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1 Supplementary Materials

- Text S1
- 3 Figure and table captions
- 4 Figures S1 to S7
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6 Text S1

7 River profile analysis

8 For graded, steady-state river profiles, where the rock uplift rate, *U*, is balanced by the long term 9 erosion rate, *E*, the relationship between local channel slope, *S*, and the upstream drainage area, *A*, can 10 be described by a power function (Flint, 1974):

$$S = k_s A^{-\theta} \tag{S1},$$

11 where k_s is the channel steepness index, θ is the channel concavity index (Kirby and Whipple, 2012). The 12 covariation of k_s and θ requires normalization that is typically done by fixing θ to a reference value (i.e., 13 reference concavity index, θ_{ref}) that is ~0.3-0.7 globally to calculate the normalized channel steepness 14 index k_{sn} (Kirby and Whipple, 2012; Lague, 2014; Harel et al., 2016). Equation S1 can be used to derive k_s 15 or k_{sn} empirically from regression through *S* and *A* data. However, such calculation introduces noise and 16 requires a large amount of smoothing (Snyder et al., 2000; Wobus et al., 2006). Thus, it is preferable to 17 use χ , a path-dependent integral parameter of the inverse of *A* raised to θ_{ref} (Royden and Perron, 2013):

$$\chi = \int_{x_b}^{x} \left(\frac{A_0}{A(x')}\right)^{\theta_{ref}} dx'$$
(S2),

where x_b is the referenced distance at the drainage network outlet, and A_0 is the referenced upstream drainage area, usually chosen as unity ($A_0 = 1$). Equation S1, where S = dz/dx, can be integrated with respect to distance to generate a χ -elevation plot (typically referred to as a χ -plot):

$$z(x) = z(x_b) + k_{sn}\chi$$
(S3),

where the slope of a linear regression of $\chi - z$ is k_{sn} . We calculate χ by integrating A or drainage area weighted by the spatial distribution of mean annual precipitation, $MAP \times A$, in equation S2, using ChiProfiler (Gallen and Wegmann, 2017). We use χ and the precipitation-weighted χ to calculate k_{sn} and

 k_{sn0} (cf. Adams et al., 2020), respectively, via linear regressions through basin-wide χ -z data (Equation 24 S3; Fig. 1, inset in the main text). We calculate the R^2 for each of these linear regressions to determine 25 the basin morphological steady state, where higher R^2 values reflect basins that appear to be 26 27 morphologically in steady state (i.e., roughly linear χ -z plots; e.g., Fig. 1, inset in the main text), and low R^2 values reflect basins that are in a transient state of adjustment. We seek to avoid these transient basins 28 29 because many bedrock rivers in them adjust their width in addition to slope during channel adjustment 30 and affect assumed scaling relationships in the calculations above to bias the relationship between E and 31 k_{sn} in equation 1 in the main text (e.g., Whittaker et al., 2007). We conduct this analysis for all basins with 32 published ¹⁰Be cosmogenic nuclide basin averaged erosion rates in detrital quartz from the Octopus 33 archive (Codilean et al., 2018, 2022).

34 Binning, regressions, and sensitivity analyses

35 We use existing global climate data of mean annual precipitation (MAP), mean annual temperature (MAT) (Fick and Hijmans, 2017), aridity index (AI=MAP/Mean annual evapotranspiration) 36 37 (Trabucco and Zomer, 2009), and SRTM 3-arc second digital elevation model (DEM) tiles (ME) (OpenTopography) to calculate mean climate proxy values for each basin (Figs. 1 in the main text, S1; 38 39 Table S1). We convert all global data to rasters, project them to WGS 84 geographic coordinate system, 40 and crop them to basin geometries using Arc Pro 2.0.8. We project the basin rasters to UTM coordinate system, resample them to 90 m cell size, calculate their mean MAP, MAT, AI, and ME values using Matlab 41 and TopoToolbox (Schwanghart and Scherler, 2014), and compile this data with E, k_{sn} , and k_{snQ} data for 42 each basin (Table S1). To control for bedrock variability, we restrict our analysis to bedrock rivers that 43 44 drain \geq 90% crystalline rocks (plutonic and metamorphic units) based on a global composite geological map (Fig. S2A; 'GLiM'; Hartmann and Moosdorf, 2012). We calculate the distribution and dominance of 45 plutonic and metamorphic units and compare them to MAP, MAT, AI, and ME in each basin to ensure that 46 47 there is no global relationship between rock type and climate variability (Fig. S2B; Table S1).

We bin the $E - k_{sn}$ and $E - k_{sn}Q$ datasets based on increments of MAP, MAT, AI, and ME (Fig. S3). We select the bins to have an equal number of data points, with at least 15 data points in each bin. For each climate bin, we conduct a linear regression through log-transformed $E - k_{sn}$ and $E - k_{snQ}$ data using total least squares to determine the power-law exponent, p, and its constant, C, in equation 1 (main text) (Fig. S4). We assess the relationships between these parameters and the climate proxy data to evaluate if systematic patterns exist (Fig. S4). We attempt to account for the general nonlinearity (i.e., p) in the global dataset by conducting normalized regressions through the data under a fixed $p \sim 2.1$ (i.e., the 55 global value of our dataset; cf. Harel et al., 2016; Adams et al., 2020) to determine the normalized yintercept, C_{ne} and C_{ne0} , for the $E - k_{sn}$ and $E - k_{sn0}$ data sets, respectively (Fig. S5; Table S2). For all 56 57 modeled regressions, we calculate the statistical goodness-of-fit metrics of r-square, chi-square, and 58 Kolmogorov-Smirnov two-sided p-value at the 90% significance level to evaluate the significance of p and C_{ne} in our analysis (Table S2). We calculate the uncertainties for p and C in each regression by conducting 59 60 10⁵ Monte Carlo simulations, where each simulation is for a set of random realizations from E and k_{sn} or E and k_{snO} and their standard error using a truncated normal distribution based for each data point (Figs. 61 62 S3, S5; Table S2).

We conduct several sensitivity analyses to evaluate the robustness of p in our modeled 63 64 regressions, namely testing the impact of the number of bins with an equal number of data points per bin 65 (e.g., for the dataset in Table S1, 4 bins with 107 points per bin, 6 bins with 71 points per bin, 8 bins with 53 points per bin, etc.; Fig. S6A); testing for the impact of transience in our basin analysis by changing the 66 minimum R^2 morphological threshold value from $R^2 = 0.75$ to $R^2 = 0.95$ (Figs. 1, inset in main text, S6B; 67 Table S2); and testing for the impact of the chosen reference concavity index, by calculating k_{sn} under 68 $\theta_{ref} = 0.3, 0.4, \text{ and } 0.5$ (equation S2, S3; Fig. S6C). Generally, we find that p is robust to the number of 69 bins and changes in θ_{ref} and that the increase of R^2 improve the overall fit for the modeled regressions 70 71 (Table S2).

72 Threshold stochastic stream power incision models (STIMs) and discharge variability

Bedrock channel incision rate is often modeled as a function of the magnitude of the shear stress (or stream power) exerted on a river bed (Howard, 1994). Approximations of this general concept simulate the instantaneous channel incision rate, I^* , as a function of the channel slope, S, raised to an exponent n, upstream drainage area, A, raised to an exponent m, and an erodibility coefficient, K, which captures rock type, climate and changes in channel hydraulic geometry, and a term for threshold for channel incision, ψ (Lague et al., 2005; Lague, 2014):

$$I^* = KA^m S^n - \psi \tag{S4},$$

where $\psi = k_e \tau_c^{\ a}$, in which k_e and a are constants that depend on substrate properties and τ_c is the critical shear stress for channel incision. Assuming that ψ is negligible and that $I^* \cong E$ in steady-state basins (i.e., the instantaneous channel incision rate is steady over time and in equilibrium with the longterm basin-averaged erosion rate), equation S4 is reduced to the constant effective discharge stream power incision model (SPIM) solution (Howard, 1994; Kirby and Whipple, 2012; Lague, 2014):

$$E = KA^m S^n \tag{S5}.$$

However, when the second right hand term in equation S4 is significant, the critical discharge needed to overcome the threshold shear stress, Q_c (which is typically defined by the effective bedload grain size), and the distribution of floods are important. Stochastic threshold stream power incision models (i.e., STIM) account for this via the calculation of E as the integral of the product of I (equation S4) and the probability of threshold-breaching floods (i.e., floods large enough to generate shear stress capable of exceeding ψ) for specific normalized discharges, $pdf(Q^*)$:

$$Q^* = Q/\overline{Q} \tag{S6}$$

$$E = \bar{I} = \int_{Q_c}^{Q_{max}} I^*(Q^*) p df(Q^*) dQ$$
 (S7),

where \overline{I} is the river erosion rate, and Q, \overline{Q} , Q^* , and Q_{max} are the actual, mean, normalized, and maximum discharges, respectively (Lague et al., 2005; DiBiase and Whipple, 2011). Lague et al. (2005) present a model where $pdf(Q^*)$ is represented by an inverse gamma distribution:

$$pdf(Q^{*}) = \frac{k^{k+1}}{\Gamma(k+1)} exp\left(-\frac{k}{Q^{*}}\right) Q^{*-(2+k)}$$
(S8),

where Γ is the inverse gamma function, and k is a shape parameter that describes discharge variability.
In this model, low to high k reflect heavier-tailed, higher-variability flood distributions to lighter-tailed,
lower-variability flood distributions (Lague et al., 2005; Lague, 2014).

96 This model predicts that in threshold-dominated bedrock river systems, the nonlinearity between E and k_{sn} (equation 1 in main text) systematically increases with decreasing discharge variability (i.e., 97 98 higher k in equation S8; Figure 4 in main text). This behavior arises because for steeper channels, smaller 99 magnitude floods are capable of overcoming incision thresholds, while for shallow to moderate grade 100 channels, small floods are less effective, allowing only larger floods to overcome bedrock incision 101 thresholds (Lague et al., 2005; DiBiase and Whipple, 2011; Deal et al., 2018). The integral of discharge 102 events that breach this incision threshold is related to the erosional efficiency in a STIM framework, where 103 more threshold breaching events increase erosional efficiency (erosion rate at a given slope and drainage 104 area). Thus, as channel steepness increases, the flood size needed to breach thresholds declines, and more 105 erosive floods are included in a lighter-tailed, lower-discharge flood distribution relative to a heavier-106 tailed, higher-discharge flood distribution system.

107 To empirically determine k from discharge records, it is easier to use the complementary 108 cumulative distribution function, $ccdf(Q^*)$, to avoid binning complexities when comparing actual 109 discharge data (DiBiase and Whipple, 2011):

$$ccdf(Q^*) = \Gamma(k/Q^*, k+1)$$
(S9).

110 Empirical studies and theory suggest k to systematically increase with increasing MAP and AI 111 (Lague, 2014; Rossi et al., 2016; Deal et al., 2018). To demonstrate this general pattern at the global scale, 112 we use equation S9 along with several discharge records near some of our studied basins to empirically 113 determine the shape parameter k for each of these stations and compare it with MAP and AI patterns. 114 We gather mean daily discharge records of ~20-50 years from several gauges near some of our analyzed 115 basins that span a large range of climate conditions to calculate their exceedance probability plots (Fig. 116 S7A; see locations in Figs. 1 in main text, S1). We calculate similar exceedance probability plots using Lague 117 et al.'s STIM and equation S9 to verify that STIM predictions generally fit the recorded data (Fig. S7B), and 118 that predicted k values are consistent with MAP and AI patterns at these gauge stations (Fig. 4 in main 119 text; Fig. S7C; DiBiase and Whipple, 2011; Lague, 2014; Deal et al., 2018).

120 Figure and table captions

121 Figure S1: Global climate rasters used for our analysis. (A) Mean annual precipitation (MAP; 'WorldClim 2'; Fick and Hijmans, 2017) (B) Mean annual temperature (MAT; 'WorldClim 2'; Fick and 122 123 Hijmans, 2017); (C) Aridity index (AI; 'CGIAR-CSI'; Trabucco and Zomer, 2009); (D) Elevation (ME; 124 'ETOPO1'; Amanter and Eakins, 2009). Marked are locations of analyzed basin regions (red stars). IDH = 125 Idaho, USA; DEN = Denver, USA; App = Appalachians, USA; SGM = San Gabriel Mountains, USA; GUA = 126 Guatemala; TRI = Trinidad and Tobago; AND = Chilean/Bolivian Andes; BRA = Florianopolis, Brazil; ALPS = 127 European Alps; SAF = South Africa; KEN = Kenya; NAM = Namibia; MAD = Madagascar; SIND = Southwest 128 India; HIM = Himalayas; MYA = Myanmar; CHE = Chengdu, China; TAW = Taiwan; JAP = Japan; AUS = 129 Australia.

Figure S2: (A) Global rock type data (after 'GLiM'; Hartmann and Moosdorf, 2012) classified by crystalline, volcanic, and sedimentary rocks, where crystalline rocks consist of plutonic and metamorphic units. All analyzed basins in this study consist of at least 90% crystalline units. (B) Binned data of the percentages of plutonic and metamorphic units for 428 basins under a steady state threshold of $R^2 > 0.9$, versus MAP, MAT, AI, and ME (Table S1). At the global scale, no relationship is observed between changes in the percentage of plutonic or metamorphic units and climate. **Figure S3:** (A) Modeled regressions for k_{sn} versus *E* (equation 1 in main text) for 428 basins under a steady state threshold of $\mathbb{R}^2 > 0.9$, where each data point represents one basin. Data is classified and binned by (from top left clockwise) MAP, MAT, AI, and ME (see Table S1). Statistical goodness-of-fit parameters (R-square, chi-square, KS test p-value) for each climate bin modeled regression are summarized in Table S2. (B) Same as (A) but for k_{snQ} , where MAP across the basin is weighted in drainage area prior to calculation of χ (see text). Regressions with low or negative R-square or KS test p-value < 0.1 are dashed (see also Table S2). Inset figures show log-log plots of the main figures.

Figure S4: (A) Values of p in equation 1 for each modeled regression for k_{sn} and k_{snQ} under changes in MAP, MAT, AI, and ME. Note a systematic increase in p with increasing MAP (i.e., wetter) and AI (i.e., higher humidity). (B) Same as (A) but for C in equation 1. Note a systematic decrease in C with increasing MAP and AI. For all regressions, k_{snQ} does not significantly change systematic patterns. Most uncertainties in p and C are small and thus not visible.

Figure S5: (A) Modeled normalized regressions for k_{sn} versus E (equation 1 in main text) under a fixed p = 2.1, which is the global value from best fit regression through the entire $E - k_{sn}$ dataset. Regressions and associated data points with low or negative R-square or p-value < 0.1 are dashed (see also Table S2). (B) Changes in C_{ne} and C_{neQ} under a fixed p = 2.1. Note that C_{ne}/C_{neQ} decreases over a small range ~1-8 X 10⁻⁹yr⁻¹ with all climate proxies. Inset figures show log-log plots of the main figures.

Figure S6: Sensitivity analyses for modeled regressions under **(A)** changes of the number of bins; **(B)** changes of the R^2 threshold used to define morphological steady state (e.g., Figure 1, inset in main text); **(C)** changes in θ_{ref} . *p* (and hence *C* that covary with it; Figure S4) is statistically robust to changes in MAP and AI under different number of bins, R^2 threshold values, and θ_{ref} values. Inset figures in (A) and (B) show all *p* values (i.e., also ones that are > 5).

Figure S7: (A) Exceedance probability plots of recorded mean daily discharge (m³/s), Q (equation S6), from six stream gauge stations with records spanning ~20-50 yrs near some of our analyzed basins (for locations, see Figs. 1 in main text, S1). (B) Exceedance probability plots following equation S9, where low to high k represents high to low discharge variability (Lague et al., 2005; DiBiase and Whipple, 2011; Deal et al., 2018). Note a general systematic decrease in discharge variability (higher k) with increasing MAP and AI. (C) Scatter plot of MAP and AI as a function of calculated k in (B).

164 **Table S1:** Locations, climate proxy values, erosion rates, k_{sn} , $k_{sn}Q$, basin area, and percentage of 165 crystalline units for 428 analyzed basins under a morphological steady-state threshold of $R^2 > 0.9$.

- 166**Table S2:** p, C, C_{ne} and statistical goodness-of-fit metrics of R-square, chi-square, and167Kolmogorov-Smirnov two-sided p-value at the 90% significance level for six climate bins of MAP, MAT, AI,168and ME under different R^2 morphological steady-state thresholds from 0.75 to 0.95. Marked in red are
- low or negative r-square values or Kolmogorov-Smirnov p-value < 0.1 of poorly fit regressions.

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90°E

Hot

90°E

120°E

150°E

IAP

TAW

120°E

150°E



Metamorphic percentage (%)









