Progress In Electromagnetics Research, Vol. 147, 171–202, 2014

Cloaking and Invisibility: A Review

Romain Fleury and Andrea Alù*

(*Invited Review*)

Abstract—Invisibility has been a tantalizing concept for mankind over several centuries. With recent developments in metamaterial science and nanotechnology, the possibility of cloaking objects to incoming electromagnetic radiation has been escaping the realm of science fiction to become a technological reality. In this article, we review the state-of-the-art in the science of invisibility for electromagnetic waves, and examine the different available technical concepts and experimental investigations, focusing on the underlying physics and the basic scientific concepts. We discuss the available cloaking methods, including transformation optics, plasmonic and mantle cloaking, transmission-line networks, parallel-plate cloaking, anomalous resonance methods, hybrid methods and active schemes, and give our perspective on the subject and its future. We also draw a parallel with cloaking research for acoustic and elastodynamic waves, liquid waves, matter waves and thermal flux, demonstrating how ideas initiated in the field of electromagnetism have been able to open groundbreaking venues in a variety of other scientific fields. Finally, applications of cloaking to non-invasive sensing are discussed and reviewed.

1. INTRODUCTION

During the past few years, there has been a considerable interest in studying theoretically and experimentally the electromagnetic properties of artificial media made with "atom-like" inclusions, engineered to interact with impinging waves in anomalous ways, inducing properties otherwise impossible with natural materials. These *meta*materials [1–11] are well known today for opening exotic possibilities to manipulate electromagnetic waves, including negative refraction [12–16], spatial localization and sub-wavelength focusing [13], spontaneous emission control [17–20], and anomalous tunneling effects [21–25].

Perhaps one of the most appealing applications of metamaterials consists in the possibility of making objects invisible to the impinging radiation. In practical terms, invisibility, or the cloaking effect, aims at cancelling the electromagnetic field that is scattered by an object, by placing it inside a cover (cloak) that makes the ensemble undetectable to electromagnetic sensors. Related concepts, such as invisible particles [26, 27], non-radiating source distributions [28–32], low-scattering antennas [33–35], and the non-uniqueness of the inverse scattering problem [36–39] have been investigated by the scientific community over the last century, yet the recent surge of metamaterials applied to invisibility and transparency has surely revamped the interest in these concepts and made significant advances both from the theoretical and the experimental point of view.

Ideally, cloaking an object implies total scattering suppression from all angles, independently of the environment, of the position of the observer, and over a wide frequency range, the ultimate goal being all-angle invisibility over the entire visible range. Several challenges, however, arise along the way to realizing such an ideal device. In the following, we review the available techniques to achieve

---

Accepted 14 January 2015, Accepted 4 February 2015, Scheduled 7 February 2015

* Corresponding author: Andrea Alù (alu@mail.utexas.edu).

The authors are with the Department of Electrical and Computer Engineering, The University of Texas at Austin, 1 University Station C0803, Austin, TX 78712, USA.

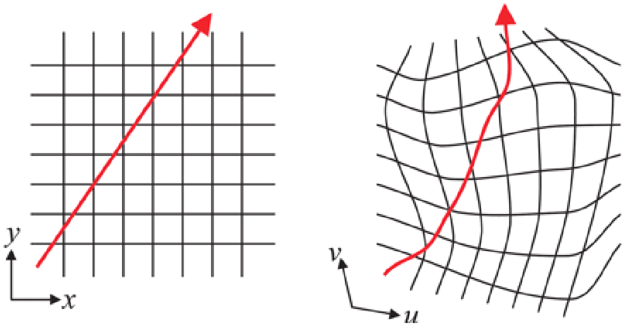
some form of transparency and invisibility, and discuss their intrinsic advantages and constraints. We point out current and future research trends in the field of invisibility, namely: (i) a simplification of the invisibility requirements to obtain more practical, yet functional designs; (ii) a drive towards the ultimate limits in terms of bandwidth as a function of size and overall scattering suppression, following recent findings on ultimate bounds of the cloaking operation based on passivity and causality [40, 41]; (iii) extension of the science of invisibility to other physical systems for which cloaks may be simpler to realize; (iv) application of cloaking to technologies with less stringent specifications than invisibility cloaking of human-scale objects in the visible; (v) a realistic path towards active or nonlinear cloaks, that may relax many of the current limitations.

In this context, we will review the few initial proposals for cloaking and discuss how these proposals evolved towards more realistic designs, underlining the performance compromises made in the process. We will also illustrate the progress of the science of cloaking in the fields of acoustics, quantum mechanics, thermodynamics, and liquid surface waves, comparing the available solutions with the case of electromagnetic waves. Finally, we will show that invisibility cloaks can have groundbreaking impact in the field of non-invasive sensing, where they can be used to render a sensor invisible to an external field, while maintaining its ability to receive the incoming signals. Such application possesses less stringent specifications than total invisibility of electrically large objects illuminated by broadband light, in particular in terms of bandwidth, scattering reduction, and size of the object to cloak. Overall, our analysis aims at drawing a clear snapshot of the upcoming challenges in the science and technology associated with invisibility and cloaking.

2. TRANSFORMATION-BASED CLOAKING

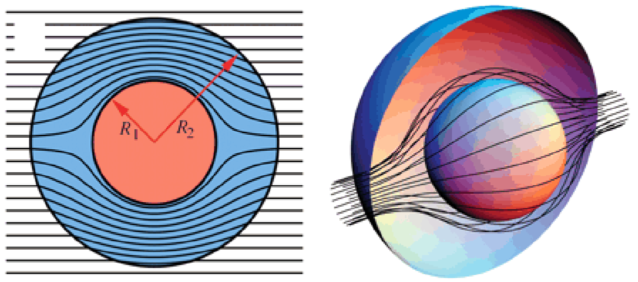
2.1. Transformation Electrodynamics

Transformation-based cloaking applies the concept of transformation electrodynamics to manipulate the flow of electromagnetic energy using a transformation that stretches the coordinate grid of space, as sketched in Fig. 1. In the left panel, the red arrow represent a ray of light (or the Poynting vector) going through a homogeneous material. By stretching and compressing the Cartesian space (panel b), one can bend the ray at will and control its propagation, similar to what is originally speculated in Fermat's principle. Because Maxwell's equations are form-invariant under a coordinate transformation, it is possible to re-interpret electromagnetic propagation in the transformed system as a propagation in the untransformed coordinate system with a specific functional distribution of the permeability μ and permittivity ε tensors. Essentially, inhomogeneity and anisotropy in the background plays the role of a geometrical distortion, leading to a practical way to effectively create any coordinate transformation in the physical space, providing one possesses full control over ε and μ. Metamaterials may therefore, at least in principle, be used to mimic coordinate transformations, and force light to follow curved coordinates.



(a) (b)

Figure 1. Illustration of the light manipulation possibilities offered by transformation optics. ©2006 AAAS. Reprinted with permission [57].



(a) (b)

Figure 2. Principle of operation of transformation-based spherical cloaks. ©2006 AAAS. Reprinted with permission [57].

The basis of transformation electrodynamics was established by pioneering works in the 20th century [42–47], which noticed the relationship between light propagation in an effective space-time geometry, and the invariance of Maxwell equations under orthogonal transformation of space. Transformation electrodynamics was later formulated for the general case of non-orthogonal transformations [48] and applied to a variety of problems including solving lattice band structure where different scales are at play [48], solving propagation in irregular lattices [49], implementing perfectly matched layers [50], describing negative refraction [51], and designing unique microwave [52], optical components [53] and plasmonic devices [54–56].

2.2. The Spherical Transformation Cloak

Spherical transformation cloaks [57–64] were proposed simultaneously and independently by Pendry et al. [57] and by Leonhardt [58] in 2006. A spherical transformation cloak can be obtained using transformation optics (TO) by finding a transformation that takes a free-space spherical region of radius R_2 and maps all the contained volume into an annulus $R_1 < r < R_2$, essentially expanding the center point of the sphere into a finite spherical region of radius R_1 isolated from external electromagnetic radiation (see Fig. 2(a) and Fig. 3). Propagation of electromagnetic waves can be pictured using either the initial coordinate system, or the transformed one: the transformation maps free-space into a curved space with a hole.

Applying this procedure to the radial transformation $r' = R_1 + r(R_2 - R_1)/R_1$, one obtains the required constitutive parameter tensors for the cloak [57]

$$\begin{aligned} \varepsilon_{rr} = \mu_{rr} &= \frac{R_2}{R_2 - R_1}\frac{(r - R_1)^2}{r^2} \\ \varepsilon_{\theta\theta} = \mu_{\theta\theta} &= \frac{R_2}{R_2 - R_1} \\ \varepsilon_{\phi\phi} = \mu_{\phi\phi} &= \frac{R_2}{R_2 - R_1} \end{aligned} \tag{1}$$

Similar expressions can be obtained in the 2D case to realize a cylindrical cloak. Such cloaks, by design, provide identically zero scattering at the design frequency, and re-route the power flow around the concealed volume, which is electromagnetically isolated from the outside world [65], as represented in Fig. 2(b).

The operation of this ideal cloak has been confirmed by follow-up studies using the ray picture [66–68]. One problem of using geometrical optics to validate the operation of the cloak is that since the transformation is singular (it transforms a point to a finite area surface), the central ray does not know whether to go up or down, right or left [57]. Therefore, the behavior of this critical ray is undetermined. The only way to prove directly that the cloak has identically zero scattering is to exactly solve Maxwell equations. This task was undertaken numerically [69, 70], and the operation of the cloak confirmed. However, numerical studies are unable to prove that the scattering is identically zero because of numerical discretization. By solving analytically the scattering problem using Mie theory, it was shown that the scattering in all the scattering channels is made identically zero, for both the ideal cylindrical [71, 72] and spherical [73] cases. As an alternative, it is also possible to derive rigorously the cloak profiles directly from the requirement that the scattering must be zero for all possible incident fields [74].

Although theoretically very elegant, the transformation-based cloak is impossible to fabricate in this ideal scenario, and it remains very challenging even after some approximations are considered. In general, the application of transformation optics requires spatially inhomogeneous and anisotropic constitutive magnetic and dielectric tensors, as exemplified in (1). The electric and magnetic responses of the cloak material must also be identical to guarantee impedance matching, and therefore no reflection from the interfaces, as well as to guarantee no polarization dependence of the optical response. At the same time superluminal propagation and extreme values of the constitutive parameters are required in the cloak [75], i.e., one requires phase velocities larger than the speed of light in vacuum. For instance, a ray of light (the red line in Fig. 3(a)) that takes normally no time to cross the origin (blue point) of the untransformed coordinate system, is required to propagate in zero time along the finite-length blue circle in Fig. 3(b). This explains why the phase velocity in the cloak at the inner radius must be infinite, according to (1). The cloak therefore requires superluminal propagation because any ray that goes through it has more distance to travel in an equal time than it would have in the associated direct path in vacuum, since it has to avoid the cloaked region.

In principle, metamaterials can provide large anisotropy and extreme constitutive parameters [76–78], with either very small [21–23, 79–84], or very large values [85]. However, as a consequence of Kramers-Kronig relations, which are satisfied for any passive causal medium, superluminal propagation is only attainable near material resonances, which drastically limit the bandwidth of operation and are fundamentally associated with material losses. Material loss is a source of unwanted scattering, since, by virtue of the optical theorem, any absorption is associated with forward scattering. On top of that, it is extremely difficult to achieve strong magnetic response at optical frequencies. The implementation of the cloak is made even more difficult by the extreme sensitivity of transformation cloaks to tiny perturbations, as discussed in [71].

To address these issues, Schurig et al. proposed a simplified version of the transformation cloak, and experimentally verified the concept [86]. First, for simplicity, they focused on a 2D cylindrical cloak, for which it can be shown that all constitutive tensor components have only radial dependence. Microwave frequencies were chosen because of the ease of fabrication and the possibility of achieving the required strong magnetic responses. Because of the nature of their experiment, limited to only one polarization for which the electric field is polarized along the cylinder axis, only ε_{zz}, μ_{rr} and $\mu_{\theta\theta}$ matter. Finally, they approximated the requirements of transformation-optics by following the functional dependence with the radial coordinate only in terms of the index $\sqrt{\varepsilon\mu}$, choosing constant values for ε_{zz} and $\mu_{\theta\theta}$, but keeping the required radial dependency for μ_{rr}. This eikonal approximation maintains the ray path distortion as in the ideal case, but sacrifices the impedance $\sqrt{\mu/\varepsilon}$, introducing mismatch and reflections, i.e., unwanted scattering.

Figure 4 shows a picture of their first-of-its-kind experimental cloak for electromagnetic waves, where the μ_{rr} radial dependency is obtained using split-ring resonators with varying geometry. The results from measurements and simulations are shown in Fig. 5, with incidence from the left. The cloaked object is a copper cylinder. Panel a shows the simulated field and power stream lines for the ideal cloak, while panel b shows the simulation of the simplified eikonal cloak, for which some mismatch is expected and result in some scattering (notice the shadow). Panel c shows the measured field for the bare cylinder. When the cylinder is covered, the wave fronts closely resemble the one from panel b, and are much less disturbed than in the case of the bare object (panel c), demonstrating the operation of the cloak.

Similar microwave experiments were realized later for the other polarization of interest [87].

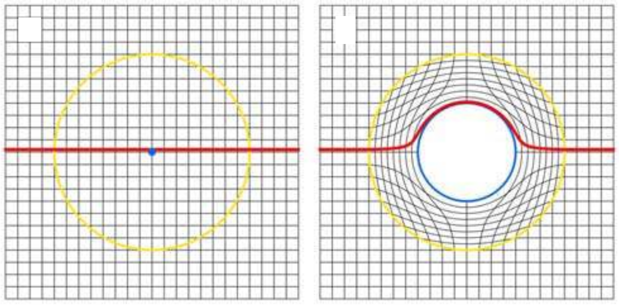

(a) (b)

Figure 3. Any transformation cloak requires superluminal propagation. ©2011 IOP Publishing. Reprinted with permission [75].

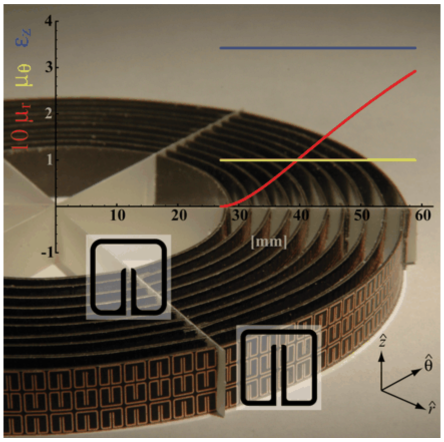


Figure 4. First experimental verification of transformation-based cloaking at microwave frequencies. ©2006 AAAS. Reprinted with permission [86].

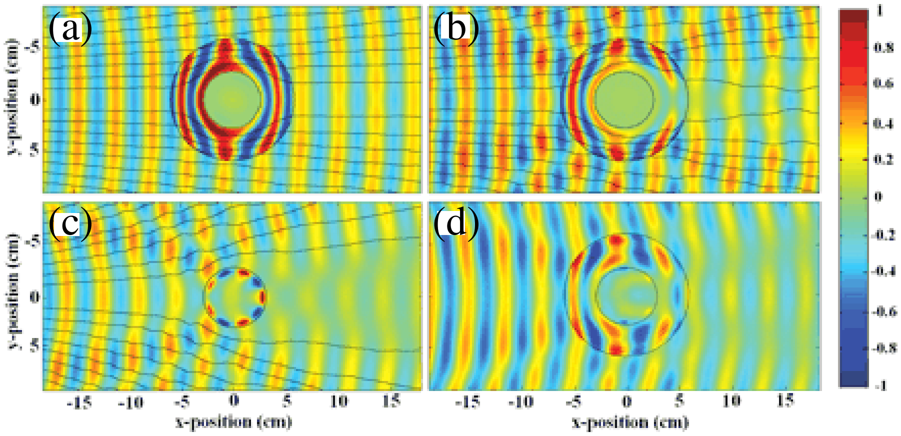


Figure 5. Electric field simulations and measurements for the cylindrical transformation cloak. (a) Simulation, ideal cloak. (b) Simulation, eikonal cloak. (c) Measurements, bare cylinder. (d) Measurements, covered cylinder. ©2006 AAAS. Reprinted with permission [86].

Exciting proposals have been put forward to realize a similar experiment at optical frequencies, without the need for magnetic resonators, for a specific polarization of interest and with some residual mismatch [88]. Using different types of transformation rules may ease the design of non-magnetic cloaks at optical frequencies [89, 90]. However, as discussed above, all these simplified versions of invisibility cloaks introduce some residual scattering [91]. We also note the interesting possibility of realizing "anti-cloaks" [92–94], i.e., covers to be placed around a cloaked object that undoits effect by realizing an anti-transformation that neutralizes the transformation in the invisibility cloak.

The spherical and cylindrical transformation cloaks are definitely elegant solutions to realize an invisibility effect. In principle, they are not limited by the size of the object, and can perfectly cancel scattering from all viewing angles and for all polarizations. Because their fabrication poses severe challenges in this ideal scenario, researchers have been exploring simplified versions of these cloaks, for which only one constitutive parameter is tailored. As discussed above, the response necessarily becomes polarization dependent and not impedance matched, which hinders the broad applicability of these concepts. In the following we outline solutions to simplify the cloak requirements to achieve invisibility. Most of the new concepts that have been proposed to improve TO-based cloaks start with the same observation: the drawbacks of the spherical or cylindrical ideal cloaks are due to the necessity of having superluminal propagation, especially near the inner radius of the cloak where extreme values for the constitutive parameters are required. By relaxing this requirement it is possible to bring invisibility cloaks much closer to reality, as detailed in the following subsections.

2.3. Non-Euclidean Transformation Cloaking

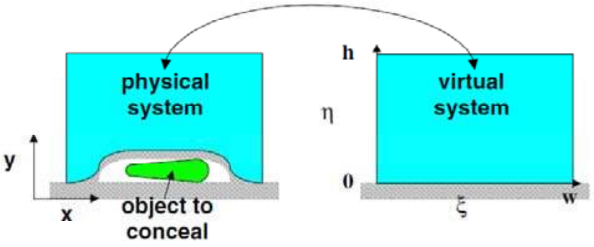
Leonhardt and Tyc [95] proposed to use a curved non-Euclidian transformation (such as the one obtained on the surface of a sphere rather than on a plane) to avoid infinite expansions and singularities. This approach can in principle avoid superluminal propagation, making this concept extremely promising for all-angle broadband cloaking in the visible [75]. The compromise made here is that cloaking is accompanied by an added phase delay, which corresponds to the additional propagation time that the wave takes to propagate in the associated curved space. In other words, this approach leads to all-angle broadband cloaking for rays, not for waves. This is an expected trade-off for an invisibility device built without singularities for phase velocity [96]. Such cloaks are detectable by, for instance, performing time of flight measurements [97], or using interferometric techniques. This is not a problem if one aims at concealing objects to the human eye, which is not sensitive to phase differences[†].

[†] The human eye is also insensitive to polarization, which may also be used as an advantage to simplify the cloaking scheme, as discussed below.

2.4. Carpet Cloaking and Its Variants

Another approach, called carpet cloaking, was introduced by Li and Pendry [98], which proposed to use quasi-conformal mapping in transformation optics. It can be shown that quasi-conformal mapping minimizes the anisotropy in a two dimensional coordinate transformation. Li and Pendry showed, by applying an optimization procedure, that certain types of transformation media can be achieved with dielectric-only materials and very reasonable values of permittivity. Such transformations may be implemented in a broadband way at visible frequencies. As a by-product, the very weak anisotropy of the resulting media may be neglected, obtaining isotropic graded index cloaks. They showed that this concept could be applied, for instance to make a mirror with a bump appear totally flat, by covering the bump with a carpet cloak, also known as a ground plane cloak. The method was applied to a new cloaking strategy, represented in Fig. 6. The blue region represents the medium on which the transformation is applied. The grey regions are filled with a perfect electric conductor (PEC). Essentially, the physical system is a mirror with a bump. The object to hide is embedded in the bump, surrounded by PEC. Finding the transformation that makes the mirror appear flat gives the required graded index profile for the cloak.

Full wave simulations confirm the operation of the device, as shown in Fig. 7. Without the cloak (panel b), the bump scatters the incident beam in different directions. When the cloak tops the bump (panel a), everything happens as if the mirror were flat. Obviously, the main drawback of this method is to work only in a 2D geometry. In other words, the bump would be immediately detectable in the



(a) (b)

Figure 6. Principle of operation of a carpet cloak. ©2008 APS. Reprinted with permission [98].

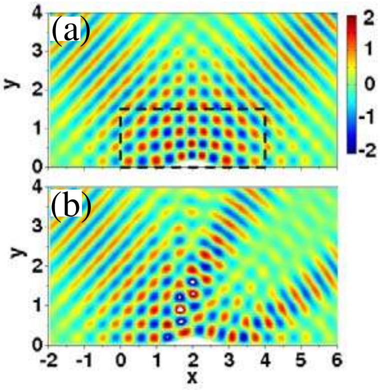


Figure 7. Full wave simulation of carpet cloaking. (a) Electric field distribution for the reflection by PEC mirror with a bump covered by the carpet cloak (contained within the dashed rectangle) and (b) corresponding situation without the cloak. ©2008 APS. Reprinted with permission [98].

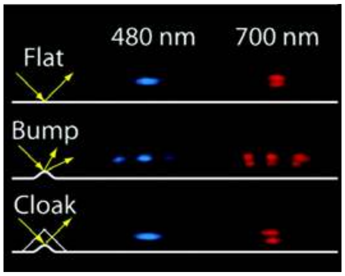


Figure 8. Experimental validation of carpet cloaking in the visible. ©2011 ACS. Reprinted with permission [113].

third dimension. To address the robustness of the method in the third dimension, it is possible to look at photorealistic images of objects hidden under a carpet cloak in three-dimensional scenarios, generated using ray-tracing algorithms. This technique has been extensively used to investigate the performance of most of the broadband cloaking methods discussed in this section [99–104]. In the case of the carpet cloak, robustness in the third dimension is also observed, although image distortions are inevitable [100]. An additional drawback is the requirement to operate in reflection from a mirror, which obviously limits the applicability of these concepts. A more fundamental issue is the presence of a lateral shift that accompanies isotropic ground-plane cloaking [105]. This lateral shift of the reflected ray, comparable to the height of the bump, is due to the fact that the anisotropy requirement of an ideal carpet cloak is typically relaxed in practical realizations. This is a serious issue, because it means that such an isotropic cloak is not a better solution than using instead a second ground plane on top of the bump. Thus, an observer can detect that something is there because there is a shift of the reflected ray, although it cannot know what is there. The case of a 3D axisymmetric bump has also been treated [106]. It was shown that three-dimensional quasi-conformal mapping is possible, but it requires some anisotropy in the cloak.

Carpet cloaking was first validated at microwave frequencies, using non-resonant metallic elements [107]. This experiment was not directly scalable to optical frequencies, because the employed metallic elements would be somewhat lossy. Several subsequent works reported simultaneously the experimental validation of broadband carpet cloaking at infrared frequencies, observing 2D invisibility [108–110]. The carpet cloak was also built, still for a 2D bump, but characterized in 3D at infrared frequencies, with good scattering reduction for large out-of-plane viewing angles up to 60 degrees [111]. A ground plane cloak was built and characterized also for a fully 3D axisymmetric bump at microwave frequencies [112]. Finally, carpet cloaks were experimentally validated at visible frequencies over a broadband range [113]. Fig. 8 summarizes the results of the experiment in [113], validating the invisibility of a bump over a mirror for the extreme cases of blue and red light.

It is worth noting also the elegant possibility of achieving carpet cloaking with smart materials that acquire the correct gradient-index profile naturally from the elastic deformation induced by the bump [114]. In order to avoid the lateral shift problem, it is necessary to retain the weak anisotropy in the carpet cloak [105]. Therefore, perfect carpet cloaking still requires anisotropy. In 2009, researchers managed to emulate anisotropy using tapered waveguides [115]. This indeed led to the observation of broadband cloaking based on a coordinate transformation at optical frequencies. Remarkably, the cloaked region was two orders of magnitude larger than the wavelength of light. However, this cloaking strategy only works in a specifically shaped waveguide. Another possible route is to modify the carpet cloaking transformation in order to obtain a variant of carpet cloaking that can be realized with directly available natural anisotropic materials, but does not suffer from the lateral shift problem, making it possible to use carpet cloaking to hide bigger objects [116, 117]. Such modified carpet cloaks were implemented with affine transformations using bi-refringent natural dielectric crystals, calcite in the visible [118, 119] and saphire at THz [120]. With this method, 2D carpet cloaking of macroscopic objects is possible in a wide range of incidence angles and without lateral shift, but only for one polarization of light.

Other proposals include building free-standing variants of the carpet cloak, which do not require a ground plane. These different kinds of carpet cloaks can be invisible either unidirectionally or in multiple directions [121–123], and implemented exactly with natural bi-refringent dielectric crystals [121] or by other means at microwaves [122, 123]. The compromise here is to sacrifice all-angle invisibility to gain some simplicity in the design. Note that some other proposals went even further in simplification, by looking for unidirectional, or multidirectional cloaking, and also dropping the requirement that the phase of the external field must be recovered [124, 125]. The solution to this problem is trivial: it can be realized using mirrors or lenses [125], as described in a book from the famous magician R. Houdin written more than 150 years ago [126]. Therefore, there is no particular scientific challenge in hiding an object only in one or a few directions and without phase preservation. If one drops the phase requirement, the real challenge is to cloak from all directions, as in non-Euclidean transformation cloaking [95]. On the other hand, if we drop the all-angle requirements and perform unidirectional cloaking, the true scientific challenge is to preserve the phase, as discussed in [122].

2.5. Transformation Cloaking at Zero Frequency

A final approach to simplify the invisibility requirement of the transformation methods but still achieve functional and realizable transformation cloaks is to work at zero frequency [127]. In the DC case, the electric and magnetic components of the electromagnetic field decouple, which enables cloaking from DC magnetic/electric fields only with permeability/permittivity cloaks. Still, anisotropic and position-dependent constitutive parameters are required, but it is possible to implement them in simple configurations. DC magnetic cloaks, i.e., "anti-magnets", have been discussed and studied both theoretically and experimentally, using superconductors [128–131]. Cloaks for DC electric fields have also been fabricated using resistor networks [132–134].

3. PLASMONIC AND MANTLE CLOAKING

3.1. The Scattering Cancellation Approach

Transformation electrodynamics is the ideal approach to realize perfect cloaking. However, a transformation cloak can only be built in an approximate way, leaving some residual scattering in the process. For instance, the permittivity and permeability profiles can never match in reality the requirements of becoming exactly infinite at the inner radius of a transformation cloak. Any practical realization of a transformation cloak will be somehow limited, and its extreme sensitivity to tiny imperfections [71] will result in some scattering. This residual scattering is not easily controllable, and may become large, as it is due to small imperfections in the cloak implementation that are not controllable.

Conversely, plasmonic and mantle cloaking are based on a radically different approach, based on scattering cancellation. This technique does not aim at totally cancelling the scattering from a given region of space, but instead only cancels out the dominant scattering terms in the multipole expansion of the scattered field. Since it relies on a non-resonant cancellation approach, the residual scattering is usually controlled. In addition, the design can be based on homogeneous and isotropic materials, in contrast to the very complicated material properties that one obtains with transformation cloaks. Due to their robustness, these cloaks can be built with less constraints, making their realization and experimental verification more at hand.

Scattering-cancellation cloaks are typically built to hide a specific object, and do not necessarily work for another object. This is not necessarily a drawback, if interested in making a cloak applicable to arbitrary objects we simply need to cloak a reflective (conducting) surface with the desired shape, which isolates its interiors to any electromagnetic wave. Having more flexibility in tailoring the cloak for the object of choice is actually desirable, as it leads to broader bandwidths and better robustness for penetrable objects [135]. More relevant is the fact that the scattering cancellation technique is not well suited for objects that are very large when compared with the wavelength, because the number of dominant scattering harmonics generally grows fast with the size of the object, increasing the required number of layers in the cloak and the complexity of the design. However, a variety of applications do not aim at cloaking very large objects. We will discuss one example in Section 5, where we will show that plasmonic and mantle cloaks open new vistas in the field of non-invasive sensing.

To formally approach the problem, consider the scattering of a monochromatic $\exp(j\omega t)$ wave propagating in a given background material with permittivity ε_0 and permeability μ_0 by an arbitrary object placed at the origin of a spherical coordinate system (r, θ, φ). The impinging plane wave can be expanded in spherical harmonics as [136]

$$\begin{aligned}\vec{E}_i &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} a_{nm}\nabla\times\nabla\times\left(\vec{r}\psi_n^m\right) - j\omega\mu_0 b_{nm}\nabla\times\left(\vec{r}\psi_n^m\right)\\ \vec{H}_i &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} b_{nm}\nabla\times\nabla\times\left(\vec{r}\psi_n^m\right) - j\omega\varepsilon_0 a_{nm}\nabla\times\left(\vec{r}\psi_n^m\right)\end{aligned} \tag{2}$$

where $\vec{r}$ is the radial vector, and ψ_n^m are scalar spherical harmonics, which are solutions of the Helmholtz equation in the considered coordinate system. Each term in the sum corresponds to a transverse magnetic (TM) or transverse electric (TE) spherical wave, which stands for the incident (n, m) multipole with respective amplitudes a_{nm} and b_{nm}. Because Maxwell equations are linear and the harmonics are orthogonal functions, one can solve the scattering problem separately for each harmonic, writing the total scattering field as a superposition of the contribution of each multipoles, i.e., [137]

$$\begin{aligned}\vec{E}_s &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} c_n^{TM} a_{nm} \nabla\times\nabla\times(\vec{r}\psi_n^m) - j\omega\mu_0 c_n^{TE} b_{nm}\nabla\times(\vec{r}\psi_n^m) \\ \vec{H}_s &= \sum_{n=1}^{\infty}\sum_{m=-n}^{n} c_n^{TE} b_{nm} \nabla\times\nabla\times(\vec{r}\psi_n^m) - j\omega\varepsilon_0 c_n^{TM} a_{nm}\nabla\times(\vec{r}\psi_n^m)\end{aligned} . \tag{3}$$

The coefficients c in (3) quantify how much scattering is associated with a given TM or TE wave, and they are independent of the excitation. These scattering coefficients depend only on the particular object and on frequency. For a given object of characteristic size a, contributions up to order $n \approx k_0 a$, with $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, dominate the sums in (3) [138, 139]. The overall visibility of the object is quantified by the total scattering cross-section σ_S [137]:

$$\sigma_s = \frac{2\pi}{|k_0|^2}\sum_{n=1}^{+\infty}\sum_{m=-n}^{n}(2n+1)\left(\left|c_{nm}^{TE}\right|^2 + \left|c_{nm}^{TM}\right|^2\right). \tag{4}$$

Therefore, the coefficients c determine the possibility for an observer to detect the presence of an object. By designing suitable covers that cancel the dominant scattering coefficients in the sum (4), one can induce invisibility for a given object. This is the concept of dominant scattering cancellation.

3.2. Plasmonic Cloaking

Plasmonic cloaking implements the scattering cancellation technique using cloaks or shells made out of homogeneous and isotropic materials [140–155]. The term 'plasmonic cloaking' stems from the fact that in general, to cloak most objects, a cloak with permittivity below unity is ideal, which requires the use of materials with plasma-like dispersion, called plasmonic materials.

As an example, let's take the simple case of scattering of a plane wave from a dielectric sphere. Let us assume that the radius of the sphere is a and its permittivity ε, and this sphere is embedded in a spherical dielectric shell of radius $a_c > a$ and permittivity ε_c. Because the system is spherically symmetric, we can take $m = 0$ in Eqs. (2)–(4) and drop the index m. The ensemble forms a core-shell structure whose scattering coefficients c_n^{TE} and c_n^{TM} may be calculated exactly by enforcing the boundary conditions. Because the shell only adds one degree of freedom, it can only cancel the scattering from one multipole. We therefore make the additional assumption $k_0 a_c \ll 1$ to keep the number of dominant scattering harmonics to a minimum. This assumption on the size is not necessary in general, but is made here to ease the demonstration. Several layers are in general required to extend the technique to big objects or to cloak at multiple frequencies [146]. In the quasistatic limit, we obtain $c_1^{TE} \approx 0$ and

$$c_1^{TM} \approx -j(k_0 a_c)^3 f(\varepsilon_c, a/a_c), \tag{5}$$

where f is a real valued function of ε_c and a/a_c. The factor $f(\varepsilon_c, a/a_c)$ can be canceled under the following invisibility condition [140]

$$\left(\frac{a}{a_c}\right)^3 = \frac{(\varepsilon_c - \varepsilon_0)(2\varepsilon_c + \varepsilon)}{(\varepsilon_c - \varepsilon)(2\varepsilon_c + \varepsilon_0)}. \tag{6}$$

Because the left-hand side of (6) is necessarily bounded between 0 and 1, this invisibility condition cannot be fulfilled for any values of ε and ε_c. Fig. 9 summarizes in a contour plot the values of permittivity for which (6) is fulfilled, if one sets $n = 1$ in the figure. In particular, it is apparent in Fig. 9 that for any value of permittivity ε, we can find a cloak (ε_c, a_c) to make the sphere invisible. In addition, we can infer from the figure that the induced dipole moments in the cloak and in the particle are always opposite. This remark enables the interpretation of plasmonic cloaking illustrated in Fig. 10. Take the example of a dielectric double-positive (DPS) sphere with $\varepsilon > \varepsilon_0$, i.e., with a positive polarizability. Such a sphere requires, according to Fig. 9, a cloak with $\varepsilon_c < \varepsilon_0$, i.e., an epsilon negative medium (ENG) that has negative polarizability. Both the sphere and its cloak, when taken separately, scatter the incident field and are therefore detectable. However, when they are combined in a single scatterer, their polarizabilities cancel each other to create an invisible system with zero net induced dipole moment.

Such a method has been widely studied also in more complex situations [142, 143]. It was shown to be robust to geometry imperfections and material losses [144]. As shown in Fig. 11, cloaking for a collection of particles was demonstrated in very stringent scenarios due to the excellent near-field scattering cancellation capability of the method [145]. The use of several homogeneous and isotropic layers was proposed to achieve cloaking at multiple frequencies or for bigger objects [146, 147]. The effect of size and frequency dispersion was studied to evaluate the limitations of the method [135], showing that it leads to larger bandwidth than transformation cloaks. Plasmonic cloaking was shown to work even for irregularly shaped objects with anisotropic scattering properties [148]. The behavior of plasmonic cloaks under non-monochromatic excitation were studied to address the behavior of the cloak in time-domain [149]. Scattering from cylinders covered with plasmonic cloaks was thoroughly studied and the method was shown to be robust to the effect of finite length, even under oblique illumination [150]. Satellite cloaking was also proposed, a method analogous to plasmonic cloaking in which four spherical plasmonic particles symmetrically surround the dielectric object to be concealed [151].

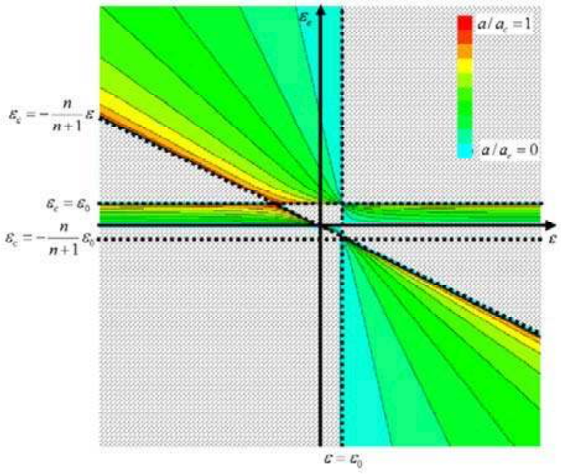


Figure 9. Range of permittivity for which the invisibility condition (6) admits physical solutions. Here, we have taken the particular case $n = 1$. ©2005 APS. Reprinted with permission [140].

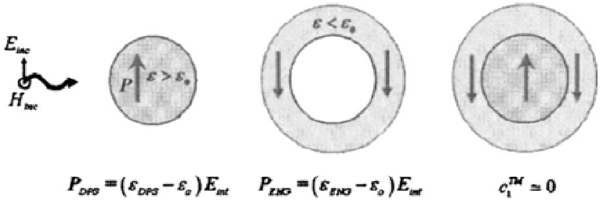


Figure 10. Principle of plasmonic cloaking. ©2005 APS. Reprinted with permission [140].

At optical frequencies, plasmonic cloaking may be implemented by using natural plasmonic materials such as metals. When natural materials are not available, the method needs to be implemented with metamaterials. For instance, plasmonic covers can be realized with parallel plate metamaterials in the 2D cylindrical case [152, 153], for one polarization. Fig. 12 shows the scattering cross-section for a dielectric cylinder surrounded by a cloak made of dielectric material and 8 metallic fins, and compares it with the one of the object in free-space. This technique can, in principle, be used to implement plasmonic cloaking in the 2D case at optical, infrared or microwave frequencies.

Such plasmonic metamaterials were used to experimentally validate plasmonic cloaking at microwave frequencies in a waveguide set-up [154]. Later, the same technique was employed to observe plasmonic cloaking for a free-standing object in free-space, and verified its functionality from all angles, both in the near- and far-field [155]. A picture of the cloaked cylinder used in this experiment is shown in Fig. 13, and near-field measurements are presented in Fig. 14, demonstrating invisibility at the design frequency (3.1 GHz).

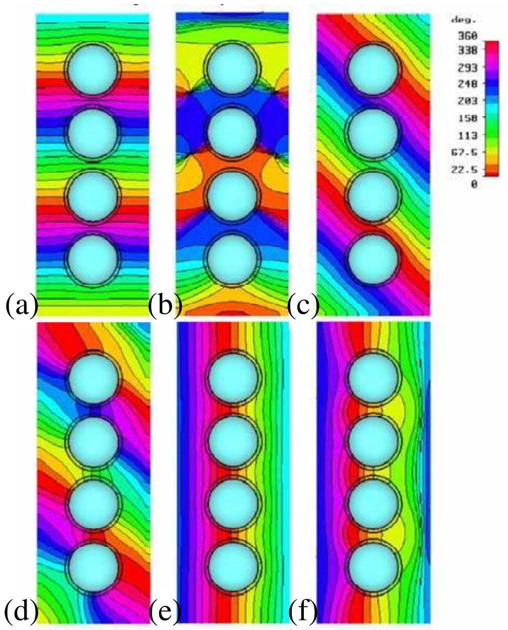


Figure 11. Field distribution (snapshot in time) for plasmonic cloaking of a collection of PEC spheres at various incidence angle (a) 0 deg, cloaked, (b) 0 deg, bare, (c) 45 deg, cloaked, (d) 45 deg, bare, (e) 90 deg, cloaked, (f) 90 deg, bare. ©2007 OSA. Reprinted with permission [145].

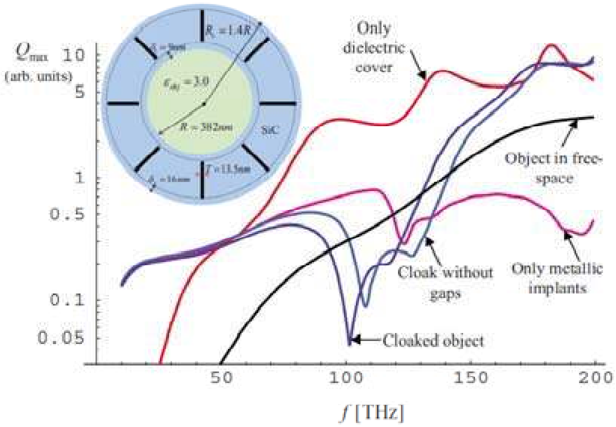


Figure 12. Scattering cross-section for a dielectric cylinder covered by a parallel plate metamaterial with plasmonic-like dispersion. ©2008 APS. Reprinted with permission [153].

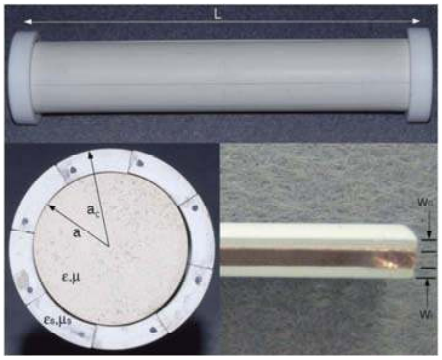


Figure 13. Experimental validation of plasmonic cloaking in free-space: cloaked cylinder. ©2012 IOP Publishing. Reprinted with permission [155].

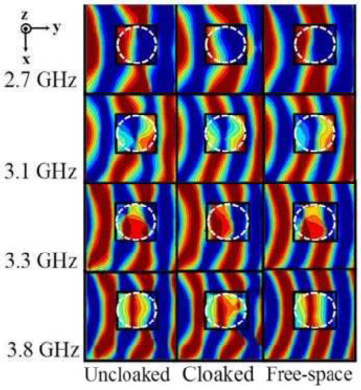
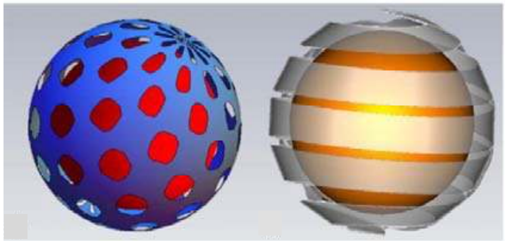


Figure 14. Experimental validation of plasmonic cloaking in free-space: near field measurements (snapshot in time). ©2012 IOP Publishing. Reprinted with permission [155].

3.3. Mantle Cloaking

The mantle cloaking technique implements the scattering cancellation technique using a different approach compared to plasmonic cloaking. Instead of using a homogeneous isotropic layer of material, it uses an ultrathin isotropic frequency selective surface [156, 157]. By tailoring the surface impedance of the mantle cloak, it is possible to cancel the dominant scattering from the object. Examples of mantle covers that may be used to tailor scattering are represented in Fig. 15. The advantage of this method is that it leads to ultrathin, light weight and conformal designs, and it tends to provide broader bandwidths.



(a) (b)

Figure 15. Examples of possible cloak designs that may realize mantle cloaking. ©2009APS. Reprinted with permission [158].

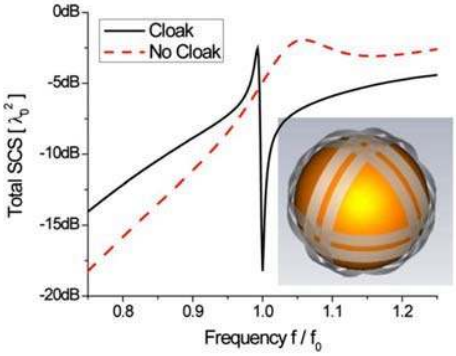


Figure 16. Scattering cancellation by mantle cloaking. ©2009 APS. Reprinted with permission [158].

When dealing with mantle cloaking, the only difference in the calculation of the scattering coefficients is the boundary condition on the surface. In the case of an infinitely thin metascreen covering a sphere, the boundary condition must describe the discontinuity of the tangential magnetic field imposed by the induced surface current. Mantle covers are typically composed of subwavelength elements, and described by an averaged surface impedance which link the induced average surface current density $\mathbf{J}_\mathrm{S}$ to the tangential electric field $\mathbf{E}_{\tan} = Z_S^{TM}\mathbf{J}_\mathrm{S}$. At radio-frequencies it is usually assumed that the ultrathin cover is low-loss, thus $Z_S^{TM} = jX_s$. Usually, for the polarization of interest, Z_S^{TM} may be considered a scalar quantity, i.e., the surface does not cause polarization or harmonic coupling.

The boundary conditions on the metascreen interface are the continuity of the tangential electric field, and the discontinuity of the magnetic field,

$$\mathbf{E}_{\tan}|_{r=a^\pm} = \mathrm{Z}_s^{TM}\hat{\mathbf{r}} \times \left(\mathbf{H}_{\tan}|_{r=a^+} - \mathbf{H}_{\tan}|_{r=a^-}\right). \tag{7}$$

Repeating the same procedure as in the case of plasmonic cloaking, it is possible to find the correct value of surface reactance for a spherical cloak of radius a_c surrounding a dielectric sphere. With the definition $\gamma = a/a_c$, we find [158]

$$X_s = \frac{2[2+\varepsilon-\gamma^3(\varepsilon-1)]}{3\gamma^3\omega a\varepsilon_0(\varepsilon-1)}. \tag{8}$$

Using thin patterned metasurfaces, we have theoretically and numerically demonstrated that mantle cloaking can result in a substantial scattering reduction for cylindrical and spherical objects, from all viewing angles [159–161]. A numerical example is given in Fig. 16, where we plot the scattering cross section of a dielectric sphere with $\varepsilon = 10$, covered by a quasi-isotropic inductive FSS made with six orthogonal interconnected conductive stripes with $a_c/a = 1.1$. The diameter of the object is $\lambda_0/3$, and the surface is engineered to provide the required surface impedance to cancel the dipolar contribution to the scattering at the design frequency. A scattering reduction of more than 15 dB is observed for this realistic design. Engineering recipes for designing a variety of surface impedances for mantle cloaks are given in [162]. Graphene may also be used to realize ultrathin mantle cloaks at terahertz frequencies [163]. Mantle cloaking was experimentally validated by cloaking a free-standing object in free space, and from all angles, for one polarization [164]. The object shown in Fig. 17 is a dielectric rod covered by a specifically designed frequency selective surface. The measured near-field patterns are shown in Fig. 18, demonstrating good performance over a relatively broad frequency range. Future research will focus on designing mantle cloaks that work for both polarizations, and improved omnidirectional mantle cloaks taking into account the anisotropy of the surface and cross-polarization coupling. Note that mantle cloaking leads to extremely thin and conformal cloak designs at microwaves, in contrast with all the transformation-based method and plasmonic cloaks. However, plasmonic cloaks may be more advantageous at visible frequencies, using natural materials, because of the difficulty to realize patterned mantle surfaces with subwavelength resolution.

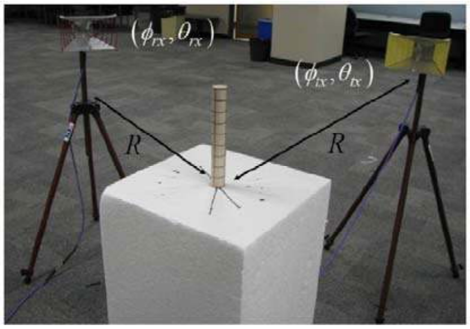


Figure 17. Experimental set-up used to measure all-angle scattering cancellation by mantle cloaking. ©2013 IOP Publishing. Reprinted with permission [164].

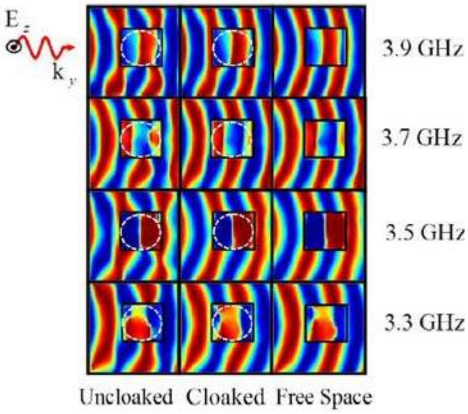


Figure 18. Experimental validation of mantle cloaking in free-space: near field measurements (snapshot in time). ©2013 IOP Publishing. Reprinted with permission [164].

4. ALTERNATIVE CLOAKING TECHNIQUES

There are other cloaking methods that do not directly fit into the above categories. We briefly review them in this section.

4.1. Cloaking Using Transmission-Line Networks

Transmission-line networks can be used to hide objects from electromagnetic waves [165–169]. Cloaking with a transmission-line network is illustrated in Fig. 19. The idea is to force the incident wave to pass through a network of transmission lines that is tailored to have very low scattering cross-section. Therefore, this method can hide objects that fit inside the fishnet formed by the network, since there is virtually no field there. Because the power is not required to go around the object, superluminal propagation is not in principle required. Thus, the method is inherently broadband and suitable for cloaking from short signals. The main drawbacks of this technique is that the cloaked object is required to fit between the network lines, which implies cloaking only objects whose geometry is complementary to the networks, or very small objects. A numerical example showing the operation of a transmission-line network cloak is given in Fig. 20.

4.2. Parallel-plate Cloaking

Parallel-plate cloaking [170–172] aims at guiding the incident electromagnetic energy around an object by using a cloak made of parallel metallic fins with adiabatically decreasing separation (cf. Fig. 21). In a sense, the principle of operation is similar to a transformation cloak, although it is not designed using a coordinate transformation. The underlying idea is that in free space, the propagation of a plane electromagnetic wave is not disturbed by the presence of parallel perfectly conducting sheets that are perpendicular to the electric field. This is true regardless of the distance between the sheets. Here, the thickness of the parallel plates is increased gradually until they are in contact and create a short, preventing the electromagnetic energy to reach the center of the cloak, where the object is hidden. Instead, the energy prefers to go around the concealed object, as can be inferred from the numerical simulations of Fig. 22, showing the electric field distribution around the cloak (panel a) as well as the Poynting vector (panel b). This method shows exceptionally broadband cloaking capabilities, for the considered polarization.

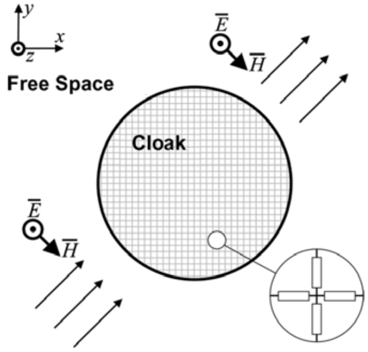
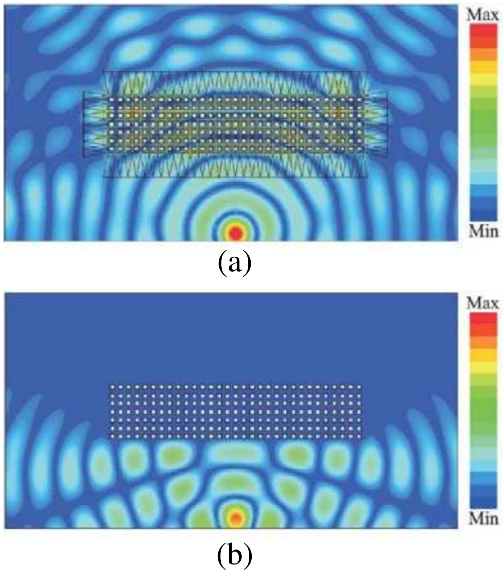


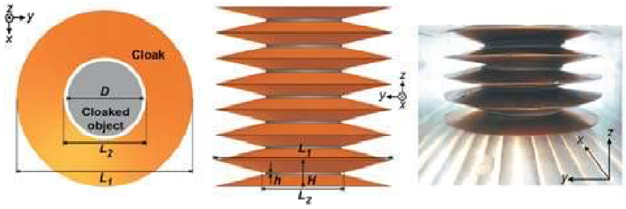
Figure 19. Principle of cloaking using transmission-line networks. The incident wave is squeezed into a transmission line network that is specially designed to have very low scattering. Any object that fit in between the lines of the network is not traversed by the incident wave, and does not scatter. ©2008 IEEE. Reprinted with permission, from IEEE Transactions on Antennas and Propagation [166].



(a)

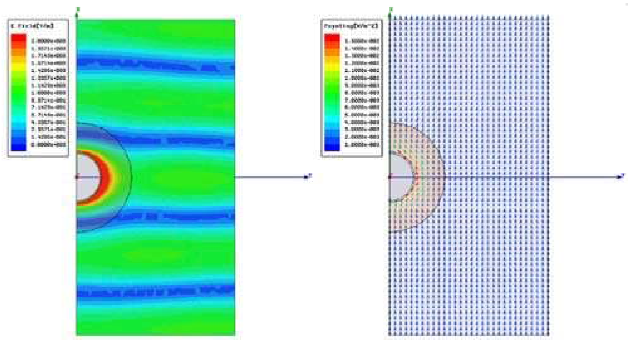
(b)

Figure 20. (a) A transmission-line network cloak renders an array of rods invisible. (b) Without it, the object would be almost opaque. ©2008 IEEE. Reprinted with permission, from IEEE Transactions on Antennas and Propagation [165].



(a) (b) (c)

Figure 21. A parallel-plate cloak used for broadband cloaking of a conducting cylinder. ©2009 APS. Reprinted with permission [170].



(a) (b)

Figure 22. Numerical simulations for a parallel plate cloak. (a) Electric field and (b) Poynting vector. ©2009 APS. Reprinted with permission [170].

4.3. Anomalous Resonances

An alternative method for inducing invisibility is based on localized anomalous resonances [173–178]. The method exploits resonances, such as plasmonic resonances or resonances at the interface between complementary media, to induce invisibility in a finite region outside the cloak. The main drawback of such a technique is the extreme sensitivity of plasmonic resonances to material losses. We also mention that similar cloaking effects at a distance outside the cloaking shell can be induced using complementary media like pairs of right handed and left handed media [177, 178].

4.4. Active Cloaking

It is generally believed that linearity, causality and passivity together limit the operation of any invisibility cloak in bandwidth, and that there exist a general trade-off between bandwidth and total scattering reduction [40, 179]. In particular, any scattering reduction at a particular frequency achieved with a passive cloak is necessarily accompanied by an increase of the scattering when integrated over the entire frequency spectrum [40]. While an active cloaking system is still forced to abide by causality, its limitation in terms of bandwidth may be less restrictive than a passive system. The potentials of active cloaks or interactions between active systems and passive cloaks have been discussed and theoretically studied [180, 181], and it was proposed to use cloaks made of layers of sensors and sources to create broadband invisibility cloaks [180]. An active scheme for cloaking the Laplace and the Helmholtz equations have been put forward in [182, 183]. Experiments on active cloaking have been conducted at DC frequencies with interesting results [184]. The problem of active cloaking at non-zero frequency is very challenging because the external wave leaves very little time for an active system to perform calculation and react. Another possibility is to work with active self-sensing elements. Active mantle cloaking may be realized using non-Foster metasurfaces [185]. The implementation of active cloaking may be significantly easier for acoustic waves however [186].

A totally different active scheme, that does not require sensors, is provided by the use of parity-time symmetric structures or metamaterials [187–194]. It can be shown that structures possessing specific arrangements of gain and lossy media respecting PT symmetry may be designed to induce unidirectional invisibility [193]. An example of such structures is shown in Fig. 23. It is a two ports, PT symmetric, reciprocal, active system that possesses a unitary transmission coefficient from both sides, and a zero reflection coefficient only from one side. In the direction for which the reflection is zero, the wave is transmitted as if no object was here, even for its phase. From the other side, transmission is still one, but there is an amplified reflected wave (in green).

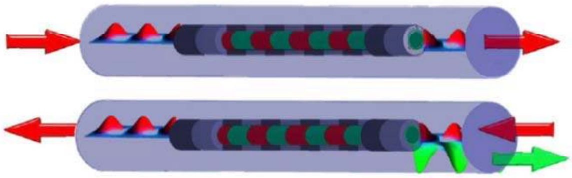


Figure 23. A PT-symmetric optical structure can be designed to be invisible from one side. ©2011 APS. Reprinted with permission [193].

4.5. Hybrid Methods and Miscellaneous

A recent article presented some promising experimental results on a hybrid method that aims at taking advantage of both transformation optics and dominant scattering cancellation method [195]. A cylindrical object is rendered invisible by using a cloak with several layers. The cloak operates without superluminal propagation by design. Some layers are used to cancel the dominant scattering multipoles, but anisotropic materials are used to try to take advantage of the transformation approach. The cloak is designed numerically using optimization procedures. Because the operation is non-resonant, it is quite broadband. However, the reported total scattering reduction is not very large. Another approach to cloaking used the mirage effect with thermally modulated transparent carbon nanotubes sheets [196]. In another work, cloaking has been achieved in the diffusive regime of light propagation, by applying a dominant scattering cancellation technique to Fick's diffusion equation [197].

5. CLOAKING BEYOND ELECTROMAGNETISM

As mentioned in the introduction as a general trend, the science of invisibility is expanding towards other physical systems were energy transmission is either due to wave propagation in forms different from electromagnetism, or to diffusion. This section is devoted to cloaking for non-electromagnetic systems such as elastic waves, acoustic waves, matter-waves, thermal flux, and liquid surface waves.

5.1. Elastic and Acoustic Waves

Cloaking of acoustic waves is highly desirable in a variety of applications, for instance invisibility from sonars. Acoustic metamaterials and sonic crystals with unusual values of bulk modulus and density [198–209] have been the subject of intense research for more than a decade now. They provide increased flexibility in designing media with specific acoustic properties, which may lead to the realization of acoustic cloaks.

Pioneering work by Milton et al. examined the form-invariance of the general elasto-dynamic equations, in an attempt to formulate a theory of transformation acoustics [210]. They reached the conclusion that the linear elasto-dynamic equations are not form-invariant under coordinate transformation, even in the particular case of acoustic (i.e., longitudinal-only) waves in gases or liquids. This was followed by investigations of metamaterials that could be used to cloak elastic waves in some particular cases [211]. Subsequent works showed that the transformation method could be applied for acoustic waves in 2D [212], based on an equivalence between the 2D acoustic equations and Maxwell equations for a single polarization. But still, at that time, it was believed that the transformation method could not provide acoustic invisibility cloak designs in the fully 3D case, even in the scalar case.

Yet, two different papers [213, 214] showed, using different indirect methods, that the cloaking solution can be obtained in the case of 3D acoustic waves, and result in an anisotropic effective mass and bulk modulus cloak with functional dependency as in the electromagnetic case. Later, the problem of form-invariance of the elasto-dynamic equations was re-examined [215]. It was shown that the elasto-dynamic equations are not in general form-invariant, but however, the particular case of acoustic waves is governed by form-invariant equations. The velocity vector in acoustics transforms differently than the E and H vectors in transformation optics, a point that was initially missed in the original study [215]. The use of transformation acoustics indeed leads to the cloak profile previously obtained by direct scattering analysis [214] and analogy to the DC conductivity equation [213]. It should be also recognized that the solution for the transformation acoustic cloak was already contained in a prior work from Greenleaf et al. on general Helmholtz equation cloaking analysis with respect to Riemannian metrics [216].

Transformation acoustics can lead to various cloak geometries [217], but possesses the drawback of requiring metamaterials with anisotropic density. Anisotropic versions of pentamode structures may be a solution for transformation elastodynamics [218–220]. Efforts to obtain artificial materials with anisotropic mass [221] led to promising proposals to build the acoustic transformation cloak out of multilayered structures made with two types of acoustic isotropic metamaterials, whose parameters change as a function of the radial distance [222–224]. Practical realization of the radial dependence may be achieved using sonic crystals [222]. Simplified cloaks based on the eikonal approximation were also proposed, and sensitivity to the presence of a non-vanishing shear modulus was studied [225]. The first experimental realization of a 2D cylindrical transformation acoustic cloak used a network of acoustic resonators based on the transmission-line approach [226]. An acoustic version of the carpet cloak was also fabricated [227]. Cloaking for elasto-dynamic waves was also proposed and observed in simplified scenarios [228–230].

The scattering cancellation method was also transposed to acoustic waves, demonstrating the acoustic equivalent of plasmonic [231–233] and mantle [234] cloaking. The realization of the first 3D acoustic cloak was recently reported, using a dominant scattering cancellation method, but unidirectional [235]. Finally, we note the possibility of increasing the bandwidth of acoustic cloaks by using hybrid methods mixing transformation acoustics and scattering techniques [236].

5.2. Thermal Waves

The general idea that phonons, just like photons or electrons, can be manipulated to control the heat flow and create components like rectifiers, transistors or diodes for heat flow is very appealing [237]. We ask the question, can we build a thermodynamic cloak such that a conductive object is not only isolated from heat, but also does not perturb the heat flow? The answer is yes. Application of coordinate transformations to the form-invariant heat diffusion equation has led to the formulation of the ideal thermal cloak and promising theoretical proposals for molding the flow of heat [238–245]. Interesting experiments on thermal cloaking implementing these concepts in a 2D planar structure have been reported [246, 247]. Thermal cloaking have also been realized in 3D, based on a simple static design, using thin cloaks made of two layers of homogeneous materials [248–250].

5.3. Quantum Matter Waves

Any particle, according to quantum mechanics, can be described by a wave function, or probability amplitude, the square modulus of which gives the probability of presence of the particle at any particular point. Such wave functions describe matterwaves that obey a wave equation, the Schrodinger equation, mathematically analogous to electromagnetic waves, which obey Maxwell's equations. Because any particle has wave-like properties at the quantum scale, it may be possible to cloak them, effectively rendering matter invisible to these waves, or particle beams. In a pioneering paper [251], it was shown that a potential distribution of finite extent can be cloaked from matter waves providing the potential distribution is surrounded by a quantum cloak made of a medium with specific anisotropic inhomogeneous effective mass and potential [251–255]. This is obtained by applying the transformation method to the effective-mass Schrodinger equation, which is form-invariant under coordinate transformations. Because such a transformation cloak may be extremely challenging to realize in practice, the scattering cancellation technique was instead proposed [256], showing that potential wells in realistic semiconductors can be cloaked from incident electrons by using homogeneous and isotropic layers [256, 257]. This may be used to realize furtive quantum sensors [258, 259]. Graphene may also be used to implement electron cloaks [260]. Matter-wave metamaterials are currently under active investigation, and could also be of interest to build electron cloaks [261–264].

5.4. Surface Liquid Waves

We briefly mention the case of transformation cloaks that have been theoretically proposed and experimentally realized for surface waves at the interface between a liquid and a gas [265, 266]. This may have application for shielding floating bodies from surface waves.

6. APPLICATIONS OF CLOAKING TO NON-INVASIVE SENSING

We illustrate now what we believe to be a particularly promising perspective and application in the science and technology of cloaking. As discussed in the above sections, all-angle invisibility for human-scale bodies in the visible is extremely challenging to achieve, even if the phase requirement is dropped. This is because human-scale objects are considerably larger than the wavelength of light, and visible frequencies correspond to quite a broadband window to cover. However, very promising applications of the science of cloaking can be found if these stringent requirements of size, bandwidth and frequency range are relaxed. A relevant research direction is therefore to look for applications that do not require these severe specifications. In this part we give an example of such an application, and highlight the potentials of using cloaks to build invisible sensors that do not perturb the field that they measure. Sensors can be relatively small and narrowband. They make perfect candidates for transitioning the science of cloaking to industrial applications.

6.1. Cloaking a Sensor

There is a long history [267–274] of the science of low-interference communications systems and furtive measurement devices, dating back to the concept of minimum-scattering antennas [33]. Invisibility cloaks may open promising new venues to these questions. The concept of cloaked sensors [275, 276], proposed in 2009, is illustrated in Fig. 24. Panel a shows a simple sketch of a bare sensor, capable of receiving some power from the impinging radiation to perform a sensing operation. This power is associated with the absorption cross-section of the sensor. At the same time, such a receiving system also scatters a portion of the incident power, which is associated with its scattering cross-section. This scattered wave disturbs the probed field, potentially perturbing the measurement, and makes the sensor detectable to its surroundings. As a result, the higher the ratio between absorption cross-section and

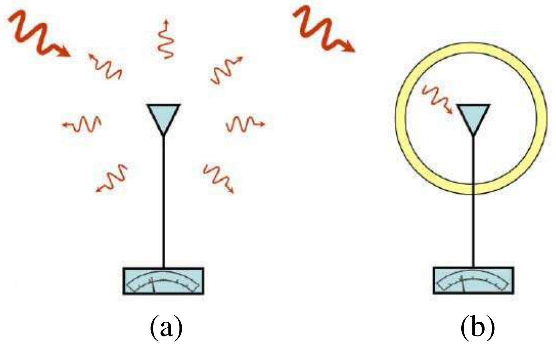


Figure 24. (a) A conventional sensor may be able to sense a signal, but this is associated with scattering, which disturbs the probed field and make the sensor detectable. (b) The sensor can be cloaked in order to cancel or reduce its scattering but maintain its ability to receive a signal. ©2009 APS. Reprinted with permission [275].

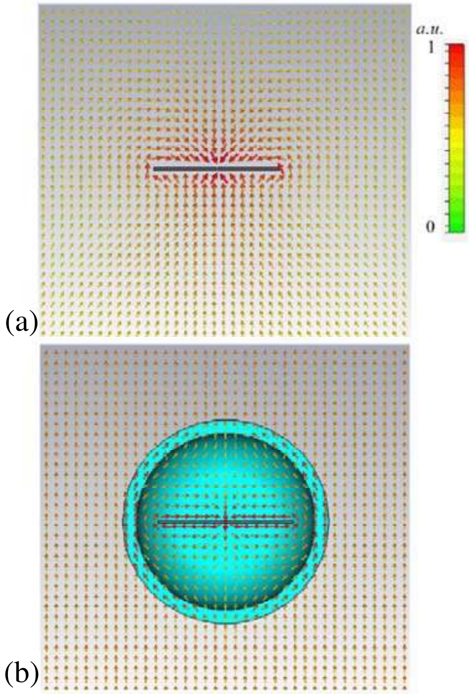


Figure 25. (a) Average power in the near field of a dipole antenna in receiving mode. The antenna is illuminated by a plane wave. It is able to receive power, but strongly disturbs the straight power flow lines of the incident wave. (b) When the cloak is employed, the antenna still receive power, but the straight power flow stream lines are restored. ©2009 APS. Reprinted with permission [275].

the scattering cross-section, the less-intrusive the sensor and the better the measurement is expected to be. The purpose of the cloak is to drastically increase this ratio, called absorption efficiency, as represented in panel b, by strongly reducing the scattered field.

Figure 25(a) shows a numerical simulation for a realistic RF dipole antenna, on which a plane wave is incident from bottom to top. The arrow plot represents the average power flow. The antenna clearly picks up some power, as indicated by the inward directed Poynting vector near the gap of the dipole. At the same time, one can notice that the Poynting vector arrows far from the antenna do not describe parallel stream lines, as one would expect for the incident field alone. This means that the sensing operation is accompanied by strong scattering. Panel b shows the same antenna, but surrounded by a properly designed plasmonic cloak that aims at canceling most of the dominant dipolar scattering from the antenna. One can see, looking inside the cloak, that the antenna is still able to receive power. However, outside of the cloak, the arrows indicating the power flow are all parallel and of equal magnitude, indicating that the scattering is now negligible. This realistic numerical example shows that the concept of non-invasive cloaked sensor is viable.

For RF antennas, there are also other strategies to control scattering and absorption, such as proper loading with an electrical circuit [267, 277]. Therefore at RF, the use of a cloak is an alternative technique that may bring more flexibility in designing low-scattering antennas [278–281], or antenna arrays with reduced mutual blockage [282, 283]. At infrared or optical frequencies however, this technique opens new venues for realizing furtive sensing systems. Fig. 26 illustrates the possibility of using plasmonic cloaks to cover the tip of near field scanning optical microscopes (NSOM) operating in the aperture mode [284, 285]. The top panel represents the field associated with a surface plasmon emitted from the far left side of the figure and propagating towards the right at the interface between a silver metallic

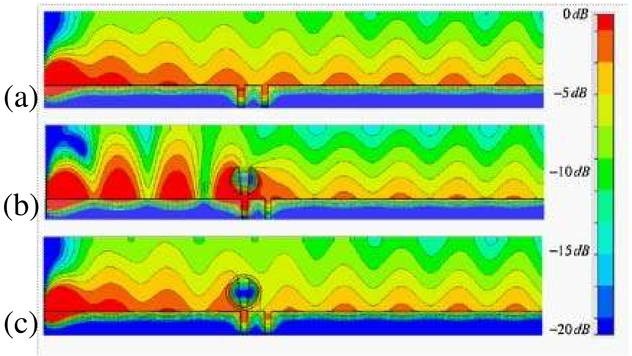


Figure 26. (a) Field associated with a surface plasmon propagating at the planar surface of a silver metallic block is emitted on the far left of the figure and incident on two slits. (b) The same plasmon when an aperture mode NSOM tip is performing a measurement. The field is greatly disturbed by the presence of the sensor. (c) When the NSOM tip is cloaked, the measured field is the same as in absence of sensor. ©2010 APS. Reprinted with permission [276].

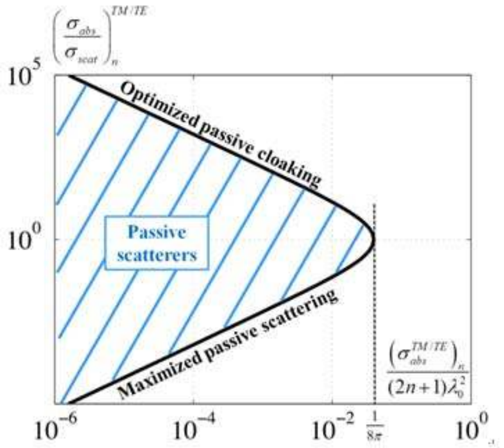


Figure 27. Fundamental bounds in absorption and scattering for spherically symmetric passive scatterers, quantifying the possibilities associated with cloaked sensors. ©2014 APS. Reprinted with permission [287].

block and free-space. This plasmon is incident on two-slits, and one desires to measure the near field around these slits. When the NSOM tip is employed (middle panel) the plasmon is strongly disturbed by the presence of the sensor, and as a result the field exciting the entrance of the aperture is much different from what it would be in absence of the sensor. When the tip is surrounded by a plasmonic cloak however, the plasmon is much less disturbed, and the field exciting the tip is almost identical to what it is in absence of the sensor.

The concept of invisible sensors has been recently validated experimentally at optical frequencies [286]. The sensor was made of a semiconductor rod connected to two electrodes. Light incident on the sensor generates an electric photocurrent that is used for sensing. By covering the sensor with a plasmonic layer, it was shown that for a given polarization, the total scattering from the sensor was strongly reduced, while absorption was maintained.

6.2. Fundamental Limitations of Passively Cloaked Sensors

In a recent contribution [287], the authors have studied the fundamental limitations associated with the concept of cloaked sensors. From the optical theorem [288], we already know that it is not allowed for a passive linear causal system to absorb some energy and at the same time have identically zero scattering cross-section. We have quantified upper and lower bounds on the absorption efficiency associated with a given TE or TM scattering harmonic of order n. This fundamental limit is only valid for passive systems, which necessarily belong to the blue hachured region of Fig. 27.

To illustrate these fundamental limitations, we show in Fig. 28 the particular example of a 40 nm silicon nanoparticle in the visible, which is a totally passive object that must abide by the physical bound. Because of losses inside the silicon, this nanoparticle absorbs some energy as it scatters light. Over the visible range, its scattering and absorption cross-sections have the same order of magnitude (panel a). If turned into a sensor, the object would be pretty intrusive. By covering the nanoparticle with a properly designed silver shell, it is possible to significantly boost the ratio between absorption and scattering over the visible range (panel b), and make the absorber considerably more efficient. In panel c, we take the data from panels a and b and represent it inside the fundamental TM_1 bound. It

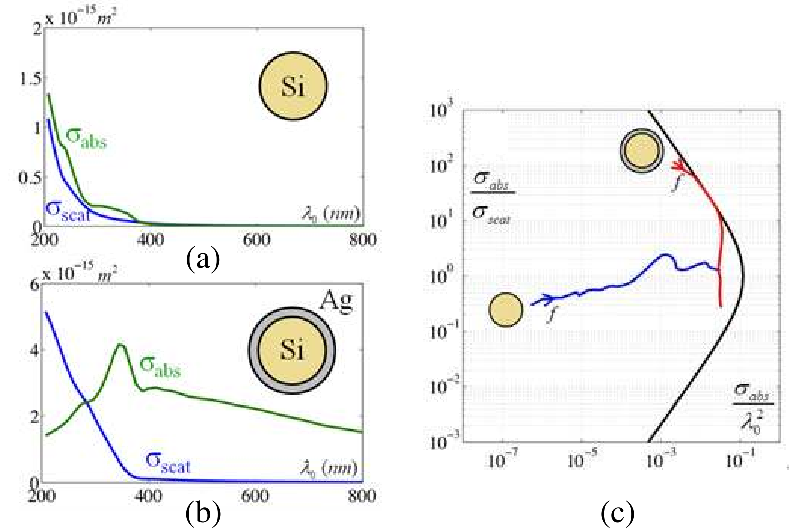


Figure 28. Fundamental bounds in absorption and scattering for spherically symmetric passive scatterers, quantifying the possibilities associated with cloaked sensors. ©2014 APS. Reprinted with permission [287].

can be inferred that the effect of the cloak is to bring the system up to the high absorption efficiency region, but it can never go higher than the black line, which essentially quantifies the limits of passively cloaked dipolar absorbers and sensors.

Cloaked sensors have also been proposed in designs based on transformation optics or cloak/anti-cloak interactions [289]. The concept has been extended to acoustic [290] and matter waves [258]. Note that the above-mentioned fundamental limitations only apply to passive systems. We conjecture that active cloaking schemes can be employed to achieve identically zero scattering in a system that is still able to perform a sensing operation.

7. CONCLUSIONS

In this paper, we have reviewed the recent developments in the science of invisibility and cloaking. We have highlighted what we believe to be dominant trends in this research area. A first trend, simplification of the cloak designs with calculated compromises on the cloaking performance, was illustrated as we described the evolution from the initial proposal for perfect scattering cancellation via transformation optics to more practical designs, sacrificing for instance the phase requirement (non-Euclidean transformation cloaking), omnidirectionality (carpet cloaking) or perfect scattering cancellation (plasmonic, mantle, TL networks, etc.). A second important trend consists of translating the science of cloaking to other physical systems where cloaks may be easier to realize, as illustrated in our review of acoustic, thermal flux, matter-waves and liquid wave cloaking. The final trend, finding applications of cloaking with less stringent requirement than invisibility at visible frequencies, was exemplified by our review of the new possibilities enabled by cloaks in the field of non-invasive sensing.

ACKNOWLEDGMENT

This works has been partially supported by the AFOSR award No. FA9550-13-1-0204, by the NSF CAREER award No. ECCS-0953311, and the DTRA YIP award No. HDTRA1-12-1-0022.

REFERENCES

1. Caloz, C. and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*, John Wiley & Sons, 2005.
2. Eleftheriades, G. V. and K. G. Balmain, *Negative-refraction Metamaterials: Fundamental Principles and Applications*, John Wiley & Sons, 2005.

3. Engheta, N. and R. W. Ziolkowski, *Metamaterials: Physics and Engineering Explorations*, John Wiley & Sons, 2006.
4. Sarychev, A. K. and V. M. Shalaev, *Electrodynamics of Metamaterials*, World Scientific, 2007.
5. Cai, W. and V. M. Shalaev, *Optical Metamaterials: Fundamentals and Applications*, Springer, 2009.
6. Cui, T. J., D. R. Smith, and R. Liu, *Metamaterials: Theory, Design, and Applications*, Springer, 2009.
7. Capolino, F., *Theory and Phenomena of Metamaterials*, CRC Press, 2009.
8. Capolino, F., *Applications of Metamaterials*, CRC Press, 2009.
9. Marqués, R., F. Martín, and M. Sorolla, *Metamaterials with Negative Parameters: Theory, Design and Microwave Applications*, John Wiley & Sons, 2011.
10. Shvets, G. and I. Tsukerman, *Plasmonics and Plasmonic Metamaterials: Analysis and Applications*, World Scientific, 2012.
11. Craster, R. V. and S. Guenneau, *Acoustic Metamaterials: Negative Refraction, Imaging, Lensing and Cloaking*, Springer, 2012.
12. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics Uspekhi*, Vol. 10, No. 4, 509–514, Apr. 1968.
13. Pendry, J. B., "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, Oct. 2000.
14. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, Vol. 84, No. 18, 4184–4187, May 2000.
15. Pendry, J., "Optics: Positively negative," *Nature*, Vol. 423, No. 6935, 22–23, May 2003.
16. Smith, D. R., J. B. Pendry, and M. C. K. Wiltshire, "Metamaterials and negative refractive index," *Science*, Vol. 305, No. 5685, 788–792, Aug. 2004.
17. Noginov, M. A., H. Li, Y. A. Barnakov, D. Dryden, G. Nataraj, G. Zhu, C. E. Bonner, M. Mayy, Z. Jacob, and E. E. Narimanov, "Controlling spontaneous emission with metamaterials," *Opt. Lett.*, Vol. 35, No. 11, 1863–1865, Jun. 2010.
18. Jacob, Z., J.-Y. Kim, G. V. Naik, A. Boltasseva, E. E. Narimanov, and V. M. Shalaev, "Engineering photonic density of states using metamaterials," *Appl. Phys. B*, Vol. 100, No. 1, 215–218, Jul. 2010.
19. Alù, A. and N. Engheta, "Boosting molecular fluorescence with a plasmonic nanolauncher," *Phys. Rev. Lett.*, Vol. 103, No. 4, 043902, Jul. 2009.
20. Fleury, R. and A. Alù, "Enhanced superradiance in epsilon-near-zero plasmonic channels," *Phys. Rev. B*, Vol. 87, No. 20, 201101, May 2013.
21. Silveirinha, M. and N. Engheta, "Tunneling of electromagnetic energy through subwavelength channels and bends using ε-near-zero materials," *Phys. Rev. Lett.*, Vol. 97, No. 15, 157403, Oct. 2006.
22. Edwards, B., A. Alù, M. E. Young, M. Silveirinha, and N. Engheta, "Experimental verification of epsilon-near-zero metamaterial coupling and energy squeezing using a microwave waveguide," *Phys. Rev. Lett.*, Vol. 100, No. 3, 033903, Jan. 2008.
23. Fleury, R. and A. Alù, "Extraordinary sound transmission through density-near-zero ultranarrow channels," *Phys. Rev. Lett.*, Vol. 111, No. 5, 055501, Jul. 2013.
24. Alù, A. and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: Resonance, tunneling and transparency," *IEEE Trans. Antennas Prop.*, Vol. 51, No. 10, 2558–2571, Oct. 2003.
25. Alù, A., G. D'Aguanno, N. Mattiucci, and M. J. Bloemer, "Plasmonic brewster angle: Broadband extraordinary transmission through optical gratings," *Phys. Rev. Lett.*, Vol. 106, No. 12, 123902, Mar. 2011.
26. Kerker, M., "Invisible bodies," *J. Opt. Soc. Am.*, Vol. 65, No. 4, 376–379, Apr. 1975.
27. Chew, H. and M. Kerker, "Abnormally low electromagnetic scattering cross sections," *J. Opt. Soc. Am.*, Vol. 66, No. 5, 445–449, May 1976.

28. Hertz, P., "Die Bewegung eines Elektrons unter dem Einflusse einer stets gleich gerichteten Kraft," *Math. Ann.*, Vol. 65, No. 1, 1–86, Mar. 1907.
29. Bohm, D. and M. Weinstein, "The self-oscillations of a charged particle," *Phys. Rev.*, Vol. 74, No. 12, 1789–1798, Dec. 1948.
30. Goedecke, G. H., "Classically radiationless motions and possible implications for quantum theory," *Phys. Rev.*, Vol. 135, No. 1B, B281–B288, Jul. 1964.
31. Hoenders, B. J., "Existence of invisible nonscattering objects and nonradiating sources," *J. Opt. Soc. Am. A*, Vol. 14, No. 1, 262–266, Jan. 1997.
32. Boardman, A. D., K. Marinov, N. Zheludev, and V. A. Fedotov, "Dispersion properties of nonradiating configurations: Finite-difference time-domain modeling," *Phys. Rev. E*, Vol. 72, No. 3, 036603, Sep. 2005.
33. Kahn, W. K. and H. Kurss, "Minimum-scattering antennas," *IEEE Trans. Antennas Prop.*, Vol. 13, No. 5, 671–675, 1965.
34. Alexopoulos, N. G. and N. K. Uzunoglu, "Electromagnetic scattering from active objects: Invisible scatterers," *Applied Optics*, Vol. 17, No. 2, 235–239, 1978.
35. Kildal, P.-S., A. A. Kishk, and A. Tengs, "Reduction of forward scattering from cylindrical objects using hard surfaces," *IEEE Trans. Antennas Prop.*, Vol. 44, No. 11, 1509–1520, 1996.
36. Devaney, A. J. and G. Sherman, "Nonuniqueness in inverse source and scattering problems," *IEEE Trans. Antennas Prop.*, Vol. 30, No. 5, 1034–1037, 1982.
37. Devaney, A. J., "Nonuniqueness in the inverse scattering problem," *Journal of Mathematical Physics*, Vol. 19, No. 7, 1526–1531, Aug. 2008.
38. Greenleaf, A., M. Lassas, and G. Uhlmann, "On nonuniqueness for Calderón's inverse problem," *Mathematical Research Letters*, Vol. 10, No. 5, 685–693, 2003.
39. Greenleaf, A., M. Lassas, and G. Uhlmann, "Anisotropic conductivities that cannot be detected by EIT," *Physiol. Meas.*, Vol. 24, No. 2, 413, May 2003.
40. Monticone, F. and A. Alù, "Do cloaked objects really scatter less?," *Phys. Rev. X*, Vol. 3, No. 4, 041005, Oct. 2013.
41. Monticone, F. and A. Alù, "On the physical bounds of cloaking and invisibility," *7th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics — Metamaterials 2013*, Bordeaux, France, Sep. 16–21, 2013.
42. Tamm, I. Y., "Electrodynamics of an anisotropic medium in the special theory of relativity," *J. Russ. Phys. Chem. Soc.*, Vol. 56, 248, 1924 (in Russian).
43. Tamm, I. Y., "Crystal-optics of the theory of relativity pertinent to the geometry of a bi-quadratic form," *J. Russ. Phys. Chem. Soc.*, Vol. 56, 1, 1925 (in Russian).
44. Van Dantzig, D., "The fundamental equations of electromagnetism, independent of metrical geometry," *Mathematical Proceedings of the Cambridge Philosophical Society*, Vol. 30, No. 04, 421–427, 1934.
45. Dolin, L. S., "On a possibility of comparing three-dimensional electromagnetic systems with inhomogeneous filling," *Izv. Vyssh. Uchebn. Zaved., Radiofiz.*, Vol. 4, 964–967, 1961.
46. Post, E. G., *Formal Structure of Electromagnetics: General Covariance and Electromagnetics*, Interscience Publishers, New York, 1962.
47. Lax, M. and D. F. Nelson, "Maxwell equations in material form," *Phys. Rev. B*, Vol. 13, No. 4, 1777–1784, Feb. 1976.
48. Ward, A. J. and J. B. Pendry, "Refraction and geometry in Maxwell's equations," *Journal of Modern Optics*, Vol. 43, No. 4, 773–793, 1996.
49. Teixeira, F. L. and W. C. Chew, "Lattice electromagnetic theory from a topological viewpoint," *Journal of Mathematical Physics*, Vol. 40, No. 1, 169–187, Jan. 1999.
50. Teixeira, F. L. and W. C. Chew, "Differential forms, metrics, and the reflectionless absorption of electromagnetic waves," *Journal of Electromagnetic Waves and Applications*, Vol. 13, No. 5, 665–686, 1999.
51. Leonhardt, U., "Notes on waves with negative phase velocity," *IEEE Journal of Selected Topics in*

Quantum Electronics, Vol. 9, No. 1, 102–105, 2003.

52. Chen, H., B. Hou, S. Chen, X. Ao, W. Wen, and C. T. Chan, "Design and experimental realization of a broadband transformation media field rotator at microwave frequencies," *Phys. Rev. Lett.*, Vol. 102, No. 18, 183903, May 2009.
53. Schurig, D., J. B. Pendry, and D. R. Smith, "Transformation-designed optical elements," *Opt. Express*, Vol. 15, No. 22, 14772–14782, Oct. 2007.
54. Liu, Y., T. Zentgraf, G. Bartal, and X. Zhang, "Transformational plasmon optics," *Nano Lett.*, Vol. 10, No. 6, 1991–1997, Jun. 2010.
55. Alù, A., F. Bilotti, and L. Vegni, "Generalized transmission line equations for bianisotropic materials," *IEEE Trans. Antennas Prop.*, Vol. 51, No. 11, 3134–3141, Nov. 2003.
56. Alù, A., F. Bilotti, and L. Vegni, "Method of lines numerical analysis of conformal antennas," *IEEE Trans. Antennas Prop.*, Vol. 52, No. 6, 1530–1540, Jun. 2004.
57. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, No. 5781, 1780–1782, Jun. 2006.
58. Leonhardt, U., "Optical conformal mapping," *Science*, Vol. 312, No. 5781, 1777–1780, Jun. 2006.
59. Shalaev, V. M., "Transforming light," *Science*, Vol. 322, No. 5900, 384–386, Oct. 2008.
60. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Cloaking devices, electromagnetic wormholes, and transformation optics," *SIAM Review*, Vol. 51, No. 1, 3–33, Feb. 2009.
61. Chen, H., C. T. Chan, and P. Sheng, "Transformation optics and metamaterials," *Nat. Mater.*, Vol. 9, No. 5, 387–396, May 2010.
62. Zhang, B., "Electrodynamics of transformation-based invisibility cloaking," *Light Sci. Appl.*, Vol. 1, No. 10, e32, Oct. 2012.
63. Leonhardt, U., "To invisibility and beyond," *Nature*, Vol. 471, No. 7338, 292–293, Mar. 2011.
64. Wood, B., "Metamaterials and invisibility," *Comptes Rendus Physique*, Vol. 10, No. 5, 379–390, Jun. 2009.
65. Sheng, P., "Waves on the Horizon," *Science*, Vol. 313, No. 5792, 1399–1400, Sep. 2006.
66. Leonhardt, U., "Notes on conformal invisibility devices," *New J. Phys.*, Vol. 8, No. 7, 118, Jul. 2006.
67. Schurig, D., J. B. Pendry, and D. R. Smith, "Calculation of material properties and ray tracing in transformation media," *Opt. Express*, Vol. 14, No. 21, 9794–9804, Oct. 2006.
68. Leonhardt U. and T. G. Philbin, "General relativity in electrical engineering," *New J. Phys.*, Vol. 8, No. 10, 247, Oct. 2006.
69. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, and J. Pendry, "Full-wave simulations of electromagnetic cloaking structures," *Phys. Rev. E*, Vol. 74, No. 3, 036621, Sep. 2006.
70. Zolla, F., S. Guenneau, A. Nicolet, and J. B. Pendry, "Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect," *Opt. Lett.*, Vol. 32, No. 9, 1069–1071, May 2007.
71. Ruan, Z., M. Yan, C. W. Neff, and M. Qiu, "Ideal cylindrical cloak: Perfect but sensitive to tiny perturbations," *Phys. Rev. Lett.*, Vol. 99, No. 11, 113903, Sep. 2007.
72. Chen, H., B.-I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic wave interactions with a metamaterial cloak," *Phys. Rev. Lett.*, Vol. 99, No. 6, 063903, Aug. 2007.
73. Zhang, B., H. Chen, B.-I. Wu, Y. Luo, L. Ran, and J. A. Kong, "Response of a cylindrical invisibility cloak to electromagnetic waves," *Phys. Rev. B*, Vol. 76, No. 12, 121101, Sep. 2007.
74. Yaghjian, A. D. and S. Maci, "Alternative derivation of electromagnetic cloaks and concentrators," *New J. Phys.*, Vol. 10, No. 11, 115022, Dec. 2008.
75. Perczel, J., T. Tyc, and U. Leonhardt, "Invisibility cloaking without superluminal propagation," *New J. Phys.*, Vol. 13, No. 8, 083007, Aug. 2011.
76. Sihvola, A., S. Tretyakov, and A. de Baas, "Metamaterials with extreme material parameters," *J. Commun. Technol. Electron.*, Vol. 52, No. 9, 986–990, Sep. 2007.
77. Alù, A. and N. Engheta, "Extremely anisotropic boundary conditions and their optical applications," *Radio Science, Special Issue for URSI EMTS 2010*, Vol. 46, RS0E11, Sep. 2011.
78. Alù, A. and N. Engheta, "Optical nanoswitch: An engineered plasmonic nanoparticle with extreme

parameters and giant anisotropy," *New J. Phys.*, Vol. 11, No. 1, 013026, Jan. 2009.

79. Alù, A., M. G. Silveirinha, A. Salandrino, and N. Engheta, "Epsilon-near-zero metamaterials and electromagnetic sources: Tailoring the radiation phase pattern," *Phys. Rev. B*, Vol. 75, No. 15, 155410, Apr. 2007.
80. Alù, A., M. G. Silveirinha, and N. Engheta, "Transmission-line analysis of ε-near-zero–filled narrow channels," *Phys. Rev. E*, Vol. 78, No. 1, 016604, Jul. 2008.
81. Ziolkowski, R. W., "Propagation in and scattering from a matched metamaterial having a zero index of refraction," *Phys. Rev. E*, Vol. 70, No. 4, 046608, Oct. 2004.
82. Alù, A. and N. Engheta, "Coaxial-to-waveguide matching with -near-zero ultranarrow channels and bends," *IEEE Trans. Antennas Prop.*, Vol. 58, No. 2, 328–339, 2010.
83. Alù, A. and N. Engheta, "Optical 'shorting wires'," *Opt. Express*, Vol. 15, No. 21, 13773–13782, Oct. 2007.
84. Liu, R., Q. Cheng, T. Hand, J. J. Mock, T. J. Cui, S. A. Cummer, and D. R. Smith, "Experimental demonstration of electromagnetic tunneling through an epsilon-near-zero metamaterial at microwave frequencies," *Phys. Rev. Lett.*, Vol. 100, No. 2, 023903, Jan. 2008.
85. Choi, M., S. H. Lee, Y. Kim, S. B. Kang, J. Shin, M. H. Kwak, K.-Y. Kang, Y.-H. Lee, N. Park, and B. Min, "A terahertz metamaterial with unnaturally high refractive index," *Nature*, Vol. 470, No. 7334, 369–373, Feb. 2011.
86. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, No. 5801, 977–980, Nov. 2006.
87. Kanté, B., D. Germain, and A. de Lustrac, "Experimental demonstration of a nonmagnetic metamaterial cloak at microwave frequencies," *Phys. Rev. B*, Vol. 80, No. 20, 201104, Nov. 2009.
88. Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nat. Photon.*, Vol. 1, No. 4, 224–227, Apr. 2007.
89. Cai, W., U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, "Nonmagnetic cloak with minimized scattering," *Applied Physics Letters*, Vol. 91, No. 11, 111105, Sep. 2007.
90. Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Designs for optical cloaking with high-order transformations," *Opt. Express*, Vol. 16, No. 8, 5444–5452, Apr. 2008.
91. Yan, M., Z. Ruan, and M. Qiu, "Cylindrical invisibility cloak with simplified material parameters is inherently visible," *Phys. Rev. Lett.*, Vol. 99, No. 23, 233901, Dec. 2007.
92. Chen, H., X. Luo, H. Ma, and C. T. Chan, "The anti-cloak," *Opt. Express*, Vol. 16, No. 19, 14603–14608, Sep. 2008.
93. Castaldi, G., I. Gallina, V. Galdi, A. Alù, and N. Engheta, "Analytical study of spherical cloak/anti-cloak interactions," *Wave Motion*, Vol. 48, No. 6, 455–467, Sep. 2011.
94. Castaldi, G., I. Gallina, V. Galdi, A. Alù, and N. Engheta, "Cloak/anti-cloak interactions," *Opt. Express*, Vol. 17, No. 5, 3101–3114, Mar. 2009.
95. Leonhardt, U. and T. Tyc, "Broadband invisibility by non-euclidean cloaking," *Science*, Vol. 323, No. 5910, 110–112, Jan. 2009.
96. Hendi, A., J. Henn, and U. Leonhardt, "Ambiguities in the scattering tomography for central potentials," *Phys. Rev. Lett.*, Vol. 97, No. 7, 073902, Aug. 2006.
97. Halimeh, J. C. and M. Wegener, "Time-of-flight imaging of invisibility cloaks," *Opt. Express*, Vol. 20, No. 1, 63–74, Jan. 2012.
98. Li, J. and J. B. Pendry, "Hiding under the carpet: A new strategy for cloaking," *Phys. Rev. Lett.*, Vol. 101, No. 20, 203901, Nov. 2008.
99. Dolling, G., M. Wegener, S. Linden, and C. Hormann, "Photorealistic images of objects in effective negative-index materials," *Opt. Express*, Vol. 14, No. 5, 1842–1849, Mar. 2006.
100. Halimeh, J. C., T. Ergin, J. Mueller, N. Stenger, and M. Wegener, "Photorealistic images of carpet cloaks," *Opt. Express*, Vol. 17, No. 22, 19328–19336, Oct. 2009.
101. Ergin, T., J. C. Halimeh, N. Stenger, and M. Wegener, "Optical microscopy of 3D carpet cloaks: Ray-tracing calculations," *Opt. Express*, Vol. 18, No. 19, 20535–20545, Sep. 2010.

102. Danner, A. J., "Visualizing invisibility: Metamaterials-based optical devices in natural environments," *Opt. Express*, Vol. 18, No. 4, 3332–3337, Feb. 2010.
103. Halimeh, J. C. and M. Wegener, "Photorealistic ray tracing of free-space invisibility cloaks made of uniaxial dielectrics," *Opt. Express*, Vol. 20, No. 27, 28330–28340, Dec. 2012.
104. Halimeh, J. C. and M. Wegener, "Photorealistic rendering of unidirectional free-space invisibility cloaks," *Opt. Express*, Vol. 21, No. 8, 9457–9472, Apr. 2013.
105. Zhang, B., T. Chan, and B.-I. Wu, "Lateral shift makes a ground-plane cloak detectable," *Phys. Rev. Lett.*, Vol. 104, No. 23, 233903, Jun. 2010.
106. Landy, N. I., N. Kundtz, and D. R. Smith, "Designing three-dimensional transformation optical media using quasiconformal coordinate transformations," *Phys. Rev. Lett.*, Vol. 105, No. 19, 193902, Nov. 2010.
107. Liu, R., C. Ji, J. J. Mock, J. Y. Chin, T. J. Cui, and D. R. Smith, "Broadband ground-plane cloak," *Science*, Vol. 323, No. 5912, 366–369, Jan. 2009.
108. Valentine, J., J. Li, T. Zentgraf, G. Bartal, and X. Zhang, "An optical cloak made of dielectrics," *Nat. Mater.*, Vol. 8, No. 7, 568–571, Jul. 2009.
109. Lee, J. H., J. Blair, V. A. Tamma, Q. Wu, S. J. Rhee, C. J. Summers, and W. Park, "Direct visualization of optical frequency invisibility cloak based on silicon nanorod array," *Opt. Express*, Vol. 17, No. 15, 12922–12928, Jul. 2009.
110. Gabrielli, L. H., J. Cardenas, C. B. Poitras, and M. Lipson, "Silicon nanostructure cloak operating at optical frequencies," *Nat. Photon.*, Vol. 3, No. 8, 461–463, Aug. 2009.
111. Ergin, T., N. Stenger, P. Brenner, J. B. Pendry, and M. Wegener, "Three-dimensional invisibility cloak at optical wavelengths," *Science*, Vol. 328, No. 5976, 337–339, Apr. 2010.
112. Ma, H. F. and T. J. Cui, "Three-dimensional broadband ground-plane cloak made of metamaterials," *Nat. Commun.*, Vol. 1, 21, Jun. 2010.
113. Gharghi, M., C. Gladden, T. Zentgraf, Y. Liu, X. Yin, J. Valentine, and X. Zhang, "A carpet cloak for visible light," *Nano Lett.*, Vol. 11, No. 7, 2825–2828, Jul. 2011.
114. Shin, D., Y. Urzhumov, Y. Jung, G. Kang, S. Baek, M. Choi, H. Park, K. Kim, and D. R. Smith, "Broadband electromagnetic cloaking with smart metamaterials," *Nat. Commun.*, Vol. 3, 1213, Nov. 2012.
115. Smolyaninov, I. I., V. N. Smolyaninova, A. V. Kildishev, and V. M. Shalaev, "Anisotropic metamaterials emulated by tapered waveguides: Application to optical cloaking," *Phys. Rev. Lett.*, Vol. 102, No. 21, 213901, May 2009.
116. Luo, Y., J. Zhang, H. Chen, L. Ran, B.-I. Wu, and J.-A. Kong, "A rigorous analysis of plane-transformed invisibility cloaks," *IEEE Trans. Antennas Prop.*, Vol. 57, No. 12, 3926–3933, 2009.
117. Xi, S., H. Chen, B.-I. Wu, and J.-A. Kong, "One-directional perfect cloak created with homogeneous material," *IEEE Microwave and Wireless Components Letters*, Vol. 19, No. 3, 131–133, 2009.
118. Zhang, B., Y. Luo, X. Liu, and G. Barbastathis, "Macroscopic invisibility cloak for visible light," *Phys. Rev. Lett.*, Vol. 106, No. 3, 033901, Jan. 2011.
119. Chen, X., Y. Luo, J. Zhang, K. Jiang, J. B. Pendry, and S. Zhang, "Macroscopic invisibility cloaking of visible light," *Nat. Commun.*, Vol. 2, 176, Feb. 2011.
120. Liang, D., J. Gu, J. Han, Y. Yang, S. Zhang, and W. Zhang, "Robust large dimension terahertz cloaking," *Advanced Materials*, Vol. 24, No. 7, 916–921, 2012.
121. Chen, H. and B. Zheng, "Broadband polygonal invisibility cloak for visible light," *Sci. Rep.*, Vol. 2, 255, 2012.
122. Landy, N. and D. R. Smith, "A full-parameter unidirectional metamaterial cloak for microwaves," *Nat. Mater.*, Vol. 12, No. 1, 25–28, Jan. 2013.
123. Urzhumov, Y., N. Landy, T. Driscoll, D. Basov, and D. R. Smith, "Thin low-loss dielectric coatings for free-space cloaking," *Opt. Lett.*, Vol. 38, No. 10, 1606–1608, May 2013.
124. Chen, H., B. Zheng, L. Shen, H. Wang, X. Zhang, N. I. Zheludev, and B. Zhang, "Ray-optics cloaking devices for large objects in incoherent natural light," *Nat. Commun.*, Vol. 4, Oct. 2013.

125. Howell, J. C. and J. B. Howell, “Simple, broadband, optical spatial cloaking of very large objects,” arXiv e-print 1306.0863, Jun. 2013.
126. Houdin, R., *The Secrets of Stage Conjuring*, Wildside Press LLC, 2008.
127. Wood, B. and J. B. Pendry, “Metamaterials at zero frequency,” *J. Phys.: Condens. Matter*, Vol. 19, No. 7, 076208, Feb. 2007.
128. Sanchez, A., C. Navau, J. Prat-Camps, and D.-X. Chen, “Antimagnets: Controlling magnetic fields with superconductor–metamaterial hybrids,” *New J. Phys.*, Vol. 13, No. 9, 093034, Sep. 2011.
129. Narayana, S. and Y. Sato, “DC magnetic cloak,” *Advanced Materials*, Vol. 24, No. 1, 71–74, 2012.
130. Gömöry, F., M. Solovyov, J. Šouc, C. Navau, J. Prat-Camps, and A. Sanchez, “Experimental realization of a magnetic cloak,” *Science*, Vol. 335, No. 6075, 1466–1468, Mar. 2012.
131. Souc, J., M. Solovyov, F. Gömöry, J. Prat-Camps, C. Navau, and A. Sanchez, “A quasistatic magnetic cloak,” *New J. Phys.*, Vol. 15, No. 5, 053019, May 2013.
132. Yang, F., Z. L. Mei, T. Y. Jin, and T. J. Cui, “dc electric invisibility cloak,” *Phys. Rev. Lett.*, Vol. 109, No. 5, 053902, Aug. 2012.
133. Liu, M., Z. L. Mei, X. Ma, and T. J. Cui, “dc illusion and its experimental verification,” *Applied Physics Letters*, Vol. 101, No. 5, 051905, Aug. 2012.
134. Mei, Z. L., Y. S. Liu, F. Yang, and T. J. Cui, “A dc carpet cloak based on resistor networks,” *Opt. Express*, Vol. 20, No. 23, 25758–25765, Nov. 2012.
135. Alù, A. and N. Engheta, “Effects of size and frequency dispersion in plasmonic cloaking,” *Phys. Rev. E*, Vol. 78, No. 4, 045602, Oct. 2008.
136. Papas, C. H., *Theory of Electromagnetic Wave Propagation*, Courier Dover Publications, 2013.
137. Jackson, J. D., *Classical Electrodynamics*, Wiley, 1998.
138. Bohren, C. F. and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, John Wiley & Sons, 2008.
139. Alù, A. and N. Engheta, “Polarizabilities and effective parameters for collections of spherical nanoparticles formed by pairs of concentric double-negative, single-negative, and/or double-positive metamaterial layers,” *Journal of Applied Physics*, Vol. 97, No. 9, 094310, Apr. 2005.
140. Alù, A. and N. Engheta, “Achieving transparency with plasmonic and metamaterial coatings,” *Phys. Rev. E*, Vol. 72, No. 1, 016623, Jul. 2005.
141. Ball, P., “Engineers devise invisibility shield,” *Nature News*, Feb. 2005.
142. Alù, A. and N. Engheta, “Plasmonic and metamaterial cloaking: Physical mechanisms and potentials,” *J. Opt. A: Pure Appl. Opt.*, Vol. 10, No. 9, 093002, Sep. 2008.
143. Chen, P.-Y., J. Soric, and A. Alù, “Invisibility and cloaking based on scattering cancellation,” *Advanced Materials*, Vol. 24, No. 44, OP281–OP304, 2012.
144. Alù, A. and N. Engheta, “Plasmonic materials in transparency and cloaking problems: Mechanism, robustness, and physical insights,” *Opt. Express*, Vol. 15, No. 6, 3318–3332, Mar. 2007.
145. Alù, A. and N. Engheta, “Cloaking and transparency for collections of particles with metamaterial and plasmonic covers,” *Opt. Express*, Vol. 15, No. 12, 7578–7590, Jun. 2007.
146. Alù, A. and N. Engheta, “Multifrequency optical invisibility cloak with layered plasmonic shells,” *Phys. Rev. Lett.*, Vol. 100, No. 11, 113901, Mar. 2008.
147. Alù, A. and N. Engheta, “Theory and potentials of multi-layered plasmonic covers for multi-frequency cloaking,” *New J. Phys.*, Vol. 10, No. 11, 115036, Nov. 2008.
148. Tricarico, S., F. Bilotti, A. Alù, and L. Vegni, “Plasmonic cloaking for irregular objects with anisotropic scattering properties,” *Phys. Rev. E*, Vol. 81, No. 2, 026602, Feb. 2010.
149. Kallos, E., C. Argyropoulos, Y. Hao, and A. Alù, “Comparison of frequency responses of cloaking devices under nonmonochromatic illumination,” *Phys. Rev. B*, Vol. 84, No. 4, 045102, Jul. 2011.
150. Alù, A., D. Rainwater, and A. Kerkhoff, “Plasmonic cloaking of cylinders: Finite length, oblique illumination and cross-polarization coupling,” *New J. Phys.*, Vol. 12, No. 10, 103028, Oct. 2010.
151. Silveirinha, M. G., A. Alù, and N. Engheta, “Cloaking mechanism with antiphase plasmonic satellites,” *Phys. Rev. B*, Vol. 78, No. 20, 205109, Nov. 2008.

152. Silveirinha, M. G., A. Alù, and N. Engheta, "Parallel-plate metamaterials for cloaking structures," *Phys. Rev. E*, Vol. 75, No. 3, 036603, Mar. 2007.
153. Silveirinha, M. G., A. Alù, and N. Engheta, "Infrared and optical invisibility cloak with plasmonic implants based on scattering cancellation," *Phys. Rev. B*, Vol. 78, No. 7, 075107, Aug. 2008.
154. Edwards, B., A. Alù, M. G. Silveirinha, and N. Engheta, "Experimental verification of plasmonic cloaking at microwave frequencies with metamaterials," *Phys. Rev. Lett.*, Vol. 103, No. 15, 153901, Oct. 2009.
155. Rainwater, D., A. Kerkhoff, K. Melin, J. C. Soric, G. Moreno, and A. Alù, "Experimental verification of three-dimensional plasmonic cloaking in free-space," *New J. Phys.*, Vol. 14, No. 1, 013054, Jan. 2012.
156. Munk, B. A., *Frequency Selective Surfaces: Theory and Design*, John Wiley & Sons, 2005.
157. Tretyakov, S., *Analytical Modeling in Applied Electromagnetics*, Artech House, 2003.
158. Alù, A., "Mantle cloak: Invisibility induced by a surface," *Phys. Rev. B*, Vol. 80, No. 24, 245115, Dec. 2009.
159. Chen, P.-Y. and A. Alù, "Mantle cloaking using thin patterned metasurfaces," *Phys. Rev. B*, Vol. 84, No. 20, 205110, Nov. 2011.
160. Chen, P.-Y., F. Monticone, and A. Alù, "Suppressing the electromagnetic scattering with an helical mantle cloak," *IEEE Antennas and Wireless Propagation Letters*, Vol. 10, 1598–1601, 2011.
161. Padooru, Y. R., A. B. Yakovlev, P.-Y. Chen, and A. Alù, "Line-source excitation of realistic conformal metasurface cloaks," *Journal of Applied Physics*, Vol. 112, No. 10, 104902, Nov. 2012.
162. Padooru, Y. R., A. B. Yakovlev, P.-Y. Chen, and A. Alù, "Analytical modeling of conformal mantle cloaks for cylindrical objects using sub-wavelength printed and slotted arrays," *Journal of Applied Physics*, Vol. 112, No. 3, 034907, Aug. 2012.
163. Chen, P.-Y. and A. Alù, "Atomically thin surface cloak using graphene monolayers," *ACS Nano*, Vol. 5, No. 7, 5855–5863, Jul. 2011.
164. Soric, J. C., P. Y. Chen, A. Kerkhoff, D. Rainwater, K. Melin, and A. Alù, "Demonstration of an ultralow profile cloak for scattering suppression of a finite-length rod in free space," *New J. Phys.*, Vol. 15, No. 3, 033037, Mar. 2013.
165. Alitalo, P., O. Luukkonen, L. Jylha, J. Venermo, and S. A. Tretyakov, "Transmission-line networks cloaking objects from electromagnetic fields," *IEEE Trans. Antennas Prop.*, Vol. 56, No. 2, 416–424, 2008.
166. Alitalo, P., O. Luukkonen, L. Jylha, J. Venermo, and S. A. Tretyakov, "Correction to 'transmission-line networks cloaking objects from electromagnetic fields' [Feb. 08, 416–424]," *IEEE Trans. Antennas Prop.*, Vol. 56, No. 3, 918–918, 2008.
167. Alitalo, P., S. Ranvier, J. Vehmas, and S. Tretyakov, "A microwave transmission-line network guiding electromagnetic fields through a dense array of metallic objects," *Metamaterials*, Vol. 2, No. 4, 206–212, Dec. 2008.
168. Alitalo, P. and S. Tretyakov, "Electromagnetic cloaking with metamaterials," *Materials Today*, Vol. 12, No. 3, 22–29, Mar. 2009.
169. Alitalo, P., F. Bongard, J.-F. Zürcher, J. Mosig, and S. Tretyakov, "Experimental verification of broadband cloaking using a volumetric cloak composed of periodically stacked cylindrical transmission-line networks," *Applied Physics Letters*, Vol. 94, No. 1, 014103, Jan. 2009.
170. Tretyakov, S., P. Alitalo, O. Luukkonen, and C. Simovski, "Broadband electromagnetic cloaking of long cylindrical objects," *Phys. Rev. Lett.*, Vol. 103, No. 10, 103905, Sep. 2009.
171. Alitalo, P. and S. A. Tretyakov, "Electromagnetic cloaking of strongly scattering cylindrical objects by a volumetric structure composed of conical metal plates," *Phys. Rev. B*, Vol. 82, No. 24, 245111, Dec. 2010.
172. Alitalo, P., A. E. Culhaoglu, A. V. Osipov, S. Thurner, E. Kemptner, and S. A. Tretyakov, "Bistatic scattering characterization of a three-dimensional broadband cloaking structure," *Journal of Applied Physics*, Vol. 111, No. 3, 034901–034901–5, 2012.
173. Milton, G. W. and N.-A. P. Nicorovici, "On the cloaking effects associated with anomalous

localized resonance," *Proc. R. Soc. A*, Vol. 462, No. 2074, 3027–3059, Oct. 2006.

174. Milton, G. W., N.-A. P. Nicorovici, R. C. McPhedran, and V. A. Podolskiy, "A proof of superlensing in the quasistatic regime, and limitations of superlenses in this regime due to anomalous localized resonance," *Proc. R. Soc. A*, Vol. 461, No. 2064, 3999–4034, Dec. 2005.
175. Nicorovici, N. A., G. W. Milton, R. C. McPhedran, and L. C. Botten, "Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance," *Opt. Express*, Vol. 15, No. 10, 6314–6323, May 2007.
176. Nicorovici, N.-A. P., R. C. McPhedran, S. Enoch, and G. Tayeb, "Finite wavelength cloaking by plasmonic resonance," *New J. Phys.*, Vol. 10, No. 11, 115020, Nov. 2008.
177. Lai, Y., H. Chen, Z.-Q. Zhang, and C. T. Chan, "Complementary media invisibility cloak that cloaks objects at a distance outside the cloaking shell," *Phys. Rev. Lett.*, Vol. 102, No. 9, 093901, Mar. 2009.
178. Zheng, G., X. Heng, and C. Yang, "A phase conjugate mirror inspired approach for building cloaking structures with left-handed materials," *New J. Phys.*, Vol. 11, No. 3, 033010, Mar. 2009.
179. Chen, H., Z. Liang, P. Yao, X. Jiang, H. Ma, and C. T. Chan, "Extending the bandwidth of electromagnetic cloaks," *Phys. Rev. B*, Vol. 76, No. 24, 241104, Dec. 2007.
180. Miller, D. A. B., "On perfect cloaking," *Opt. Express*, Vol. 14, No. 25, 12457–12466, Dec. 2006.
181. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Full-wave invisibility of active devices at all frequencies," *Commun. Math. Phys.*, Vol. 275, No. 3, 749–789, Nov. 2007.
182. Vasquez, F. G., G. W. Milton, and D. Onofrei, "Active exterior cloaking for the 2D laplace and Helmholtz equations," *Phys. Rev. Lett.*, Vol. 103, No. 7, 073901, Aug. 2009.
183. Guevara Vasquez, F., G. W. Milton, and D. Onofrei, "Broadband exterior cloaking," *Opt. Express*, Vol. 17, No. 17, 14800–14805, Aug. 2009.
184. Ma, Q., Z. L. Mei, S. K. Zhu, T. Y. Jin, and T. J. Cui, "Experiments on active cloaking and illusion for Laplace equation," *Phys. Rev. Lett.*, Vol. 111, No. 17, 173901, Oct. 2013.
185. Chen, P.-Y., C. Argyropoulos, and A. Alù, "Broadening the cloaking bandwidth with non-foster metasurfaces," *Phys. Rev. Lett,* Vol. 111, No. 23, 233001, Dec. 2013.
186. Friot, E., R. Guillermin, and M. Winninger, "Active control of scattered acoustic radiation: A real-time implementation for a three-dimensional object," *Acta Acustica United with Acustica*, Vol. 92, No. 2, 278–288, Mar. 2006.
187. Bender, C. M. and S. Boettcher, "Real spectra in non-hermitian Hamiltonians having PT symmetry," *Phys. Rev. Lett.*, Vol. 80, 5243–5246, 1998.
188. Makris, K. G., R. El-Ganainy, D. N. Christodoulides, Z. H. Musslimani, "Beam dynamics in PT symmetric optical lattices," *Phys. Rev. Lett.*, Vol. 100, 103904, 2008.
189. Kulishov, M. and J. Laniel, N. Bélanger, J. Azaña, D. Plant, "Nonreciprocal waveguide Bragg gratings," *Opt. Express*, Vol. 13, 3068–3078, 2005.
190. Razzari, L. and R. Morandotti, "Optics: Gain and loss mixed in the same cauldron," *Nature*, Vol. 488, 163–164, 2012.
191. Mostafazadeh, A., "Spectral singularities of complex scattering potentials and infinite reflection and transmission coefficients at real energies," *Phys. Rev. Lett.*, Vol. 102, 220402, 2009.
192. Schomerus, H., "Quantum noise and self-sustained radiation of PT-symmetric systems," *Phys. Rev. Lett.*, Vol. 104, 233601, 2010.
193. Lin, Z., et al., "Unidirectional invisibility induced by PT-symmetric periodic structures," *Phys. Rev. Lett.*, Vol. 106, 213901, 2011.
194. Fleury, R., D. L. Sounas, and A. Alù, "Negative refraction and planar focusing based on parity-time symmetric metasurfaces," *Phys. Rev. Lett.*, Vol. 113, No. 2, 023903, Jul. 2014.
195. Xu, S., X. Cheng, S. Xi, R. Zhang, H. O. Moser, Z. Shen, Y. Xu, Z. Huang, X. Zhang, F. Yu, B. Zhang, and H. Chen, "Experimental demonstration of a free-space cylindrical cloak without superluminal propagation," *Phys. Rev. Lett.*, Vol. 109, No. 22, 223903, Nov. 2012.
196. Aliev, A. E., Y. N. Gartstein, and R. H. Baughman, "Mirage effect from thermally modulated transparent carbon nanotube sheets," *Nanotechnology*, Vol. 22, No. 43, 435704, Oct. 2011.

197. Schittny, R., M. Kadic, T. Bückmann, and M. Wegener, "Invisibility cloaking in a diffusive light scattering medium," *Science*, Vol. 345, No. 6195, 427–429, Jun. 2014.
198. Liu, Z., X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. T. Chan, and P. Sheng, "Locally Resonant Sonic Materials," *Science*, Vol. 289, No. 5485, 1734–1736, 2000.
199. Li, J. and C. T. Chan, "Double-negative acoustic metamaterial," *Phys. Rev. E*, Vol. 70, No. 5, 055602, Nov. 2004.
200. Fang, N., D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, and X. Zhang, "Ultrasonic metamaterials with negative modulus," *Nature Materials*, Vol. 5, No. 6, 452–456, 2006.
201. Lee, S. H., C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, "Acoustic metamaterial with negative density," *Physics Letters A*, Vol. 373, No. 48, 4464–4469, Dec. 2009.
202. Bongard, F., H. Lissek, and J. R. Mosig, "Acoustic transmission line metamaterial with negative/zero/positive refractive index," *Phys. Rev. B*, Vol. 82, No. 9, 094306, 2010.
203. Lee, S. H., C. M. Park, Y. M. Seo, Z. G. Wang, and C. K. Kim, "Composite acoustic medium with simultaneously negative density and modulus," *Phys. Rev. Lett.*, Vol. 104, No. 5, 054301, Feb. 2010.
204. Zhou, X. and G. Hu, "Superlensing effect of an anisotropic metamaterial slab with near-zero dynamic mass," *Applied Physics Letters*, Vol. 98, No. 26, 263510–263510-3, Jul. 2011.
205. Liang, Z. and J. Li, "Extreme acoustic metamaterial by coiling up space," *Phys. Rev. Lett.*, Vol. 108, No. 11, 114301, Mar. 2012.
206. Xie, Y., B.-I. Popa, L. Zigoneanu, and S. A. Cummer, "Measurement of a broadband negative index with space-coiling acoustic metamaterials," *Phys. Rev. Lett.*, Vol. 110, No. 17, 175501, Apr. 2013.
207. Frenzel, T., J. D. Brehm, T. Bückmann, R. Schittny, M. Kadic, and M. Wegener, "Three-dimensional labyrinthine acoustic metamaterials," *Applied Physics Letters*, Vol. 103, No. 6, 061907, Aug. 2013.
208. Yang, M., G. Ma, Z. Yang, and P. Sheng, "Coupled membranes with doubly negative mass density and bulk modulus," *Phys. Rev. Lett.*, Vol. 110, No. 13, 134301, Mar. 2013.
209. Fleury, R. and A. Alù, "Extraordinary sound transmission through density-near-zero ultranarrow channels," *Phys. Rev. Lett.*, Vol. 111, No. 5, 055501, Jul. 2013.
210. Milton, G. W., M. Briane, and J. R. Willis, "On cloaking for elasticity and physical equations with a transformation invariant form," *New J. Phys.*, Vol. 8, No. 10, 248, Oct. 2006.
211. Milton, G. W., "New metamaterials with macroscopic behavior outside that of continuum elastodynamics," *New J. Phys.*, Vol. 9, No. 10, 359, Oct. 2007.
212. Cummer, S. A. and D. Schurig, "One path to acoustic cloaking," *New J. Phys.*, Vol. 9, No. 3, 45, Mar. 2007.
213. Chen, H. and C. T. Chan, "Acoustic cloaking in three dimensions using acoustic metamaterials," *Applied Physics Letters*, Vol. 91, No. 18, 183518, Nov. 2007.
214. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, J. Pendry, M. Rahm, and A. Starr, "Scattering theory derivation of a 3D acoustic cloaking shell," *Phys. Rev. Lett.*, Vol. 100, No. 2, 024301, Jan. 2008.
215. Cummer, S. A., M. Rahm, and D. Schurig, "Material parameters and vector scaling in transformation acoustics," *New J. Phys.*, Vol. 10, No. 11, 115025, Nov. 2008.
216. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Full-wave invisibility of active devices at all frequencies," *Commun. Math. Phys.*, Vol. 275, No. 3, 749–789, Nov. 2007.
217. Farhat, M., S. Guenneau, S. Enoch, A. Movchan, F. Zolla, and A. Nicolet, "A homogenization route towards square cylindrical acoustic cloaks," *New J. Phys.*, Vol. 10, No. 11, 115030, Nov. 2008.
218. Milton, G. W. and A. V. Cherkaev, "Which elasticity tensors are realizable?," *J. Eng. Mater. Technol.*, Vol. 117, No. 4, 483–493, Oct. 1995.
219. Kadic, M., T. Bückmann, N. Stenger, M. Thiel, and M. Wegener, "On the practicability of pentamode mechanical metamaterials," *Applied Physics Letters*, Vol. 100, No. 19, 191901, May 2012.

220. Norris, A. N., "Acoustic metafluids," *The Journal of the Acoustical Society of America*, Vol. 125, No. 2, 839–849, Feb. 2009.
221. Torrent, D. and J. Sánchez-Dehesa, "Anisotropic mass density by two-dimensional acoustic metamaterials," *New J. Phys.*, Vol. 10, No. 2, 023004, Feb. 2008.
222. Torrent, D. and J. Sánchez-Dehesa, "Acoustic cloaking in two dimensions: A feasible approach," *New J. Phys.*, Vol. 10, No. 6, 063015, Jun. 2008.
223. Cheng, Y., F. Yang, J. Y. Xu, and X. J. Liu, "A multilayer structured acoustic cloak with homogeneous isotropic materials," *Applied Physics Letters*, Vol. 92, No. 15, 151913, Apr. 2008.
224. Munteanu, L. and V. Chiroiu, "On three-dimensional spherical acoustic cloaking," *New J. Phys.*, Vol. 13, No. 8, 083031, Aug. 2011.
225. Urzhumov, Y., F. Ghezzo, J. Hunt, and D. R. Smith, "Acoustic cloaking transformations from attainable material properties," *New J. Phys.*, Vol. 12, No. 7, 073014, Jul. 2010.
226. Zhang, S., C. Xia, and N. Fang, "Broadband acoustic cloak for ultrasound waves," *Phys. Rev. Lett.*, Vol. 106, No. 2, 024301, Jan. 2011.
227. Popa, B.-I., L. Zigoneanu, and S. A. Cummer, "Experimental acoustic ground cloak in air," *Phys. Rev. Lett.*, Vol. 106, No. 25, 253901, Jun. 2011.
228. Farhat, M., S. Guenneau, and S. Enoch, "Ultrabroadband elastic cloaking in thin plates," *Phys. Rev. Lett.*, Vol. 103, No. 2, 024301, Jul. 2009.
229. Brun, M., S. Guenneau, and A. B. Movchan, "Achieving control of in-plane elastic waves," *Applied Physics Letters*, Vol. 94, No. 6, 061903, Feb. 2009.
230. Stenger, N., M. Wilhelm, and M. Wegener, "Experiments on elastic cloaking in thin plates," *Phys. Rev. Lett.*, Vol. 108, No. 1, 014301, Jan. 2012.
231. Guild, M. D., A. Alù, and M. R. Haberman, "Cancellation of acoustic scattering from an elastic sphere," *The Journal of the Acoustical Society of America*, Vol. 129, No. 3, 1355–1365, Mar. 2011.
232. Guild, M. D., M. R. Haberman, and A. Alù, "Plasmonic cloaking and scattering cancelation for electromagnetic and acoustic waves," *Wave Motion*, Vol. 48, No. 6, 468–482, 2011.
233. Guild, M. D., M. R. Haberman, and A. Alù, "Plasmonic-type acoustic cloak made of a bilaminate shell," *Phys. Rev. B*, Vol. 86, No. 10, 104302, Sep. 2012.
234. Farhat, M., P.-Y. Chen, S. Guenneau, S. Enoch, and A. Alù, "Frequency-selective surface acoustic invisibility for three-dimensional immersed objects," *Phys. Rev. B*, Vol. 86, No. 17, 174303, Nov. 2012.
235. Sanchis, L., V. M. García-Chocano, R. Llopis-Pontiveros, A. Climente, J. Martínez-Pastor, F. Cervera, and J. Sánchez-Dehesa, "Three-dimensional axisymmetric cloak based on the cancellation of acoustic scattering from a sphere," *Phys. Rev. Lett.*, Vol. 110, No. 12, 124301, Mar. 2013.
236. Martin, T. P. and G. J. Orris, "Hybrid inertial method for broadband scattering reduction," *Applied Physics Letters*, Vol. 100, No. 3, 033506, Jan. 2012.
237. Li, N., J. Ren, L. Wang, G. Zhang, P. Hänggi, and B. Li, "Colloquium: Phononics: Manipulating heat flow with electronic analogs and beyond," *Rev. Mod. Phys.*, Vol. 84, No. 3, 1045–1066, Jul. 2012.
238. Fan, C. Z., Y. Gao, and J. P. Huang, "Shaped graded materials with an apparent negative thermal conductivity," *Applied Physics Letters*, Vol. 92, No. 25, 251907, Jun. 2008.
239. Chen, T., C.-N. Weng, and J.-S. Chen, "Cloak for curvilinearly anisotropic media in conduction," *Applied Physics Letters*, Vol. 93, No. 11, 114103, Sep. 2008.
240. Li, J. Y., Y. Gao, and J. P. Huang, "A bifunctional cloak using transformation media," *Journal of Applied Physics*, Vol. 108, No. 7, 074504, Oct. 2010.
241. Guenneau, S., C. Amra, and D. Veynante, "Transformation thermodynamics: Cloaking and concentrating heat flux," *Opt. Express*, Vol. 20, No. 7, 8207–8218, Mar. 2012.
242. Narayana, S. and Y. Sato, "Heat flux manipulation with engineered thermal materials," *Phys. Rev. Lett.*, Vol. 108, No. 21, 214303, May 2012.
243. Narayana, S., S. Savo, and Y. Sato, "Transient heat flux shielding using thermal metamaterials,"

Applied Physics Letters, Vol. 102, No. 20, 201904, May 2013.

244. Dede, E. M., T. Nomura, P. Schmalenberg, and J. S. Lee, "Heat flux cloaking, focusing, and reversal in ultra-thin composites considering conduction-convection effects," *Applied Physics Letters*, Vol. 103, No. 6, 063501, Aug. 2013.
245. Hin Ooi, E. and V. Popov, "Transformation thermodynamics for heat flux management based on segmented thermal cloaks," *The European Physical Journal Applied Physics*, Vol. 63, No. 1, 10903, Jul. 2013.
246. Schittny, R., M. Kadic, S. Guenneau, and M. Wegener, "Experiments on transformation thermodynamics: Molding the flow of heat," *Phys. Rev. Lett.*, Vol. 110, No. 19, 195901, May 2013.
247. Leonhardt, U., "Applied physics: Cloaking of heat," *Nature*, Vol. 498, No. 7455, 440–441, Jun. 2013.
248. Xu, H., X. Shi, F. Gao, H. Sun, and B. Zhang, "Ultrathin three-dimensional thermal cloak," *Phys. Rev. Lett.*, Vol. 112, No. 5, 054301, Feb. 2014.
249. Alù, A., "Thermal cloaks get hot," *Physics*, Vol. 7, No. 12 (3 pages), Feb. 3, 2014.
250. Han, T., X. Bai, D. Gao, J. T. L. Thong, B. Li, and C.-W. Qiu, "Experimental demonstration of a bilayer thermal cloak," *Phys. Rev. Lett.*, Vol. 112, No. 5, 054302, Feb. 2014.
251. Zhang, S., D. A. Genov, C. Sun, and X. Zhang, "Cloaking of matter waves," *Phys. Rev. Lett.*, Vol. 100, No. 12, 123002, Mar. 2008.
252. Greenleaf, A., Y. Kurylev, M. Lassas, and G. Uhlmann, "Approximate quantum cloaking and almost-trapped states," *Phys. Rev. Lett.*, Vol. 101, No. 22, 220404, Nov. 2008.
253. Lin, D.-H., "Cloaking spin-1/2 matter waves," *Phys. Rev. A*, Vol. 81, No. 6, 063640, Jun. 2010.
254. Lin, D.-H., "Cloaking two-dimensional fermions," *Phys. Rev. A*, Vol. 84, No. 3, 033624, 2011.
255. Lin, D.-H. and P.-G. Luan, "Cloaking matter waves around a Dirac monopole," *Physics Letters A*, Vol. 376, No. 5, 675–678, Jan. 2012.
256. Fleury, R. and A. Alù, "Quantum cloaking based on scattering cancellation," *Phys. Rev. B*, Vol. 87, No. 4, 045423, Jan. 2013.
257. Liao, B., M. Zebarjadi, K. Esfarjani, and G. Chen, "Cloaking core-shell nanoparticles from conducting electrons in solids," *Phys. Rev. Lett.*, Vol. 109, No. 12, 126806, 2012.
258. Fleury, R. and A. Alù, "Furtive quantum sensing using matter-wave cloaks," *Phys. Rev. B*, Vol. 87, No. 20, 201106, May 2013.
259. Greenleaf, A., Y. Kurylev, M. Lassas, U. Leonhardt, and G. Uhlmann, "Cloaked electromagnetic, acoustic, and quantum amplifiers via transformation optics," *PNAS*, Vol. 109, No. 26, 10169–10174, Jun. 2012.
260. Liao, B., M. Zebarjadi, K. Esfarjani, and G. Chen, "Isotropic and energy-selective electron cloaks on graphene," *Phys. Rev. B*, Vol. 88, No. 15, 155432, Oct. 2013.
261. Fleury, R. and A. Alù, "Exotic properties and potential applications of quantum metamaterials," *Appl. Phys. A*, Vol. 109, No. 4, 781–788, Dec. 2012.
262. Silveirinha, M. G. and N. Engheta, "Effective medium approach to electron waves: Graphene superlattices," *Phys. Rev. B*, Vol. 85, No. 19, 195413, May 2012.
263. Silveirinha, M. G. and N. Engheta, "Transformation electronics: Tailoring the effective mass of electrons," *Phys. Rev. B*, Vol. 86, No. 16, 161104, Oct. 2012.
264. Silveirinha, M. G. and N. Engheta, "Metamaterial-inspired model for electron waves in bulk semiconductors," *Phys. Rev. B*, Vol. 86, No. 24, 245302, Dec. 2012.
265. Chen, H., J. Yang, J. Zi, and C. T. Chan, "Transformation media for linear liquid surface waves," *EPL*, Vol. 85, No. 2, 24004, Jan. 2009.
266. Farhat, M., S. Enoch, S. Guenneau, and A. B. Movchan, "Broadband cylindrical acoustic cloak for linear surface waves in a fluid," *Phys. Rev. Lett.*, Vol. 101, No. 13, 134501, Sep. 2008.
267. Harrington, R. F., "Theory of loaded scatterers," *Proc. IEEE*, Vol. 3, No. 4, 617–623, Apr. 1964.
268. Schindler, J. K., R. B. Mack, and P. Blacksmith, "The control of electromagnetic scattering by

impedance loading," *Proc. IEEE*, Vol. 53, No. 8, 993–1004, Aug. 1965.

269. Green, R. B., "Scattering from conjugate-matched antennas," *IEEE Trans. Antennas Prop.*, Vol. 14, No. 1, 17–21, Jan. 1966.
270. Hansen, R. C., "Relationships between antennas as scatters and as radiators," *Proc. IEEE*, Vol. 77, No. 5, 659–662, May 1989.
271. Andersen, J. B. and A. Frandsen, "Absorption efficiency of receiving antennas," *IEEE Trans. Antennas Prop.*, Vol. 53, No. 9, 2843–2849, Sep. 2005.
272. Kwon, D. H. and D. M. Pozar, "Optimal characteristics of an arbitrary receive antenna," *IEEE Trans. Antennas Prop.*, Vol. 57, No. 12, 3720–3727, Dec. 2009.
273. Kwon, D. H. and D. M. Pozar, "Design of received and scattered powers for dipole arrays using load optimization," *2010 IEEE Antennas and Propagation Society International Symposium (APSURSI)*, Jul. 2010.
274. Karilainen, A. O. and S. A. Tretyakov, "Circularly polarized receiving antenna incorporating two helices to achieve low backscattering," *IEEE Trans. Antennas Prop.*, Vol. 60, No. 7, 3471–3475, Jul. 2012.
275. Alù, A. and N. Engheta, "Cloaking a sensor," *Phys. Rev. Lett.*, Vol. 102, 233901, Jun. 2009.
276. Alù, A. and N. Engheta, "Cloaking a receiving antenna or a sensor with plasmonic metamaterials," *Metamaterials*, Vol. 4, No. 4, 153–159, Mar. 2010.
277. Alù, A. and S. Maslovski, "Power relations and a consistent analytical model for receiving wire antennas," *IEEE Trans. Antennas Prop.*, Vol. 58, No. 5, 1436–1448, May 2010.
278. Soric, J. C., R. Fleury, A. Monti, A. Toscano, F. Bilotti, and A. Alù, "Controlling scattering and absorption with metamaterial covers," *IEEE Trans. Antennas Prop.*, Vol. 62, No. 8, 4220–4229, Jul. 2014.
279. Kwon, D. H. and D. H. Werner, "Restoration of antenna parameters in scattering environments using electromagnetic cloaking," *Appl. Phys. Lett.*, Vol. 92, No. 11, 113507, Mar. 2008.
280. Monti, A., A. Toscano, and F. Bilotti, "Metasurface mantle cloak for antenna applications," *2012 IEEE Antennas and Propagation Society International Symposium (APSURSI)*, Jul. 2012.
281. Valagiannopoulos, C. A. and N. L. Tsitsas, "Integral equation analysis of a low-profile receiving planar microstrip antenna with a cloaking substrate," *Radio Sci.*, Vol. 47, RS004878, Apr. 2012.
282. Vehmas, J., P. Alitalo, and S. Tretyakov, "Experimental demonstration of antenna blockage reduction with a transmission-line cloak," *IET Microw. Antennas Propag.*, Vol. 6, No. 7, 830–834, Jan. 2012.
283. Monti, A., J. Soric, A. Alù, F. Bilotti, A. Toscano, and L. Vegni, "Overcoming mutual blockage between neighboring dipole antennas using a low-profile patterned metasurface," *IEEE Antennas and Wireless Propagation Letters*, Vol. 11, 1414, 2012.
284. Alù, A. and N. Engheta, "Cloaked near-field scanning optical microscope tip for noninvasive near-field imaging," *Phys. Rev. Lett.*, Vol. 105, No. 26, 263906, Dec. 2010.
285. Bilotti, F., S. Tricarico, F. Pierini, and L. Vegni "Cloaking apertureless near-field scanning optical microscopy tips," *Optics Lett.*, Vol. 36, No. 26, 211–213, Jan. 2011.
286. Fan, P., U. K. Chettiar, L. Cao, F. Afshinmanesh, N. Engheta, and M. L. Brongersma, "An invisible metal-semiconductor photodetector," *Nature Photonics*, Vol. 8, 380–385, May 2012.
287. Fleury, R., J. Soric, and A. Alù, "Physical bounds on absorption and scattering for cloaked sensors,"*Phys. Rev. B*, Vol. 89, No. 4, 045122, 12 pages, Jan. 15, 2014.
288. Alù, A. and N. Engheta, "How does zero forward-scattering in magnetodielectric nanoparticles comply with the optical theorem?," *J. Nanophoton.*, Vol. 4, No. 1, 041590, May 2010.
289. Zhang, B., H. Chen, B. I. Wu, and J. A. Kong, "Extraordinary surface voltage effect in the invisibility cloak with an active device inside," *Phys. Rev. Lett.*, Vol. 100, 063904, Feb. 2008.
290. Xu, T., X. F. Zhu, B. Liang, Y. Li, X. Y. Zou, and J. C. Cheng, "Scattering reduction for an acoustic sensor using a multilayer shell comprising a pair of homogenous isotropic single-negative media," *Appl. Phys. Lett.*, Vol. 101, 033509, Jul. 2012.