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CLOSED-LOOP CONVEYOR SYSTEMS WITH MULTIPLE POISSON INPUT AND MULTIPLE SERVERS.

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by

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ABSTRACT

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The objective of this research is to investigate the multi-item, multi-loading, multi-unloading closed-loop conveyor systems, and to evaluate their performances as queueing systems, where an ordered discipline is considered. There are two types of arrivals - singlets and doublets - each governed by a separate independent Poisson distribution.

The conveyor systems studied are structured according to the possible alternative destinations of an arrival, as it may be 'lost', recirculated, or stored.

In the first part of the study, a mathematical model is presented for both homogeneous and heterogeneous serviced conveyors, in which the arrivals denied service at the last channel are considered 'lost' to the system. The service times are exponentially distributed. The steady-state probabilities of 'n' items in the system are determined. Also, three measures of the system's performance are developed, namely: (i) probability of the system being idle; (ii) expected number of units in the system; and (iii) probability of a lost arrival.

The second part of this research is presented in a mathematical context, involving the closed-loop conveyors when recirculation of lost units is permitted. The steady-state probabilities, and the system's measures of performance are discussed for cases where either homogeneous or heterogeneous servers are allowed.

The third part of this study is the formulation of a mathematical model for a two-channel closed-loop conveyor where storage of infinite capacity exists at the last channel. The general solutions for the measures of the system's performance are derived.

The last part of this research is a simulation analysis of closed-loop conveyors. Conveyors, with either storage or recirculation, are discussed. Also, the transient solution of a two-channel conveyor without storage and lost arrivals is included. The systems are analyzed through the use of Fortran IV and G.P.S.S. simulation language.

This research has clearly demonstrated the feasibility of solving the multi-item, multi-channel conveyor system through the application of queueing theory. Also, the closed-loop conveyor systems with multiple-inputs are more efficient than those with singlet input.

TO MY PARENTS AND LINDA

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α	fraction of the recirculated arrivals
Θ_i	ratio between average service rate of channel 'i' to the average service rate of the first channel
λ_1	average arrival rate of the singlet units
λ_2	average arrival rate of the doublet units
μ	average service rate
ρ_1	'Traffic Intensity' of the singlet arrivals
ρ_2	'Traffic Intensity' of the doublet arrivals
ϕ	service time ratio between a doublet and a singlet unit
χ	probability of a channel being busy

CHAPTER I

INTRODUCTION

Conveyor Systems

The conveyor is the key material mover in most high volume manufacturing operations. The conveyor itself, incoming and outgoing material, loading and unloading points, and work stations, form a system that poses a number of interesting problems for the analyst. As a result, a body of knowledge has been emerging under the heading, 'Conveyor Theory'(9).

There are more than fifty types of conveyors classified by the American Materials Handling Society (AMHS), such as gravity conveyors, belt conveyors, endless-chain conveyors, pneumatic conveyors, screw conveyors and vibrating conveyors. Among this variety, we might distinguish four important conveyor systems in which any of the conveyors can be used. These systems are:

1. Controlled movement systems which are reversible: these typically move at the command of an operator, being indexed away from a work station as material is loaded for storage and reversed when the material is to be recovered.

2. The fixed conveyor system which is used to link together two production centers: it is exemplified by a group of workers placing units of production, onto a

gravity fed roller conveyor, to be transported to another location where a second group of workers will remove them.

3. Power and free systems: these consist of part carriers which can be connected to and disconnected from the moving portion of the conveyor at will.

4. Closed-Loop: are irreversible, continuous operating systems with part carriers which can not be removed.

The closed-loop conveyor systems are generally much simpler and lower in cost per unit length or per unit capacity than the open-loop systems(18), but, they are not nearly as flexible. The low cost of the closed-loop systems resulted in the wide application of them. Thus, in this research, we shall deal with such conveyor systems.

Statement of The Problem

The mechanical design of closed-loop conveyor systems is very well understood, but their operating characteristics as part of integrated manufacturing systems have not until recently, been researched. A typical closed-loop conveyor is shown in Figure 1.

The conveyor, together with incoming material, loading and unloading stations, work stations, and processed units, appears to possess the properties of a queueing system. Arrival times of incoming units

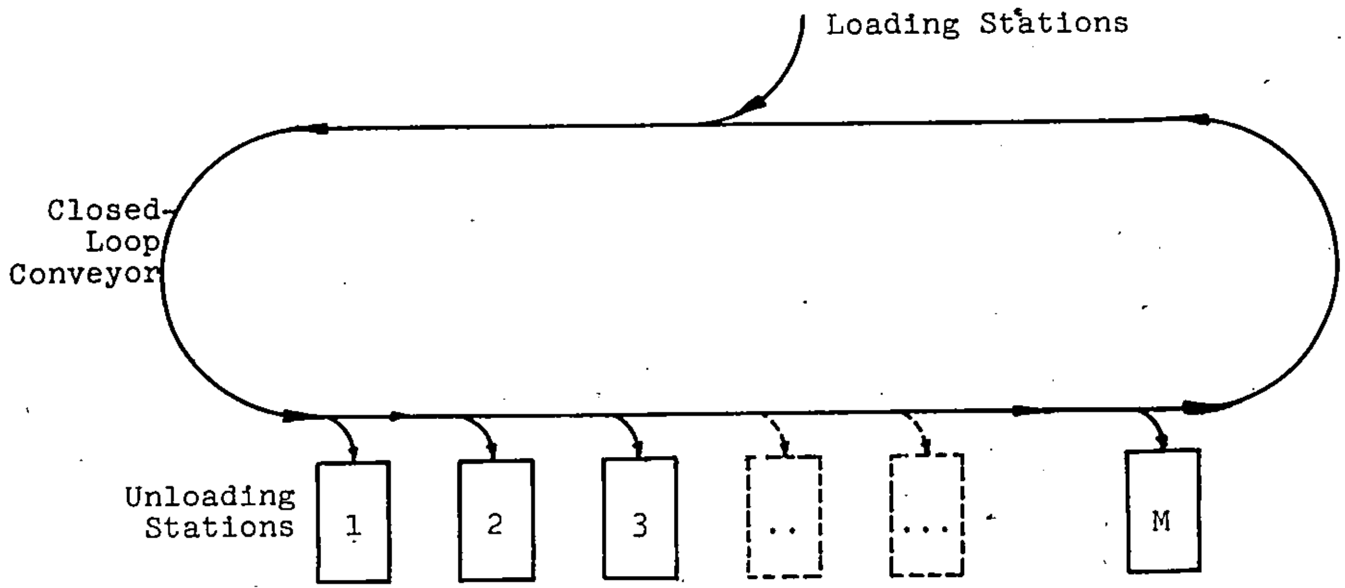


Figure 1. A Typical Closed-Loop Conveyor.

can follow deterministic or probabilistic distributions. Poisson, negative exponential, general, and Erlangian distributions are examples of the loading and unloading stations arrival patterns.

Incoming units can be transferred to the service stations according to a predetermined queueing discipline. Also, allowing unlimited or truncated storage at either the loading or unloading stations is a factor, which should be considered in studying conveyors. The mathematical analysis of conveyor systems under varying conditions of the above constraints has led to increasingly complex queueing situations.

Past research has always concentrated mainly on the case where all incoming units are homogeneous with respect to the required operations and the service time. However, no work has been carried out on the multi-item case, where more than one type of unit is placed on the conveyor and each type requires operations different from the other types, thus resulting in different service times. A typical example(16) is that of the repair and maintenance of large assemblies, each consisting of several identical units. Sometimes, only one unit is in need of repair, at other times, more than one unit (in one assembly) is in need of repair. In this case, the arrivals can be governed by a multiple Poisson distribution.

This research focuses mainly on the closed-loop conveyor as a queueing system. The system under consideration has the following properties:

1. Two types of arrivals; each type is independent and has a Poisson distribution.

2. M-Channels; where the service rate at each channel has a negative exponential distribution.

3. The service rates at all channels can be homogeneous or heterogeneous.

4. The service time of a unit from the first type of arrivals is ϕ times that of a unit from the second type.

5. Queue discipline is ordered entry; i.e. an arrival has to check with the first channel first, if busy, then to the second, and so on, until all the M-channels are exhausted.

6. The system has uniform loading and unloading rates and the conveyor travels at a constant speed.

With the above focus in mind, the specific objectives of the study is to evaluate the performance of the closed-loop conveyor system under certain conditions. The system's performance is measured by means of certain criteria known as measures of performance or effectiveness, derived from Queueing Theory literature. These measures are:

1. Steady-state probabilities of the system,

2. P_{00}: the probability that the system is idle,

3. E[n]: the expected number of units in the system,

4. $P_r(w=0)$: the probability that an arrival will have no wait prior to service, and

5. P_{rec}: the probability that an arrival will be recirculated.

Specific conditions are structured according to the possible alternative destinations of an arrival as it may be 'lost', recirculated, or stored. Hence, these conditions are:

1. Lost Arrivals: an arrival that can not be serviced at the last channel.

2. Recirculation: an arrival that can not be serviced at the last channel is allowed to recirculate and enter the system as a new arrival.

3. Storage: an arrival that can not be serviced at the second last channel is allowed to enter a storage allocated at the last channel and then wait to receive service.

4. Each one of the above conditions will be treated twice; first assuming that the servers have homogeneous service rates, and second, assuming heterogeneous service rates.

For each of the above conditions, the relationship between the measures of performance and the conveyor system's parameters will be studied. Such parameters include: (a) the number of the service channels; (b) the traffic intensities of the arrivals; (c) the service time ratio between arrivals; (d) the proportion of recirculated units; and (e) the storage capacity.

Research Design

The queueing systems in this research are dealt with, by two basic methods of attack, those being the theoretical-mathematical approach and the techniques of simulation analysis. The mathematical approach of the systems considered two cases:

1. Steady-State Case: assuming that the service channels are capable of serving at a faster average rate than units arrive, i.e. $\rho = \frac{\lambda}{\mu} < 1$ (ρ is referred to as the 'Load Factor' or the 'Traffic Intensity', λ is the average arrival rate and μ is the average service rate), then the steady-state is reached when the queue behaves independently of the initial state of the system and the probability of having a given number 'n' in the queue remains constant with time. A formulation of a set of differential-difference equations for every case is also derived. Solutions of these equations are given.

2. Transient or Time-Dependent Case: situations at which the probability of having a given number 'n'

in the queue is not constant since being dependent on time is referred to as a transient case. The transient solution of the two-channel closed-loop conveyor with ordered entry and multiple Poisson input is derived using the Runge-Kutta method.

A simulation analysis was undertaken to analyze the two and three-channel conveyor systems with recirculation and storage at each channel. Effect of the recirculation time and the capacity of the storage on the utilization of the service channels is also studied. Systems with lost arrivals and without storage are simulated. Simulation programming was conducted in G.P.S.S. III/360.

Importance of This Study

This study has three main potential contributions. First, this study enriches the literature of conveyor theory by presenting results about cases that were not researched adequately in the past. An example is a system involving multiple input and recircualtion. Second, is the extension of Queueing Theory to the treatment of closed-loop conveyors, especially the multiple-item case. This case is not frequently investigated due to its complexity. Third, with respect to applications, the results of this research would hopefully enable design engineers to maximize the performance, efficiencies, and effectiveness of the conveyor systems under given constraints.

Organization of This Dissertation

The presentation will follow this pattern: After the introductory Chapter I, Chapter II is devoted to a review of the literature on queueing and conveyor theories. In Chapter III, the M-channel closed-loop conveyor, with homogeneous servers and without storage at any of the channels, will be mathematically analyzed. Chapter IV presents the two and three channel conveyor systems with ordered entry allowing heterogeneous servers at the service channels. In Chapter V, the results of an analysis of the recirculation problem with homogeneous or heterogeneous servers will be presented. Chapter VI contains the analysis of the two-channel closed-loop conveyor system model with storage of different capacities at the second channel, while recirculation is not permitted.

Chapter VII is concerned with the simulation analysis of two and three-channel conveyors with lost arrivals. The effect of recirculation time and capacities of storage on the performance of the system are also presented. Finally, Chapter VIII is reserved for the summary, conclusions, and recommendations for future research.

CHAPTER II

CLOSED-LOOP CONVEYORS AS A QUEUEING SYSTEM: A LITERATURE REVIEW

In Chapter I, it was established that the conveyor together with related elements can be perceived as a queueing system. The purpose of this chapter is to examine past research on the subject. The scope of examination is limited to studies that treat the conveyor system as a queueing problem. This is the main frame of reference of the present study. It seems logical to review two types of past studies: foremost, the studies within the domain of 'Conveyor Theory', and secondly, queueing theory literature that can be valuable for treating conveyor problems.

The presentation in this chapter parallels the issues outlined in the statement of the problem in Chapter I. Accordingly, the problem of lost arrivals is presented first. Next, literature pertinent to recirculation is reviewed. Then, the matter of storage will be discussed. Finally, literature relevant to the multi-item problem will be assessed.

Conveyors With Lost Arrivals

Disney's technical note (5) appears to be the first published work in which a conveyor is treated as a multi-channel queueing system with ordered entries.

He chose a power and free conveyor system with 'n' identical work stations. Every entering loaded hook (Pendant) tests one or more sensors, placed before each station, in a serial order. If a sensor indicates that the station is idle, then the pendant is switched so as to enter the first such station it tests, and awaits service. If the pendant tests all switches and if all are in a position indicating a fully loaded station, then the unit is 'lost' to the input system. In queueing terms, the problem is viewed as multi-channel, in which arrivals must enter the first empty channel. A system of equations for a two-service station without storage at either of the channels, were developed. Disney then determined such performance measures as the probabilities of: (a) an idle system, (b) lost arrivals, and (c) station one(1) being busy. Also, the expected number of items in the system was evaluated.

Pritsker (34) viewed the conveyor system as a specialized queueing problem characterized by the following: (1) no storage facilities existing at the channels; (2) the output lines from the channels do not interact with the input lines; (3) all channels have equal service rates; and (4) there is no feedback of items, and those that cannot be served are lost to the system.

Pritsker analyzed and compared the performance of two cases of different arrivals and service distributions for 'm' channels (Poisson arrival / exponential service and constant arrival rate / general service system). He found that the probability of a lost item and the idle time, to be greater when arrivals are Poisson, than when arrivals are at predetermined instants. This agrees with intuition. Scheduling of arrivals is more important than scheduling of service time. Since fewer items are lost, and there is less idle time when arrivals are determined, the expected number of units in the system will be greater. The major conclusion of Pritsker's study is that there are many parameters associated with the type of conveyor systems studied. These parameters do not significantly affect the steady-state probabilistic performance of the system. Examples of such variables are: (1) the delay (distance) between service channels, and (2) the form of the service distribution if inter-arrival distribution is exponential.

Phillips(32) extended Pritsker's work and investigated the m-channel ordered entry conveyor serviced queueing system, with lost arrivals, where either homogeneous or heterogeneous service rates exist at the service channels. Steady-state equations are derived which give the probability of 'n' items in the system at any arbitrary time 't'.

Phillips obtained some results for two cases: (1) Poisson arrivals with exponential service rates; and (2) gamma distribution time between arrivals with exponential service rates.

Muth(30) extended Kwo's work (26) on closed-loop conveyor systems. Muth gave a formal mathematical description of the time and space dependence of material flow in a closed-loop conveyor system. The solution of the resulting equations provides conditions of feasibility as well as criteria affecting the design and the operation of conveyors. His work consists of a study on continuous material flow, such as belt conveyors. It is specifically assumed, that material flows into the loading station, and out of the unloading station. This is not of a constant nature, but follows a fixed pattern which repeats itself periodically with time.

An important result of his work is that incompatibility depends on, the ratio (T/P) of conveyor period to work-cycle period, and on the presence of harmonics in input and output flow rates.

Muth(31) in another paper, analyzed a closed-loop conveyor system having a single loading station, a single unloading station, and discrete time varying input and output flows. The ratio (r/p), which is the remainder of the ratio conveyor period (r) to work-cycle period (p), is shown to be an important criterion for compatibility and optimization.

Helgeson(18) analyzed overhead monorail non-reversing loop type conveyor systems. He was able to develop some monographs with which designers are able to determine the number of spaces and other parameters of the conveyor.

Gregory and Litton (13) studied a conveyor system consisting of equally-spaced hooks passing before a number of work stations. The work pieces are processed by the first available work station after which they are transferred to a separate system. In their study, they assumed that the processing times are independently and exponentially distributed. They found that the incidence of missed units is minimized, if the operators are placed in descending order of work rate along the conveyor.

In another paper, Gregory and Litton (14) presented an approach for the solution of the discrete conveyor model for service time distributions which are general but bounded, where there is no storage at the work stations and no recirculation, i.e. a system with lost arrivals. The service or processing times are random variables, which have a general distribution with the exception of the assumption that there is a finite upper bound on the value that the random variables can take. The model was then solved by using a Markovian analysis. Therefore, the steady-state probabilities and the expected number of units can be evaluated.

An approach that has been used extensively in studying the closed-loop single hook system has been the study of the individual work station. Reis, Dunlap and Schneider (36) investigated the effects of changing the banking disciplines of the individual stations. The effects usually considered were, the expected delay at a station, and the expected production for the station.

Heikes (17) developed an approximated model for predicting the performance of a conveyor system consisting of single unit carriers or 'hooks'. These move past a series of loading stations, where attempts are made to place them onto the hooks, then past a series of unloading stations, where attempts are made to remove them from the hooks.

Conveyors With Recirculation

Pritsker (34) studied the problem of recirculation for m-channel closed loop conveyors. He considered a mathematical and a simulated approach for studying recirculation. The mathematical approach was conducted under cost constraints. One of the major conclusions of Pritsker's study is that the feedback delay (distance) does not significantly affect the steady-state probabilistic performance of the system. Because of the complexity of the distribution of recycled items, Pritsker constructed a simulation model using the SIMSCRIPT programming language for studying conveyor systems with feedbacks.

Phillips (32) studied the m-channel ordered entry conveyor systems with lost arrivals. The problem of recirculation was analyzed by using only simulation techniques, and thus, some results were obtained for the following cases: (1) Poisson arrivals with exponential service rates; and (2) gamma distribution time between arrivals with exponential service rates. Phillips found that feedback delays appear to have no effect on the expected queue length or the probability that each channel is busy.

Phillips and Skeith (33) extended Phillips' work (32) and presented the results of a simulation analysis of a conveyor-serviced ordered entry system with recirculation of the lost items. Their results can be summarized as:

1. The recirculation traffic can be reduced by either increasing the feedback delay constant or increasing the storage capacity of the system.

2. Slower recirculation times will decrease

conveyor traffic and help to reduce the competition for space at the input side of the system.

Burbridge (3) studied the problem of recirculation for closed-loop conveyor systems. Analytical and experimental approaches were presented to study the effects of the conveyor parameters on the flow of traffic in a conveyor system. The entire conveyor system is analyzed by means of GERT (Graphical Evaluation and Review Technique).

Conveyors With Storage

Disney (7) studied a two-service station conveyor without storage at the first station, but 'n' units can be stored at the second station. For the two service case, with an equal amount of storage at both the service channels (M=N=1, M is the maximum amount of storage allowed before service one(1) and N is the maximum amount of storage allowed before service two(2)), Disney illustrated the probability of both stations being busy, as follows:

$$P_{11} = \frac{\rho^2}{2} / \sum_{i=0}^{2} \frac{\rho^i}{i!}$$

where ρ = the load factor (λ/μ),

λ = the average arrival rate,

and μ = the average service time per service.

Disney found that the imbalance, caused by the lack of storage facilities, can be corrected by allowing limited storage. The amount of storage required to achieve balance depends on the load factor, and generally the greater the load factor, the less the storage is required to achieve balance. Also, the service facilities farthest from the point of input, should be given more storage capacity. Disney concluded his research by noting that the extension to more than two channels is theoretically feasible, but not computationally attractive.

Gupta (15) extended Disney's studies to a general case where the maximum number of units allowed in channels one(1) and two(2) are M and N, respectively. This differed from Disney's model where the maximum amount of storage in channel one (1), was only one unit. Using the generating functions technique, Gupta obtained the queue size distribution in the steady-state case.

Pritsker (34) studied the m-channel closed-loop conveyor without storage at the first (m-1) channels and an infinite storage at the m^{th} channel. This corresponds to the case where all items are removed from the conveyor for processing by the last channel. In this case, no item is lost. Pritsker found, for the 'm'-channel conveyor with Poisson arrival and exponential service, that the last channel will be busier when infinite storage is provided.

Phillips (32) studied the m-channel conveyors with ordered entry where storage was permitted. He found that the allocation of additional storage to servers, at which the utilization is low, will tend to balance the work load and correct the bias of the ordered entry system. This observation was previously noted by Disney, for an ordered entry system without storage.

Phillips and Skeith (33) presented the results of a simulation analysis of a conveyor-serviced ordered entry queueing system with storage at each channel and recirculation of lost items. They found that there are complex interactions between, the storage at each channel, the recirculation delay constant, and the number of items which must be recirculated. The exact form of this interaction was not determined, but the effects of this interaction have been made evident.

Multi-Item Conveyors

Conveyors with multi-loading points and multi-unloading points were studied by Morris (28), who showed that by increasing the speed of the conveyor beyond the loading and unloading rates, the amount of interference at both the loading and unloading stations increased. He also solved the problem of multi-item carriers by increasing the capacity of each carrier to two units (similar units that require the same service time). By doing so, the amount of interference at the loading station was sharply decreased.

This literature review would not be complete without due consideration to other published material of queueing theory, relevant to this research. This section examines these studies, although they were not conducted in a conveyor context.

Harris (16) derived and plotted the steady-state probabilities of arrivals and whether the arrivals will have to wait for service in a single channel, first come - first served queueing system, in which the arrival discipline is a multiple Poisson process. His discussion is restricted to batches of sizes one and two for algebraic simplicity. The steady-state probability of 'n' units in the system is derived and plotted with varying ρ_1 and ρ_2 (ρ_1 and ρ_2 are the traffic intensities of the singlet and doublet arrivals respectively). The average queue length, and the probability that an arrival will not have to wait prior to service, are given. However, Harris' case is a special one, later considered by Ancker and Gafarian (1). They studied the case of a single server queueing system for 'm' different types of customers having different Poisson arrivals and exponential service times where the queue discipline is FCFS. The characteristic equations of the system are derived; such as the mean value of the waiting time and the expected number of tasks in the system.

Sharma (42) studied the case of various input sources as did Gafarian, but Sharma considered a phase type service. There is one server at the first phase while two parallel servers are at the second phase. In another model by Sharma (42) the number of parallel servers at the second phase can take any value from one(1) to L. The probability distribution functions, for the number of units waiting for service in each queue, as well as the mean number of units in the system, are obtained.

Truemper and Liittschwager (45) considered a case similar to that of Harris. They assumed that the Poisson arrivals are from both finite and infinite populations. The only difference between Harris' work and that of Truemper and Liitschwager, is that of allowing a Poisson arrival from a finite population. Characteristic equations of the system were derived.

Kotiak and Slater (23) studied a queueing system with two types of customers served by two desks.

They assumed that the customer's arrival is a mixed Poisson stream, having exponential service-time distribution, with different means characterizing the two types of customers. The queue discipline is FCFS. Kotiak and Slater considered two schemes. In Scheme I, each type of customer has a particular desk, for which he queues on arrival. In Scheme II, all customers keep in a single queue and process indifferently to either desk. It was shown that Scheme II is more efficient than Scheme I.

The two-channel queueing system with heterogeneous servers has been investigated by Morse (29), Saaty (39), and Krishnamoorthi (24). Morse (29) studied the problem in order to illustrate the approach of using the hyper-exponential distribution to represent tandom queueing service. Saaty (39) discussed the problem in order to illustrate a technique for determining the transient probabilities for time dependent queues. Krishnamoorthi examined the problem in order to overcome the objections to a random selection queueing system.

Conclusions

The preceeding review of the literature suggests the following conclusions:

1. The m-channel closed-loop conveyors without storage at any channel and allowing a single input, have been carefully studied.

2. Conveyors with recirculation were not adequately treated analytically. However, some recirculation problems were analyzed by simulation techniques.

3. Solutions of the two-channel closed-loop conveyors with storage at either the first or the second channel were possible.

4. The multi-item closed-loop conveyor systems have not been dealt with.

CHAPTER III

CLOSED-LOOP CONVEYOR WITH HOMOGENEOUS SERVERS AND LOST ARRIVALS

The conveyor system of interest in the remainder of this dissertation is called the 'closed-loop conveyor with multiple inputs'(Figure 1). One application of this conveyor is the repair of large assemblies, e.g. cross-bar frames and telephone exchanges, each consisting of several identical units; sometimes, only one unit is in need of repair, at other times, more than one unit (in one assembly) is in need of repair, where, the units are being transferred to the service stations by using the conveyor. This is a situation of multiple inputs. Another application of this conveyor is the case of transferring multi-items which need to be assembled, and take different times to do so.

In studying the multi-item conveyors with ordered entry, the arrival systematically checks station one(1) through M-1, and if service is denied at station M, then one of the following three possible situations occurs: (1) the arrival is 'lost' to the system; (2) the arrival joins a storage at the service channel where it remains until the channel becomes available for service; or (3) the arrival recirculates and again becomes a new arrival to the first station.

At this time, let us turn our attention to the first alternative, and study the M-channel closed-loop conveyors with homogeneous servers at the service channels - using automatic machines at the service channels to serve the arrival with equal service rates (a case of using homogeneous servers) - and no storage is allowed at any of the service channels. Two, three, four, and M-channel conveyors will be examined in this chapter. This system consists of a closed-loop conveyor, several loading stations, and unloading stations.

General Assumptions

The assumptions made concerning the conveyor system under consideration are as follows:

1. There are two types of input arrivals. Each unit of the first type requires only one operation to be processed at a service channel; henceforth, referred to as a singlet unit. By contrast, each unit of the second type requires more than one operation; henceforth referred to as a doublet unit.

2. Each type of the above arrivals, is independent from the other and is governed by a different Poisson distribution.

3. The service time of each channel has a negative

exponential distribution.

4. The service time of a doublet unit is ϕ times that of a singlet unit.

5. The queue discipline is ordered entry, i.e. each arrival first checks with the first channel, if occupied, then with the second, and so on, until it is serviced at the first available channel.

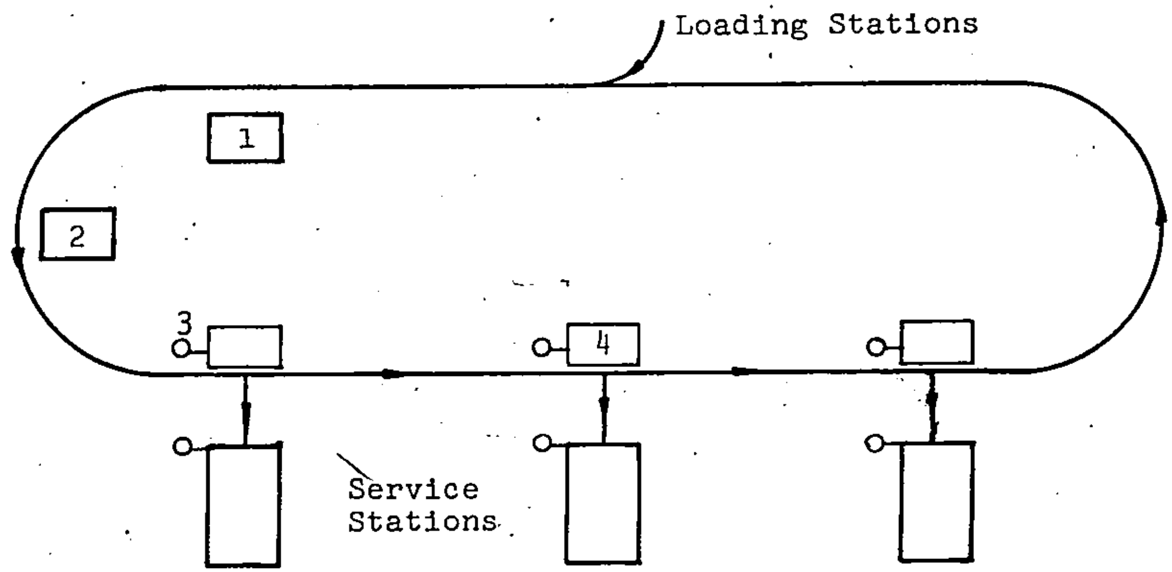
6. No random fluctuations in either the loading or the unloading rates.

7. No storage is assumed at the loading stations, i.e. every arrival finds a place on the conveyor, it does not have to wait to be placed on the conveyor.

8. After the units are being serviced at the service channels, they leave the system by other means than the conveyor used in the system.

9. The conveyor travels at a uniform speed.

The conveyor model was set up in the Department of Industrial Engineering, the University of Windsor, Windsor, Ontario. The model is shown in Figure 2. There are two types of arrivals -singlets and doublets - each type has a particular kind of sticker. Before seeking service, the arrival being placed on the conveyor passes through a laser beam, at which time the laser beam unit recognizes it, as either a singlet or a doublet arrival. The arrival then passes through a light beam of a photoelectric cell, where an



(1) Laser Beam unit
(2) Electronic Counter
(3) Photoelectric Cell
(4) Pneumatic Diverter

Figure 2. A Typical Set-up of the Closed-Loop Conveyor System.

electric unit counts the numbers of the arrivals and the average time between arrivals. A photoelectric cell is connected to each channel, in order to indicate whether the channel is busy or idle. If the channel is busy, the light of the photoelectric cell goes off and vice-versa. Each of the photoelectric cells that are connected to the service channels, operates a pneumatic diverter. The arrival seeking service at a service channel will be pushed by the diverter to the first available channel.

Two-Channel Closed-Loop Conveyor with Homogeneous Service Rates

In addition to the general assumptions, it is assumed that:

1. There are only two service channels.
2. The two channels have equal service rates.
3. No storage is allowed at any of the channels.
4. If the arrival is denied service at the second channel, it is considered 'lost' to the system.

let P_{ij} = the steady-state probability that i singlet units and j doublet units are in the system;

λ_1 = arrival rate for singlets;

λ_2 = arrival rate for doublets; and

μ = service rate at the channels.

The probability of more than one arrival or more than one service in time Δt is negligible. The equilibrium probabilities can then be derived as follows:

$$P_{00}(t+\Delta t) = P_{00}(t).[1-(\lambda_1+\lambda_2)\Delta t] + P_{10}(t).\mu\Delta t + P_{01}(t).[\tfrac{1}{\phi}\mu\Delta t] \qquad \text{III - 1}$$

$$P_{01}(t+\Delta t) = P_{01}(t).[1-(\lambda_1+\lambda_2+\tfrac{\mu}{\phi})\Delta t] + \tfrac{2}{\phi}\mu\Delta t.P_{02}(t) + \mu\Delta t.P_{11}(t) + \lambda_2\Delta t.P_{00}(t) \qquad \text{III - 2}$$

$$P_{02}(t+\Delta t) = P_{02}[1-\tfrac{2}{\phi}\mu\Delta t] + \lambda_2\Delta t.P_{01}(t) \qquad \text{III - 3}$$

$$P_{10}(t+\Delta t) = P_{10}(t).[1-(\lambda_1+\lambda_2+\mu)\Delta t] + 2\mu\Delta t.P_{20}(t) + \tfrac{\mu}{\phi}\Delta t.P_{11}(t) + \lambda_1\Delta t.P_{00}(t) \qquad \text{III - 4}$$

$$P_{11}(t+\Delta t) = P_{11}(t).[1-(\tfrac{\phi+1}{\phi})\mu\Delta t] + \lambda_1\Delta t.P_{01}(t) + \lambda_2\Delta t.P_{10}(t) \qquad \text{III - 5}$$

$$P_{20}(t+\Delta t) = P_{20}(t).[1-2\mu\Delta t] + \lambda_1\Delta t.P_{10}(t) \qquad \text{III - 6}$$

Rewriting equations III - 1 through III - 6 and by dividing by Δt, the steady-state equations are then obtained by setting the derivatives equal to zero.

The steady-state equations are given as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + \mu.P_{10}(t) + \frac{\mu}{\phi}.P_{01}(t) = 0 \qquad \text{III - 7}$$

$$-(\lambda_1+\lambda_2+\frac{\mu}{\phi}).P_{01}(t) + 2\frac{\mu}{\phi}.P_{02}(t) + \mu.P_{11}(t)$$
$$+ \lambda_2.P_{00}(t) = 0 \qquad \text{III - 8}$$

$$-2\frac{\mu}{\phi}.P_{02}(t) + \lambda_2.P_{01}(t) = 0 \qquad \text{III - 9}$$

$$-(\lambda_1+\lambda_2+\mu).P_{10}(t) + 2\mu.P_{20}(t) + \frac{\mu}{\phi}.P_{11}(t)$$
$$+ \lambda_1.P_{00}(t) = 0 \qquad \text{III - 10}$$

$$-\mu(\frac{\phi+1}{\phi}).P_{11}(t) + \lambda_1.P_{01}(t) + \lambda_2.P_{10}(t) = 0 \qquad \text{III - 11}$$

$$-2\mu.P_{20}(t) + \lambda_1.P_{10}(t) = 0 \qquad \text{III - 12}$$

Set $\rho_1=\frac{\lambda_1}{\mu}$ (traffic intensity of the singlet arrivals) and $\rho_2=\frac{\lambda_2}{\mu}$ (traffic intensity of the doublet arrivals), in the above system of equations. In this set there is one dependent equation, i.e. we can assume a value for one P_{ij} and solve all others in terms of it. Using matrix notation, one can write this system as:

$$A \quad C = \bar{0} \qquad \text{III - 13}$$

where A is a square matrix, its components are shown as follows:

$$A = \begin{vmatrix} -(\rho_1+\rho_2) & \frac{1}{\phi} & 0 & 1 & 0 & 0 \\ \rho_2 & -(\rho_1+\rho_2+\frac{1}{\phi}) & \frac{2}{\phi} & 0 & 1 & 0 \\ 0 & \rho_2 & -\frac{2}{\phi} & 0 & 0 & 0 \\ \rho_1 & 0 & 0 & -(\rho_1+\rho_2+1) & \frac{1}{\phi} & 2 \\ 0 & \rho_1 & 0 & \rho_2 & -(\frac{\phi+1}{\phi}) & 0 \\ 0 & 0 & 0 & \rho_1 & 0 & -2 \end{vmatrix}$$

C is a column matrix of P_{ij} for given values of i and j.

$$C = \begin{vmatrix} P_{00} \\ P_{01} \\ P_{02} \\ P_{10} \\ P_{11} \\ P_{20} \end{vmatrix}$$

$\bar{0}$ is a null column matrix. Solving Equation III-13 one can get P_{ij}'s in terms of P_{00}. These probabilities are given below:

$$P_{01} = \phi\rho_2 P_{00}$$

$$P_{02} = \frac{\phi^2}{2}\rho_2^2 . P_{00}$$

$$P_{10} = \rho_1 . P_{00}$$

$$P_{11} = \phi\rho_1\rho_2 . P_{00}$$

$$P_{20} = \frac{1}{2}\rho_1^2 . P_{00}$$

By defining the boundary condition,

$$\sum_{i=0}^{2} \sum_{j=0}^{2} P_{ij} = 1 \qquad i+j \leq 2 \qquad \text{III - 14}$$

one can then obtain all values of P_{ij}. Rewriting equations III - 7 through III - 12 using matrix notations,

$$B \;\; C + C_0 = \bar{0} \qquad \text{III - 15}$$

where C_0 is a column matrix having zero elements, except the last element, which has a value of -1.0; B is the same as the matrix A except that each element in the last row of A is set as 1.0.

By solving Equation III - 15, one gets:

$$P_{00} = \frac{1}{1 + (\rho_1 + \phi\rho_2) + \frac{1}{2}(\rho_1 + \phi\rho_2)^2} \qquad \text{III - 16}$$

Consequently, all the other values of P_{ij} can be calculated.

The expected number of units in the system can be determined by using the following formula:

$$E[n] = \sum_{i=0}^{2} \sum_{j=0}^{2} (i+j).P_{ij} \qquad i+j \leq 2 \qquad \text{III - 17}$$

Substituting by the values of P_{ij} in equation III - 17, one gets:

$$E[n] = \frac{(\rho_1+\phi\rho_2) + (\rho_1+\phi\rho_2)^2}{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2} \qquad \text{III - 18}$$

Also, the probability that an arrival will have no wait prior to service is determined by using the following formula:

$$P_r[w=0] = \sum_{i=0}^{2} \sum_{j=0}^{2} P_{ij} \qquad i+j \leq 1 \qquad \text{III - 19}$$

consequently,

$$P_r[w=0] = \frac{1 + (\rho_1+\phi\rho_2)}{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2} \qquad \text{III - 20}$$

Three-Channel Closed-Loop Conveyor with Homogeneous Service Rates

By allowing three channels without storage, one can write the steady-state probability equations as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + \mu.P_{10}(t) + \frac{\mu}{\phi}.P_{01}(t) = 0 \qquad \text{III - 21}$$

$$-(\lambda_1+\lambda_2+\frac{\mu}{\phi}).P_{01}(t) + 2\frac{\mu}{\phi}.P_{02}(t) + \mu.P_{11}(t)$$

$$+ \lambda_2.P_{00}(t) = 0 \qquad \text{III - 22}$$

$$-(\lambda_1+\lambda_2+2\tfrac{\mu}{\phi}).P_{02}(t) + \mu.P_{12}(t) + \lambda_2.P_{01}(t)$$

$$+ 3\tfrac{\mu}{\phi}.P_{03}(t) = 0 \qquad \text{III - 23}$$

$$-3\tfrac{\mu}{\phi}.P_{03}(t) + \lambda_2.P_{02}(t) = 0 \qquad \text{III - 24}$$

$$-(\lambda_1+\lambda_2+\mu).P_{10}(t) + 2\mu.P_{20}(t) + \lambda_1.P_{00}(t)$$

$$+ \tfrac{\mu}{\phi}.P_{11}(t) = 0 \qquad \text{III - 25}$$

$$-(\lambda_1+\lambda_2+\mu(\tfrac{\phi+1}{\phi})).P_{11}(t) + \lambda_2.P_{10}(t) + \lambda_1.P_{01}(t)$$

$$+ 2\tfrac{\mu}{\phi}.P_{12}(t) + 2\mu.P_{21}(t) = 0 \qquad \text{III - 26}$$

$$-(\tfrac{\phi+2}{\phi})\mu.P_{12}(t) + \lambda_1.P_{02}(t) + \lambda_2.P_{11}(t) = 0 \qquad \text{III - 27}$$

$$-(\lambda_1+\lambda_2+2\mu).P_{20}(t) + 3\mu.P_{30}(t) + \lambda_1.P_{10}(t)$$

$$+ \tfrac{\mu}{\phi}.P_{21}(t) = 0 \qquad \text{III - 28}$$

$$-(\tfrac{2\phi+1}{\phi})\mu.P_{21}(t) + \lambda_1.P_{11}(t) + \lambda_2.P_{20}(t) = 0 \qquad \text{III - 29}$$

$$-3\mu.P_{30}(t) + \lambda_1.P_{20}(t) = 0 \qquad \text{III - 30}$$

Rewriting Equations III - 21 through III - 30 in matrix form, as in III - 13 and III - 15, one can then solve these equations to get P_{ij}'s in terms of P_{00}.

$$P_{01} = \phi\rho_2 . P_{00}$$

$$P_{02} = \frac{\phi^2}{2}\rho_2^2 . P_{00}$$

$$P_{03} = \frac{\phi^3}{6}\rho_2^3 . P_{00}$$

$$P_{10} = \rho_1 . P_{00}$$

$$P_{11} = \phi\rho_1\rho_2 . P_{00}$$

$$P_{12} = \frac{\phi^2}{2}\rho_1\rho_2^2 . P_{00}$$

$$P_{20} = \frac{1}{2}\rho_1^2 . P_{00}$$

$$P_{21} = \frac{\phi}{2}\rho_1^2\rho_2 . P_{00}$$

$$P_{30} = \frac{\rho_1^3}{6} . P_{00}$$

Using the boundary condition

$$\sum_{i=0}^{3} \sum_{j=0}^{3} P_{ij} = 1 \qquad i+j \leq 3 \qquad \text{III - 31}$$

one gets the value of P_{00} as follows:

$$P_{00} = \frac{1}{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2 + \frac{1}{6}(\rho_1+\phi\rho_2)^3} \qquad \text{III - 32}$$

The expected number of units in the system can be determined by using the following formula:

$$E[n] = \sum_{i=0}^{3} \sum_{j=0}^{3} (i+j)P_{ij} \qquad i+j \leq 3 \qquad \text{III - 33}$$

Substituting the values of P_{ij} in Equation III - 27, one gets:

$$E[n] = \frac{(\rho_1+\phi\rho_2) + (\rho_1+\phi\rho_2)^2 + \frac{1}{2}(\rho_1+\phi\rho_2)^3}{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2 + \frac{1}{6}(\rho_1+\phi\rho_2)^3}$$

..... III -34

and $P_r[w=0]$ can be determined as follows:

$$P_r[w=0] = \frac{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2}{1 + (\rho_1+\phi\rho_2) + \frac{1}{2}(\rho_1+\phi\rho_2)^2 + \frac{1}{6}(\rho_1+\phi\rho_2)^3}$$

..... III - 35

Four-Channel Closed-Loop Conveyor with Homogeneous Service Rates

Following the same steps, as in that of the two and three channel cases, one can write the steady-state equilibrium probability equations for the four-channel case without storage at any of the channels.

$$-(\lambda_1+\lambda_2).P_{00}(t) + \mu.P_{10}(t) + \frac{\mu}{\phi}.P_{01}(t) = 0 \qquad \text{III - 36}$$

$$-(\lambda_1+\lambda_2+\frac{\mu}{\phi}).P_{01}(t) + 2\frac{\mu}{\phi}.P_{02}(t) + \mu.P_{11}(t)$$

$$+ \lambda_2.P_{00}(t) = 0 \qquad \text{III - 37}$$

$$-(\lambda_1+\lambda_2+2\frac{\mu}{\phi}).P_{02}(t) + \lambda_2.P_{01}(t) + \mu.P_{12}(t)$$

$$+ 3\frac{\mu}{\phi}.P_{03}(t) = 0 \qquad \text{III - 38}$$

$$-(\lambda_1+\lambda_2+3\frac{\mu}{\phi}).P_{03}(t) + \lambda_2.P_{02}(t) + \mu.P_{13}(t) + 4\frac{\mu}{\phi}.P_{04}(t) = 0 \qquad \text{III - 39}$$

$$-4\frac{\mu}{\phi}.P_{04}(t) + \lambda_2.P_{03}(t) = 0 \qquad \text{III - 40}$$

$$-(\lambda_1+\lambda_2+\mu).P_{10}(t) + 2\mu.P_{20}(t) + \lambda_1.P_{00}(t) + \frac{\mu}{\phi}.P_{11}(t) = 0 \qquad \text{III - 41}$$

$$-[\lambda_1+\lambda_2+\mu(\frac{\phi+1}{\phi})].P_{11}(t) + \lambda_2.P_{10}(t) + \lambda_1.P_{01}(t) + 2\frac{\mu}{\phi}.P_{12}(t) + 2\mu.P_{21}(t) = 0 \qquad \text{III - 42}$$

$$-[\lambda_1+\lambda_2+\mu(\frac{\phi+2}{\phi})].P_{12}(t) + \lambda_1.P_{02}(t) + \lambda_2.P_{11}(t) + 3\frac{\mu}{\phi}.P_{13}(t) + 2\mu.P_{22}(t) = 0 \qquad \text{III - 43}$$

$$-(\frac{\phi+3}{\phi})\mu.P_{13}(t) + \lambda_1.P_{03}(t) + \lambda_2.P_{12}(t) = 0 \qquad \text{III - 44}$$

$$-(\lambda_1+\lambda_2+2\mu).P_{20}(t) + 3\mu.P_{30}(t) + \lambda_1.P_{10}(t) + \frac{\mu}{\phi}.P_{21}(t) = 0 \qquad \text{III - 45}$$

$$-[\lambda_1+\lambda_2+(\frac{2\phi+1}{\phi})\mu].P_{21}(t) + \lambda_1.P_{11}(t) + \lambda_2.P_{20}(t) + 3\mu.P_{31}(t) + 2\frac{\mu}{\phi}.P_{22}(t) = 0 \qquad \text{III - 46}$$

$$-(\frac{2\phi+2}{\phi})\mu.P_{22}(t) + \lambda_1.P_{12}(t) + \lambda_2.P_{21}(t) = 0 \qquad \text{III - 47}$$

$$-(\lambda_1+\lambda_2+3\mu).P_{30}(t) + \lambda_1.P_{20}(t) + \frac{\mu}{\phi}.P_{31}(t)$$

$$+ 4\mu.P_{40}(t) = 0 \qquad \text{III - 48}$$

$$-(\frac{3\phi+1}{\phi})\mu.P_{31}(t) + \lambda_1.P_{21}(t) + \lambda_2.P_{30}(t) = 0 \qquad \text{III - 49}$$

$$-4\mu.P_{40}(t) + \lambda_1.P_{30}(t) = 0 \qquad \text{III - 50}$$

Solving the above equations, one gets the following values for P_{ij}:

$$P_{01} = \phi\rho_2.P_{00}$$

$$P_{02} = \frac{\phi^2}{2}\rho_2^2.P_{00}$$

$$P_{03} = \frac{\phi^3\rho_2^3}{6}.P_{00}$$

$$P_{04} = \frac{\phi^4\rho_2^4}{24}.P_{00}$$

$$P_{10} = \rho_1.P_{00}$$

$$P_{11} = \phi\rho_1\rho_2.P_{00}$$

$$P_{12} = \frac{\phi^2}{2}\rho_1\rho_2^2.P_{00}$$

$$P_{13} = \frac{\phi^3}{6}\rho_1\rho_2^3.P_{00}$$

$$P_{20} = \tfrac{1}{2}\rho_1^2.P_{00}$$

$$P_{21} = \frac{\phi}{2}\rho_1^2\rho_2 . P_{00}$$

$$P_{31} = \frac{\phi}{6}\rho_1^3\rho_2 . P_{00}$$

$$P_{30} = \frac{\rho_1^3}{6} . P_{00}$$

$$P_{41} = \frac{\phi}{24}\rho_1^4\rho_2 . P_{00}$$

$$P_{40} = \frac{\rho_1^4}{24} . P_{00}$$

Using the boundary condition, one can get:

$$P_{00} = \frac{1}{1+(\rho_1+\phi\rho_2)+\frac{1}{2}(\rho_1+\phi\rho_2)^2+\frac{1}{6}(\rho_1+\phi\rho_2)^3+\frac{1}{24}(\rho_1+\phi\rho_2)^4} \qquad \text{..... III - 51}$$

The expected number of units in the system can be expressed as:

$$E[n] = \frac{(\rho_1+\phi\rho_2)+(\rho_1+\phi\rho_2)^2+\frac{1}{2}(\rho_1+\phi\rho_2)^3+\frac{1}{6}(\rho_1+\phi\rho_2)^4}{1+(\rho_1+\phi\rho_2)+\frac{1}{2}(\rho_1+\phi\rho_2)^2+\frac{1}{6}(\rho_1+\phi\rho_2)^3+\frac{1}{24}(\rho_1+\phi\rho_2)^4} \qquad \text{..... III - 52}$$

and

$$P_r[w=0] = \frac{1+(\rho_1+\phi\rho_2)+\frac{1}{2}(\rho_1+\phi\rho_2)^2+\frac{1}{6}(\rho_1+\phi\rho_2)^3}{1+(\rho_1+\phi\rho_2)+\frac{1}{2}(\rho_1+\phi\rho_2)^2+\frac{1}{6}(\rho_1+\phi\rho_2)^3+\frac{1}{24}(\rho_1+\phi\rho_2)^4} \qquad \text{..... III - 53}$$

M-Channel Closed-Loop Conveyor with Homogeneous Service Rates

There could be as many as 'M' channels in the system, and without storage at any of these channels, one can write the steady-state equilibrium probability equations as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + \mu.P_{10}(t) + \frac{\mu}{\phi}.P_{01}(t) = 0 \qquad \text{III - 54}$$

$$-(\lambda_1+\lambda_2+i\mu).P_{i0}(t) + \lambda_1.P_{i-1,0}(t) + \frac{\mu}{\phi}.P_{i1}(t)$$
$$+ (i+1)\mu.P_{i+1,0}(t) = 0$$
$$\text{where} \quad i = 1,2,3,\ldots.,M-1 \qquad \text{III - 55}$$

$$-M\mu.P_{M0}(t) + \lambda_1.P_{M-1,0}(t) = 0 \qquad \text{III - 56}$$

$$-(\lambda_1+\lambda_2+\frac{j}{\phi}\mu).P_{0j}(t) + \mu.P_{1j}(t) + (\frac{j+1}{\phi})\mu.P_{0,j+1}(t)$$
$$+ \lambda_2.P_{0,j-1}(t) = 0$$
$$\text{where} \quad j = 1,2,3,\ldots.,M-1 \qquad \text{III - 57}$$

$$-M\frac{\mu}{\phi}.P_{0M}(t) + \lambda_2.P_{0,M-1}(t) = 0 \qquad \text{III - 58}$$

$$-[\lambda_1+\lambda_2+(\frac{\phi i+j}{\phi})\mu].P_{ij}(t) + (i+1)\mu.P_{i+1,j}(t)$$
$$+(\frac{j+1}{\phi})\mu.P_{i,j+1}(t) + \lambda_1.P_{i-1,j}(t)$$

$$+ \lambda_2 . P_{i,j-1}(t) = 0$$

where $i = 1,2,3,\ldots,M-1$

$j = 1,2,3,\ldots,M-1$

and $i+j \leq M-1$ III - 59

$$-\left(\frac{\phi i+j}{\phi}\right)\mu . P_{ij}(t) + \lambda_1 . P_{i-1,j}(t) + \lambda_2 . P_{i,j-1}(t) = 0$$

where $i = 1,2,3,\ldots,M-1$

$j = 1,2,3,\ldots,M-1$

and $i+j = M$ III - 60

By induction, the general term of the probability of having 'i' singlet units and 'j' doublet units in the system is found as:

$$P_{ij} = \frac{\phi^j}{i!\ j!} \rho_1^i \rho_2^j . P_{00}$$

where $i = 0,1,2,\ldots,M$

$j = 0,1,2,\ldots,M$

and $i+j \leq M$ III - 61

For evaluating the system's performance, three measures are proposed, as follows:

1. The probability that all channels are idle (P_{00}):

Using the boundary condition

$$\sum_{i=0}^{M} \sum_{j=0}^{M} P_{ij} = 1 \qquad i+j \leq M \qquad \text{III - 62}$$

one gets P_{00}:

$$P_{00} = \frac{1}{\sum_{s=0}^{M} \frac{1}{s!}(\rho_1+\phi\rho_2)^s} \qquad \text{III - 63}$$

2. The expected number of units in the system:

$$E[n] = \frac{\sum_{s=1}^{M} \frac{1}{(s-1)!} (\rho_1+\phi\rho_2)^s}{\sum_{s=0}^{M} \frac{1}{s!} (\rho_1+\phi\rho_2)^s} \qquad \text{III - 64}$$

3. The probability of a lost item (the probability that all channels are busy at the time of arrival of an item), i.e. $1-P_r[w=0]$:

$$P_r[w=0] = \frac{\sum_{s=0}^{M-1} \frac{1}{s!}(\rho_1+\phi\rho_2)^s}{\sum_{s=0}^{M} \frac{1}{s!} (\rho_1+\phi\rho_2)^s} \qquad \text{III - 65}$$

From the above presentation, there are four important independent variables: i) ρ_1 - traffic intensity of singlet units; ii) ρ_2 - traffic intensity of double units; iii) ϕ - service time ratio of the doublet unit to that of the singlet unit; and iv) M -

the number of channels.

There are three measures of performance of the system. These measures are: i) P_{00} - the probability of the system being idle; ii) E[n] - the expected number of units in the system; and iii) $P_r[w=0]$ - is the probability of a unit having no wait before being serviced.

The effect of the independent variables on the measures of performance of the conveyor can be summarized as follows:

1. Effect of ρ_1 and ρ_2: By fixing the value of $\phi=2$ (i.e., the service time of a doublet unit is <u>twice</u> that of a singlet unit) and by fixing the number of the channels in the system equal to two(2), one gets:

$$P_{00} = \frac{1}{1 + (\rho_1+2\rho_2) + \frac{1}{2}(\rho_1+2\rho_2)^2} \qquad \text{III - 66}$$

$$E[n] = \frac{(\rho_1+2\rho_2) + (\rho_1+2\rho_2)^2}{1 + (\rho_1+2\rho_2) + \frac{1}{2}(\rho_1+2\rho_2)^2} \qquad \text{III - 67}$$

$$P_r[w=0] = \frac{1 + (\rho_1+2\rho_2)}{1 + (\rho_1+2\rho_2) + \frac{1}{2}(\rho_1+2\rho_2)^2} \qquad \text{III - 68}$$

By changing the values of ρ_1 and ρ_2, the above measures of performance are plotted in Figures 3,4,5,6,7, and 8. It is apparent that P_{00}(probability that all channels

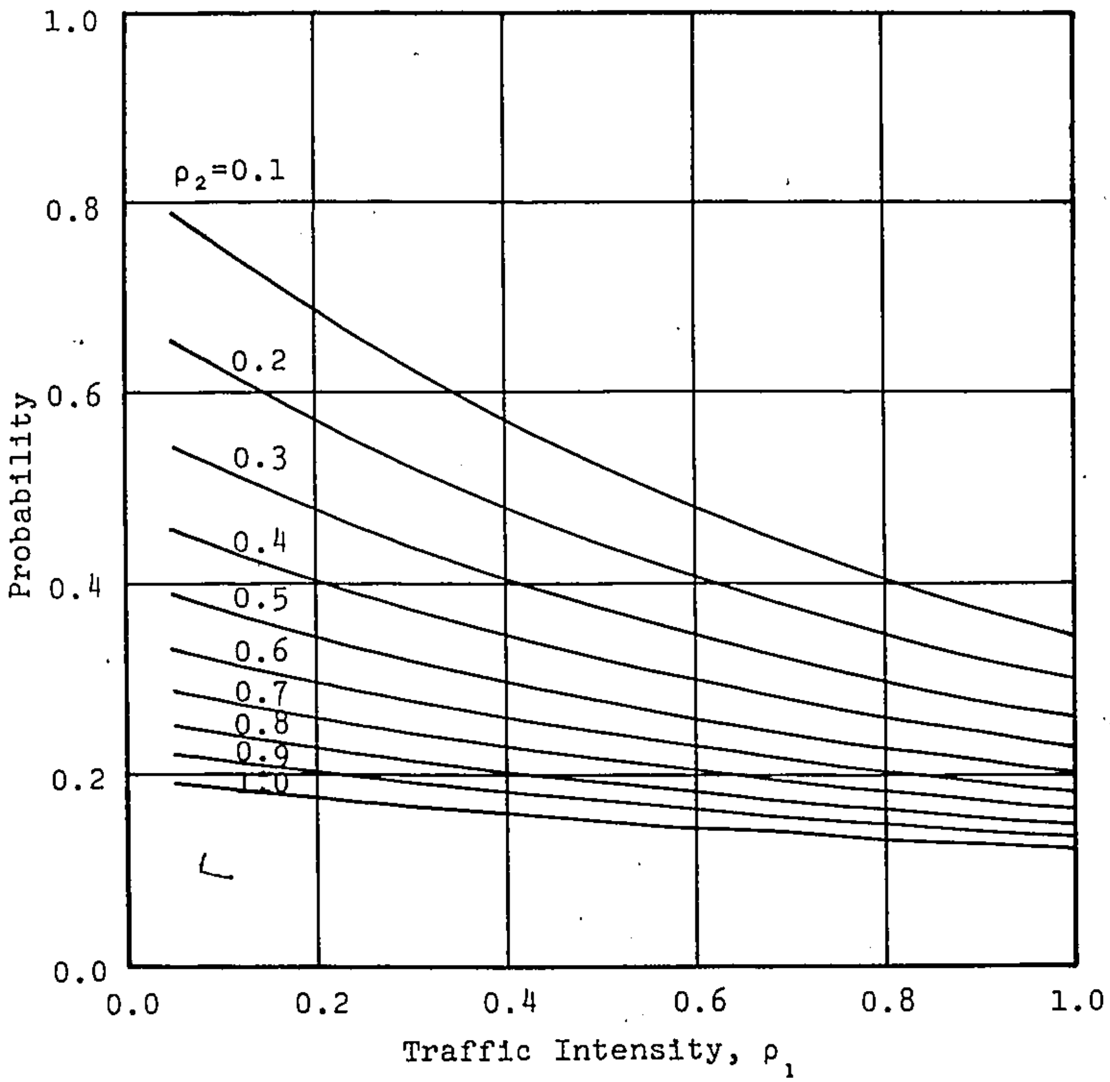


Figure 3. Change of Probability P_{00} with ρ_1, ρ_2 Fixed; for a two-Channel Case.

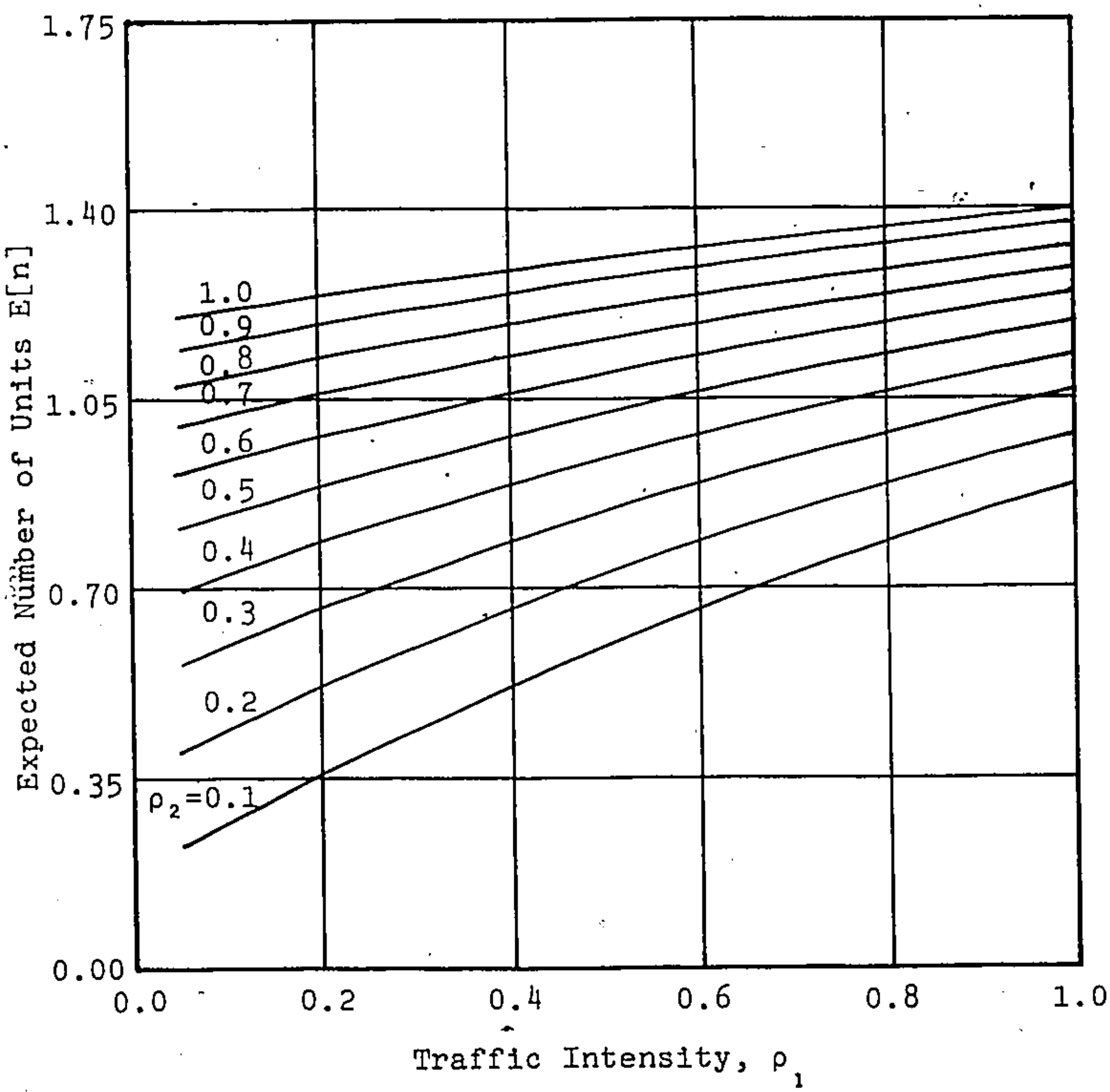


Figure 4. Expected number of units E[n] with ρ_1, ρ_2 fixed; for the two-channel case.

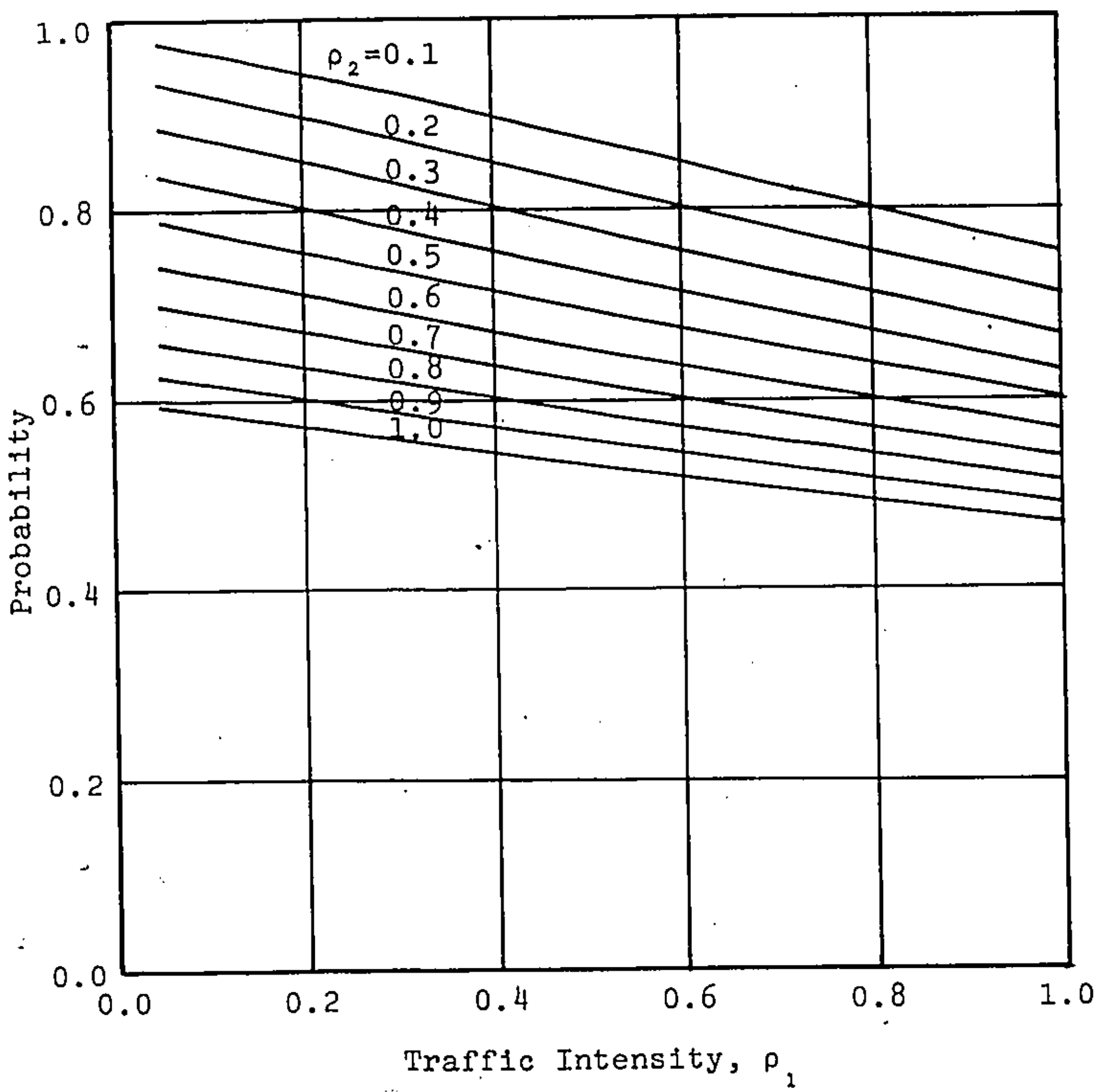


Figure 5. Change of probability $P_r(w=0)$ with ρ_1, ρ_2 fixed; for the two-channel case.

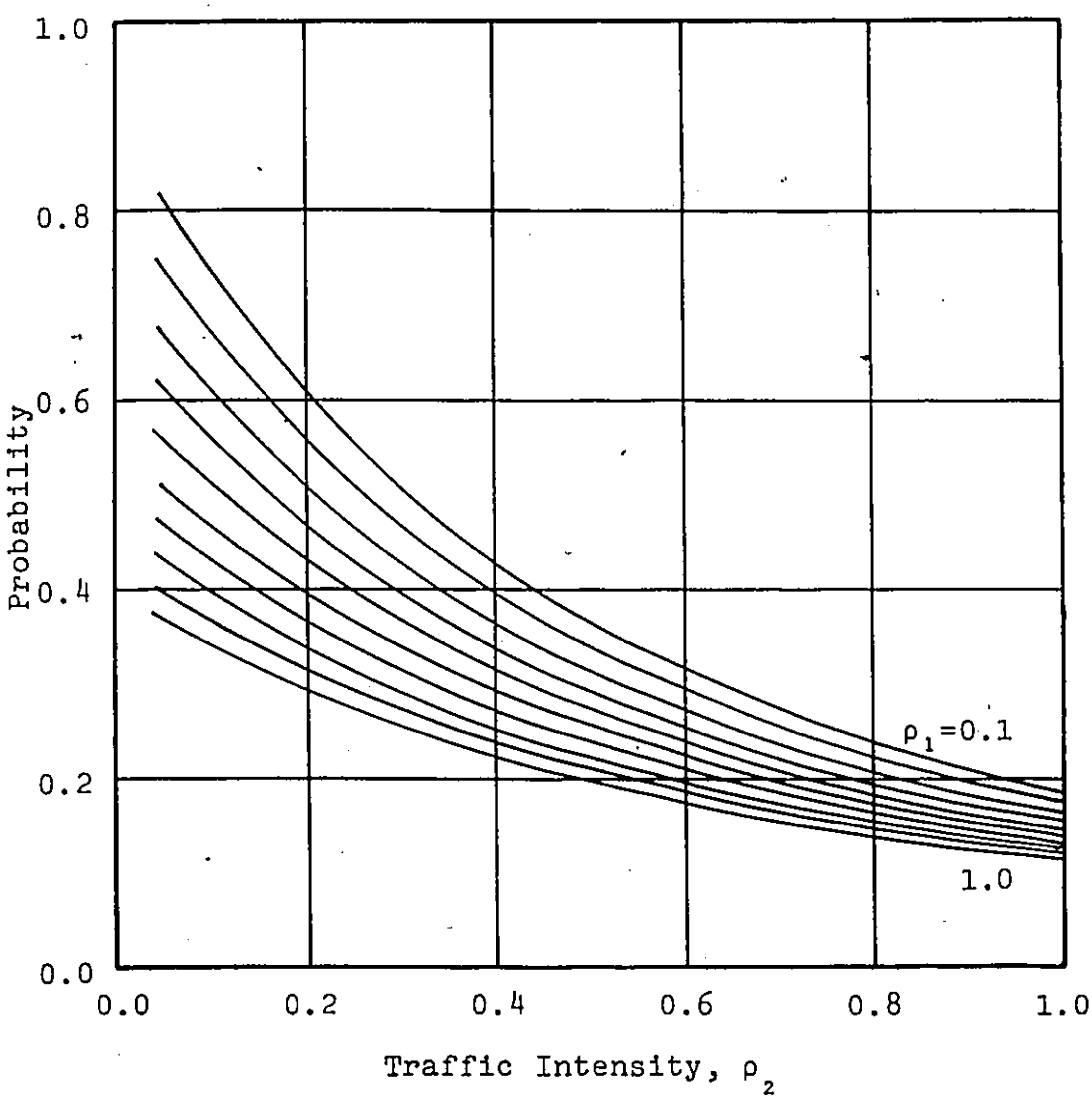


Figure 6. Change of Probability P_{00} with ρ_2, ρ_1 fixed; for the two-channel case.

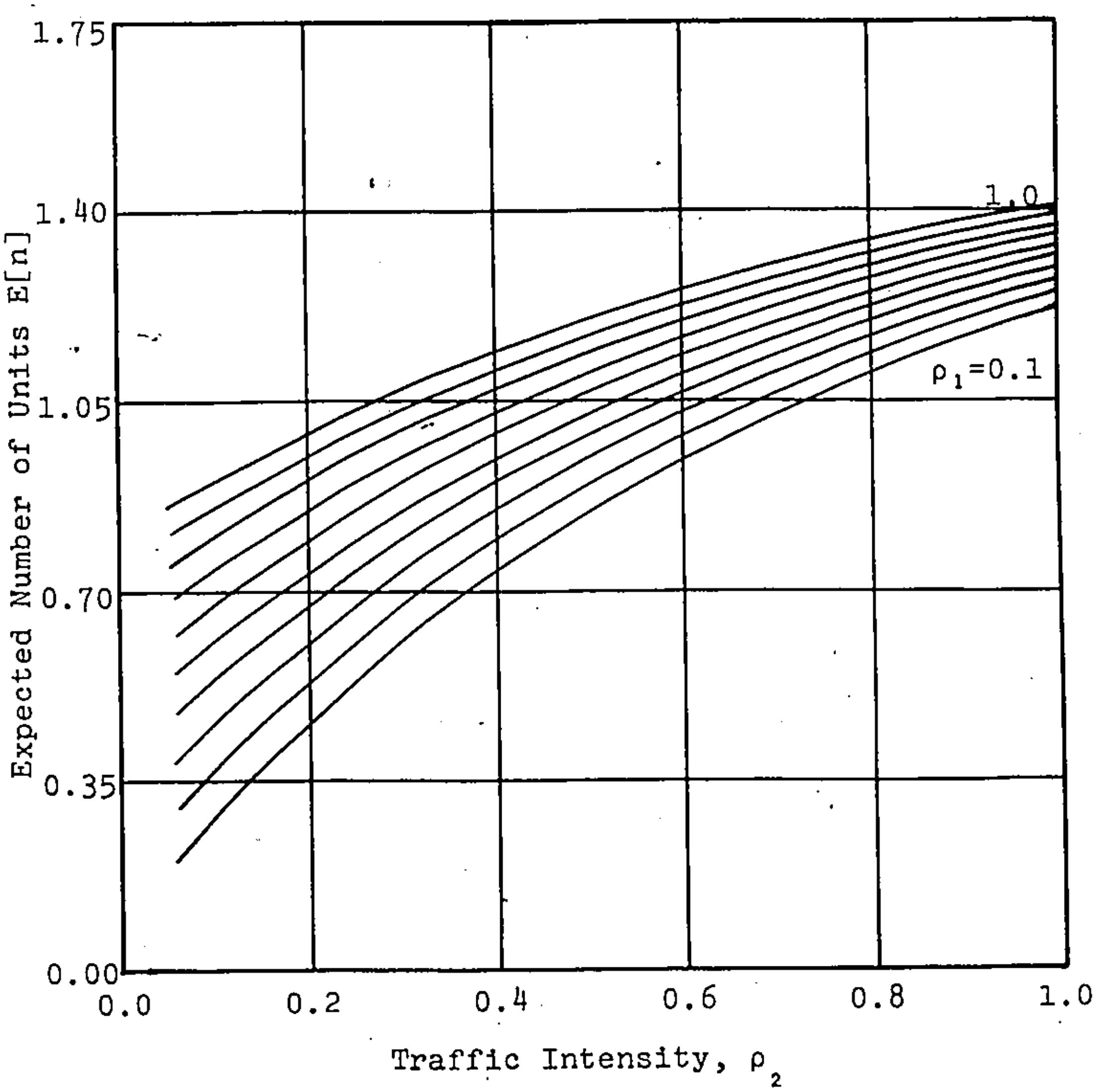


Figure 7. Expected number of Units E[n] with ρ_2, ρ_1 fixed; for the two-channel case.

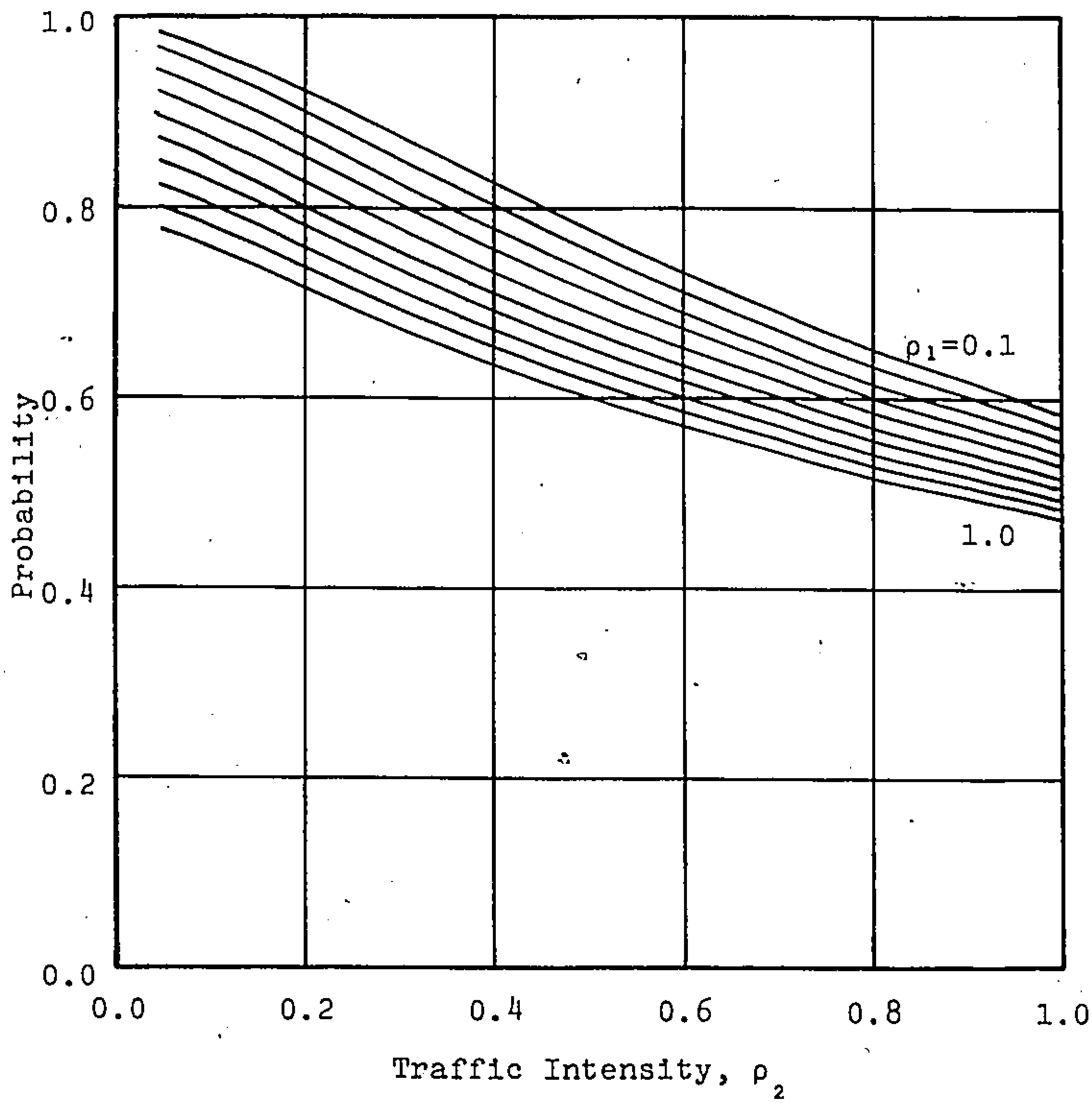


Figure 8. Change of probability $P_r(w=0)$ with ρ_2, ρ_1 fixed; for the two-channel case.

are idle) is minimum, when $\rho_1=\rho_2=1.0$. Also, keeping ρ_2 constant, at a high value, and increasing ρ_1, will result in a decrease in the value of P_{00}. So too, maintaining a high value of ρ_2, while increasing the value ρ_1, will result in an increase in the expected number of units in the system (E[n]). .

However, maintaining a high value for ρ_1, while increasing the value of ρ_2, will not result in a high expected number of units in the system. The maximum value of E[n] is reached when $\rho_1=\rho_2=1.0$.

In order to minimize the probability of lost arrivals, one needs to maintain a constant high value of ρ_2 while decreasing the value of ρ_1.

2. Effect of the number of the channels: By fixing the value of $\phi=2$ and by fixing the values of ρ_1 and ρ_2, while changing M (number of service channels), one gets the effect of the number of channels on the system's measures of performance, as shown in Figures 9,10, and 11.

As ρ_1 increases, M increases up to a certain value to allow P_{00} to approach a constant value. Beyond that M value, M does not affect P_{00}.

E[n] increases with the increase of ρ_1. Furthermore, E[n] requires, as it approaches a constant value, a higher M value, at increasing values of ρ_1, more so than at lower values of ρ_1.

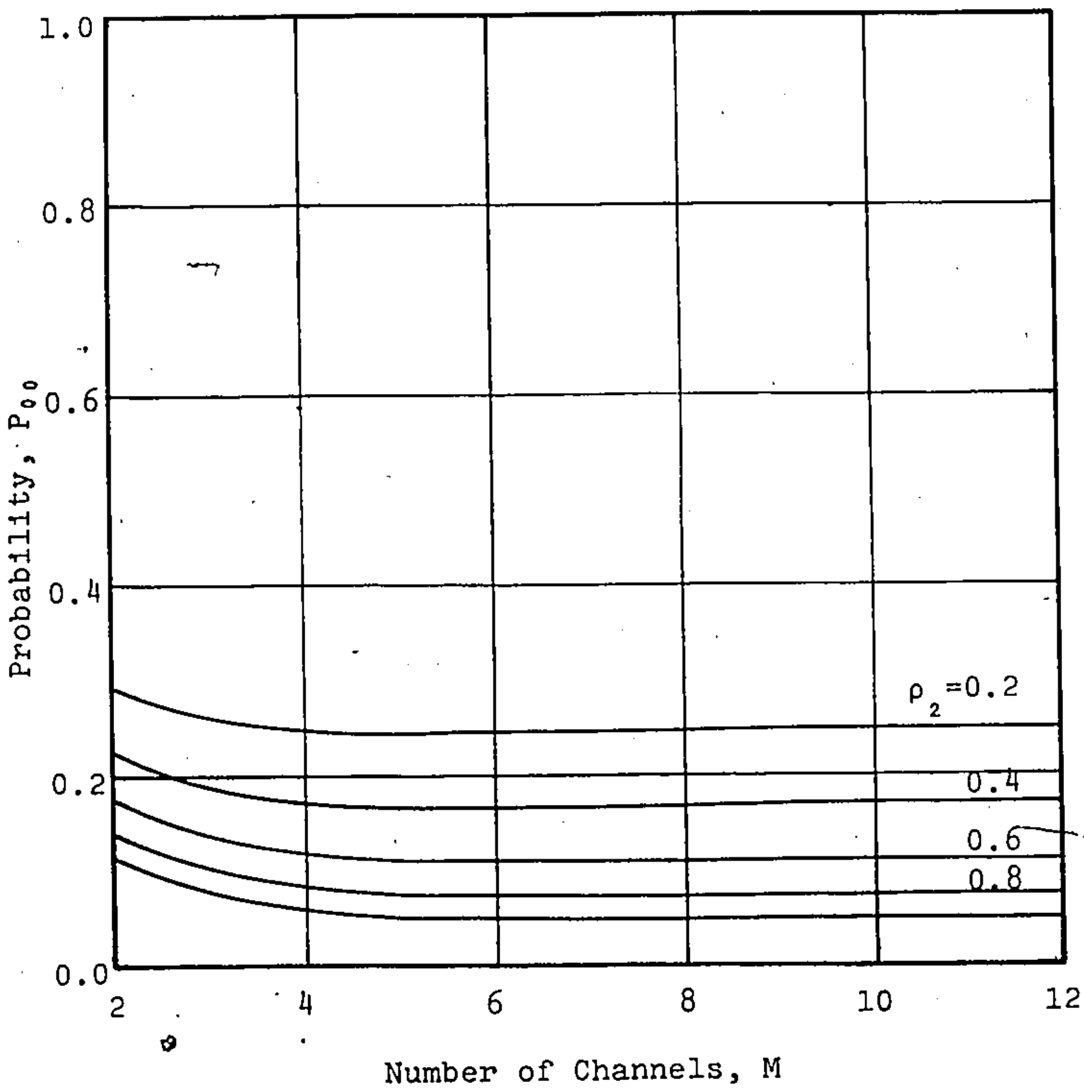


Figure 9. Relationship between the number of channels and the probability of the system being idle; ρ_1 fixed at 1.0.

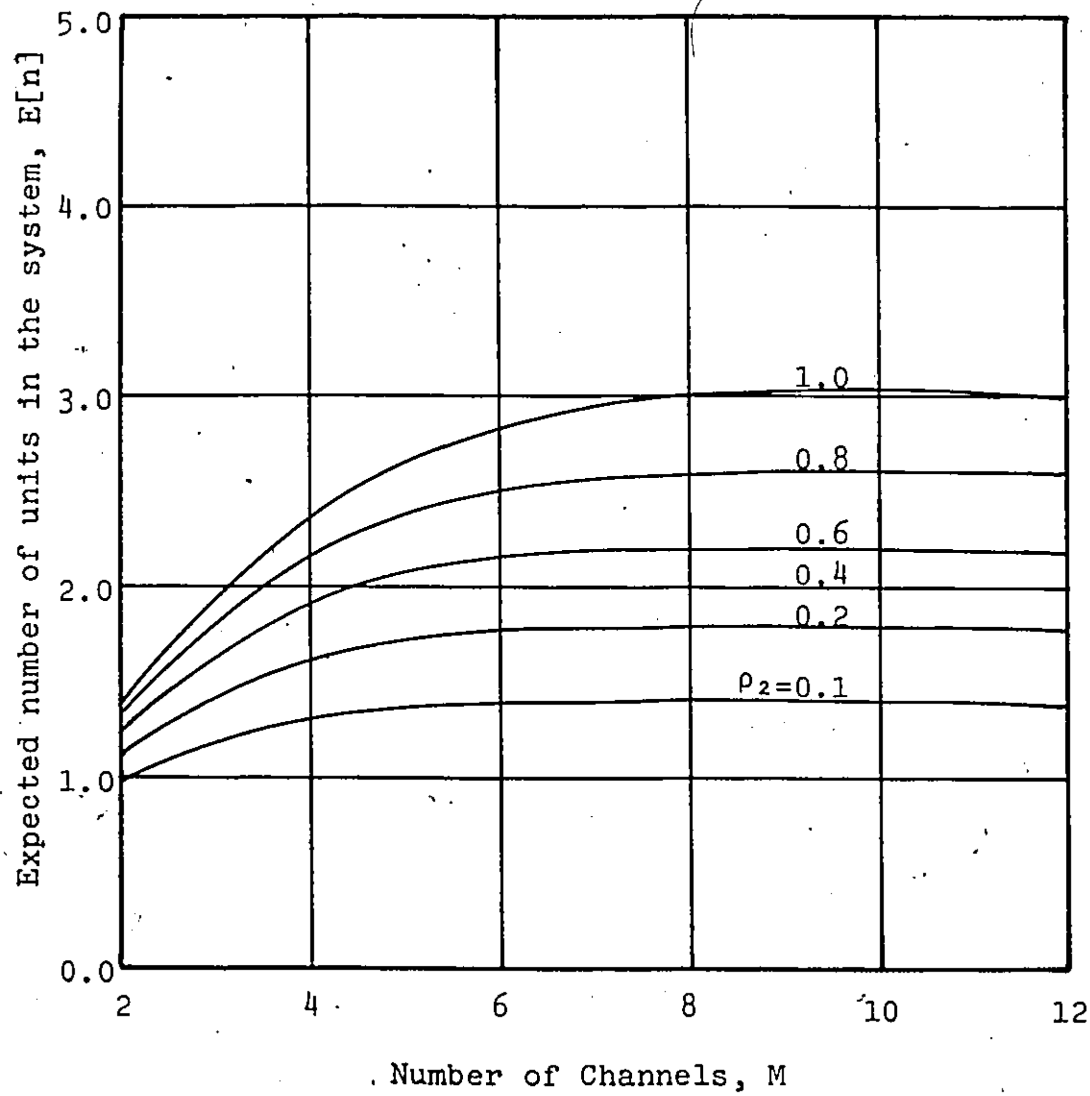


Figure 10. Relationship between M and E[n]; ρ_1 fixed at 1.0.

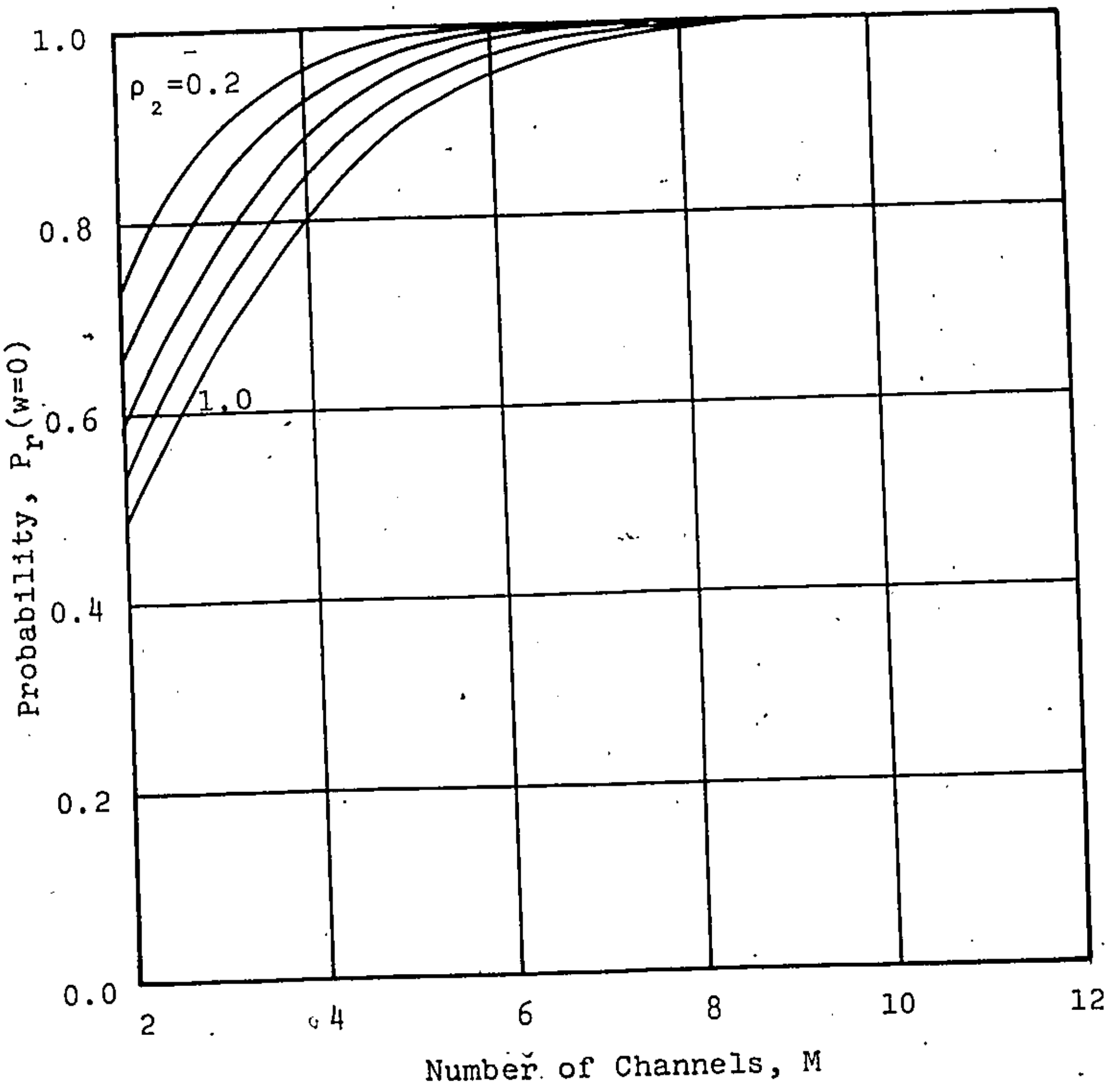


Figure 11. Relationship between M and $P_r(w=0)$; ρ_1 fixed at 1.0.

$P_r[w=0]$ approaches a maximum value as ρ_1 decreases. For $P_r[w=0]$ to approach a constant value, it requires a higher M value, at higher values of ρ_1, more so than at lower values of ρ_1.

3. Effect of ϕ: By fixing the values of ρ_1 and ρ_2 and M (the number of service channels is 2) and by changing the value of ϕ, one gets the effect of the service ratio of a doublet unit, to that of a singlet unit on the system's performance, as shown in Figures 12, 13, and 14.

From these figures, it is obvious that, in order to minimize the probability of the system being idle (P_{00}), one should consider the following points:

1. Keep ρ_1 at a high value, while increasing the value of ρ_2. However, the opposite will not result in minimum values of P_{00}.

2. As ϕ (the service time ratio) increases, the values of P_{00} will decrease. However, as ϕ increases up to a certain value, P_{00} will approach a constant value. Beyond that ϕ value, ϕ does not affect P_{00}.

Following the opposite of the above procedure, will result in increasing the probability, that an arrival will have no wait prior to service. Also, increasing ϕ will result in a slight increase of the expected number of units in the system.

In order to make the results more useful - in a

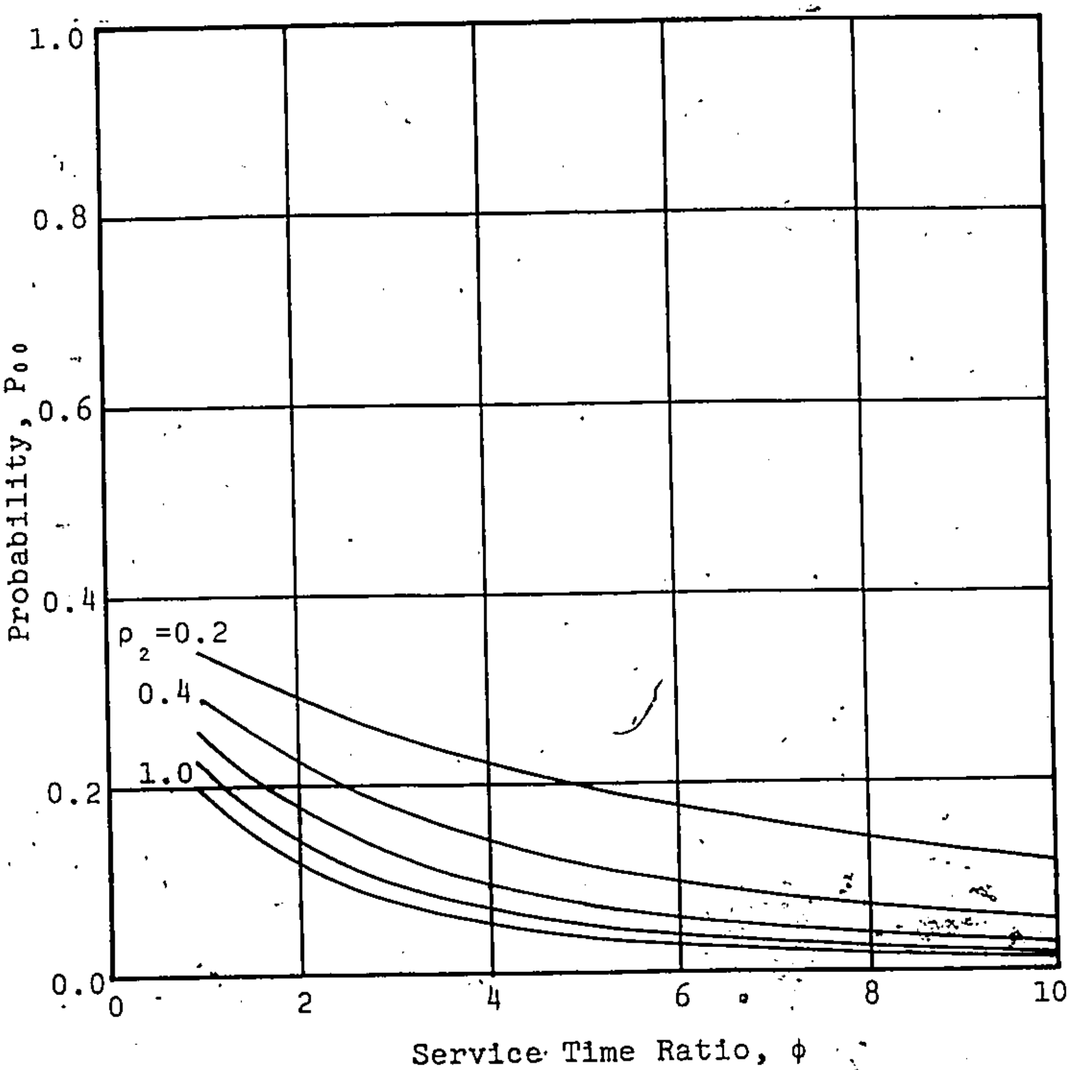


Figure 12. Change of probability P_{00} with ϕ, ρ_1 fixed at 1.0, for the two-channel case.

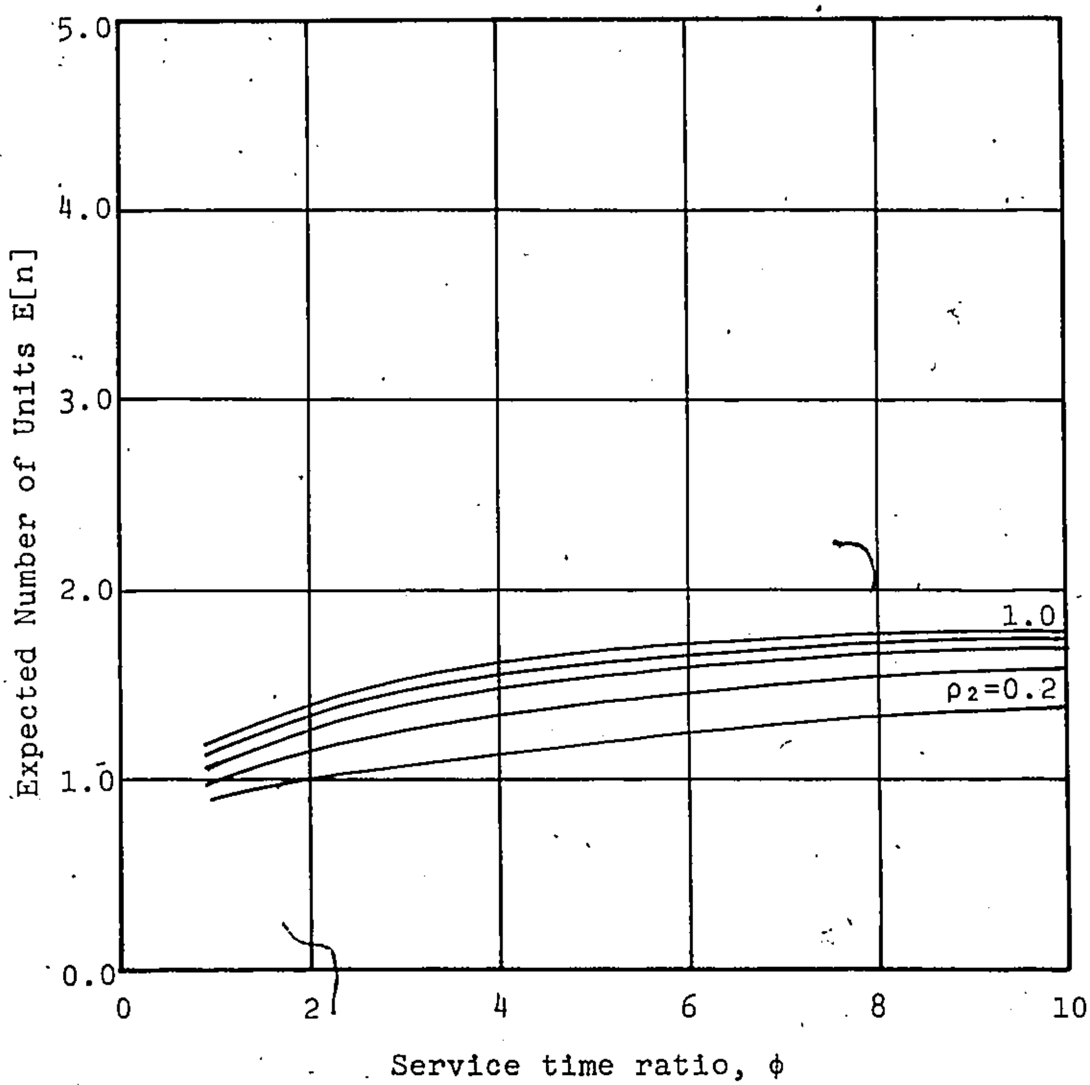


Figure 13. Expected number of units E[n] with ϕ, ρ_1 fixed at 1.0, for the two-channel case.

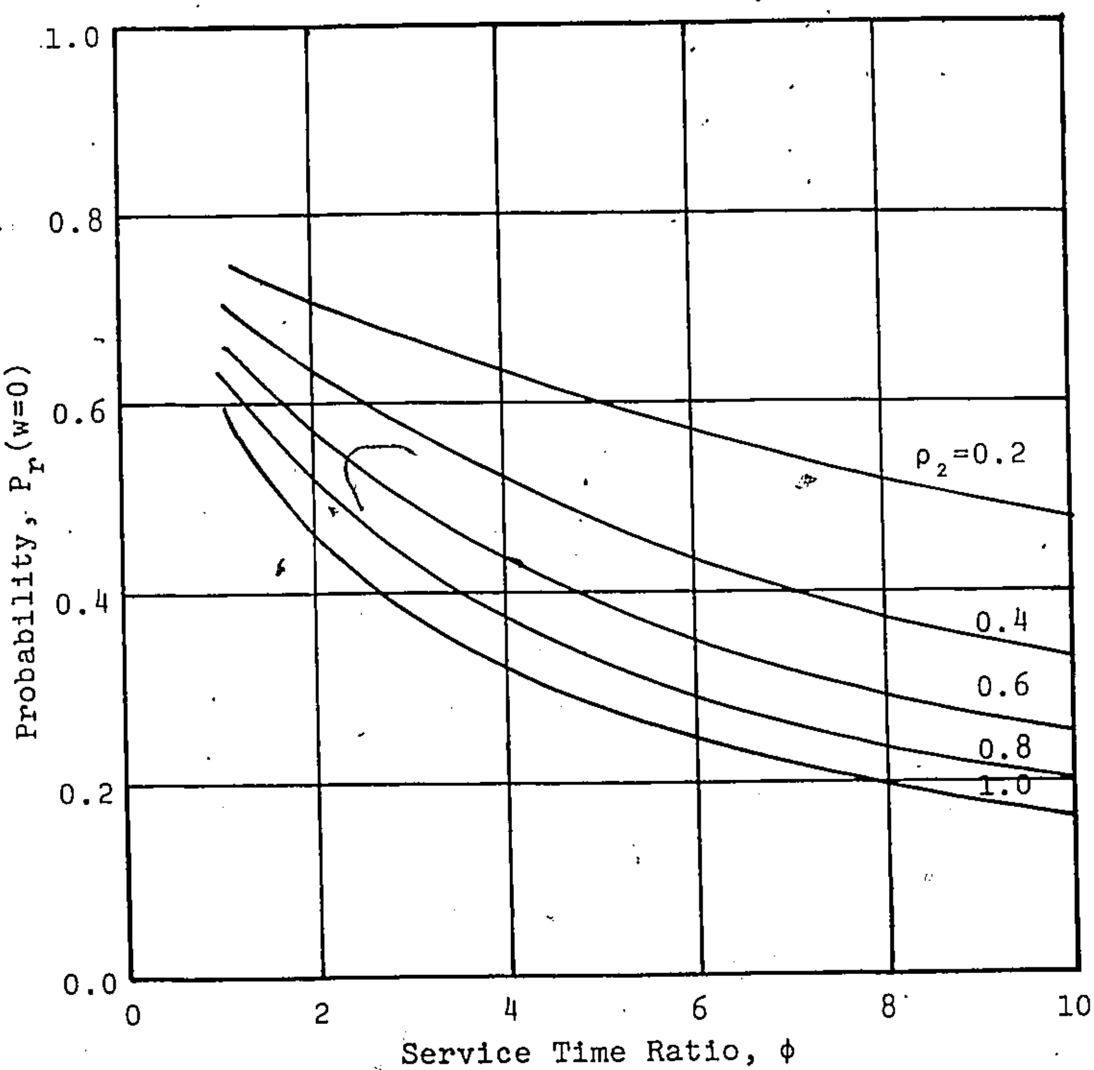


Figure 14. Change of probability $P_r(w=0)$ with ϕ, ρ_1 fixed at 1.0, for the two-channel case.

practical sense - these results have been coded in Fortran IV. The programme is given in Figure 15 and is completely self-contained with the following features: (i) the probability matrix is set for any number of channels (M); and (ii) the solution of this probability matrix is given in the output, where all the probabilities are given.

Also, computer programmes to determine and plot the probabilities for two, three and M-channel conveyors are given in the appendices.

FIGURE 15

```
C     CLOSED LOOP CONVEYOR SYSTEM WITH M CHANNELS AND
C     NO STORAGE AT EACH OF THEM, ALLOWING MULTIPLE
C     POISSON INPUTS
      DIMENSION A(120,120),B(120,1),MN(120),MM(120),
     1MM3(120),MN1(120)
      DIMENSION AA(14400)
      MM1=0.0
      DO 100 M=2,100
      IF(M.EQ.2)GOTO 10
      M2=M-1
      MM1=MM1+M2
10    N=(M+1)*M-MM1
      WRITE(6,50)M,N
50    FORMAT(20X,'NUMBER OF CHANNELS',I3,10X,'NUMBER OF
     1EQUATIONS',I3)
      DO 800 IZ=4,20,4
      PS=IZ
      R2=PS/20.
      DO 800 JZ=4,20,4
      PT=JZ
      R1=PT/20.
      WRITE(6,250)R1,R2
250   FORMAT(6X,'R1=',F10.4,6X,'R2',F10.4)
      WRITE(6,1)
1     FORMAT(6X,'*****      ******')
      SALAM=0.0
      ISUMN=0.0
      ISUM=0.0
      SADAT=0.0
      SABRY=0.0
      SUM=0.0
      SUMR=1.
      SAFER=0.0
      DO 150 II=1,N
      DO 150 JJ=1,N
      A(II,JJ)=0.0
150   B(II,1)=0.0
      IC=M+1
      DO 70 IJ=1,IC
70    MN(IJ)=2-IJ
      DO 80 JKL=1,IC
80    MM(JKL)=-JKL
      DO 90 IJK=1,M
90    MN1(IJK)=IJK
      DO110 JKS=1,IC
110   MM3(JKS)=1-JKS
C     PROBABILITY P(5,0)
```

```
      ID=M-1
      DO 120 K=1,ID
      ISUM=ISUM+MN(K)
      ISUMN=ISUMN+MN(K+1)
      KK=ISUM+1+K*M
C     PROBABILITY P K,O
      A(K,KK)=-(R1+R2+K)
      WRITE(6,132)K,KK,A(K,KK)
132   FORMAT(6X,'PROBABILITY P(K,0)=','A(',I3,',',I3,')
     1=',F10.5)
      JJJ=ISUMN+2+(K+1)*M
C     PROBABILITY P(K+1,0)
      A(K,JJJ)=K+1
      WRITE (6,550) K,JJJ,A(K,JJJ)
550   FORMAT(6X,'PROBABILITY P(K+1,0)=','A(",I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(K,1)
      KK1=ISUM+2+K*M
      A(K,KK1)=0.5
      WRITE(6,133)K,KK1,A(K,KK1)
133   FORMAT(6X,'PROBABILITY P(K,1)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(0,0)
      A(1,1)=R1
      IF(K.EQ.1)GOTO120
      SUMR=SUMR+MN(K-1)
      KK2=SUMR+(K-1)*M
      A(K,KK2)=R1
      WRITE(6,134)K,KK2,A(K,KK2)
134   FORMAT(6X,'PROBABILITY P(K-1,0)=','A(',I3,',',I3,')
     1=,F10.5)
120   CONTINUE
      DO 140 KL=1,IC
      SUM=SUM+MN(KL)
140   KK3=SUM+KL*M
C     PROBABILITY P(M,0)
      A(M,KK3)=-M
      WRITE(6,135)M,KK3,A(M,KK3)
135   FORMAT(6X,'PROBABILITY P(M,0)=','A(",I3,',',I3,')
     1=',F10.5)
      KU=KK3-2
      A(M,KK3-2)=R1
      WRITE(6,136)M,KU,A(M,KU)
136   FORMAT(6X,'PROBABILITY P(M-1,0)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(0,L) EQUATION NO (4)
      IE=2*M-1
      DO 700 K1=IC,IE
      JR=K1-M
      JR1=JR+1
      JR2=JR+2
```

```
      JR3=K1+2
      F1=JR/2.
C     PROBABILITY P(0,L)
      A(K1,JR1)=-(R1+R2+F1)
      WRITE(6,137)K1,JR1,A(K1,JR1)
137   FORMAT(6X,'PROBABILITY P(0,L)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(0,L+1)
      F3=JR
      A(K1,JR2)=(1+F3)/2.
      WRITE(6,138)K1,JR2,A(K1,JR2)
138   FORMAT(6X,'PROBABILITY P(0,L+1)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(1,L)
      A(K1,JR3)=1.
      WRITE(6,139)K1,JR3,A(K1,JR3)
139   FORMAT(6X,'PROBABILITY P(1,L)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(0,L-1)
      A(K1,JR)=R2
      WRITE(6,141)K1,JR,A(K1,JR)
141   FORMAT(6X,'PROBABILITY P(0,L-1)=','A(',I3,',',I3,')
     1=',F10.5)
700   CONTINUE
      F2=-M/2.
      A(2*M,M+1)=F2
      A(2*M,M)=R2
C     EQUATION NO (6)
      RSUM=0.0
      IS=1
      SS=0.0
      SADAT=0.
      SALAM=0.
      DO 160 IJ=1,ID
      DO 170 IK=1,ID
      F4=IJ
      IF((IJ+IK).GT.M-1)GOTO 170
      SALAM=SALAM+MN(IK)
      LZ=SALAM+1+IJ+IK*M
      IL=2*M+IS
      A(IL,LZ)=-(R1+R2+IK+F4/2.)
      WRITE(6,142)IL,LZ,A(IL,LZ)
142   FORMAT(6X,'PROBABILITY P(K,L)=','A(',I3,',',I3,')
     1=',F10.5)
      SADAT=SADAT=MM3(IK)
      LZ1=SADAT+2+IJ+(IK+1)*M
      A(IL,LZ1)=IK+1
      WRITE(6,143)IL,LZ1,A(IL,LZ1)
143   FORMAT(6X,'PROBABILITY P(K+1,L)=','A(',I3,',',I3,')
     1=',F10.5)
      LZ2=SALAM+2+IJ+IK*M
```

```
      A(IL,LZ2)=(F4+1)/2.
      WRITE(6,144)IL,LZ2,A(IL,LZ2)
144   FORMAT(6X,'PROBABILITY P(K,L+1)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(K-1,L)
      IF(IK.NE.1)GOTO 500
      RSUM=RSUM+IK
      IL1=2*M+IS
      IL2=IJ+1
      A(IL1,IL2)=R1
      WRITE(6,145)IL1,IL2,A(IL1,IL2)
145   FORMAT(6X,'PROBABILITY P(K-1,L)=','A(',I3,',',I3,')
     1=',F10.5)
500   IF(IK.EQ.1)GOTO401
400   LZ3=SALAM+IK+IJ-1+(IK-1)*M
      A(IL,LZ3)=R1
      WRITE (6,146)IL,LZ3,A(IL,LZ3)
146   FORMAT(6X,'PROBABILITY P(K-1,L)=','A(",I3,',',I3,')
     1=',F10.5)
401   LZ4=IK*M+IJ+SALAM
C     PROBABILITY P(K,L-1)
      A(IL,LZ4)=R2
      WRITE(6,147)IL,LZ4,A(IL,LZ4)
147   FORMAT(6X,'PROBABILITY P(K,L-1)=','A(',I3,',',I3,')
     1=',F10.5)
      IS+IS+1
170   CONTINUE
      SS=0.0
      SALAM=0.
      SADAT=0.
160   CONTINUE
C     PROBABILITY OF EQUATION (7)
      SABRY=0.
      SADAT=0.
      SALAM=0.
      SAFER=0.
      DO 1000 IIJ=1,ID
      DO 180 IIK=1,ID
      FS=IIJ
      IF(IIK.EQ.1)GOTO 111
      SAFER=SAFER+MN(IIK-1)
111   SALAM=SALAM+MN(IIK)
      SABRY=SABRY+MN(IIK)
      IF((IIJ+IIK).LT.M)GOTO 180
      IF(IIJ+IIK.GT.M)GOTO 180
      LZ5=(IIK)*M+IIJ+SABRY+1
      IL4=2*M+((M-2)*(M-1))/2+IIK
      IL5=IIJ+1
      A(IL4,LZ5)=-(IIK+F5/2.)
      WRITE(6,154)IL4,LZ5,A(IL4,LZ5)
154   FORMAT(6X'PROBABILITY P(K,L)=','A(',I3,',',I3,')
```

```
     1=',F10.5)
      IF(IIK.NE.1)GOTO 200
      A(IL4,IL5)=R1
      WRITE(6,148)IL4, IL5,A(IL4,IL5)
148   FORMAT(6X,'PROBABILITY P(K-1,L)=','A(',I3,',',I3,')
     1=',F10.5)
200   LZ6=SAFER+1+IIJ+(IIK-1)*M
      LZ7=IIK*M+IIJ+SALAM
      A(IL4,LZ7)=R2
      WRITE(6,152)IL4,LZ7,A(IL4,LZ7)
152   FORMAT(6X,'PROBABILITY P(K,L-1)=','A(',I3,',',I3,')
     1=',F10.5)
      IF(IIK.EQ.1)GOTO 180
      A(IL4,LZ6)=R1
      WRITE(6,149)IL4,LZ6,A(IL4,LZ6)
149   FORMAT(6X,'PROBABILITY P(K-1,L)=','A(',I3,',',I3,')
     1=',F10.5)
C     PROBABILITY P(K,L-1)
180   CONTINUE
      SABRY=0.
      SADAT=0.
      SAFER=0.
      SALAM=0.
1000  CONTINUE
      DO 210 IIR=1,N
      A(N,IIR)=1.
      B(IIR,1)=0.
210   CONTINUE
      B(N,1)=1.
      WRITE(6,221)
221   FORMAT(6X,'PROBABILITY MATRIX')
      WRITE(6,230)((B(IH,JH),JH=1,1),IH=1,N)
230   FORMAT('RIGHT HAND SIDE',10(F10.5))
      NN=N*N
      MNF=0
      DO 2000 I=1,N
      DO 2000 J=1,N
      AA(MNF+1)=A(J,I)
      MNF=MNF+1
2000  CONTINUE
      CALL SIMQ(AA,B,N,KS)
      WRITE(6,240)(B(I,1),I=1,N)
240   FORMAT(6X,'***  S O L U T I O N  ***',6(F15.9))
800   CONTINUE
      PRINT 900
900   FORMAT(6X,'**********  E N D  O F  C A S E  ****
     1******')
      SUMR=0.
      SALAM=0.
      ISUMN=0.
      ISUM=0.
```

```
      SADAT=0.
      SABRY=0.
      SUM=0.
      SAFER=0.
100   CONTINUE
      STOP
      END
```

CHAPTER IV

CLOSED-LOOP CONVEYORS WITH HETEROGENEOUS SERVERS AND LOST ARRIVALS

Chapter III dealt with closed-loop conveyors having homogeneous servers. It was assumed that the servers had equal service rates. This situation is exemplified by the case of using automated machines as servers.

There are many situations in practice, where the servers have unequal service rates. This is illustrated in the case where operators are at the service channels. In this situation, the operators work with unequal service rates, due to the physical and mental differences between them.

This chapter deals with situations where the servers have unequal service rates. The steady-state probability equations of two and three channel closed-loop conveyors and the system's measures of performance are evaluated. The two-channel conveyors having more than two input sources are also dealt with.

In addition to the general assumptions of the system, given in Chapter III, one assumes that:

1. The service rates of the first, second, third, and M^{th} channel are μ_1, μ_2, μ_3,...,μ_M, respectively.

2. The service rate ratio between the service

rate of the i^{th} channel to that of the 1^{st} channel is Θ_i, i.e., $\Theta_i = \frac{\mu_i}{\mu_1}$.

3. The traffic intensity of the singlet arrivals is $\rho_1 = \frac{\lambda_1}{\mu_1}$ and that of the doublet arrivals is $\rho_2 = \frac{\lambda_2}{\mu_1}$.

One can now proceed to develop the steady-state probability equations.

Two-Channel Closed-Loop Conveyor With Heterogeneous Servers

Consider the case of a two-channel conveyor without storage at any of the service channels. The service rates at the first and the second channel are μ_1 and μ_2, respectively.

Let P(i,j) equal the steady-state probability that channel 1 has i units and channel 2 has j units, with i=0,1 and j=0,1. One can now proceed to derive the equilibrium probability equations as follows:

$$P_{0,0}(t+\Delta t) = P_{0,0}(t)[1 - (\lambda_1+\lambda_2)\Delta t] + [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}]\Delta t . P_{1,0}(t) + [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}]\Delta t . P_{0,1}(t) \quad \text{..... VI - 1}$$

$$P_{0,1}(t+\Delta t) = P_{0,1}(t)[1 - (\lambda_1+\lambda_2 + (\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2}$$

$$+ \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}))]\Delta t$$

$$+ P_{1,1}(t)\ (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)})\Delta t$$

..... IV - 2

$$P_{1,0}(t+\Delta t) = P_{1,0}(t)[1 - (\lambda_1+\lambda_2 + (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2}$$

$$+ \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)})]\Delta t + P_{1,1}(t)\ (\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2}$$

$$+ \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)})\Delta t + (\lambda_1+\lambda_2)\Delta t . P_{0,0}(t)$$

..... IV - 3

$$P_{1,1}(t+\ t) = P_{1,1}(t)[1 - (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}$$

$$+ \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}]\Delta t$$

$$+ (\lambda_1+\lambda_2)\Delta t . P_{1,0}(t) + (\lambda_1+\lambda_2)\Delta t . P_{0,1}(t)$$

..... IV - 4

Follwoing the same steps as in Chapter III, one can obtain the steady-state probability equations as follows:

$$-(\lambda_1+\lambda_2).P(0,0) + [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,0)$$

$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(0,1) = 0$$

..... IV - 5

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)})].P(0,1)$$

$$+ [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,1) = 0 \qquad \ldots\ldots \text{ IV - 6}$$

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)})].P(1,0)$$

$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(1,1)$$

$$+ (\lambda_1+\lambda_2).P(0,0) = 0 \qquad \text{IV - 7}$$

$$-[\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(1,1)$$

$$+ (\lambda_1+\lambda_2).P(1,0) + (\lambda_1+\lambda_2).P(0,1) = 0 \qquad \ldots\ldots \text{ IV - 8}$$

Solving the above system of equations as it was followed in the solution of the two-channel conveyor with homogeneous servers and using the boundary condition

$$\sum_{i=0}^{1} \sum_{j=0}^{1} P(i,j)=1$$

one obtains the following:

$$P(0,1) = [(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})(\rho_1+\rho_2)^2]$$

$$/ \{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})(\rho_1+\rho_2)^2 \ (1+2\theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \ (\rho_1+\rho_2)\ (3+\Theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 \ (1+\Theta_2)\} \qquad \text{IV - 9}$$

$$P(1,0) = \{[\Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\ (\rho_1+\rho_2)]\ [(\rho_1+\rho_2)$$

$$+ \frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2} + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\Theta_2]\}$$

$$/ \{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\ (\rho_1+\rho_2)^2(1+2\Theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \ (\rho_1+\rho_2)\ (3+\Theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 \ (1+\Theta_2)\} \qquad \text{IV - 10}$$

$$P(1,1) = \{(\rho_1+\rho_2)^2\ [\rho_1+\rho_2+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\Theta_2]\}$$

$$/ \{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\ (\rho_1+\rho_2)^2\ (1+2\Theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \ (\rho_1+\rho_2)\ (3+\Theta_2)$$

$$+ \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 \ (1+\Theta_2)\} \qquad \text{IV - 11}$$

The measures of the system's performance can be evaluated as follows:

1. P(0,0); Probability of the system being idle:

$$P(0,0) = \{[\Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2]\ [2(\rho_1+\rho_2) + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})$$

$$+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\Theta_2]\} \ / \ \{(\rho_1+\rho_2)^3$$

$$+ \left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right) (\rho_1+\rho_2)^2 (1+2\Theta_2)$$

$$+ \Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)^2 (\rho_1+\rho_2) (3+\Theta_2)$$

$$+ \Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)^3 (1+\Theta_2)\} \qquad \text{IV - 12}$$

2. E[n]; The expected number of units in the system:

$$E[n] = \{[\Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right) (\rho_1+\rho_2)] [(\rho_1+\rho_2)$$

$$+ \frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2} + \left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)\Theta_2] + (\rho_1+\rho_2)^2$$

$$\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right) + 2(\rho_1+\rho_2)^2 [(\rho_1+\rho_2)$$

$$+ \Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)]\} / \{(\rho_1+\rho_2)^3$$

$$+ \left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right) (\rho_1+\rho_2)^2 (1+2\Theta_2)$$

$$+ \Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)^2 (\rho_1+\rho_2) (3+\Theta_2)$$

$$+ \Theta_2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)^3 (1+\Theta_2)\} \qquad \text{IV - 13}$$

3. P(1,1); The probability of a lost item: (see equation IV-11).
4. The probability that the first server is busy is given as $P_1(\text{busy}) = \chi_1$, where

$$\chi_1 = \{(\rho_1+\rho_2)^2 [(\rho_1+\rho_2) + 2\left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)\Theta_2]$$

$$+ \left(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}\right)^2 (\rho_1+\rho_2)\Theta_2 (1+\Theta_2)\} /$$

$$\{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\rho_1+\rho_2)^2 (1+2\Theta_2)$$

$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 (\rho_1+\rho_2) (3+\Theta_2)$$

$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 (1+\Theta_2)\} \qquad \text{IV - 14}$$

5. The probability that the second server is busy is given as $P_2(\text{busy}) = \chi_2$ where

$$\chi_2 = \{(\rho_1+\rho_2)^2 [(\rho_1+\rho_2) + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (1+\Theta_2)]\}$$

$$/ \{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\rho_1+\rho_2)^2 (1+2\Theta_2)$$

$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 (\rho_1+\rho_2) (3+\Theta_2)$$

$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 (1+\Theta_2)\} \qquad \text{IV - 15}$$

Verification of the Results

Setting the arrival rate of the doublet units equal to zero, i.e., $\lambda_2=0$, in equation IV-12, one obtains:

$$P(0,0) = \{\Theta_2(1+2\rho_1+\Theta_2)\} / \{\Theta_2+3\rho_1\Theta_2+\Theta_2^2+2\rho_1^2\Theta_2$$

$$+\rho_1\Theta_2^2+\rho_1^2+\rho_1^3\} \qquad \text{IV - 16}$$

which is the same as P(0,0), evaluated for one type of arrival. Also, setting $\Theta_2=1$, in equation IV-16, one obtains:

$$P(0,0) = 2/\{\rho_1^2 + 2\rho_1 + 2\} \qquad \text{IV - 17}$$

Equation IV-17 is the same as that developed by Disney (5) for a two-channel conveyor with homogeneous servers.

Effect of Service Rate Ratio on the Measures of the System's Performance

The effect of ρ_1, ρ_2, and ϕ on the performance of the closed-loop conveyor system, was investigated in Chapter III. To study the effect of the service rate ratio(Θ) on the performance of the two-channel closed-loop conveyor, the value of the following parameters are kept constant:

(i) ρ_1=1.0 (traffic intensity of the singlets equals to unity); and

(ii) ϕ=2.0 (service time of a doublet arrival is twice that of a singlet arrival).

Substituting the above values in equations IV-11, IV-12, IV-13, and IV-15, the effect of ϕ can then be evaluated. Figures 16, 17, 18, and 19 illustrate the effect of the service rate ratio on the performance of the system. The probability of the system being idle (P_{00}) increases as Θ increases; while the expected number of units in the system (E[n]), the probability of a lost arrival (P_{11}) and the utilization of the

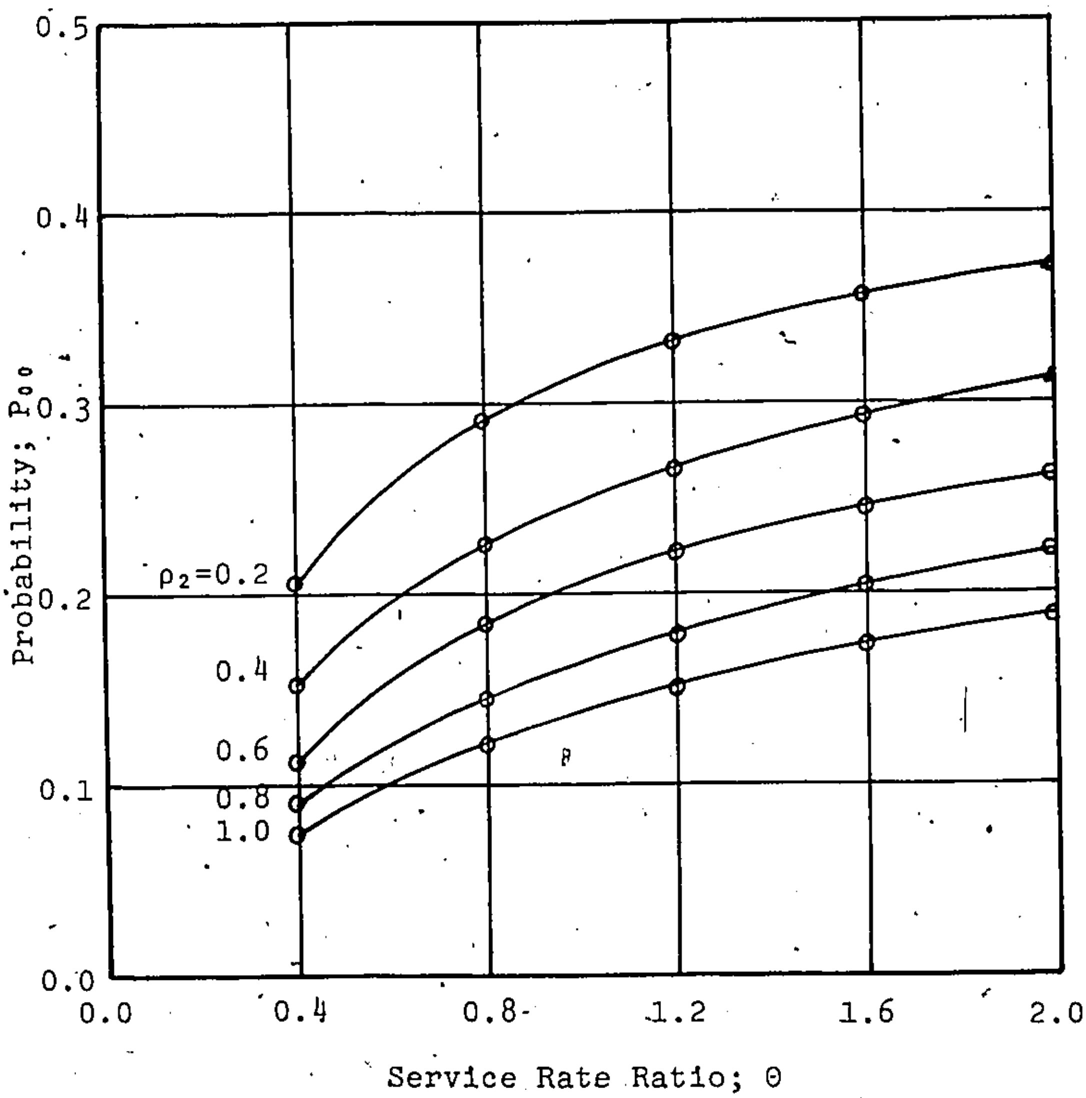


Figure 16. Effect of Θ on P_{00} for the two-channel conveyor with heterogeneous servers; ρ_1=1.0

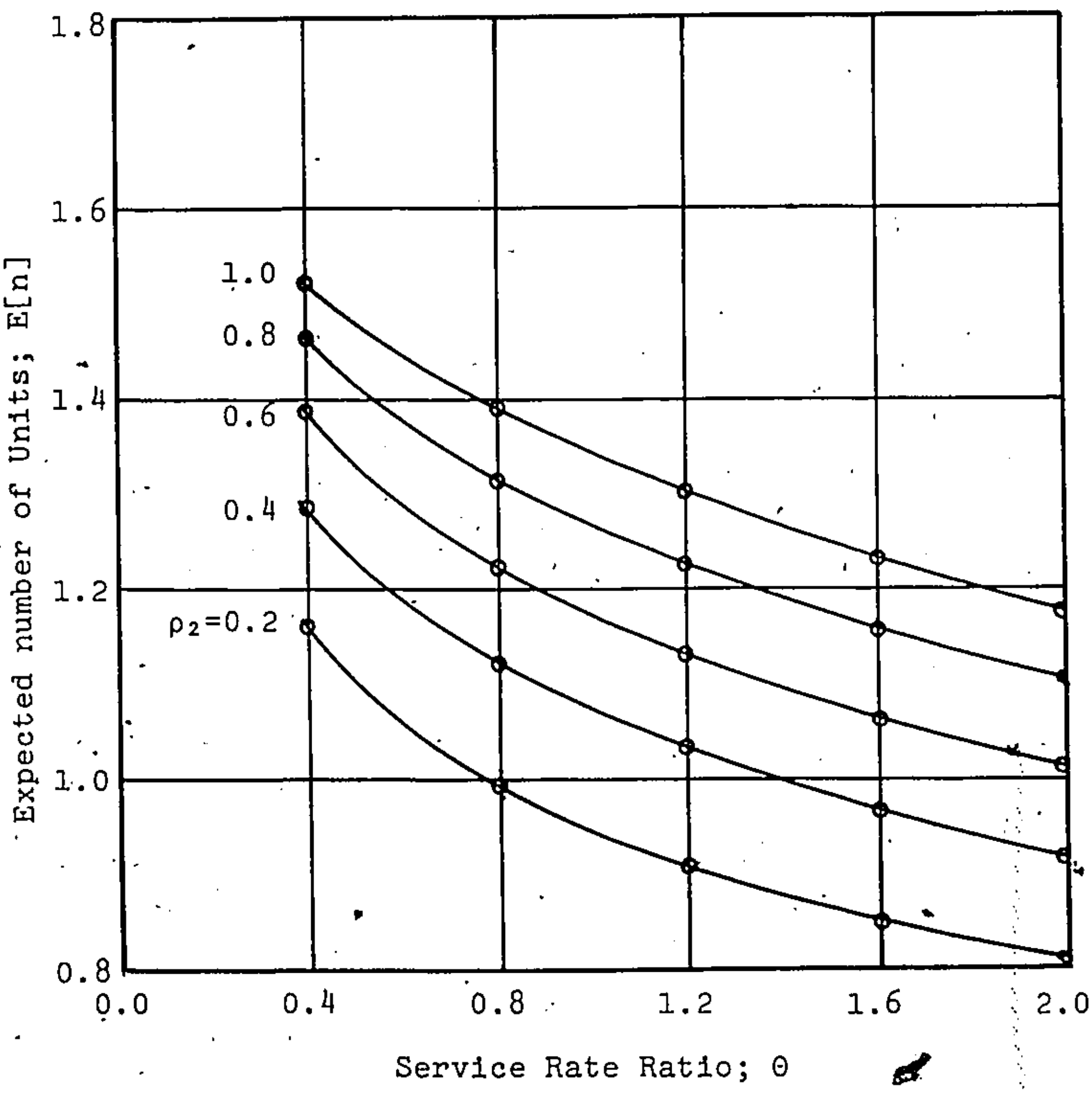


Figure 17. Effect of Θ on E[n] for the two-channel conveyor with heterogeneous servers; ρ_1=1.0

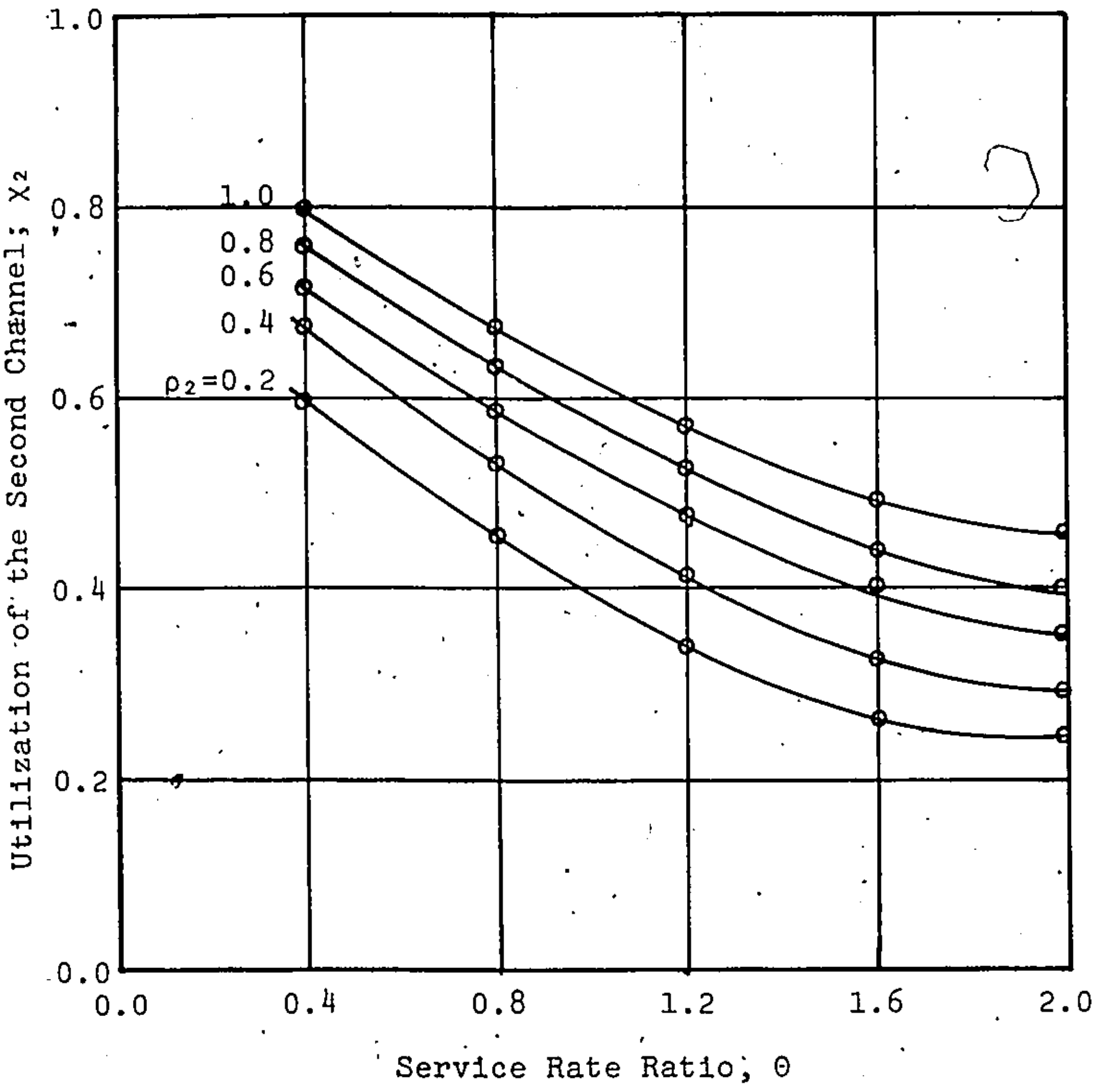


Figure 18. Effect of Θ on the utilization of the second channel χ_2 for the two-channel conveyor; $\rho_1=1.0$

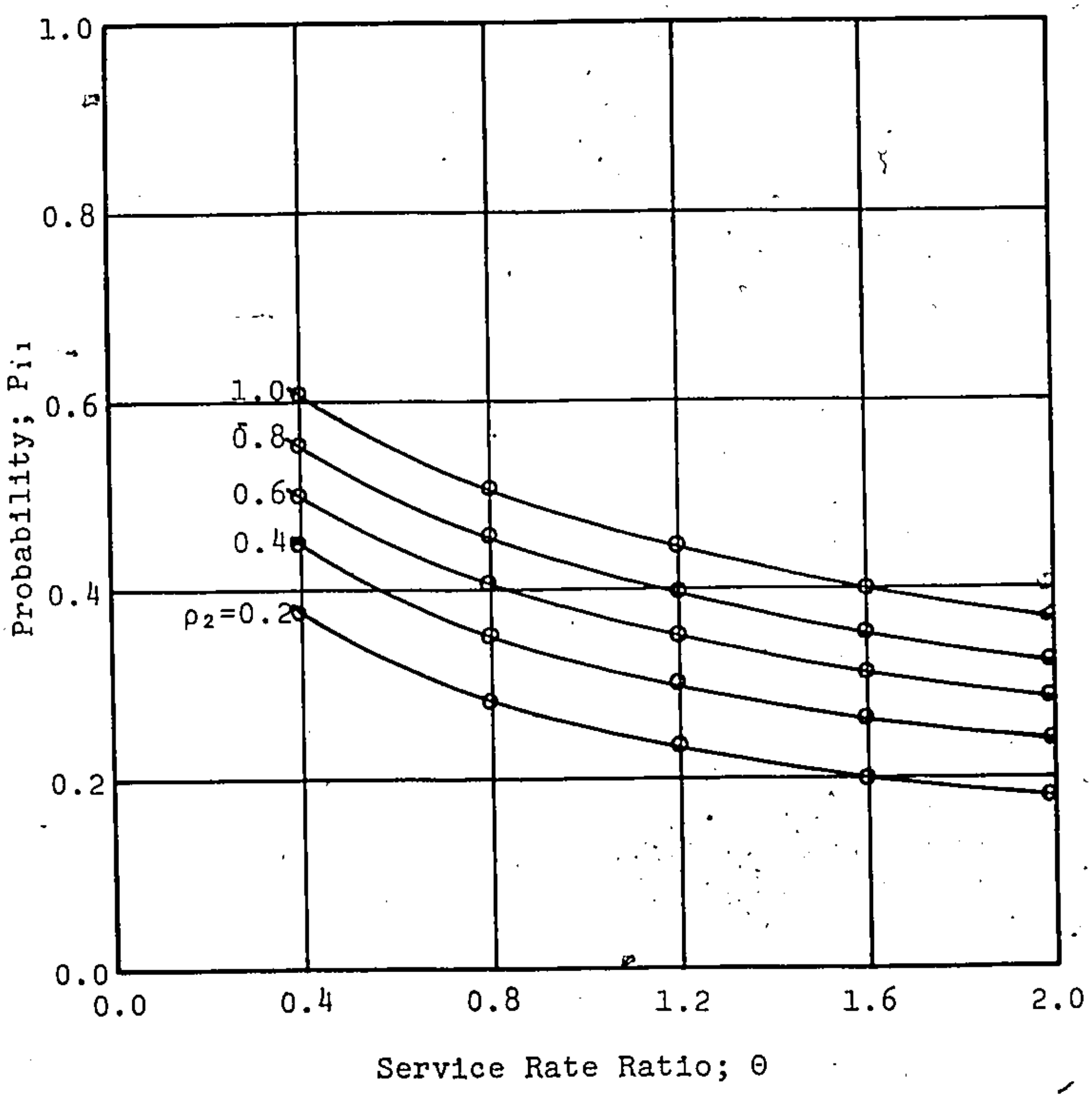


Figure 19. Effect of Θ on the probability of a lost arrival, for the two-channel conveyor; $\rho_1=1.0$

second channel decrease with the increase of Θ.

However, it was found that when Θ=0.4, that the probability of the first channel being busy (χ_1) and the probability of the second channel being busy (χ_2) have almost equal values.

Three-Channel Closed-Loop Conveyor
With Heterogeneous Servers

Let μ_1, μ_2, and μ_3 be the service rates at the first, second, and the third channel, respectively. P(i,j,k) equal the steady-state probability that channel 1 has 'i' units, channel 2 has 'j' units and channel 3 has 'k' units, with i=0,1; j=0,1; and k=0,1. The steady-state probability equations are derived as follows:

$$-(\lambda_1+\lambda_2).P(0,0,0) + [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,0,0)$$
$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(0,1,0)$$
$$+ [\frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}].P(0,0,1) = 0$$

..... IV - 18

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)})].P(1,0,0)$$
$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(1,1,0)$$
$$+ [\frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}].P(1,0,1)$$

$$+ (\lambda_1+\lambda_2).P(0,0,0) = 0 \qquad \text{IV - 19}$$

$$-[(\lambda_1+\lambda_2) + [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}]].P(0,1,0)$$

$$+ [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,1,0)$$

$$+ [\frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}].P(0,1,1) = 0$$

..... IV - 20

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)})].P(0,0,1)$$

$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(0,1,1)$$

$$+ [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,0,1) = 0$$

..... IV - 21

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2}$$

$$+ \frac{-\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)})].P(1,1,0)$$

$$+ [\frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}].P(1,1,1)$$

$$+ (\lambda_1+\lambda_2).P(1,0,0) + (\lambda_1+\lambda_2).P(0,1,0) = 0$$

..... IV - 22

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_3}{\lambda_1+\lambda_2}$$

$$+ \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}].P(1,0,1) + (\lambda_1+\lambda_2).P(0,0,1)$$

$$+ \left[\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}\right].P(1,1,1) = 0$$

..... IV - 23

$$-\left[(\lambda_1+\lambda_2) + \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_1\mu_2}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_3}{\phi(\lambda_1+\lambda_2)}\right].P(0,1,1)$$

$$+ \left[\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}\right].P(1,1,1) = 0$$

..... IV - 24

$$-\left[\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)} + \frac{\lambda_1\mu_3}{\lambda_1+\lambda_2} + \frac{\lambda_1\mu_3}{\phi(\lambda_1+\lambda_2)}\right].P(1,1,1)$$

$$+ (\lambda_1+\lambda_2).P(0,1,1) + (\lambda_1+\lambda_2).P(1,0,1)$$

$$+ (\lambda_1+\lambda_2).P(1,1,0) = 0 \qquad \text{IV - 25}$$

Writing the above equations in matrix form, one can obtain the values of P(i,j,k) in terms of P(0,0,0), as follows:

$$P(0,0,1) = \xi\, P(0,0,0)$$

$$P(0,1,0) = \xi_1 P(0,0,0)$$

$$P(1,0,0) = \xi_2 P(0,0,0)$$

$$P(0,1,1) = \xi_3 P(0,0,0)$$

$$P(1,0,1) = \xi_4 P(0,0,0)$$

$$P(1,1,0) = \xi_5 P(0,0,0)$$

$$P(1,1,1) = \xi_6 P(0,0,0)$$

where:

$$\xi = \{[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}]^3 \; [1+\frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3]$$

$$[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2]^2\} \; / \; \{\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}$$

$$[(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3]$$

$$[(\Theta_2+\Theta_3) \; (1 + \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2+\Theta_3)$$

$$- \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}] \; [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2)^2 + 2\Theta_2]$$

$$- \Theta_2(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)})^2 \; [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1+\Theta_2) + \Theta_2\Theta_3] \; [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}$$

$$+ \Theta_2)^2 + 2\Theta_2 - \Theta_3] + \Theta_3[1 + \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}]$$

$$[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}] \; [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3]$$

$$[1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + 2\Theta_2 + \Theta_3] + \Theta_3(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)})$$

$$[(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2)^2 + \Theta_2] \; [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1+\Theta_2) + \Theta_2\Theta_3 + (1$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}) \; (1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3)]\}$$

$$\xi_1 = \xi\{[[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (\Theta_2+2\Theta_3) + (\Theta_2+\Theta_3)$$

$$(1+\Theta_2+\Theta_3)] \, [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3]$$

$$- \Theta_2\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} \, [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1+\Theta_2) + \Theta_2\Theta_3]]\}$$

$$/ \{[\Theta_2\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3)]\}$$

$$\xi_2 = \{\xi[(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2]$$

$$+ \Theta_3] \, [(\Theta_2+\Theta_3) \, (\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + 1 + \Theta_2 + \Theta_3)]$$

$$- \xi\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} \, [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1+\Theta_2) + \Theta_2\Theta_3] \, [\Theta_2+\Theta_3]$$

$$+ (\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)})^2 \, (1+ \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3)\}$$

$$/ \{[1 + \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}] \, [\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}]$$

$$[1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3]\}$$

$$\xi_3 = \xi[\ \{(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3\}$$

$$/ \{\Theta_2(1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3)\}\ ]$$

$$\xi_4 = \xi\{\ \{[\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3]^2 + \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}$$

$$(1+\Theta_2) + \Theta_2\Theta_3\} / \{(1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2+2\Theta_3)\}\}$$

$$\xi_5 = \xi\{[(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3] [(\Theta_2+\Theta_3)$$

$$(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + 1 + \Theta_2 + \Theta_3)]$$

$$- \Theta_2(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}) [(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2$$

$$+ \frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} (1+\Theta_2) + \Theta_2\Theta_3]\}$$

$$/ \{\Theta_2(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}) (1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}$$

$$+ \Theta_2 + 2\Theta_3)\}$$

$$\xi_6 = \xi\{[(\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_3)^2 + \Theta_3] [\frac{(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)}$$

$$+ \Theta_2 + \Theta_3]\} / \{\Theta_2(1 + \frac{2(\lambda_1+\lambda_2)^2}{\mu_1(\lambda_1+\lambda_2/\phi)} + \Theta_2 + 2\Theta_3)\}$$

By imposing the boundary condition:

$$\sum_{i,j,k=0}^{1} P(i,j,k)=1$$

the measures of the system's performance can be evaluated as follows:

1. Probability of the system being idle; $P(0,0,0)$:

$$P(0,0,0) = 1 / (1 + \xi + \sum_{s=1}^{6} \xi_s) \qquad \text{IV - 26}$$

2. Expected number of units in the system; $E[n]$:

$$E[n] = \sum_{i,j,k=0}^{1} (i+j+k)\, P(i,j,k)$$

which gives:

$$E[n] = [\xi+\xi_1+\xi_2+2(\xi_3+\xi_4+\xi_5)+3\xi_6] \,/\, [1+\xi+\sum_{s=1}^{6}\xi_s]$$

3. The probability of a lost item; P(1,1,1):

$$P(1,1,1) = \xi_6/(1+\xi+\sum_{s=1}^{6}\xi_s)$$

4. The probability that the first server is busy; $P_1(\text{busy}) = \chi_1$, where

$$\chi_1 = [\xi_2+\xi_4+\xi_5+\xi_6] \,/\, [1+\xi+\sum_{s=1}^{6}\xi_s]$$

Verification of the Results

Set the arrival rate of the doublets, equal to zero (i.e., $\lambda_2=0$), $\Theta_2=\Theta_3=1$ (system of homogeneous servers) and $\rho_1=1.0$. One then obtains the following:

$\xi = 0.06667$

$\xi_1 = 0.21111$

$\xi_2 = 0.72222$

$\xi_3 = 0.05556$

$\xi_4 = 0.07778$

$\xi_5 = 0.36667$

$\xi_6 = 0.16667$

Substitute in equation IV-26, one obtains

$$P(0,0,0) = \frac{1}{2.6666} = 0.37500$$

Substitute in equation III-32 for ρ_2=0 and ρ_1=1, one obtains:

$$P_{00} = \frac{6}{16} = 0.37500$$

which is the same as that calculated from equation IV-26.

The generalization of a set of equations describing the system in terms of the channels, appears most formidable. So too, the special case of channels numbering more than three, appears most unattractive from the mathematical analysis point of view, as the number of equations required to describe the system having 'M' service channels is 2^M. For the three-channel system, eight equations were required. Sixteen and thirty-two equations are required for four and five channels, respectively. Solution of any of these sets is numerically possible by using computer programming, while its analytical solution is not considered analytically feasible.

Two-Channel Conveyor Having More Than Two Input Sources and Heterogeneous Servers

In addition to the general assumptions of the system given in Chapter III, one considers that:

1. There are N types of arrivals, each type is governed by a different independent Poisson distribution with mean arrival rates $\lambda_1, \lambda_2, \ldots, \lambda_N$.

2. The service time needed for an arrival from type 'i' is ϕ_i times that of a unit from the first type of arrival (note: $\phi_1=1.0$).

3. The service rate ratio between the service rate of the second channel and the first is Θ_2, i.e., $\Theta_2=\mu_2/\mu_1$.

4. Traffic intensity $\rho_i=\lambda_i/\mu_1$

One can now proceed to write the steady-state probability equations as follows:

$$-(\sum_{i=1}^{N}\lambda_i).P(0,0) + \mu_1((\sum_{i=1}^{N}\lambda_i/\phi_i)/(\sum_{i=1}^{N}\lambda_i)).P(1,0)$$

$$+ \mu_2((\sum_{i=1}^{N}\lambda_i/\phi_i)/(\sum_{i=1}^{N}\lambda_i)).P(0,1) = 0 \qquad \ldots\ldots \text{ IV - 27}$$

$$-[(\sum_{i=1}^{N}\lambda_i) + \mu_2((\sum_{i=1}^{N}\lambda_i/\phi_i)/(\sum_{i=1}^{N}\lambda_i))].P(0,1)$$

$$+(\mu_1(\sum_{i=1}^{N}\lambda_i/\phi_i)/(\sum_{i=1}^{N}\lambda_i)).P(1,1) = 0 \qquad \ldots\ldots \text{ IV - 28}$$

$$-[(\sum_{i=1}^{N} \lambda_i) + \mu_1((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i)].P(1,0)$$

$$+ \mu_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i)).P(1,1)$$

$$+ (\sum_{i=1}^{N} \lambda_i).P(0,0) = 0 \qquad \text{IV - 29}$$

$$-[\mu_1((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i)) + \mu_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))]$$

$$.P(1,1) + (\sum_{i=1}^{N} \lambda_i).P(0,1) = 0$$

$$\ldots\ldots \quad \text{IV - 30}$$

Solving the above system of equations and using the boundary condition:

$$\sum_{i,j=0}^{1} P(i,j)=1$$

one obtains the following:

$$P(0,1) = [(\sum_{i=1}^{N} \rho_i)^2 \ ((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))]$$

$$/ \{(\sum_{i=1}^{N} \rho_i)^3 + ((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))$$

$$(\sum_{i=1}^{N} \rho_i)^2 \ (1+2\Theta_2) + \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)$$

$$/ \left(\sum_{i=1}^{N} \lambda_i\right)\Big)^2 \left(\sum_{i=1}^{N} \rho_i\right) (3+\Theta_2)$$

$$+ \Theta_2\left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right)^3 (1+\Theta_2)\Big\}$$

..... IV - 31

$$P(1,0) = \Big\{\Big[\Theta_2\left(\sum_{i=1}^{N} \rho_i\right) \left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right)\Big]$$

$$\Big[\left(\sum_{i=1}^{N} \rho_i\right) + \left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right)$$

$$(1+\Theta_2)\Big]\Big\} \Big/ \Big\{\left(\sum_{i=1}^{N} \rho_i\right)^3 + \Big(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)$$

$$/ \left(\sum_{i=1}^{N} \lambda_i\right)\Big) \left(\sum_{i=1}^{N} \rho_i\right)^2 (1+2\Theta_2)$$

$$+ \Theta_2\left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right)^2 \left(\sum_{i=1}^{N} \rho_i\right)$$

$$(3+\Theta_2) + \Theta_2\left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right)^3$$

$$(1+\Theta_2)\Big\}$$

IV - 32

$$P(1,1) = \Big\{\left(\sum_{i=1}^{N} \rho_i\right)^2 \Big[\left(\sum_{i=1}^{N} \rho_i\right) + \Theta_2\Big(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)$$

$$/ \left(\sum_{i=1}^{N} \lambda_i\right)\Big)\Big]\Big\} \Big/ \Big\{\left(\sum_{i=1}^{N} \rho_i\right)^3$$

$$+ \left(\left(\sum_{i=1}^{N} \lambda_i/\phi_i\right)/\left(\sum_{i=1}^{N} \lambda_i\right)\right) \left(\sum_{i=1}^{N} \rho_i\right)^2 (1+2\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^2 \ (\sum_{i=1}^{N} \rho_i) \ (3+\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^3 \ (1+\Theta_2)\}$$

..... IV - 33

The measures of the system's performance can be derived as follows:

1. The probability of the system being idle; P_{00}:

$$P_{00} = \{[\Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^2] \ [2(\sum_{i=1}^{N} \rho_i)$$

$$+ (\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i) + \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)$$

$$/ (\sum_{i=1}^{N} \lambda_i))]\} \ / \ \{(\sum_{i=1}^{N} \rho_i)^3 + ((\sum_{i=1}^{N} \lambda_i/\phi_i)$$

$$/ (\sum_{i=1}^{N} \lambda_i)) \ (\sum_{i=1}^{N} \rho_i)^2 \ (1+2\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^2 \ (\sum_{i=1}^{N} \rho_i) \ (3+\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^3 \ (1+\Theta_2)\}$$

..... IV - 34

2. The expected number of units in the system; $E[n]$:

$$E[n] = \{[\Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i)) \ (\sum_{i=1}^{N} \rho_i)]$$

$$[(\sum_{i=1}^{N} \rho_i) + (\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))]$$

$$+ (\sum_{i=1}^{N} \rho_i)^2 \; ((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))$$

$$+ 2(\sum_{i=1}^{N} \rho_i)^2 \; [(\sum_{i=1}^{N} \rho_i) + \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)$$

$$/ (\sum_{i=1}^{N} \lambda_i))]\} \quad / \quad \{(\sum_{i=1}^{N} \rho_i)^3 + ((\sum_{i=1}^{N} \lambda_i/\phi_i)$$

$$/ (\sum_{i=1}^{N} \lambda_i)) \; (\sum_{i=1}^{N} \rho_i)^2 \; (1+2\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^2 \; (\sum_{i=1}^{N} \rho_i) \; (3+\Theta_2)$$

$$+ \Theta_2((\sum_{i=1}^{N} \lambda_i/\phi_i)/(\sum_{i=1}^{N} \lambda_i))^3 \; (1+\Theta_2)\}$$

..... IV - 35

3. The probability of a lost arrival (P(1,1)) as in Equation IV-11.

CHAPTER V

CLOSED-LOOP CONVEYOR SYSTEMS WITH RECIRCULATION

The previous chapters of this paper dealt only, with the case of closed-loop conveyors with lost arrivals, where homogeneous or heterogeneous servers were allowed in the system. The analysis carried out so far, considered an arrival - denied service at the last service channel - as lost to the system. In real life situations, this might not occur. When the arrival is denied service at the last service channel, it does not leave the system, but rather recirculates along the recirculation line, and reenters the system with the new arrivals. If the item, after having recirculated, finds a service station idle, it enters the service facility. If that condition does not exist, the item once again recirculates. The procedure is repeated until the item can enter the service facility.

Pritsker (34) studied the steady-state condition of a conveyor system with feedback. The input rate to the service channels (λ') is the sum of the arrival rate (λ) and the proportion that are fed back, say $P_m\lambda'$, therefore,

$$\lambda' = \lambda + P_m\lambda'$$

and

$$\lambda' = \frac{\lambda}{1-P_m}$$

Pritsker found that the expected number of busy channels is not directly a function of the number of channels, nor of the input distribution. The feedback delay constant does not affect the probabilities associated with the system's performance. The reason for this, is that, the feedback delay causes recycled items to arrive at the first channel at a later time, but with the same inter-arrival distribution.

Phillips (32) simulated an ordered entry closed-loop conveyor with recirculation. He found that the recirculation traffic can be reduced by either increasing the feedback delay constant or by increasing the storage capacity of the system. Also, optimum results with respect to the expected number in the system can be attained by setting the feedback delay constant approximately equal to the average service rate.

An extension of Pritsker's work was the simulation study done by Phillips and Skeith (33). The system analyzed by Phillips and Skeith is exactly the same as that studied by Pritsker, except that storage is allowed at any service station. Conclusions of their study agreed with Pritsker that the recirculation time has little effect upon the probabilistic properties of the queueing system at the service station.

Disney and D'Avignon (46), studied a single server where the units after being served either immediately join the queue again with some probability or depart permanently with the complementary probability. The feedback mechanism depends on the 'state' of the system, as well as on the amount of the service time expended on the item and, in a Markov manner, the 'history' of the previous feedback decisions. The input to the server consists of two streams: (i) a stream of new arrivals, which is taken to be a Poisson process, and (ii) a stream of feedback items, which, in general, is not Poisson.

Burbridge (3) studied a closed-loop conveyor where recirculation is permitted. Two basic approaches were utilized in studying the conveyor systems. The first of these approaches was the analytical approach, where attention was focused on the finite queueing problem that exists at the service facility. The problem was represented by a GERT (Graphical Evaluation and Review Technique) network, using the concept of imbedded Markov chains. Burbridge estimated and approximated the steady-state probabilities and the probabilities associated with recirculating units. The second approach considered by Burbridge was the experimental approach, where conveyors with recirculation and storage were analyzed. Burbridge derived the distribution of 'T', the time between successive arrivals in a stationary branching Poisson process. He showed that the probability density function for T can be given by:

$$f_T(t) = \begin{cases} \lambda(1+a)e^{-\lambda(1+a)t} & 0 \le t < k \\ \{a/(1+a)\}e^{-\lambda(1+a)k} & t = k \\ \{\lambda/(1+a)\}e^{-\lambda t - \lambda a k} & t > k \end{cases}$$

where λ = the arrival rate

$a = r/(1-P)$, where

r = probability a primary arrival will recirculate

P = probability a recirculating arrival will recirculate

k = the recirculation time

The recirculated unit will be stored on the conveyor until it finds any of the service channels unoccupied. Then, it can be serviced. If the input rate is λ and the recirculated proportion is $P_r\lambda$, the effective input will be more than the original input, consequently the recirculated proportion will increase with the time until the conveyor is packed. Therefore, the proportion of the recirculated units can be kept constant, for a fixed arrival and service rates, by decreasing the original input by a portion equal to the recirculated proportion.

The purpose of this chapter is to investigate the problem of recirculation for the M-channel closed-loop conveyor and to investigate the two-channel conveyor having heterogeneous servers with recirculation. Results are given for these systems with no storage at any of the channels.

M-Channel Conveyors With Homogeneous Servers and Recirculation

The case studied here is similar to the case of the M-channel conveyor that was studied in Chapter III, of this paper. The servers have equal service rates (μ), and no storage is allowed at any of the service channels. When the arrival checks all the channels and finds they are occupied, the arrival then recirculates and enters the system as a new arrival. The recirculated arrival might be a singlet or a doublet unit. However, the recirculated arrivals follow a Poisson distribution.

Let λ_{e_1} = the effective arrival rate of the singlets

λ_{e_2} = the effective arrival rate of the doublets

α = the proportion of the recirculated arrivals

P_{ij} = the probability of having 'i' singlet and 'j' doublets in the system.

Then $\lambda_{e_1} = \lambda_1 + \alpha\lambda_{e_1}$ V - 1

and $\lambda_{e_2} = \lambda_2 + \alpha\lambda_{e_2}$ V - 2

The proportion of the recirculated arrivals can be determined either analytically, or by simulation. It was shown in Chapter III that the probability of an item being recirculated (P_{rec}) is given as:

$$P_{rec} = 1 - P_r(w=0) \text{ , where}$$

$$P_r(w=0) = \left\{ \sum_{s=0}^{M-1} \frac{1}{s!} (\rho_1+\phi\rho_2)^s \right\} / \left\{ \sum_{s=0}^{M} \frac{1}{s!} (\rho_1+\phi\rho_2)^s \right\}$$

or simply, P_{rec} can be expressed as:

$$P_{rec} = \left\{ \frac{1}{M!} (\rho_1+\phi\rho_2)^M \right\} / \left\{ \sum_{s=0}^{M} \frac{1}{s!} (\rho_1+\phi\rho_2)^s \right\}$$

..... V - 3

The arrival rate of the recirculated singlet units is given by:

$$\frac{\frac{\lambda_{e_1}}{M!}(\rho_1+\phi\rho_2)^M}{\sum_{s=0}^{M}\frac{1}{s!}(\rho_1+\phi\rho_2)^s} \quad \text{and}$$

the arrival rate of the recirculated units is

$$\frac{\frac{\lambda_{e_2}}{M!}(\rho_1+\phi\rho_2)^M}{\sum_{s=0}^{M}\frac{1}{s!}(\rho_1+\phi\rho_2)^M}$$

The proportion of the recirculated units can then be determined by equation V-3, consequently, λ_{e_1} and λ_{e_2} can be evaluated. One can now derive the steady-state probability equations for the M-channel case, as it was followed in Chapter III.

$$-[\frac{\lambda_1}{1-\alpha}+\frac{\lambda_2}{1-\alpha}].P_{00}(t)+\mu.P_{10}(t)+\frac{\mu}{\phi}.P_{01}(t)=0$$

..... V - 4

$$-[\frac{\lambda_1}{1-\alpha}+\frac{\lambda_2}{1-\alpha}+i\mu].P_{i0}(t)+\frac{\lambda_1}{1-\alpha}.P_{i-1,0}(t)+\frac{\mu}{\phi}.P_{i1}(t)$$

$$+(i+1)\mu.P_{i+1,0}(t)=0$$

where $i=1,2,3...,M-1$ V - 5

$$-[M\mu].P_{M0}(t) + \frac{\lambda_1}{1-\alpha}.P_{M-1}(t) = 0 \qquad \text{V - 6}$$

$$-[\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha} + \frac{j}{\phi}].P_{0j}(t) + \mu.P_{1j}(t)$$

$$+ (\frac{j+1}{\phi})\mu.P_{0,j+1}(t) + \frac{\lambda_2}{1-\alpha}.P_{0,j-1}(t) = 0$$

where $j=1,2,3,...,M-1$ V - 7

$$-\frac{M\mu}{\phi}.P_{0M}(t) + \frac{\lambda_2}{1-\alpha}.P_{0,M-1}(t) = 0 \qquad \text{V - 8}$$

$$-[\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha} + (i+\frac{j}{\phi})\mu].P_{ij}(t) + (i+1) .P_{i+1,j}(t)$$

$$+(\frac{j+1}{\phi})\mu.P_{i,j+1}(t) + \frac{\lambda_1}{1-\alpha}.P_{i-1,j}(t)$$

$$+ \frac{\lambda_2}{1-\alpha}.P_{i,j-1}(t) = 0$$

where $i=1,2,3,...,M-1$

$j=1,2,2,...,M-1$

and $i+j \leq M-1$ V - 9

$$-[(i+\frac{j}{\phi})\mu].P_{ij}(t) + \frac{\lambda_1}{1-\alpha}.P_{i-1,j}(t)$$

$$+ \frac{\lambda_2}{1-\alpha}.P_{i,j-1}(t) = 0$$

where $i=1,2,3,...,M-1$

$j=1,2,3,...,M-1$

and $i+j = M$ V - 10

Setting $\rho_1=\lambda_1/\mu$ and $\rho_2=\lambda_2/\mu$ and writing the above system of equations in matrix notations, one can then solve these equations in terms of P_{00}, and the general term of the probability of having 'i' singlet and 'j' doublet units in the system is given by:

$$P_{ij} = \frac{\phi^j}{i!j!} \left[\frac{\rho_1}{1-\alpha}\right]^i \left[\frac{\rho_2}{1-\alpha}\right]^j . P_{00}$$

where $i=0,1,2,3,\ldots,M$

$j=0,1,2,3,\ldots,M$

and $i+j \le M$ V - 11

The value of P_{00} can be evaluated by using the boundary condition:

$$\sum_{i=0}^{M} \sum_{j=0}^{M} P_{ij}=1 \qquad i+j\le M$$

in the equilibrium equations. P_{00} is given as:

$$P_{00} = 1 \Big/ \left\{ \sum_{s=0}^{M} \frac{1}{s!} \left[\frac{\rho_1}{1-\alpha} + \phi\frac{\rho_2}{1-\alpha}\right]^s \right\} \qquad \text{V - 12}$$

The expected number of units in the system can be evaluated as:

$$E[n] = \left\{ \sum_{s=1}^{M} \frac{1}{(s-1)!} \left[\frac{\rho_1}{1-\alpha} + \phi\frac{\rho_2}{1-\alpha}\right]^s \right\} \Big/ \left\{ \sum_{s=0}^{M} \frac{1}{s!}\right.$$

$$\left[\frac{\rho_1}{1-\alpha} + \frac{\rho_2}{1-\alpha}\right]^s \} \qquad V - 13$$

The probability that an arrival will have no wait prior to service, or the probability that an arrival being recirculated, (P_{rec}) is derived as:

$$P_{rec} = \{ \sum_{s=0}^{M-1} \frac{1}{s!}\left[\frac{\rho_1}{1-\alpha} + \frac{\rho_2}{1-\alpha}\right]^s \}$$

$$/ \{ \sum_{s=0}^{M} \frac{1}{s!}\left[\frac{\rho_1}{1-\alpha} + \frac{\rho_2}{1-\alpha}\right]^s \} \qquad V - 14$$

Two-Channel Conveyors With Heterogeneous Servers And Recirculation

In Chapter IV, the two and three channel conveyor serviced queueing systems with no storage at any channel and heterogeneous servers were considered allowing multiple-Poisson inputs of singlet and doublet arrivals. The situation described and dealt with in Chapter IV considered the case of lost arrivals; i.e., arrivals that find all the servers busy will never return to the system, so it leaves the system by other means than the conveyor under study. However, in practicality, the system can be economically feasible if either storage or recirculation is allowed at the service channels.

The second alternative will be considered in the analysis conducted in this problem, while the first alternative will be dealt with later. Consider a two-service channel conveyor with service rates μ_1 and μ_2 at the first and second channel, respectively. The recirculated singlet and doublet units follow Poisson distributions, with mean arrival rates $\alpha\lambda_1$ and $\alpha\lambda_2$, respectively, where α is the proportion of the recirculated units. The value of α can be determined either analytically, or by simulation. It was shown in Chapter IV, that the probability of an item being recirculated P_{rec}=P(1,1); where

$$P(1,1) = \{(\rho_1+\rho_2)^2\ [\rho_1+\rho_2+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\Theta_2]\}$$
$$/\ \{(\rho_1+\rho_2)^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\ (\rho_1+\rho_2)^2\ (1+2\Theta_2)$$
$$+\ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2\ (\rho_1+\rho_2)\ (3+\Theta_2)$$
$$+\ \Theta_2(1+\Theta_2)\ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3\} \qquad \text{V - 15}$$

Let P(i,j) = probability of having 'i' and 'j' units at the first and second channel, respectively.

λ_{e_1} = effective arrival rate of the singlets

λ_{e_2} = effective arrival rate of the doublets

α = proportion of the recirculated arrivals

The arrival rate of the recirculated singlet units is given by λ_{e_1}.P(1,1) and that of the doublets is given by $\lambda_{e_2}.P(1,1)$. The proportions of the recirculated units can then be evaluated using equation V-15.

Following previously established procedures, the steady-state equilibrium equations can be derived as follows:

$$-[\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha}].P(0,0) + [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,0)$$

$$+ [\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(0,1) = 0 \qquad \text{V - 16}$$

$$-[\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha} + \frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(0,1)$$

$$+ [\frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,1) = 0 \qquad \text{V - 17}$$

$$-[\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha} + \frac{\lambda_1\mu_1}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_1}{\phi(\lambda_1+\lambda_2)}].P(1,0)$$

$$+[\frac{\lambda_1\mu_2}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu_2}{\phi(\lambda_1+\lambda_2)}].P(1,1)$$

$$+ (\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha}).P(0,0) = 0 \qquad \text{V - 18}$$

$$-[(\mu_1+\mu_2)\ (\frac{\lambda_1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\phi(\lambda_1+\lambda_2)})].P(1,1)$$

$$+ (\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha}).P(1,0)$$

$$+ (\frac{\lambda_1}{1-\alpha} + \frac{\lambda_2}{1-\alpha}).P(0,1) = 0 \qquad \text{V - 19}$$

Setting $\rho_1=\frac{\lambda_1}{\mu_1}$, $\rho_2=\frac{\lambda_2}{\mu_1}$, and $\Theta_2=\frac{\mu_2}{\mu_1}$ and solving the above system of equations using the boundary condition

$$\sum_{i,j=0}^{1} P(i,j)=1$$

one obtains the following:

$$P(0,1) = [(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\frac{\rho_1+\rho_2}{1-\alpha})^2] / \{(\frac{\rho_1+\rho_2}{1-\alpha})^3$$
$$+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\frac{\rho_1+\rho_2}{1-\alpha})^2 (1+2\Theta_2)$$
$$+\Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 (\frac{\rho_1+\rho_2}{1-\alpha}) (3+\Theta_2)$$
$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 (1+\Theta_2)\}$$

$$P(1,0) = \{[\Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\frac{\rho_1+\rho_2}{1-\alpha})] [(\frac{\rho_1+\rho_2}{1-\alpha})$$
$$+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) + \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})]\}$$
$$/ \{(\frac{\rho_1+\rho_2}{1-\alpha})^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\frac{\rho_1+\rho_2}{1-\alpha})^2(1+2\Theta_2)$$
$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 (\frac{\rho_1+\rho_2}{1-\alpha}) (3+\Theta_2)$$
$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 (1+\Theta_2)\}$$

$$P(1,1) = \{(\frac{\rho_1+\rho_2}{1-\alpha})^2 (\frac{\rho_1+\rho_2}{1-\alpha} + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\Theta_2)\}$$
$$/ \{(\frac{\rho_1+\rho_2}{1-\alpha})^3+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) (\frac{\rho_1+\rho_2}{1-\alpha})^2 (1+2\Theta_2)$$
$$+ \Theta_2(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 (\frac{\rho_1+\rho_2}{1-\alpha}) (3+\Theta_2)$$
$$+ (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 (1+\Theta_2)\}$$

The probability of the system being idle is given as:

$$
\begin{aligned}
P_{00} = \{ & \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \; (2(\frac{\rho_1+\rho_2}{1-\alpha}) + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}))\} \; / \; \{(\frac{\rho_1+\rho_2}{1-\alpha})^3 \\
& + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) \; (\frac{\rho_1+\rho_2}{1-\alpha})^2 \; (1+2\Theta_2) \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \; (\frac{\rho_1+\rho_2}{1-\alpha}) \; (3+\Theta_2) \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 \; (1+\Theta_2)\} \qquad \text{V - 20}
\end{aligned}
$$

The expected number of units in the system can be evaluated as:

$$
\begin{aligned}
E[n] = \{ & \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) \; (\frac{\rho_1+\rho_2}{1-\alpha}) \; (\frac{\rho_1+\rho_2}{1-\alpha} + \frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2} \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})) + (\frac{\rho_1+\rho_2}{1-\alpha})^2 \; (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) \\
& + 2(\frac{\rho_1+\rho_2}{1-\alpha})^2 \; (\frac{\rho_1+\rho_2}{1-\alpha} + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}))\} \\
& / \; \{(\frac{\rho_1+\rho_2}{1-\alpha})^3 + (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2}) \; (\frac{\rho_1+\rho_2}{1-\alpha})^2 \; (1+2\Theta_2) \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^2 \; (\frac{\rho_1+\rho_2}{1-\alpha}) \; (3+\Theta_2) \\
& + \Theta_2 (\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})^3 \; (1+\Theta_2)\} \qquad \text{V - 21}
\end{aligned}
$$

Effect of the Recirculated Proportions on the System's Performance

The effect of ρ_1, ρ_2, ϕ, and M on the system's performance was discussed in Chapter III. To study the effect of the recirculated proportions (α) on the system's performance for the M-channel conveyor with homogeneous servers, the values of the following parameters are kept constant:

1. ρ_1=1.0 (Traffic intensity of the singlet units equals unity.)

2. ϕ=2.0 (The time needed to serve a doublet unit is twice that of the singlet unit.)

3. M=2.0 (There are two service channels.)

Substituting the above values in equations V-12, V-13, and V-14, one gets:

$$P_{00} = 1 \,/\, \{1+ \left(\frac{1+2\rho_2}{1-\alpha}\right) + \tfrac{1}{2}\left(\frac{1+2\rho_2}{1-\alpha}\right)^2 \} \qquad \text{V - 22}$$

$$E[n] = \left(\frac{1+2\rho_2}{1-\alpha}\right) \left(1 + \frac{1+2\rho_2}{1-\alpha}\right) \,/\, \{1 + \frac{1+2\rho_2}{1-\alpha} + \tfrac{1}{2}\left(\frac{1+2\rho_2}{1-\alpha}\right)^2\} \qquad \text{V - 23}$$

and

$$P_{rec} = \left(1 + \frac{1+2\rho_2}{1-\alpha}\right) \,/\, \{1 + \frac{1+2\rho_2}{1-\alpha} + \tfrac{1}{2}\left(\frac{1+2\rho_2}{1-\alpha}\right)^2\} \qquad \text{..... V - 24}$$

The effect of the recirculated proportions on the performance of the system is shown in Figures 20, 21,

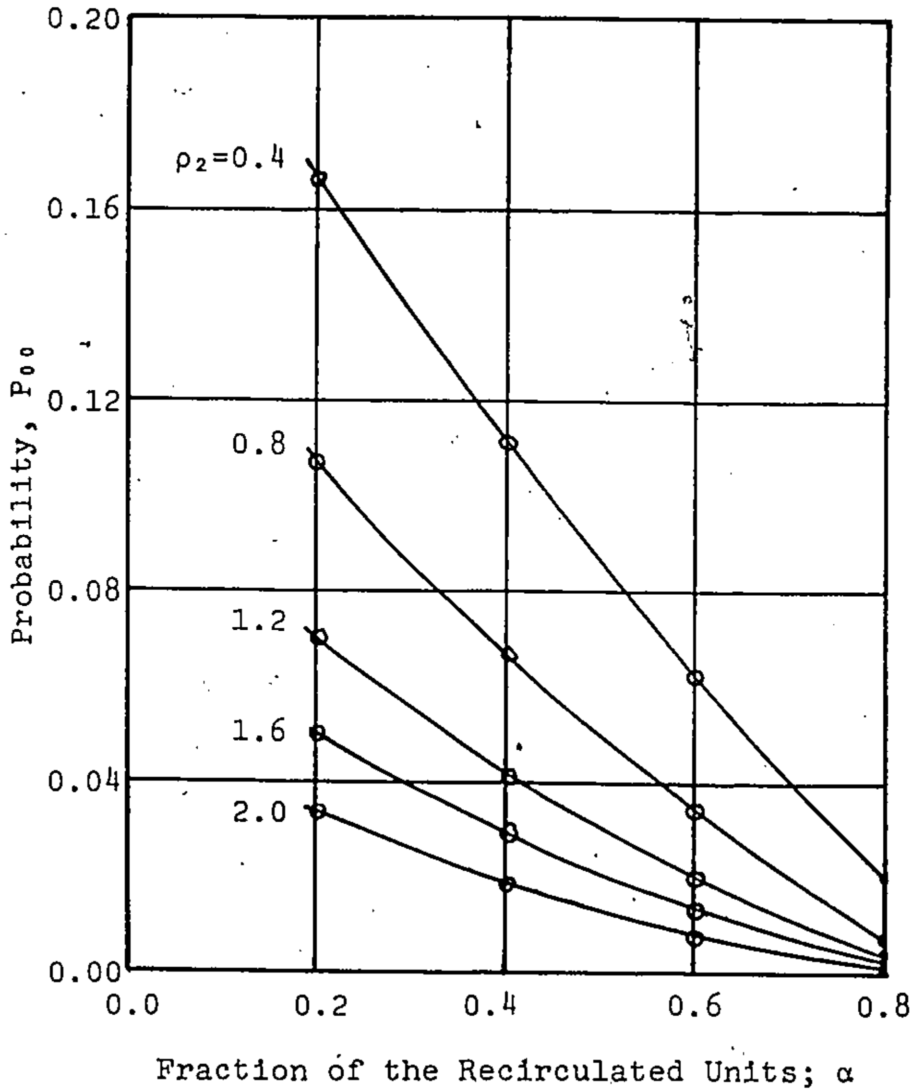


Figure 20. Effect of α on P_{00} for the two-channel conveyor with homogeneous servers; $\rho_1=1.0$

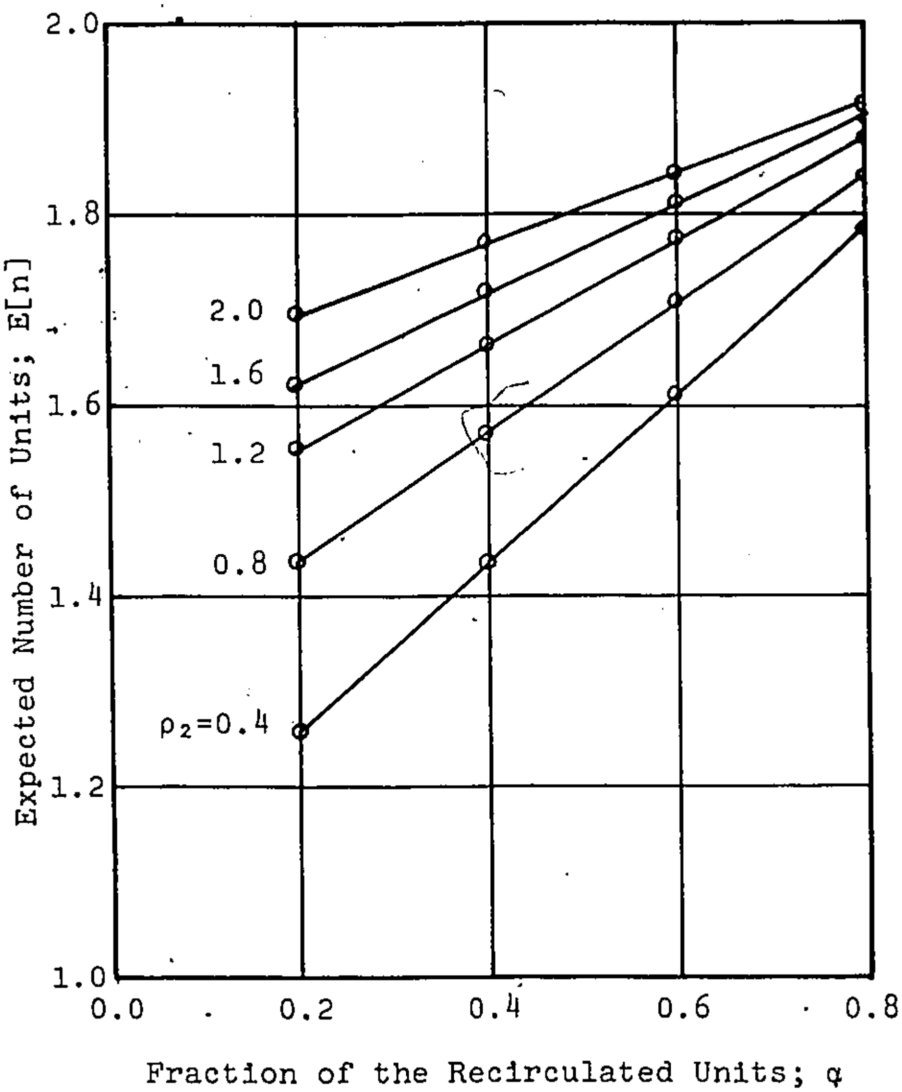


Figure 21. Effect of α on $E[n]$ for the two-channel conveyor with homogeneous servers; $\rho_1=1.0$

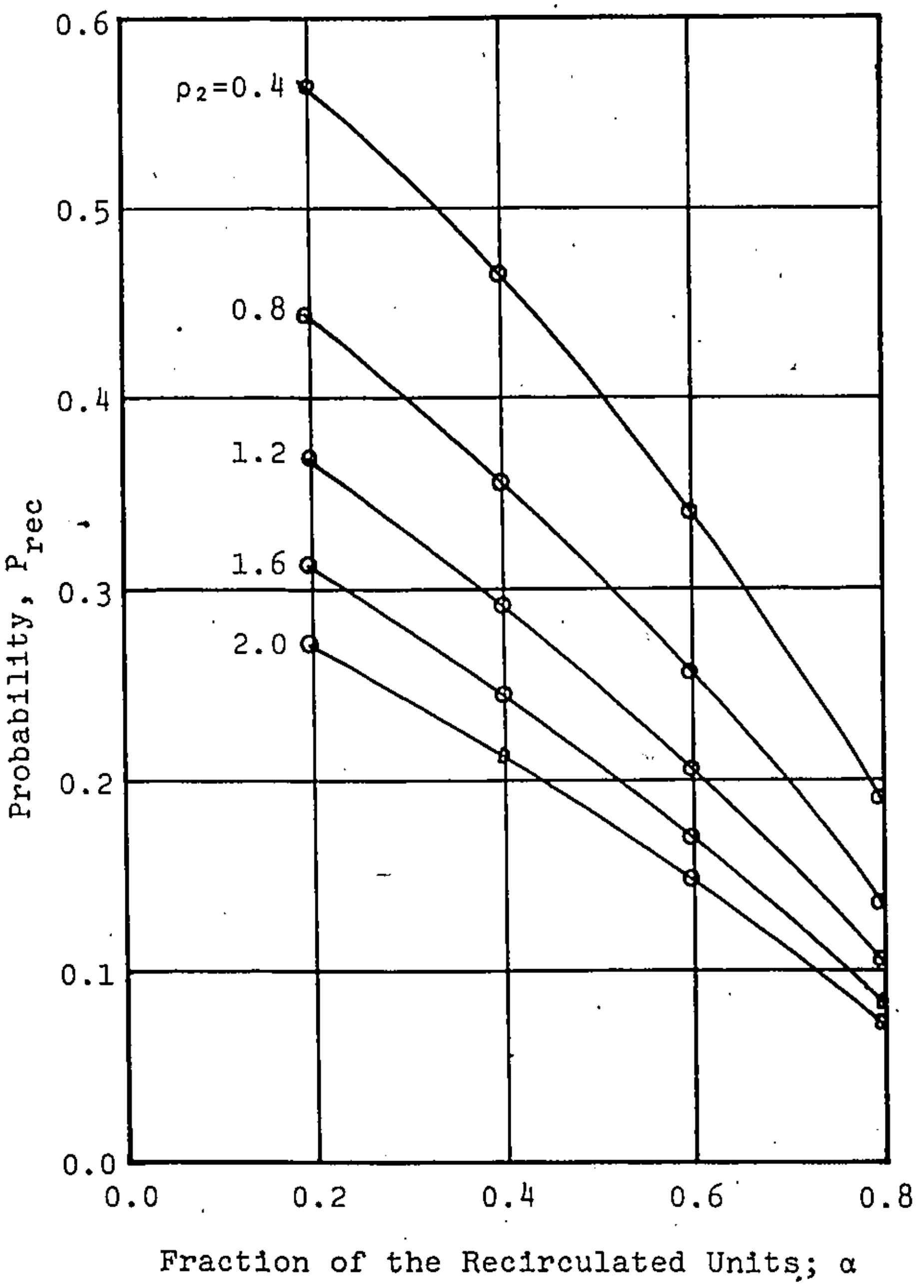


Figure 22. Effect of α on P_{rec} for the two-channel conveyor with homogeneous servers; ρ_1=1.0

and 22. It is concluded that the probability of the system being idle (P_{00}) and the probability that an arrival will be recirculated, decrease with the increase of the fraction of the recirculated arrivals; while the expected number of units in the system E[n] increases with the increase of α. To study the effect of the recirculated proportions (α) on the system's performance for the two-channel conveyor with heterogeneous servers, the values of the following are kept fixed:

1. ϕ=2.0 (The time needed to serve a doublet unit is twice that of the singlet unit.)

2. ρ_1=1.0 (Traffic intensity of the singlet units equals unity.)

3. Θ_2=0.4 (Service rate of the server at the second channel is 0.4 times that of the server at the first channel.)

Substituting the above values in equations V-20 and V-21, one obtains:

$$P_{00} = \{0.4\left(\frac{1+\rho_2/2}{1+\rho_2}\right)^2 \left(2\left(\frac{1+\rho_2}{1-\alpha}\right) + \left(\frac{1+\rho_2/2}{1+\rho_2}\right)\right. + 0.4\left(\frac{1+\rho_2/2}{1+\rho_2}\right)\} / \{\left(\frac{1+\rho_2}{1-\alpha}\right)^3 + 1.8\left(\frac{1+\rho_2/2}{1+\rho_2}\right) \left(\frac{1+\rho_2}{1-\alpha}\right)^2 + 0.136\left(\frac{1+\rho_2/2}{1+\rho_2}\right)^2 \left(\frac{1+\rho_2}{1-\alpha}\right) + 0.56\left(\frac{1+\rho_2/2}{1+\rho_2}\right)^2\}$$

and

$$E[n] = \{0.4(\frac{1+\rho_2/2}{1+\rho_2}) \ (\frac{1+\rho_2}{1-\alpha}) \ (\frac{1+\rho_2}{1-\alpha} + \frac{1+\rho_2/2}{1+\rho_2}$$

$$+ 0.4(\frac{1+\rho_2/2}{1+\rho_2})) \quad + (\frac{1+\rho_2}{1-\alpha})^2 \ (\frac{1+\rho_2/2}{1+\rho_2})$$

$$+ 2(\frac{1+\rho_2}{1-\alpha})^2 \ (\frac{1+\rho_2}{1-\alpha} + 0.4(\frac{1+\rho_2/2}{1+\rho_2}))\}$$

$$/ \ \{(\frac{1+\rho_2}{1-\alpha})^3 + 1.8(\frac{1+\rho_2/2}{1+\rho_2}) \ (\frac{1+\rho_2}{1-\alpha})^2$$

$$+ 0.136(\frac{1+\rho_2/2}{1+\rho_2})^2 \ (\frac{1+\rho_2}{1-\alpha}) + 0.56(\frac{1+\rho_2/2}{1+\rho_2})^3 \ \}$$

The effect, of the fraction of the recirculated arrivals on the probability of the system being idle and the expected number of units in the system for the two-channel conveyor with heterogeneous servers, is shown in Figures 23a and b, respectively. It is apparent that the probability of the system being idle decreases with the increase of α, while the expected number of units in the system increases as α increases.

The results of this chapter provide the designers of closed-loop conveyors with important relationships between the parameters which are involved in the design of such conveyors.

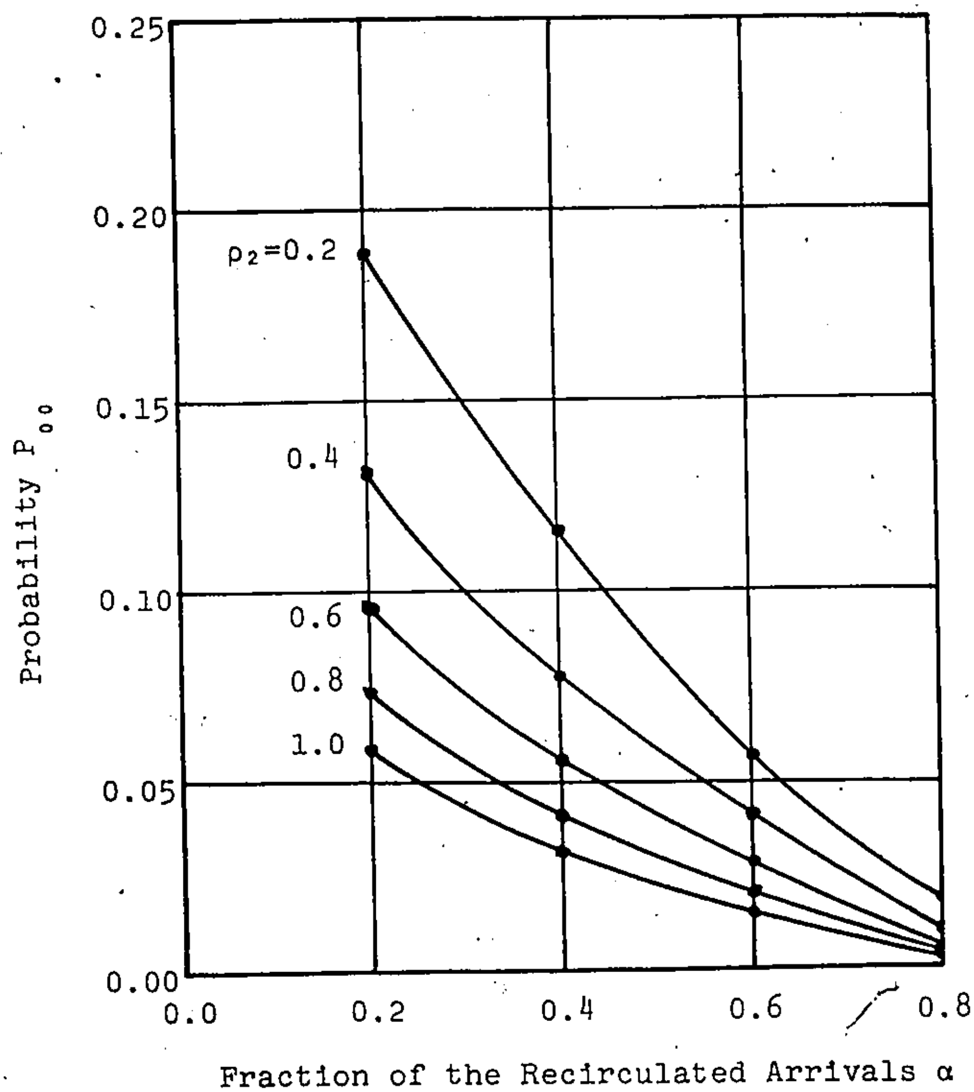


Figure 23a. Effect of α on P_{00} for the two-channel conveyor with heterogeneous servers; ρ_1=1.0

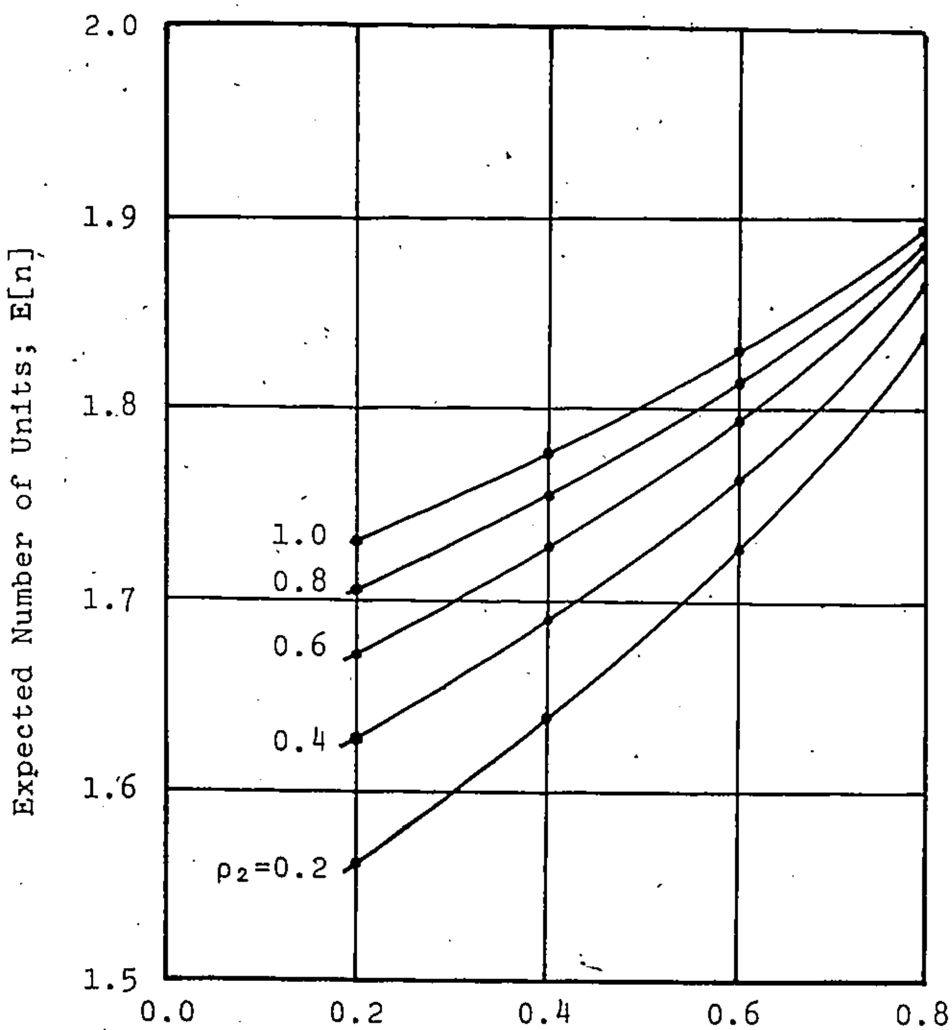


Figure 23b. Effect of α on $E[n]$ for the two-channel conveyor with heterogeneous servers; $\rho_1=1.0$

CHAPTER VI

THE TWO-CHANNEL CONVEYOR SERVICED HOMOGENEOUS QUEUEING SYSTEM WITH STORAGE AT LAST CHANNEL

The analyses of the conveyors studied previously, in Chapters III, IV, and V, are only applicable to systems where no storage was allowed at any of the channels.

The situation of no storage exists in such cases where the units are unloaded from the conveyor, as soon as they arrive at the unloading stations. There are situations where storage is allowed before service facilities. Suppose an arrival seeks service at any of the service channels and finds all the channels are busy - instead of being lost to the system or recirculated - the arrival will then enter any of the storages that are available at the service channels and then wait to be serviced. In cases where all the storages at the service channels are full, the arrival is either considered lost to the system or it recirculates. Allowing storages, at the service facilities: (i) reduces the amount of lost units and the recirculated units; (ii) the delay time between one service and the next is reduced to zero; and (iii) there is more conveyor space available for items seeking service because of off-line storage.

Disney (7) and Gupta (15) investigated the ordered entry conveyor system with homogeneous servers, while storage is allowed at each channel. Disney found in the case where storage is allowed at the service channels, that the storage facilities should not be allocated evenly. Rather, <u>to achieve balance, the servicers farthest from the input must be given the greatest amount of storage</u>.

Phillips and Skeith (33) in a simulation study showed that in most cases, when storage was allocated evenly, the utilization of Channel one (1) increased even more, while the utilization of the subsequent channels decreased slightly. Based on these findings, a general rule should be noted: the maximum balance and the overall efficiency can best be obtained by allocating extra storage to the last channel in the ordered queueing system with storage at each channel. Disney (7) examined the homogeneous many-server queueing system with storage allowed at each channel and reported that the solution of this case appears unfeasible.

Based on the above findings, the derivations to follow represent a two-channel homogeneous ordered entry conveyor with no storage at the first channel and a storage of variable capacity is allowed at the second channel.

The following cases of conveyors with storage at the second channel were studied.

Case 1: Conveyors With Homogeneous Servers and Storage at the Second Channel

In addition to the general assumptions made in Chapter III, one can assume that (1) the storage at the second channel is of different capacities and (2) the servers at the channels have equal service rates. In the derivations which follow, the term P_{ij} is the probability of having 'i' units at the first channel (i=0,1) and 'j' units at the second channel (j=0,1,2,..,N).

Two-Channel Conveyor With Storage of Unit Capacity at the Second Channel

Following the procedures outlined in Chapters III, IV, and V, one can derive the steady-state probability equations to be as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{10}(t) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{01}(t) = 0 \qquad \text{..... VI - 1}$$

$$-\{(\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\}.P_{01}(t) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{11}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{02}(t) = 0$$

..... VI - 2

$$-\{(\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\}.P_{02}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t) = 0$$

..... VI - 3

$$-\{(\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\}.P_{10}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{11}(t)$$

$$+ (\lambda_1+\lambda_2).P_{00}(t) = 0 \qquad \text{VI - 4}$$

$$-\{(\lambda_1+\lambda_2) + 2\left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\}.P_{11}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t)$$

$$+ (\lambda_1+\lambda_2).P_{01}(t)$$

$$+ (\lambda_1+\lambda_2).P_{10}(t) = 0 \qquad \text{VI - 5}$$

$$-2\left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t) + (\lambda_1+\lambda_2).P_{11}(t)$$

$$+ (\lambda_1+\lambda_2).P_{02}(t) = 0 \qquad \text{VI - 6}$$

In the above set of equations, there is one dependent equation. Meaning we can assume a value for one P_{ij} and solve all others in terms of this. Using matrix notation, one can write this system of equations as:

$$A \cdot P = \bar{0}$$

Here, P is a column vector for all values of P_{ij}, for given values of 'i' and 'j'. $\bar{0}$ is the null column matrix. A is 6 x 6 matrix with elements of the above equations.

Solving the above system of equations, one obtains P_{ij} in terms of P_{00}. These probabilities are given below:

$$P_{01} = \{2(\rho_1+\rho_2)^2 . P_{00}\} \;/\; \{(\frac{\phi\rho_1+\rho_2}{\phi(\rho_1+\rho_2)}(\frac{4\phi\rho_1+4\rho_2}{\phi(\rho_1+\rho_2)} + 3(\rho_1+\rho_2))\} \qquad \text{VI - 7}$$

$$P_{02} = \{(\rho_1+\rho_2)^3 . P_{00}\} \;/\; \{(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2 (\frac{4\rho_1+4\rho_2/\phi}{(\rho_1+\rho_2)} + 3(\rho_1+\rho_2))\} \qquad \text{VI - 8}$$

$$P_{10} = \{(\rho_1+\rho_2) \; (\frac{4\rho_1+4\rho_2/\phi}{\rho_1+\rho_2} + (\rho_1+\rho_2)).P_{00}\} \;/\; \{(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}) \; (\frac{4\rho_1+4\rho_2/\phi}{\rho_1+\rho_2} + 3(\rho_1+\rho_2))\} \qquad \text{VI - 9}$$

$$P_{11} = \{(\rho_1+\rho_2)^2 \; (\frac{2\rho_1+2\rho_2/\phi}{\rho_1+\rho_2} + (\rho_1+\rho_2)).P_{00}\}$$

$$/ \{(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2 (\frac{4\rho_1+4\rho_2/\phi}{\rho_1+\rho_2} + 3(\rho_1+\rho_2))\} \qquad \text{VI - 10}$$

$$P_{12} = \{(\rho_1+\rho_2)^3 (\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2} + (\rho_1+\rho_2)).P_{00}\} / \{(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3 (\frac{4\rho_1+4\rho_2/\phi}{\rho_1+\rho_2} + 3(\rho_1+\rho_2))\} \qquad \text{VI - 11}$$

The value of P_{00} can be determined by imposing the boundary condition:

$$\sum_{i=0}^{1} \sum_{j=0}^{2} P_{ij}=1$$

P_{00} is given by:

$$P_{00} = \{(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3 (\frac{4\rho_1+4\rho_2/\phi}{\rho_1+\rho_2} + 3(\rho_1+\rho_2))\} / \{4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^4 + 7(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3 (\rho_1+\rho_2) + 5(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2 (\rho_1+\rho_2)^2 + 3(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}) (\rho_1+\rho_2)^3 + (\rho_1+\rho_2)^4\} \qquad \text{VI - 12}$$

The expected number of units in the system E[n] is given by:

$$E[n] = \{ 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^3 (\rho_1+\rho_2) + 7\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^2 (\rho_1+\rho_2)^2$$
$$+ 8\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right) (\rho_1+\rho_2)^3$$
$$+ 3(\rho_1+\rho_2)^4 \} \; / \; \{ 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^4$$
$$+ 7\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^3 (\rho_1+\rho_2)$$
$$+ 5\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^2 (\rho_1+\rho_2)^2$$
$$+ 3\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right) (\rho_1+\rho_2)^3$$
$$+ (\rho_1+\rho_2)^4 \} \qquad \text{VI - 13}$$

The probability of a lost item is given as:

$$P(\text{lost}) = \{ (\rho_1+\rho_2)^3 \left((\rho_1+\rho_2) + \left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)\right) \}$$
$$/ \{ 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^4 + 7\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^3 (\rho_1+\rho_2)$$
$$+ 5\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^2 (\rho_1+\rho_2)^2$$
$$+ 3\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right) (\rho_1+\rho_2)^3$$
$$+ (\rho_1+\rho_2)^4 \} \qquad \text{VI - 14}$$

Let the probability of the first channel being busy, be represented by $P_1(\text{busy}) = \chi$; where

$$\chi = \sum_{j=0}^{2} P_{1j} = \{ (\rho_1+\rho_2)/\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)$$

$$- (\rho_1+\rho_2)^2/(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2$$

$$+ (\rho_1+\rho_2)^3/(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3$$

$$- (\rho_1+\rho_2)^4/(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^4$$

$$+ \;.....\; \}$$

or simply,

$$\chi = (\rho_1+\rho_2)/((\frac{\rho_1+\rho_2\ \phi}{\rho_1+\rho_2}) + (\rho_1+\rho_2)) \qquad \text{VI - 15}$$

<u>Two-Channel Conveyor With Storage</u>

<u>of Two Unit Capacity at the Second Channel</u>

No storage is allowed at the first channel, while a storage of two unit capacity is permitted at the second channel. The derived steady-state probability equations are as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{10}(t)$$
$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{01}(t) = 0$$

$$..... \qquad \text{VI - 16}$$

$$-((\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})).P_{01}(t)$$
$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{11}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{02}(t) = 0$$

..... VI - 17

$$-((\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})).P_{02}(t)$$
$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t)$$
$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{03}(t) = 0$$

..... VI - 18

$$-((\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})).P_{03}(t)$$
$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{13}(t) = 0$$

..... VI - 19

$$-((\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})).P_{10}(t)$$
$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{11}(t)$$
$$+ (\lambda_1+\lambda_2).P_{00}(t) = 0 \qquad \text{VI - 20}$$

$$-((\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})).P_{11}(t)$$
$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t)$$
$$+ (\lambda_1+\lambda_2).P_{01}(t)$$
$$+ (\lambda_1+\lambda_2).P_{10}(t) = 0 \qquad \text{VI - 21}$$

$$-((\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{12}(t)$$
$$+ (\lambda_1+\lambda_2).P_{02}(t) + (\lambda_1+\lambda_2).P_{11}(t)$$
$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{13}(t) = 0$$

..... VI - 22

$$-2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{13}(t) + (\lambda_1+\lambda_2).P_{03}(t)$$
$$+ (\lambda_1+\lambda_2).P_{12}(t) = 0$$

VI - 23

Solving the above system of equations, one obtains the steady-state probabilities (P_{ij}) in terms of P_{00} as follows:

$$P_{01} = \{ (\rho_1+\rho_2)^2(3(\rho_1+\rho_2) + 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \}.P_{00}$$
$$/ \{ 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}) ((\rho_1+\rho_2)$$
$$+ (\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) ((\rho_1+\rho_2)$$
$$+ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \}$$

VI - 24

$$P_{02} = \{ (\rho_1+\rho_2)^3 \}.P_{00} / \{ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2((\rho_1+\rho_2)$$
$$+ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \}$$

VI - 25

$$P_{03} = \{ (\rho_1+\rho_2)^4 \}.P_{00} / \{ 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3((\rho_1+\rho_2)$$

$$+ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \} \qquad \text{VI - 26}$$

$$P_{10} = \{ (\rho_1+\rho_2) ((\rho_1+\rho_2)^2 + 8(\rho_1+\rho_2)$$
$$+ 8(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2) \}.P_{00}$$
$$/\{4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}) ((\rho_1+\rho_2)$$
$$+ 2(\frac{\rho_1+\rho_2/}{\rho_1+\rho_2})) ((\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})$$
$$+ (\rho_1+\rho_2)) \} \qquad \text{VI - 27}$$

$$P_{11} = \{ (\rho_1+\rho_2)^2((\rho_1+\rho_2) + 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \}.P_{00}$$
$$/ \{ 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^2((\rho_1+\rho_2).$$
$$+ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \} \qquad \text{VI - 28}$$

$$P_{12} = \{ (\rho_1+\rho_1)^3 \}.P_{00} / \{ 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^3 \} \qquad \text{VI - 29}$$

$$P_{13} = \{ (\rho_1+\rho_2)^4 ((\rho_1+\rho_2) + (\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \}.P_{00}$$
$$/ \{ 4(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})^4 ((\rho_1+\rho_2)$$
$$+ 2(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2})) \} \qquad \text{VI - 30}$$

The value of P_{00} can be determined by using the boundary condition:

$$\sum_{i=0}^{1} \sum_{j=0}^{3} P_{ij}=1$$

P_{00} is given by:

$$P_{00} = \{\ 8\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^6 + 12\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^5(\rho_1+\rho_2) + 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^4(\rho_1+\rho_2)^2\}$$
$$/\ \{\ 8\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^6 + 20\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^5(\rho_1+\rho_2) + 20\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^4(\rho_1+\rho_2)^2 + 13\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^3(\rho_1+\rho_2)^3$$
$$+ 8\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^2(\rho_1+\rho_2)^4 + 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)(\rho_1+\rho_2)^5 + (\rho_1+\rho_2)^6\ \} \qquad \text{VI - 31}$$

The expected number of units in the system E[n] is given by:

$$E[n] = \{\ 8\nu^5\eta + 20\nu^4\eta^2 + 24\nu^3\eta^3 + 26\nu^2\eta^4 + 14\nu\eta^5 + 4\eta^6\ \} / \{\ 8\nu^6 + 20\nu^5\eta + 20\nu^4\eta^2 + 13\nu^3\eta^3 + 8\nu^2\eta^4 + 4\nu\eta^5 + \eta^6\} \qquad \text{VI - 32}$$

where $\nu = \dfrac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}$

and $\qquad \eta = \rho_1 + \rho_2$

The probability of a lost item is given by:

$$P(\text{lost}) = \left\{ (\rho_1+\rho_2)^4 \left((\rho_1+\rho_2) + \left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right) \right) \right\} \Big/ \left\{ 4\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right)^4 \left((\rho_1+\rho_2) + 2\left(\frac{\rho_1+\rho_2/\phi}{\rho_1+\rho_2}\right) \right) \right\} \qquad \text{VI - 33}$$

The probability that the first channel is busy is given by $P_1(\text{busy}) = \chi$; where

$$\chi = \sum_{j=0}^{3} P_{1j}$$

or simply,

$$\chi = \frac{\eta}{\nu} - \frac{\eta^2}{\nu^2} + \frac{\eta^3}{\nu^3} - \ldots\ldots$$

which gives:

$$\chi = \frac{\eta}{\nu+\eta} \qquad \text{VI - 34}$$

Verification of Results

Let the arrival rate of the doublets equal zero, i.e., $\lambda_2 = 0$, in equation VI - 31. One can then obtain the following:

$$P_{00} = (8 + 12\rho_1 + 4 \rho_1^2) / (8 + 20\rho_1 + 20\rho_1^2 + 13\rho_1^3 + 8\rho_1^4 + 4\rho_1^5 + \rho_1^6)$$

which can be rewritten as:

$$\text{(i)} \quad P_{00} = \{ 4(\rho_1+1) (\rho_1+2) \} / \{ (8 + 20\rho_1 + 20\rho_1^2 + 13\rho_1^3 + 8\rho_1^4 + 4\rho_1^5 + \rho_1^6 \}$$

Equation (i) is the same equation given by Disney (5) in equation (ii) which follows.

$$\text{(ii)} \quad P_{00} = \{ 4(\rho+1) (\rho+2) \} / S$$

where $S = \rho^6 + 4\rho^5 + 8\rho^4 + 13\rho^3 + 20\rho^2 + 20\rho + 8$

It is obvious that equations (i) and (ii) are identical.

Two-Channel Conveyor With Storage of Three Unit Capacity at the Second Channel

A storage of three unit capacity is allowed at the second channel. Following the same procedures as in the case of the two unit capacity storage, one can develop the steady-state probability equations as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2}+\frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{10}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{01}(t)$$

$$= 0 \qquad \text{VI - 35}$$

$$-((\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)).P_{01}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{11}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{02}(t) = 0$$

$$\ldots\ldots \quad \text{VI - 36}$$

$$-((\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)).P_{02}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{12}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{03}(t) = 0$$

$$\ldots\ldots \quad \text{VI - 37}$$

$$-((\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)).P_{04}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{14}(t)$$

$$= 0 \qquad \text{VI - 38}$$

$$-((\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)).P_{10}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{11}(t)$$

$$+ (\lambda_1+\lambda_2).P_{00}(t) = 0 \qquad \text{VI - 39}$$

$$-[(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{11}(t)$$

$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{12}(t)$$

$$+ (\lambda_1+\lambda_2).P_{01}(t)$$

$$+ (\lambda_1+\lambda_2).P_{10}(t) = 0 \qquad \text{VI - 40}$$

$$-[(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{12}(t)$$

$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{13}(t)$$

$$+ (\lambda_1+\lambda_2).P_{11}(t)$$

$$+ (\lambda_1+\lambda_2).P_{02}(t) = 0 \qquad \text{VI - 41}$$

$$-[(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{13}(t)$$

$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{14}(t)$$

$$+ (\lambda_1+\lambda_2).P_{12}(t)$$

$$+ (\lambda_1+\lambda_2).P_{03}(t) = 0 \qquad \text{VI - 42}$$

$$-2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{14}(t)$$

$$+ (\lambda_1+\lambda_2).P_{13}(t)$$

$$+ (\lambda_1+\lambda_2).P_{04}(t) = 0 \qquad \text{VI - 43}$$

Solving the above equations, one can obtain the values of P_{ij} in terms of P_{00}, as follows:

$$P_{01} = \{ 2\eta^2(2\eta^2 + 5\nu\eta + 4\nu^2) \}.P_{00}$$

$$/ \{ \nu(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\nu^3) \}$$

..... VI - 44

$$P_{02} = \{ \eta^3(3\eta + 4\nu) (\nu + \eta) \}.P_{00}$$

$$/ \{ \nu^2(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3)\}$$

..... VI - 45

$$P_{03} = \{ 2\eta^4(\eta + \nu)^2 \}.P_{00}$$

$$/ \{ \nu^3(16\nu^3 + 32\nu^3\eta + 19\nu\eta^2 + 3\eta^3)\}$$

..... VI - 46

$$P_{04} = \{ \eta^5(\eta + \nu)^2 \}.P_{00}$$

$$/ \{ \nu^4(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3)\}$$

..... VI - 47

$$P_{10} = \{ \eta(16\nu^3 + 24\nu^2\eta + 9\nu\eta^2 - \eta^3) \} \cdot P_{00}$$

$$/ \ \{ \nu(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \}$$

..... VI - 48

$$P_{11} = \{ \eta^2(\eta + \nu) (\eta^2 + 6\nu\eta + 8\nu^2) \} \cdot P_{00}$$

$$/ \ \{ \nu^2(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \}$$

..... VI - 49

$$P_{12} = \{ \eta^3(\eta + 4\nu) (\eta + \nu)^2 \} \cdot P_{00}$$

$$/ \ \{ \nu^3(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \}$$

..... VI - 50

$$P_{13} = \{ \eta^4(\eta + 2\nu) (\eta + \nu)^2 \} \cdot P_{00}$$

$$/ \ \{ \nu^4(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \}$$

..... VI - 51

$$P_{14} = \{ \eta^5(\eta + \nu)^3 \} \cdot P_{00}$$

$$/ \ \{ \nu^5(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \}$$

..... VI - 52

where $\nu = (\rho_1 + \rho_2/\phi) \ / \ (\rho_1 + \rho_2)$

and $\eta = \rho_1 + \rho_2$

The measures of performance can be evaluated as follows:
The probability of the system being idle (P_{00}):

$$P_{00} = \{ \nu^5(16\nu^3 + 32\nu^2\eta + 19\nu\eta^2 + 3\eta^3) \} \cdot P_{00}$$
$$/ \{ 16\nu^8 + 48\nu^7\eta + 59\nu^6\eta^2 + 44\nu^5\eta^3$$
$$+ 30\nu^4\eta^4 + 21\nu^3\eta^5 + 12\nu^2\eta^6 + 5\nu\eta^7 + \eta^8) \}$$

..... VI - 53

The expected number of units in the system, E[n]:

$$E[n] = \{ 16\nu^7\eta + 48\nu^6\eta^2 + 67\nu^5\eta^3 + 72\nu^4\eta^4$$
$$+ 67\nu^3\eta^5 + 48\nu^2\eta^6 + 23\nu\eta^7 + 5\eta^8 \}$$
$$/ \{ 16\nu^8 + 48\nu^7\eta + 59\nu^6\eta^2 + 44\nu^5\eta^3$$
$$+ 30\nu^4\eta^4 + 21\nu^3\eta^5 + 12\nu^2\eta^6 + 5\nu\eta^7 + \eta^8 \}$$

..... VI - 54

The probability of a lost item is given as:

$$P(\text{lost}) = \{ \eta^5(\eta + \nu)^3 \}$$
$$/ \{ 16\nu^8 + 48\nu^7\eta + 59\nu^6\eta^2 + 44\nu^5\eta^3$$

$$+ 30\nu^4\eta^4 + 21\nu^3\eta^5 + 12\nu^2\eta^6 + 5\nu\eta^7$$

$$+ \eta^8 \} \qquad \text{VI - 55}$$

The probability that the first channel is busy is given by $P_1(\text{busy}) = \chi$, where

$$\chi = \frac{\eta}{\nu} [1 - \frac{\eta}{\nu} + \frac{\eta^2}{\nu^2} - \frac{\eta^3}{\nu^3} + \ldots]$$

or simply,

$$\chi = \frac{\eta}{\nu + \eta} \qquad \text{VI - 56}$$

Two-Channel Conveyor With Storage of N Units Capacity at the Second Channel

No storage is allowed at the first channel and storage of N units capacity is allowed at the second channel. One can write the steady-state probability equations as follows:

$$-(\lambda_1+\lambda_2).P_{00}(t) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{10}(t)$$

$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).P_{01}(t) = 0$$

$$\ldots\ldots \quad \text{VI - 57}$$

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{0j}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{1j}(t)$$

$$+ \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right).P_{0,j+1}(t) = 0$$

where $j = 1,2,3,\ldots N-1$ VI - 58

$$-\left[(\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\right].P_{0N}(t)$$

$$+ \left[\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right].P_{1N}(t) = 0 \qquad \text{VI - 59}$$

$$-\left[(\lambda_1+\lambda_2) + \left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\right].P_{10}(t)$$

$$+ \left[\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right].P_{11}(t)$$

$$+ (\lambda_1+\lambda_2).P_{00}(t) = 0 \qquad \text{VI - 60}$$

$$-\left[(\lambda_1+\lambda_2) + 2\left(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right)\right].P_{1j}(t)$$

$$+ \left[\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right].P_{1,j+1}(t)$$

$$+ (\lambda_1+\lambda_2).P_{0,j}(t) + (\lambda_1+\lambda_2).P_{1,j-1}(t) = 0$$

where $j = 1,2,3,\ldots,N-1$ VI - 61

$$-2\left[\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}\right].P_{1N}(t) + (\lambda_1+\lambda_2).P_{0N}(t)$$

$$+ (\lambda_1+\lambda_2).P_{1,N-1}(t) = 0 \qquad \text{VI - 62}$$

There is one dependent equation in the above set of equations, which enables us to obtain all the values of P_{ij} in terms of one P_{ij}. Using

$$\sum_{i=0}^{1} \sum_{j=0}^{N} P_{ij}=1$$

we can obtain the values of the probabilities.

It was found that the above system of equations could not be solved recursively. As a consequence, two methods were used to solve these equations: (i) a computer programme method, and (ii) a generating functions method.

(i) The Computer Programme Method: Input to this programme is as follows:

1. The capacity of the storage N;

2. The traffic intensities of the singlets 'ρ_1' and 'ρ_2': These are referred to as R1 and R2, respectively, in the computer programme; and

3. Service time ratio 'ϕ' denoted by "PHI' in the computer programme.

Output of this computer programme consists of the following, as in Figure 24:

1. The number of the steady-state probability equations corresponding to the storage capacity N;

2. The values of P_{ij} for $i=0,1$, $j=0,1,2,\ldots,N$; and

3. The expected number of units in the system: E[n].

(ii) The Generating Functions Method: Let the generating function be defined as:

FIGURE 24

```
C       CLOSED LOOP CONVEYOR SYSTEM WITH STORAGE OF
C       CAPACITY M AT THE SECOND CHANNEL
        DIMENSION A(30,30),B(30,1),AA(900)
        DO 10 M=1,20
        M1=M+1
        N=M1*2+2
        WRITE(6,20) M,N
20      FORMAT(6X,'STORAGE CAPACITY M =',I2,6X,
       1'NUMBER OF EQUATIONS N=',I2)
        DO 200 IZ=4,20,4
        PS=IZ
        R2=PS/20.
        DO 200 JZ=4,20,4
        PT=JZ
        R1=JZ/20.
        PHI=2.
        WRITE(6,30)R1,R2
30      FORMAT(6X,'R1=',F10.5,6X,'R2=',F10.5)
        BS=R1+R2
        AS=(R1+R2/PHI)/BS
        WRITE(6,35)BS,AS
35      FORMAT(6X,'BS=',F10.5,6X,'AS=',F10.5)
        DO 40 II=1,N
        DO 40 JJ=1,N
        A(II,JJ)=0.0
40      B(II,1)=0.0
        A(1,1)=-BS
        A(1,2)=AS
        A(1,M+3)=AS
        MK=M+1
        DO 50 K=2,MK
        A(K,K)=-(BS+AS)
        A(K,K+M+2)=AS
        A(K,K+1)=AS
50      CONTINUE
        A(M+2,M+2)=-(BS+AS)
        A(M+2,N)=AS
        A(M+3,M+3)=-(BS+AS)
        A(M+3,M+4)=AS
        A(M+3,1)=BS
        DO 60 IK=1,M
        A(IK+M+3,IK+M+3)=-(BS+2*AS)
        A(IK+M+3,IK+M+4)=AS
        A(IK+M+3,IK+M+2)=BS
        A(IK+M+3,IK+1)=BS
60      CONTINUE
        A(2*M+4,N)=-2*AS
```

```
      A(2*M+4,M+2)=BS
      A(2*M+4,N-1)=BS
      DO 70 ILS=1,N
      A(N,ILS)=1.0
      B(ILS,1)=0.0
70    CONTINUE
      B(N,1)=1.0
      WRITE(6,80)
80    FORMAT(6X,'PROBABILITY MATRIX')
      WRITE(6,90)((A(IH,JH),JH=1,N),IH=1,N)
90    FORMAT(6X,6(F15.5))
      NN=N*N
      MNF=0.0
      DO 2000 I=1,N
      DO 2000 J=1,N
      AA(MNF+1)=A(J,I)
      MNF=MNF+1
2000  CONTINUE
      CALL SIMQ(AA,B,N,KS)
      WRITE(6,250)(B(I,1),I=1,N)
250   FORMAT(6X,'*** S O L U T I O N ***',6(F15.9))
      X=0.0
      KI=M+1
      DO 254 I=1,KI
      X=X+B(I+1,1)*I
254   CONTINUE
      Y=0.0
      JIK=M+2
      DO 256 I=1,JIK
      Y=Y+B(I+M+2,1)*I
256   CONTINUE
      EN=X+Y
      WRITE(6,22) EN
22    FORMAT(6X,'********** EXPECTED NUMBER OF UNITS**
     1********='F15.7)
200   CONTINUE
      PRINT 900
900   FORMAT(6X,'***** END OF CASE *****')
      PRINT 24
24    FORMAT('*************************************')
10    CONTINUE
      STOP
      END
```

$$F_j(x) = \sum_{k=1}^{N} P(j,k)x^k \qquad \text{VI - 63}$$

From equations VI-57, VI-58 and VI-59

$$-(\lambda_1+\lambda_2).P_{00} + [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].P_{10}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].P_{01} = 0$$

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})]x.P_{10}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}]x.P_{11}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}]x.P_{02} = 0$$

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$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})]x^{N-1}.P_{0,N-1}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x^{N-1}.P_{1,N-1}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x^{N-1}.P_{0,N} = 0$$

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].x^{N}.P_{0N}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x^{N}.P_{1N} = 0$$

These equations give:

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].x.F_0(x)$$

$$- (\lambda_1+\lambda_2).x.P_{00}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x.F_1(x)$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x.P_{10}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].F_0(x) = 0 \qquad \text{VI - 64}$$

From VI-60, VI-61, and VI- 62

$$-[(\lambda_1+\lambda_2) + (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{10}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].P_{11}$$

$$+ (\lambda_1+\lambda_2).P_{00} = 0$$

$$-[(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].x.P_{11}$$

$$+ (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}).x.P_{12}$$

$$+ (\lambda_1+\lambda_2).x.P_{01} + (\lambda_1+\lambda_2).x.P_{10} = 0$$

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$$-[(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].x^{N-1}.P_{1,N-1}$$

$$+ [\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x^{N-1}.P_{1N}$$

$$+ (\lambda_1+\lambda_2).x^{N-1}.P_{0,N-1}$$

$$+ (\lambda_1+\lambda_2).x^{N-1}.P_{1,N-2} = 0$$

$$-2[\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}].x^{N}.P_{1N} + (\lambda_1+\lambda_2).x^{N}.P_{0N}$$

$$+ (\lambda_1+\lambda_2).x^{N}.P_{1,N-1} = 0$$

which give:

$$\{ [\frac{1}{x}(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)}) + (\lambda_1+\lambda_2)x]$$

$$- [(\lambda_1+\lambda_2) + 2(\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})]\}.F_1(x)$$

$$+ (\lambda_1+\lambda_2).F_0(x) + (\lambda_1+\lambda_2).P_{00}$$

$$+ x^N.P_{1N}[(\lambda_1+\lambda_2)\ (1-x)]$$

$$+ [x(\lambda_1+\lambda_2) - (\lambda_1+\lambda_2)$$

$$- (\frac{\lambda_1\mu}{\lambda_1+\lambda_2} + \frac{\lambda_2\mu}{\phi(\lambda_1+\lambda_2)})].P_{10} = 0 \qquad \text{VI - 65}$$

From VI-64 and VI-65 one obtains:

$$F_0(x) = \{ [\mu(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})]^2\ (1-x)x.P_{10}$$

$$- \mu(\lambda_1+\lambda_2/\phi)\ (1-x)x^{N+2}.P_{1N}$$

$$- (\lambda_1+\lambda_2)x \{(\lambda_1+\lambda_2)x^2 - [(\lambda_1+\lambda_2) + \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}]x + \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}\} \cdot P_{00} \}$$

$$/ \{ [(\lambda_1+\lambda_2) + \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}]$$

$$\{(\lambda_1+\lambda_2)x - (\lambda_1+\lambda_2) [(\lambda_1+\lambda_2) + 2\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}]\}x^2$$

$$+ \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2} [x(3\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2} + 2(\lambda_1+\lambda_2)) - \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}] \} \qquad \text{VI - 66}$$

The generating function solution requires the roots of the denominator in equation VI-66. The roots x_1, x_2, and x_3 were found to be:

$$x_1 = \mu\frac{(\lambda_1+\lambda_2/\phi)}{(\lambda_1+\lambda_2)^2} [1 + \sqrt{[\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}] / [\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2} + (\lambda_1+\lambda_2)]}]$$

$$x_2 = \frac{(\lambda_1+\lambda_2/\phi)}{(\lambda_1+\lambda_2)^2} [1 - \sqrt{[\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}] / [\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2} + (\lambda_1+\lambda_2)]}]$$

$$x_3 = 1$$

The solution equations of the probabilities are given as:

$$\gamma_1 P_{10} - \sigma_1 P_{1N} - \psi_1 P_{00} = 0 \qquad \text{VI - 67}$$

$$\gamma_2 P_{10} - \sigma_2 P_{1N} \quad \psi_2 P_{00} = 0 \qquad \text{VI - 68}$$

where $\gamma_i = \mu(\frac{\lambda_1+\lambda_2/\phi}{\lambda_1+\lambda_2})\ (1-x_i)x_i \quad i=1,2$

$$\sigma_i = (\lambda_1+\lambda_2/\phi)\ (1-x_i)x_i^{N+2} \quad i=1,2$$

$$\psi_i = (\lambda_1+\lambda_2)x_i\ \{(\lambda_1+\lambda_2)x_i^2 - [(\lambda_1+\lambda_2) + \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}]x_i + \frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2}\} \quad i=1,2$$

Solving VI-67 and VI-68 we obtain:

$$P_{1N} = \frac{\psi_1\ \gamma_2 - \psi_2\gamma_1}{\gamma_1\ \sigma_2 - \gamma_2\sigma_1} . P_{00} \qquad \text{VI - 69}$$

$$P_{10} = \frac{\psi_1\ \sigma_2 - \psi_2\sigma_1}{\gamma_1\ \sigma_2 - \gamma_2\sigma_1} . P_{00} \qquad \text{VI - 70}$$

Using the boundary condition:

$$\sum_{j=0}^{N} \sum_{i=0}^{1} P_{ij} = 1$$

we obtain

$$\chi = \sum_{j=0}^{N} P_{1j} = (\lambda_1+\lambda_2)\ /\ \{(\frac{\mu(\lambda_1+\lambda_2/\phi)}{\lambda_1+\lambda_2})$$

$$+ (\lambda_1+\lambda_2) \} \qquad \text{VI - 71}$$

From equations VI-69, VI-70, and VI-71, one can obtain the probability P_{00}, i.e., the probability of the system being idle.

Case 2: Conveyors Having More Than Two Input Sources With a Storage Capacity of N Units at the Second Channel

This problem has a finite number of arrivals, each governed by an independent Poisson distribution, with mean arrival rates λ_1, λ_2, λ_3, ..., λ_k, ..., λ_M. The service time of an arrival from type k is ϕ_k times that of the first type of arrival. Hence, it should be noted that $\phi_1 = 1$. The case of the system with homogeneous servers was studied.

Following the same procedures outlined in Case 1, probability reasoning led to the following differential - difference equations, characterizing the model:

$$-[\sum_{k=1}^{M} \lambda_k].P_{00}(t) + [(\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{10}(t)$$
$$+ [(\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{01}(t)$$
$$= 0 \qquad \text{VI - 72}$$

$$-[\sum_{k=1}^{M} \lambda_k + (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{0j}(t)$$

$$+ (\tilde{\mu} \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k) .P_{1j}(t)$$

$$+ (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k).P_{0,j+1}(t) = 0$$

where $j=1,2,3,\ldots,N-1$ VI - 73

$$-[\sum_{k=1}^{M} \lambda_k + (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{0N}(t)$$

$$+ (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k).P_{1N}(t) = 0$$

..... VI - 74

$$-[\sum_{k=1}^{M} \lambda_k + (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{10}(t)$$

$$+ (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k).P_{11}(t)$$

$$+ (\sum_{k=1}^{M} \lambda_k).P_{00}(t) = 0$$ VI - 75

$$-[\sum_{k=1}^{M} \lambda_k + (2\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)].P_{1j}(t)$$

$$+ (\mu \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k).P_{1,j+1}(t)$$

$$+ (\sum_{k=1}^{M} \lambda_k).P_{0j}(t)$$

$$+ \left(\sum_{k=1}^{M} \lambda_k \right) . P_{1,j-1}(t) = 0$$

where $j=1,2,3,\ldots,N-1$ VI - 76

$$\left(\left(\mu \sum_{k=1}^{M} \lambda_k/\phi_k \right) / \left(\sum_{k=1}^{M} \lambda_k \right) \right) . P_{1N}(t)$$

$$+ \left(\sum_{k=1}^{M} \lambda_k \right) . P_{0N}(t)$$

$$+ \left(\sum_{k=1}^{M} \lambda_k \right) . P_{1,N-1}(t) = 0 \qquad \text{VI - 77}$$

The solution of the above system of equations was obtained by using a computer programme developed for this purpose (see Appendix A). This computer printout provides the conveyor designer with relevant values of the measures of performance for any specific values of arrival rates, service rates, service time ratios, and for the storage capacity.

Equations VI-72 through VI-77 can also be solved by using the generating function techniques as in Case 1. The roots of the generating function denominator are given by:

$$x_1 = \left(\mu \sum_{k=1}^{M} \lambda_k/\phi_k \right) / \left(\sum_{k=1}^{M} \lambda_k \right) [1 +$$

$$\sqrt{\left[\frac{\mu \sum_{k=1}^{M} \lambda_k/\phi_k}{\sum_{k=1}^{M} \lambda_k}\right] \Big/ \left[\frac{\mu \sum_{k=1}^{M} \lambda_k/\phi_k}{\sum_{k=1}^{M} \lambda_k} + \sum_{k=1}^{M} \lambda_k\right]} \;\;]$$

$$x_2 = \left(\mu \sum_{k=1}^{M} \lambda_k/\phi_k \right) \Big/ \left(\sum_{k=1}^{M} \lambda_k \right) \Big[1 - \sqrt{\left[\frac{\mu \sum_{k=1}^{M} \lambda_k/\phi_k}{\sum_{k=1}^{M} \lambda_k}\right] \Big/ \left[\frac{\mu \sum_{k=1}^{M} \lambda_k/\phi_k}{\sum_{k=1}^{M} \lambda_k} + \sum_{k=1}^{M} \lambda_k\right]} \;\Big]$$

$$x_3 = 1$$

Substituting in the generating function equation, one obtains the following:

$$P_{1N} = \left(\frac{\psi_1\gamma_2 - \psi_2\gamma_1}{\gamma_1\sigma_2 - \gamma_2\sigma_1} \right).P_{00} \qquad \text{VI - 78}$$

$$P_{10} = \left(\frac{\psi_1\sigma_2 - \psi_2\sigma_1}{\gamma_1\sigma_2 - \gamma_2\sigma_1} \right).P_{00} \qquad \text{VI - 79}$$

where

$$\gamma_i = \left([\mu \sum_{k=1}^{M} \lambda_k/\phi_k]/[\sum_{k=1}^{M} \lambda_k] \right) (1-x_i)x_i \qquad i=1,2$$

$$\sigma_i = \left([\mu \sum_{k=1}^{M} \lambda_k/\phi_k]/[\sum_{k=1}^{M} \lambda_k] \right) (1-x_i)x_i^{N+2} \qquad i=1,2$$

and

$$\psi_i = \left(\sum_{k=1}^{M} \lambda_k\right) x_i \; \{ \left(\sum_{k=1}^{M} \lambda_k\right) x_i^2 - [\sum_{k=1}^{M} \lambda_k$$

$$+ \left(\mu \sum_{k=1}^{M} \lambda_k/\phi_k\right) / \left(\sum_{k=1}^{M} \lambda_k\right)] x_i$$

$$+ \left(\mu \sum_{k=1}^{M} \lambda_k/\phi_k\right) / \left(\sum_{k=1}^{M} \lambda_k\right) \}$$

where $i=1,2$

Using the boundary condition

$$\chi = \sum_{j=0}^{N} P_{1j} = \{ \sum_{k=1}^{M} \lambda_k \} / \{ \left(\sum_{k=1}^{M} \lambda_k/\phi_k\right) / \left(\sum_{k=1}^{M} \lambda_k\right)$$

$$+ \sum_{k=1}^{M} \lambda_k \}$$

with equations VI-78 and VI-79, the system's performance can be evaluated.

Case 3: Conveyors Having More Than Two Input Sources and Heterogeneous Servers Where Storage of Variable Capacity is Allocated at the Second Channel

The purpose of the following analysis is to study a two-channel conveyor with storage of capacity N at the second channel and no storage at the first channel. The service rates are μ_1, and μ_2 for the first and the second channel respectively. Also, the input arrivals have the same characteristics as that studied in Case 2. The Kolmogorov-Chapman birth-death equations describing the system for the steady-state case were found to be as follows:

$$-\left(\sum_{k=1}^{M}\lambda_k\right).P_{00}(t) + \left[\left(\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{10}(t)$$

$$+ \left[\left(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{01}(t) = 0$$

..... VI - 80

$$-\left[\left(\sum_{k=1}^{M}\lambda_k\right) + \left(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{0j}(t)$$

$$+ \left[\left(\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{1j}(t)$$

$$+ \left[\left(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{0,j+1}(t)$$

$$= 0$$

where $j=1,2,\ldots,N-1$ VI - 81

$$-\left[\left(\sum_{k=1}^{M}\lambda_k\right) + \left(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{0N}(t)$$

$$+ \left[\left(\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M}\lambda_k\right)\right].P_{1N}(t) = 0$$

..... VI - 82

$$-[(\sum_{k=1}^{M}\lambda_k) + (\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].P_{10}(t)$$

$$+ [(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].P_{11}(t)$$

$$+ (\sum_{k=1}^{M}\lambda_k).P_{00}(t) = 0 \qquad \text{VI - 83}$$

$$-[(\sum_{k=1}^{M}\lambda_k) + \{(\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)\}(\mu_1+\mu_2)].P_{1j}(t)$$

$$+ [(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].P_{1,j+1}(t)$$

$$+ (\sum_{k=1}^{M}\lambda_k).P_{1,j-1}(t)$$

$$+ (\sum_{k=1}^{M}\lambda_k).P_{0j}(t) = 0 \qquad \text{VI - 84}$$

$$-[(\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)](\mu_1+\mu_2).P_{1M}(t)$$

$$+ (\sum_{k=1}^{M}\lambda_k).P_{0M}(t) + (\sum_{k=1}^{M}\lambda_k).P_{1,M-1}(t) = 0$$

..... VI - 85

The above system of equations was solved by the development of a computer programme.
This programme provides the conveyor designer with the measures of performance.
(Note: The service rate ratio (μ_2/μ_1) is referred to

as Θ 'THETA' in the computer programme.)

The previous equations can also be solved by using the generating function technique as it was developed for Cases 1 and 2. Define

$$F_j(x) = \sum_{k=1}^{M} P(j,k)x^k$$

From equations VI-80, VI-81 and VI-82 one obtains the following:

$$-[(\sum_{k=1}^{M}\lambda_k) + (\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].F_0(x)$$

$$- (\sum_{k=1}^{M}\lambda_k).P_{00}$$

$$+ [(\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].F_1(x)$$

$$+ [(\mu_1\sum_{k=1}^{M}\lambda_k/\phi_k)/(\sum_{k=1}^{M}\lambda_k)].P_{10}$$

$$+ [(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k)/(x\sum_{k=1}^{M}\lambda_k)].F_0(x) = 0$$

..... VI - 86

From equations VI-83, VI-84, and VI - 85, one obtains:

$$-\{(\mu_2\sum_{k=1}^{M}\lambda_k/\phi_k)/(x\sum_{k=1}^{M}\lambda_k) + (x\sum_{k=1}^{M}\lambda_k) - [(\sum_{k=1}^{M}\lambda_k)$$

$$+ (\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k) +$$

$$+ (\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)]\}.F_1(x)$$

$$+ (\sum_{k=1}^{M} \lambda_k)x^N(1-x).P_{1N} - [(\sum_{k=1}^{N} \lambda_k)$$

$$+ (\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k) - (\sum_{k=1}^{M} \lambda_k)x].P_{10}$$

$$+ (\sum_{k=1}^{M} \lambda_k).F_0(x) + (\sum_{k=1}^{M} \lambda_k).P_{00} = 0$$

..... VI - 87

From equations VI-86 and VI-87, we obtain:

$$F_0(x) = \{\mu_2[(\sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)]\ x(1-x).P_{10}$$

$$- \mu_1(\sum_{k=1}^{M} \lambda_k)\ (1-x)x^{N+2}.P_{1N}$$

$$- [(\sum_{k=1}^{M} \lambda_k)^2 x^3 - ((\sum_{k=1}^{M} \lambda_k)^2 + (\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k))x^2$$

$$- (x\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)].P_{00}\}$$

$$/\ \{x^3[(\sum_{k=1}^{M} \lambda_k)\ (\sum_{k=1}^{M} \lambda_k + (\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)/$$

$$(\sum_{k=1}^{M} \lambda_k))] - x^2[(\sum_{k=1}^{M} \lambda_k)^2 + (3\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)$$

$$+ \left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2] + x[(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)$$

$$2\left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2 + (\mu_1+\mu_2)$$

$$\left(\left(\sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2]$$

$$- \left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2\} \qquad \text{VI - 88}$$

The roots of the generating function's denominator of equation VI-88 are given by:

$$x_1 = \Bigg[[2\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k + \left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2$$

$$+ \mu_1\mu_2\left(\left(\sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2]$$

$$+ \text{SQUARE ROOT OF } \{\left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2$$

$$[4\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k + \left(\left(\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2$$

$$+ 2\mu_1\mu_2\left(\left(\sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2$$

$$+ \left(\left(\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k\right)/\left(\sum_{k=1}^{M} \lambda_k\right)\right)^2]\}\Bigg]$$

$$/\ 2[(\sum_{k=1}^{M} \lambda_k)^2 + \mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k]$$

$$x_2 = \Bigg[[2\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k + ((\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2$$

$$+ \mu_1\mu_2((\sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2]$$

$$- \text{SQUARE ROOT OF } \{((\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2$$

$$[4\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k + ((\mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2$$

$$+ 2\mu_1\mu_2((\sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2$$

$$+ ((\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k))^2]\} \Bigg]$$

$$/ \; 2[(\sum_{k=1}^{M} \lambda_k)^2 + \mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k]$$

$x_3 = 1$.

The solution equations are derived as follows:

$$P_{1N} = \frac{\psi_1\gamma_2 - \psi_2\gamma_1}{\gamma_1\sigma_2 - \gamma_2\sigma_1} . P_{00} \qquad \text{VI - 89}$$

$$P_{10} = \frac{\psi_1\sigma_2 - \psi_2\sigma_1}{\gamma_1\sigma_2 - \gamma_2\sigma_1} . P_{00} \qquad \text{VI - 90}$$

where

$$\gamma_i = \mu_2[(\sum_{k=1}^{M} \lambda_k/\phi_k)/(\sum_{k=1}^{M} \lambda_k)]x_i(1-x_i) \qquad i=1,2$$

$$\sigma_i = \mu_1 \left(\sum_{k=1}^{M} \lambda_k\right)(1-x_i)x_i^{N+2} \qquad i=1,2$$

and $$\psi_i = \{\left(\sum_{k=1}^{M} \lambda_k\right)^2 x_i^3 - [\left(\sum_{k=1}^{M} \lambda_k\right)^2 + \mu_2 \sum_{k=1}^{M} \lambda_k/\phi_k] x_i^2$$

$$- \mu_2 \left(\sum_{k=1}^{M} \lambda_k/\phi_k\right) x_i\} \qquad i=1,2$$

Equations VI-89, VI-90, and VI-91

$$\chi = \sum_{j=0}^{N} P_{1j} = \left(\sum_{k=1}^{M} \lambda_k\right) / [(\mu_1 \sum_{k=1}^{M} \lambda_k/\phi_k)/\left(\sum_{k=1}^{M} \lambda_k\right)$$

$$+ \left(\sum_{k=1}^{M} \lambda_k\right)] \qquad \text{VI - 91}$$

yield the measures of the system's performance.

It should be noted that the probability of the first channel being busy is given by equation VI-91 which is independent of the storage capacity at the second channel.

Effect of Storage Capacity on the Performance of the System

The effect of ρ_1, ρ_2, ϕ, and M on the performance of the closed-loop conveyor system was investigated in Chapters III, and IV. To study the effect of the storage capacity on the performance of the two-channel closed-loop conveyor, the values of the following parameters are kept constant:

(i) ρ_2=1.0 (traffic intensity of the doublet units equals unity); and

(ii) ϕ=2.0 (the service time of a doublet arrival is twice that of a singlet arrival).

The effect of the storage capacity on the performance of the system is shown in Figures 25a and b. It is apparent that, P_{00} approaches zero, as the storage capacity increases. Also, the expected number of units in the system increases with the increase of the storage capacity.

It was found that the capacity of the storage does not affect the utilization of the first channel. The effect of the traffic intensities on the utilization of the first channel is shown in Figure 25c. As the traffic intensities increase, the utilization of the first channel increases.

When comparing the results of Chapters III, IV, and V for the two channel case, it is advisable to allocate storage at the second channel, rather than allowing the lost arrivals to recirculate. This will result in a more efficient performance of the system.

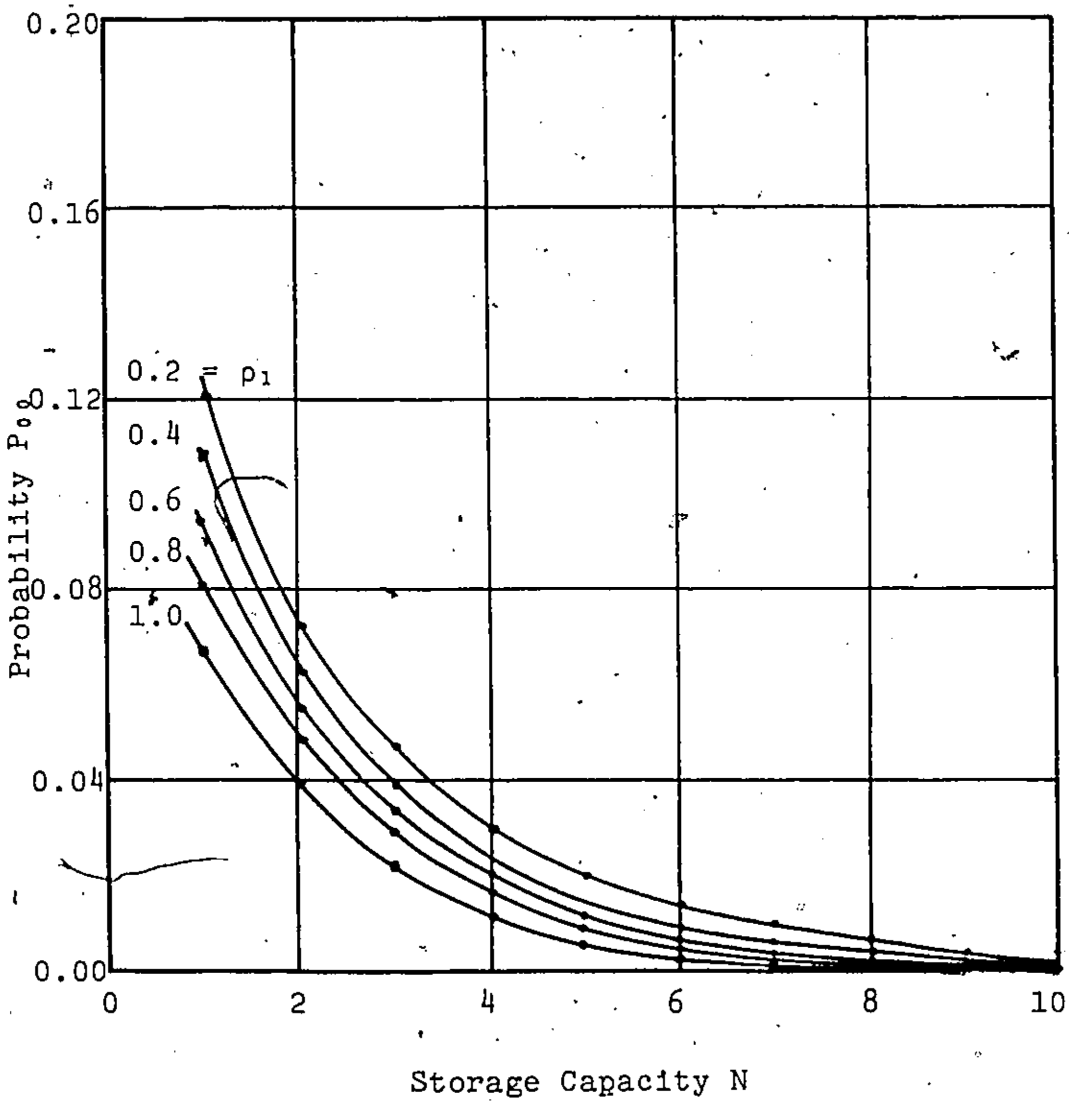

Figure 25a. Effect of storage capacity on the probability of the system being idle P_{00}; ρ_2=1.0

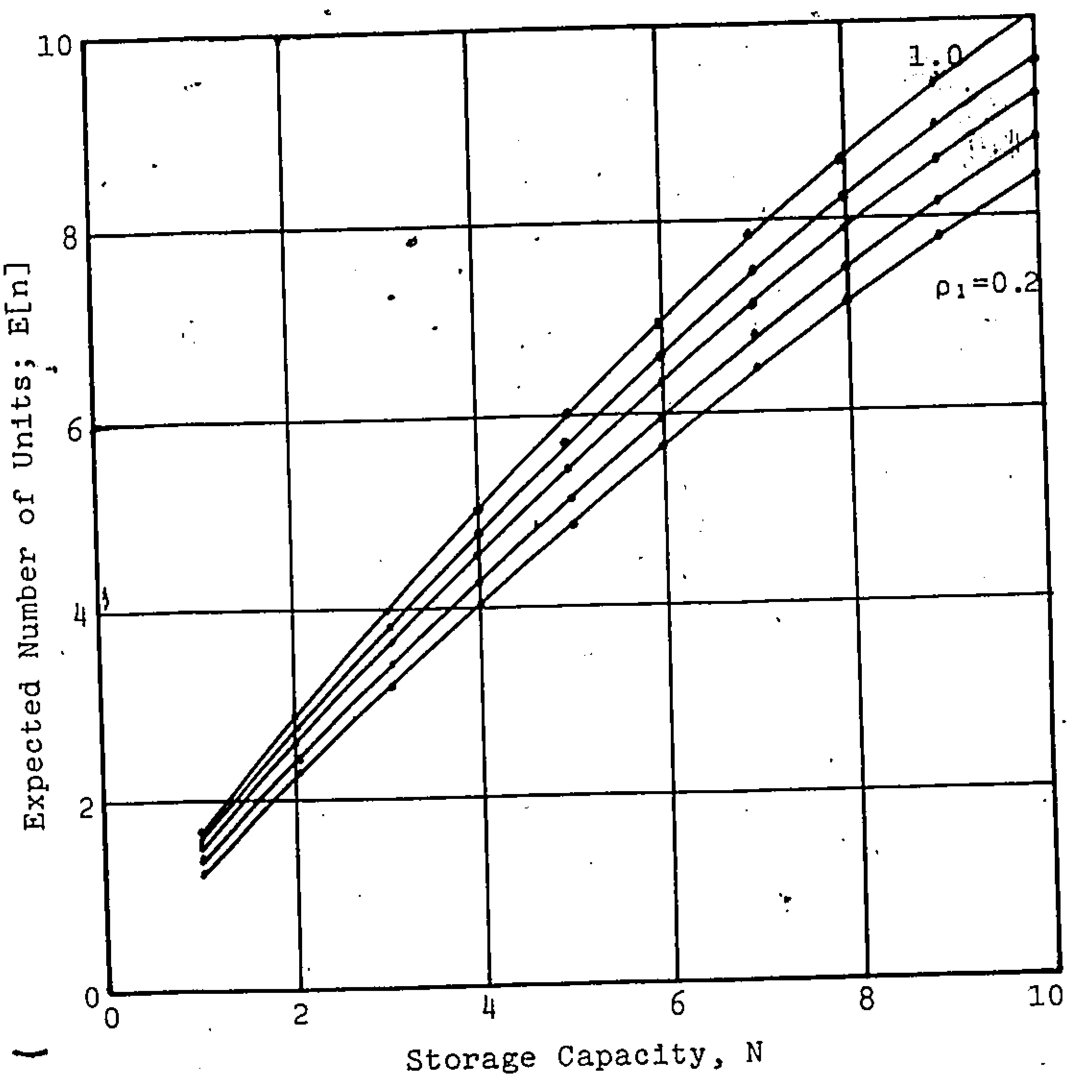

Figure 25b. Effect of storage capacity on the expected number of units in the system E[n]; ρ_2=1.0

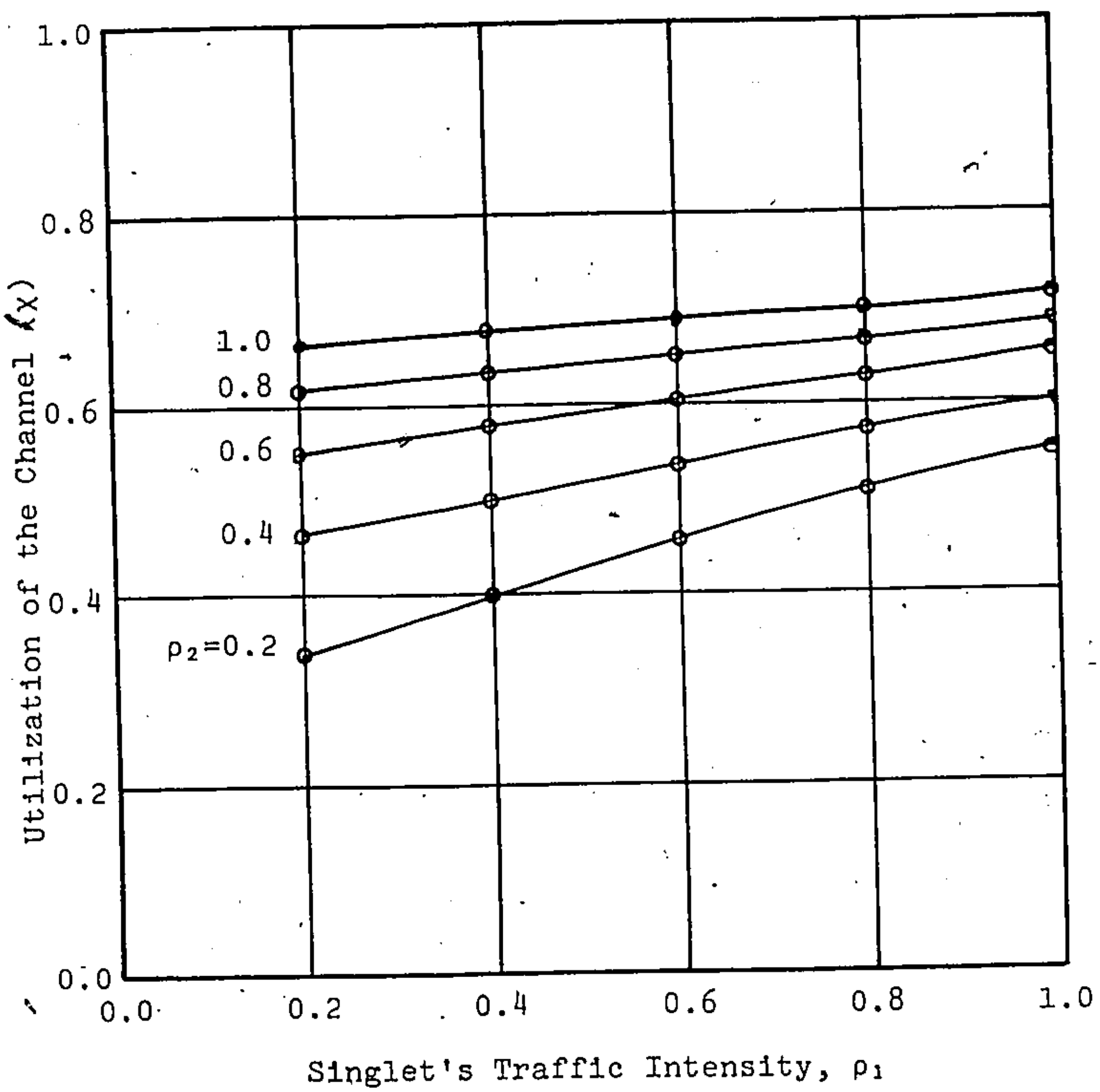

Figure 25c. Effect of traffic intensities on the utilization of the first channel for a two-channel conveyor with storage at the second channel.

CHAPTER VII

SIMULATION ANALYSIS

In the previous chapters of this dissertation an analytical approach was taken to solve different cases regarding the loading and unloading of the ordered-entry closed-loop conveyors.

The purpose of this chapter is to present the results of the simulation analysis, and to compare them with theoretically developed results. Another objective is to analyze other cases - which are not mathematically feasible.

The following cases for two and three channel conveyors with homogeneous servers have been examined:

1. Conveyors with lost arrivals and no storage is allowed at any of the service channels.

2. Conveyors with lost arrivals and a storage of different capacities was allocated at any of the service channels.

3. Conveyors with recirculation, and the recirculated units have different recirculation times.

4. Distributions of the recirculated units; and

5. Transient-solution of the two-channel closed-loop conveyor system without storage at any of the service channels.

G.P.S.S., (General Purpose Simulation System) was chosen for this study because of its availability and flexibility of structure. By employing simulation, a computer model was developed. Then, it was actuated by generating input data. The system's behaviour was then recorded.

Conveyors With Lost Arrivals

The cases, of two and three-channel conveyors with lost arrivals and no storage at any of the service channels, were studied first. Values of ρ_2 were kept constant while that of ρ_1 were increased by decreasing the service rate (μ) and keeping the arrival rate constant all through the simulation.

A G.P.S.S. flow chart for the two-channel conveyor is shown in Figure 26. The simulation was carried out for ten thousand (10,000), twenty thousand (20,000) and fifty thousand (50,000) transactions. It was found that the steady-state could be reached at less than ten thousnad (10,000) transactions. Accordingly, simulations for ten thousand (10,000) transactions were carried out for all different values of ρ_1 and ρ_2. The relationship, between the total number of entries to the channels and ρ_1 for different values of ρ_2, was plotted from the simulated results for both the two and three-channel conveyor systems, as shown in Figures 27 and 28.

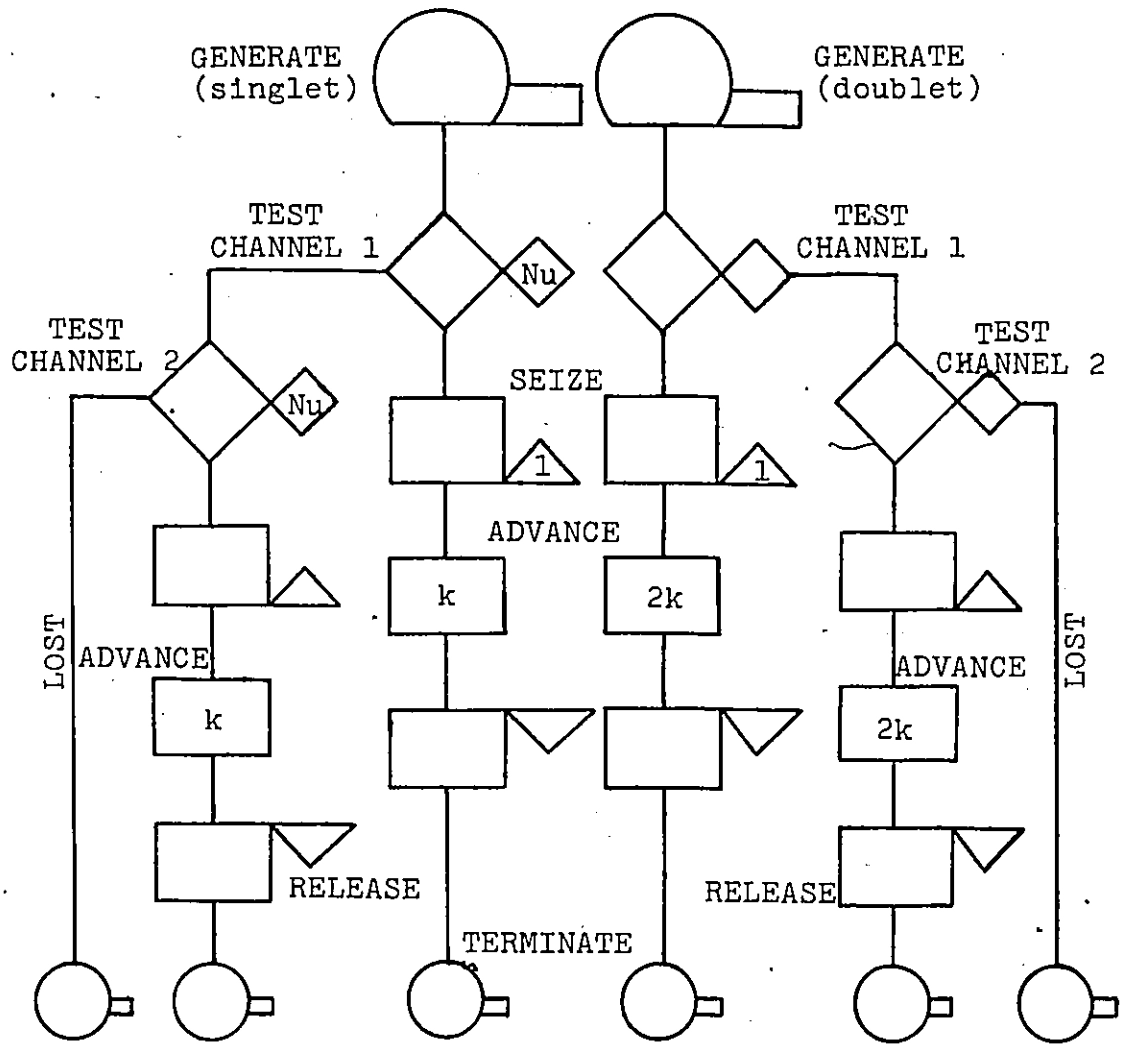

Figure 26. Simplified GPSS Flow Chart for a Two-Channel Ordered Entry Queueing System with Multiple-Poisson Input and No Storage at each Channel

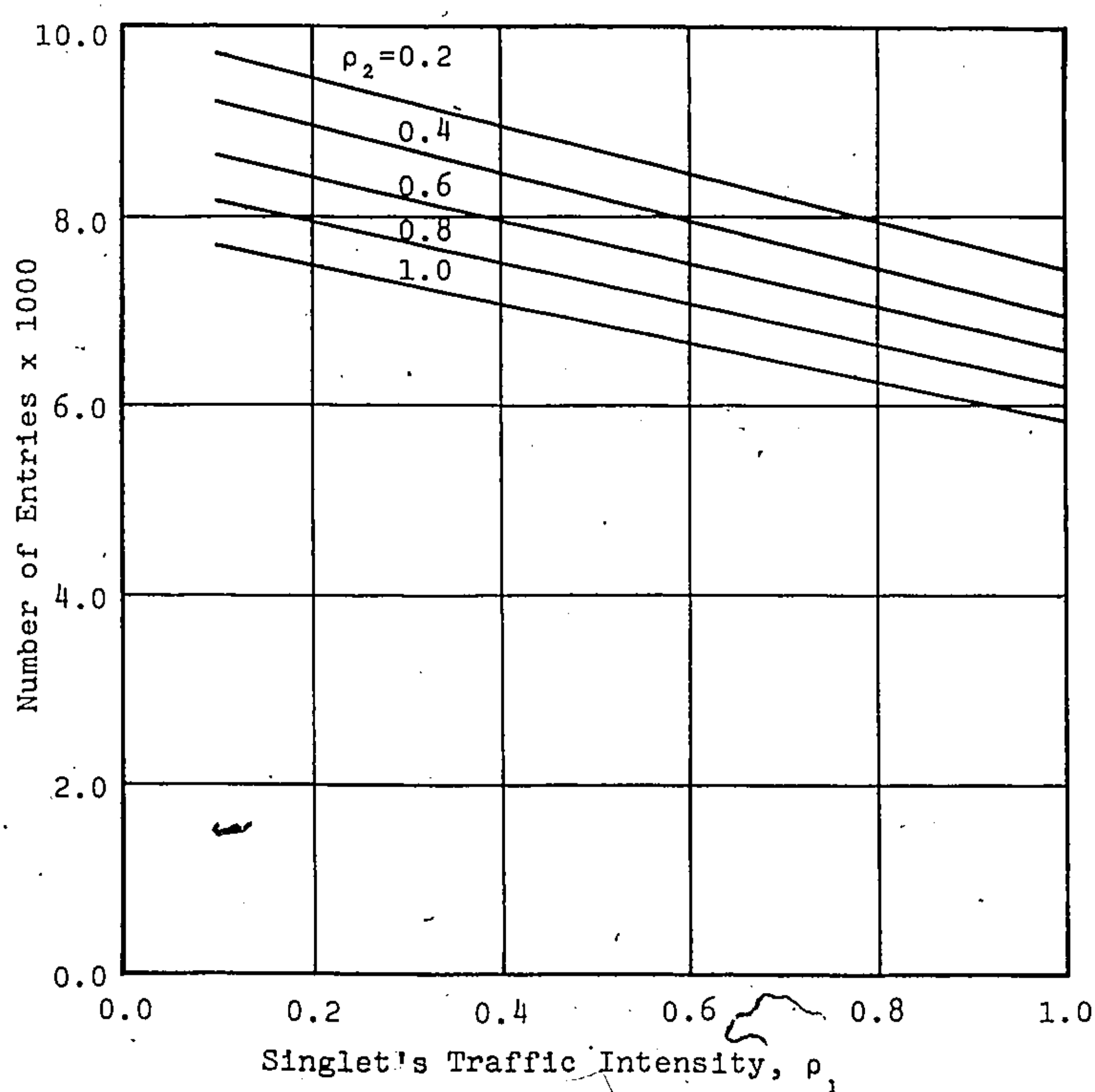

Figure 27. Number of entries to the first channel with ρ_1; ρ_2 fixed for the two-channel case.

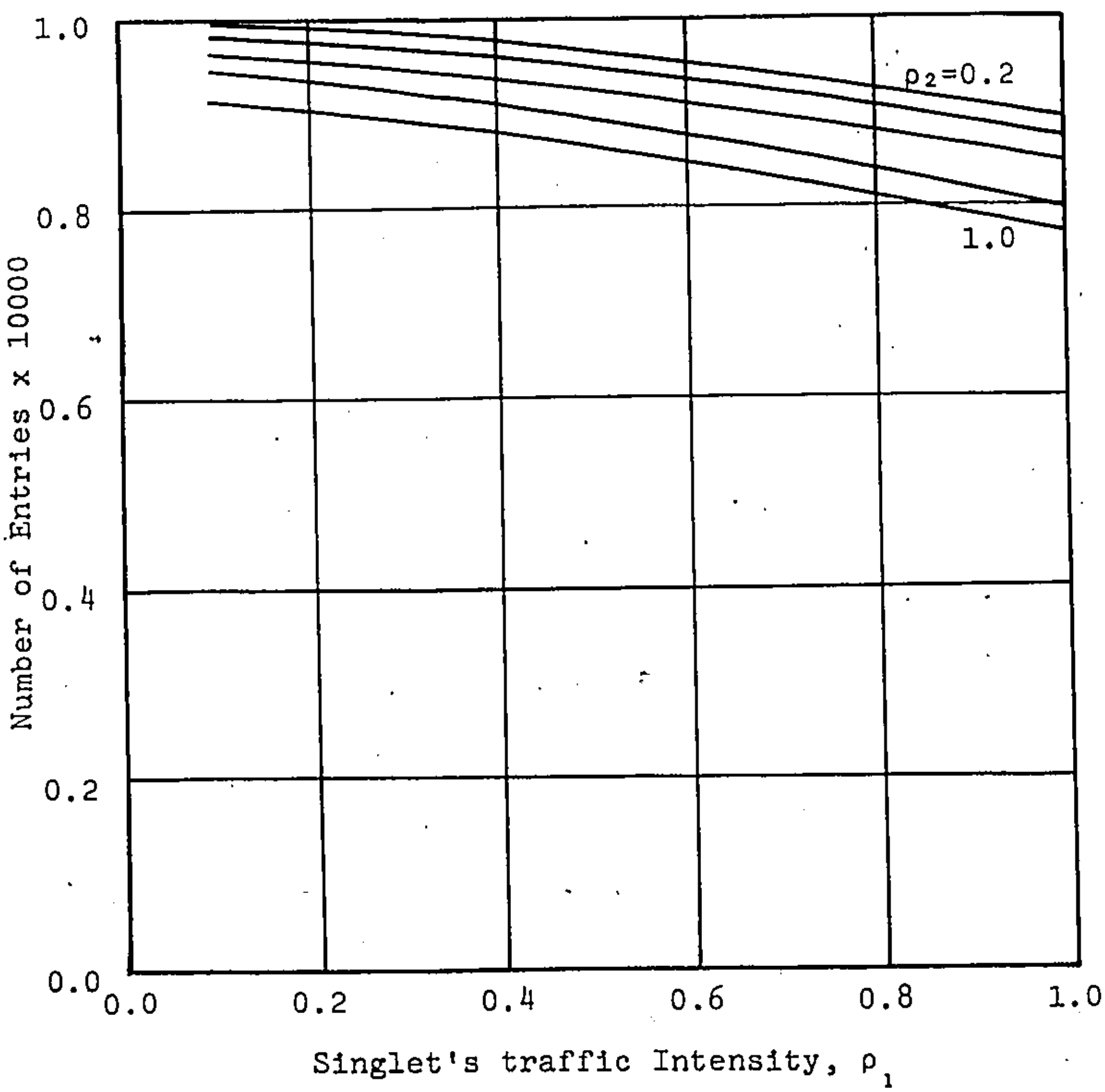

Figure 28. Number of entries to the first channel with ρ_1; ρ_2 fixed for the three channel case.

It is obvious, that the number of entries to the channels decrease with the increase of ρ_1. Also, the number of entries, per channel, decrease as the order of the channels succeed from the first; e.g. the number of entries to the first channel is greater than that of the second channel, and the number of entries to the second channel is greater than that of the third channel and so on.

The utilization of the service channels for different values of ρ_1, and ρ_2, for both cases of the two and three-channel conveyors, without storage is shown in Figures 29 and 30. It is apparent, that the utilization of any channel increases with the increase of ρ_1 and ρ_2. However, it should be noted, that the utilization of the first channel is always greater than the utilization of the second channel, and that of the second channel is greater than the utilization of the third channel, for the same values of ρ_1 and ρ_2. This is true for both the two and three-channel conveyor systems. A sample of the input data and the output results is shown in Table 1.

Conveyors With Storage

Additional simulation models were constructed to study the second case, where storage of different capacities were allowed at the service channels. Based on Disney and Phillips' results, it is

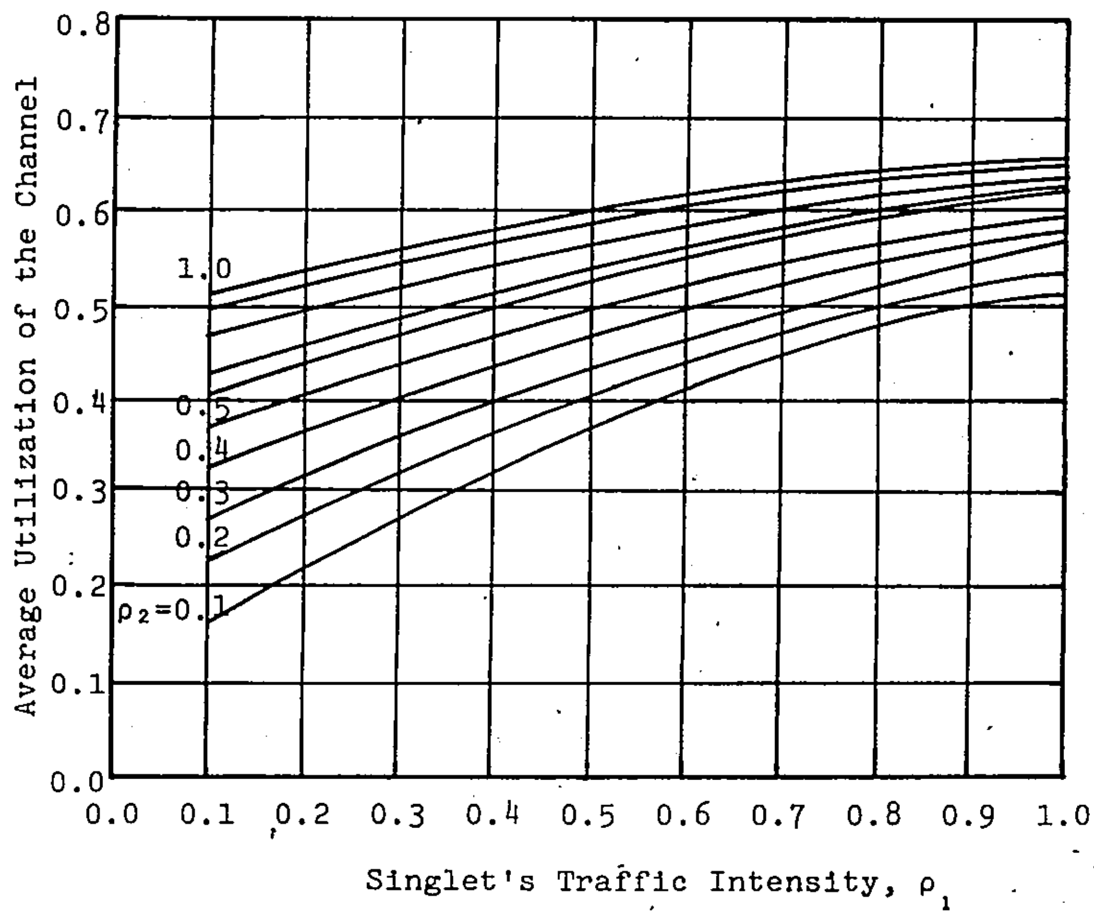

Figure 29. Utilization of the first service channel with ρ_1, ρ_2 fixed; two channel conveyor without storage.

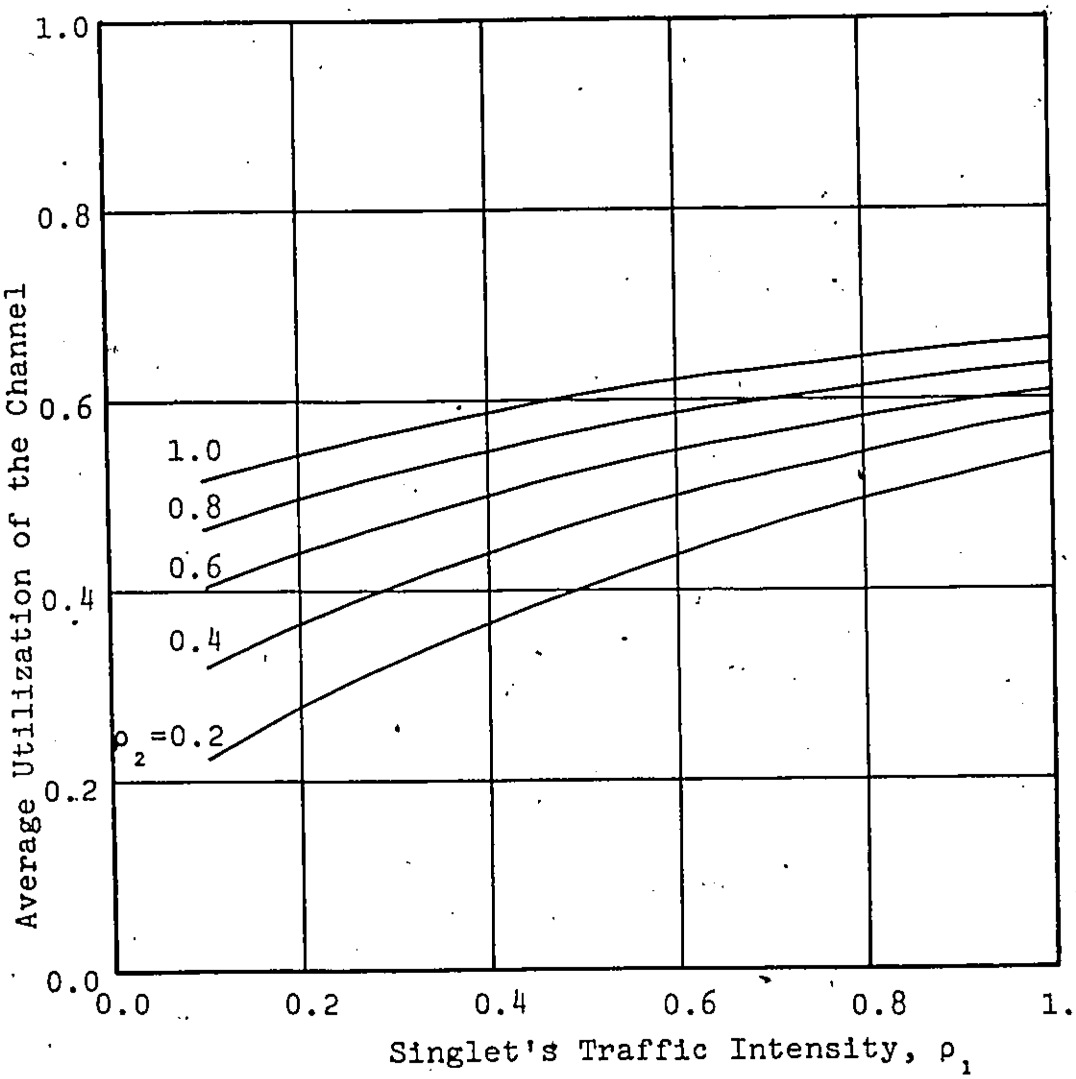

Figure 30. Utilization of the first service channel with ρ_1, ρ_2 fixed; three channel conveyor without storage.

TABLE 1

Singlet's interarrival time $1/\lambda_1$	Singlet's service time $1/\mu_1$	Singlet's traffic intensity ρ_1	Doublet's interarrival time $1/\lambda_2$	Doublet's service time $1/\mu_2$	Doublet's traffic intensity ρ_2	Utilization of the first channel	Utilization of the second channel	Utilization of the third channel	Simulated E[n] for 10,000 transactions	Theoretical Values of E[n]
100	10	0.1	200	20	0.1	0.161	0.030	0.002	9993	9966
100	20	0.2	400	40	0.1	0.218	0.056	0.007	9966	9928
100	30	0.3	600	60	0.1	0.283	0.089	0.015	9940	9873
100	40	0.4	800	80	0.1	0.335	0.121	0.031	9874	9801
100	50	0.5	1000	100	0.1	0.376	0.163	0.045	9822	9714
100	60	0.6	1200	120	0.1	0.409	0.197	0.071	9733	9613
100	70	0.7	1400	140	0.1	0.441	0.235	0.090	9647	9499
100	80	0.8	1600	160	0.1	0.469	0.262	0.117	9501	9375
100	90	0.9	1800	180	0.1	0.493	0.295	0.139	9414	9242
100	100	1.0	2000	200	0.1	0.523	0.331	0.160	9262	9102
100	10	0.1	100	20	0.2	0.225	0.060	0.008	9966	9873
100	20	0.2	200	40	0.2	0.283	0.092	0.018	9918	9801
100	30	0.3	300	60	0.2	0.329	0.125	0.033	9879	9714
100	40	0.4	400	80	0.2	0.363	0.160	0.047	9805	9613
100	50	0.5	500	100	0.2	0.408	0.197	0.065	9720	9499
100	60	0.6	600	120	0.2	0.445	0.230	0.089	9626	9375
100	70	0.7	700	140	0.2	0.463	0.262	0.111	9518	9242
100	80	0.8	800	160	0.2	0.491	0.289	0.136	9374	9102
100	90	0.9	900	180	0.2	0.520	0.316	0.160	9288	8957
100	100	1.0	1000	200	0.2	0.548	0.366	0.189	9057	8808

recommended to allocate storage at the last channel to achieve balance and maximum overall efficiency. Storages of different capacities were allocated at the service channel for the two-channel conveyor system. Effect of storage capacity on the percentage utilization of the channels is shown in Figures 31, 32, and 33. It seems that the percentage of utilization of the channels increases as the values of both ρ_1 and ρ_2 increase. Storage capacity does not seem to have any effect on the channel utilization, but the utilization of the storage decreases as their capacities increase according to the data in Figure 34.

Conveyors With Recirculation

Simulation models, to study the third case, were developed. In this case, where a two-channel closed-loop conveyor was considered, the arrival first checks Channel one (1), and if it is occupied, the arrival then checks Channel two (2). If it is also busy, the arrival then recirculates and enters the system as a new arrival and repeats the above procedure, until it receives service at either of the channels.

The objective of this analysis is to investigate the effect of the recirculation time on the percentage utilization of the channels. For fixed values of ρ_1 and ρ_2 and for different values of the recirculation

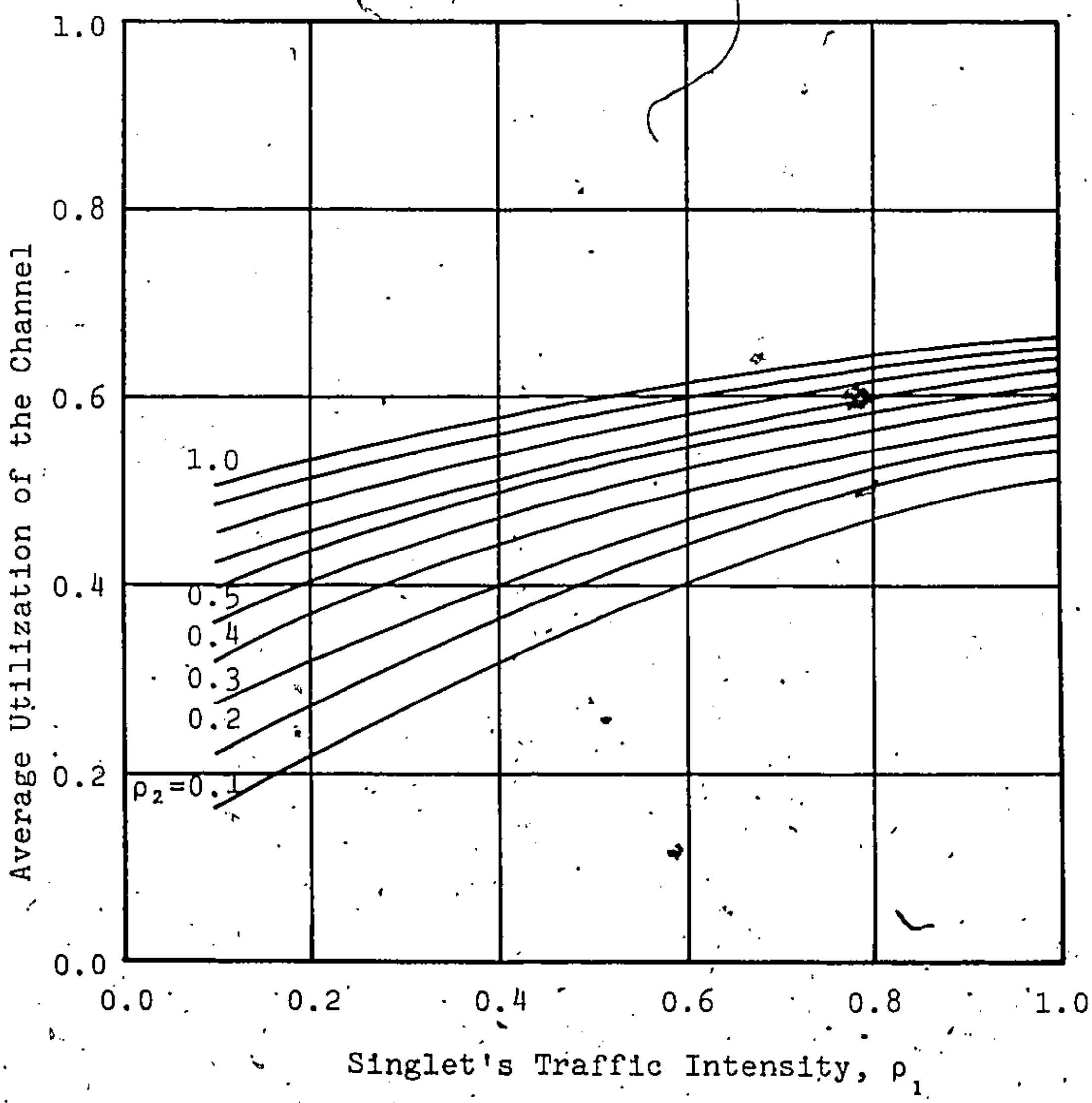

Figure 31. Effect of the traffic intensities on the average utilization of the first channel for a two-channel conveyor with a storage capacity of one(1) at the second channel.

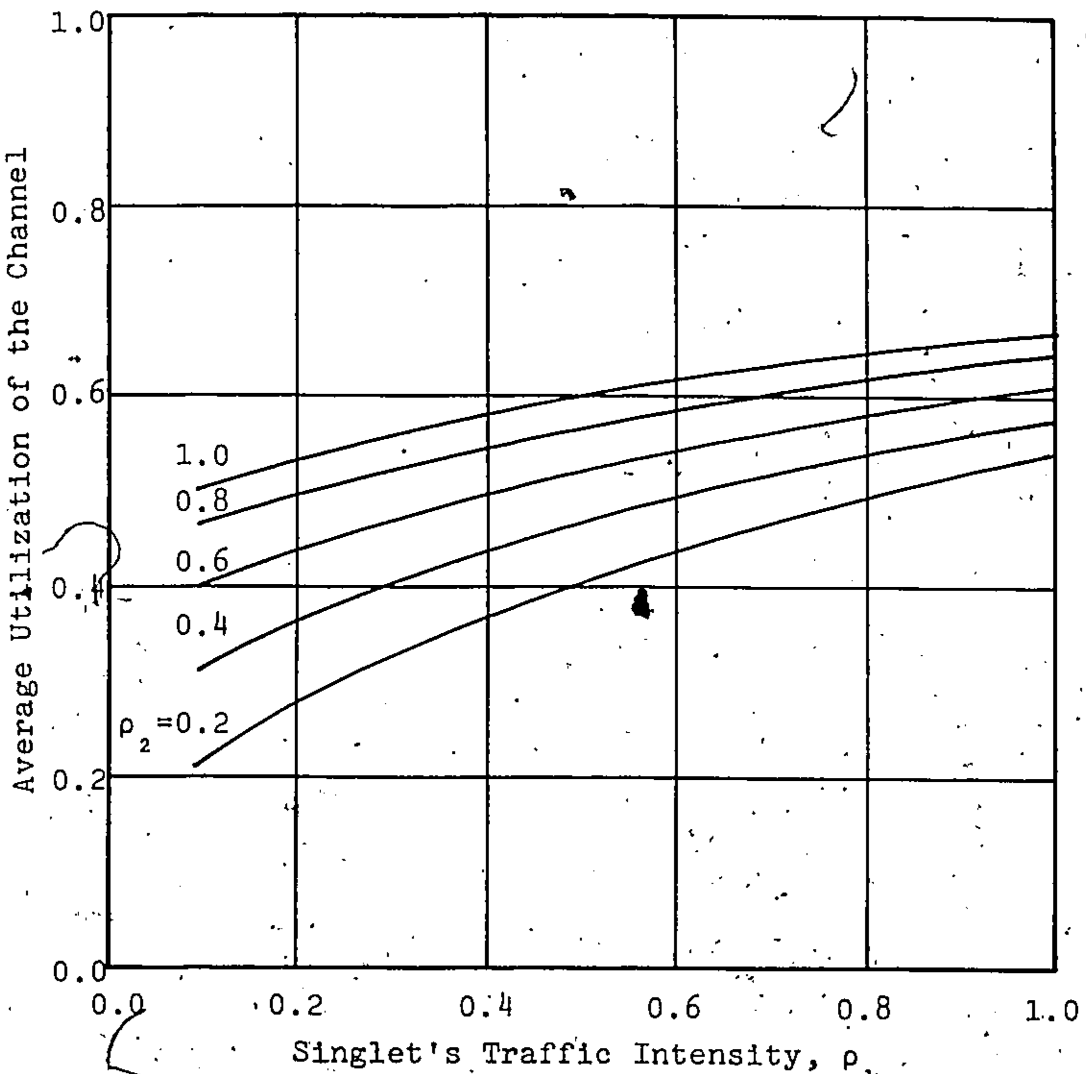

Figure 32. Effect of the traffic intensities on the average utilization of the first channel for a two-channel conveyor with a storage capacity of five(5) at the second channel.

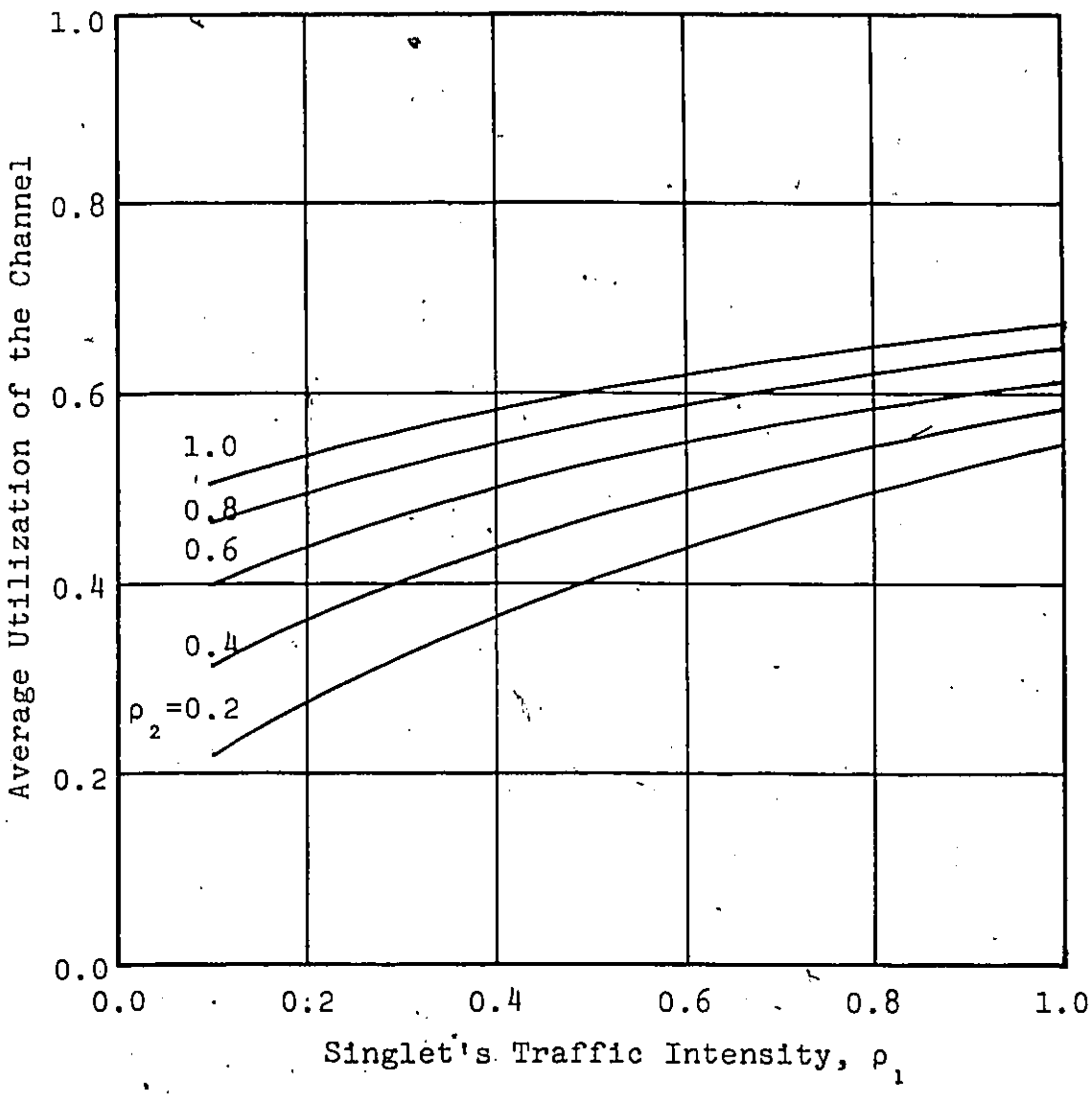

Figure 33. Effect of the traffic intensities on the average utilization of the first channel for a two-channel conveyor with a storage capacity of ten(10) at the second channel.

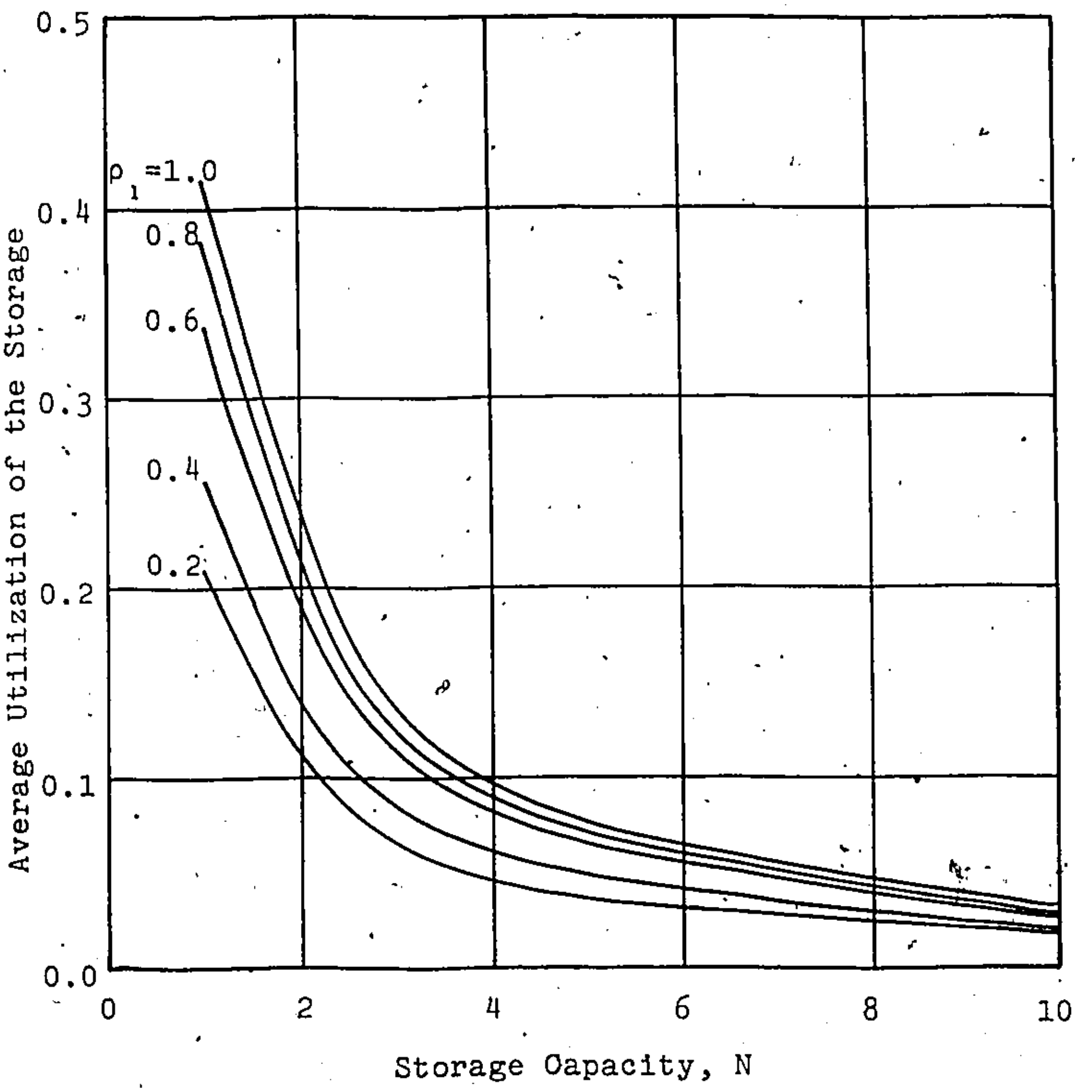

Figure 34. Relationship between the storage capacity and its utilization; ρ_2 fixed at 1.0.

time, the utilization of the second service channel is higher in the case when there is recirculation, than if there is lost arrivals. The two service channels have almost the same values of the percentage utilization. These values approach similar values with the increase of ρ_1 and p_2. Also, it seems that the recirculation time does not affect the utilization of the channel, as shown in Figures 35 and 36. A sample of these results is shown in Table 2.

Arrival Distributions of The Recirculated Units

It is difficult to determine mathematically, the distribution of the recirculated units. Therefore, the fourth case of simulation was considered to determine the distribution of the recirculated singlet and doublet units. A computer programme (see appendices) was developed and the times between the recirculated units, after they were denied service at the last channel, were recorded. Also, the frequency distribution curves of the recirculated units were plotted. It was found that the arrival rates of the recirculated units follow a Poisson distribution, with means different from the input Poisson distributions. The frequency distribution of the recirculated units is shown in Figure 37.

Transient-Solution of The Two-Channel Conveyor

The fifth case of simulation deals with a transient-

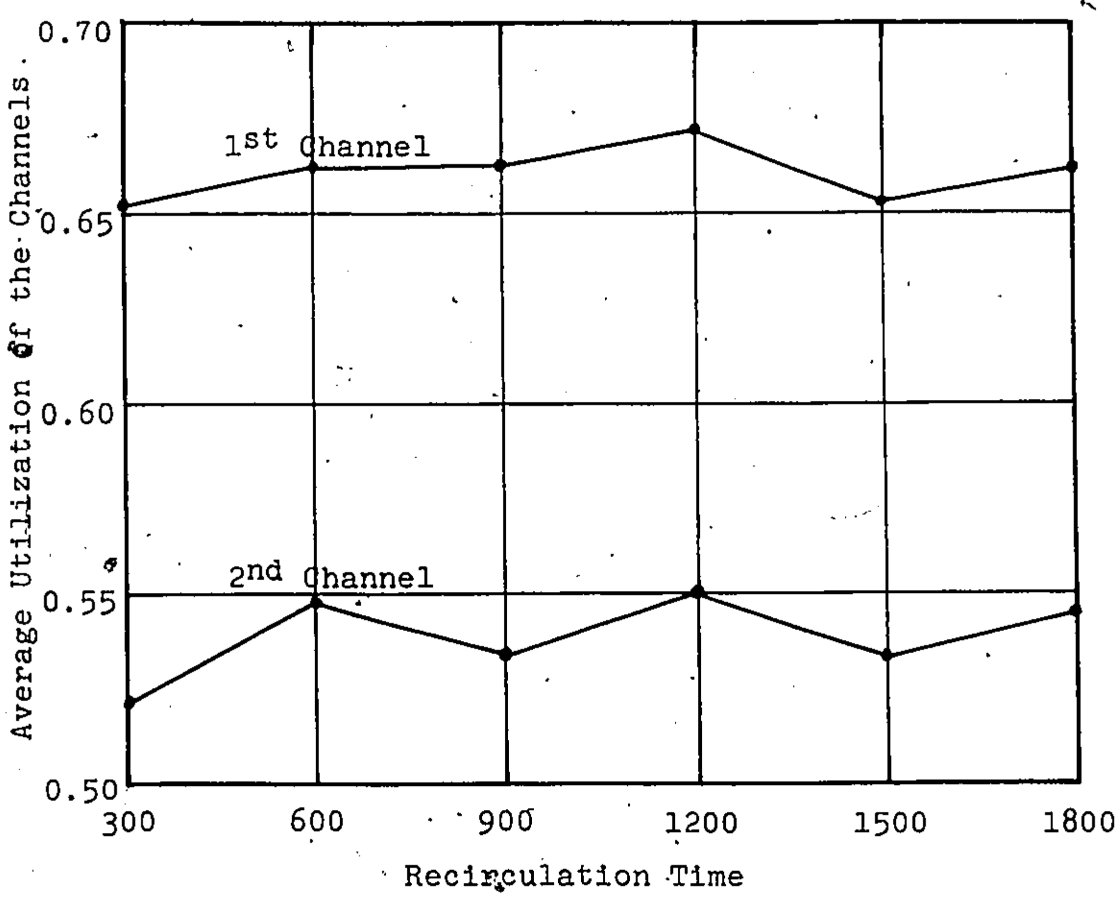

Figure 35. Effect of recirculation time on the utilization of the service channels, ρ_1 and ρ_2 fixed at 1.0 and 0.2, respectively.

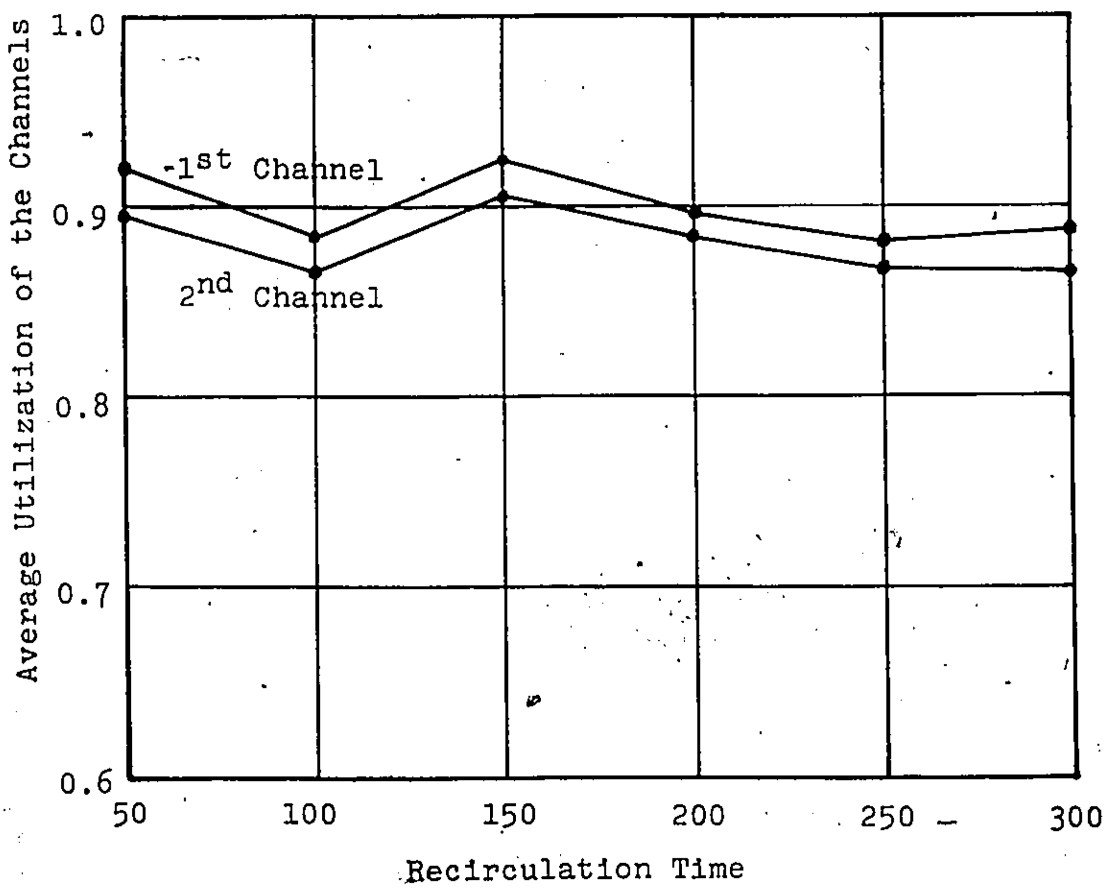

Figure 36. Effect of recirculation time on the utilization of the service channels, ρ_1 and ρ_2 fixed at 1.0 and 0.8, respectively.

TABLE 2

Two-Channel Conveyor With Recirculation

Recirculation time	Utilization of the first channel	Utilization of the second channel	Remarks
300 600 900 1200 1500 1800	0.651 0.668 0.665 0.670 0.658 0.665	0.522 0.549 0.538 0.552 0.532 0.545	where: $1/\lambda_1 = 100$ $1/\mu_1 = 100$ $\rho_1 = 1.0$ $1/\lambda_2 = 1000$ $1/\mu_2 = 200$ $\rho_2 = 0.2$
120 240 360 480 600 720	0.744 0.751 0.728 0.738 0.729 0.729	0.656 0.671 0.636 0.648 0.642 0.636	$1/\lambda_1 = 100$ $1/\mu_1 = 100$ $\rho_1 = 1.0$ $1/\lambda_2 = 500$ $1/\mu_2 = 200$ $\rho_2 = 0.4$
75 150 225 300 375 450	0.847 0.826 0.816 0.817 0.819 0.825	0.796 0.776 0.768 0.773 0.783 0.787	$1/\lambda_1 = 100$ $1/\mu_1 = 100$ $\rho_1 = 1.0$ $1/\lambda_2 = 330$ $1/\mu_2 = 200$ $\rho_2 = 0.6$
50 100 150 200 250 300	0.917 0.887 0.918 0.899 0.887 0.889	0.893 0.867 0.908 0.886 0.867 0.867	$1/\lambda_1 = 100$ $1/\mu_1 = 100$ $\rho_1 = 1.0$ $1/\lambda_2 = 250$ $1/\mu_2 = 200$ $\rho_2 = 0.8$

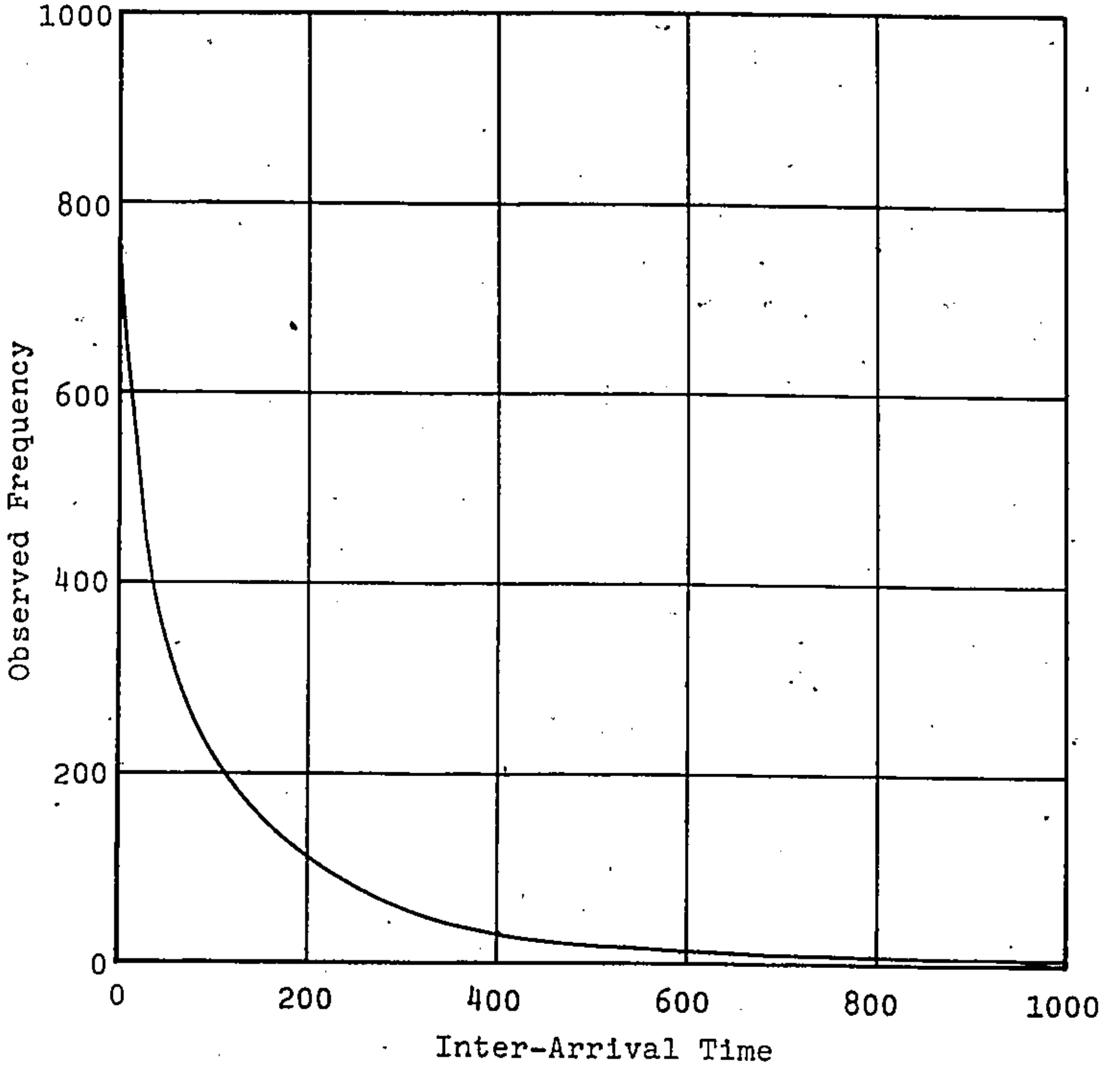

Figure 37. Observed frequency and the arrival time of the recirculated units.

solution of the two-channel conveyor without storage at any of the channels, allowing multiple Poisson inputs. Homogeneous servers were allowed at the service channels. The time-dependent equations can be derived as follows:

$$\frac{dP_{00}(t)}{dt} = -(\lambda_1+\lambda_2).P_{00}(t) + \mu.P_{10}(t) + \frac{\mu}{\phi}.P_{01}(t) \quad ..... \quad VII - 1$$

$$\frac{dP_{01}(t)}{dt} = -(\lambda_1+\lambda_2+\frac{\mu}{\phi}).P_{01}(t) + 2\frac{\mu}{\phi}.P_{02}(t) + \mu.P_{11}(t) + \lambda_2.P_{00}(t) \quad VII - 2$$

$$\frac{dP_{02}(t)}{dt} = -2\frac{\mu}{\phi}.P_{02}(t) + \lambda_2.P_{01}(t) \quad VII - 3$$

$$\frac{dP_{10}(t)}{dt} = -(\lambda_1+\lambda_2+\mu).P_{10}(t) + 2\mu.P_{20}(t) + \frac{\mu}{\phi}.P_{11}(t) + \lambda_1.P_{00}(t) \quad VII - 4$$

$$\frac{dP_{11}(t)}{dt} = -(\frac{\phi+1}{\phi})\mu.P_{11}(t) + \lambda_2.P_{10}(t) + \lambda_1.P_{01}(t) \quad ..... \quad VII - 5$$

$$\frac{dP_{20}(t)}{dt} = -2\mu.P_{20}(t) + \lambda_1.P_{10}(t) \quad VII - 6$$

Dividing the above equations by μ and using the matrix notations, equations VII - 1 to VII - 6 can be written in the following form:

$$(1/\mu).\dot{P} = A.P \qquad \text{VII - 7}$$

$\dot{P}$ is a column vector. Its elements are:

$$\begin{vmatrix} \dot{P}_{00}(t) \\ \dot{P}_{01}(t) \\ \dot{P}_{02}(t) \\ \dot{P}_{10}(t) \\ \dot{P}_{11}(t) \\ \dot{P}_{20}(t) \end{vmatrix}$$

P is another column matrix. Its elements are:

$$\begin{vmatrix} P_{00}(t) \\ P_{01}(t) \\ P_{02}(t) \\ P_{10}(t) \end{vmatrix}$$

$$\begin{vmatrix} P_{11}(t) \\ P_{20}(t) \end{vmatrix}$$

and A is a 6x6 square matrix. Its elements are:

$$\begin{vmatrix} -(\rho_1+\rho_2) & \frac{1}{\phi} & 0 & 1 & 0 & 0 \\ \rho_2 & -(\rho_1+\rho_2+\frac{1}{\phi}) & \frac{2}{\phi} & 0 & 1 & 0 \\ 0 & \rho_2 & -\frac{2}{\phi} & 0 & 0 & 0 \\ \rho_1 & 0 & 0 & -(\rho_1+\rho_2+1) & \frac{1}{\phi} & 2 \\ 0 & \rho_1 & 0 & \rho_2 & -(\frac{\phi+1}{\phi}) & 0 \\ 0 & 0 & 0 & \rho_1 & 0 & -2 \end{vmatrix}$$

The analytical solution of equation VII - 6 is feasible. Since we are interested in the numerical solution, a computer programme (see appendices) was run to calculate the values of the probabilities and to plot the time for the different values of ρ_1 and ρ_2. Let $\phi=2$, i.e. the service time of the doublet unit is twice that of the singlet unit. Samples of the transient-solutions are shown in Figures 38 and 39. It is important to note, that the time needed to reach the steady-state is reduced by increasing the values of the traffic intensities ρ_1 and ρ_2.

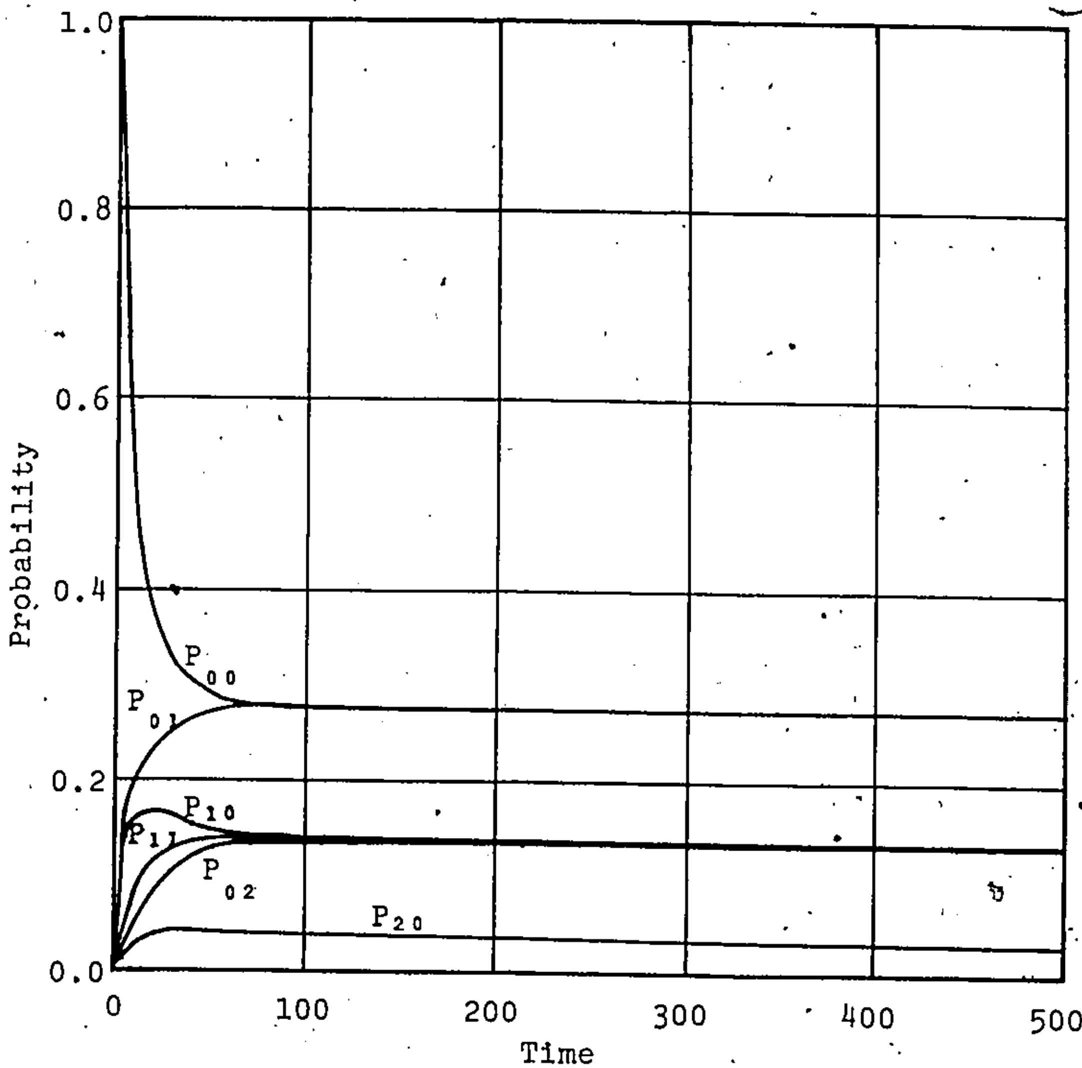

Figure 38. Transient solution of the probabilities for the two-channel conveyor with lost arrivals; $\rho_1 = \rho_2 = 0.5$.

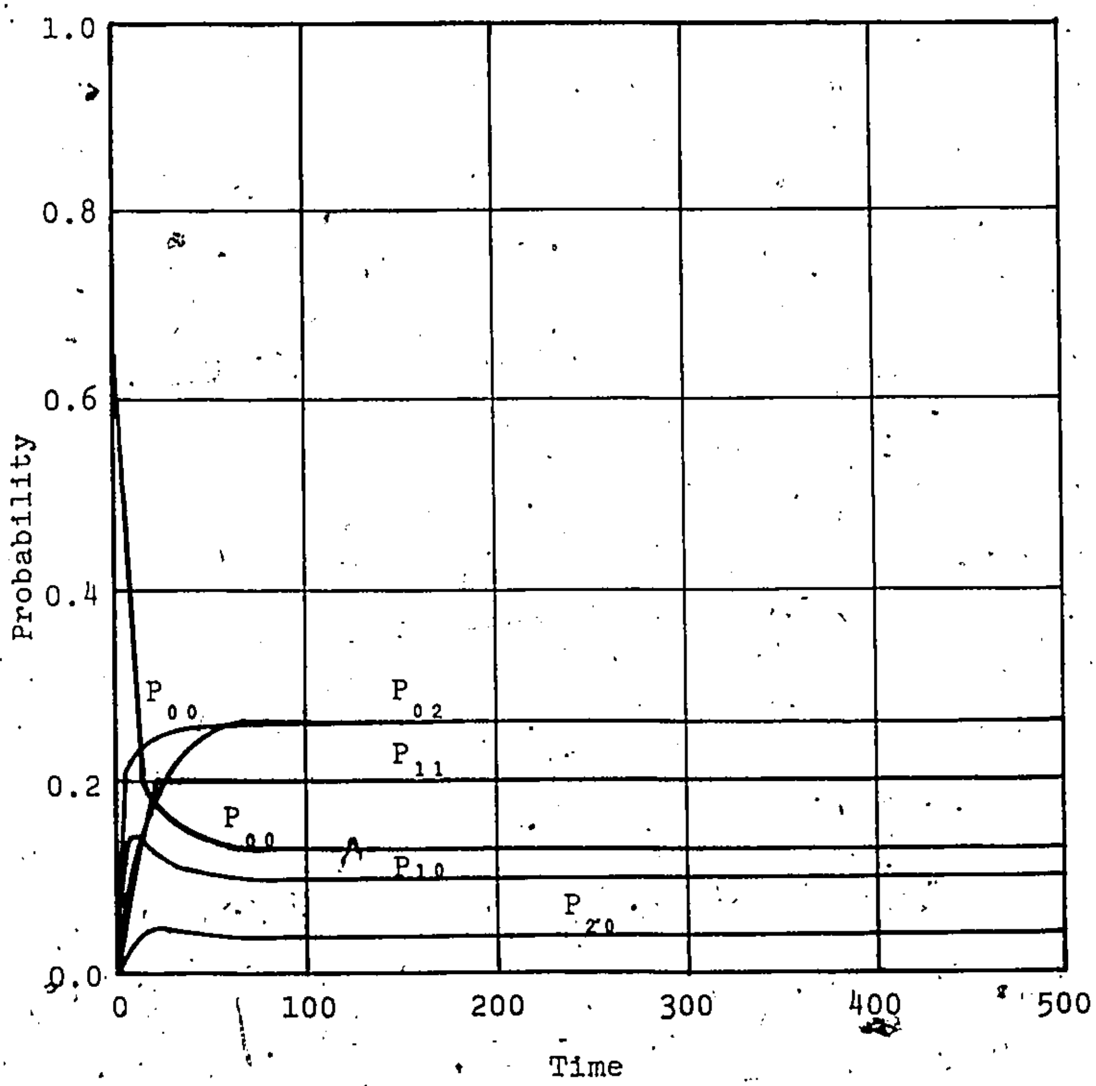

Figure 39. Transient solution of the probabilities for the two-channel conveyor with lost arrivals; $\rho_1 = \rho_2 = 1.0$

Comparison Between the Theoretical and Simulated Results

Figures 40a and b show the relationship between the traffic intensities and the probability of an arrival having no wait prior to service for the two and three channel conveyors with homogeneous servers.

The values of ρ_2 (doublet's traffic intensity) are kept constant, while the vlaues of ρ_1 (singlet's traffic intensity) are increased by an increment of 0.1.

From the graphs, it is apparent that there is an agreement between the theoretical and simulated results. Also, the simulated results possess higher values than the theoretical ones. One explanation , of this, is that this difference is due to the nature of the simulation analysis; when calling the random numbers to generate values for the arrival and service rates. These values of the random numbers do not fall in the same range as those values from theoretically assumed distributions.

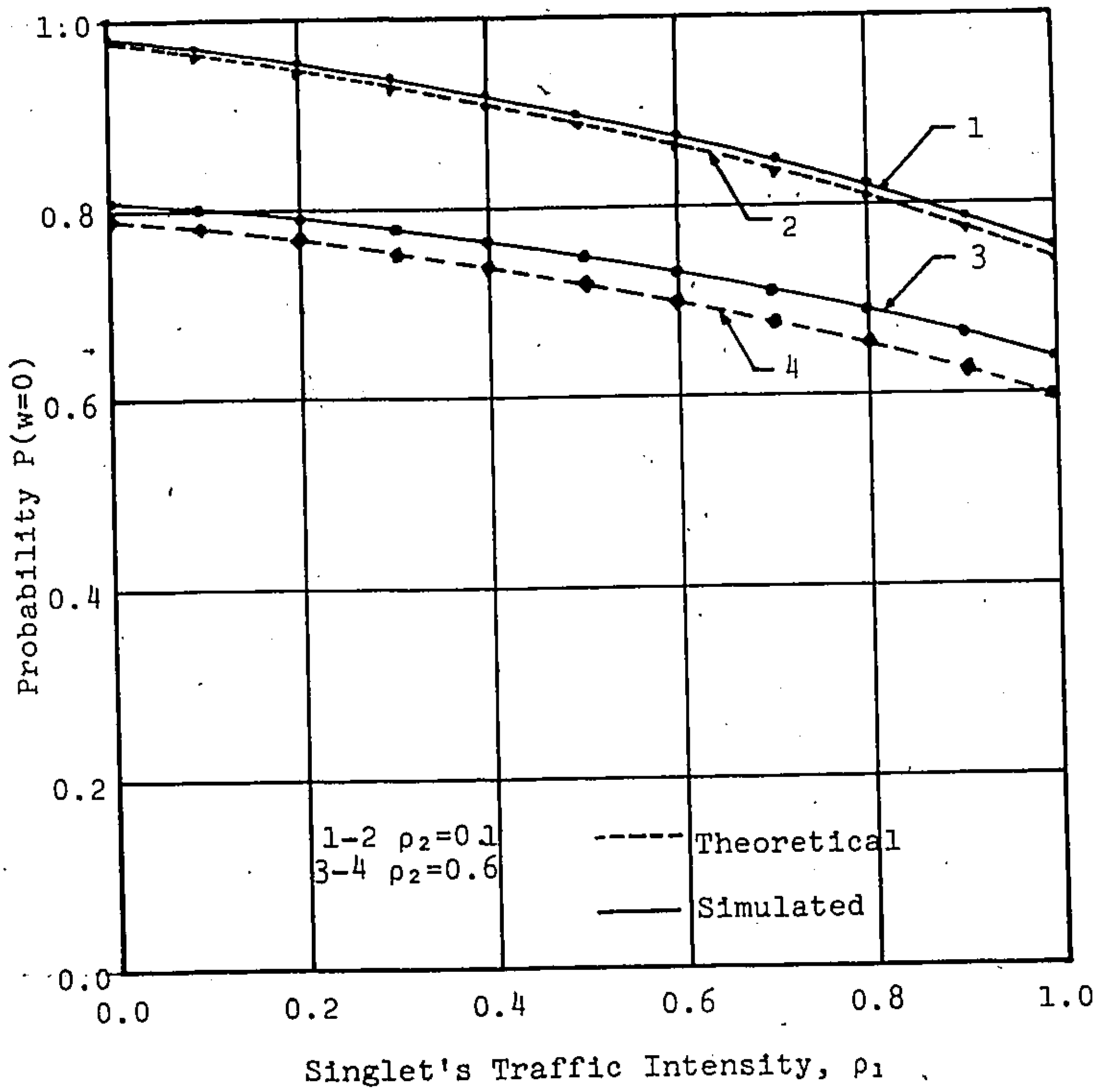

Figure 40a. Relationship between P(w=0) and $\ddot{\rho}_1$, for both the simulated and theoretical results. A three-channel conveyor with homogeneous servers.

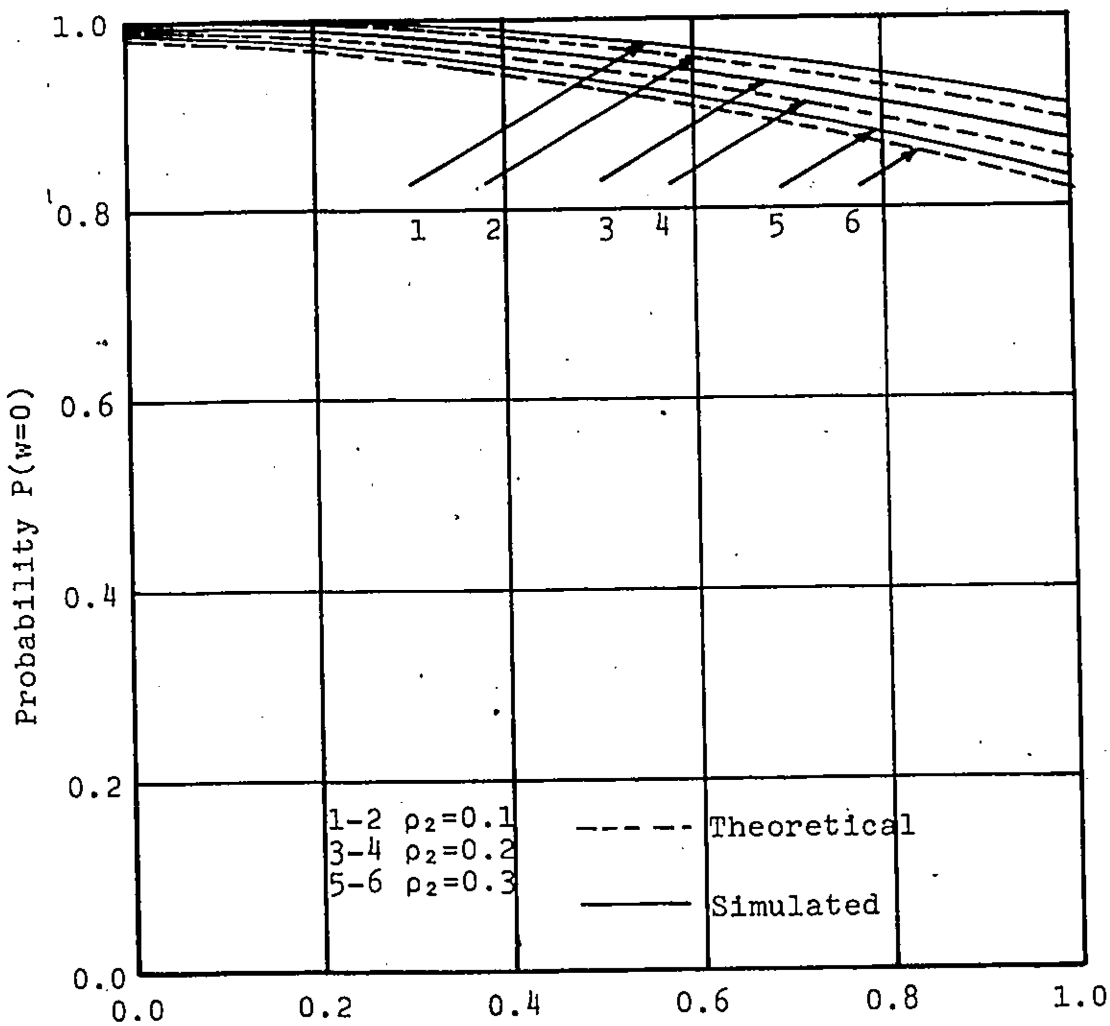

Figure 40b. Relationship between $P(w=0)$ and ρ_1 for both the simulated and theoretical results. A three-channel conveyor with homogeneous servers.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

This research investigated the multi-item, multi-loading and multi-unloading conveyor systems under various conditions. The analyses and findings of this research can be summarized as follows:

1. Conveyor systems with lost arrivals: These conveyors are without storage at any of the channels, with either homogeneous or heterogeneous servers. It was found that the probability of the system being idle can be minimized, by setting the doublet's traffic intensity at a high level, while increasing the value of the singlet's traffic intensity until the level of probability specified by the production system is achieved. This will also maximize the expected number of units in the system. The reverse, however, is not as efficient.

As the number of service channels increases, the performance of the system increases. However, there is a certain value of the number of channels beyond which the performance of the system does not seem to be affected.

Computer programmes were developed to solve the steady-state probability equations and to determine the values of these probabilities for any number of channels (M) and for any value of the traffic intensities ρ_1 and ρ_2.

2. Conveyor systems with recirculation: These conveyors are without storage at any of the channels with either homogeneous or heterogeneous servers. Arrivals that are denied service at the service channels are allowed to recirculate. It was found that an increase of the percentage of the recirculated singlet or doublet units leads to an improvement in the system's performance.

3. Conveyor systems with storage: Storage was allocated at the last channel where homogeneous servers were allowed. It was found that as the storage capacity increases, the system's performance is more efficient.

The findings of this research suggest a number of conclusions:

1. This research has clearly demonstrated the feasibility of solving the multi-item, multi-channel conveyor systems through the application of queueing theory.

2. Closed-loop conveyor systems with multiple-inputs are more efficient than those with a single input.

3. Allowing units on the conveyor system with a high service time ratio (the service time of the

doublet unit to that of the singlet) will lead to improvement in the system's performance.

4. Allocating storage at the last channel will result in a more efficient conveyor system, than allowing the lost arrivals to recirculate. Generally, closed-loop conveyors with storage at the last channel are more efficient than those without storage.

5. Increasing the number of service channels leads to an improvement in the performance of the system. However, there is a certain value of the number of channels beyond which the performance does not seem to be affected.

6. Traffic intensity of the doublet arrivals has a significant effect on the performance of the system. As the doublets traffic intensity increases, the performance of the system will improve.

7. In the two-channel conveyor systems, the utilization of the second service channel increases by allocating storage at that channel.

8. The utilization of the first service channel for the closed-loop conveyor system with lost arrivals is always greater than the utilization of the second channel, and that of the second channel is greater than the utilization of the third channel, and so on.

9. The utilization of the first service channel for the closed-loop conveyor system with storage at

the second channel, is independent of the capacity of the storage.

10. Allowing recirculation will improve the system's performance. However, the recirculation time has no effect on the utilization of the service facilities.

11. The findings are relevant for conveyor designers as they attempt to determine the optimal parameters of the conveyor that can maximize the performance and efficiency under given cost constraints.

Suggestions For Future Research

There are many facets of the ordered entry conveyor serviced queueing systems which need to be analyzed. The following areas are suggested for future research:

1. Solution of M-channel closed-loop conveyor systems with multiple Poisson input where storage is allowed at each channel;

2. M-channel closed-loop conveyor systems with more than two types of arrivals where homogeneous or heterogeneous servers are allowed at the service channels;

3. Analytical analysis for the distribution of the recirculated units;

4. Solution of the above cases with different

arrival distributions other than Poisson arrivals;

5. Applying queueing theory to analyze conveyor systems with multiple inputs other than the closed-loop conveyor systems; and

6. Cost analysis involving the relationships which exist between the addition of extra servers, extra storage and allowing recirculation on the system's performance.

APPENDIX A

This appendix consists of computer programmes which study and simulate the closed-loop conveyor system.

Appendix A is composed of the following programmes:

FIGURE 41

```
C      CLOSED LOOP CONVEYOR SYSTEM WITH M CHANNELS
C      MEASURES OF PERFORMANCE
       DIMENSION A(15),B(15),P00(15),EN(15),PRW0(15),
      1FAC(15)
       DIMENSION P00S(20,20,20),ENS(20,20,20),PRW0S(20,
      1,20,20)
       DIMENSION MM1(20,20,20),Y(20),X(20)
       AMOVE=-1.
       CALL PLOTID('SAYED ABDELRAZIK','U14900X298')
       CALL NLIMIT(200.)
       FAC(1)=1.
       DO 100 II=2,14
100    FAC(II)=FAC(II-1)*II
       A(1)=1.0
       B(1)=0.
       DO 50 I=4,20,4
       PS=I
       R2=PS/20.
       DO 70 J=4,20,4
       QS=J
       R1=QS/20
       WRITE(6,60)R1,R2
60     FORMAT(6X,'TRAFFIC INTENSITY R1=',F10.5,6X,'TRAFFIC
      1INTENSITY R2=',F10.5)
       C=R1+2*R2
       DO 10 M=2,13
       ID = M -1
       A(M)=A(M-1)+(1/(FAC(M-1))*(C**ID))
       IF(M.EQ.2)GOTO 25
       B(M)=B(M-1)+(1./FAC(M-2))*(C**ID)
       IF(M.GT.2)GOTO 10
25     B(M)=B(M-1)+C**ID
10     CONTINUE
       DO 30 M1=2,12
       P00(M1)=1./A(M1+1)
       EN(M1)=B(M1+1)*P00(M1)
       PRW0(M1)=A(M1)*P00(M1)
       WRITE(6,40)M1,P00(M1),EN(M1),PRW0(M1)
40     FORMAT(6X,'NUMBER OF THE CHANNELS=',I2,6X,'P00=',
      1F10.7,6X,'EN=',F10.7,6X,'PRW0=',F10.7)
       P00S(I,J,M1)=P00(M1)
       ENS(I,J,M1)=EN(M1)
       PRW0S(I,J,M1)=PRW0(M1)
       MM1(I,J,M1)=M1
30     CONTINUE
70     CONTINUE
       A(1)=0.0
```

```
      B(1)=0.0
50    CONTINUE
      MM=0
      DO 901 I=4,20,4
      DO 90  J=4,20,4
      DO 91  M1=2,12
      Y(M1-1)=POOS(I,J,M1)
      X(M1-1)=MM1(I,J,M1)
91    CONTINUE
      CALL CALCO2(X,Y,13,AMOVE,6.,5.,0.,2.,0.,0.2,0,1,2)
      YY=Y(I)/Y(13)
      XX=X(I)/X(13)
      IF(MM.NE.0)GOTO 111
      CALL SYMBOL(XX,YY,0.14,3HR2=,0.,3)
      MM=MM+1
111   CONTINUE
      AMOVE=0.
90    CONTINUE
      AMOVE=10.
901   CONTINUE
      AMOVE=15.
      MM=0
      DO 920 I=4,20,4
      DO 92  J=4,20,4
      DO 93  M1=2,12
      Y(M1-1)=ENS(I,J,M1)
      X(M1-1)=MM1(I,J,M1)
93    CONTINUE
      CALL CALCO2(X,Y,13,AMOVE,6.,5.,0.,2.,0.,1.0,0,1,2)
      YY=Y(I)/Y(13)
      XX=X(I)/X(13)
      IF(MM.NE.0)GOTO 112
      CALL SYMBOL(XX,YY,0.14,3HR2=,0.,3)
      MM=MM+1
112   CONTINUE
      AMOVE=0.
92    CONTINUE
      AMOVE=10.
920   CONTINUE
      AMOVE=10.
      MM=0
      DO 940 I=4,20,4
      DO 94  J=4,20,4
      DO 95  M1=2,12
      Y(M1-1)=PRWOS(I,J,M1)
      X(M1-1)=MM1(I,J,M1)
95    CONTINUE
      CALL CALCO2(X,Y,13,AMOVE,6.,5.,0.,2.,0.,0.2,0,1,2)
      YY=Y(I)/Y(13)
      XX=X(I)/X(13)
      IF(MM.NE.0)GOTO 113
```

```
      CALL SYMBOL(XX,YY,0.14,3HR2=,0.,3)
      MM=MM+1
113   CONTINUE
      AMOVE=0.
94    CONTINUE
      AMOVE=10.
940   CONTINUE
      CALL PLTEND(15.)
      STOP
      END
```

FIGURE 42

```
C         CLOSED LOOP CONVEYOR SYSTEM HAVING MORE THAN TWO
C         INPUT SOURCES WITH STORAGE OF CAPACITY M AT THE
C         SECOND CHANNEL .
          DIMENSION A(30,30),B(30,1),AA(900)
          DO 10 M=1,20
          M1=M+1
          N=M1*2+2
          WRITE (6,20) M,N
20        FORMAT(6X,'STORAGE CAPACITY M=',I2,6X,'NUMBER OF
         1EQUATIONS N=',I2)
          S=5
          DO 15 I=1,S
          R(I)=I/20.0
          BSS(1)=0.0
15        BSS(I)=BSS(I)+R(I)
          BS=BSS(S)
          R1(1)=0.05
          DO 16 J=2,S
          SR=J
          R2(J)=J/20.0
          PHI(J)=S
16        ASS(J)=ASS(J)+R2(J)/PHI(J)
          AS=ASS(S)/BS
          WRITE(6,35)BS,AS
35        FORMAT(6X,'BS=',F10.5,6X,'AS=',F10.5)
          DO 40 II=1,N
          DO 40 JJ=1,N
          A(II,JJ)=0.0
40        B(II,1)=0.0
          A(1,1)=-BS
          A(1,2)=AS
          A(1,M+3)=AS
          MK=M+1
          DO 50 K=2,MK
          A(K,K)=-(BS+AS)
          A(K,K+M+2)=AS
          A(K,K+1)=AS
50        CONTINUE
          A(M+2,M+2)=-(BS+AS)
          A(M+2,N)=AS
          A(M+3,M+3)=-(BS+AS)
          A(M+3,M+4)=AS
          A(M+3,1)=BS
          DO 60 IK=1,M
          A(IK+M+3,IK+M+3)=-BS+2*AS)
          A(IK+M+3,IK+M+4)=AS
          A(IK+M+3,IK+M+2)=BS
```

```
      A(IK+M+3,IK+1)=BS
60    CONTINUE
      A(2*M+4,N)=-2*AS
      A(2*M+4,M+2)=BS
      A(2*M+4,N-1)=BS
      DO 70 ILS=1,N
      A(N,ILS)=1.0
      B(ILS,1)=0.0
70    CONTINUE
      B(N,1)=1.0
      WRITE(6,80)
80    FORMAT(6X,'PROBABILITY MATRIX')
      WRITE(6,90)((A(IH,JH),JH=1,N),IH=1,N)
90    FORMAT(6X,6(F15.5))
      NN=N*N
      MNF=0
      DO 2000 I=1,N
      DO 2000 J=1,N
      AA(MNF+1)=A(J,I)
      MNF=MNF+1
2000  CONTINUE
      CALL SIMQ(AA,B,N,KS)
      WRITE(6,250)(B(I,1),I=1,N)
250   FORMAT(6X,'*** SOLUTION ***',6(F15.9))
      X=0.
      KI=M+1
      DO 254 I=1,KI
      X=X+B(I+1,1)*I
254   CONTINUE
      Y=0.
      JIK=M+2
      DO 256 I=1,JIK
      Y=Y+B(1+M+2,1)*I
256   CONTINUE
      EN=X+Y
      WRITE(6,22) EN
22    FORMAT(6X,'********* EXPECTED NUMBER OF UNITS ***
     1*****=',F15.7)
      PRINT 900
900   FORMAT(6X,'***** END OF CASE *****')
      PRINT 24
24    FORMAT('*************************************')
10    CONTINUE
      STOP
      END
```

FIGURE 43

```
      EXTERNAL VECTOR
      DIMENSION YY(1000),XX(1000),ZZ(1000),RR(1000),
     1SS(1000),UU(1000),SUM(1000),TT(1000)
      REAL *8X(25),XDOT(25)
      COMMON R1,R2
      REAL PLT(200)
      NN=0.
      AMOVE=-1.
      CALL PLOTID('SAYED ABDELRAZIK','G14900U332')
      CALL NLIMIT(350.)
      N=6
      H=0.09
      T=0.0
      DO 16 JJ=10,20,10
      R2=JJ/20.
      DO 15 II=10,20,5
      R1=II/20
      WRITE(6,50)R1,R2
50    FORMAT(5X,'R1=',F12.5,5X,'R2=',F12.5)
      X(1)=1.0
      X(2)= 0.0D0
      X(3)= 0.0D0
      X(4)= 0.0D0
      X(5)= 0.0D0
      X(6)= 0.0D0
      DO 1 I=1,500
      TT(I)=T
      CALL RKINT(T,X,N,G,VECTOR)
      WRITE(6,44)T,X(1),X(2),X(3),X(4),X(5),X(6),SUM(I)
44    FORMAT(8(1X,E15.7))
      XX(I)=X(1)
      YY(I)=X(2)
      ZZ(I)=X(3)
      RR(I)=X(4)
      SS(I)=X(5)
      UU(I)=X(6)
1     CONTINUE
      CALL CALCO2(TT,XX,502,AMOVE,5.0,5.0,0.,100.,0.,
     10.2,0,1,2)
      TT1=TT(50)*(1./TT(502))
      XX1=XX(50)*(1./XX(502))
      CALL SYMBOL (TT1,XX1,0.14,3HP00,0.0,3)
      YY(501)=XX(501)
      YY(502)=XX(502
      CALL CALCO2(TT,YY,502,0.00,5.0,5.0,0.,100.,0.,0.2,
     10,1,2)
      TT2=TT(100)*(1./TT(502))
      YY2=YY(100)*(1./YY(502))
      CALL SYMBOL (TT2,YY2,0.14,3HP01,0.0,3)
      ZZ(501)=YY(501)
      ZZ(502)=YY(502)
```

```
      CALL CALCO2(TT,ZZ,502,0.00,5.0,5.0,0.,100.,0.,0.2,
     10,1,2)
      TT3=TT(150)*(1./TT(502))
      ZZ3=ZZ(150)*(1./ZZ(502))
      CALL SYMBOL (TT3,ZZ3,0.14,3HP02,0.0,3)
      RR(501)=ZZ(501)
      RR(502)=ZZ(502)
      CALL CALCO2(TT,RR,502,0.00,5.0,5.0,0.,100.,0.,0.2,
     10,1,2)
      TT4=TT(200)*(1./TT(502))
      RR4=RR(200)*(1./RR(502))
      CALL SYMBOL(TT4,RR4,0.14,3HP10,0.,3)
      SS(501)=RR(501)
      SS(502)=RR(502)
      CALL CALCO2(TT,SS,502,0.00,5.0,5.0,0.,100.,0.,0.2,
     10,1,2)
      TT5=TT(250)*(1./TT(502))
      SS5=SS(250)*(1./SS(502))
      CALL SYMBOL (TT5,SS5,0.14,3HP11,0.,3)
      UU(501)=SS(501)
      UU(502)=SS(502)
      CALL CALCO2(TT,UU,502,0.00,5.0,5.0,0.,100.,0.,0.2,
     10,1,2)
      TT6=TT(350)*(1./TT(502))
      UU6=UU(350)*(1./UU(502))
      CALL SYMBOL (TT6,UU6,0.14,3HP20,0.,3)
      NN=NN+1
      AMOVE=15.
      T=0.0
15    CONTINUE
16    CONTINUE
      CALL PLTEND(45.)
      STOP
      END


      SUBROUTINE RKINT(T,X,M,H,VECTOR)
      EXTERNAL VECTOR
      REAL*8 X(25),XDOT(25),K1(25),K2(25),K3(25),K4(25),
     1SAVEX(25)
      DO 10 J=1,N
      SAVEX(J)=X(J)
10    CONTINUE
      T=T+1
      CALL VECTOR(T,X,XDOT,N)
      DO 11 J=1,N
      K1(J)=XDOT(J)
11    X(J)=SAVEX(J)+0.5*H*K1(J)
      T=T+0.5*H
```

```
      CALL VECTOR(T,X,XDOT,N)
      DO 12 J=1,N
      K2(J) = XDOT(J)
12    X(J)=SAVE(J)+0.5*H*K2(J)
      CALL VECTOR(T,X,XDOT,N)
      DO 13 J=1,N
      K3(J)=XDOT(J)
13    X(J)=SAVEX(J)+H*K3(J)
      T=T+0.5*H
      CALL VECTOR(T,X,XDOT,N)
      DO 14 J=1,N
      K4(J)=XDOT(J)
14    X(J)=SAVEX(J)+(H/6)*(K1(J)+2*K2(J)+2*K3(J)+K4(J))
      RETURN
      END


      SUBROUTINE VECTOR (T,X,XDOT,N)
      REAL *8 XDOT(25),X(25)
      COMMON R1,R2
      XDOT(1)=-(R1+R2)*X(1)+X(4)+X(2)/2.
      XDOT(2)=-(R1+R2+1./2.)*X(2)+X(3)+R2*X(1)+X(5)
      XDOT(3)=-X(3)+R2*X(2)
      XDOT(4)=-(R1+R2+1.)*X(4)+2*X(6)+X(5)/2.+R1*X(1)
      XDOT(5)=-1.5*X(5)+R1*X(2)+R2*X(4)
      XDOT(6)=-2.*X(6)+R1*X(4)
      RETURN
      END
```

FIGURE 44

```
          SIMULATE
*         SIMULATION OF A CLOSED-LOOP CONVEYOR SYSTEM
*         WITH TWO CHANNELS, EACH HAVING NO STORAGE AND
*         ALLOWING MULTIPLD POISSON INPUTS AND THE
*         SERVICE RATE AT EACH CHANNEL IS EXPONENTIALLY
*         DISTRIBUTED.
*         DEFINITIONS:
*         CHA1 AND CHA2 ARE THE NAMES OF CHANNELS 1 & 2
*         RESPECTIVELY
*         XPDIS IS THE EXPONENTIAL DISTRIBUTION FUNCTION
 UNIFO FUNCTION      RN1,C11
0,0/0.1,1/0.2,2/0.3,3/0.4,4/0.5,5/0.6,6/0.7,7/0.8,8/0.9,
9/1.0,10
 XPDIS   FUNCTION    RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2/.75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/
.94,2.81/.95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/.995,
5.3/.998,6.2/.999,7/.9998,8
 LUC      FVARIABLE  FN$UNIFO*1
          GENERATE   100,FN$XPDIS
          GATE NU    CHA1,CHEC       CHECK CHA1
          SEIZE      CHA1            SEEK SERVICE CHA1
          ADVANCE    70,FN$XPDIS
          RELEASE    CHA1            LEAVE CHA1
          TABULATE   BTIME
          TERMINATE  1
 CHEC     GATE NU    CHA2,BYBYE      CHECK CHA2
          SEIZE      CHA2            SEEK SERVICE CHA2
          ADVANCE    70,FN$XPDIS
          RELEASE    CHA2            LEAVE CHA2
          TABULATE   BTIME
          TERMINATE  1
 BTIME    TABLE      M1,5,5,30
 BYBYE    TERMINATE  1
          GENERATE   280,FN$XPDIS
          GATE NU    CHA1,LEA        CHECK CHA1
          SEIZE      CHA1            SEEK SERVICE CHA1
          ADVANCE    140,FN$XPDIS
          RELEASE    CHA1            LEAVE CHA1
          TABULATE   CTIME
          TERMINATE  1
 LEA      GATE NU    CHA2,SALAM      CHECK CHA2
          SEIZE      CHA2            SEEK SERVICE CHA2
          ADVANCE    140,FN$XPDIS
          RELEASE    CHA2            LEAVE CHA2
          TABULATE   CTIME
          TERMINATE  1
```

```
         TERMINATE  1
CTIME    TABLE      M1,5,5,30
SALAM    TERMINATE  1
         START      200,NP
         RESET
         START      10000
         END
```

FIGURE 45

```
          SIMULATE
*         SIMULATION OF A CLOSED-LOOP CONVEYOR SYSTEM WITH
*         TWO CHANNELS AND MULTIPLE POISSON INPUTS WHERE
*         THE SERVICE RATE AT EACH CHANNEL IS EXPONENTIALLY
*         DISTRIBUTED.  A STORAGE CAPACITY (4) AND (1) ARE
*         ALLOWED AT THE FIRST AND SECOND CHANNELS,
*         RESPECTIVELY.
*         DEFINITIONS:
*         CHA1 AND CHA2 ARE THE NAMES OF CHANNELS 1 & 2
*         RESPECTIVELY.
*         XPDIS IS THE EXPONENTIAL DISTRIBUTION FUNCTION
 UNIFO FUNCTION     RN1,C11
0,0/0.1,1/0.2,2/0.3,3/0.4,4/0.5,5/0.6,6/0.7,7/0.8,8/0.9,
9/1.0,10
 XPDIS FUNCTION     RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2/.75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,
2.81/.95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/.995,5.3/
.998,6.2/.999,7/.9998,8
 LUC      FVARIABLE   FN$UNIFO*1
          GENERATE    100,FN$XPDIS
          GATE NU     CHA1,NOT
          SEIZE       CHA1
          ADVANCE     100,FN$XPDIS
          RELEASE     CHA1
          TABULATE    BTIME
          TERMINATE   1
 NOT      GATE SE     STRS,MASH
          ENTER       STRS
          GATE NU     CHA1
          LEAVE       STRS
          SEIZE       CHA1
          ADVANCE     100,FN$XPDIS
          RELEASE     CHA1
          TABULATE    BTIME
          TERMINATE   1
 MASH     GATE NU     CHA2,SMASH
          SEIZE       CHA2
          ADVANCE     100,FN$XPDIS
          RELEASE     CHA2
          TABULATE    BTIME
          TERMINATE   1
 SMASH    GATE SE     STR,NOTT
          ENTER       STR
          GATE NU     CHA2
          LEAVE       STR
          SEIZE       CHA2
```

```
        ADVANCE     100,FN$XPDIS
        RELEASE     CHA2
        TABULATE    BTIME
        TERMINATE   1
BTIME   TABLE       M1,5,5,30
NOTT    TERMINATE   1
        GENERATE    1000,FN$XPDIS
        GATE NU     CHA1,LOT
        SEIZE       CHA1
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA1
        TABULATE    CTIME
        TERMINATE   1
LOT     GATE SE     STRS,NAS
        ENTER       STRS
        GATE NU     CHA1
        LEAVE       STRS
        SEIZE       CHA1
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA1
        TABULATE    CTIME
        TERMINATE   1
NAS     GATE NU     CHA2,SSM
        SEIZE       CHA2
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA2
        TABULATE    CTIME
        TERMINATE   1
SSM     GATE SE     STR,NOTL
        ENTER       STR
        GATE NU     CHA2
        LEAVE       STR
        SEIZE       CHA2
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA2
        TABULTAE    CTIME
        TERMINATE   1
CTIME   TABLE       M1,5,5,30
NOTL    TERMINATE   1
STRS    STORAGE     4
STR     STORAGE     1
        START       200,NP
        RESET
        START       10000
        END
```

FIGURE 46

```
         SIMULATE
*        SIMULATION OF A CLOSED-LOOP CONVEYOR SYSTEM WITH
*        TWO CHANNELS AND ALLOWING MULTIPLE POISSON
*        INPUTS WHERE THE SERVICE RATE AT EACH CHANNEL IS
*        EXPONENTIALLY DISTRIBUTED.
*        THIS PROGRAMME IS TO DETERMINE THE FREQUENCY
*        DISTRIBUTION OF THE RECIRCULATED UNITS.
*        DEFINITIONS:
*        CHA1 AND CHA2 ARE THE NAMES OF CHANNELS 1 & 2
*        RESPECTIVELY.
*        XPDIS IS THE EXPONENTIAL DISTRIBUTION FUNCTION
 UNIFO FUNCTION    RN1,C11
0,0/0.1,1/0.2,2/0.3,3/0.4,4/0.5,5/0.6,6/0.7,7/0.8,8/0.9,
9/1.0,10
 XPDIS   FUNCTION    RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2/.75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/
.94,2.81/.95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/.995,
5.3/.998,6.2/.999,7/.9998,8
 LUC     FVARIABLE   FN$UNIFO*1
         GENERATE    100,FN$XPDIS
         GATE NU     CHA1,CHEC          CHECK CHA1
         SEIZE       CHA1               SEEK SERVICE CHA1
         ADVANCE     60,FN$XPDIS
         RELEASE     CHA1               LEAVE CHA1
         TABULATE    BTIME
         TERMINATE   1
 CHEC    GATE NU     CHA2,BYBYE         CHECK CHA2
         SEIZE       CHA2               SEEK SERVICE CHA2
         ADVANCE     60,FN$XPDIS
         RELEASE     CHA2               LEAVE CHA2
         TABULATE    BTIME
         TERMINATE   1
 BYBYE   GATE SE     STR,ROWH
         ENTER       STR
         GATE NU     CHA2
         LEAVE       STR
         SEIZE       CHA2
         ADVANCE     60,FN$XPDIS
         RELEASE     CHA2
         TABULATE    BTIME
         TERMINATE   1
 BTIME   TABLE       M1,5,5,30
 ROWH    ENTER       LINE
         QUEUE       ONE
         SEIZE       RIP
         DEPART      ONE
```

```
         ADVANCE    100,FN$XPDIS
         RELEASE    RIP
         LEAVE      LINE
         TERMINATE  1
         GENERATE   120,FN$XPDIS
         GATE NU    CHA1,LEA          CHECK CHA1
         SEIZE      CHA1              SEEK SERVICE CHA1
         ADVANCE    120,FN$XPDIS
         RELEASE    CHA1              LEAVE CHA1
         TABULATE   CTIME
         TERMINATE  1
LEA      GATE NU    CHA2,SALAM        CHECK CHA2
         SEIZE      CHA2              SEEK SERVICE CHA2
         ADVANCE    120,FN$XPDIS
         RELEASE    CHA2              LEAVE CHA2
         TABULATE   CTIME
         TERMINATE  1
SALAM    GATE SE    STR,COWH
         ENTER      STR
         GATE NU    CHA2
         LEAVE      STR
         SEIZE      CHA2
         ADVANCE    120,FN$XPDIS
         RELEASE    CHA2
         TABULATE   CTIME
         TERMINATE  1
CTIME    TABLE      M1,5,5,30
COWH     TERMINATE  1
RLIME    TABLE      RT,10,10,50,100
INQUE    QTABLE     ONE,0,100,20
LINE     SOTRAGE    190
STR      STORAGE    10
         START      100,NP
         RESET
         START      10000
         END
```

FIGURE 47

```
         SIMULATE
*        SIMULATION OF A CLOSED-LOOP CONVEYOR WITH TWO
*        CHANNELS AND ALLOWING MULTIPLE POISSON INPUTS
*        WHERE THE SERVICE RATE AT EACH CHANNEL IS
*        EXPONENTIALLY DISTRIBUTED.  NO STORAGE IS AT
*        THE FIRST CHANNEL AND A STORAGE CAPACITY (2) IS
*        AT THE SECOND CHANNEL.
*        DEFINITIONS:
*        CHA1 AND CHA2 ARE THE NAMES OF CHANNELS 1 & 2,
*        RESPECTIVELY
*        XPDIS IS THE EXPONENTIAL DISTRIBUTION FUNCTION
 UNIFO   FUNCTION    RN1,C11
0,0/0.1,1/0.2,2/.3,3/0.4,4/0.5,5/0.6,6/0.7,7/0.8,8/0.9,
9/1.0,10
 XPDIS   FUNCTION    RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2/.75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/
.94,2.81/.95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/.995,
5.3/.998,6.2/.999,7/.9998,8
 LUC     FVARIABLE   FN$UNIFO*1
         GENERATE    100,FN$XPDIS
         GATE NU     CHA1,CHEC          CHECK CHA1
         SEIZE       CHA1               SEEK SERVICE CHA1
         ADVANCE     40,FN$XPDIS
         RELEASE     CHA1               LEAVE CHA1
         TABULATE    BTIME
         TERMINATE   1
 CHEC    GATE NU     CHA2,BYBYE         CHECK CHA2
         SEIZE       CHA2               SEEK SERVICE CHA2
         ADVANCE     40,FN$XPDIS
         RELEASE     CHA2               LEAVE CHA2
         TABULATE    BTIME
         TERMINATE   1
 BYBYE   GATE SE     STR,ROWH
         ENTER       STR
         GATE NU     CHA2
         LEAVE       STR
         SEIZE       CHA2
         ADVANCE     40,FN$XPDIS
         RELEASE     CHA2
         TABULATE    BTIME
         TERMINATE   1
 BTIME   TABLE       M1,5,5,30
 ROWH    TERMINATE   1
         GENERATE    200,FN$XPDIS
         GATE NU     CHA1,LEA           CHECK CHA1
```

```
          SEIZE       CHA1              SEEK SERVICE CHA1
          ADVANCE     80,FN$XPDIS
          RELEASE     CHA1              LEAVE CHA1
          TABULATE    CTIME
          TERMINATE   1
LEA       GATE NU     CHA2,SALAM        CHECK CHA2
          SEIZE       CHA2              SEEK SERVICE CHA2
          ADVANCE     80,FN$XPDIS
          RELEASE     CHA2              LEAVE CHA2
          TABULATE    CTIME
          TERMINATE   1
SALAM     GATE SE     STR,COWH
          ENTER       STR
          GATE NU     CHA2
          LEAVE       STR
          SEIZE       CHA2
          ADVANCE     80,FN$XPDIS
          RELEASE     CHA2
          TABULATE    CTIME
          TERMINATE   1
CTIME     TABLE       M1,5,5,30
COWH      TERMINATE   1
STR       STORAGE     2
          START       200,NP
          RESET
          START       10000
          END
```

FIGURE 48

```
        SIMULATE
*       SIMULATION OF A CLOSED-LOOP CONVEYOR SYSTEM
*       WITH TWO CHANNELS AND ALLOWING MULTIPLE POISSON
*       INPUTS, WHERE THE SERVICE RATE AT EACH CHANNEL
*       IS EXPONENTIALLY DISTRIBUTED.  RECIRCULATION
*       IS ALLOWED.
*       DEFINITIONS:
*       CHA1 AND CHA2 ARE THE NAMES OF CHANNELS 1 & 2
*       RESPECTIVELY
*       XPDIS IS THE EXPONENTIAL DISTRIBUTION FUNCTION
 UNIFO  FUNCTION    RN1,C11
0.0/0.1,1/0.2,2/0.3,3/0.4,4/0.5,5/0.6,6/0.7,7/0.8,8/0.9,
9/1.0,10
 XPDIS  FUNCTION    RN1,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2/.75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52,
/.94,2.81/.95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/
.995,5.3/.998,6.2/.999,7/.9998,8
        GENERATE    100,FN$XPDIS
 CKL    GATE NU     CHA1,CHEC
        SEIZE       CHA1            SEEK SERVICE CHA1
        ADVANCE     100,FN$XPDIS
        RELEASE     CHA1            LEAVE CHA1
        TABULATE    BTIME
        TERMINATE   1
 CHEC   GATE NU     CHA2,BYBYE      CHECK CHA2
        SEIZE       CHA2            SEEK SERVICE CHA2
        ADVANCE     100,FN$XPDIS
        RELEASE     CHA2            LEAVE CHA2
        TABULATE    BTIME
        TERMINATE   1
 BTIME  TABLE       M1,5,5,30
 BYBYE  ADVANCE     300
        TRANSFER    ,CKL
        GENERATE    660,FN$XPDIS
 CRF    GATE NU     CHA1,LEA
        SEIZE       CHA1            SEEK SERVICE CHA1
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA1            LEAVE CHA1
        TABULATE    CTIME
        TERMINATE   1
 LEA    GATE NU     CHA2,SALAM      CHECK CHA2
        SEIZE       CHA2            SEEK SERVICE CHA2
        ADVANCE     200,FN$XPDIS
        RELEASE     CHA2            LEAVE CHA2
        TABULATE    CTIME
        TERMINATE   1
```

```
CTIME   TABLE      M1,5,5,30
SALAM   ADVANCE    300
        TRANSFER   ,CRF
        START      200,NP
        RESET
        START      10000
        END
```

APPENDIX B

Two-channel closed-loop conveyor system with storage of unit capacity at the second channel.

Appendix B shows the effect of traffic intensities on the steady-state probabilities and the measures of the system's performance.

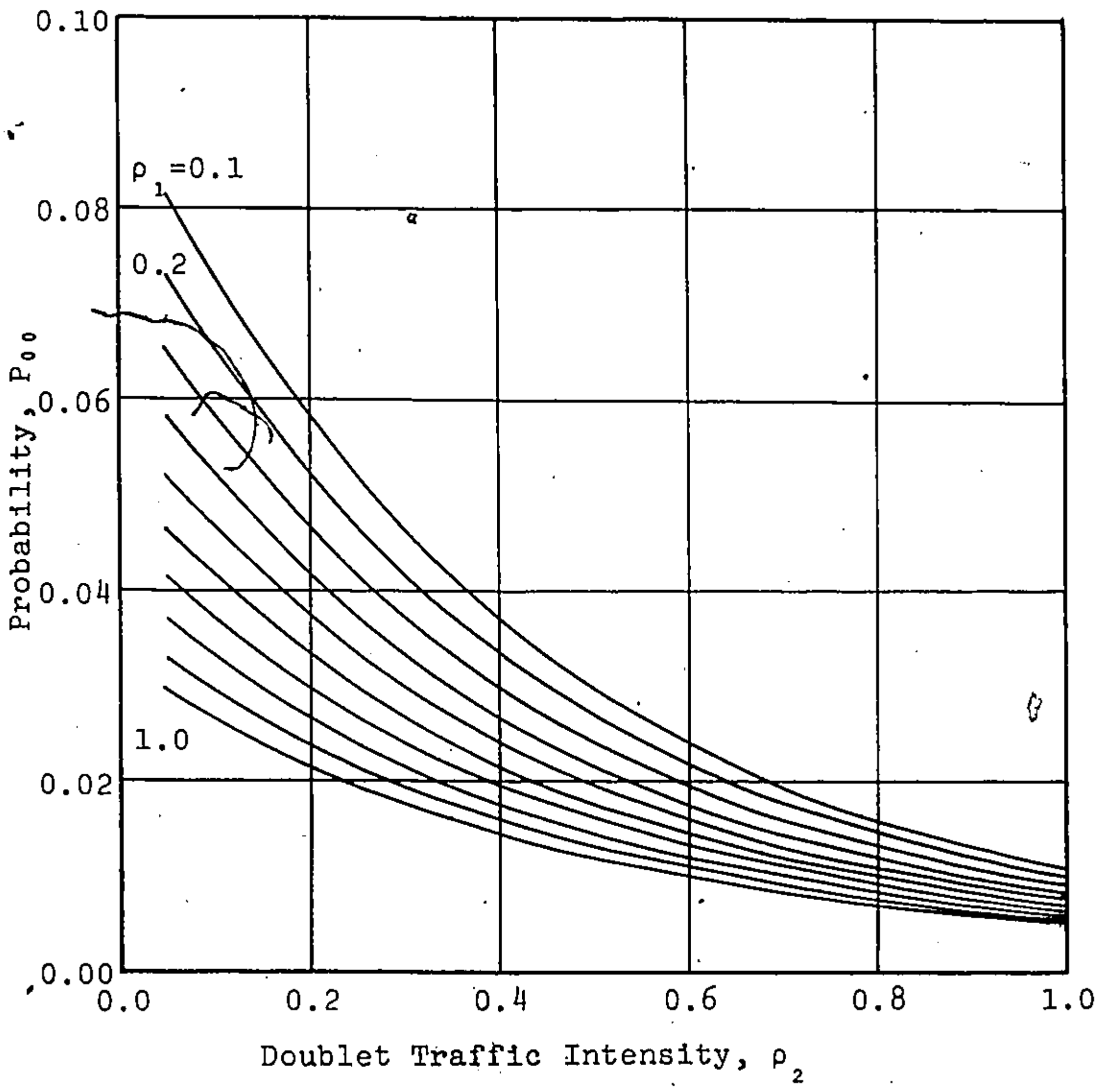

Figure 49. Change of probability P_{00} with ρ_2; ρ_1 fixed.

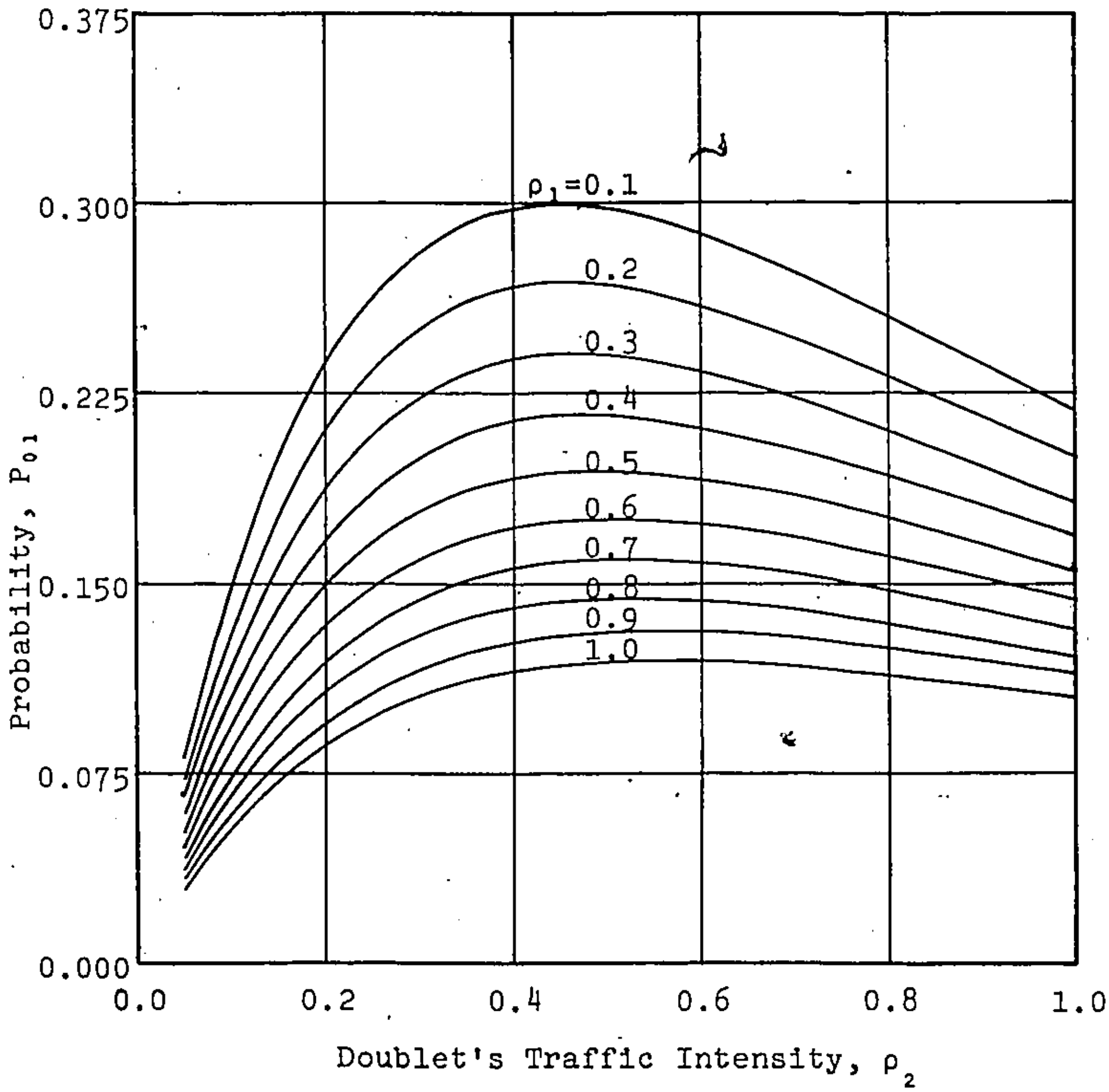

Figure 50. Change of probability P_{01} with ρ_2; ρ_1 fixed.

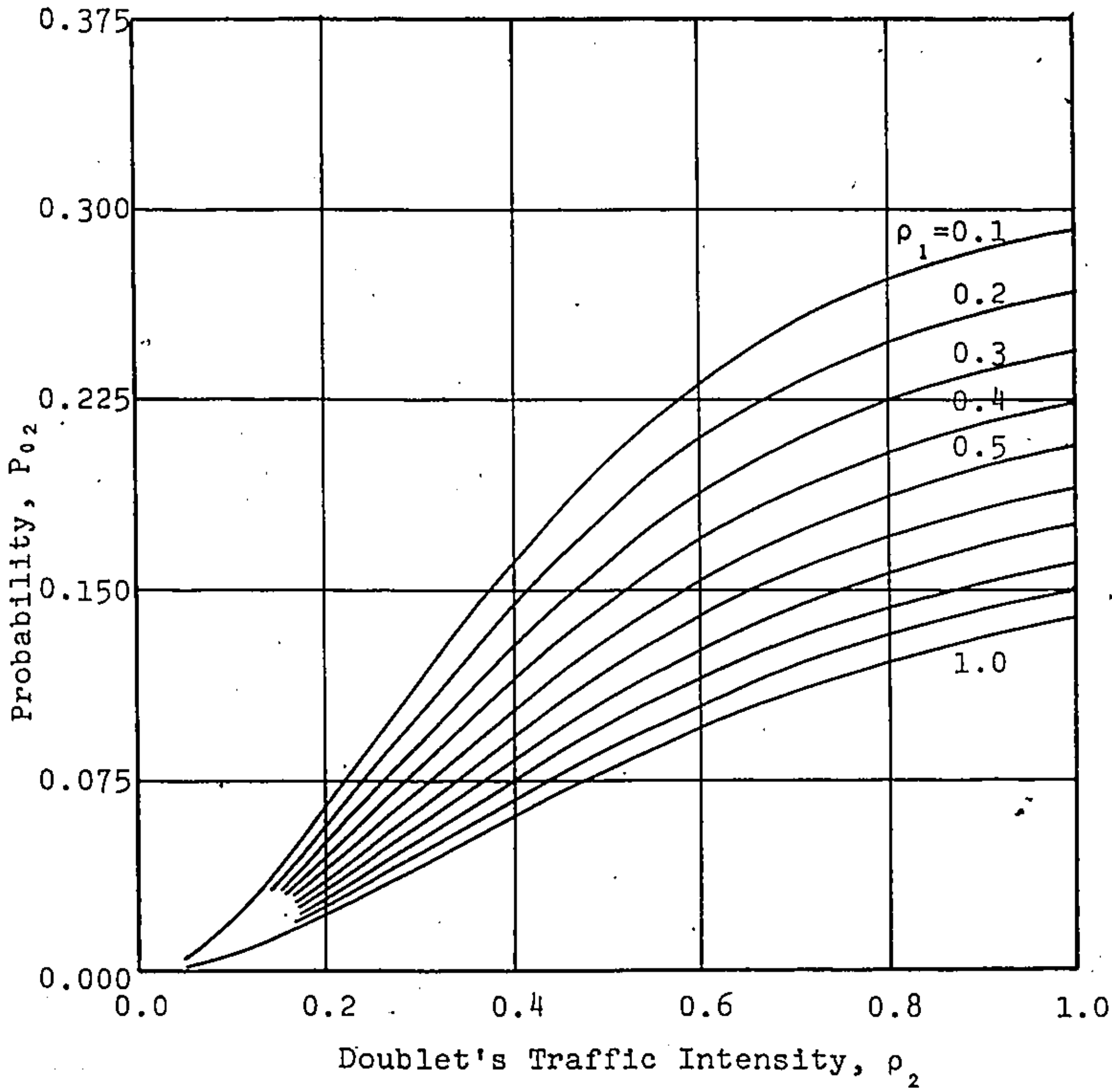

Figure 51. Change of probability P_{02} with ρ_2; ρ_1 fixed.

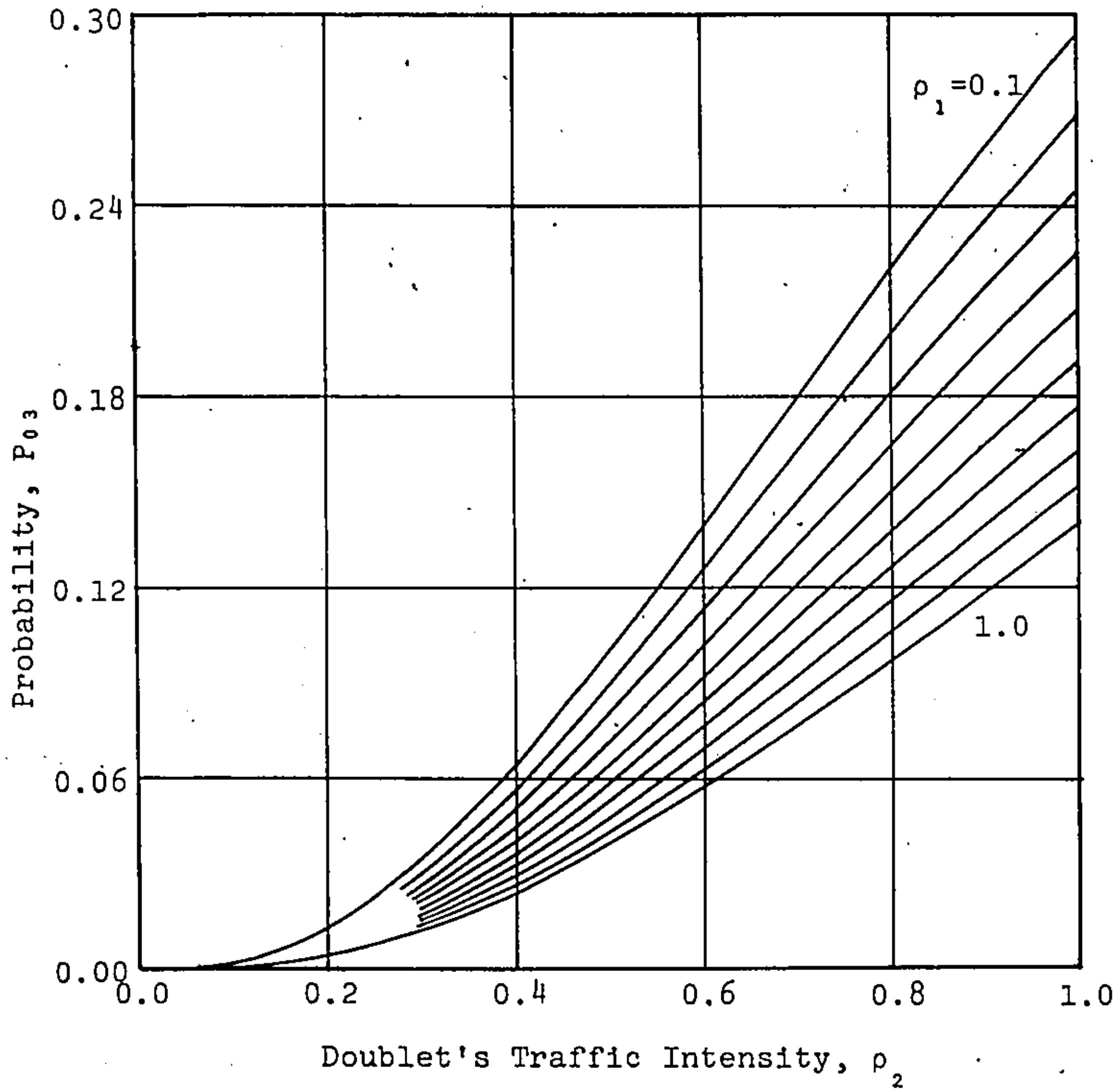

Figure 52. Change of probability P_{03} with ρ_2; ρ_1 fixed.

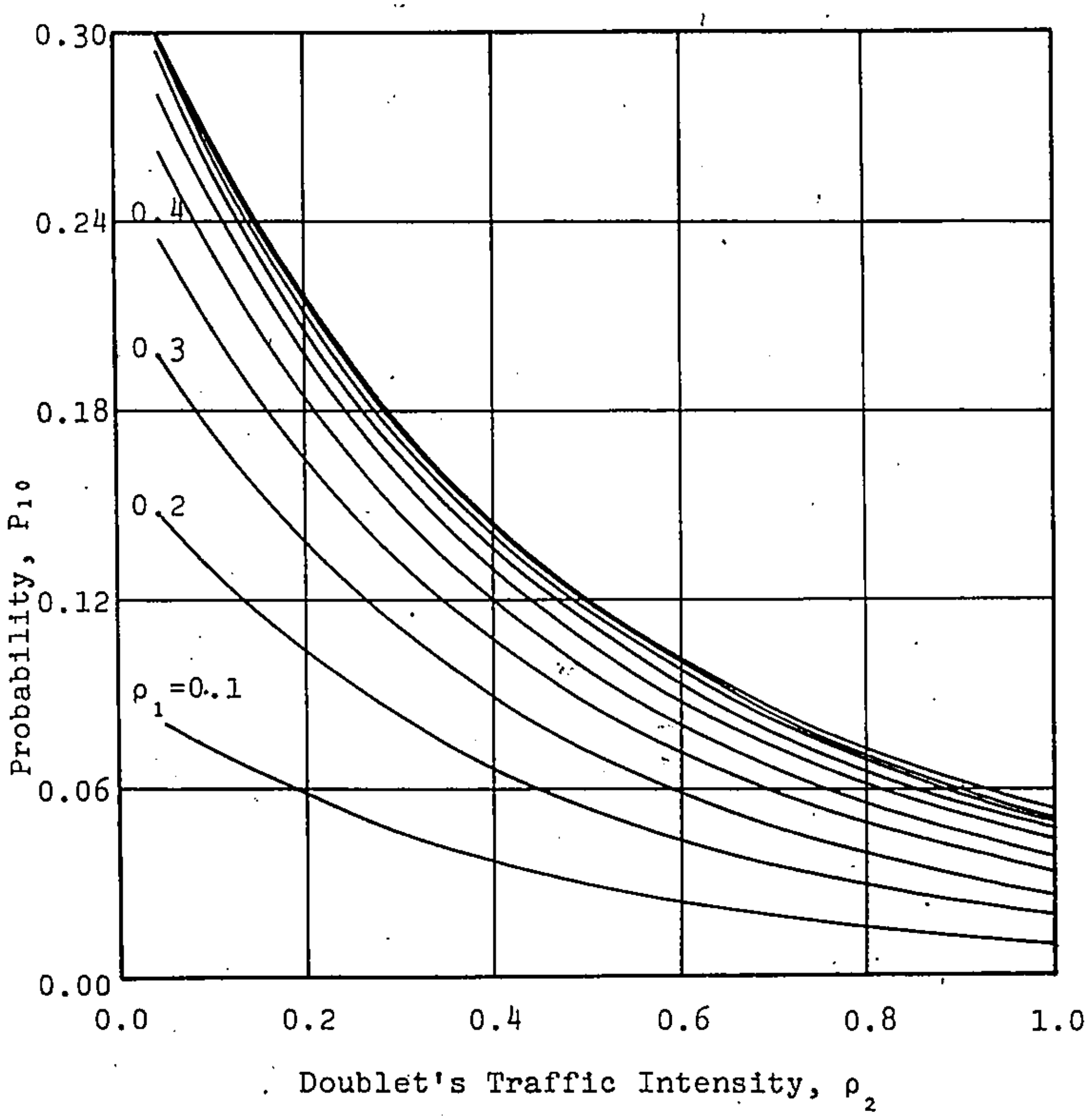

Figure 53. Change of probability P_{10} with ρ_2; ρ_1 fixed.

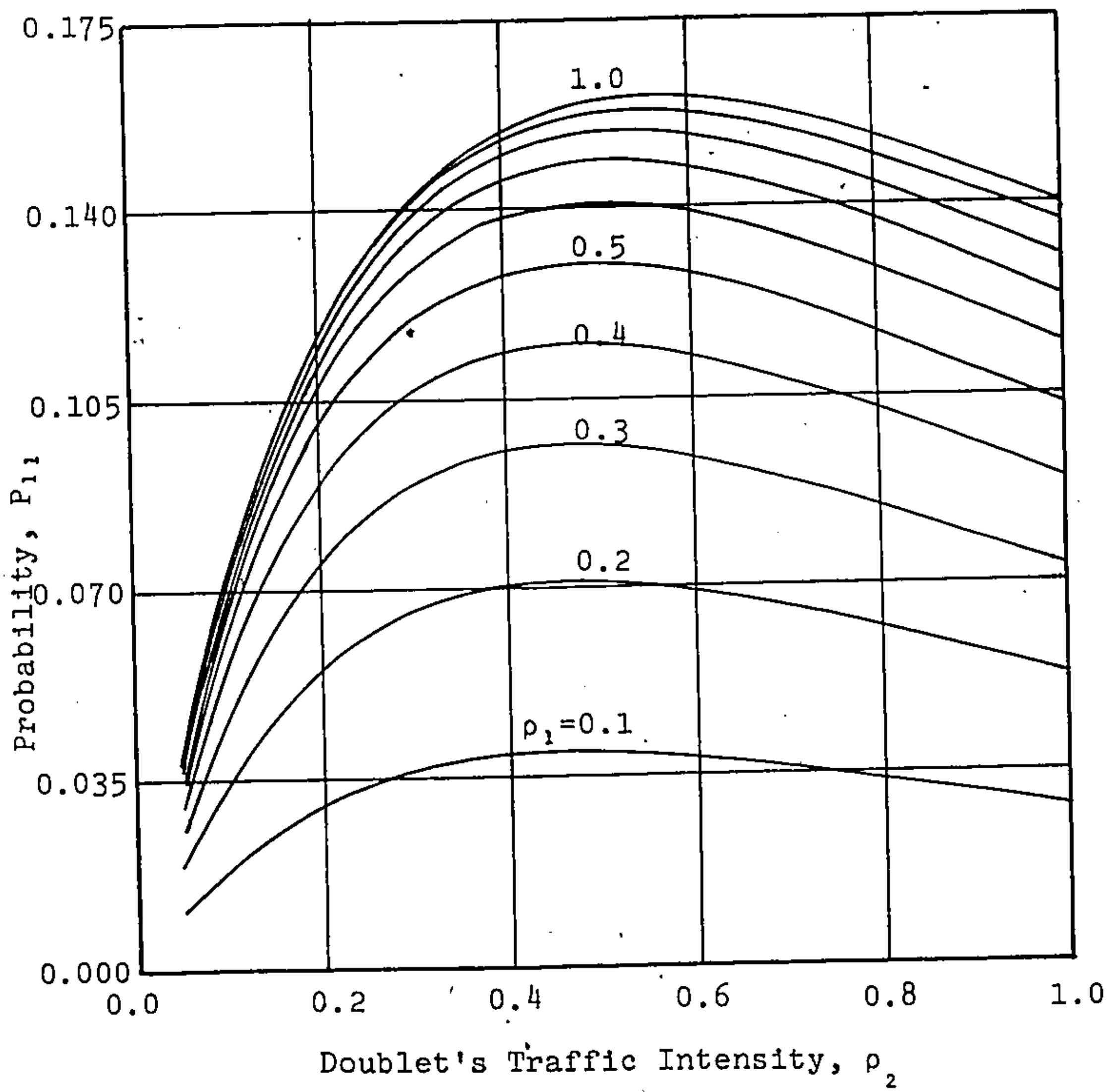

Figure 54. Change of probability P_{11} with ρ_2; ρ_1 fixed.

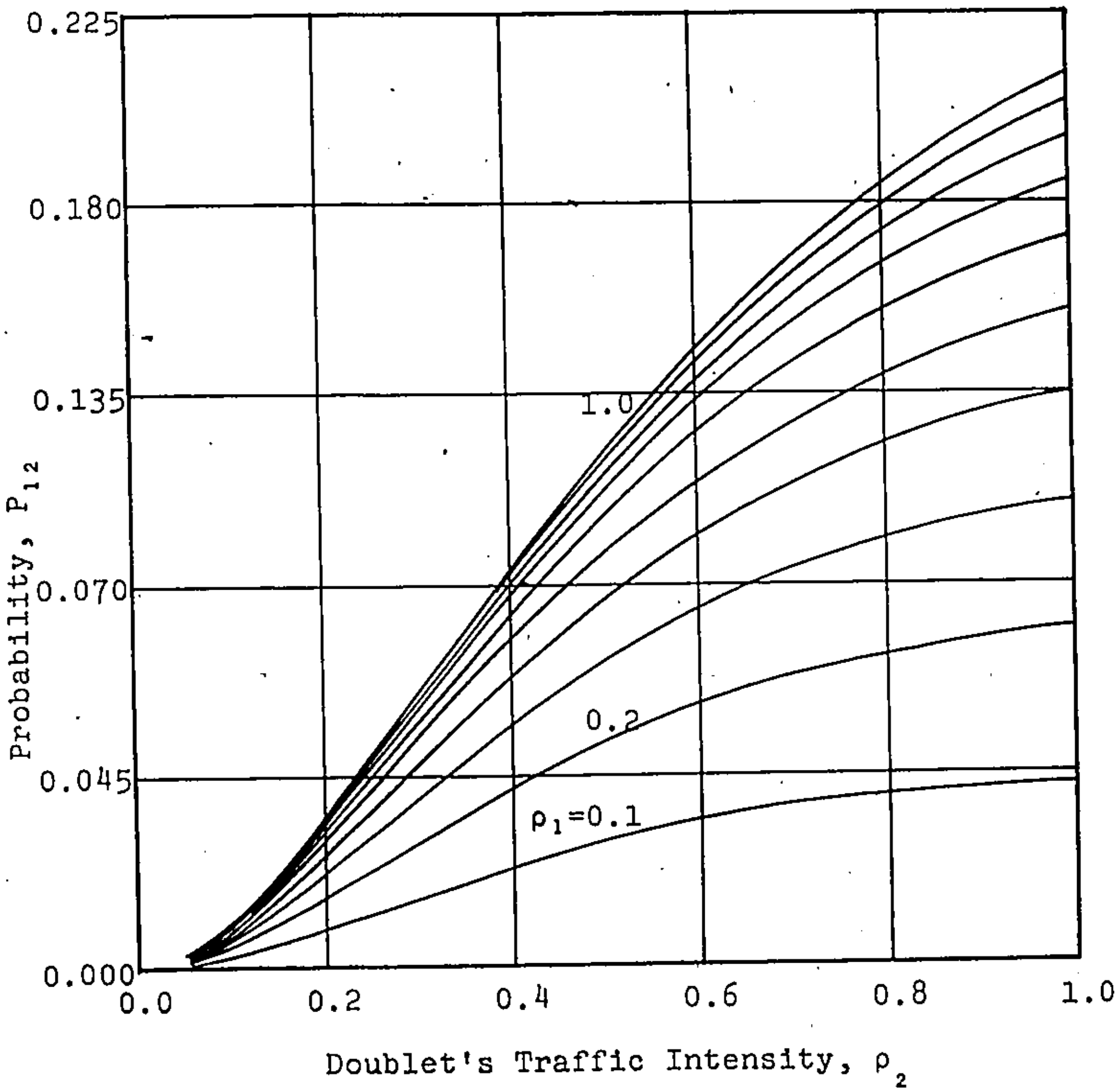

Figure 55. Change of probability P_{12} with ρ_2; ρ_1 fixed.

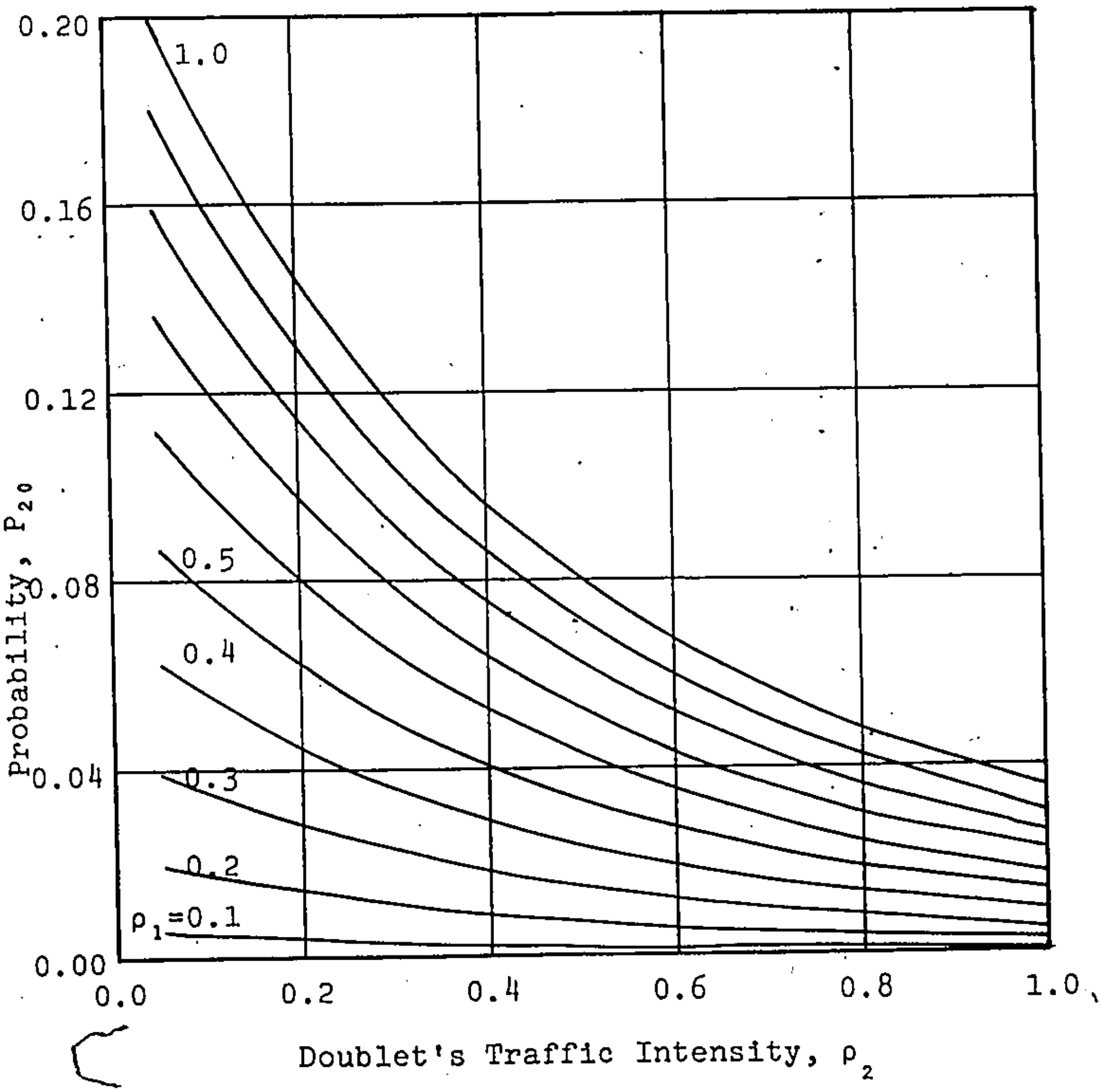

Figure 56. Change of probability P_{20} with ρ_2; ρ_1 fixed.

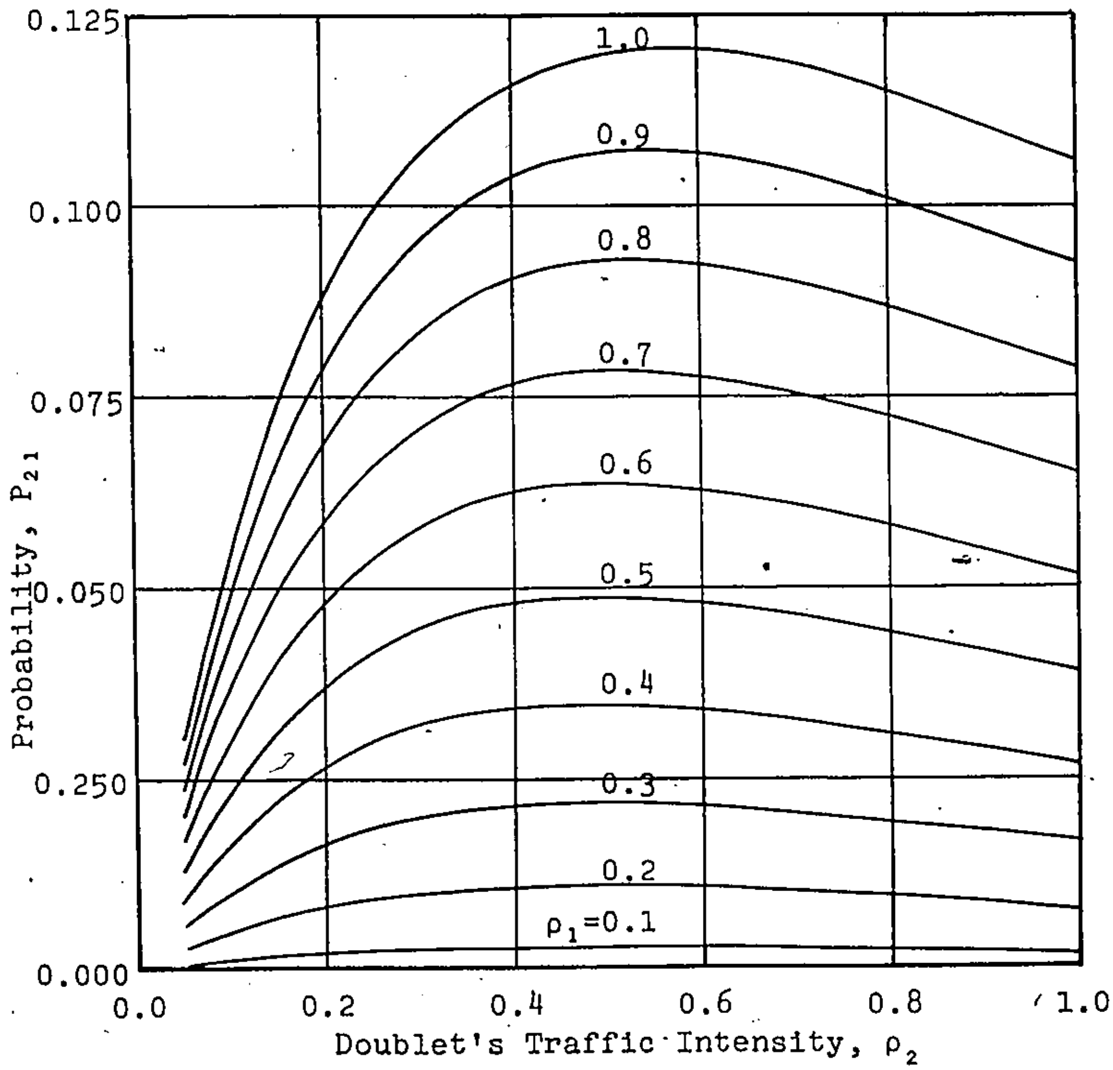

Figure 57. Change of probability P_{21} with ρ_2; ρ_1 fixed.

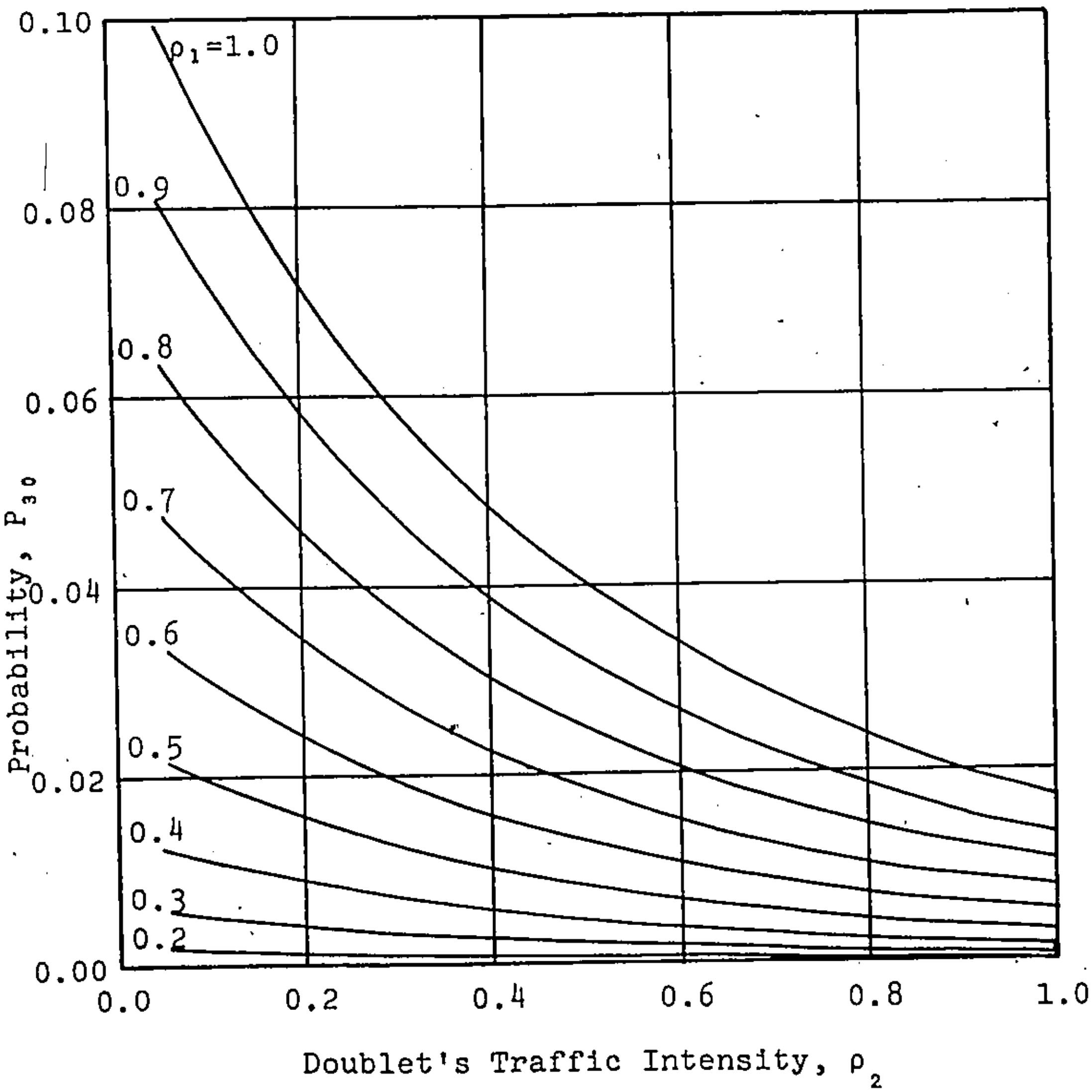

Figure 58. Change of probability P_{30} with ρ_2; ρ_1 fixed.

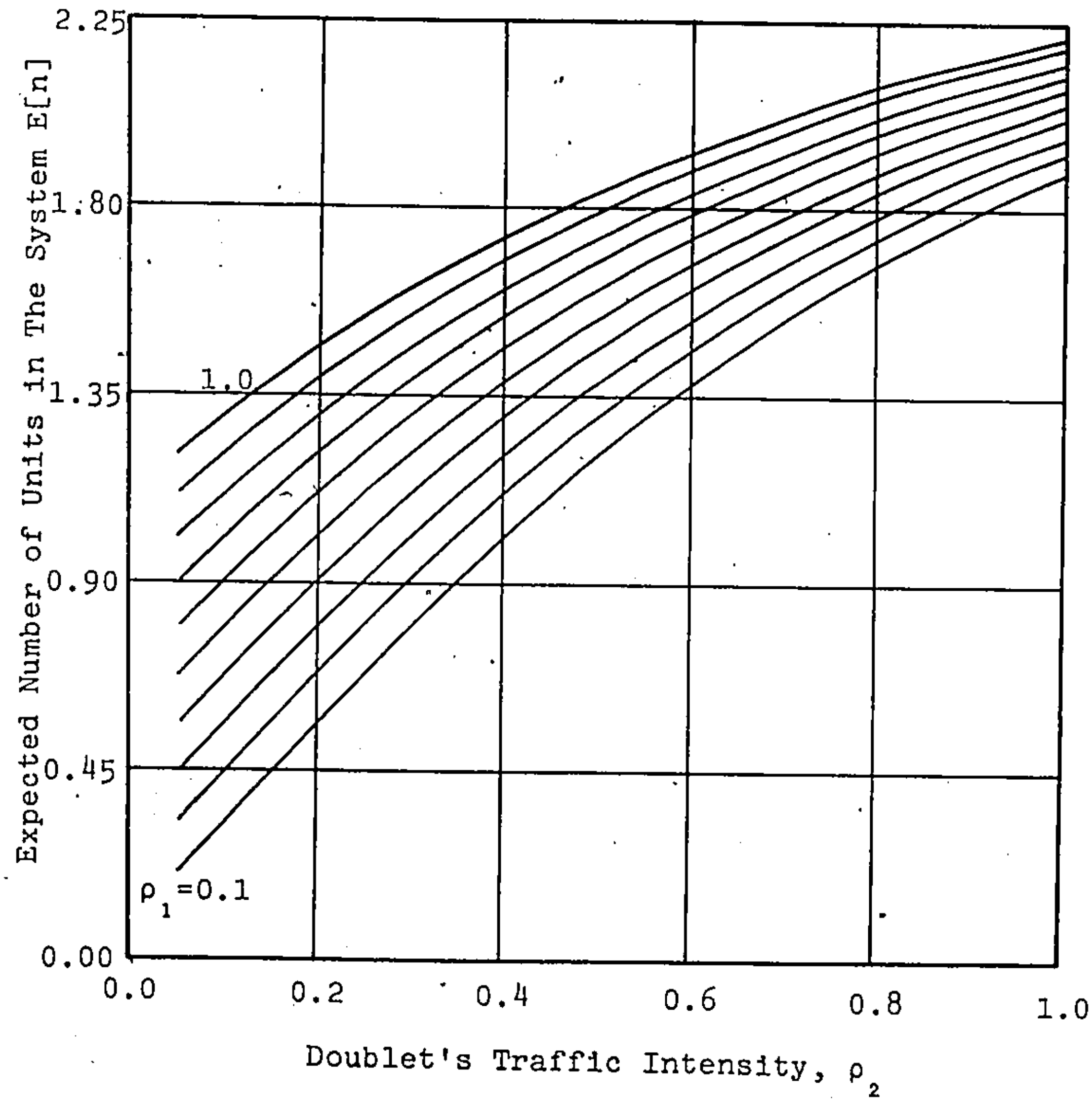

Figure 59. Expected number of units in the system with ρ_2, ρ_1 fixed.

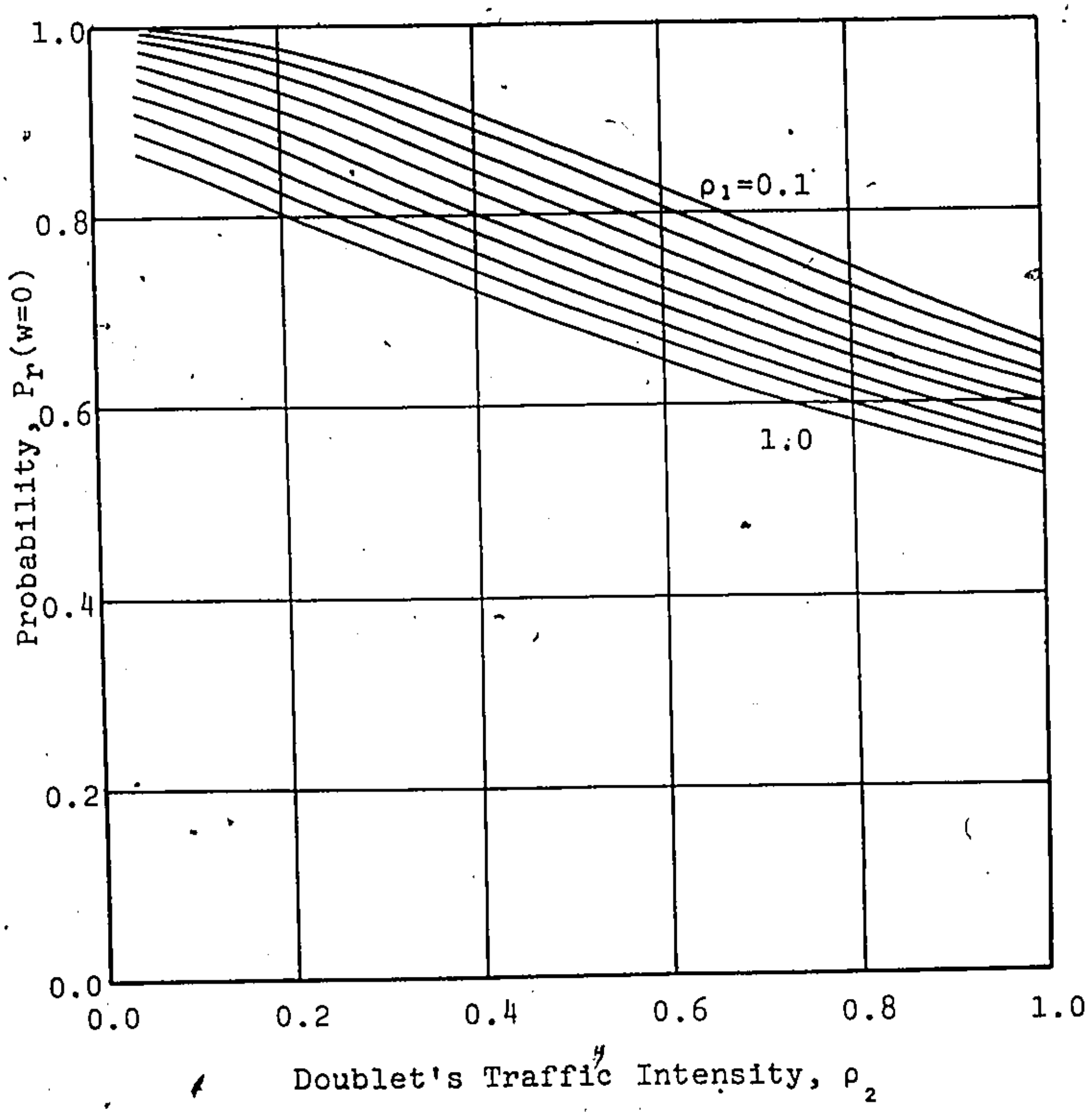

Figure 60. Change of probability, $P_r(w=0)$ with ρ_2; ρ_1 fixed.

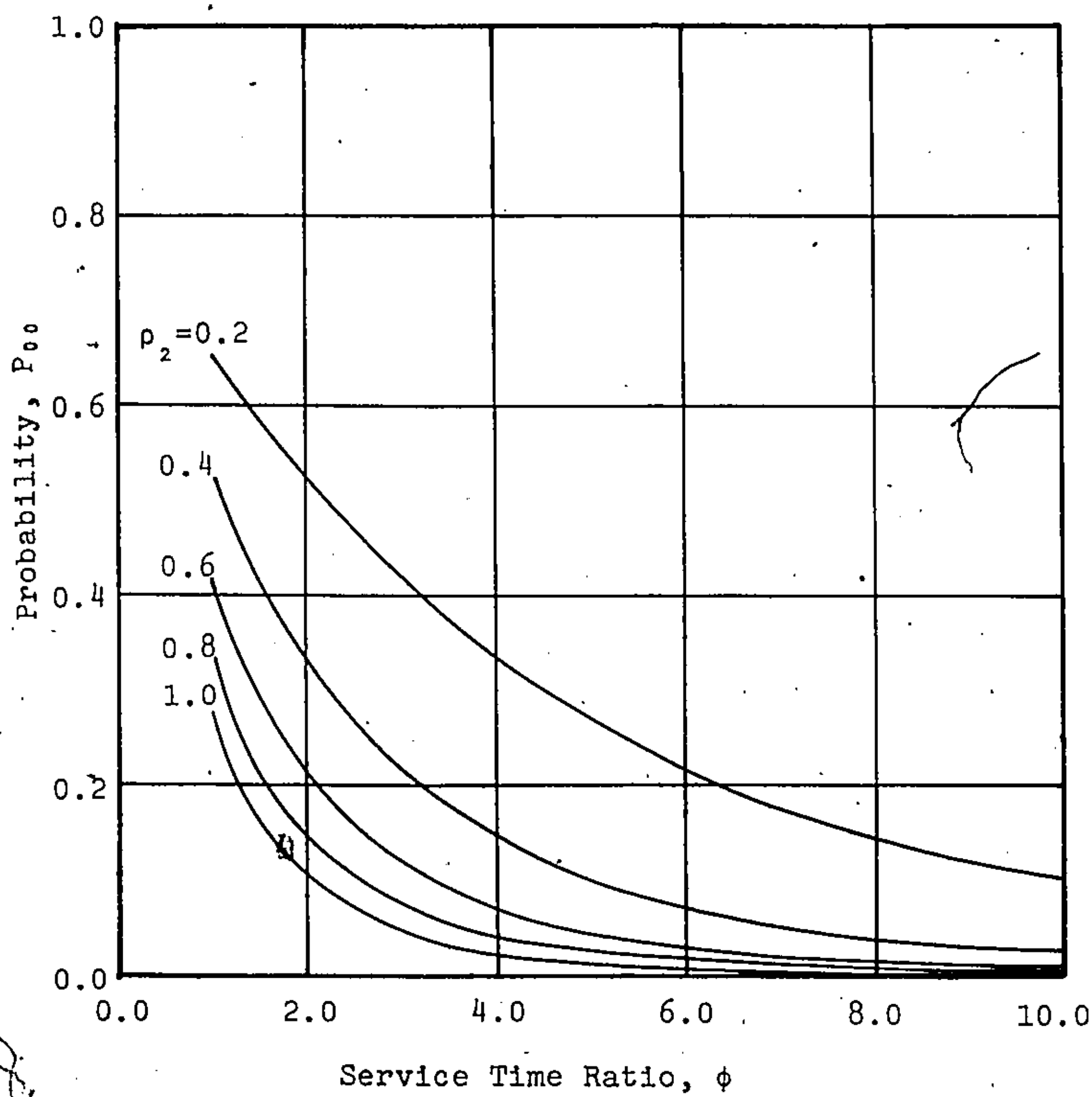

Figure 61. Change of probability P_{00} with ϕ; ρ_1 fixed at 0.2.

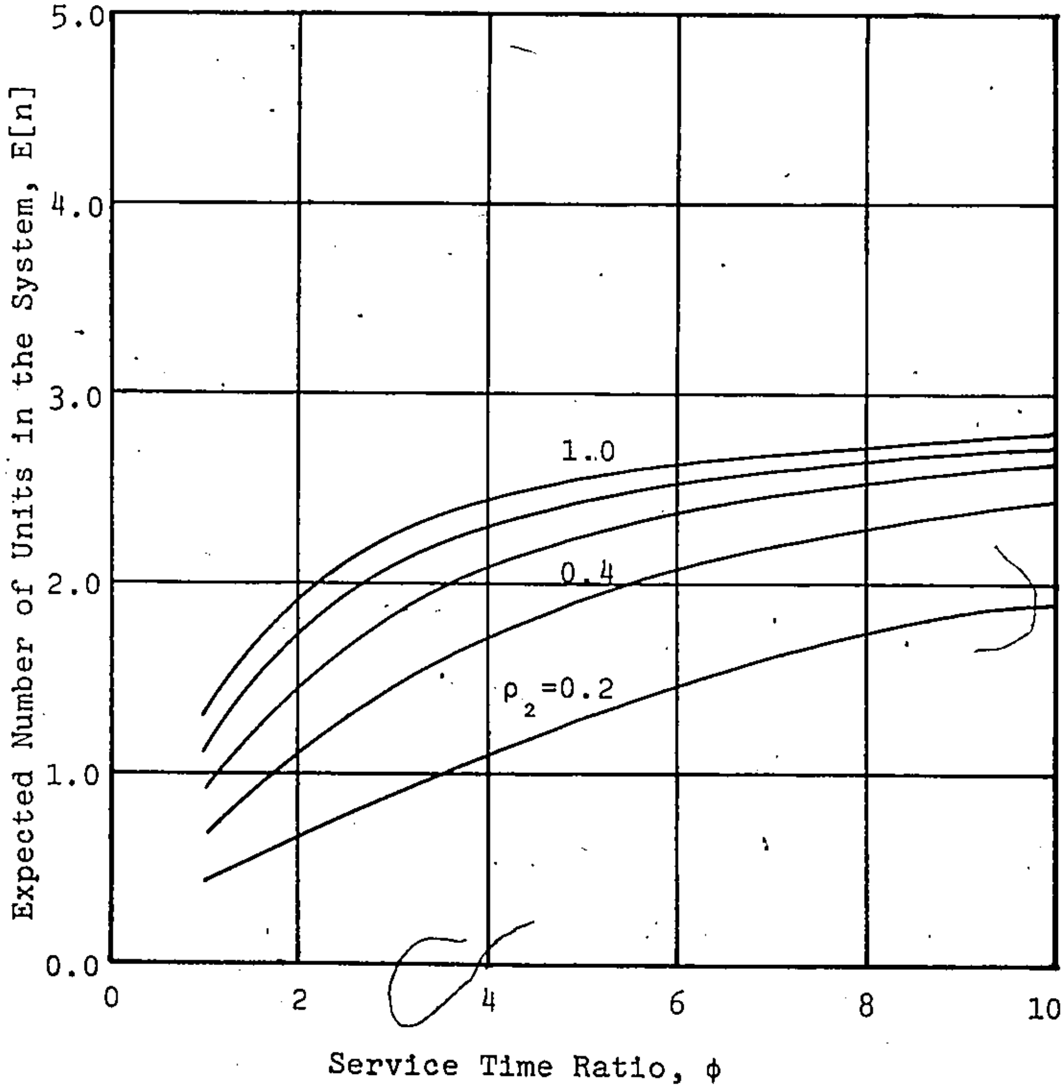

Figure 62. Expected number of units in the system E[n] with ϕ, ρ_1 fixed at 0.2.

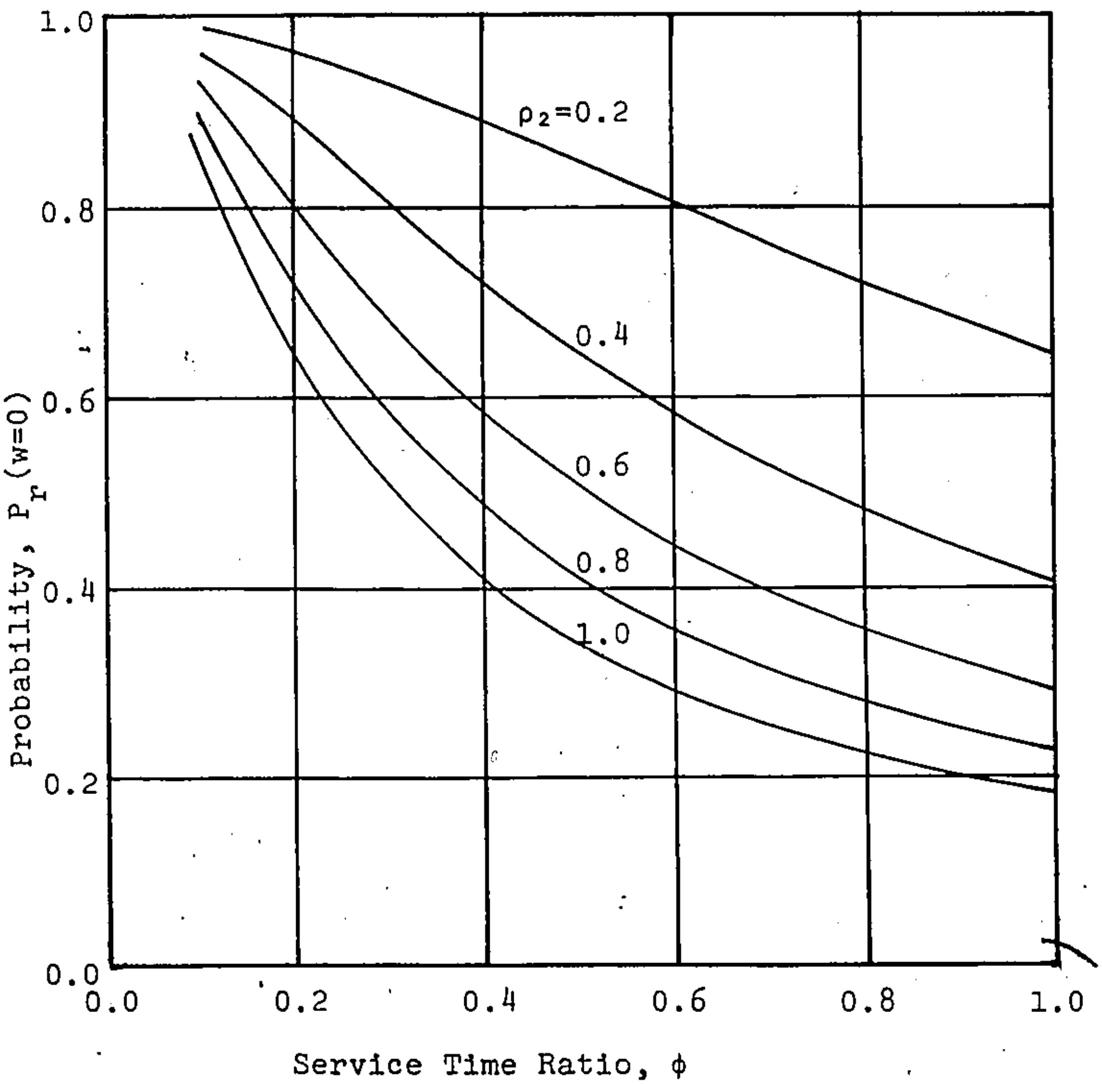

Figure 63. Change of Probability $P_r(w=0)$ with ϕ, ρ_1 fixed at 0.2.

APPENDIX C

Three-channel closed-loop conveyor with lost arrivals.

Appendix C contains figures to show the effect of traffic intensities on the steady-state probabilities and the measures of the system's performance. Also, the relationships between the traffic intensities and the average utilization of the service channels are included.

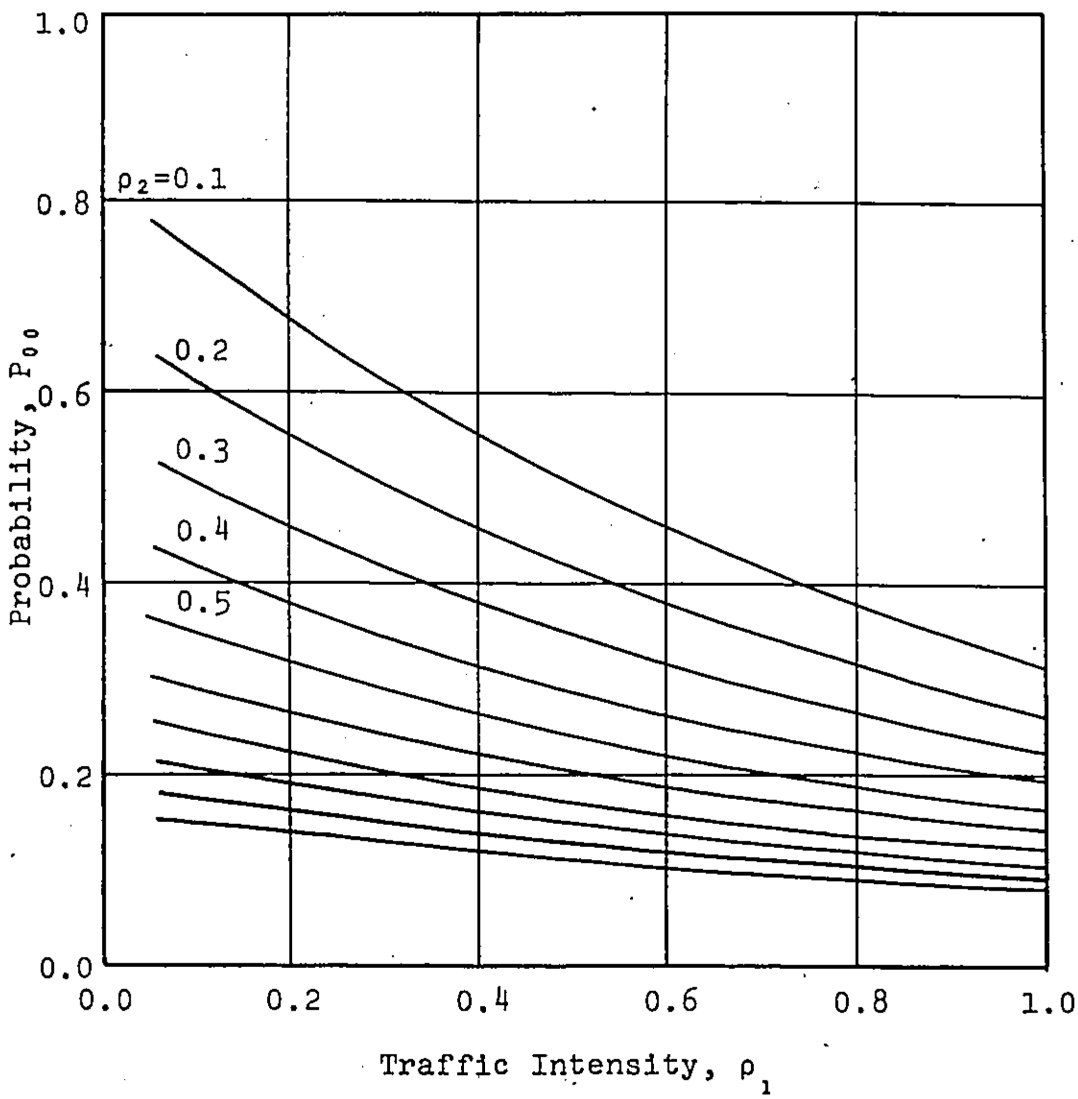

Figure 64. Change of probability P_{00} with ρ_1, ρ_2 fixed.

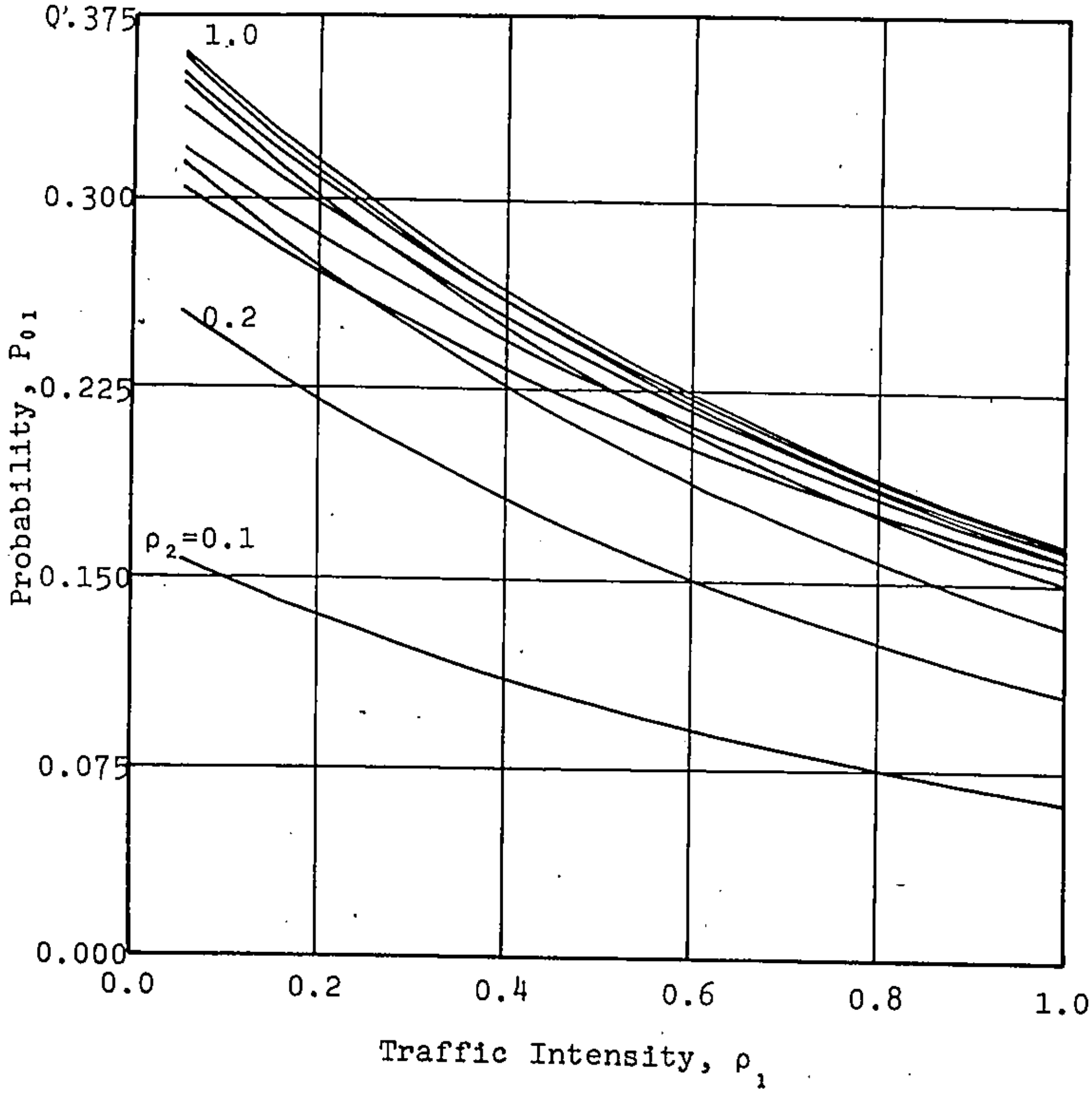

Figure 65. Change of probability P_{01} with ρ_1; ρ_2 fixed.

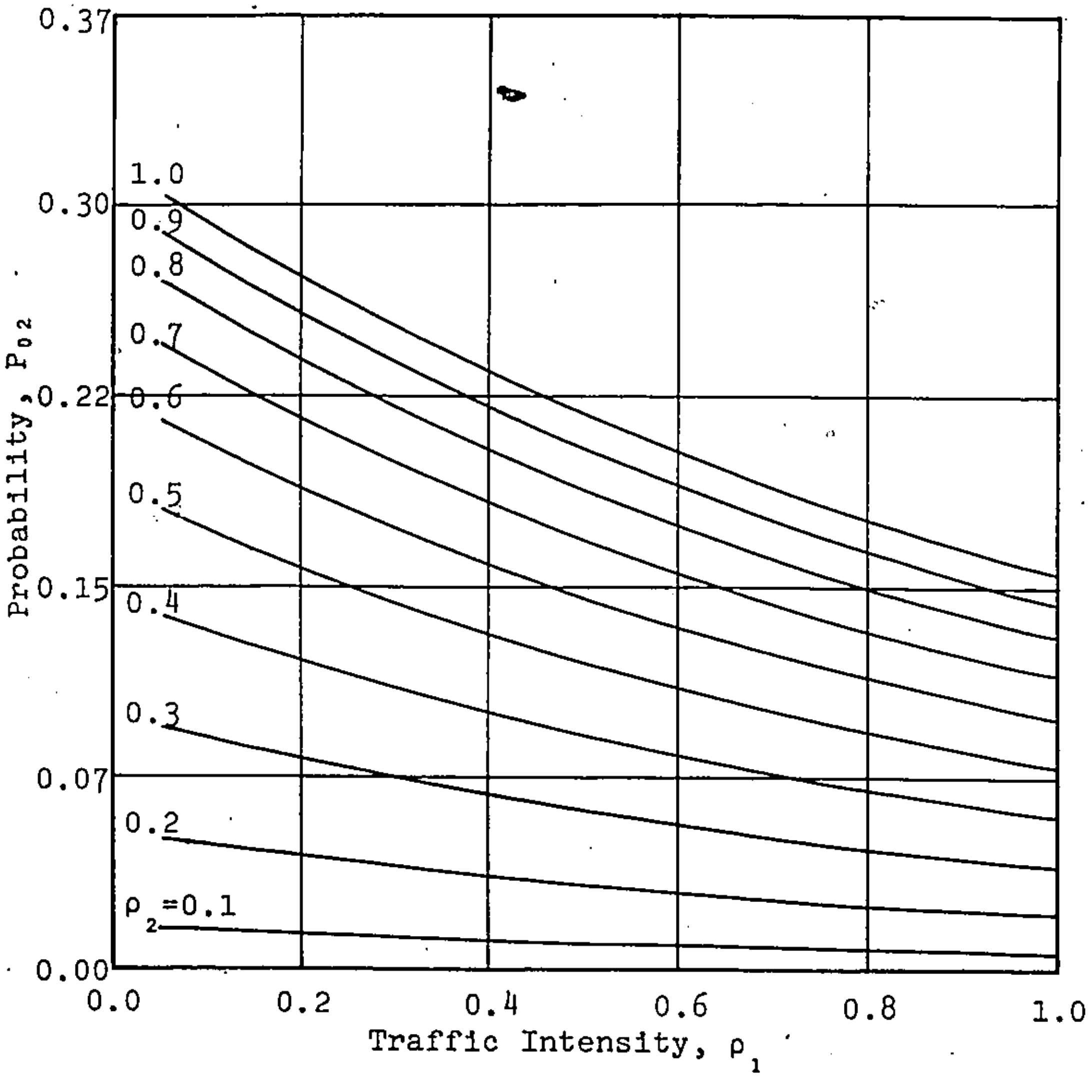

Figure 66. Change of probability P_{02} with ρ_1, ρ_2 fixed.

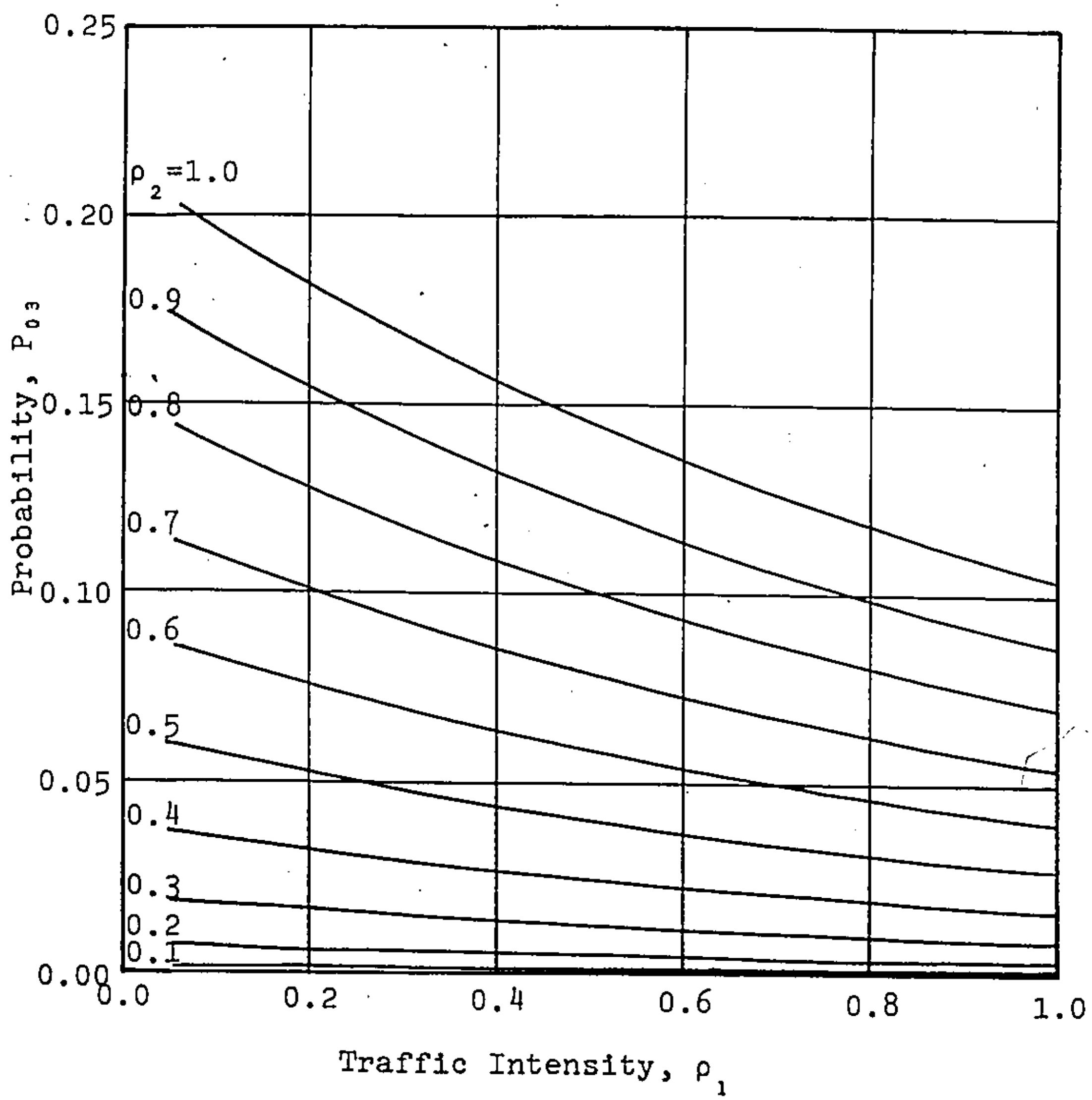

Figure 67. Change of probability P_{03} with ρ_1; ρ_2 fixed.

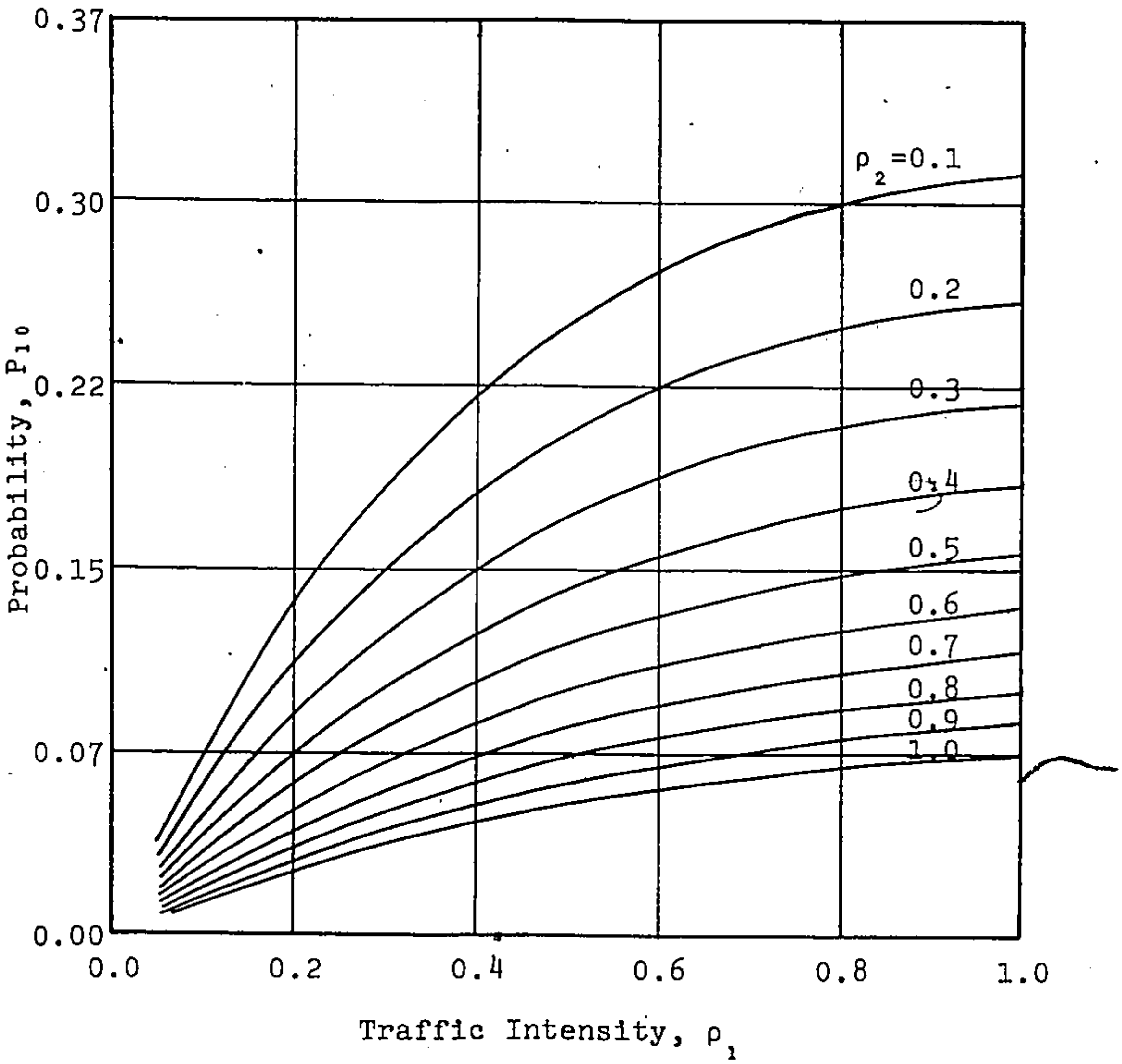

Figure 68. Change of probability P_{10} with ρ_1; ρ_2 fixed.

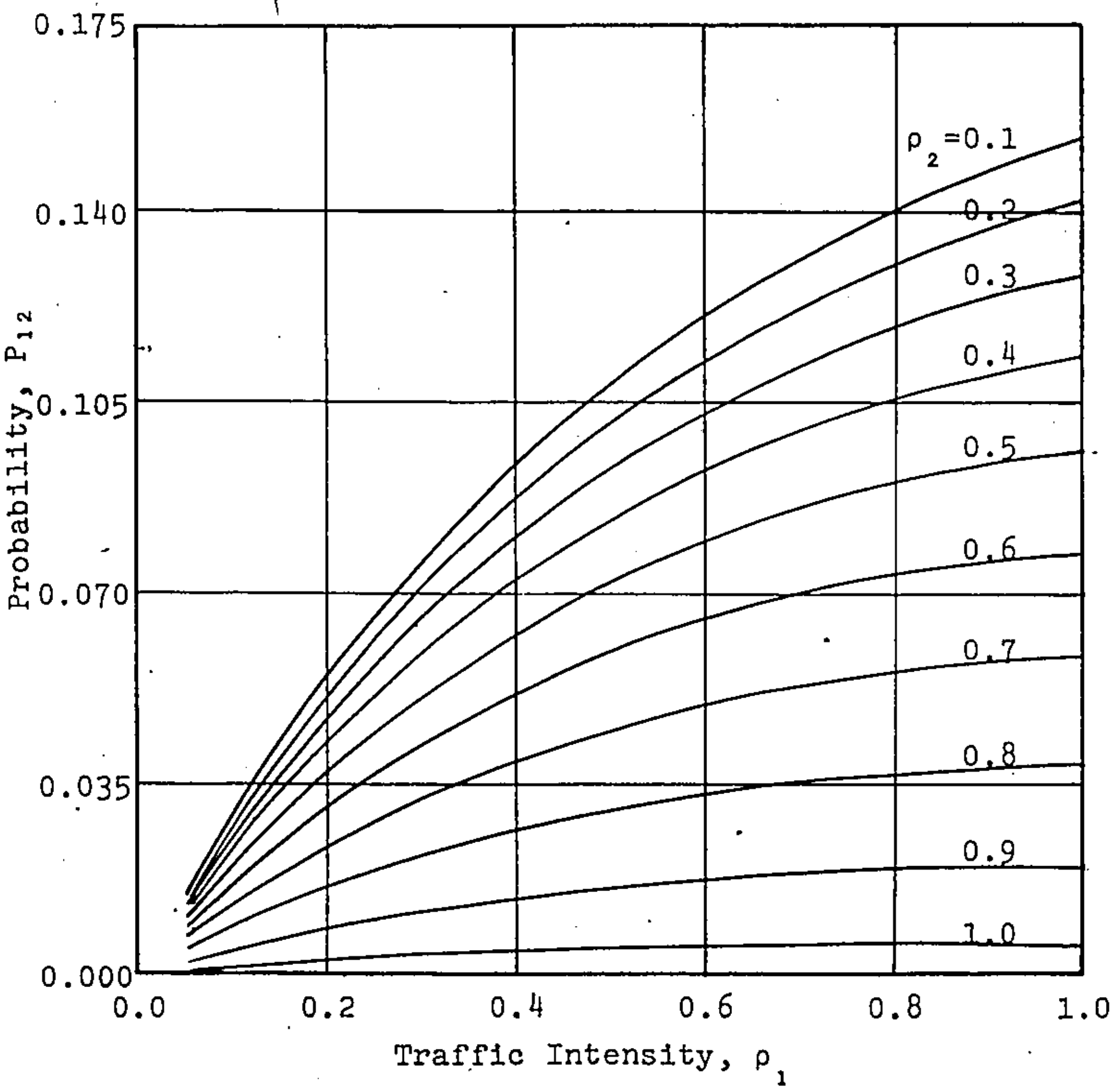

Figure 69. Change of probability P_{12} with ρ_1; ρ_2 fixed.

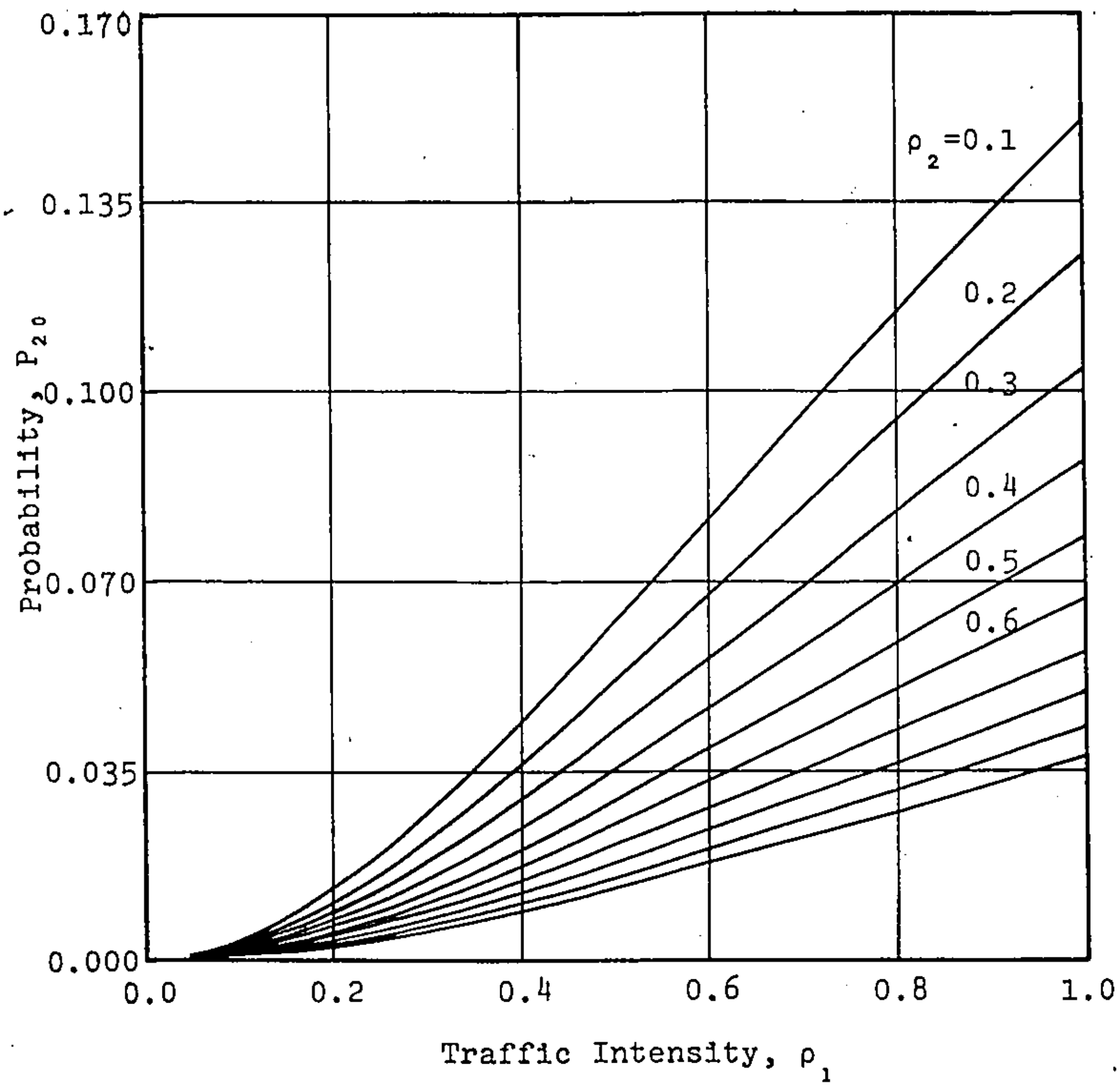

Figure 70. Change of the probability P_{20} with ρ_1; ρ_2 fixed.

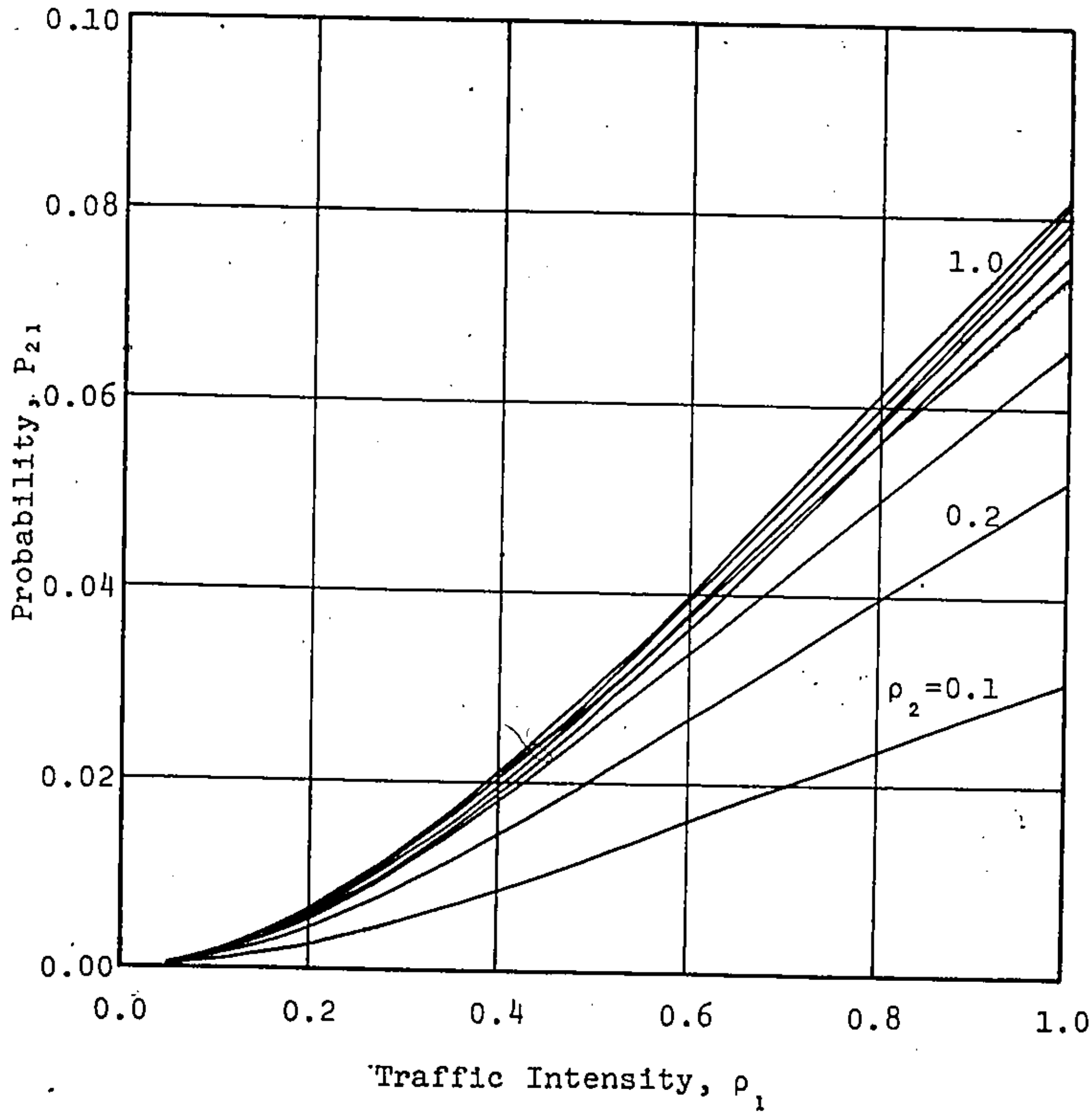

Figure 71. Change of probability P_{21} with ρ_1; ρ_2 fixed.

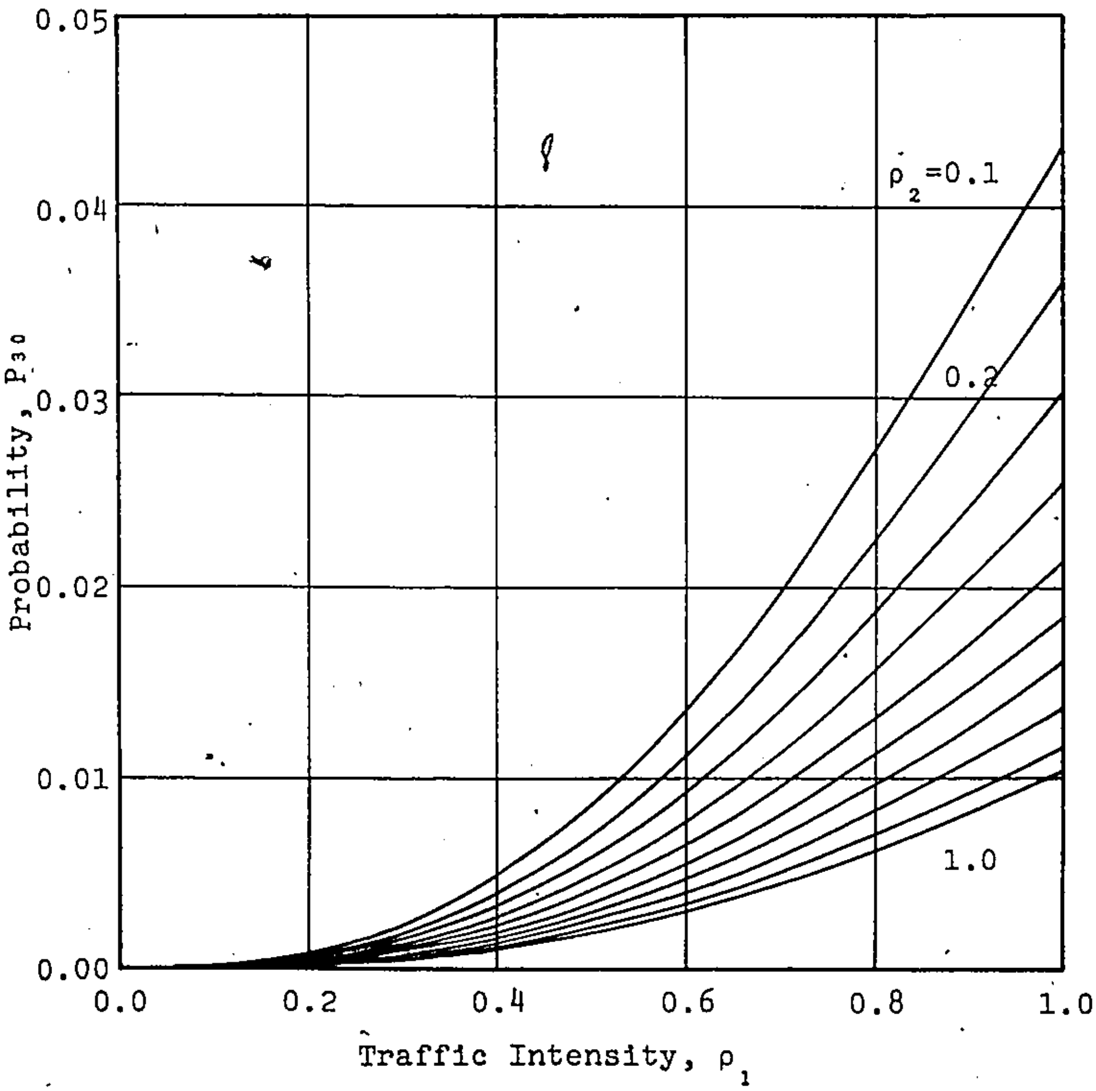

Figure 72. Change of probability P_{30} with ρ_1; ρ_2 fixed.

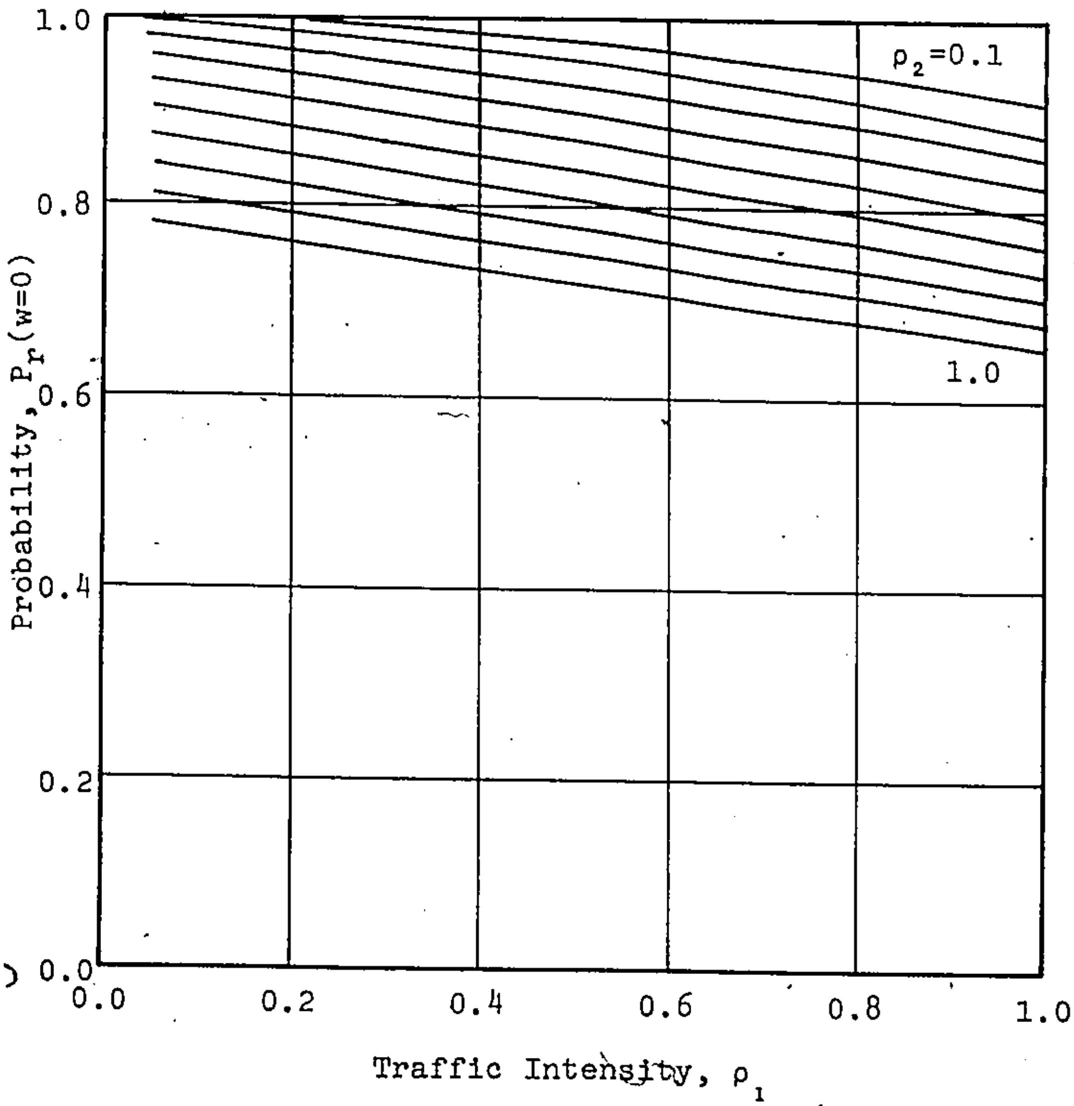

Figure 73. Change of probability $P_r(w=0)$ with ρ_1; ρ_2 fixed.

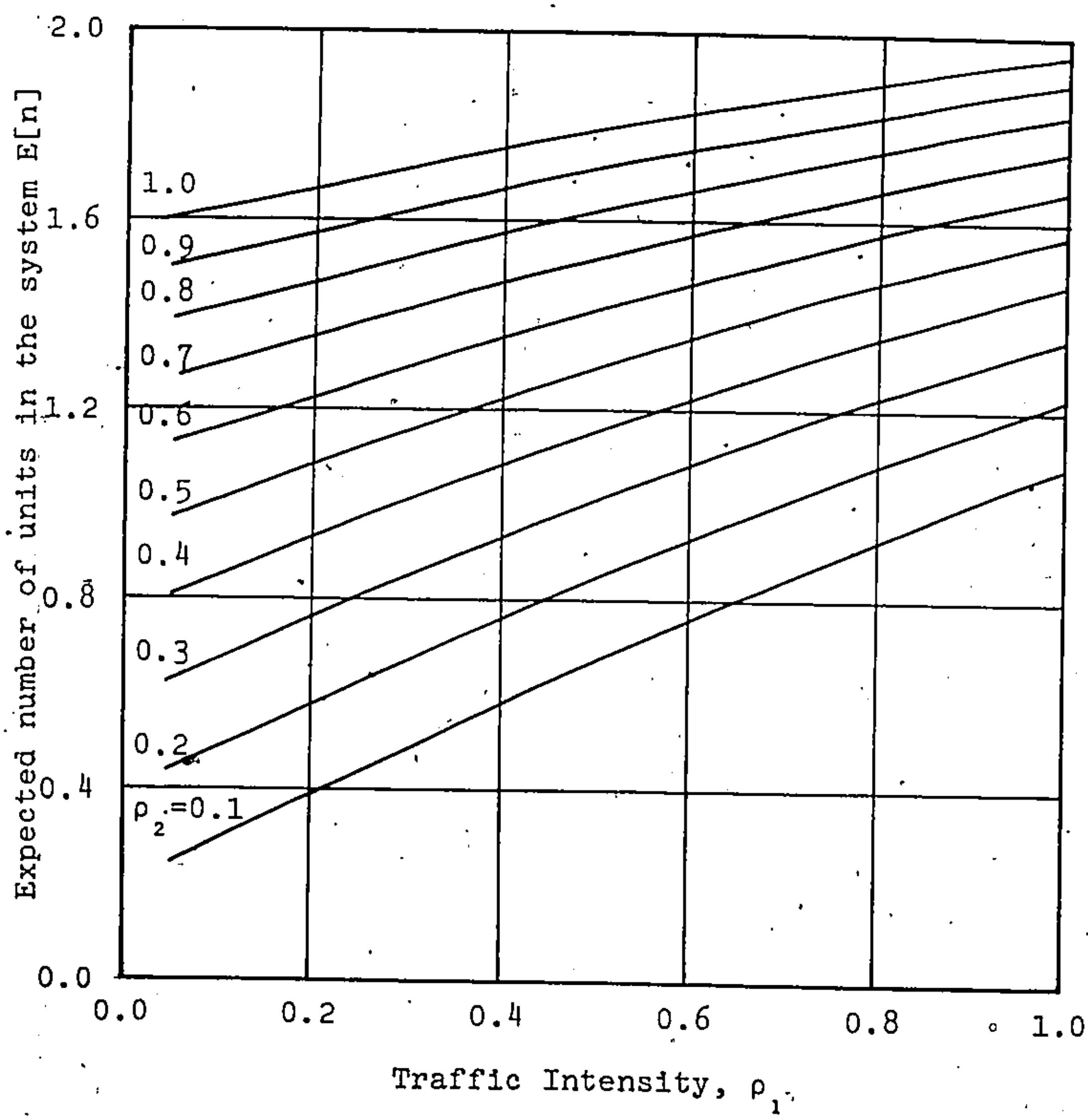

Figure 74. Expected number of units E[n] with ρ_1; ρ_2 fixed.

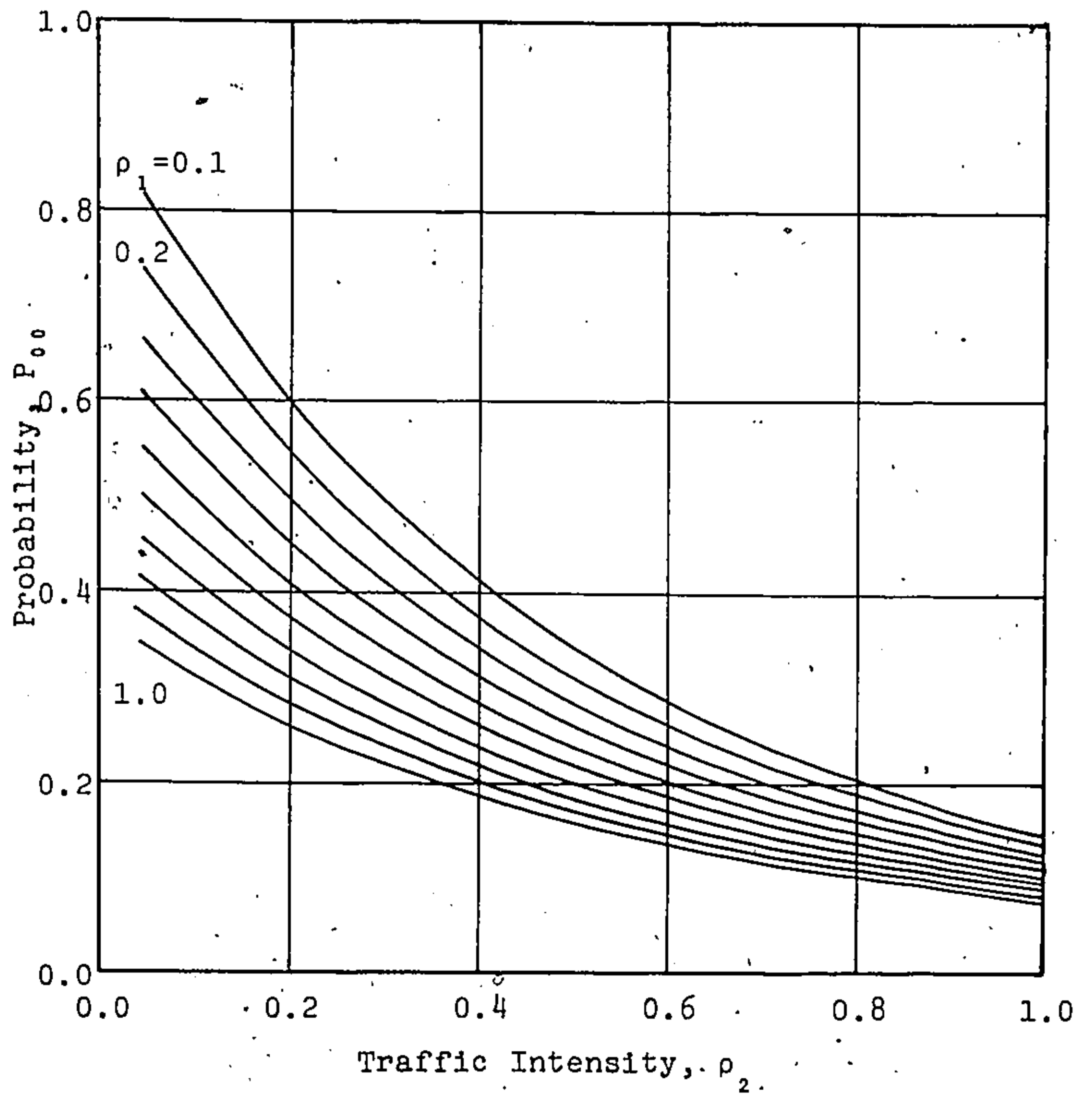

Figure 75. Change of Probability P_{00} with ρ_2; ρ_1 fixed.

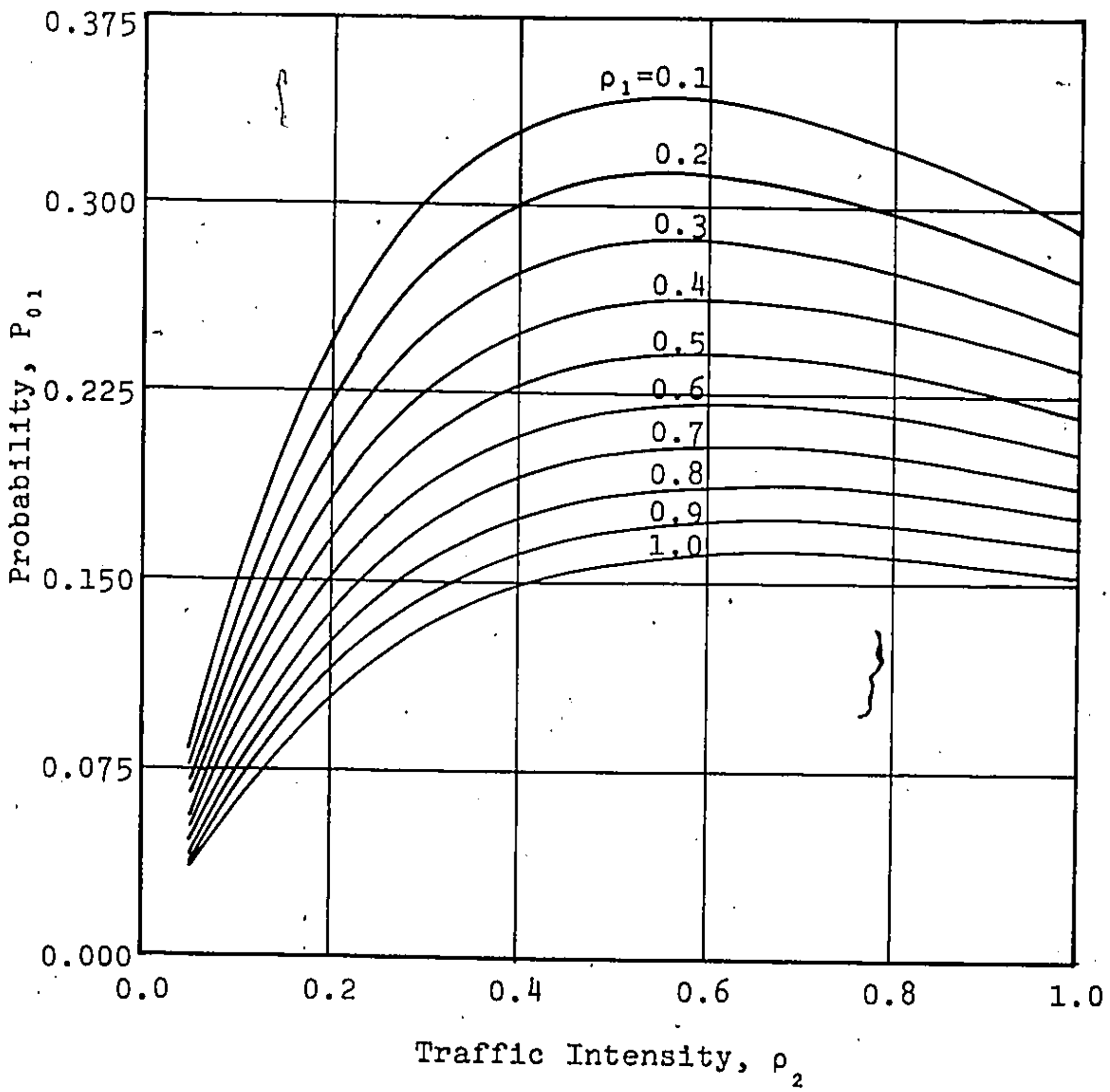

Figure 76. Change of probability P_{01} with ρ_2; ρ_1 fixed.

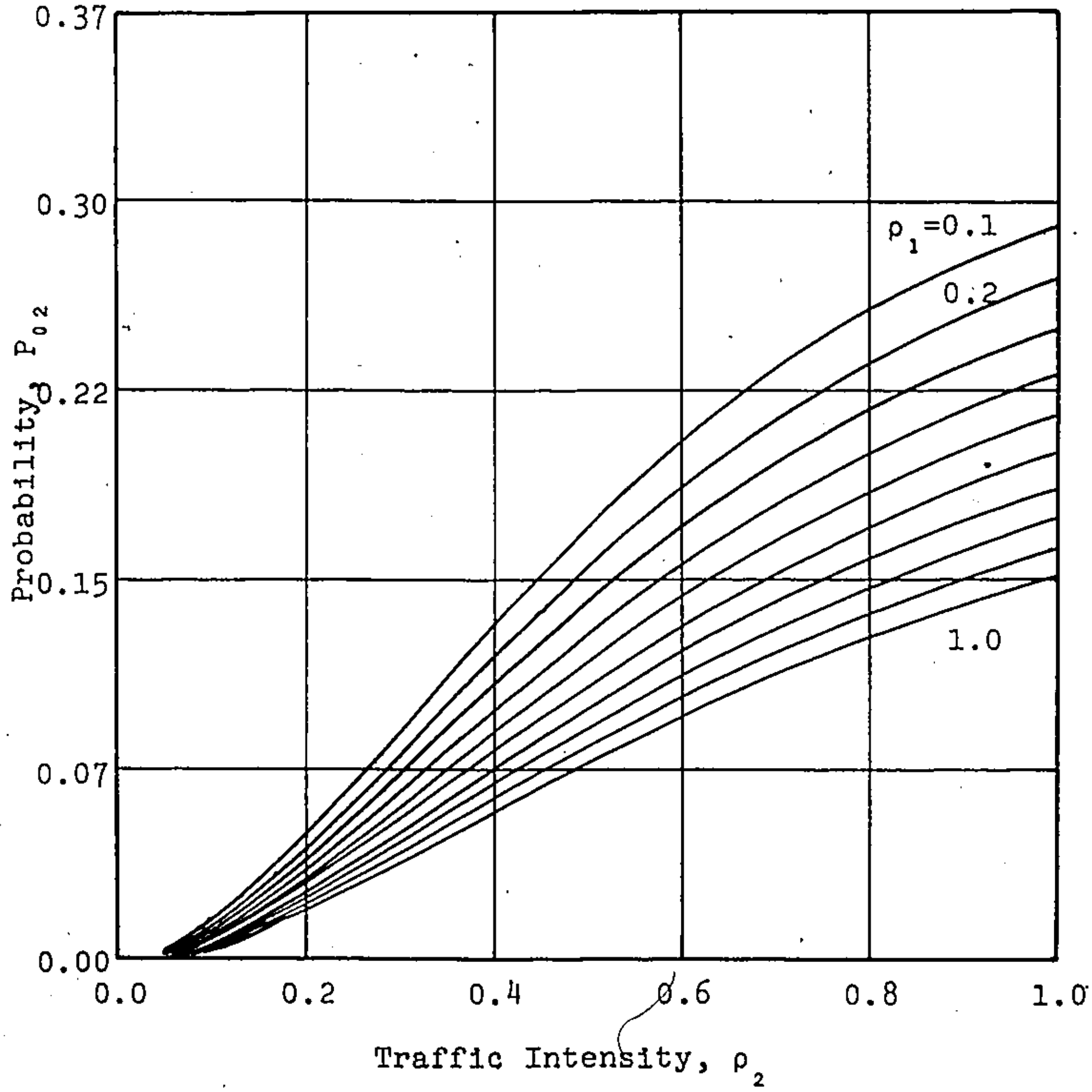

Figure 77. Change of the probability P_{02} with ρ_2; ρ_1 fixed.

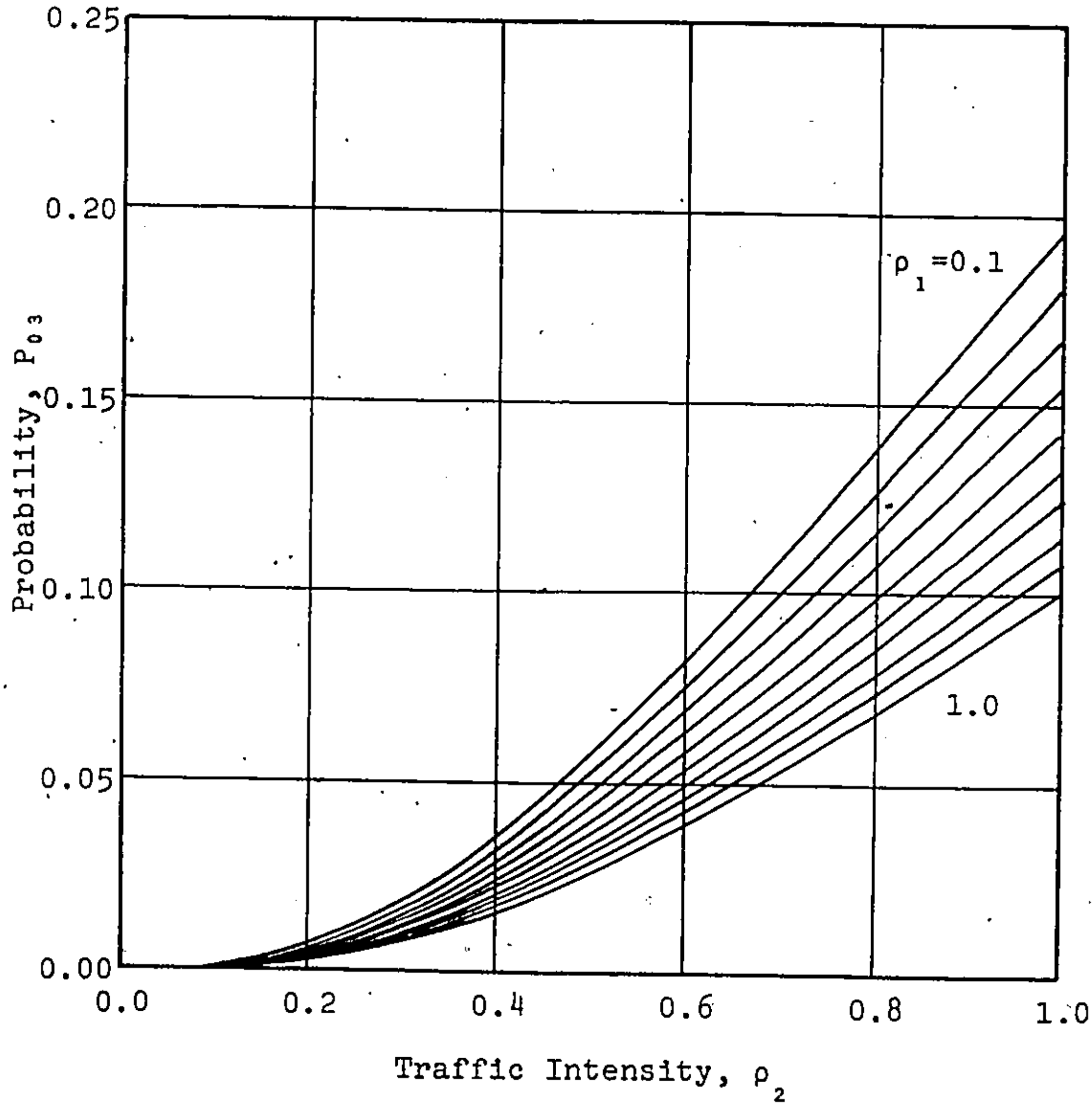

Figure 78. Change of probability P_{03} with ρ_2; ρ_1 fixed.

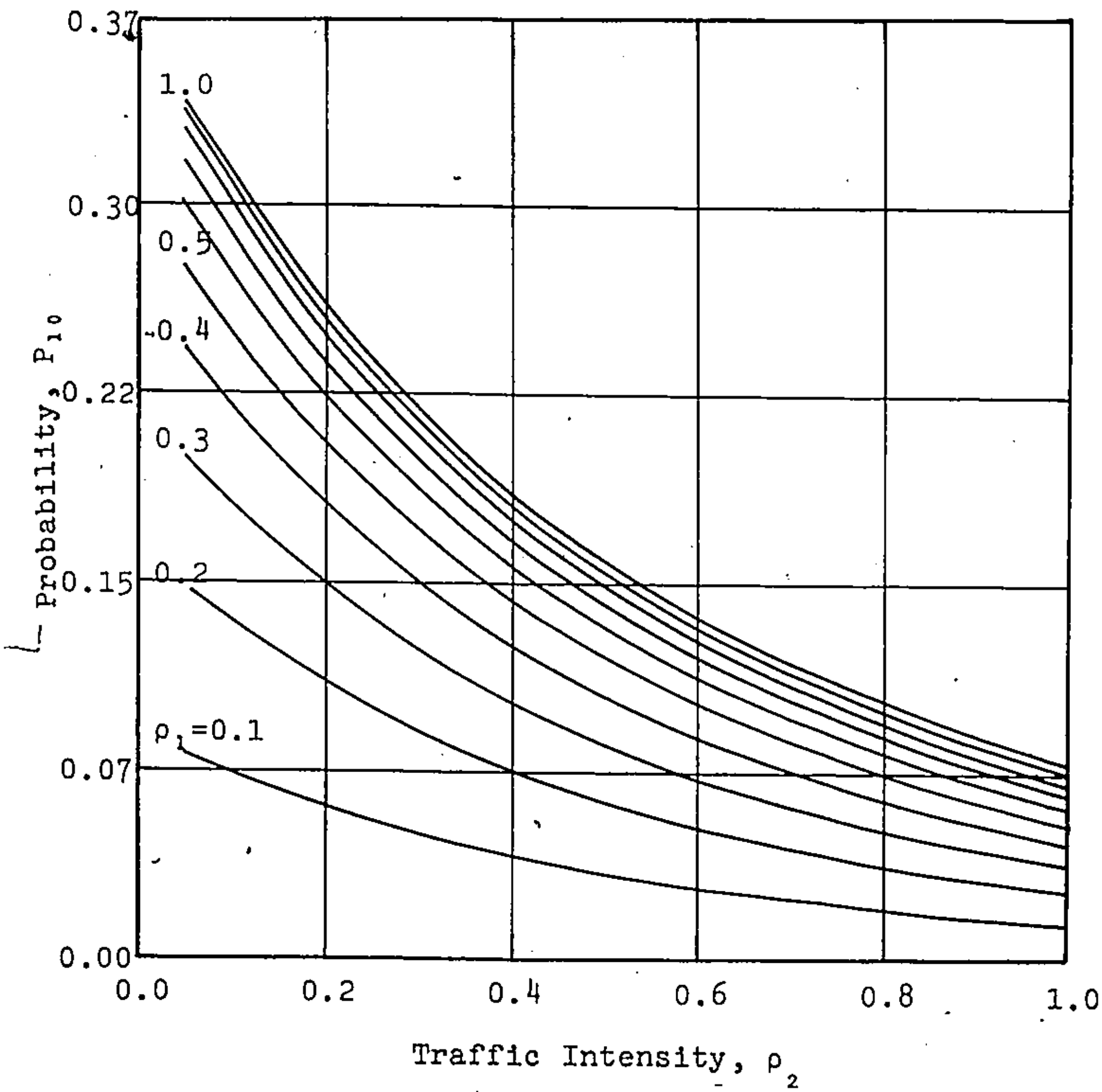

Figure 79. Change of probability P_{10} with ρ_1; ρ_2 fixed.

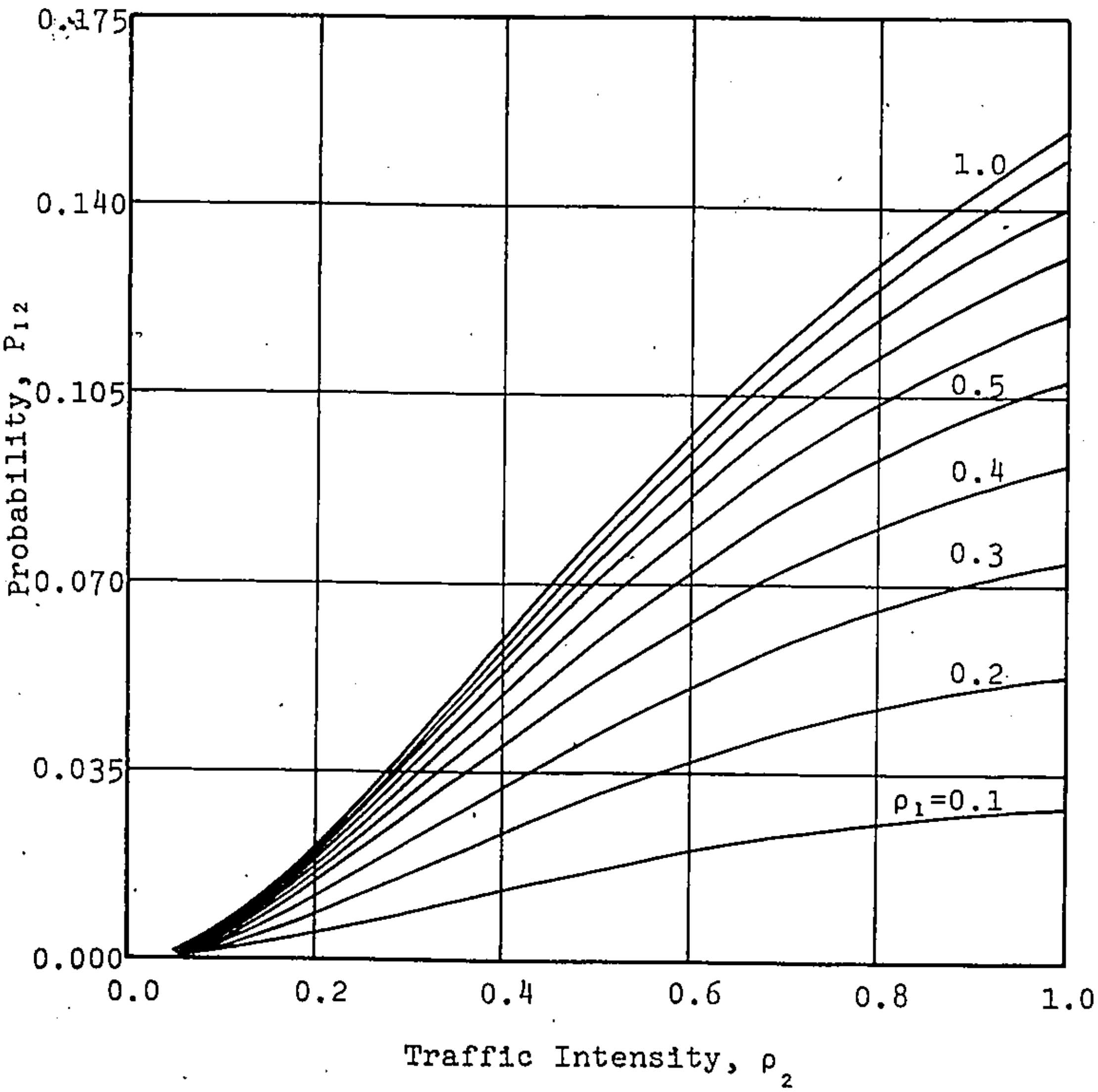

Figure 80. Change of the probability P_{12} with ρ_2; ρ_1 fixed.

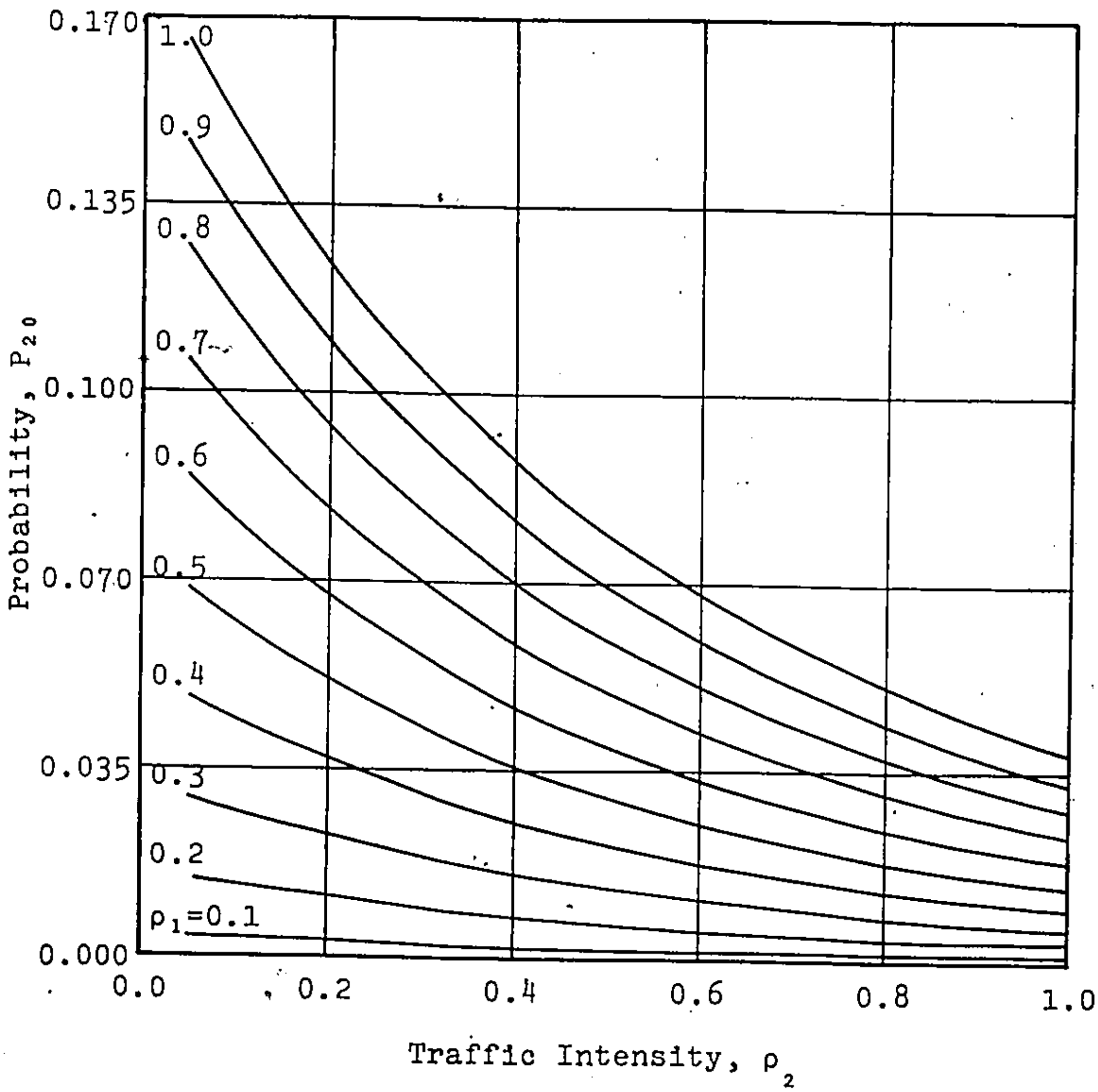

Figure 81. Change of the probability P_{20} with ρ_2; ρ_1 fixed.

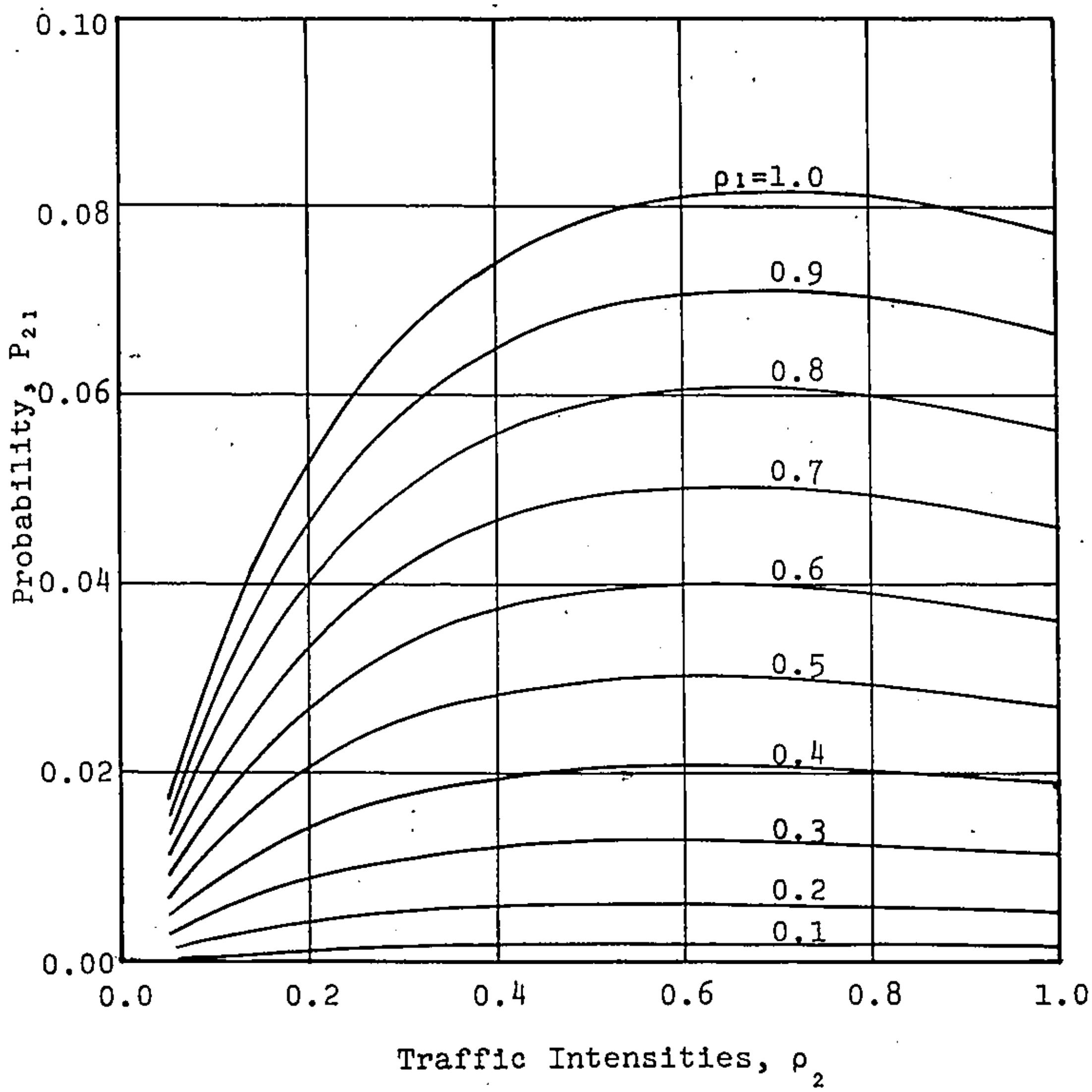

Figure 82. Change of probability P_{21} with ρ_2; ρ_1 fixed.

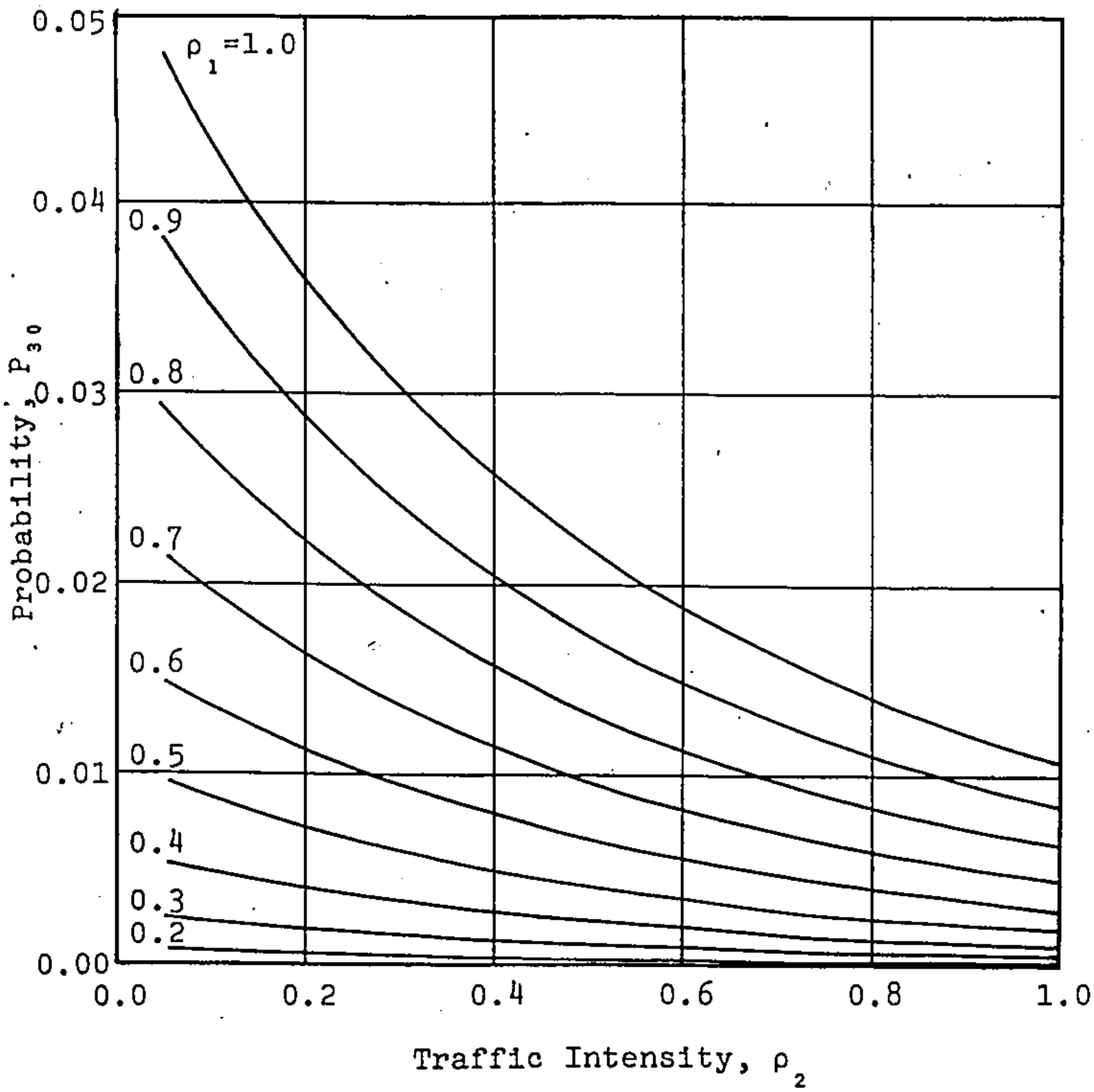

Figure 83. Change of probability P_{30} with ρ_2; ρ_1 fixed.

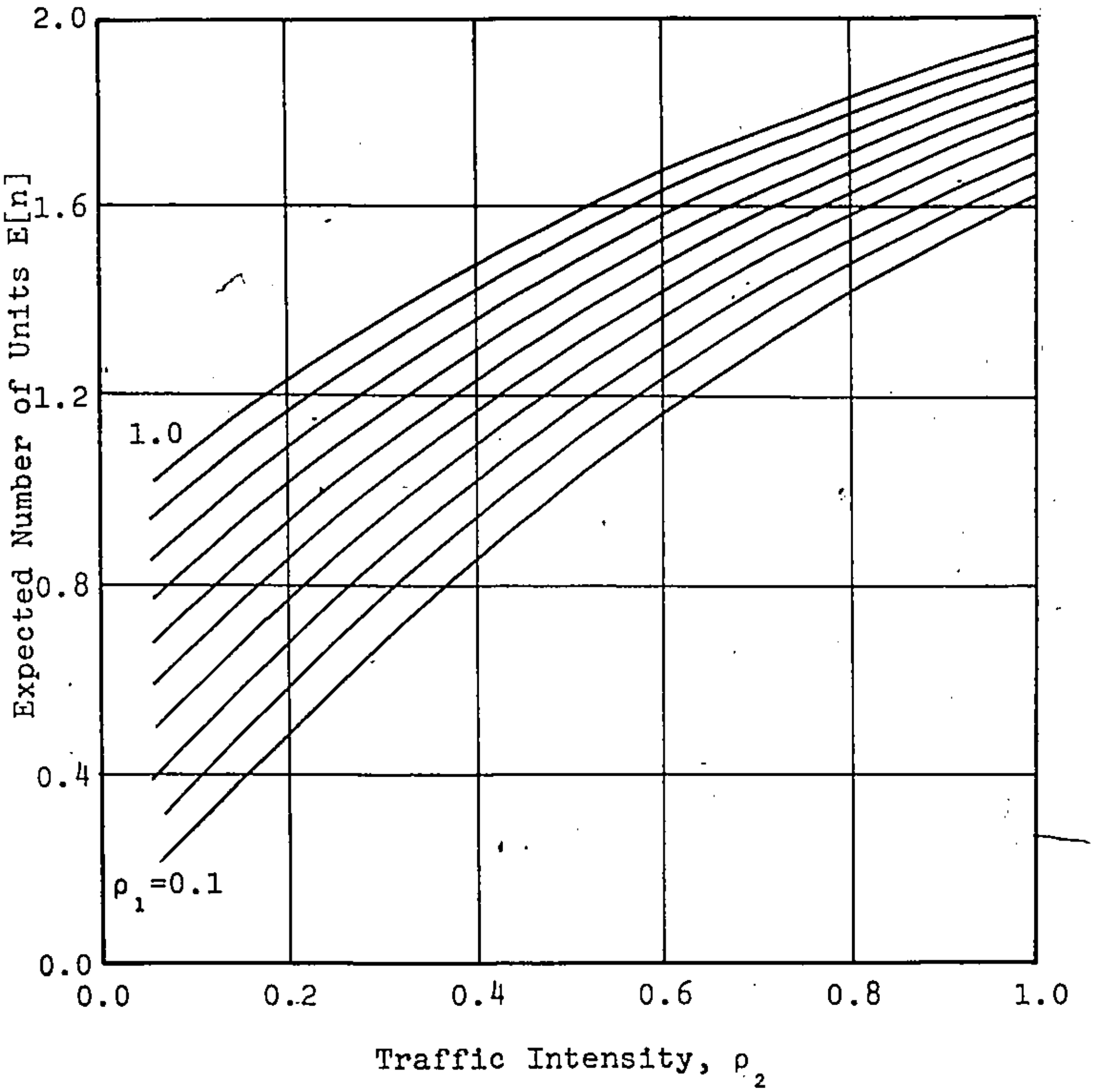

Figure 84. Expected number of units E[n] with ρ_2; ρ_1 fixed.

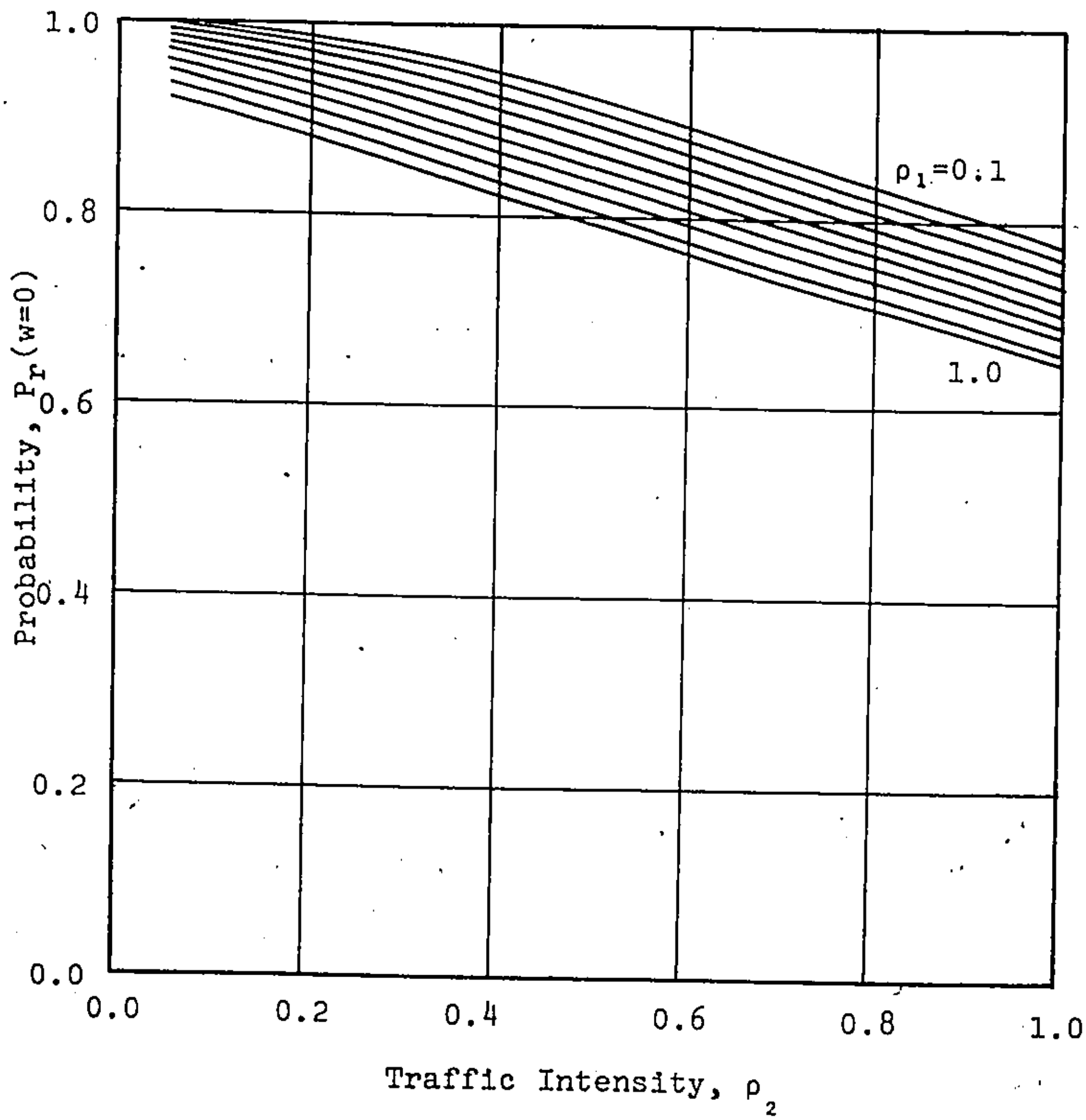

Figure 85. Change of probability $P_r(w=0)$ with ρ_2; ρ_1 fixed.

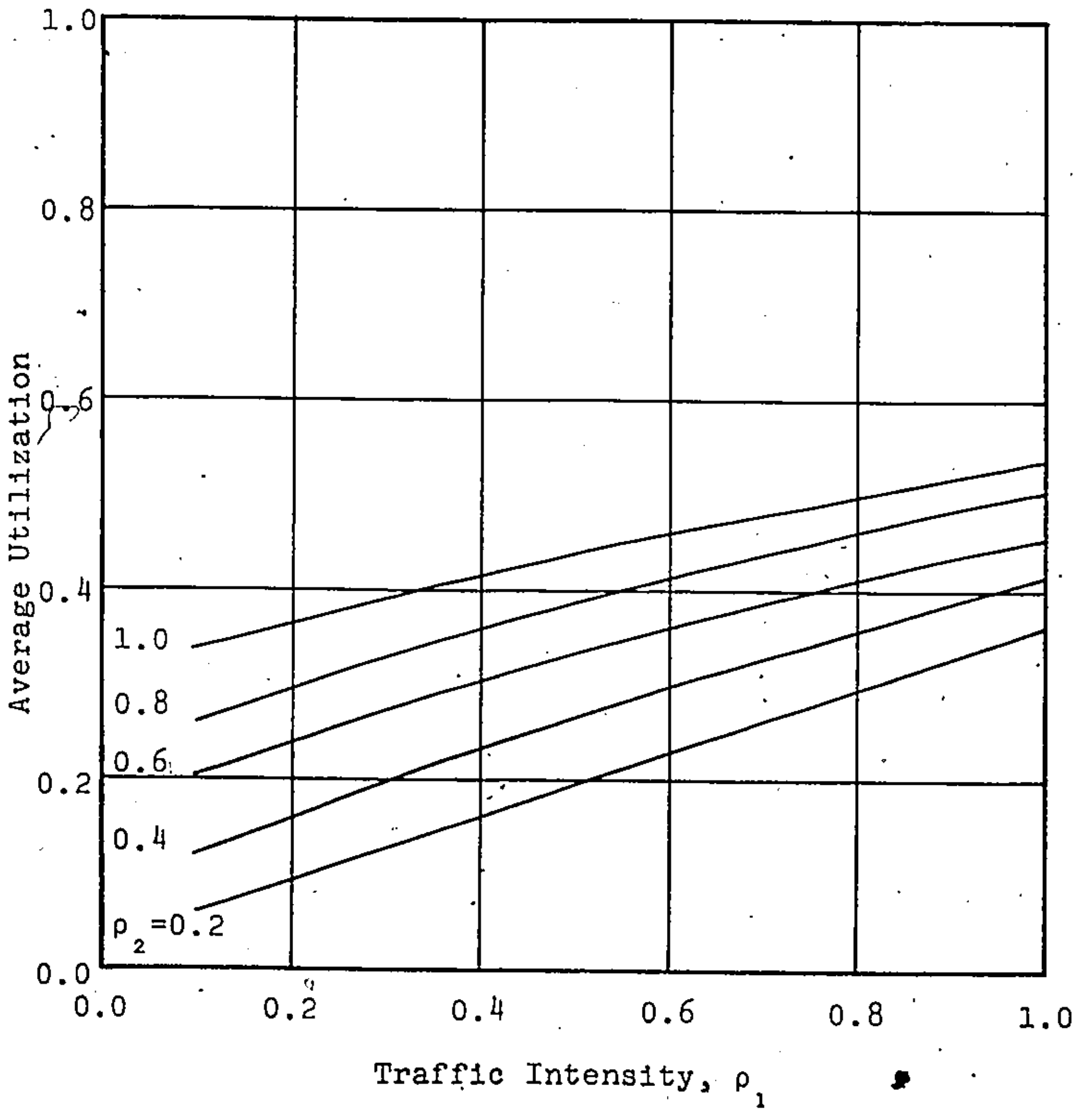

Figure 86. Effect of ρ_1 and ρ_2 on the utilization of the second channel.

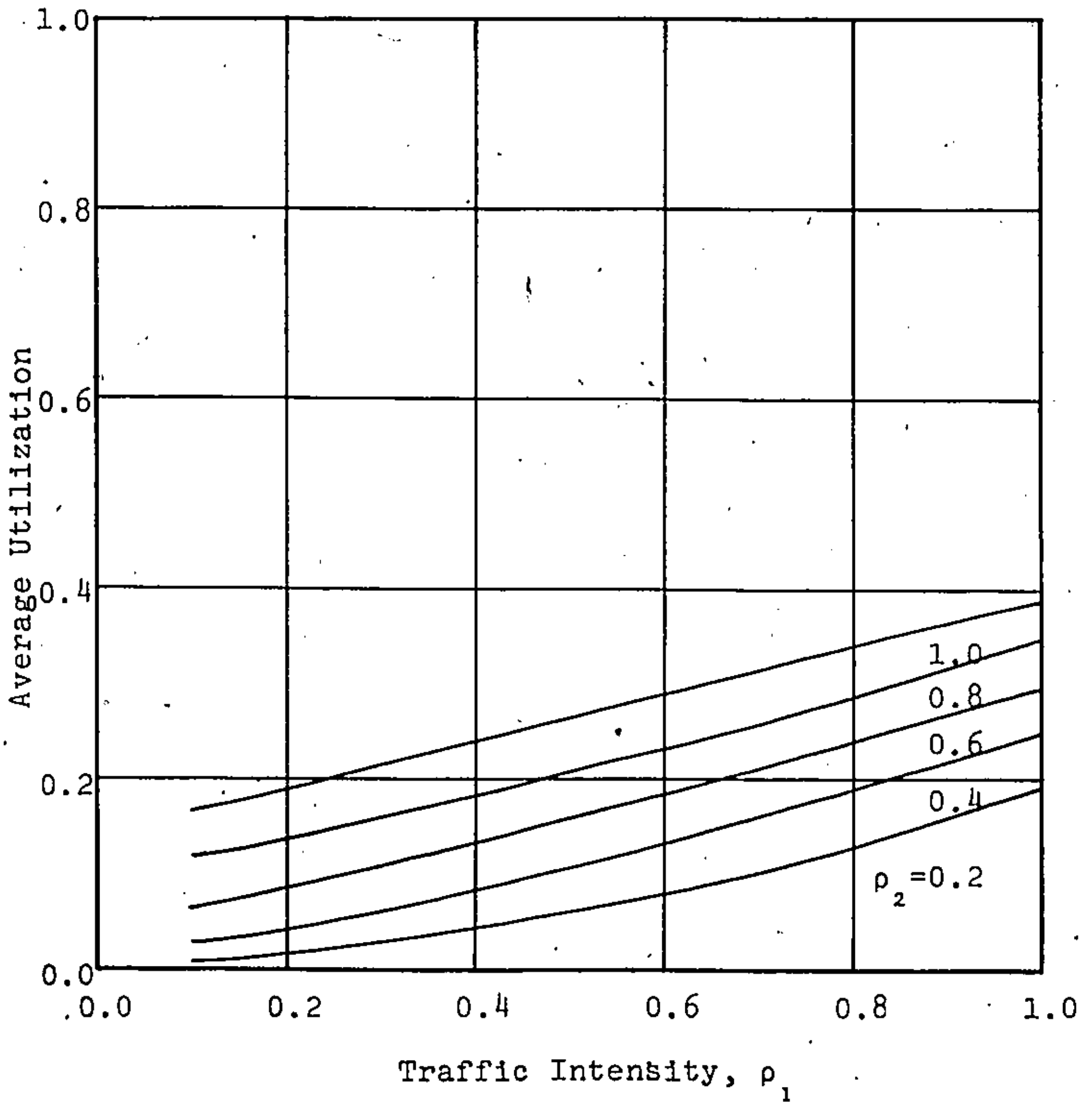

Figure 87. Effect of ρ_1 and ρ_2 on the utilization of the third channel.

APPENDIX D

M-channel closed-loop conveyor with lost arrivals.

Appendix D illustrates the effect of the number of service channels on the system's measures of performance.

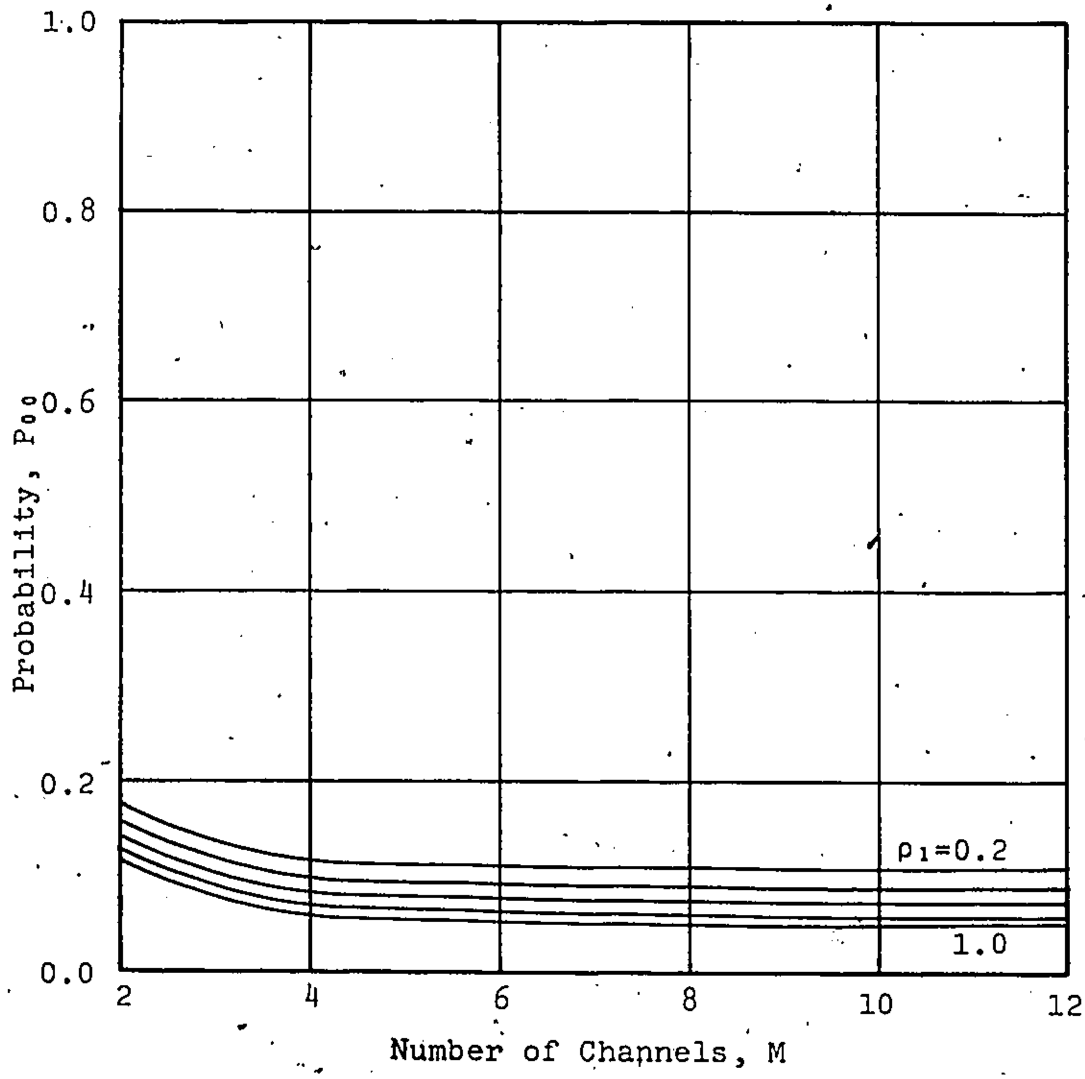

Figure 89. Relationship between the number of channels and the probability of the system being idle; ρ_2 fixed at 1.0.

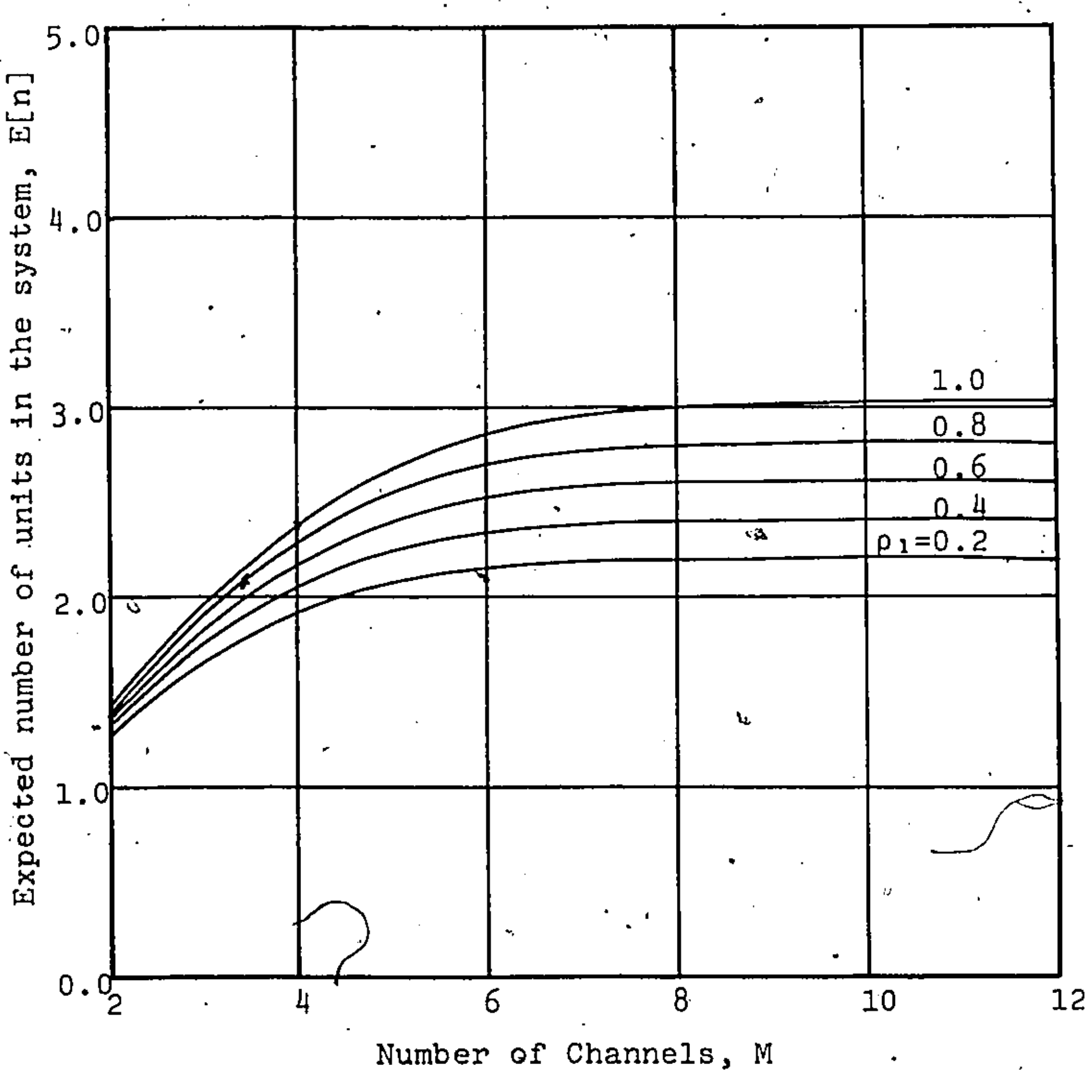

Figure 90. Relationship between M and E[n]; ρ_2 fixed at 1.0

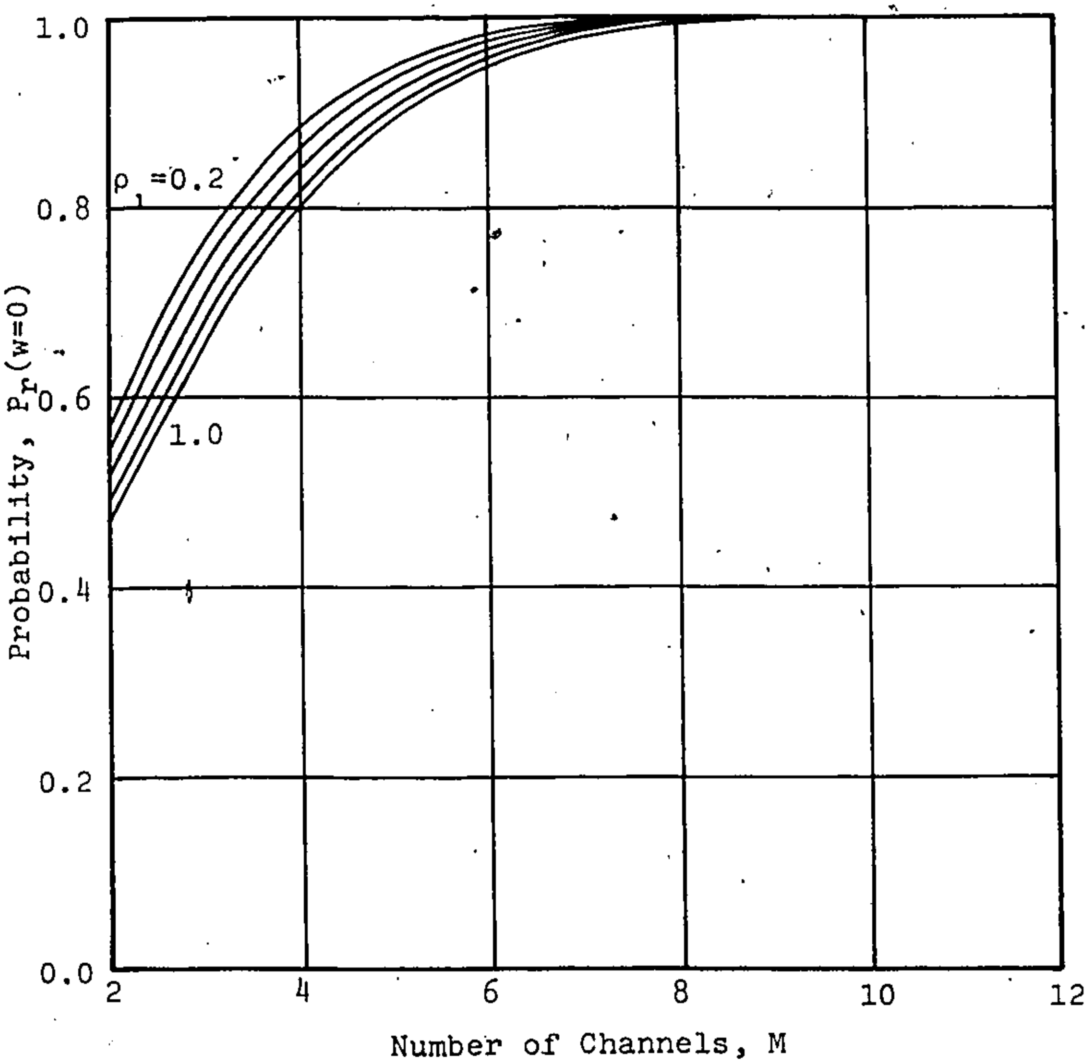

Figure 91. Relationship between M and $P_r(w=0)$; ρ_2 fixed at 1.0.

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