# Climate policy and wealth distribution

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# Climate policy and wealth distribution*

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**Abstract**

We set up a model with inter-generational bequest transfers and a climate damage on the wealth of heterogeneous households. We show that, under the imperfections of credit markets and depending on the wealth distribution across households, a balanced budget climate policy may widen the wealth inequality between the rich and the poor class. A climate policy may create positive effects on the wealth of households but these effects are asymmetric across households in terms of both magnitudes and the transmissions of the gains from climate policy within households. The poor's gains from the climate policy are mainly transmitted into improving the living standard and then to invest in human capital because of the higher marginal return to education investment. In contrary, the rich's gains from the climate policy are transmitted biasedly into physical capital accumulation and enhance their monopoly position in producing intermediate inputs. We show that, for any climate policy, there exists a corresponding threshold of aggregate physical capital. When the aggregate physical capital of the economy exceeds this threshold then the corresponding climate policy may widen the inter-generational bequest transfers among heterogeneous households, therefore, contributing enlarge the wealth gap between the rich and the poor class in the long run.

**JEL**: D62, D63, O15, Q52, Q54.

**Keywords**: Climate policy, balanced budget policy, imperfections of credit markets, inter-generational bequest transfer, wealth inequality.

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# 1 Introduction

Two issues emerging in the twenty first century are climate change and income/wealth inequality. The Paris Climate Agreement on limiting global warming to well below 2°C above pre-industrial level indeed requires strict transitions to clean economies. These transitions can be implemented through a carbon tax policy in which the government imposes a Pigouvian tax on the dirty production sector, and use the tax revenue to subsidy for the clean production sector (Acemoglu et al. 2012, 2016). Such a climate policy may improve the environmental quality and living conditions for households. However, these effects of climate policy may be asymmetric to heterogeneous households that is need to be investigated. The attention on wealth inequality was addressed since long and has been recently put in the center of debate among economists and social scientists after the publications of "*Capital in the twenty-first century*" by Piketty (2014). However, the important link between climate policy and wealth/income inequality has not been sufficiently investigated in the literature. This paper aims to contribute a theoretical work linking the climate policy and wealth inequality to identify the condition under which a climate policy may widen the wealth gap between the rich and the poor class.

There has been a huge literature considering the optimal paths of global emission and optimal taxation to decentralize such the optimal global emission (Nordhaus 1992, 1993; Nordhaus and Boyer 2000; Pizer 1999; Acemoglu et al. 2012; Golosov et al. 2014; among others). These literature, however, ignore the asymmetric effects of tax policy on the wealth of households in an economy. In addition, while a large body of existing literature focuses on the relationship between climate policy and inequality between countries, the theoretical consideration about the relationship between climate policy and inequality between households are very limited. This paper aims to fill this gap in the literature.

There is also a sizable literature explaining the relationship between wealth/income inequality and development as well as explaining the persistent wealth/income inequality (Galor and Zeira 1993; Galor and Moav 2004, 2006; Piketty 1997; Piketty and Zucman 2014; Piketty et al. 2019; Lakner and Milanovic 2013; Liberati 2015, among many others). These literature stress that the wealth inequality can be persistence due to the imperfections of credit markets and the inter-generational bequest transfers within households. The presence of imperfections of credit markets under the wealth inequality creates a long-lasting effect on investment in human capital and entrepreneurial activities, contributing to the persistence of wealth inequality.

*How is about the link between climate change policy and wealth/income inequality*? Surprisingly, there is quite small literature, particularly from theoretical aspect, on the interaction and dynamics of this link. In an alternative approach, Vasconcelos et al. (2014) study climate policies under wealth inequality in which the authors focus on the conflicting policies between the rich and poor countries, as well as the challenges in achieving the cooperation between the rich and the poor. Blonz et al. (2011) and William III et al. (2014) study empirically the near term effects of climate policy on welfare of heterogeneous households in which they consider such the effects on population with different age groups. Dennig et al. (2015) advance a DICE model to stress the importance of accounting inequality within regions and point out the asymmetric effects of climate policy on welfare of households. Ravallion et al. (2000) empirically show that higher inequality in income, both between and within countries, is associated with lower carbon emission at given average incomes. Grunewald et al. (2011) provide a U-shaped relationship between carbon emission and income inequality. Chancel and Piketty (2015) argue that the rich should be responsible for climate bills because, in terms of consumption-based pollution, they contribute more to global warming.

Differing from the related literature, we consider theoretically in this paper not only the effect of a climate policy on the wealth distribution but also the conditions under which the climate policy will widen the wealth gap between the rich and poor classes, while the poor cannot escape the poverty trap. We set up an overlapping generations model with heterogeneous households under imperfections of credit markets and an climate externality on the wealth of households. In particular, we identify a threshold of aggregate capital corresponding to the each climate policy. Whenever the aggregate physical capital exceeds this threshold then such the climate policy tend to widen the gap in wealth between the rich and poor classes. That is because the gain from the climate policy may be transmitted inter-generationally into human capital investment for the poor or it just purely improves the poor's living condition without inter-generational transfer within a poor household, while it is transmitted biasedly into physical capital accumulation for the rich. The improvement in human capital of the whole economy benefits biasedly for the profits of intermediate producing firms owned by the rich class, probably amplifying the wealth inequality and enhancing physical capital accumulation. The greater physical capital accumulation leads to the more physical capital allocated in producing dirty intermediate inputs, generating more burden for future climate policy.

The rest of paper is organized as follows. Section 2 introduces all components of the benchmark model. Section 3 characterizes the equilibria and the dynamics of the model. The effects of climate policy on macroeconomic variables and the wealth of households are presented in section 4. Section 5 provides some discussion and conclude the paper.

## 2 The benchmark model

We consider a discrete time overlapping generations economy with constant population. Following Acemoglu et al. (2012), we assime that there is one homogeneous final output which is produced out by human capital and intermediate inputs. In each period $t \in \mathbb{N}$, we define $I_t$ and $J_t$ are nonempty sets of adult (or working) individuals/households and intermediate inputs, respectively, i.e. each adult individual $i$ in period $t$ belongs to the set $I_t$ and each intermediate $j$ in period $t$ belongs to the set $J_t$. Each individual lives for two periods as childhood and adulthood. Along with choosing optimal education investment, individuals allocate their incomes (when they are adult), which come from labor income, capital income and monopoly profit, between consumption and bequest transfer for their children so as to maximize their life-time utility.

## 2.1 Final good sector

There is one homogeneous final output which is produced out by human capital and intermediate inputs under the following (aggregate) production function[1]

$$Y_t = H_t^{1-\alpha} \left( \int_{J_t} a_{cj}^{1-\alpha} x_{cjt}^{\alpha} dj + \int_{J_t} a_{dj}^{1-\alpha} x_{djt}^{\alpha} dj \right); \quad \alpha \in (0,1) \tag{1}$$

where $Y_t$ is aggregate final output and $H_t$ is aggregate human capital employed in final good production in period $t$, i.e.

$$H_t = \int_{I_t} h_t^i di$$

in which $h_t^i$ is human capital of individual $i \in I_t$.

The subscripts "$c$" and "$d$" denote for "clean" and "dirty" respectively. So the $x_{cjt}$ and $x_{djt}$ are the amounts of the clean and dirty inputs $j \in J_t$ employed in the final good production. The $a_{cj} > 0$ and $a_{dj} > 0$ are quality (or productivity) indexes of the corresponding intermediate inputs $j \in J_t$.

Suppose that the final good sector operates under the perfectly competitive environment. The profit maximization problem of producing firms in this sector is

$$\max_{H_t, \{x_{vjt}\}_{vj \in \{c,d\} \times J_t}} H_t^{1-\alpha} \sum_{v \in \{c,d\}} \int_{J_t} a_{vj}^{1-\alpha} x_{vjt}^{\alpha} dj - w_t H_t - \sum_{v \in \{c,d\}} \int_{J_t} p_{vjt} x_{vjt} dj$$

where $w_t$ is return on human capital and $p_{vjt}$ is price of intermediate input $vj \in \{c,d\} \times J_t$ in the period $t$. The first-order conditions with respect to $w_t$ and $p_{vjt}$ give us

$$w_t = \frac{1-\alpha}{H_t^{\alpha}} \sum_{v \in \{c,d\}} \int_{J_t} a_{vj}^{1-\alpha} x_{vjt}^{\alpha} dj \tag{2}$$

$$p_{vjt} = \alpha H_t^{1-\alpha} a_{vj}^{1-\alpha} x_{vjt}^{\alpha-1}; \tag{3}$$

## 2.2 Intermediate sectors and climate policy

For the sake of simplicity, we assume that each intermediate input indexed by $vj \in \{c,d\} \times J_t$ is produced in period $t$ according to the following production function

[1] Such a final good production function is introduced extensively in Aghion and Howitt (2009).

$$x_{vjt} = k_{vjt} \tag{4}$$

where $k_{vjt}$ is amount of physical capital used as input in the intermediate sector $vj$. We assume that the physical capital fully depreciates in each period $t$ of use. So the cost of producing $x_{vjt}$ units of intermediate input $vj$ in period $t$ is $r_t k_{vjt}$, where $r_t$ is the rental rate of physical capital in period $t$. The producer of the intermediate good $vj$ in period $t$ decides the quantity $x_{vjt}$ to be produced so as to maximize its monopolist profit. Therefore, the monopolist profit of the entrepreneur $vj$ in period $t$ is

$$\pi_{vjt} = \max_{k_{vjt}} (1 - \tau_{vt}) p_{vjt} k_{vjt} - r_t k_{vjt} \tag{5}$$

given $r_t$ and $\tau_{vt}$, where $\tau_{vt} < 1$ is Pigouvian tax rate (or subsidy if negative) imposed by the government on the production of each intermediate good $v \in \{c, d\}$ in period $t$. These tax rates represent the climate policy of the government. In this paper we will consider the climate policy in which $\tau_{dt} \in [0, 1)$ and $\tau_{ct} \leq 0$, i.e. in any period $t$ the government impose a Pigouvian tax rate $\tau_{dt} \in [0, 1)$ on the production of dirty intermediate sectors and use this tax revenue to subsidy for the production of clean sector at a rate $-\tau_{ct} \geq 0$. The extreme values $(\tau_{ct}, \tau_{dt}) = (0, 0)$ implies the case no any climate policy is carried out.

Substituting (3) and (4) into (5) we have

$$\pi_{vjt} = (1 - \tau_{vt}) \alpha H_t^{1-\alpha} a_{vj}^{1-\alpha} k_{vjt}^{\alpha} - r_t k_{vjt} \quad \text{with} \quad k_{vjt} \in \arg\max_{k_{vjt}} (1 - \tau_{vt}) \alpha H_t^{1-\alpha} a_{vj}^{1-\alpha} k_{vjt}^{\alpha} - r_t k_{vjt}$$

Hence,

$$k_{vjt} = \left[ \frac{\alpha^2 (1 - \tau_{vt})}{r_t} \right]^{\frac{1}{1-\alpha}} H_t a_{vj} \tag{6}$$

From (6), we can compute the aggregate physical capital in period $t$ is

$$K_t = \sum_{v \in \{c,d\}} \int_{J_t} k_{vjt} dj = \left( \frac{\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} H_t \sum_{v \in \{c,d\}} (1 - \tau_{vt})^{\frac{1}{1-\alpha}} A_v \tag{7}$$

where

$$A_v = \int_{J_t} a_{vj} dj$$

is defined as the aggregate quality/productivity of all intermediate inputs type $v \in \{c, d\}$ in the intermediate input set $J_t$.

From (7), the rental rate of capital is determined by

$$r_t = \alpha^2 \left[ \frac{H_t}{K_t} \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} \tag{8}$$

Let substitute (8) into (6) we determine the allocation rule of physical capital across intermediate sectors $vj$ in period $t$

$$k_{vjt} = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}} a_{vj}}{\sum_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} K_t \tag{9}$$

The monopolist profit $\pi_{vjt}$ is determined by substituting (9) and (8) into (5), that is

$$\pi_{vjt} = \alpha(1-\alpha) H_t^{1-\alpha} (1-\tau_{vt})^{\frac{1}{1-\alpha}} a_{vj} \left( \frac{K_t}{\sum_{v' \in \{c,d\}} (1-\tau_{v't})^{\frac{1}{1-\alpha}} A_{v'}} \right)^{\alpha}$$

And the aggregate monopolist profit in period $t$ is

$$\Pi_t = \sum_{v \in \{c,d\}} \int_{J_t} \pi_{vjt} dj = \alpha(1-\alpha) \left[ H_t \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{1-\alpha} K_t^{\alpha} \tag{10}$$

By substituting (9) into (2), with a note that $x_{vjt} = k_{vjt}$ for all $v \in \{c,d\}$ and $j \in J_t$, we can determine the return on human capital is

$$w_t = (1-\alpha) \frac{\sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v}{\left[ \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right]^{\alpha}} \left( \frac{K_t}{H_t} \right)^{\alpha} \tag{11}$$

The government balanced budget constraint requires

$$\sum_{v \in \{c,d\}} \tau_{vt} \int_J p_{vjt} x_{vjt} dj = 0 \quad \Longleftrightarrow \quad \sum_{v \in \{c,d\}} \tau_{vt} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0 \tag{12}$$

**Lemma 1.** *Under the government balanced budget constraint of the climate policy $(\tau_{ct}, \tau_{dt}) \in \Re_- \times [0,1)$, the following relation holds*

$$\sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v = \sum_{v \in \{c,d\}} (1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v$$

*Proof.*

Indeed, from government balanced budget condition (12) we have

$$\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v=\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v+\sum_{v\in\{c,d\}}\tau_{vt}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}}A_v=\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}}A_v$$

□

Under the government balanced budget constraint (12), and from lemma 1, the return on human capital (11) becomes

$$w_t=(1-\alpha)\left[\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v\right]^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha} \tag{13}$$

## 2.3 Dynamics of pollution stock

We assume that only the use of dirty intermediate inputs degrade the environment. So the dynamics of pollution stock is characterized by

$$P_t=\bar{P}+(1-\delta)(P_{t-1}-\bar{P})+\xi\int_{J_t}x_{djt}dj$$

where $P_t$ is pollution stock in period $t$ which can be view as the carbon concentration in the atmosphere. the term $\xi\int_{J_t}x_{djt}dj$ is the flow of pollution in period $t$ coming from the uses of dirty intermediate inputs $j\in J_t$; the $\xi>0$ is dirtiness coefficient of dirty intermediate input. $\bar{P}$ is the natural state of carbon concentration in the atmosphere, i.e. the state of ecological system without any human activity; $\delta\in(0,1]$ is the decay rate of pollution which measures the convergent speed of pollution stock to the natural state $\bar{P}$. For sake of simplicity without loss of any generality we normalize $\bar{P}=0$. Note that $x_{djt}=k_{djt}$ and with the allocation rule of physical capital in (9), the dynamics of pollution stock can be written as follows

$$P_t=(1-\delta)P_{t-1}+\xi\frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}}A_d}{\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}K_t \tag{14}$$

where

$$\frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}}A_d}{\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}K_t=K_{dt}$$

is aggregate physical capital which is allocated in producing intermediate inputs $\{dj\}_{j\in J_t}$.

## 2.4 Individuals/Households

We assume that in each period $t \in \mathbb{N}$ the economy is populated by a constant and continuum population of working individuals who are identical in innate talent and preference. Without lost of any generality, we normalize the size of population by 1. Each working individual in any period $t$ has a single parent and a single offspring, and lives for two periods, say $t-1$ and $t$. In the first period of life, period $t-1$ (say childhood period), individuals spend full time for human capital formation. In the second period of life, period $t$ (say adulthood period), they supply their efficiency units of labor to the market in-elastically, and allocate their wealth (coming from labor income and inheritance from their parents) between consumption and inter-generational bequest transfers to their offspring so as to maximize their utilities. The human capital increases if the time for education is supplemented with capital investment in education. The human capital of adult individual $i \in I_t$, $h_t^i$, is

$$h_t^i = h(e_t^i)$$

where $e_t^i \geq 0$ is capital investment in education for individual $i$ who becomes an adult in period $t$.

**Assumption 1.** $h(0) = 1$, $h'(e) > 0$, $h''(e) < 0$, $\lim_{e \to 0^+} h'(e) = +\infty$ and $\lim_{e \to +\infty} h'(e) = 0$.

The assumption 1 implies that without capital investment in education, each individual is endowed one unit of labor. The human capital formation is an increasing and concave function in education investment and satisfies Inada conditions.

The utility of individual $i \in I_t$ comes from the consumption $c_t^i$ and the bequest to its offspring $b_{t+1}^i$, obeying the following utility function

$$U(c_t^i, b_{t+1}^i) = (1-\gamma)\ln c_t^i + \gamma \ln(\theta_t + b_{t+1}^i); \quad \theta_t > 0 \tag{15}$$

where $\gamma \in (0,1)$ is preference weight towards the individual's offspring.[2] The budget constraint of individual $i \in I_t$ is

$$c_t^i + b_{t+1}^i \leq W_t^i \phi(P_t) \tag{16}$$

where $W_t^i$ is the wealth owned by individual $i \in I_t$. The function $\phi(P_t)$ in (16) is the damage effect of pollution stock $P_t$ in period $t$ and satisfies $\phi'(P_t) < 0$, $\phi(P_t) \in (0,1)$. We assume that the pollution stock in each period damages the wealth of individuals and it plays as a negative externality. We define

$$\psi(W_t^i, P_t) = W_t^i \phi(P_t) \quad \text{and} \quad \psi(w_t, P_t) = w_t \phi(P_t)$$

[2] We follow the utility function from Galor and Moav (2004, 2006) that allows a corner solution for bequest transfer, i.e. $b_{t+1}^i = 0$, when the wealth of individual $i$ is too small.

as respectively the *disposable wealth* and and the *minimal disposable wealth* of an individual $i \in I_t$. Note that the terms $\psi(w_t, P_t)$ are identical across individuals $i \in I_t$ and the term $\psi(w_t, P_t)$ is the wealth which is constituted only by a unit of physical labor endowment of an individual.

The $\ln \theta_t$, which appears in the utility function when $b^i_{t+1} = 0$, is the minimal "expected" utility derived from the offspring of an household. So, the parameter $\theta_t$ can be viewed as the expectation of an individual $i \in I_t$ on the *minimal* disposable wealth of his/her offspring $i \in I_{t+1}$ can own in period $t+1$. This expectation is based on the individual's own *minimal* disposable wealth, i.e. $\psi(w_t, P_t) = w_t \phi(P_t)$. Indeed, the minimal disposable wealth $\psi(w_t, P_t)$ is what an adult individual $i \in I_t$ observes in period $t$. Basing on this observation, he/she assigns an expectation on the minimal disposable wealth that his/her offspring may own in the next period. So, we impose the following assumption

**Assumption 2.** $\theta_t = \theta \psi(w_t, P_t),\ \theta > 0.$

The parameter $\theta$ may change over time depending (endogenously) on other factors that are not captured in the model. However, $\theta$ is not a focus of the model and hence, for simplification without lessening the power of the model, we treat $\theta$ to be a constant.

The utility maximization problem of individual $i \in I_t$ is

$$\max_{c^i_t > 0, b^i_{t+1} \geq 0} (1-\gamma) \ln c^i_t + \gamma \ln(\theta_t + b^i_{t+1})$$

subject to

$$c^i_t + b^i_{t+1} \leq \psi(W^i_t, P_t)$$

given $W^i_t$ and $P_t$.

Solving the optimization problem above of individual $i \in I_t$, we obtain the optimal choice on bequest transfer is

$$b^i_{t+1} = \max\left\{0, \gamma \left[\psi(W^i_t, P_t) - \frac{\theta_t(1-\gamma)}{\gamma}\right]\right\} \tag{17}$$

The inter-generational bequest transfer $b^i_t \geq 0$ that the individual $i$ receives from his/her parent is allocated between education investment, $e^i_t$, and for capital saving, $s^i_t$, which is lent to the capital market in period $t$ to earn capital income. Hence,

$$b^i_t = e^i_t + s^i_t$$

and the wealth of individual $i \in I_t$ is

$$W^i_t = w_t h(e^i_t) + (b^i_t - e^i_t) r_t + \rho^i_t \Pi_t$$

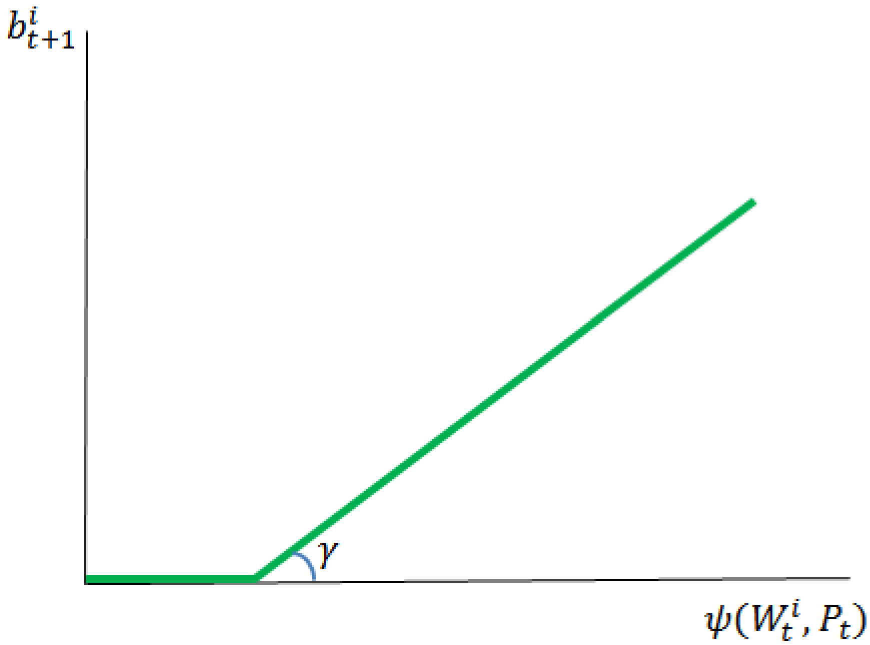


Figure 1: Bequest transfer

in which $w_t h(e^i_t)$ is the labor income, $(b^i_t - e^i_t)r_t$ is the capital income, and $\rho^i_t \Pi_t$ is the income from the monopolist profits. The $\rho^i_t \in (0,1)$ is the share of the monopolist profits assigned for individual $i \in I_t$, hence $\{\rho^i_t\}_{i \in I_t}$ holds

$$\int_{I_t} \rho^i_t di = 1$$

In the absence of borrowing constraints, the rational household $i \in I_t$ will choose $e^i_t$ such that

$$e^i_t \in \arg\max_{e^i_t} \left[ w_t h(e^i_t) + (b^i_t - e^i_t) r_t + \rho^i_t \Pi_t \right] \tag{18}$$

Note that the term $\rho^i_t \Pi_t$ in the optimization problem (18) above does not depend on $e^i_t$. So under the assumption 1 and in the absence of borrowing constraint, the optimal level of education investment is

$$e^i_t = e^*_t \quad \forall i \in I_t \qquad \text{where} \quad h'(e^*_t) = \frac{r_t}{w_t}$$

We assume that individuals cannot borrow for education investment because of the imperfections of credit markets, i.e. the education investment $e^i_t$ of an individual $i$ is limited by the bequest transfer $b^i_t$ that the individual receives from his/her parent. So, the education investment is stated in the following proposition

**Proposition 1.**

*(i) Under the imperfections of credit markets, the optimal education investment $e_t^i$ of individual $i \in I_t$ depends on the transfer bequest $b_t^i$ that the individual receives from parent, in particular*

$$e_t^i = \min\left\{b_t^i, e_t^*\right\}$$

*(ii) For any given strictly positive aggregate stock of bequest transfer $b_t = \int_{I_t} b_t^i di > 0$, then $e_t^* > \underline{e}_t$ where $\underline{e}_t \in (0, b_t)$ is a unique solution to*

$$h'(e_t) = \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t}$$

*Moreover, $\underline{e}_t = \underline{e}(b_t)$ is an increasing function of $b_t$.*

*Proof.*

(i) Indeed, the existence of optimal solution $e_t^i$ is guaranteed by the compactness of the domain set, $0 \le e_t^i \le b_t^i \ \forall i \in I_t$, and continuity in $e_t^i$ of the objective function. When $b_t^i = 0$, it is trivially that $e_t^i = 0$. When $b_t^i \in (0, e_t^*]$, suppose a contradiction that $e_t^i < b_t^i$ and we will prove that there exists $\varepsilon \in (0, b_t^i - e_t^i)$ such that

$$w_t h(e_t^i + \varepsilon) + (b_t^i - e_t^i - \varepsilon) r_t > w_t h(e_t^i) + (b_t^i - e_t^i) r_t$$

which is equivalent to

$$\frac{h(e_t^i + \varepsilon) - h(e_t^i)}{\varepsilon} > \frac{r_t}{w_t} \quad \Longleftrightarrow \quad h(e_t^i + \varepsilon) - h(e_t^i) > \varepsilon h'(e_t^*)$$

since $h'(e_t^*) = r_t / w_t$. The last inequality trivially holds because of the increasing and concavity of function $h(e)$ and $e_t^i + \varepsilon < e_t^*$. Hence, in this case $e_t^i < b_t^i$ is not the optimal education investment. Therefore, $e_t^i = b_t^i$ when $b_t^i \in [0, e_t^*]$.

When $b_t^i > e_t^*$ then the optimal education investment is set the level which equalizes the marginal return of education investment and marginal capital income, i.e. $w_t h'(e_t^i) = r_t$, which gives us $e_t^i = e_t^*$.

(ii) We have

$$h'(e_t) = \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t} \quad \Longleftrightarrow \quad G(b_t, e_t) = h'(e_t) - \frac{\alpha^2}{1-\alpha}\frac{h(e_t)}{b_t - e_t} = 0 \tag{19}$$

Under the assumption 1,

$$G_e(b_t, e_t) = h''(e_t) - \frac{\alpha^2}{1-\alpha}\frac{h'(e_t)(b_t - e_t) + h(e_t)}{(b_t - e_t)^2} < 0$$

i.e. $G(b_t, e_t)$ is monotonically decreasing in $e_t \in (0, b_t)$. In addition, $\lim\limits_{e_t \to 0^+} G(b_t, e_t) = +\infty$ and

$\lim_{e_t \to +\infty} G(b_t, e_t) = -\infty$. Therefore, there exists a unique $\underline{e}_t \in (0, b_t)$ satisfying (19).

By applying the implicit function theorem for the function $G(b_t, \underline{e}_t) = 0$ we have, $\underline{e}_t$ is a function of $b_t$, i.e. $\underline{e}_t = \underline{e}(b_t)$, in which

$$\underline{e}'(b_t) = \frac{\alpha^2 h(\underline{e}_t)}{\alpha^2[h'(\underline{e}_t)(b_t - \underline{e}_t) + h(\underline{e}_t)] - (1-\alpha)(b_t - \underline{e}_t)^2 h''(\underline{e}_t)} > 0$$

Now we prove that $e_t^* \geq \underline{e}_t$. In effect, From (8) and (13) which determine $r_t$ and $w_t$ respectively, we have

$$h'(e_t^*) = \frac{r_t}{w_t} = \frac{\alpha^2}{1-\alpha}\frac{H_t}{K_t} = \frac{\alpha^2}{1-\alpha}\frac{\int_{I_t} h(e_t^i)di}{\int_{I_t}(b_t^i - e_t^i)di}$$

We suppose a contradiction that $e_t^* < \underline{e}_t$, hence

$$h(\underline{e}_t) > \int_{I_t} h(e_t^i)di \quad \text{and} \quad \underline{e}_t > \int_{I_t} e_t^i di \quad \text{since} \quad e_t^i \leq e_t^* \; \forall i \in I_t.$$

Note that, since we normalize the size of population by 1 then $\underline{e}_t$ and $h(\underline{e}_t)$ are respectively aggregate education investment and human capital of economy when each adult individual $i \in I_t$ has an education investment $\underline{e}_t$. So, it would hold that

$$h'(e_t^*) = \frac{\alpha^2}{1-\alpha}\frac{\int_{I_t} h(e_t^i)di}{\int_{I_t}(b_t^i - e_t^i)di} < \frac{\alpha^2}{1-\alpha}\frac{h(\underline{e}_t)}{b_t - \underline{e}_t} = h'(\underline{e}_t) \quad \Longleftrightarrow \quad e_t^* > \underline{e}_t$$

which contradicts the assumption $e_t^* < \underline{e}_t$. Therefore, $e_t^* \geq \underline{e}_t$.

□

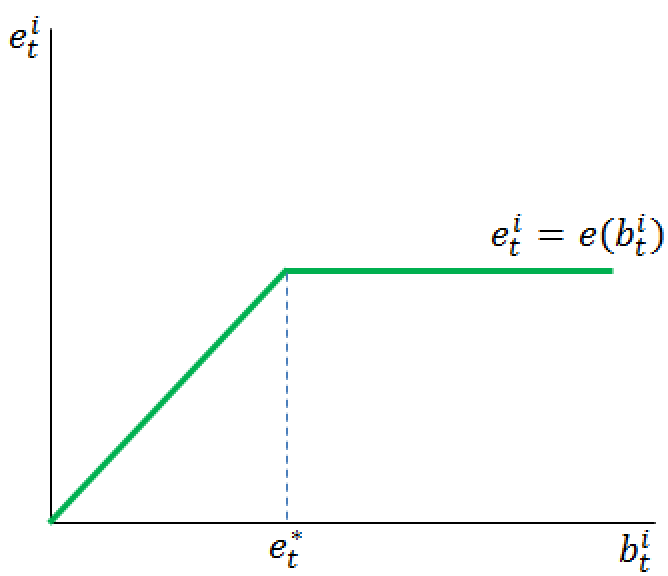


Figure 2: Education investment

We need to impose essential assumptions about the ownership of monopoly firms. We assume that in each period $t$, a fraction $\lambda \in (0,1)$ of population are monopolists of intermediate goods. We denote $I_t^\lambda$ to be the set of monopolists in period $t$, then $I_t^\lambda \subset I_t$. The owners of the monopoly firms own the monopolist profits.

**Assumption 3.** The share of monopoly profits is as follows

$$\rho_t^i \begin{cases} = 0 & \text{if} \quad b_t^i \in [0, e_t^* + \hat{k}) \\ > 0 & \text{if} \quad b_t^i \geq e_t^* + \hat{k} \end{cases} ; \hat{k} > 0.$$

The assumption 3 implies that in order to own monopoly firms and earn monopoly profit, the bequest transfer $b_i^i$ that individual $i$ receives from his/her parent must exceeds a threshold at which after investing in education at level $e_t^*$, the individual $i$ must have a sufficient physical capital, i.e. $b_t^i - e_t^* \geq \hat{k}$ holds, to run a monopoly firm. This assumption is plausible because of widely observed fact that when a potential entrepreneur access the capital market to borrow capital for operating a monopoly producing firm, the banks or financial intermediate institutions *always* require an initial capital capacity beside the entrepreneurial skills of the applicant.[3]

# 3 Equilibria and Dynamics

In this section we define the equilibria of the economy and then we characterize the dynamics of the model and obtain the reduced dynamics of inter-generational bequest transfers for any household $i \in I_t$.

## 3.1 Equilibria

**Definition 1.** *In this economy, under the balanced budget climate policy $(\tau_{ct}, \tau_{dt}) \in \Re_- \times [0,1)$, the competitive equilibria in any period $t$ is characterized by: (i) the determinations of returns on production factors; (ii) optimal choices of each individual $i \in I_t$; (iii) the allocation rule of physical capital in intermediate sectors; and (iv) the dynamics of pollution stock. Analytically, the equilibria is characterized by following system of equations:*

*(i) Determinations of returns on production factors:*

[3]In this model, we assume that individuals have identical innate talent. Hence, the difference in education investment creates the difference in entrepreneurial skills.

$$w_t = (1-\alpha)\left[\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v\right]^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha}$$

$$r_t = \alpha^2\left[\frac{H_t}{K_t}\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_{vt}\right]^{1-\alpha}$$

*(ii) Optimal choices of each individual* $i \in I_t$*:*

$$e_t^i = \min\left\{b_t^i, e_t^*\right\}$$

$$c_t^i = \min\left\{W_t^i\phi(P_t), (1-\gamma)\left[W_t^i\phi(P_t)+\frac{\theta_t}{\gamma}\right]\right\}$$

$$b_{t+1}^i = \max\left\{0, \gamma\left[W_t^i\phi(P_t)-\frac{\theta_t(1-\gamma)}{\gamma}\right]\right\}$$

$$W_t^i = w_t h(e_t^i) + (b_t^i - e_t^i)r_t + \rho_t^i\Pi_t$$

where

$$\Pi_t = \alpha(1-\alpha)\left[H_t\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_{vt}\right]^{1-\alpha}K_t^{\alpha}$$

*(iii) The allocation rule of physical capital in intermediate sectors* $j \in J_t$*:*

$$k_{vj} = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}}a_{vj}}{\sum_{v'\in\{c,d\}}(1-\tau_{v't})^{\frac{1}{1-\alpha}}A_{v'}}K_t$$

*(iv) The dynamics of pollution stock*

$$P_t = (1-\delta)P_{t-1} + \xi\frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}}A_d}{\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}K_t$$

given $K_t$, $H_t$, $P_{t-1}$, and $\{b_t^i, \rho_t^i\}_{i\in I_t}$.

## 3.2 Dynamics of bequest transfer

The evolution of inter-generational bequest transfer within each family $i$, as follow from (17), is

$$b^i_{t+1} = \max\left\{0, \gamma\phi(P_t)\left[w_t h(e^i_t) + (b^i_t - e^i_t)r_t + \rho^i_t\Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right]\right\} \tag{20}$$

We study the dynamics of inter-generational bequest transfer under the following assumption

**Assumption 4.** $h(\underline{e}(\lambda\hat{k})) > \theta(1-\gamma)/\gamma > 1$.

The first inequality in assumption 4 implies that $w_t h(\underline{e}(\lambda\hat{k}))\phi(P_t) > \theta w_t\phi(P_t)(1-\gamma)/\gamma$, i.e. the disposable wealth of a non-monopolist individual at the education level $\underline{e}(\lambda\hat{k}) \in (0, e^*_t)$ is high enough at which that individual will leave a part of wealth, as bequest transfer, for his/her offspring. Note that by construction,

$$\lambda\hat{k} < \inf_{t\in\mathbb{N}} \int_{I_t} b^i_t di$$

i.e. $\lambda\hat{k}$ is a lower bound of bequest stock $b_t = \int_{I_t} b^i_t di$ of the economy.

The second inequality in assumption 4 trivially holds for some plausible values of $\theta$ and $\gamma$, for instance $\theta \geq 1$ (i.e. individuals put an expectation on the minimal disposable wealth that their offspring can hold at least as much as that of them) and $\gamma$ is around 1/4 (that is widely employed for numerical exercises in the literature).

**Proposition 2.** *Under the assumptions 1, 2, 3 and 4, there exists a unique threshold $\underline{b} \in (0, e^*_t)$ such that*

$$b^i_{t+1}\begin{cases} = 0 & \text{if} \quad b^i_t \in [0, \underline{b}] \\ > 0 & \text{if} \quad b^i_t > \underline{b}\end{cases} \qquad \textit{where} \quad \underline{b} = h^{-1}\left(\tfrac{\theta(1-\gamma)}{\gamma}\right)$$

*Proof.*

In effect, when $b^i_t \in (0, e^*_t)$, under assumptions 2 and 3, the disposable wealth of individual $i \in I_t$ is $\phi(P_t)w_t h(b^i_t)$. From (20), we have

$$b^i_{t+1} > (=)\, 0 \iff \phi(P_t)w_t h(b^i_t) > (\leq) \frac{\theta w_t\phi(P_t)(1-\gamma)}{\gamma} \iff h(b^i_t) > (\leq) \frac{\theta(1-\gamma)}{\gamma}$$

Under assumption 1, the last inequalities are equivalent to

$$b^i_t > (\leq)\, h^{-1}\left(\frac{\theta(1-\gamma)}{\gamma}\right) \equiv \underline{b}$$

Under the assumption 4, we have

$$h(\underline{b}) < h(\underline{e}(\lambda\hat{k})) \implies h(\underline{b}) < h(e^*_t) \quad \text{since} \quad e^*_t \geq \underline{e}(\lambda\hat{k})$$

Therefore, $\underline{b} \in (0, e_t^*)$.

□

The statements in propositions 1 and 2 allow us to represent the dynamics of bequest transfers as follows

$$b_{t+1}^i = \begin{cases} 0 & \text{if} \quad b_t^i \in [0, \underline{b}) \\ \gamma\phi(P_t)\left[w_t h(b_t^i) - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \in [\underline{b}, e_t^*) \\ \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \in [e_t^*, e_t^* + \hat{k}) \\ \gamma\phi(P_t)\left[w_t h(e_t^*) + (b_t^i - e_t^*)r_t + \rho_t^i \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] & \text{if} \quad b_t^i \geq e_t^* + \hat{k} \end{cases} \tag{21}$$

It would be interesting to study the evolution of bequest transfer in the space $(b_t^i, b_{t+1}^i) \subset \Re_+^2$. We call this *evolution* in this space is *conditional evolution* in the sense that we have to fix other variables which indeed have their own evolution in the interaction with the evolution of the bequest transfers. This simplification would be helpful for us to focus on the mechanism leading to the wealth inequality. We would note, however, that this approach is completely valid when we focus on any two successive periods of time $t$ and $t+1$. Therefore, the whole evolution of bequest transfers, indeed, is the replication of what we try to do in this section. We will study this *conditional evolution* under following conditions which guarantee the existence of multiple *conditional steady states*.

(i) $\phi(P_t)w_t\left[\gamma h(e_t^*) - \theta(1-\gamma)\right] \geq e_t^*$;

(ii) $\gamma\phi(P_t)r_t < 1$;

(iii) $\gamma\phi(P_t)\left[w_t h(e_t^*) + \hat{k}r_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] \leq e_t^* + \hat{k}$;

(iv) $\gamma\phi(P_t)\left[w_t h(e_t^*) + \hat{k}r_t + \rho_t^i \Pi_t - \frac{\theta w_t(1-\gamma)}{\gamma}\right] > e_t^* + \hat{k} \quad \forall i \in I_t^\lambda \subset I_t$.

Under these conditions, the evolution of inter-generational bequest transfers is depicted in these following figures

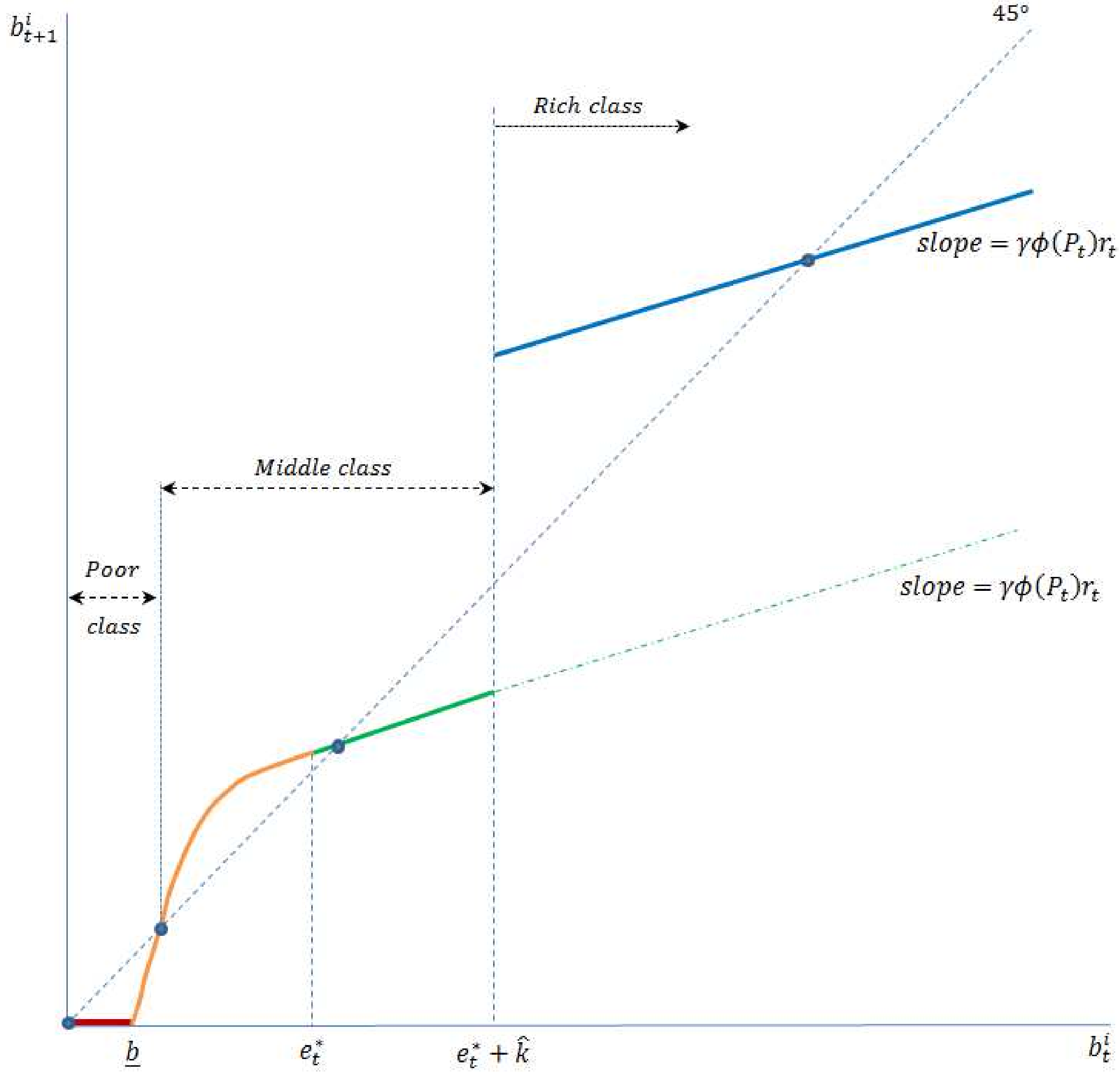


Figure 3: Conditional evolution of bequest transfers (monopoly profit shares are even across monopolists)

## 4 Effects of climate policy

In this section we study the effects of balanced budget climate policy on structure of production factors (or, more precisely, the resource allocation for constituting aggregate production factors), on decisions and the wealth of individuals, and on wealth inequality and bequest transfers across individuals.

**Definition 2.** *We define a set of the balanced budget climate policies in any period t as follows:*

$$\mathcal{C}_t \equiv \left\{ (\tau_{ct}, \tau_{dt}) \in \Re_- \times (0,1) \text{ such that } \sum_{v\in\{c,d\}} \tau_{vt}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0 \right\}$$

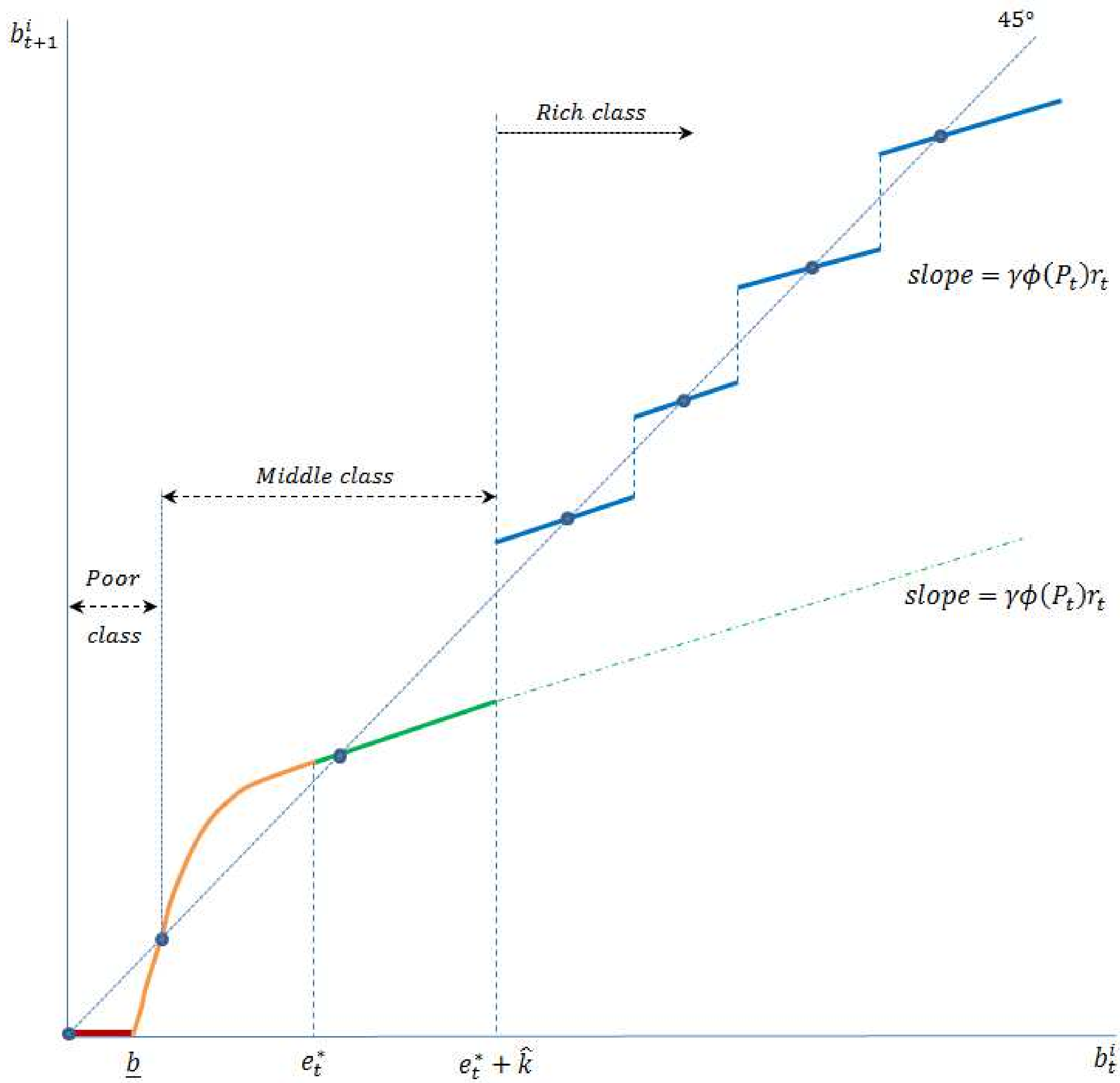


Figure 4: Conditional evolution of bequest transfers (monopoly profit shares are uneven across monopolists)

## 4.1 On resource allocation

**Proposition 3.** *In the economy set up above, in any period $t$, the allocation of (bequest) resource for constituting aggregate physical capital $K_t$ and human capital $\{e^i_t, h^i_t\}_{i\in I_t}$ is independent on the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$.*

*Proof.*

We have in any period $t$, the aggregate bequest $b_t = \int_{I_t} b^i_t di$ is given. From the equations (8) and (13) determining respectively the rental rate of physical capital and return on human capital under climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, we have

$$\frac{r_t(\tau_{ct}, \tau_{dt})}{w_t(\tau_{ct}, \tau_{dt})} = \frac{\alpha^2}{1-\alpha}\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})}$$

where $r_t(\tau_{ct}, \tau_{dt})$ is rental rate of physical capital in the period $t$ under the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$; and the analogous logic applied for other variables in period $t$. We suppose a negation that

$$\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})} < \frac{H_t(0,0)}{K_t(0,0)} \quad \text{for some } (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$$

then

$$e_t^*(\tau_{ct}, \tau_{dt}) > e_t^*(0,0)$$

Note that we prove in proposition 1 that $e_t^i(\tau_{ct}, \tau_{dt}) = \min\{b_t^i, e_t^*(\tau_{ct}, \tau_{dt})\}$, hence the last inequality implies that

$$\begin{cases} e_t^i(\tau_{ct}, \tau_{dt}) = e_t^i(0,0) & \text{if} \quad b_t^i \in [0, e_t^*(0,0)] \\ \\ e_t^i(\tau_{ct}, \tau_{dt}) > e_t^i(0,0) & \text{if} \quad b_t^i > e_t^*(0,0) \end{cases}$$

$$\Longrightarrow \begin{cases} H_t(\tau_{ct}, \tau_{dt}) = \int_{I_t} h(e_t^i(\tau_{ct}, \tau_{dt}))di > \int_{I_t} h(e_t^i(0,0))di = H_t(0,0) \\ \\ K_t(\tau_{ct}, \tau_{dt}) = b_t - \int_{I_t} e_t^i(\tau_{ct}, \tau_{dt})di < b_t - \int_{I_t} e_t^i(0,0)di = K_t(0,0) \end{cases}$$

which leads to a contradiction that $H_t(\tau_{ct}, \tau_{dt})/K_t(\tau_{ct}, \tau_{dt}) > H_t(0,0)/K_t(0,0)$. An analogous logic can be applied for the case of supposing a negation that $H_t(\tau_{ct}, \tau_{dt})/K_t(\tau_{ct}, \tau_{dt}) > H_t(0,0)/K_t(0,0)$. Therefore, it must hold

$$\frac{H_t(\tau_{ct}, \tau_{dt})}{K_t(\tau_{ct}, \tau_{dt})} = \frac{H_t(0,0)}{K_t(0,0)} \quad \forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$$

which, along with proposition 1, trivially gives us for any climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ that

$$K_t(\tau_{ct}, \tau_{dt}) = K_t(0,0), \quad e_t^i(\tau_{ct}, \tau_{dt}) = e_t^i(0,0) \quad \text{and} \quad h_t^i(\tau_{ct}, \tau_{dt}) = h_t^i(0,0) \quad \forall i \in I_t$$

□

Proposition 3 suggests us that the effects of climate policy on the individual $i$'s disposable wealth through its effects on return on human capital, on rental rate of physical capital, as well as on the pollution stock.

## 4.2 On macroeconomic variables

**Proposition 4.** *In any period $t$, the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ decreases the rental rate of physical capital, return on human capital, and aggregate monopoly profit compared to the case* *of no climate policy* $(\tau_{ct}, \tau_{dt}) = (0,0)$. *In addition, it holds*

$$\frac{r_t(\tau_{ct},\tau_{dt})}{r_t(0,0)} = \frac{w_t(\tau_{ct},\tau_{dt})}{w_t(0,0)} = \frac{\Pi_t(\tau_{ct},\tau_{dt})}{\Pi_t(0,0)} = \frac{Y_t(\tau_{ct},\tau_{dt})}{Y_t(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}{\sum\limits_{v\in\{c,d\}}A_v}\right]^{1-\alpha}$$

*Proof.*

Indeed, under the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ the allocation of physical capital $k_{vjt}(\tau_{ct}, \tau_{dt})$, as shown in (9), is

$$k_{vjt}(\tau_{ct},\tau_{dt}) = \frac{(1-\tau_{vt})^{\frac{1}{1-\alpha}}a_{vj}}{\sum\limits_{v'\in\{c,d\}}(1-\tau_{v't})^{\frac{1}{1-\alpha}}A_{v'}}K_t \neq \frac{a_{vj}}{\sum\limits_{v'\in\{c,d\}}A_{v'}}K_t = k_{vjt}(0,0); \quad vj \in \{c,d\}\times J_t$$

for given aggregate physical capital $K_t$.

By solving the following optimization problem, which maximizes the final output in any period $t$,

$$\max_{\{k_{vjt}\}_{vj\in\{c,d\}\times J_t}} H_t^{1-\alpha}\sum_{v\in\{c,d\}}\int_{J_t}a_{vj}^{1-\alpha}k_{vjt}^{\alpha}dj \quad \text{subject to} \quad \sum_{v\in\{c,d\}}\int_{J_t}k_{vjt}dj = K_t$$

given $K_t$ and $H_t$,[4] we find that the allocation rule $k^*_{vjt} = k_{vjt}(0,0)$ is the unique optimal allocation of capital.

By substituting the physical capital allocation rules $k_{vjt}(\tau_{ct}, \tau_{dt})$ under $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ and $k_{vjt}(0,0)$ into the final good production function, along with the government balanced budget constraint as mentioned in lemma 1, we have

$$Y_t(\tau_{ct},\tau_{dt}) = \left[H_t\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v\right]^{1-\alpha}K_t^{\alpha} < \left[H_t\sum_{v\in\{c,d\}}A_v\right]^{1-\alpha}K_t^{\alpha} = Y_t(0,0)$$

Hence, with $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, it holds

$$\sum_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v < \sum_{v\in\{c,d\}}A_v$$

So, now it is quite straightforward from equations (8), (10) and (13), which determine respectively rental rate of physical capital, aggregate monopoly profit and return on human capi-

[4] Note that in the final good production function we replace $x_{vjt}$ by $k_{vjt}$ because of the production function, $x_{vjt} = k_{vjt}$, in the intermediate sector $vj$.

tal, as well as the final good production that, with balanced budget climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, they hold

$$\frac{r_t(\tau_{ct}, \tau_{dt})}{r_t(0,0)} = \frac{w_t(\tau_{ct}, \tau_{dt})}{w_t(0,0)} = \frac{\Pi_t(\tau_{ct}, \tau_{dt})}{\Pi_t(0,0)} = \frac{Y_t(\tau_{ct}, \tau_{dt})}{Y_t(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v}\right]^{1-\alpha} < 1$$

□

The economic intuition supporting for proposition 4 is follows: The climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ distorts the optimal allocation rule of capital in producing intermediate inputs, therefore reduces the aggregate final output. The aggregate final output, indeed, is distributed for aggregate labor income, aggregate physical capital income, and aggregate monopoly profit. The rule of distributing the aggregate final output for labor income, physical capital income, and monopoly profit is indeed independent on climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. That is because, as proven in proposition 3, $K_t$ and $H_t$ are independent on $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. Hence, the aggregate labor income (and return on human capital), aggregate physical capital income (and rental rate of physical capital), and aggregate monopoly profit are distorted exactly the same by the distortion in the optimal allocation rule of capital in producing intermediate inputs.

### 4.3 On bequest transfers and inequality

From the dynamics equation (21) of bequest transfer we find that when $b_t^i \in [0, \underline{b}]$ then $b_{t+1}^i = 0$ whatever the climate policy $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ is. We now consider the effects of climate policy on the bequest transfer $b_{t+1}^i$ of families $i \in I_t$ and characterized by $b_t^i > \underline{b}$. Along with the equality stated in proposition 4, we can determine

$$\frac{b_{t+1}^i(\tau_{ct}, \tau_{dt})}{b_{t+1}^i(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}{\sum\limits_{v\in\{c,d\}} A_v}\right]^{1-\alpha} \frac{\phi(P_t(\tau_{ct}, \tau_{dt}))}{\phi(P_t(0,0))} \quad \text{if} \quad b_t^i > \underline{b}$$

For exposition purpose without lessening crucially the power of the model and analyses, we follow Golosov et al. (2014) to specify the functional form of the climate damage as follows[5]

$$\phi(P) = \exp(-P) \quad \text{for} \quad P \geq 0$$

For this functional form of climate damage function and from the dynamics of pollution stock characterized in (14), we have, when $b_t^i > \underline{b}$

[5]Such a damage function could be found also in Dao et al. (2017).

$$\frac{b^i_{t+1}(\tau_{ct},\tau_{dt})}{b^i_{t+1}(0,0)} = \left[\frac{\sum\limits_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}{\sum\limits_{v\in\{c,d\}}A_v}\right]^{1-\alpha} \exp\left\{\xi\left(\frac{A_d}{\sum\limits_{v\in\{c,d\}}A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}}A_d}{\sum\limits_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}\right)K_t\right\} \tag{22}$$

From the last equation, we find that the overall effect of climate policy $(\tau_{ct},\tau_{dt})\in\mathcal{C}_t$ on the direction of change in bequest transfer $b^i_{t+1}(\tau_{ct},\tau_{dt})/b^i_{t+1}(0,0)$ when $b^i_t > \underline{b}$ depends on the climate policy itself in the relation with the size of aggregate physical capital in the economy.

**Proposition 5.** *In the economy set up above,*

*(i) for any household $i\in I_t$ with $b^i_t > \underline{b}$, it holds $b^i_{t+1}(\tau_{ct},\tau_{dt}) > (=)(<)\, b^i_{t+1}(0,0)$ with climate policy $(\tau_{ct},\tau_{dt})\in\mathcal{C}_t$ if, and only if,*

$$K_t > (=)(<)\frac{1-\alpha}{\xi}\frac{\left[\tau_{ct}-\tau_{dt}\left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{\alpha}{1-\alpha}}\right](\tau_{dt}-\tau_{ct})}{\left[1-\left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{1}{1-\alpha}}\right](1-\tau_{ct})\tau_{ct}\tau_{dt}}\ln\left(\frac{\tau_{ct}-\tau_{dt}\left(\frac{1-\tau_{dt}}{1-\tau_{ct}}\right)^{\frac{\alpha}{1-\alpha}}}{(\tau_{ct}-\tau_{dt})(1-\tau_{dt})^{\frac{\alpha}{1-\alpha}}}\right) = \hat{K}_t(\tau_{ct},\tau_{dt})$$

*Moreover, $\hat{K}_t(\tau_{ct},\tau_{dt})$ is bounded for all $(\tau_{ct},\tau_{dt})\in\mathcal{C}_t$.*

*(ii) A climate policy $(\tau_{ct},\tau_{dt})\in\mathcal{C}_t$ alters the disposable wealth $\psi(W^i_t,P_t)$ of all households $i\in I_t$ by the same multiplier, and*

$$\psi(W^i_t(\tau_{ct},\tau_{dt}),P_t(\tau_{ct},\tau_{dt})) > (=)(<)\, \psi(W^i_t(0,0),P_t(0,0)) \iff K_t > (=)(<)\hat{K}_t(\tau_{ct},\tau_{dt})$$

*Proof.* (i) The proof for this statement is fairly straightforward from equation (22) when we evaluate the bequest ratio $b^i_{t+1}(\tau_{ct},\tau_{dt})/b^i_{t+1}(0,0)$ in comparison with 1. Indeed,

$$\frac{b^i_{t+1}(\tau_{ct},\tau_{dt})}{b^i_{t+1}(0,0)} > (=)(<)\, 1$$

if, and only if

$$(1-\alpha)\ln\left(\frac{\sum\limits_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}{\sum\limits_{v\in\{c,d\}}A_v}\right) + \xi\left(\frac{A_d}{\sum\limits_{v\in\{c,d\}}A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}}A_d}{\sum\limits_{v\in\{c,d\}}(1-\tau_{vt})^{\frac{1}{1-\alpha}}A_v}\right)K_t > (=)(<)\, 0$$

which is equivalent to

$$K_t > (=)(<) \frac{1-\alpha}{\xi} \left( \frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)^{-1} \ln \left( \frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right) > 0$$

Using the balanced government budget condition $\sum\limits_{v\in\{c,d\}} \tau_{vt}(1-\tau_{vt})^{\frac{\alpha}{1-\alpha}} A_v = 0$ as mentioned in (12) and substituting it into the right hand side of the last inequality, we will obtain $\hat{K}_t(\tau_{ct}, \tau_{dt})$.

We have the balanced budget condition

$$\tau_{ct}(1-\tau_{ct})^{\frac{\alpha}{1-\alpha}} A_c + \tau_{dt}(1-\tau_{dt})^{\frac{\alpha}{1-\alpha}} A_d = 0$$

By applying the implicit function theorem for the last equation with respect to $\tau_{ct}$ and $\tau_{dt}$, we have $\tau_{ct}$ is a function of $\tau_{dt}$, in which $\lim\limits_{\tau_{dt}\to 0^+} \tau_{ct} = \lim\limits_{\tau_{dt}\to 1^-} \tau_{ct} = 0$, and

$$\lim_{\tau_{dt}\to 0^+} \frac{\partial \tau_{ct}}{\partial \tau_{dt}} = -\frac{A_d}{A_c}$$

We have

$$\hat{K}_t(\tau_{ct}, \tau_{dt}) = \frac{1-\alpha}{\xi} \left( \frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)^{-1} \ln \left( \frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v} \right)$$

Since $\hat{K}_t(\tau_{ct}, \tau_{dt}) > 0$ for all $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ then we just need prove that $\hat{K}_t(\tau_{ct}, \tau_{dt})$ is bounded from above. Suppose a negation that there existed $(\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$ such that $\hat{K}_t(\tau_{ct}, \tau_{dt})$ could approach $+\infty$. We have the term $\ln \left( \sum\limits_{v\in\{c,d\}} A_v / \sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v \right) > 0$ is always bounded from above because $0 < \sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v < \sum\limits_{v\in\{c,d\}} A_v < +\infty$. Hence, in order to $\hat{K}_t(\tau_{ct}, \tau_{dt})$ approaches $+\infty$, then the only possibily is that $\frac{A_d}{\sum\limits_{v\in\{c,d\}} A_v} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_d}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}$ approaches 0 from the right. This possibility occurs only if $\tau_{dt}$ approaches 0.[6] However,

[6]It is straightforward to show that $\frac{A_{dt}}{\sum\limits_{v\in\{c,d\}} A_{vt}} - \frac{(1-\tau_{dt})^{\frac{1}{1-\alpha}} A_{dt}}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_{vt}} > 0 \ \forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$. Because it is equivalent to $\frac{1-\tau_{ct}}{1-\tau_{dt}} > 1 \ \forall (\tau_{ct}, \tau_{dt}) \in \mathcal{C}_t$, which trivially holds.

$$\tau_{dt} \to 0 \quad \Longrightarrow \quad \ln\left(\frac{\sum\limits_{v\in\{c,d\}} A_v}{\sum\limits_{v\in\{c,d\}} (1-\tau_{vt})^{\frac{1}{1-\alpha}} A_v}\right) \to 0$$

So, by applying the l'Hopital rule, and note that $\lim\limits_{\tau_{dt}\to 0^+} \frac{\partial \tau_{ct}}{\partial \tau_{dt}} = -\frac{A_d}{A_c}$, we have

$$\lim_{\tau_{dt}\to 0^+} \hat{K}_t(\tau_{ct},\tau_{dt}) = \frac{1-\alpha}{\xi} \lim_{\tau_{dt}\to 0^+} \frac{\left[A_d + \frac{\partial \tau_{ct}}{\partial \tau_{dt}} A_c\right] \sum\limits_{v\in\{c,d\}} A_v}{A_d \sum\limits_{v\in\{c,d\}} A_v - A_d \left[A_d + \frac{\partial \tau_{ct}}{\partial \tau_{dt}} A_c\right]} = 0$$

That is to say, $\hat{K}_t(\tau_{ct},\tau_{dt})$ is bounded.

□

Proposition 5 tells us that for a climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ which is carried out in period $t$, if the stock of physical capital is rather high, in particular $K_t > \hat{K}_t(\tau_{ct},\tau_{dt})$, then that climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ may increase the inequality in bequest transfers across households. That is because the climate policy improves the environmental quality and, hence, enhances the disposable wealth of households. Under the climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ where $K_t > \hat{K}_t(\tau_{ct},\tau_{dt})$, while the choice of bequest transfers to the offspring of the poor households are unchanged and set at 0 when the bequest the households receive from their parents does not exceed $\underline{b}$, the bequest transfers to the offspring of the rich households and middle-class households increase.

This proposition implies that for a very unequal economy in bequest transfers, the climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ may enhance the inequality. For a given aggregate bequest transfer, the aggregate physical capital tend to be higher in an unequal economy in bequest transfer than in a more equal one because when the bequest is owned too biasedly to the rich then a higher fraction of bequest will be transformed into physical capital, making $K_t > \hat{K}_t(\tau_{ct},\tau_{dt})$, since the education investment is bounded. Hence, in this case, the climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ enhances inequality. But when the bequests spread more equally to the poor and middle classes then they are transformed more into education investment, reducing the aggregate physical capital in the economy. Therefore, it is more likely for a climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ to satisfy $K_t < \hat{K}_t(\tau_{ct},\tau_{dt})$, which may reduce the inequality in bequest transfers $\{b^i_{t+1}\}_{i\in I_t}$ for the next generation. To assess more precisely the effect of a climate policy $(\tau_{ct},\tau_{dt}) \in \mathcal{C}_t$ on the inequality in bequest transfer for the generation $t+1$, we need the information about the distribution of $\{b^i_t\}_{i\in I_t}$.

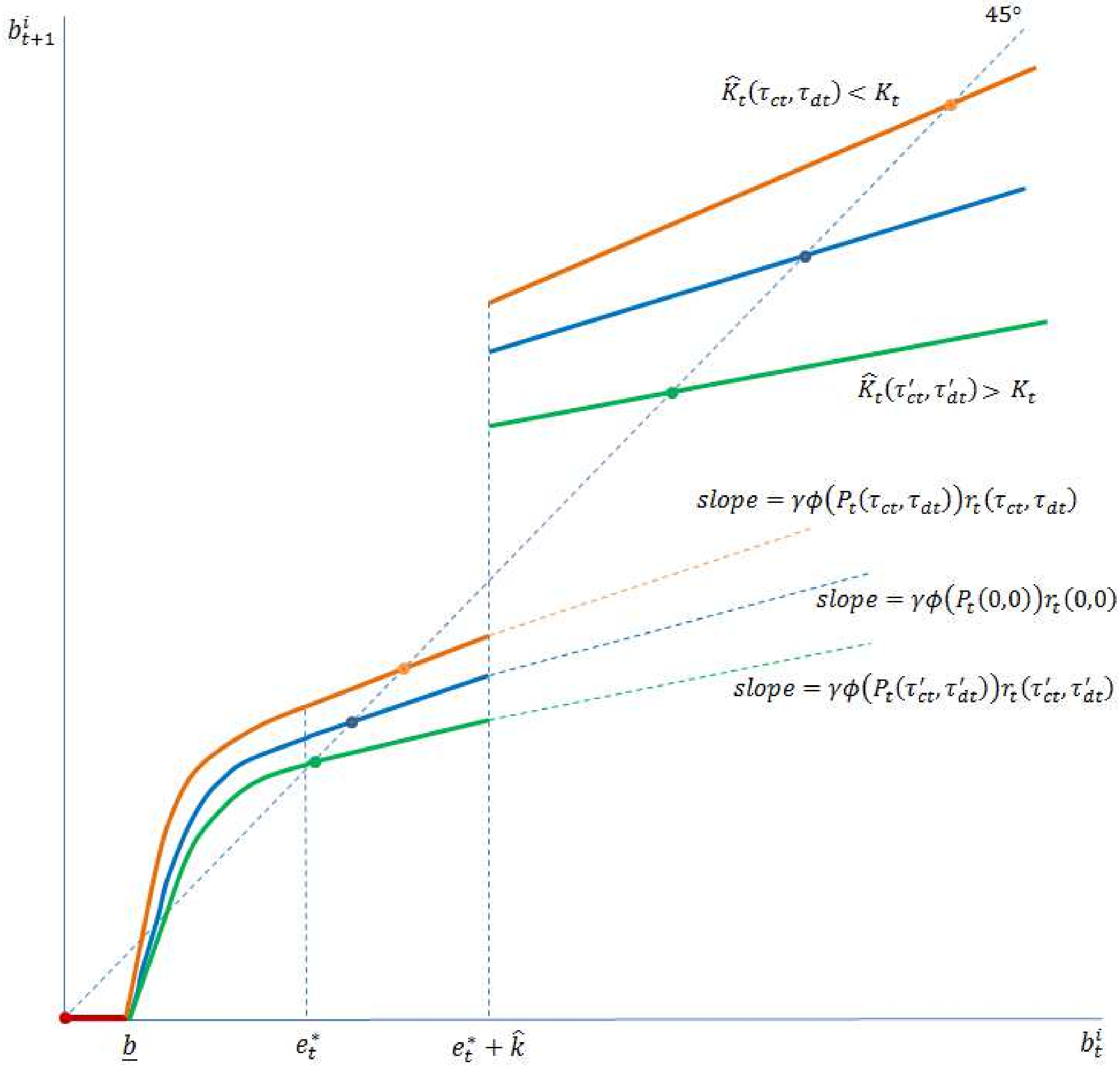


Figure 5: Effects of climate policies on dynamics of bequest transfer

## 5 Concluding remarks

We set up an overlapping generations economy with imperfections of credit markets and a climate damage on the wealth of heterogeneous households to investigate theoretically the link between two emerging issues in the 21st century, that are climate change and wealth inequality. For each balanced budget climate policy, we identify a corresponding threshold of the aggregate physical capital above which such the climate policy may enlarge the wealth gap between the rich and poor class. This theoretical results derived from our model may be helpful in investigating the effects of climate policy on wealth inequality as well as design a composite policy to improve the equity and protect environment.

The model suggests that, for a given stock of aggregate inter-generational bequest transfer, physical capital stock tends to be higher in a more unequal economy. That is because of the asymmetry in allocating the bequest transfers for human capital investment and physical capital accumulation among heterogeneous households in the presence of imperfections of credit markets. Following the statement in Proposition 5, this makes climate policy more likely benefit biasedly to the rich. In addition, while the climate policy may not help the poor escape the poverty trap, in the long run, the rich may become richer due to the positive effects of improving the aggregate human capital on monopoly profits. That contributes to widen wealth gap between the rich and the poor class. Hence, from equity perspectives, we should think about taxing monopoly profits to subsidize the clean production sector and/or subsidize education for the poor to reduce inequality in wealth distribution.

Our model so far, for simplification, ignores the role of research and development sector at which the tax revenue from dirty production sector is used to subsidy clean technology innovations (see Acemoglu et al. 2012 and 2016). Introducing innovation sectors would be challenged, interesting, and promising in our current framework. The theoretical results of our model hinge specifically on a plausible assumption of credit market imperfections. Relaxing this assumption and/or introducing policies that eliminate the imperfections of credit markets would promise important and interesting research. These ideas, among others, are left for further research in our research agenda.

## Declarations

**Conflicts of interest/Competing interests**: There are no conflicts of interest/Competing interests.

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