, involves the fueling of fast reactors with about 750,000 SQ of Pu per year. Case D, with the lowest fueling rate, corresponds to 250,000 SQ of Pu per year. Currently the IAEA standard for uncertainty in closing the material balance of a plutonium reprocessing plant is 1% (IAEA, 2001b, IPFM,
Again, magnitude matters. Even with enhanced monitoring, surveillance and containment to detect off-normal operation or diversion of materials, failure worldwide to account for 1% of 500,000 SQ per year, 5000 SQ per year, could create an unstable international environment where nations would be very concerned about the activities of others and perceive the need to take precautionary actions themselves.

Are there approaches to resolving the issue of diversion in a world with such large stocks and flows of Pu and minor actinides? Because of the magnitude of these flows, to assure against national diversion or insider-aided theft, the standards for material accountancy at reprocessing plants would need to be improved by 2 – 3 orders of magnitude. This may not be possible. A fundamental problem with the alternative solution of internationalizing the “back end” of the fuel cycle is that it necessitates – by definition – the transport of the used fuel. Because of the long cooling time needed before transporting fast reactor TRU-based fuel, this has the consequence of eliminating Case A, the most attractive from the point of view of conservation of uranium resources and reduction of uranium enrichment. It also has the consequence – again by definition – that extremely large quantities of Pu, some significant fraction of 500,000 SQ in fresh fuel, would be in transport every year, crossing international borders. This evidently creates its own set of diversion and theft risks. TRU in fast reactor fuel is not self-protecting (Kang and von Hippel, 2005), and can be rapidly chemically separated and used for weapons, in contrast to the $^{235}$U in LWR fuel that requires further isotopic enrichment for military use.

Are there approaches to resolving the issue of breakout from non-proliferation agreements? This seems at least equally problematic. Consider that the startup fuel for 1 GWe of fast reactor capacity requires ~8t of Pu or ~1000 SQ. In a world where the nuclear weapons states had disarmed to hundreds of weapons each, the temptation to use this fuel for military purposes could be very strong, particularly for a state that perceived itself to be under existential threat, even from conventional weapons. The annual fueling for a fast reactor is much greater than the annual Pu waste quantity from an LWR, ~2t (250 SQ) / GWe-yr vs. 0.3t (37 SQ) / GWe-yr, and its processing would be even easier and faster for a host nation (1 – 3 weeks vs. 1 – 3 months for irradiated LWR fuel), since it would not be burdened with highly radioactive fission products (IAEA, 2001a). Furthermore the global fueling flows in Table 2 are much greater than the Pu + MA production rates shown in Table 1.
The proliferation risks associated with fast reactors, as currently understood, appear qualitatively greater than those associated with LWRs. Thoughtful people could conclude that without a dramatic change in the proliferation resistance of fast reactor technology, the risks associated with additional long-term surface-average temperature rise of 0.3°C (subject to the caveats discussed above) would be more acceptable.

5.3 Fusion

Power can be produced by “fusing” heavy forms of hydrogen to form helium (von Hippel and Goldston, 2010). In laboratory experiments up to 16 MWt has been produced for periods of order 1 second, demonstrating the scientific feasibility of producing fusion energy using magnetic fields to confine hot fusion fuel. Based on these scientific results, the ITER fusion experiment is under construction in Cadarache, France as an international collaboration of China, Europe, India, Japan, Russia, South Korea and the United States. ITER is designed to produce hundreds of MW of thermal power from fusion for periods of up to one hour, which will demonstrate the technological feasibility of fusion energy. In the U.S. the National Ignition Facility has just come on line, with the primary mission to study the physics of nuclear weapons, but also with a mission to demonstrate the scientific feasibility of fusion using the inertia of tiny exploding pellets to confine them for long enough to provide more fusion energy than laser energy delivered to the target. The economic practicality of fusion power plants is, however, not assured, and considerable R&D is required to move from scientific feasibility to technological feasibility to commercial application. Many of the nations engaged in fusion R&D using magnetic confinement are targeting mid-century for the first commercial use of fusion.

Figure 5 shows a scenario for the application of fusion power for electricity production after mid-century. The maximum growth rate of fusion power in this scenario is 0.86%/year of the world electricity market, which is less than the growth rate of fission power 1975 - 1990, 1.2%/year of the electricity market at that time. In this scenario 15.8 Mt of uranium is mined for LWRs, equal to the IAEA/NEA projected total resource.
Fusion has significant proliferation advantages (Goldston et al., 2009). While the energetic neutrons from fusion can be used to transmute $^{238}\text{U}$ to $^{239}\text{Pu}$, or $^{232}\text{Th}$ to $^{233}\text{U}$, this is very easy to detect and even prevent. First, fusion systems are easily enough detectable due to their size, energy use and effluents that clandestine use of a small fusion facility to produce weapons materials is not a realistic threat. Next, in normal operation a fusion power plant should have no uranium, thorium, plutonium or fission products at all. The detection of these at very low levels is straightforward, so covert production and diversion of weapons materials in a declared facility would not be a serious risk. Finally, the breakout scenario for fusion is qualitatively different from that for fission. At the time of breakout a fusion plant operator does not have any weapons material at all. His threat is not to use existing materials, but rather to begin to produce such materials. Over a period of order 1 – 2 months, it would be possible to convert a pure fusion power plant to produce $\sim$1 SQ of $^{239}\text{Pu}$ or $^{233}\text{U}$ per week. However, it would be straightforward to interdict this production, for example by destroying a cooling tower, electrical power conditioning system or cryoplant, none of which would pose a threat of nuclear contamination. The possible choices for action would not be limited to land invasion or aerial attack with significant risk of radiological contamination of civilian populations.
5.4 Fusion-Fission Hybrid Transmuters

In the scenario of figure 5 with no further processing of the used nuclear fuel from the LWRs, 27,000t of TRU remain ultimately to be placed in geological repositories with capacity of ~27x the statutory limit Yucca Mtn., world wide.

It has been proposed to use accelerator-driven neutron sources to drive sub-critical fission reactors to transmute, effectively to burn, TRU. Fusion systems can also produce neutrons, in principle with much lower energy input than accelerators, so studies have been undertaken to examine this option (Freidberg and Finck, 2010). Here we consider one of the more well-developed concepts (Stacey, 2009), based on a subcritical fast reactor driven by fusion neutrons (see Appendix 4) burning the left-over TRU from LWRs. Figure 6 shows the TRU stocks and flows associated with this concept, as applied to the scenario of figure 5. For simplicity we have assumed that a constant fraction of all nominally fusion systems until 2100 would be fusion-fission hybrid TRU burners. 9.9% is the required fraction to put all of the world's used LWR nuclear fuel TRU into process by the end of the century.

This scenario shares the main proliferation risks of the fast reactor scenarios, large stocks and flows of Pu and minor actinides. The advantage in this case is that as the TRU from the original set of LWRs is burned up, no further TRU is produced. After 2100 the stock and flow of TRU both drop by a factor of 2 every 30.6 years, rather than grow as nuclear power expands. Also in this scenario at most 1 in 10 power plants is ever a TRU burner, so the burners can conceivably be less dispersed than CR = 0.5 fast reactors, which constitute ~39% of a steady-state system in which they burn the waste from LWRs. If the technology is developed to make the scenario of figure 6 an option, a judgment will be required as to whether this is safer, from a proliferation point of view, than depositing the used LWR nuclear fuel in geological repositories.

In principle, fusion-fission hybrids could instead play approximately the same role as the fast reactors in the CR = 0.5 scenarios shown in figures 4c and 4d with fewer burner reactors, but without a qualitative proliferation advantage. In steady state, burner fast reactors at CR = 0.5 must constitute ~39% of a steady-state “balanced” fleet, corresponding to a “support ratio” of LWRs to FRs of 1.6 (Appendix 3). The equivalent steady-state support ratio for the fusion-fission hybrid systems (Appendix 4) is 3.6.
6. Conclusions

Nuclear energy may be needed to provide \(~30\%\) of world electrical power production by 2100. This can be achieved through a combination of light-water reactors, fast-spectrum reactors and potentially fusion. However the magnitude of the undertaking is large, constituting a 12x increase in nuclear electric power production. The very large scale and the associated broadening of the range of nations using nuclear power bring with them serious proliferation risks. The risks associated with light-water reactors at the needed scale are large. Institutional arrangements for their management have been proposed, albeit difficult to implement. The risks associated with fast reactors appear to be much greater and more resistant to management. In effect, the calculations here set the context for judging the degree of proliferation resistance that would be needed for the risks of fission technology to be acceptable. Thoughtful people will want to weigh proliferation risks against the risks associated with the \(\sim0.64^\circ\text{C}\) additional long-term global surface-average temperature rise that is estimated to occur if coal-fired power plants without carbon sequestration were to replace all nuclear power in this scenario, with no other changes. If fusion is developed, it will provide an option with qualitatively lower proliferation risks than fission.
Acknowledgements
The author would like to thank Larry Grisham and Greg Hammett for their contributions to the discussion of carbon sequestration and intermittent renewables, Valentina Bosetti for help in accessing the EMF 22 database, Alex Glaser for insights into the LWR fuel cycle and helpful comments, and Steven Piet for help finding key references on fast reactor parameters, and for very useful discussions. He would like to thank M.V. Ramana for helpful discussions, and Frank von Hippel for bringing up the issue of Pu “mines” vs. Pu “rivers” that motivated this analysis.

This work was supported by U.S. DOE Contract # DE-AC02-76CH03073

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1 Downloaded on December 7, 2009. One model appeared to have a miscalibration, providing results for year 2000
electricity production low by more than by a factor of three, and was not included.
2 http://tonto.eia.doe.gov/cfapps/ipdbproject/IEDIndex3.cfm accessed on January 14, 2010
3 Using the 3-reservoir model in the RICE-99 spreadsheet, downloaded from http://www.econ.yale.edu/
%7Enordhaus/homepage/dice_section_V.html on 1/25/10
4 “Reactor-grade” Pu that emerges with a mixture of Pu isotopes from LWRs is considered by the IAEA to be
adequate for the production of nuclear explosives. Only mixtures with > 80% 238Pu are excluded, and since Pu is
dominantly produced and transmuted by neutron capture starting from 238U, little 238Pu is available in the nuclear
fuel cycle.
5 The maximum energy loss, \( \Delta E_n \), in an elastic collision of a neutron with a nucleus in the coolant is given by
\( \Delta E_n/E_n = \frac{4A}{(A+1)^2} \), where A is the atomic number of the target nucleus. For A >> 1 this becomes small. The hydrogen in
water is a very effective at reducing neutron energy.
APPENDICES

General notes to Appendices:
1) In all cases the designation “Mt”, accepted for use with the S.I. system, denotes millions of metric tonnes. This is to be distinguished from “MT” which is often used in the U.S. literature to denote metric tonnes.
2) In all cases power production, e.g., $P_{LWR}$ and $P_{FR}$, is measured in GWe-yr/yr, to be interpreted as actual electric power production, to be distinguished from the commonly quoted electrical power capacity. For example 111 power plants each with 1 GWe capacity, operating at capacity factor 0.9, produce 99.9 GWe-yr/yr.
3) All flows assigned to year $n$ are assumed to occur on January 1 of year $n$, and all stocks assigned to year $n$ are assumed to be assayed at mid-year, July 1, of year $n$.

Appendix 1: Committed Energy Production

Ignoring startup effects, in a system of power plants that has been operating at a nearly steady power level for a period of time long compared with plant lifetimes, the average plant will be at mid-point in its lifetime, and the amount of additional energy that is committed to be produced by these plants during their remaining lifetimes is $E_{com} = P(t_0) \tau_{PP}/2$, where $P(t_0)$ represents the power level at the time when the commitment is assessed, $t_0$, and $\tau_{PP}$ the expected power plant lifetime. This simple result can be generalized for a continuous exponentially growing or decaying system to give the remaining committed energy production after time $t_0$:

$$E_{com} = P(t_0) \tau_{PP} \left[ \frac{m \tau_{PP} - \left(1 - e^{-mt_{PP}}\right)}{m \tau_{PP} \left(1 - e^{-mt_{PP}}\right)} \right]$$

Eq. A1.1

where $m$ is the annual growth or decay rate of LWR power before $t_0$. For the estimates given in the text $\tau_{PP}$ is taken to be 60 years, and $m$ is chosen to fit the annual power production 30 years before the designated commitment point. Committed additional CO$_2$ emissions, mining and used fuel production for different types of power plants can be computed from $E_{com}$. 
Appendix 2: Equations for Stocks and Flows of Uranium, Plutonium and Minor Actinides Associated with Light Water Reactors

A2.1 TRU in Existing Used Nuclear Fuel
The TRU in existing used nuclear fuel, denoted \( TRU_{UNF}(0) \) in the following equations, is required as an initial condition in the time-dependent calculations of TRU in used nuclear fuel. It was estimated at 2580t of TRU on the basis of the IAEA Overview of Global Spent Nuclear Fuel Storage (Fukuda et al., 2003), the IAEA (2009) Nuclear Technology Review and the Global Fissile Material Report (IPFM, 2009).

A2.2 Uranium Fueling
The natural uranium consumed to produce 1 GWe-yr of nuclear electricity from LWRs, denoted \( U_c \) in the following equations, was evaluated at 204.7t, based on Figure A-4.2 in the MIT (2003) “Future of Nuclear Power” report and associated calculations, assuming a relatively aggressive 0.25% \(^{235}\text{U}\) concentration in the enrichment tails to maximize uranium utilization, and 4.51% fuel enrichment. The MIT report assumes 33% efficiency for the LWRs, which is adopted here. Adjustment was made for the assumed capacity factor of 0.9. The 204.7t of natural uranium required for 1 GWe-yr, enriched to 4.51%, with 0.25% tails, corresponds to 22.15t of initial heavy metal (iHM) fuel. 6.89 kg of Separative Work (SWU) is required for each kg of this fuel. Within this fuel is almost exactly 1t of \(^{235}\text{U}\), so the annual flow of \(^{235}\text{U}\) to LWRs, in tonnes, is almost exactly equal to \( P_{LWR} \) in GWe-yr/yr.

It was assumed that 1 year is required for processing, enrichment and fabrication between the time uranium is considered to be “mined” and when it is used as fuel. Since the residence time of fuel in LWRs is assumed to be 4.5 years, when a net new reactor is started in the calculations, 4.5 yearly loads of U are assumed to be required. For simplicity, if a reactor is decommissioned and another commissioned in the same year, it is assumed that only one year of fresh fuel is required for those reactors, that year.

A2.3 TRU Stock in LWRs
Time-averaging of the composition of LWR fuel during burn from 0 to 50 MWd/kg (Glaser, 2009) provides an estimate of the inventory of TRU in a 1 GWe LWR. Taking into account a capacity factor of 0.9, this gives the TRU stock for 1 GWe-yr/yr of electricity production of

\[
TRU_{LWRc} = 0.80t
\]

Eq. A2.1
The total stock of TRU in LWR cores is then simply $TRU_{LWRc} P_{LWR}(n)$

### A2.4 TRU Flow to LWR Used Nuclear Fuel Stock
The flow of TRU from LWRs to the total stock of LWR used nuclear fuel per GWe-yr was estimated on the basis of the MIT report, Table A4-1, assuming burnup of 50 GWd per metric tonne of initial heavy metal, and adjusting for the assumed 0.9 capacity factor. The LWR production rate of TRU, denoted $TRU_{p,LWR}$ in the following equations, equals 0.3207t/yr, of which 0.295t is Pu. When a net reactor is decommissioned it is assumed that $TRU_{LWRc}$ flows to the used nuclear fuel, but if a reactor is decommissioned in the same year that another is commissioned, then only $TRU_{p,LWR}$ of TRU is assumed to be produced from those reactors, that year, and to flow into the stock of LWR used nuclear fuel.

The statutory capacity of Yucca Mtn. is taken to be 70,000t of heavy metal of which 1.447% or 1013t, is TRU.

### A2.5 Evolution Equations for Stocks and Flows
The evolution equations are formulated as difference equations, with time-step of 1 year. As noted above, all flows($n$) are considered to occur on January 1 of year $n$, and all stocks($n$) are evaluated on July 1 of year $n$.

Taking into account the assumed one-year delay between mining and fueling, the change in the stock of mined uranium is given by
\[
U_m(n) = U_m(n-1) + U_c P_{LWR}(n+1) + 3.5 U_c Max\left[ P_{LWR}(n+1) - P_{LWR}(n), 0 \right] \quad \text{Eq. A2.2}
\]
where $U_m(-1)$ is set at zero, so that $U_m(0)$, supplying the uranium for the first year of operation in the calculation is included.

The evolution of the stock of TRU in LWR used nuclear fuel due to LWR operation is given by:
\[
TRU_{UNF}(n) = TRU_{UNF}(n-1) + TRU_{p,LWR} P_{LWR}(n) + TRU_{LWRc} Max\left[ P_{LWR}(n-1) - P_{LWR}(n), 0 \right] \quad \text{Eq. A2.3}
\]
Note that when FRs are included in Appendix 3, they will add important terms to this equation.

A3.1 TRU Fueling Flow to FRs

The burnup rate \( BU_{FR} \) and TRU mass fraction \( f_{TRU} \) for a modern TRU-burning fast reactor design has recently been calculated as a function of conversion ratio \( CR \). The annual fueling rate, in metric tonnes, required to produce 1 GWe-yr can be calculated from these as

\[
L_{FR}(t) = \frac{1 \text{ GWe}}{\eta_{th}} \frac{365.25d}{BU_{FR}(\text{GWthd} / t)} f_{TRU}
\]

Eq. A3.1

The thermal efficiency, \( \eta_{th} \), for these designs is estimated at 38%. Figures 2-20 and 2-21 in Bays et al. (2009) provide \( BU_{FR} \) and \( f_{TRU} \) as functions of \( CR \), but numerical values are not available. Figure A2-1 provides a fit to \( L_{FR} \) based on values read from these figures, used in the following calculations. Only \( CR \) values between 0.5 and 1.5 have been used in the calculations.

Figure A2-1. \( L_{FR} \), the TRU load to fast reactors in order to produce 1 GWe-yr of electrical energy, from graphically reported calculations for burnup and fraction of TRU in fast reactor heavy metal, as a function of conversion ratio, \( CR \).

The residence time of the fuel in the reactor is assumed to be \( \tau_{FR} = 4 \) years, taking into account capacity factor, and consistent with estimates of damage tolerance by Hoffman et al., 2006. (Note that in these calculations \( \tau_{FR} \) is constrained to integer values.) It is assumed that each additional GWe-yr/yr of installed FRs requires a load of \( \tau_{R,FR} L_{FR} \).
A3.2 TRU Stock
The stock of TRU in a FR is a complex calculation, due to fuel shuffling, changes in reactivity, and other effects. Since $L_{FR} \sim 2B_{FR}$ and $|CR - 1| < 0.5$ in these calculations the effects of various approximations are in the few % range. Here we take an approximation that has the benefit that it allows an accurate check of stocks against flows - i.e., in all fast reactors at all times the rate of growth of TRU is $B_{FR}(CR-1)/$GWe-yr. This gives on July 1st of any year an in-reactor inventory of $\tau_{R,FR}L_{FR} + B_{FR}(CR-1)/2$.

The total inventory of TRU in the fuel cycle depends on $\tau_F$ as discussed in the main text. We take $\tau_F$ to vary between 2 years, for on-site cooling, reprocessing and fuel fabrication, to 11 years for cooling, transportation to an centralized fuel recycling center, reprocessing, fuel fabrication, and return to the fast reactor as in Dixon et al. (2007). TRU in LWR used nuclear fuel that is to be used to fuel FRs is given an effective $\tau_F$ of 1 year.

A3.3 TRU Unload Flow from FRs
At $\eta_h = 38\%$, 1 GWe-yr of electrical energy requires 2.632 GWth-yr of thermal energy. Since the fission of 1t of heavy metal results in 1000 GWthd of thermal energy, this means that $B_{FR}$, the burned heavy metal per GWe-yr is 0.9611t. The amount of TRU unloaded after this much energy production is

$$B_{FR}CR + (L_{FR} - B_{FR}) = L_{FR} + B_{FR}(CR-1) \quad \text{Eq. A3.2}$$

It is assumed that when an FR is decommissioned, and another is commissioned, the net unload flow is not affected. The unload flow from a net decommissioned FR would be

$$\tau_{R,FR}L_{FR} + B_{FR}(CR-1) \quad \text{Eq. A3.3}$$

since it is assumed that the decommissioning would occur at the end of a burn cycle. Note, however, that in the the calculations of figures 4a - d there is never a net decrease of $P_{FR}$.

A3.4 Processing Losses
Many references assume 1% waste loss during reprocessing at the industrial scale. Consistent with these, in the calculations of this paper we take $F_w = 0.01$.

A3.5 Evolution Equations for Stocks and Flows
The total fueling flow needed for the FRs on Jan 1 of year $n$ is given by
\[ F_{\text{fr}}^{\text{tot}}(n) = L_{\text{fr}}P_{\text{fr}}(n) + (\tau_{\text{r,fr}} - 1)L_{\text{fr}}[P_{\text{fr}}(n) - P_{\text{fr}}(n-1)] \quad \text{Eq. A3.4} \]

whereas the total fueling flow available from prior operation of FRs is given by

\[
F_{\text{fr}}^p(n) = \left[ L_{\text{fr}} + B_{\text{fr}}(CR - 1) \right] P_{\text{fr}}(n - \tau_F - 1) / (1 + F_w) \\
+ (\tau_{\text{r,fr}} - 1) L_{\text{fr}} \text{Max}[P_{\text{fr}}(n - \tau_F - 1) - P_{\text{fr}}(n - \tau_F + 1), 0] / (1 + F_w) \quad \text{Eq. A3.5}
\]

including the source from net decommissioning of fast reactors. The fueling flow from the stock of LWR used nuclear fuel is just the difference between these.

To get to the full TRU evolution equation, the additional contribution to \( TRU_{\text{UNF}} \) from LWRs must be included, as must loss to waste. Furthermore, the \( TRU_{\text{UNF}} \) must be reprocessed and fabricated into fuel, requiring an assumed period of 1 year. Taking this in account, we have:

\[
TRU_{\text{UNF}}(n) = TRU_{\text{UNF}}(n-1) \\
- (1 + F_w) \left[ L_{\text{fr}} P_{\text{fr}}(n+1) + (\tau_{\text{r,fr}} - 1) L_{\text{fr}} \text{Max}[P_{\text{fr}}(n+1) - P_{\text{fr}}(n), 0] \right] \\
+ \left[ L_{\text{fr}} + B_{\text{fr}}(CR - 1) \right] P_{\text{fr}}(n - \tau_F) \\
+ (\tau_{\text{r,fr}} - 1) L_{\text{fr}} \text{Max}[P_{\text{fr}}(n - \tau_F) - P_{\text{fr}}(n - \tau_F + 1), 0] \\
+ TRU_{p,\text{LWR}} P_{\text{LWR}}(n-1) + TRU_{\text{LWRc,Max}}[P_{\text{LWR}}(n-1) - P_{\text{LWR}}(n), 0] \quad \text{Eq. A3.6}
\]

The total stock of TRU in the FR system at any time, \( n \), is given by the sum of the TRU in the FRs, plus the total fueling for year \( n + 1 \), multiplied by \((1 + F_w)\), plus all \( FR \rightarrow FR \) fueling in process for other years.

\[
TRU_{\text{FRS}}(n) = \left[ \tau_{\text{r,fr}} L_{\text{fr}} + 0.5(CR - 1)B_{\text{fr}} \right] P_{\text{fr}}(n) \\
+ (1 + F_w) L_{\text{fr}} \left\{ P_{\text{fr}}(n+1) + (\tau_{\text{r,fr}} - 1) \left\{ \text{Max}[P_{\text{fr}}(n+1) - P_{\text{fr}}(n), 0] \right\} \right\} \\
+ \left[ L_{\text{fr}} + B_{\text{fr}}(CR - 1) \right] \sum_{m=n+1-\tau_F}^{n-1} P_{\text{fr}}(m) \quad \text{Eq. A3.7}
\]

where we are not including the possibility of net reduction of FRs over time, since that does not occur in the calculations shown here, nor are we allowing for the case of overproduction from FRs, where the TRU unload from year \( n - \tau_F - 1 \) is greater than \((1 + F_w)\) times the loading requirement for year \( n \).

The flow of TRU to the waste stream is easily evaluated as \( F_w \) times the total flow to fueling FRs.
\[ TRU_w(n) = TRU_w(n-1) + F_w \left\{ L_{FR} P_{FR}(n) + (\tau_{R,FR}-1) L_{FR} \left[ P_{FR}(n) - P_{FR}(n-1) \right] \right\} \]

Eq. A3.8

again assuming no net decommissioning of fast reactors.

It is helpful, to check the numerical implementation of these equations, to evaluate the changes in stocks from the start to the end of the calculation, against the summed flows. For example, for cases with monotonically rising \( P_{FR} \):

\[
TRU_{UNF}(N) - TRU_{UNF}(0) = TRU_{p,LWR} \sum_{n=0}^{N-1} P_{LWR}(n) + TRU_{LWR}[P_{LWR}^{max}(1,N) - P_{LWR}(N)]
- (1 + F_w) L_{FR} \left[ \sum_{n=1}^{N} P_{FR}(n+1) + (\tau_{R,FR}-1) P_{FR}(N+1) \right] + [L_{FR} + B_{FR}(CR-1)] \sum_{n=1}^{N-\tau_{FR}} P_{FR}(n)
\]

Eq. A3.9

We can also sum the flows into and out of the TRU pool associated with the FR system, giving the result:

\[
TRU_{FRS}(N) = \left(1 + F_w\right) L_{FR} \left[ \sum_{n=1}^{N} P_{FR}(n+1) + (\tau_{R,FR}-1) P_{FR}(N+1) \right] - \left[L_{FR} + B_{FR}(CR-1)\right] \sum_{n=1}^{N-\tau_{FR}} P_{FR}(n)
+ B_{FR} \left( CR-1 \right) \left[ \sum_{n=1}^{N-1} P_{FR}(n) + 0.5 P_{FR}(N) \right] - F_w L_{FR} \left[ \sum_{n=1}^{N} P_{FR}(n) + (\tau_{R,FR}-1) P_{FR}(N) \right]
\]

Eq. A3.10

These and other checks have been implemented in the calculations here. The results are accurate to numerical precision.

A3.6 Fraction of FR’s in a “Balanced” Steady State System

From these equations is it straightforward to evaluate the fraction, \( f_{FR} \), of total nuclear electric power in fast reactors with \( CR < 1 \) that will burn (and dispose as waste) exactly the TRU that is produced from a fraction \( (1 - f_{FR}) \) of total nuclear electric power in thermal reactors, in a steady-state situation. This amounts to solving the evolution equation for \( TRU_{UNF} \) (Eq. 3.6) for a situation in which all terms are independent of \( n \).

\[ f_{FR} = \frac{TRU_{p,LWR}}{TRU_{p,LWR} + B_{FR}(1-CR) + F_w L_{FR}} \]

Eq. A3.11
For \( CR = 0.5, L_{FR} = 2.43t \), and \( f_{FR} = 0.388 \).

### A3.7 Growth and Decay Rates of Fast Reactors with Zero TRU Input

One can use the above equations to consider growing or decaying situations with net zero input of TRU. This again amounts to solving equation 3.6, but now making only \( TRU_{UNF} \) independent of \( n \) (and neglecting any source from LWRs). For the growth case, one arrives simply at

\[
CR_{m>0} = 1 + \frac{(1 + F_w)(1 + m\tau_{R,FR})(1 + m)^{\tau_F} - 1}{(B_{FR} / L_{FR})} \quad \text{Eq. A3.12}
\]

where \( m > 0 \) is the annual growth rate of FRs. For physical intuition, it is helpful to look at the limit of small \( m \), which gives

\[
B_{FR}(CR_{m>0} - 1) = L_{FR}[F_w + m(\tau_{R,FR} + \tau_F)(1 + F_w)] \quad \text{Eq. A3.13}
\]

The left-hand side is the amount of extra TRU produced per year per GWe-yr, while the right-hand side represents the needs for the next year in terms of sustaining the current FRs against loss to waste and the needed growth in stock of the reactors and the fuel reservoir, taking into account loss to waste.

Equation A3-12 has been tested against the time-dependent numerical calculation. Setting LWR power to zero, and using A3-12 for the relation between \( CR \) and \( m \), there is no change in LWR used nuclear fuel, to numerical precision.

Equation A3-12 however is not in good agreement with equation 2 of Piet et al., 2009:

\[
CR_{m>0} = e^{m(\tau_F + \tau_{R,FR})}[1 + m(\tau_{R,FR} - 1)]
\]

even setting \( F_w = 0 \). The derivation of this equation is not given. It is notable that the ratio \( B_{FR}/L_{FR} \) does not appear. The authors evaluate two cases with \( m = 0.0175 \). For \( \tau_{R,FR} = 4, \tau_F = 2 \), the “example for onsite recycling”, \( CR = 1.17 \) is required per their equation 2, cited above, and for the case of \( \tau_{R,FR} = 4, \tau_F = 11 \), the “example for offsite recycling”, \( CR = 1.37 \) is required.

Table A3.1 compares the results of the two equations in the limit \( F_w = 0 \).
<table>
<thead>
<tr>
<th>$m$, $\tau_R$, $\tau_F$</th>
<th>$CR_{m&gt;0}$ Eq. 2 of Piet et al. (2009)</th>
<th>$B_{FR}/L_{FR}$</th>
<th>$CR_{m&gt;0}$ eq. A3.12, $FW = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.75%, 4, 2</td>
<td>1.17</td>
<td>0.56</td>
<td>1.19</td>
</tr>
<tr>
<td>1.75%, 4, 11</td>
<td>1.37</td>
<td>0.63</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Table A3.1. Comparison of eq. A3-12 from this work with Eq. 2 of Piet et al. (2009).

Sometimes it is convenient to solve for $m$ in terms of $CR$. An iterative solution for $m > 0$, equivalent to $CR > 1 + FW_{FR}/BF_{FR}$, can be found by gathering together higher order terms in $m$. 

$$m_{>0} = \frac{(CR_{m>0} - 1)(B_{FR} / L_{FR}) - FW - (1 + FW)\left((1 + m\tau_{R,FR})(1 + m)^{\tau_r} - \left[1 + m(\tau_{R,FR} + \tau_F)\right]\right)}{(1 + FW)(\tau_{R,FR} + \tau_F)}$$

Eq. A3.14

Only a few iterations on $m$ (starting with $m = 0$) are required for accurate convergence.

Equation A3.12 is somewhat different for the decaying case, $m < 0$. Solving for the situation where no extra TRU accumulates from decommissioning FRs, but rather the TRU unloaded from operation in year $n - \tau_F - 1$ is just what is needed to fuel the FRs in year $n$, and allowing for FR decommissioning to return fuel to the stock of FR-derived TRU (consistent with the derivation of Eq. A3.6), one arrives at a slightly different formula for CR:

$$CR_{m<0} = 1 + \frac{(1 + FW)(1 + m)^{\tau_r + 1} + (\tau_{R,FR} - 1)m - 1}{B_{FR}/L_{FR}}$$

Eq. A3.15

Of course in the limit $m \to 0$, $CR_{m<0} = CR_{m>0} = 1 + FW_{FR}/BF_{FR}$.

The associated iterative solution for $m_{<0}$ is,

$$m_{<0} = \frac{(CR_{m<0} - 1)(B_{FR} / L_{FR}) - FW - (1 + FW)\left((1 + m)^{\tau_r + 1} - (\tau_F + 1)m\right)}{\tau_F + \tau_{R,FR} + FW(\tau_F + 1)}$$

Eq. A3.16
Appendix 4: Equations for Stocks and Flows of Plutonium and Minor Actinides Associated with Fusion-Fission Hybrid Systems

A4.1 TRU Fueling Flow to Fusion-Fission Hybrids
The fusion-fission hybrid (FFH) system described by Stacey (2009) uses a modest fusion system, producing 180 – 240 MW of fusion power, to drive a sub-critical fast reactor producing 3000 MWth output power by burning TRU. Since this system should be capable of producing ~1 GWe, the consumption of TRU is $B_{FFH} = 1.096t/GWe-yr$. There is no concomitant production of TRU, since no fertile material is included in the fuel loading. The calculated burn-up fraction (Sommer et al., 2010) is $BF_{FFH} \sim 23.8\%$, from which the total input load of TRU per GWe-yr can be calculated at $L_{FFH} = 4.605t$. The residence time of fuel in the system for this burnup is 2800 full-power days. Taking into account a reasonable duty factor this corresponds to $\tau_{R,FFH} \sim 9$ years.

A4.2 TRU Stock
Using the same simplified model for the TRU stock in FFH systems as in FRs, we have stock at mid-year in each FFH of

$$\tau_{R,FFH}L_{FFH} - BF_{FFH}L_{FFH}/2.$$

A4.3 TRU Unload Flow from Fusion-Fission Hybrids
The amount of TRU unloaded after 1 GWe-yr of production is just $L_{FFH}(1 - BF_{FFH})$. It is assumed that when an FFH is decommissioned, and another is commissioned, the net unload flow is not affected. When a net FFH system is decommissioned, its stock of TRU is returned to the pool of TRU.

A4.4 Processing Losses
As with FRs, processing losses are assumed to be 1%.

A4.5 Evolution Equations for Stocks and Flows
These equations are analogous with the FR equations of Appendix 3.

The total fueling flow needed for the FFHs on Jan 1 of year $n$ is given by

$$F_{FFH}^{tot}(n) = L_{FFH}P_{FFH}(n) + (\tau_{R,FFH} - 1)L_{FFH}\left[P_{FFH}(n) - P_{FFH}(n-1)\right]$$

Eq. A4.1

whereas the total fueling flow available from prior operation of FFHs is given by
\[ F_{FFH}^F(n) = \frac{L_{FFH} P_{FFH}(n - \tau_F - 1)}{1 + F_w} \left[ (1 - BF_{FFH}) P_{FFH}(n - \tau_F - 1) + (\tau_{R,FFH} - 1) \max [P_{FFH}(n - \tau_F - 1) - P_{FFH}(n - \tau_F), 0] \right] \]  

Eq. A4.2

including the source from net decommissioning of fusion-fission hybrids. (For simplicity we use the same symbol, \( \tau_F \), for the residence-time of the fuel in the reprocessing system as for FRs.) As with FRs, the fueling flow from the stock of LWR used nuclear fuel is just the difference between these.

To get to the full evolution equation, the additional contribution to \( TRU_{UNF} \) from LWRs must be included, as must loss to waste. Furthermore, the \( TRU_{UNF} \) must be reprocessed and fabricated into fuel, requiring an assumed period of 1 year. Taking these in account, we have, for the case of FFH systems, with no FR systems (we do not consider mixing the two):

\[
TRU_{UNF}(n) = TRU_{UNF}(n - 1) \\
- (1 + F_w) L_{FFH} \left[ P_{FFH}(n + 1) + (\tau_{R,FFH} - 1) \max [P_{FFH}(n + 1) - P_{FFH}(n), 0] \right] \\
+ L_{FFH} (1 - BF_{FFH}) P_{FFH}(n - \tau_F) \\
+ (\tau_{R,FFH} - 1) L_{FFH} \max [P_{FFH}(n - \tau_F) - P_{FFH}(n - \tau_F + 1), 0] \\
+ TRU_{p,LWR} P_{LWR}(n - 1) + TRU_{LWR} \max [P_{LWR}(n - 1) - P_{LWR}(n), 0] 
\]

Eq. A4.3

The total stock of TRU in the FFH system at any time, \( n \), is given by the sum of the TRU in the FFHs, plus the total fueling for year \( n + 1 \), multiplied by \((1 + F_w)\), plus all \( FFH \rightarrow FFH \) fueling in process for other years.

\[
TRU_{FFHS}(n) = L_{FFH} \left[ \tau_{R,FFH} - BF_{FFH} / 2 \right] P_{FR}(n) \\
+ (1 + F_w) L_{FFH} \left[ P_{FFH}(n + 1) + (\tau_{R,FFH} - 1) \max [P_{FFH}(n + 1) - P_{FFH}(n), 0] \right] \\
+ L_{FFH} (1 - BF_{FFH}) \sum_{m=n+1-\tau_F}^{n-1} P_{FFH}(m) 
\]

Eq. A4.4

where we are not including the possibility of net reduction of FFHs over time, since that does not occur in the calculations shown here, nor are we allowing for the case of overproduction from FFHs, where the TRU unload from year \( n - \tau_F - 1 \) is greater than \((1 + F_w)\) times the loading requirement for year \( n \), also not a case considered here.

The flow of TRU to the waste stream is easily evaluated as \( F_w \) times the total flow to fueling FFHs.
\[ TRU_w(n) = TRU_w(n-1) + F_w L_{FFH} \left\{ P_{FFH}(n) + \left( \tau_{R,FFH} - 1 \right) \left[ P_{FFH}(n) - P_{FFH}(n-1) \right] \right\} \] Eq. A4.5

again assuming no net decommissioning of FFH systems during the time of calculation.

Conservation equations can be derived to provide numerical checks, analogous to those for FRs:

\[ TRU_{UNF}(N) - TRU_{UNF}(0) = TRU_{P,LWR} \sum_{n=0}^{N-1} P_{LWR}(n) + TRU_{LWRc} [P_{LWR}^\text{max}(1,N) - P_{LWR}(N)] \]

\[ - (1 + F_w) L_{FFH} \left\{ \sum_{n=1}^{N} P_{FFH}(n+1) + \left( \tau_{R,FFH} - 1 \right) P_{FFH}(N+1) \right\} + L_{FFH} \left( 1 - BF_{FFH} \right) \sum_{n=1}^{N-\tau_F} P_{FR}(n) \]

Eq. 4.6

\[ TRU_{FFHS}(N) = (1 + F_w) L_{FFH} \left\{ \sum_{n=1}^{N} P_{FFH}(n+1) + \left( \tau_{R,FFH} - 1 \right) P_{FFH}(N+1) \right\} \]

\[ - L_{FFH} \left\{ (1 - BF_{FFH}) \sum_{n=1}^{N-\tau_F} P_{FFH}(n) + \sum_{n=1}^{N-1} BF_{FFH} P_{FFH}(n) + 0.5 BF_{FFH} P_{FFH}(N) + F_w \left[ \sum_{n=1}^{N} P_{FFH}(n) + \left( \tau_{R,FFH} - 1 \right) P_{FFH}(N) \right] \right\} \]

Eq. 4.7

These and additional numerical checks confirm the self-consistency of the given solutions for FFH systems.

**A4.6 Decay Rate of Fusion-Fission Hybrid Systems with Zero TRU Input**

Since the FFH systems described here do not produce net positive amounts of TRU, there is no analogous case to the maximum growth rate without TRU input that was considered above for FRs. However, there clearly is a decay rate of the FFH system in which individual FFH reactors are turned off as waste is burned, in just such a manner that the fuel emerging from the TRU stock at all times is just what is needed for each future year, allowing for FFH decommissioning to return fuel to the stock of FFH-derived TRU (consistent with the derivation of equation A4.3).

Starting from equation A4.3, we can solve for \( BU_{FFH} \):

\[ BF_{FFH} = 1 - (1 + F_w) (1 + m)^{\tau_F + 1} - \left( \tau_{R,FFH} - 1 \right) m \] Eq. 4.8

which has the physically intuitive limit as \( m \to 0 \) of \( BU_{FFH} = -F_w \)

We can also form an iterative solution for \( m \):

\[ m = \frac{-BF_{FFH} - F_w - \left( 1 + F_w \right) \left( \tau_F + 1 \right)} {\left( 1 + F_w \right) \tau_F + \tau_{R,FFH} + F_w} \] Eq. 4.9
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